QUBIT DESIGN

C GATTI – 6 APRILE 2022

QubIT REPORT ON TRANSMON QUBIT DESIGN

https://baltig.infn.it/qubit/qubit-wp1

QubIT Report on Transmon Qubit Design

Contents

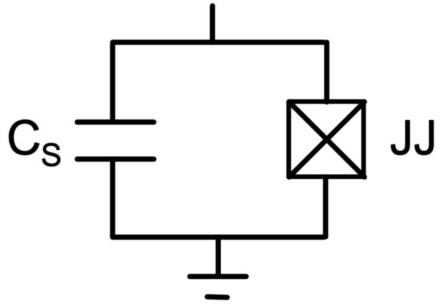
JOSEPHSON JUNCTION

Definitions

$$
\begin{aligned}
I &= I_c \sin\varphi \\
V &= \frac{\hbar}{2e}\frac{d\varphi}{dt} \\
\Phi_0 &= \frac{h}{2e} \\
\Phi &= \frac{\Phi_0}{2\pi}\varphi \\
E_j &= \frac{\Phi_0 I_c}{2\pi} \\
E_c &= \frac{e^2}{2C_j} \\
L_{0j} &= \frac{\Phi_0}{2\pi I_c} \\
\omega_{0,p} &= \sqrt{8E_jE_c}/\hbar = \frac{1}{\sqrt{L_{0j}C_j}}
\end{aligned}
$$

Hamiltonian

$$
\begin{aligned}
H &= 4E_cN^2 - E_j\left(1 - \frac{1}{2}\varphi^2 + \frac{1}{4!}\varphi^4\right) \\
&\simeq \left(4E_cN^2 + \frac{E_j}{2}\varphi^2\right) - \frac{1}{4!}E_j\varphi^4 - E_j \\
&= \hbar\omega_{0,p}\left(a^\dagger a + \frac{1}{2}\right) - \frac{1}{12}E_c\left(a + a^\dagger\right)^4 - E_j
\end{aligned}
$$



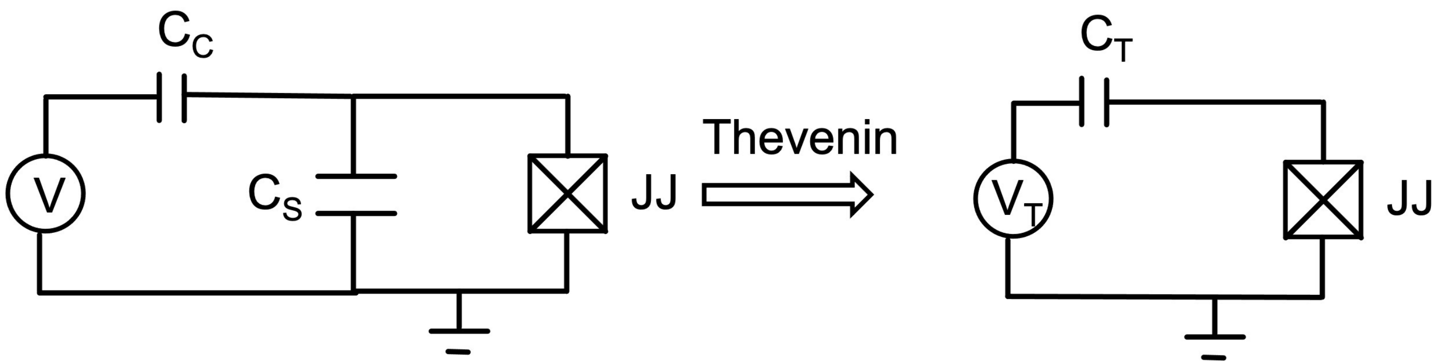
Eigenenergies

$$
E_n = \hbar\omega_{0,p}\left(n + \frac{1}{2}\right) - E_j - \frac{E_c}{12}\left(6n^2 + 6n + 3\right)
$$

Anharmonicity

$$
\begin{aligned}
\alpha &= E_{12} - E_{01} = -E_c \\
\alpha_r &= \alpha/E_{01} \simeq -\sqrt{\frac{E_c}{8E_j}}
\end{aligned}
$$

CIRCUIT QED



$$\beta = \frac{C_c}{C_s + C_c}$$

Jaynes Cummings Hamiltonian

$$H = \hbar\omega_r \left(b^\dagger b + \frac{1}{2} \right) + \frac{\hbar\omega_{01}}{2}\sigma^z + \frac{E_1 + E_0}{2} I + \hbar g_{01} \left(b^\dagger \sigma^- + b\sigma^+ \right)$$

$$\hbar g_{n,n+1} = 2\beta e V_{rms} \left(\frac{Ej}{32E_c} \right)^{1/4} \sqrt{n+1} \qquad V_{rms} = \sqrt{\hbar\omega_r / 2C_r}$$

JC HAMILTONIAN

From exact diagonalization of JC Hamiltonian:

$$E_{\pm,n} = (n+1)\hbar\omega_r \pm \frac{\hbar}{2}\sqrt{4g_{01}^2(n+1)+\Delta^2}$$

Numero critico di fotoni del risonatore

$$n_{cr} = \frac{\Delta^2}{4g_{01}^2}$$

One excitation state

$$|-,0\rangle \sim -(g_{01/\Delta})|\downarrow,0\rangle + |\uparrow,1\rangle$$ ~Cavity state

$$|+,0\rangle \sim |\downarrow,0\rangle + (g_{01/\Delta})|\uparrow,1\rangle$$ ~Qubit state

Qubit relaxation rate

$$\Gamma_{+,0} = \gamma + \left(\frac{g_{01}}{\Delta}\right)^2 k$$

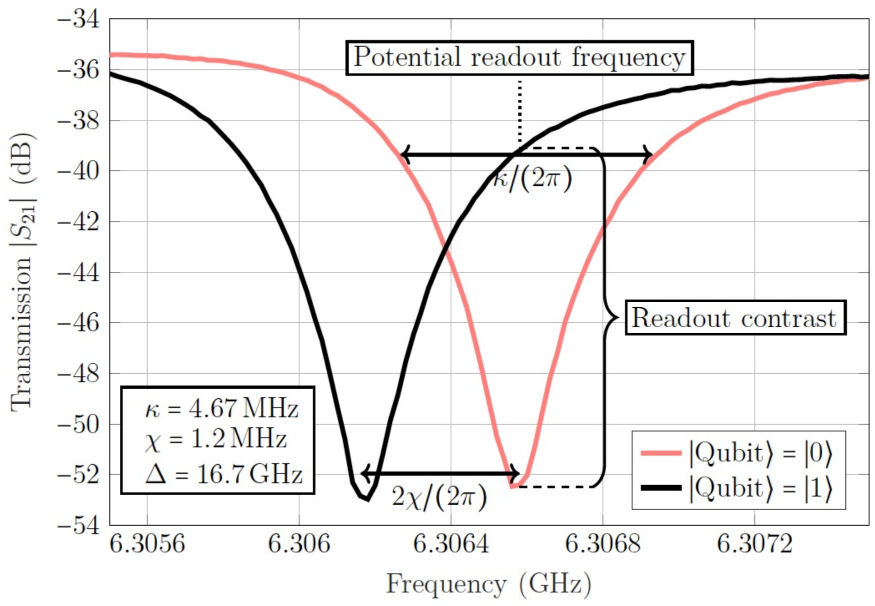
DISPERSIVE REGIME

$$H = \hbar\left[\omega_r' + \chi\sigma^z\right] b^\dagger b + \frac{\hbar}{2}\omega_{01}'\sigma^z$$

$$\chi = \chi_{01} - \chi_{12}/2$$

$$\chi_{ij} = \frac{g_{ij}^2}{\omega_{ij} - \omega_r}$$

$$\omega_{ij} = \omega_j - \omega_i$$



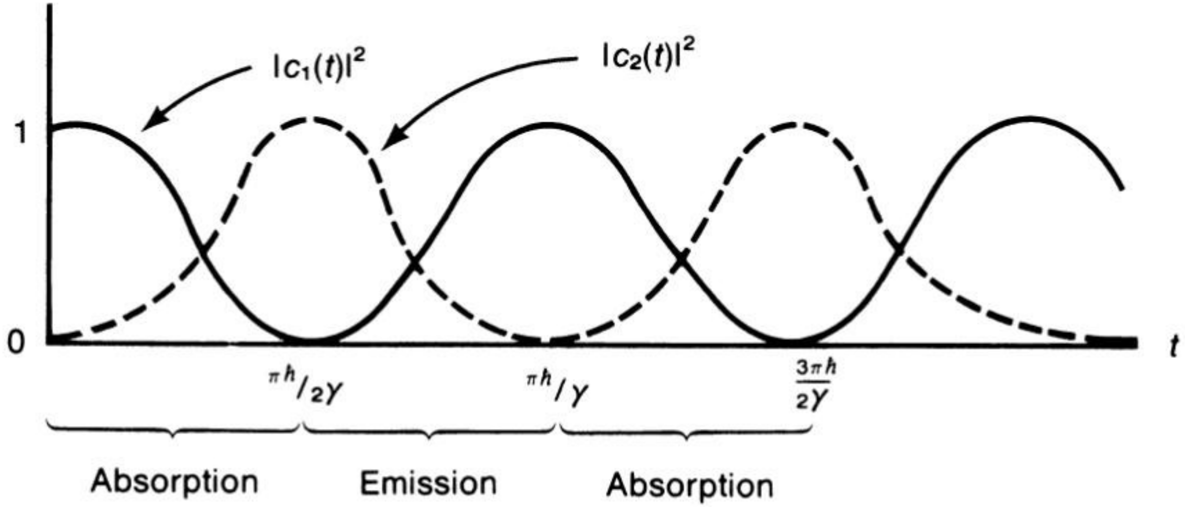
RABI OSCILLATION

$$i\hbar\frac{d}{dt}|Qubit\rangle = H_{Int}|Qubit\rangle$$

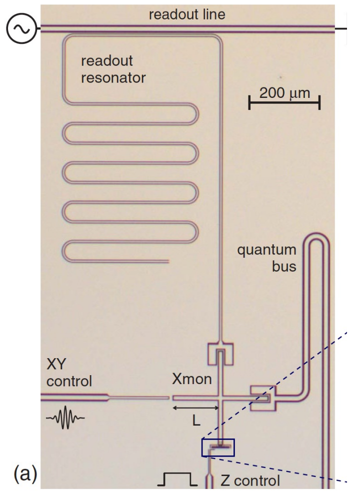
$$H_{Int} = \hbar g_{01}\left(b^{\dagger}\sigma^{-} + b\sigma^{+}\right)$$

$$\omega_{Rabi} = g_{01}\sqrt{n_r + 1}$$

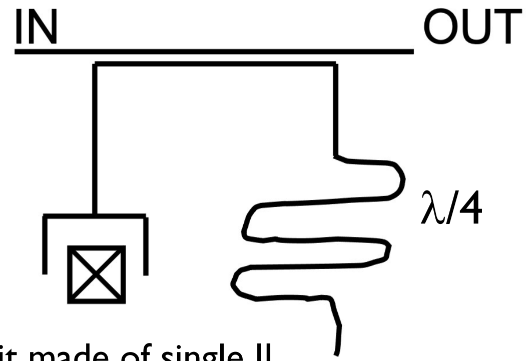
$$\begin{aligned} C_1(t) &= C_1(0)\cos\left(\omega_{Rabi}t\right) - C_0(0)\sin\left(\omega_{Rabi}t\right) \\ C_0(t) &= C_1(0)\sin\left(\omega_{Rabi}t\right) + C_0(0)\cos\left(\omega_{Rabi}t\right) \end{aligned}$$



DESIGN OF FIRST QUBIT CIRCUIT

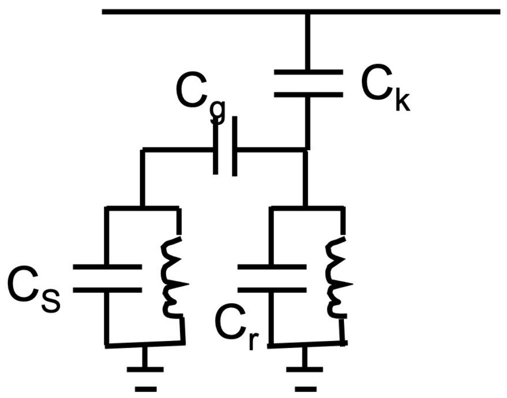


Control and Readout from the same Transmission Line



Qubit made of single JJ

Equivalent Circuit

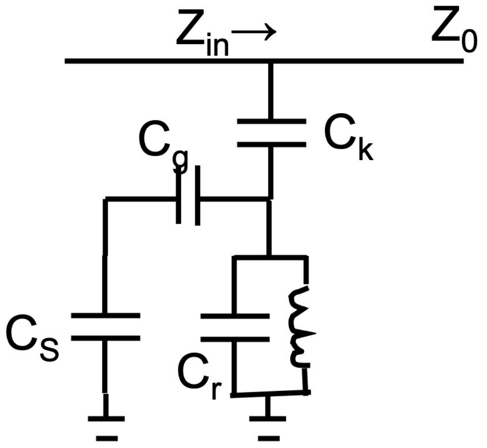


We consider in the following a $\lambda/4$ resonator with frequency $\omega_r = 6$ GHz, length 5.47 mm, and characteristic impedance $Z_0 = 50\Omega$ resulting in $C_r = 0.42$ pF and $L_r = 1.7$ nH. For the qubit we choose a frequency around 4 GHz and the transmon regime $E_j/E_c = 50$. Choosing $C_s = 80$ fF and $C_g = 10$ fF we obtain $\omega_p/2\pi = \sqrt{8E_cE_j}/\hbar = 4.3$ GHz, $E_j = 10$ GHz, $E_c = 0.215$ GHz which also provide the anharmonicity of the qubit. The frequencies of the first two levels are $\omega_{01} = 4.09$ GHz and $\omega_{12} = 3.87$ GHz. From the equations found in the previous sections we also obtain:

$$
\begin{aligned}
V_{rms} &= \sqrt{\frac{\hbar\omega_r}{2C_r}} = 2.18\ \mu\text{V} \\
\beta &= \frac{C_g}{C_g + C_s} = 0.11 \\
g_{01} &= 2\beta e V_{rms}\left(\frac{E_j}{32E_c}\right)^{1/4} = 0.824\ \text{GHz} \\
\chi &= \chi_{01} - \chi_{12}/2 = -5.73\ \text{MHz} \\
\Delta_0/2\pi &= (\omega_{01} - \omega_r)/2\pi = -1.91\ \text{GHz} \\
\Delta_1/2\pi &= (\omega_{12} - \omega_r)/2\pi = -2.12\ \text{GHz}
\end{aligned}
$$

QUBIT

READOUT



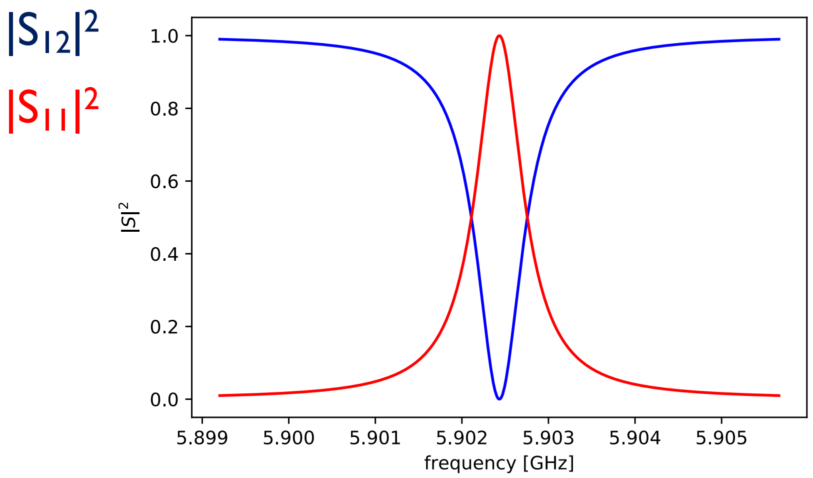
$$C_T = C_r + C_g C_s/(C_g + C_s)$$

$$C_k = 5 \text{ fF}$$

$$\xi = \omega C_k Z_0 \sim 0.01$$

$$\begin{aligned} S_{11} &= \frac{-1}{1+2\frac{1-(\omega/\omega_{Tk})^2}{j\xi(1-(\omega/\omega_T)^2)}} \\ &\simeq \frac{-1}{1-2j\frac{1-(\omega/\omega_{Tk})^2}{\xi(1-(\omega_{Tk}/\omega_T)^2)}} \\ &\simeq \frac{-1}{1+2j\frac{(\omega-\omega_{Tk})/\omega_{Tk}}{\xi C_k/(C_T+C_k)}} \\ &\simeq \frac{-1}{1+2jQ(\omega-\omega_{Tk})/\omega_{Tk}} \end{aligned}$$

$$\omega_{Tk} = 1/\sqrt{(C_T + C_k)L_r}$$

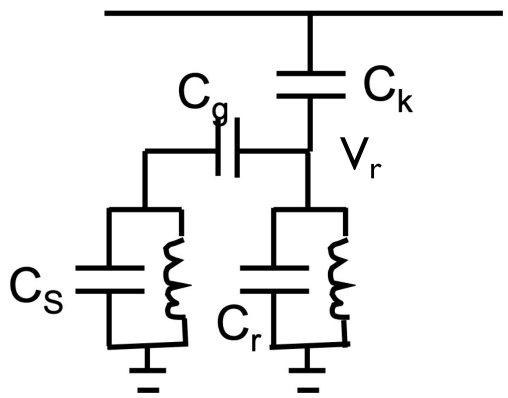


$$Q = (C_T + C_k)/C_k\xi \sim 9000$$

$$\omega_{Tk}/Q = 4 \text{ MHz}$$

READOUT: CRITICAL POWER

Per un impulso di readout, di ampiezza V^{in}, risonante con il risonatore, $\omega_{rf}=\omega_r$, la tensione ai capi del risonatore è:



$$V_r = V^{in} \frac{-2}{j\omega C_k Z_0}$$

Da cui il numero di fotoni nel risonatore è:

$$\frac{1}{2} C_r V_r^2 = n\hbar\omega_r$$

Imponendo che siano meno del numero critico di fotoni:

$$n_{cr} = \frac{\Delta^2}{4g_{01}^2}$$

Si ha il limite di potenza del segnale da utilizzare per il readout

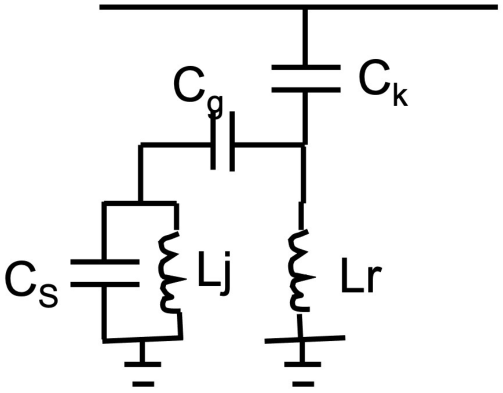
$$P^{in} < P^{cr} = \frac{n_{cr}}{4} Z_0 (\omega C_k V_{rms})^2$$

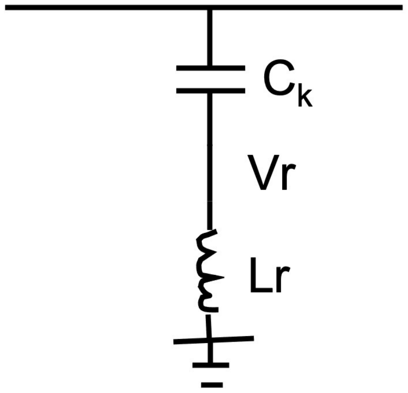
In our case we have $n_{cr} = 13$ and $P^{cr} = -131$ dbm that should be compared to a JPA noise -138 dbm and HEMT noise -128 dbm considering a bandwidth of half the cavity width.

QUBIT RELAXATION THROUGH THE RESONATOR

$$\Gamma = \left(\frac{g_{01}}{\Delta_{01}}\right)^2 \frac{\omega_r}{Q} \simeq 50\mu\mathrm{s}$$

QUBIT CONTROL

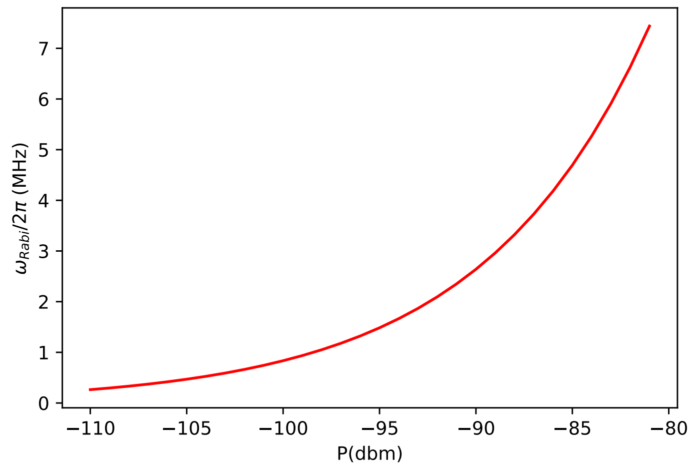




$$V_r = V_0 - V_{Ck} = V^{in} S_{12}\left(1 - \frac{1}{Z_l}\frac{1}{j\omega C_k}\right) \sim -V^{in}(\omega/\omega_k)^2$$

$$\omega_k^2 = 1/L_r C_k \sim 2\pi 40 \text{ GHz}$$

$$\omega_{Rabi} = \frac{g_{01}}{2}\frac{\sqrt{Z_0}}{V_{rms}}\left(\frac{\omega}{\omega_k}\right)^2 \sqrt{P^{in}}$$



Input Power for qubit control