Data-Driven and Equation-Informed Optimal Control of Lagrangian objects in complex flows

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Turbulent flows

https://svs.gsfc.nasa.gov/3827

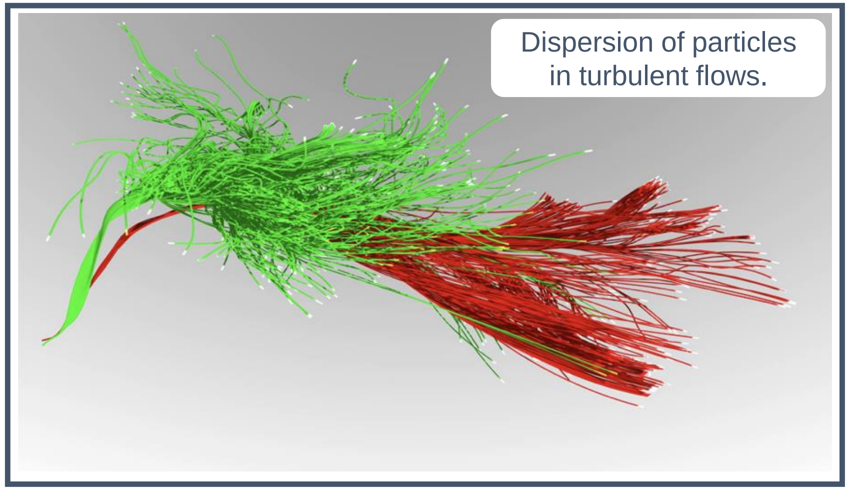
*How to exploit **coherent structures**?*

*How to avoid (or exploit) **intense fluctuations** when navigating inside the flow?*

*Which is the best **limited-control** to navigate in such complex flows?*

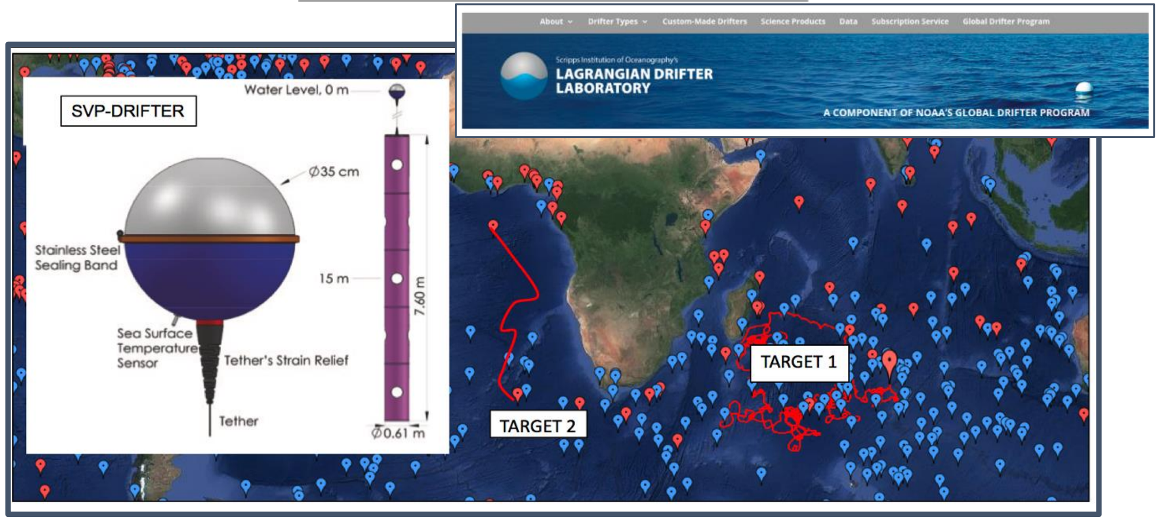
Tools:
Optimal Control (**OC**) theory
Reinforcement Learning (**RL**)

Theoretical interests:



Dispersion of particles in turbulent flows.

Engineering applications:

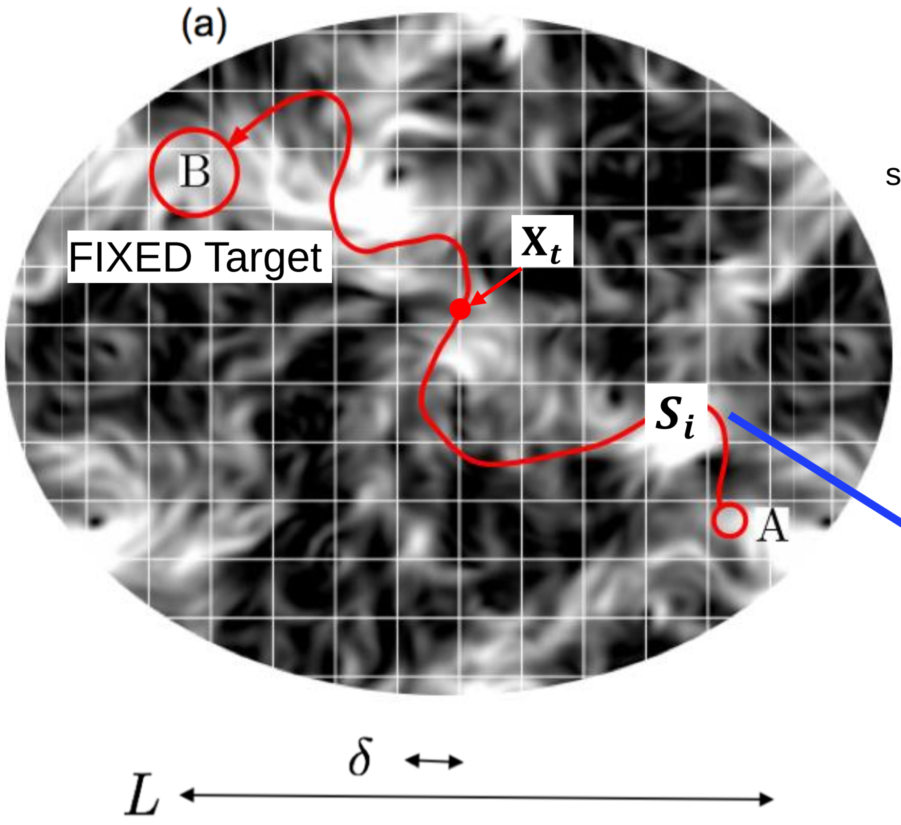


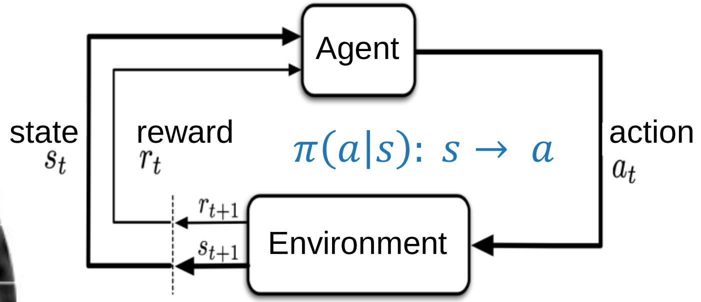
E. Zermelo. **Über das Navigations problem bei ruhender oder veränderlicher Windverteuling.** *Zeitschrift fuer Angewandte Mathematik und Mechanik 11(2) 114 (1931)*

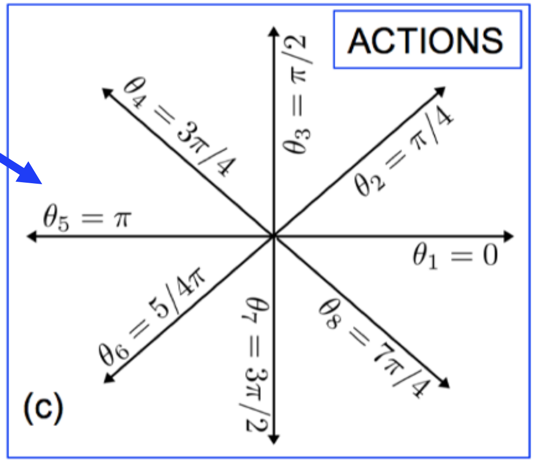
Biferale, L., Bonaccorso, F., Buzzicotti, M., Clark Di Leoni, P., & Gustavsson, K. (2019). **Zermelo's problem: Optimal point-to-point navigation in 2D turbulent flows using reinforcement learning**. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, *29*(10), 103138.

Problem setup via **RL** algorithms

Zermelo's problem







$$\begin{cases} \dot{\mathbf{X}}_t = v(\mathbf{X}_t) + \mathbf{U}(\mathbf{X}_t) \\ \mathbf{U}(\mathbf{X}_t) = V_s\, \hat{\boldsymbol{n}}\,(\mathbf{X}_t) \end{cases}$$

Small Navigation speed compared to the underling flow!

control on the **navigation direction:**

$$\hat{\boldsymbol{n}}\,(\mathbf{X}_t) = (\cos[\theta_t], \sin[\theta_t])$$

Goal: find the **minimum-time trajectory**

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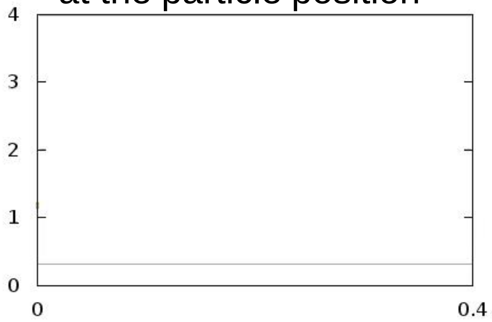
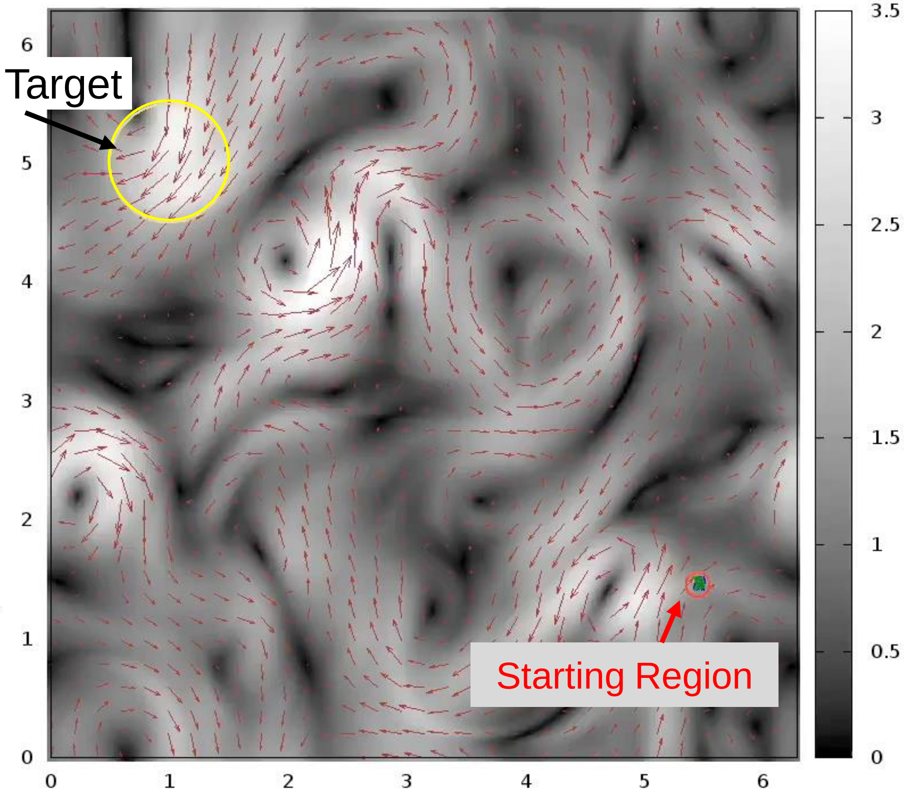
Time-Dependent 2D Turbulent Flow

Zermelo's problem

$$\frac{V_s}{\max(u)} \sim 0.2$$

Flow kinetic energy at the particle position

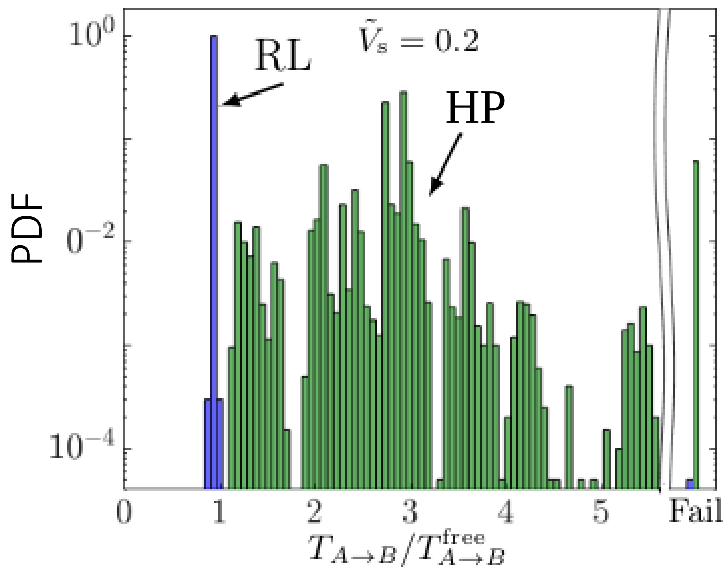




Reinforcement Learning (blue)

vs

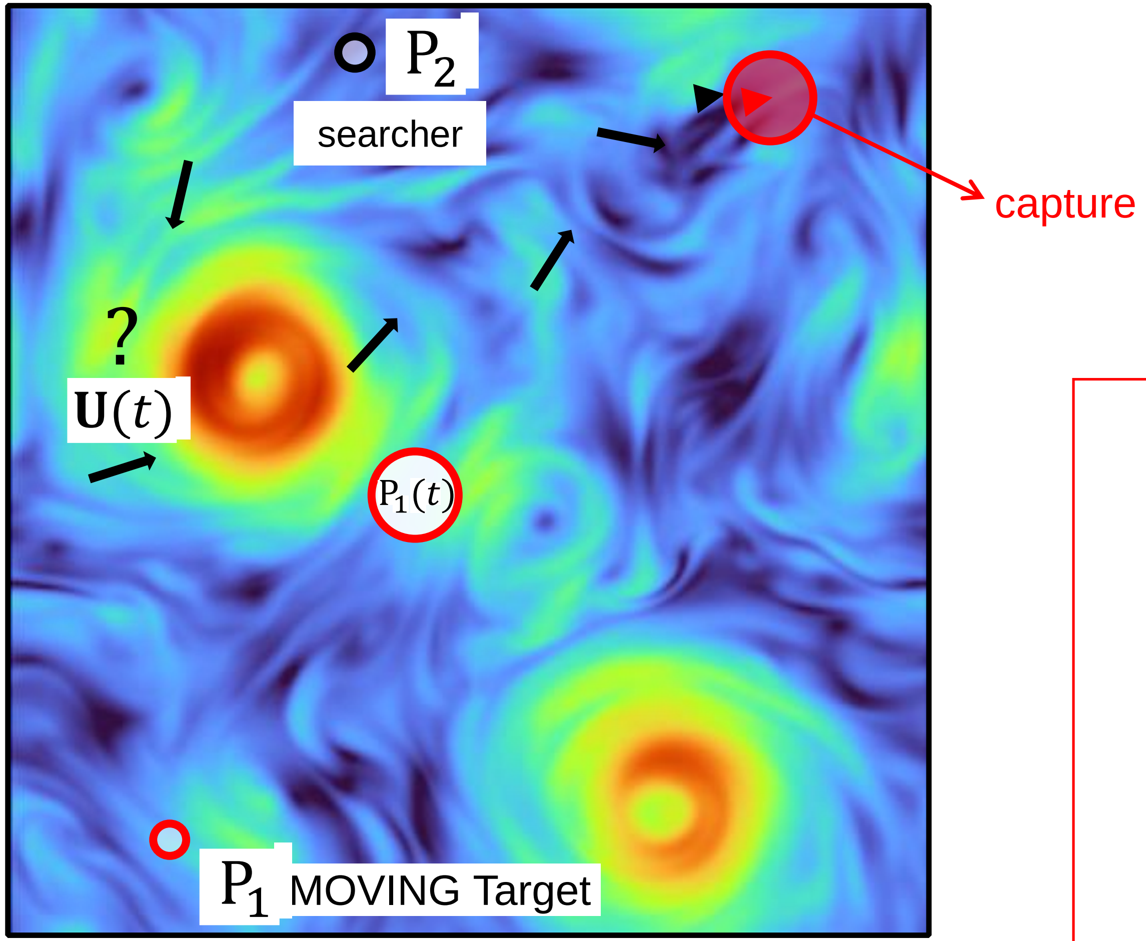
Heuristic Policy (green)



Goal: find the **minimum-time trajectory**

C. Calascibetta, L. Biferale, F. Borra, A. Celani, M. Cencini, **Optimal and Data-Driven Control of Lagrangian particles to minimize trajectories' separation in complex flows**, (*in preparation*).

2 AGENTS



$$(*)\begin{cases} \dot{\mathbf{X}}_t^1 = \boldsymbol{v}(\mathbf{X}_t^1) + \mathbf{U}'(t) \\ \dot{\mathbf{X}}_t^2 = \boldsymbol{v}(\mathbf{X}_t^2) + \mathbf{U}(t) \\ \mathbf{U}(t) = V_s\,\hat{\boldsymbol{n}}(t) \end{cases}$$

Goal: **minimize** the **separation** within a given time

Pontryagin minimum principle

Minimize $C_{t_f} = \left\| \mathbf{X}_{t_f}^1 - \mathbf{X}_{t_f}^2 \right\|^2$

assuming $(*)$ and possible constraints

$$J = C_{t_f} + \int_{t_0}^{t_f} L[\,\mathbf{X}_t^1, \mathbf{X}_t^2, \hat{\boldsymbol{n}}(t), t]\,dt$$

performance index — *Lagrangian function*

Optimal control by imposing $dJ \le 0$

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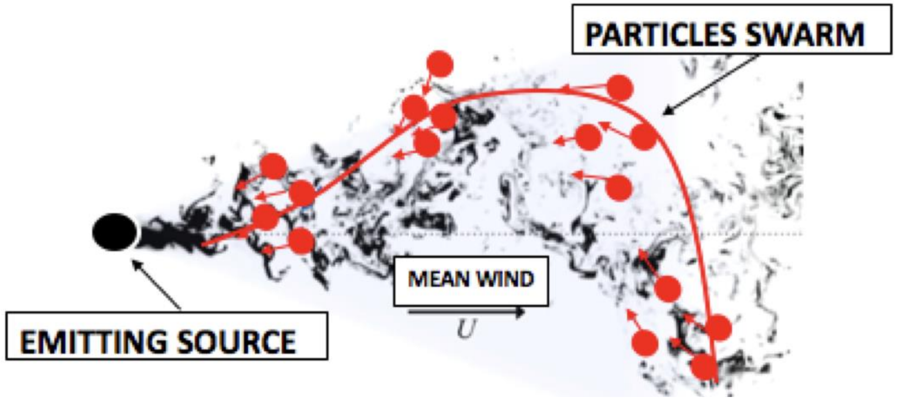
Conclusions

Open questions and future perspective:

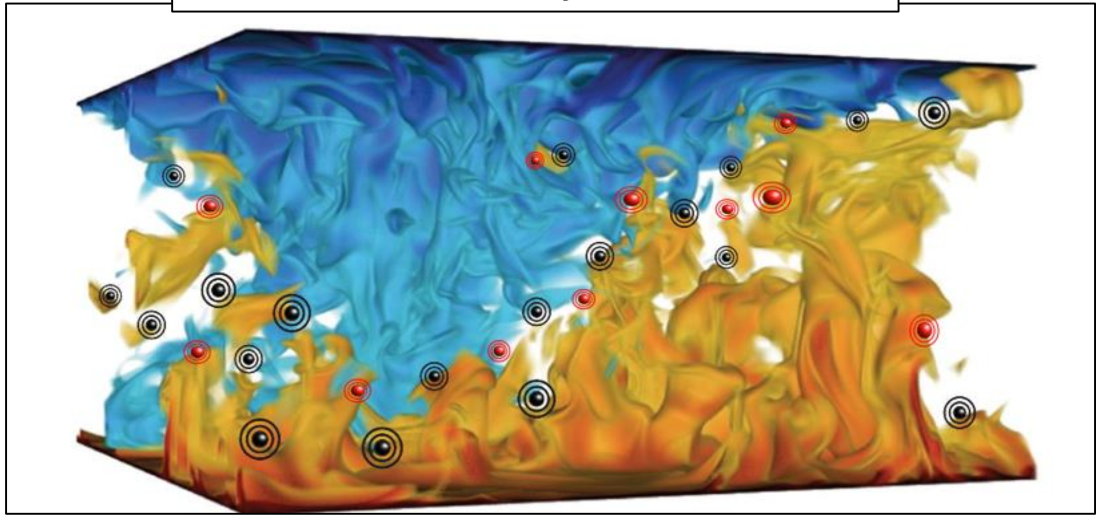
1. Can we control a multi-agent system to minimize turbulent dispersion in realistic geophysical flows?
2. Can we identify the key degrees-of-freedom to control the agents' trajectories (key flow structures)?
3. Are the agents able to collaborate with each-other during the navigation?



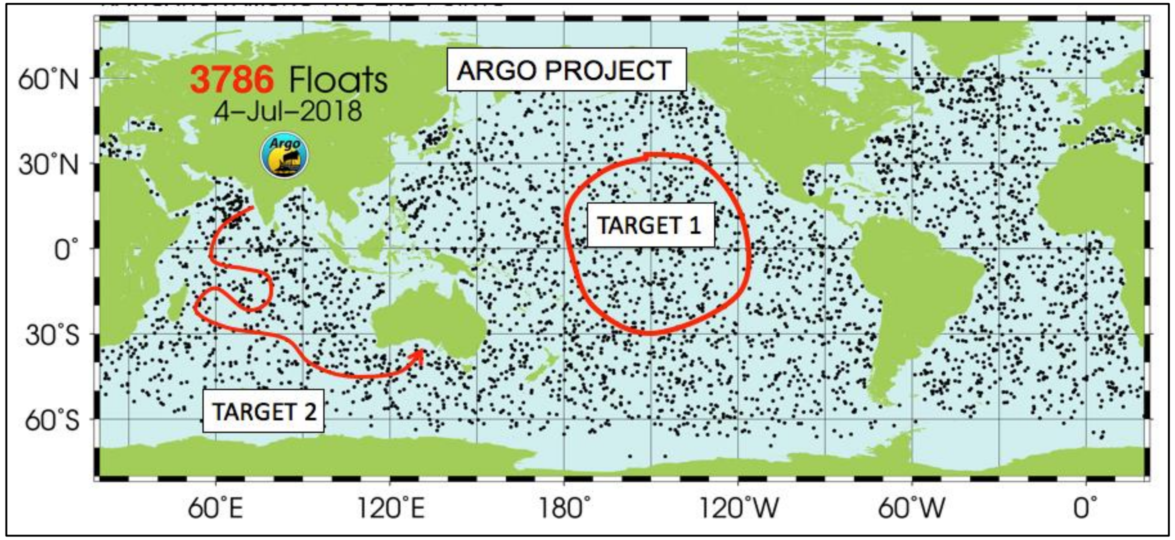
New Tools:

- We can use RL to control autonomous swimmers in a realistic way (i.e., with a limited knowledge of the underlying flow - only local or instantaneous features);
- We can use OC as a benchmark to test the RL solutions.

Smart two-way feedback



http://stilton.tnw.utwente.nl/people/stevensr/afid.html



https://argo.ucsd.edu/