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# **Chirally enhanced self-energies in the MSSM**

**Les Rencontres de Physique  
de la Vallée d'Aosta 2011**

**Effective Higgs Vertices in the generic MSSM.**  
**Andreas Crivellin**, arXiv:1012.4840 [hep-ph]

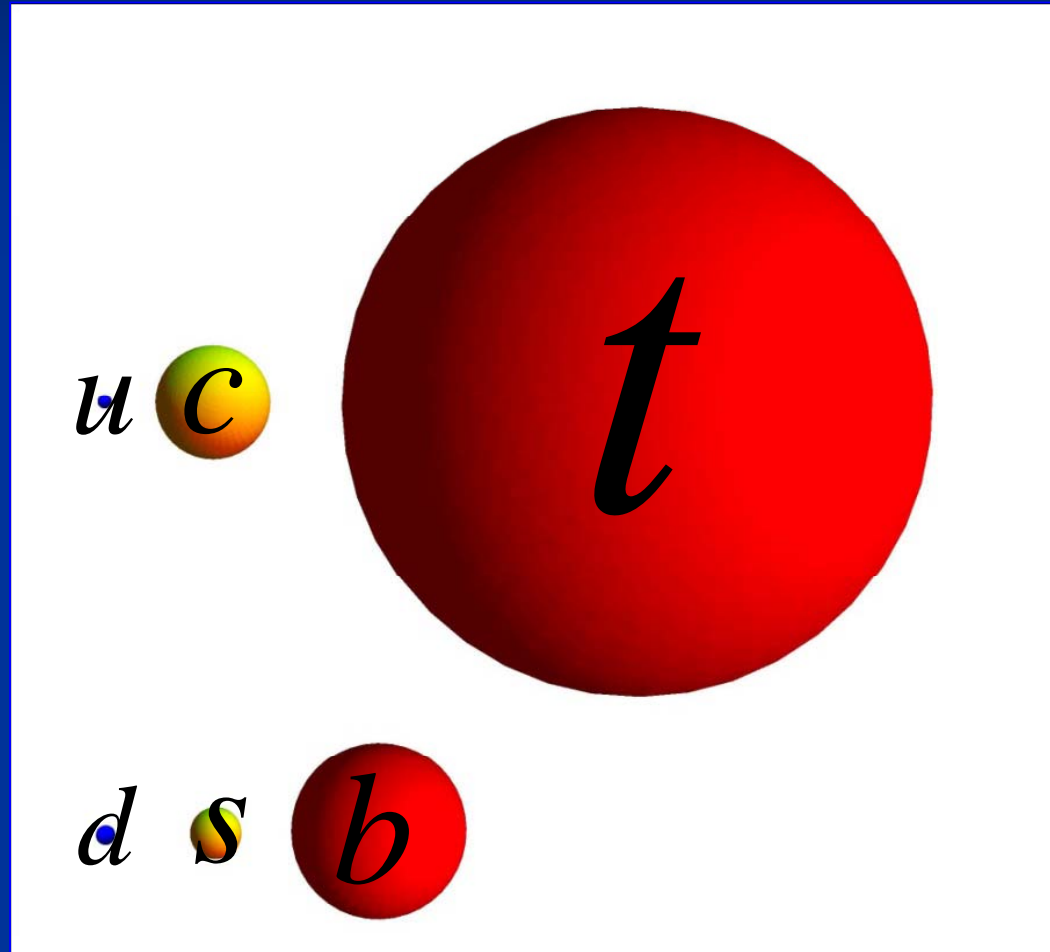
**Radiative Flavor-Violation in the MSSM**  
**Andreas Crivellin**, **Lars Hofer**, **Dominik Scherer** and **Ulrich Nierste**,  
arXiv:1103.XXXX [hep-ph]

# Outline:

- The SUSY flavor-problem
- Self-energies and the origin of chiral enhancement
- Renormalization and  $\tan(\beta)$  resummation.
- Flavor-changing neutral Higgs vertices
- Radiative flavor-violation in the MSSM

# Quark masses

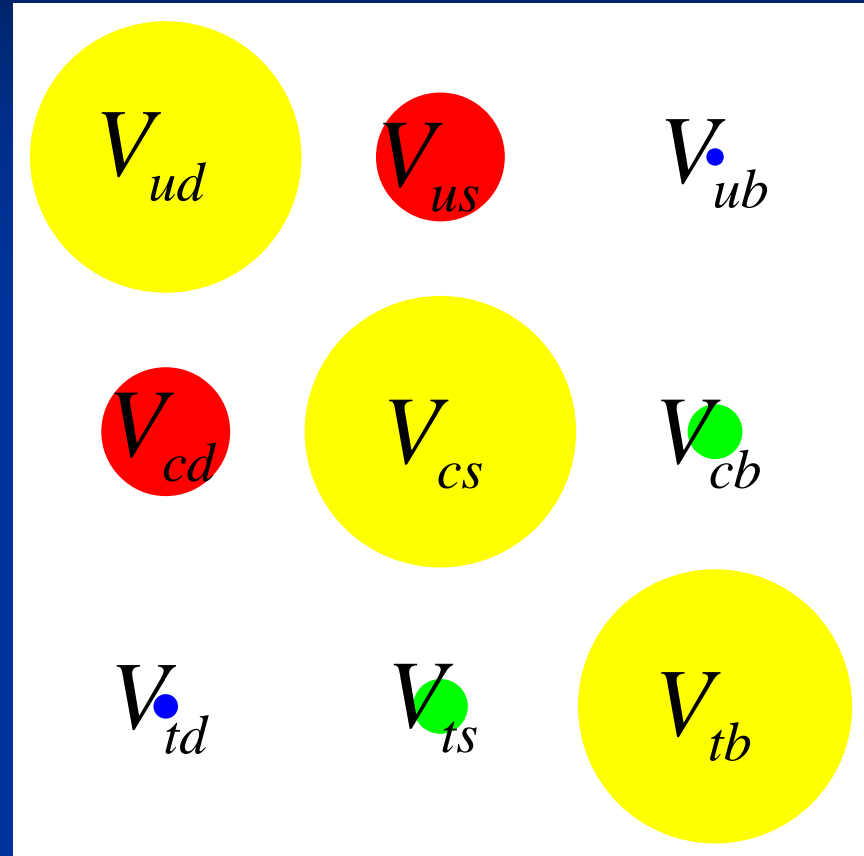
- Top quark is very heavy.  $m_t \approx v$
- Bottom quark rather light, but  $Y^b$  can be big at large  $\tan(\beta)$
- All other quark masses are very small  
➔ sensitive to radiative corrections



# CKM matrix

- CKM matrix is the only source of flavor and CP violation in the SM.
  - No tree-level FCNCs.
  - Off-diagonal CKM elements are small
- ➔ Flavor-violation is suppressed in the SM.

$$V_{\text{CKM}} =$$



# SUSY flavor (CP) problem

- The squark mass matrices are not necessarily diagonal (and real) in the same basis as the quark mass matrices.
- Especially the trilinear A-terms can induce dangerously large flavor-mixing (and complex phases) since they don't necessarily possess the suppression of the SM.
- The MSSM possesses two Higgs-doublets: Flavour-changing charged and (loop-induced) neutral Higgs interactions.
- **Possible solutions:**
  - **MFV** D'Ambrosio, Giudice, Isidori, Strumia hep-ph/0207036
  - **effective SUSY** Barbieri et al hep-ph/10110730
  - **Radiative flavour violation**

# Squark mass matrix

$$M_{\tilde{q}}^2 = \begin{pmatrix} M_{LL}^{\tilde{q}2} & \Delta^{\tilde{q}LR} \\ \Delta^{\tilde{q}LR\dagger} & M_{RR}^{\tilde{q}2} \end{pmatrix}$$

hermitian:  $\longrightarrow W^{\tilde{q}\dagger} M_{\tilde{q}}^2 W^{\tilde{q}} = M_{\tilde{q}}^{2(D)}$

$M_{LL,RR}^{\tilde{q}2}$  involves only bilinear terms (in the decoupling limit)

The chirality-changing elements are proportional to a vev

$$\Delta_{ij}^{dLR} = -v_d \left( \mu \tan(\beta) Y_i^{d(0)} \delta_{ij} + A_{ij}^d \right) \quad \tan(\beta) = \frac{v_u}{v_d}$$
$$\Delta_{ij}^{uLR} = -v_u \left( \mu \cot(\beta) Y_i^{u(0)} \delta_{ij} + A_{ij}^u \right)$$

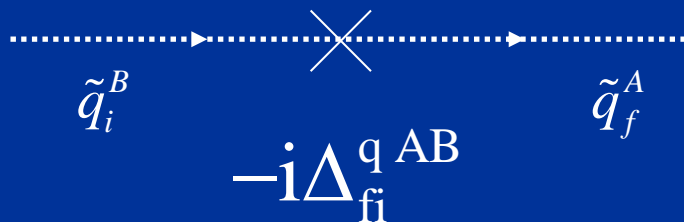
# Mass insertion approximation

(L.J. Hall, V.A. Kostelecky and S. Raby, Nucl. Phys. B 267 (1986) 415.)

- Useful to visualize flavor-changes in the squark sector

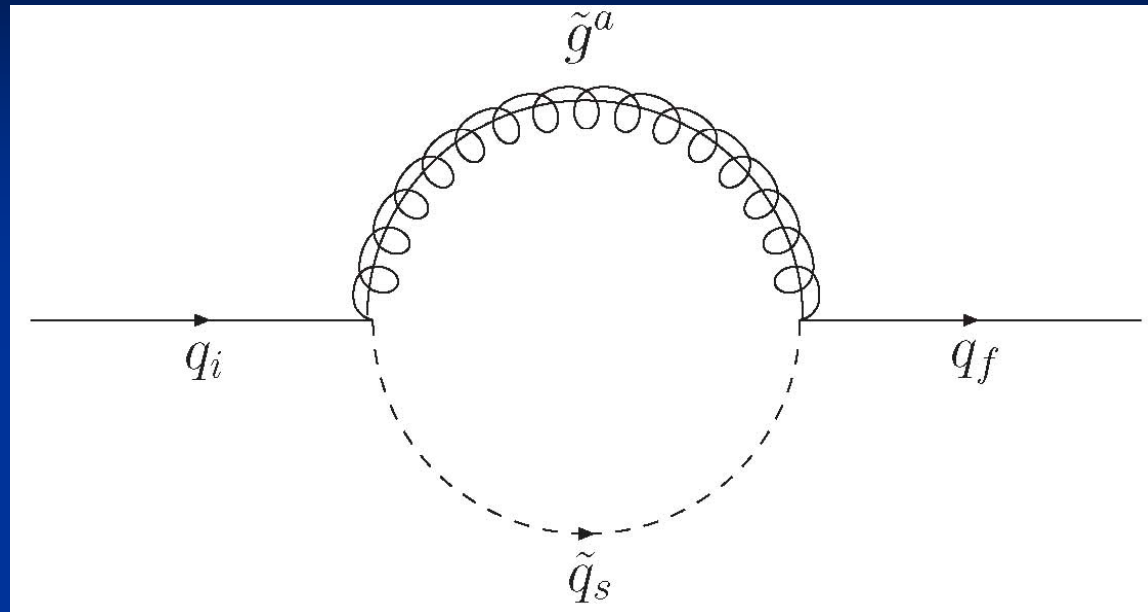
$\Delta_{ij}^{q AB}$  off-diagonal element of the squark mass matrix

- $q = u, d$
- $i, j$  flavor indices 1,2,3
- $A, B$  chiralities L,R



# SQCD self-energy:

$$-i\Sigma(0)_{fi}^{qLR} =$$



$$\Sigma_{fi}^{qLR} = \alpha_s \frac{2}{3\pi} m_{\tilde{g}} W_{fs} W_{i+3,s}^* B_0(m_{\tilde{g}}^2, m_{\tilde{q}_s}^2)$$

Finite and proportional to at least one power of  $\Delta_{fi}^{qLR}$

$$\Sigma_{fi}^{qLR} = \alpha_s \frac{2}{3\pi} m_{\tilde{g}} W_{fs}^q W_{js}^{q*} \Delta_{jl}^{qLR} W_{lt}^q W_{it}^{q*} C_0(m_{\tilde{g}}^2, m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2)$$

**decoupling limit**



# Decomposition of the self-energy

Decompose the self-energy

$$\Sigma_{ii}^{dLR} = \Sigma_{iiA}^{dLR} + \Sigma_{iiY}^{dLR}$$

into a holomorphic part proportional to an A-term

$$\Sigma_{fiA}^{dLR} = -v_d \alpha_s \frac{2}{3\pi} m_{\tilde{g}} W_{fs}^d W_{js}^{d*} A_{jl}^q W_{lt}^d W_{it}^{d*} C_0 \left( m_{\tilde{g}}^2, m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2 \right)$$

**non-holomorphic** part proportional to a Yukawa

$$\Sigma_{fiY}^{dLR} = -v_u \mu \alpha_s \frac{2}{3\pi} m_{\tilde{g}} W_{fs}^d W_{js}^{d*} Y^{dj} W_{jt}^d W_{it}^{d*} C_0 \left( m_{\tilde{g}}^2, m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2 \right)$$

Define dimensionless quantity  $\varepsilon_i^d = \frac{\Sigma_{iiY}^{dLR}}{v_u Y^{di}}$

which is dimensionless and independent of a Yukawa coupling

# Renormalization I

- All corrections are finite; counter-term not necessary.
- Minimal renormalization scheme is simplest.

## Mass renormalization

$$\begin{aligned} m_{d_i} &= v_d Y^{d_i(0)} + \sum_{ii}^{d LR} \\ &= v_d Y^{d_i(0)} + \sum_{ii A}^{q LR} + v_d \tan(\beta) Y^{d_i(0)} \varepsilon_i^d \end{aligned}$$

$$\longrightarrow Y^{d_i(0)} = \frac{m_{d_i} - \sum_{ii A}^{q LR}}{v_d (1 + \tan(\beta) \varepsilon_i^d)}$$

- $\tan(\beta)$  is automatically resummed to all orders

# Renormalization II

- Corrections to the CKM matrix:

$$V_{fi}^{CKM} = U_{jf}^{uL*} V_{jk}^{CKM(0)} U_{ki}^{dL}$$

$$U^{qL} = \begin{pmatrix} 1 - \frac{|\sum_{12}^{qLR}|^2}{2m_{q_2}^2} & \frac{1}{m_{q_2}} \sum_{12}^{qLR} & \frac{1}{m_{q_3}} \sum_{13}^{qLR} \\ \frac{-1}{m_{q_2}} \sum_{21}^{qRL} & 1 - \frac{|\sum_{12}^{qLR}|^2}{2m_{q_2}^2} & \frac{1}{m_{q_3}} \sum_{23}^{qLR} \\ \frac{-1}{m_{q_3}} \sum_{31}^{qRL} + \frac{\sum_{32}^{qRL} \sum_{21}^{qRL}}{m_{q_2} m_{q_3}} & \frac{-1}{m_{q_3}} \sum_{32}^{qRL} & 1 \end{pmatrix}$$

**important two-loop  
corrections**

A.C. Jennifer Girrbach 2010

# Chiral enhancement

$$\Sigma_{fi}^{dLR} \approx \frac{1}{50} \frac{\Delta_{fi}^{qLR}}{M_{SUSY}} = \frac{-v_d}{50} \left( \tan(\beta) Y_i^{d(0)} \delta_{ij} + \frac{A_{ij}^d}{M_{SUSY}} \right)$$

- For the bottom quark only the term proportional to  $\tan(\beta)$  is important.

➡  **$\tan(\beta)$  enhancement**

Blazek, Raby, Pokorski, hep-ph/9504364

- For the light quarks also the part proportional to the A-term is relevant.

$$\Sigma_{33Y}^{dLR} = \frac{-1}{50} v_d \tan(\beta) Y^{b(0)} \square m_b$$

$$O\left(\frac{\tan(\beta)}{50}\right)$$

$$\Sigma_{22A}^{dLR} = O(1) \square A_{22}^d \approx M_{SUSY}$$

$$\Sigma_{11A}^{dLR} = O(1) \square A_{11}^d \approx \frac{1}{50} M_{SUSY}$$

# Flavour-changing corrections

$$\frac{\sum_{fi}^q \text{LR}}{m_{q_{\max(f,i)}}} \square V_{fi}^{\text{CKM}}$$

$$V_{cb}^{\text{CKM}} : A_{23}^q \approx M_{\text{SUSY}}$$

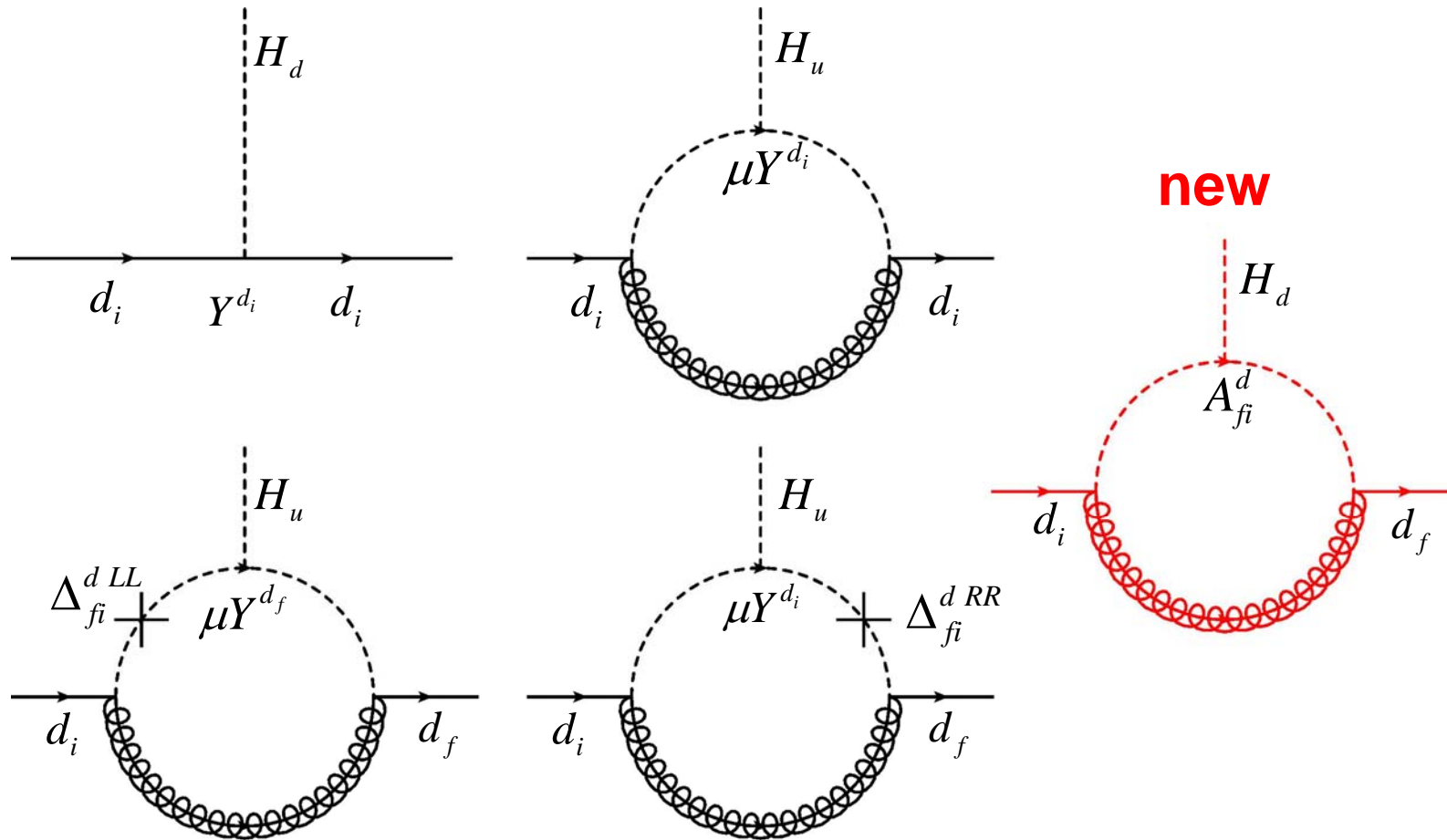
$$V_{ub}^{\text{CKM}} : A_{13}^q \approx M_{\text{SUSY}} \times 10^{-1}$$

$$V_{us}^{\text{CKM}} : A_{12}^q \approx M_{\text{SUSY}} \times 10^{-1}$$

- Flavor-changing A-term can easily lead to order one correction.

A.C., Ulrich Nierste, hep-ph/08101613

# Higgs vertices in the EFT I



# Higgs vertices in the EFT II

$$\mathcal{L}_Y^{\text{eff}} = \bar{Q}_{fL}^a \left( \left( Y_i^d \delta_{fi} + E_{fi}^d \right) \varepsilon_{ba} H_d^b + E_{fi}'^d H_u^{a*} \right) d_{iR}$$

- Non-holomorphic corrections  $E_{fi}'^d = \frac{\sum_{fi}^{dLR} Y}{V_u}$
- Holomorphic corrections  $E_{fi}^d = \frac{\sum_{fi}^{dLR} A}{V_u}$
- The quark mass matrix  $m_{fi}^d = v_d \left( Y_i^d \delta_{fi} + E_{fi}^d \right) + v_u E_{fi}'^d$  is no longer diagonal in the same basis as the Yukawa coupling

➔ Flavour-changing neutral Higgs couplings

# Effective Yukawa couplings

- Final result:

$$Y_{ij}^{\text{d eff}} = \frac{1}{V_d} \left( m_{d_i} \delta_{ij} - \tilde{\Sigma}_{ij Y}^{\text{d LR}} \right)$$

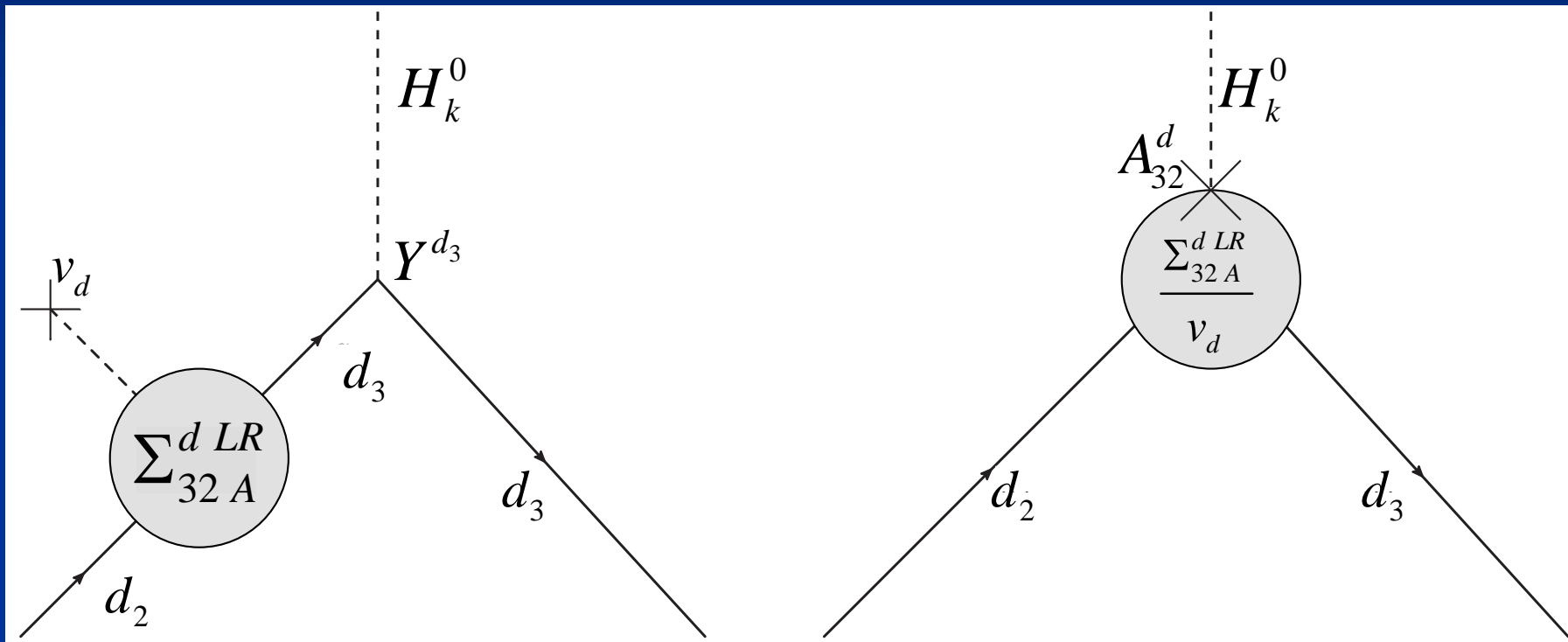
with

$$\tilde{\Sigma}_{jk Y}^{\text{d LR}} = U_{jf}^{\text{d L}*} \Sigma_{jk Y}^{\text{d LR}} U_{ki}^{\text{d R}} \approx \Sigma_{fi Y}^{\text{d LR}} - \left( \begin{array}{ccc} 0 & \frac{\Sigma_{22 Y}^{\text{d LR}}}{m_{d_2}} \Sigma_{12}^{\text{d LR}} & \frac{\Sigma_{33 Y}^{\text{d LR}}}{m_{d_3}} \Sigma_{13}^{\text{d LR}} \\ \frac{\Sigma_{22 Y}^{\text{d LR}}}{m_{d_2}} \Sigma_{21}^{\text{d LR}} & 0 & \frac{\Sigma_{33 Y}^{\text{d LR}}}{m_{q_3}} \Sigma_{23}^{\text{d LR}} \\ \frac{\Sigma_{33 Y}^{\text{d LR}}}{m_{d_3}} \Sigma_{31}^{\text{d LR}} & \frac{\Sigma_{33 Y}^{\text{d LR}}}{m_{q_3}} \Sigma_{32}^{\text{d LR}} & 0 \end{array} \right)$$

Diagrammatic explanation in the full theory:



# Higgs vertices in the full theory



- Cancellation incomplete since  $v_d Y^{d_3} \neq m_{d_3}$   
Part proportional to  $\sum_{33 Y}^{d LR}$  is left over.

➡ A-terms generate flavor-changing Higgs couplings

# Radiative flavor-violation

$SU(2)^3$  flavor-symmetry in the MSSM superpotential:

- CKM matrix is the unit matrix.
- Only the third generation Yukawa coupling is different from zero.

$$V_{\text{CKM}}^{(0)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Y^q = \frac{1}{v_q} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_{q_3} \end{pmatrix}$$

**All other elements are generated radiatively using the trilinear A-terms!**

# Features of the model

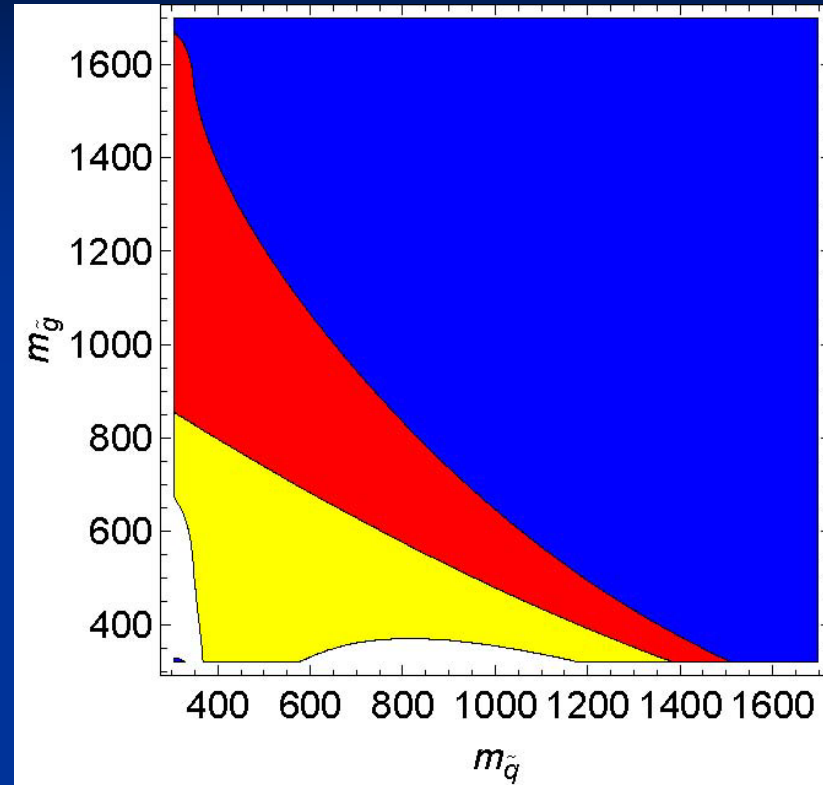
- Additional flavor symmetries in the superpotential.
- Explains small masses and mixing angles via a loop-suppression.
- Deviations from MFV if the third generation is involved.
- Solves the SUSY CP problem via a mandatory phase alignment. (Phase of  $\mu$  enters only at two loops)  
Borzumati, Farrar, Polonsky, Thomas 1999.
- The SUSY flavor problem reduces to the elements  $\delta_{32}^{qLR}, \delta_{31}^{qLR}$
- Can explain the  $B_s$  mixing phase

# CKM generation in the down-sector:

$$\sum_{13}^{\text{dLR}} = m_b V_{ub}$$

$$\sum_{23}^{\text{dLR}} = m_b V_{cb}$$

- **Constraints from  $b \rightarrow s\gamma$ .**  
Chirally enhanced corrections must be taken into account.  
A.C., Ulrich Nierste 2009
- $\delta_{31}^{\text{dLR}}, \delta_{32}^{\text{dLR}}$  less constrained since they contribute to  $C7', C8'$ .
- $\delta_{32}^{\text{dLR}}$  can explain the CP phase in  $B_s$  mixing.  
(not possible in MFV)



$$m_b \mu \tan(\beta) = 0.12 \text{TeV}^2$$



$$m_b \mu \tan(\beta) = 0 \text{TeV}^2$$

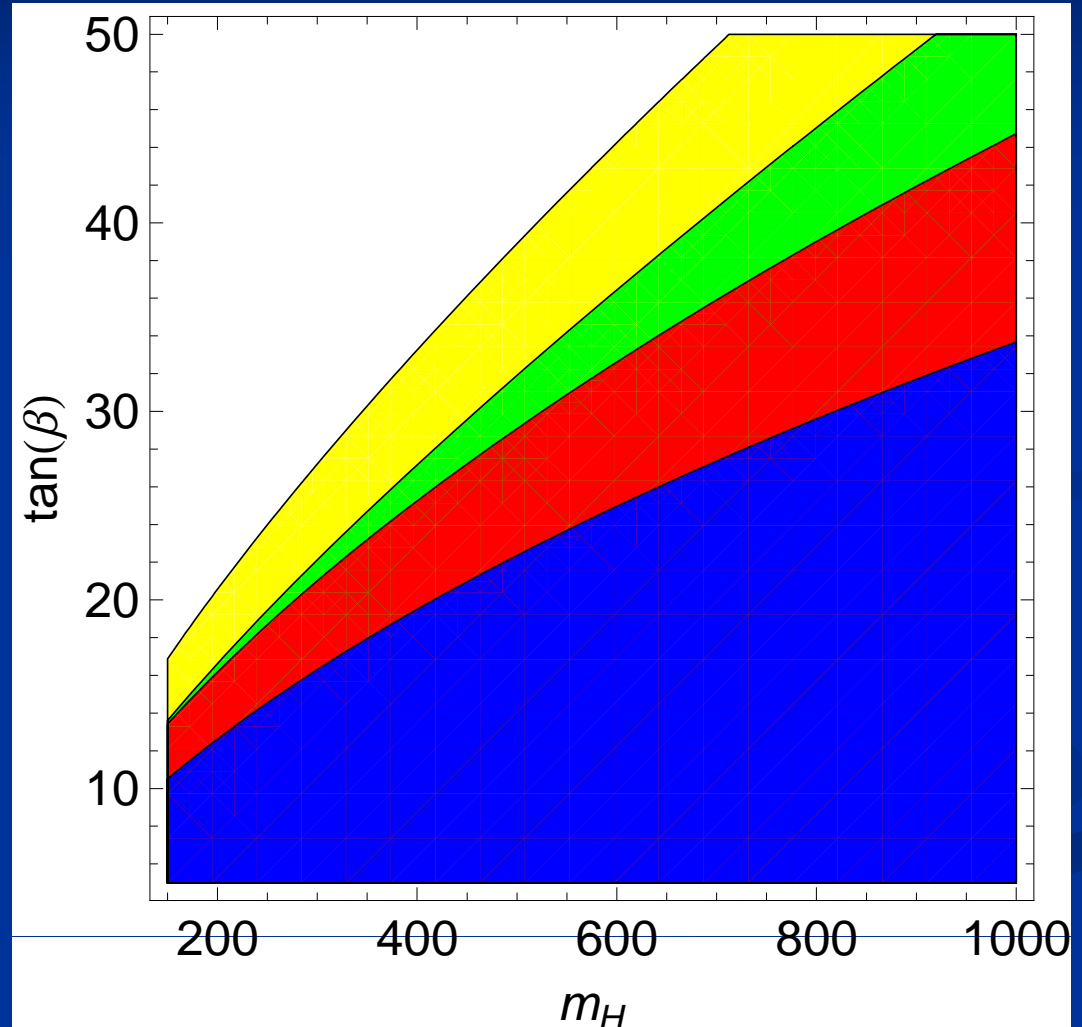
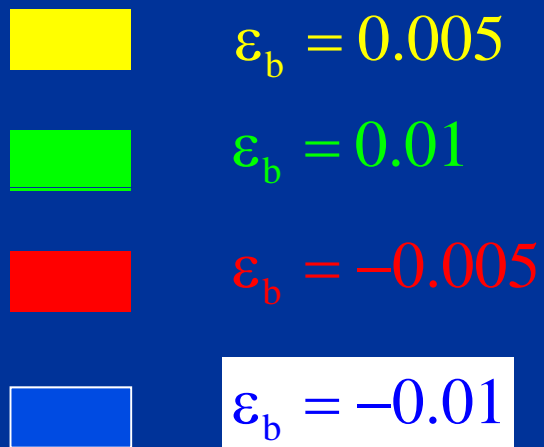


$$m_b \mu \tan(\beta) = 0.12 \text{TeV}^2$$

# Higgs effects: $B_s \rightarrow \mu\mu$

- Constructive contribution due to

$$\sum_{23}^{d LR} = m_b V_{cb}$$



# Higgs effects: $B_s$ mixing

- Contribution only if

$$V_{23}^R = \frac{\sum_{23}^{dRL}}{m_b} \neq 0$$

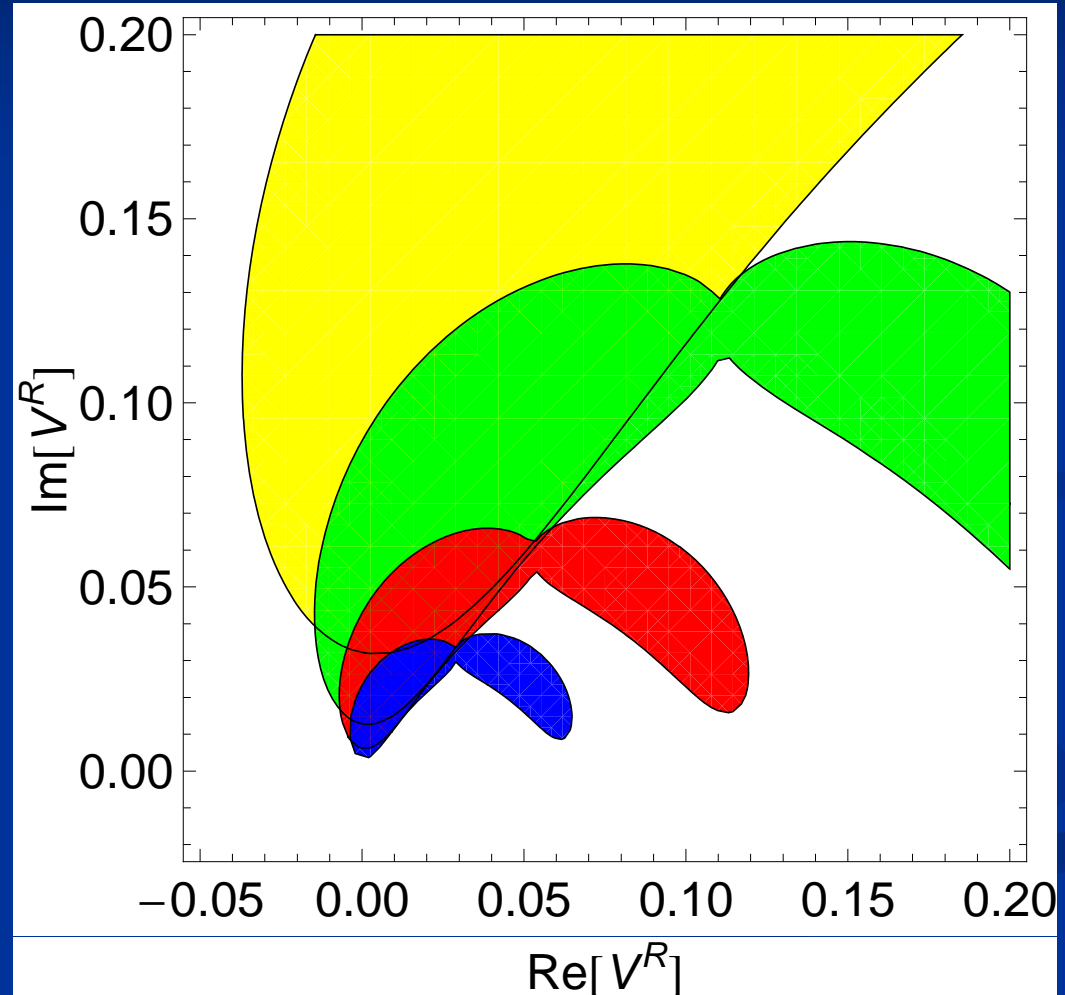
due to Pecci-Quinn symmetry

  $\tan(\beta) = 11$

  $\tan(\beta) = 14$

  $\tan(\beta) = 17$

  $\tan(\beta) = 20$



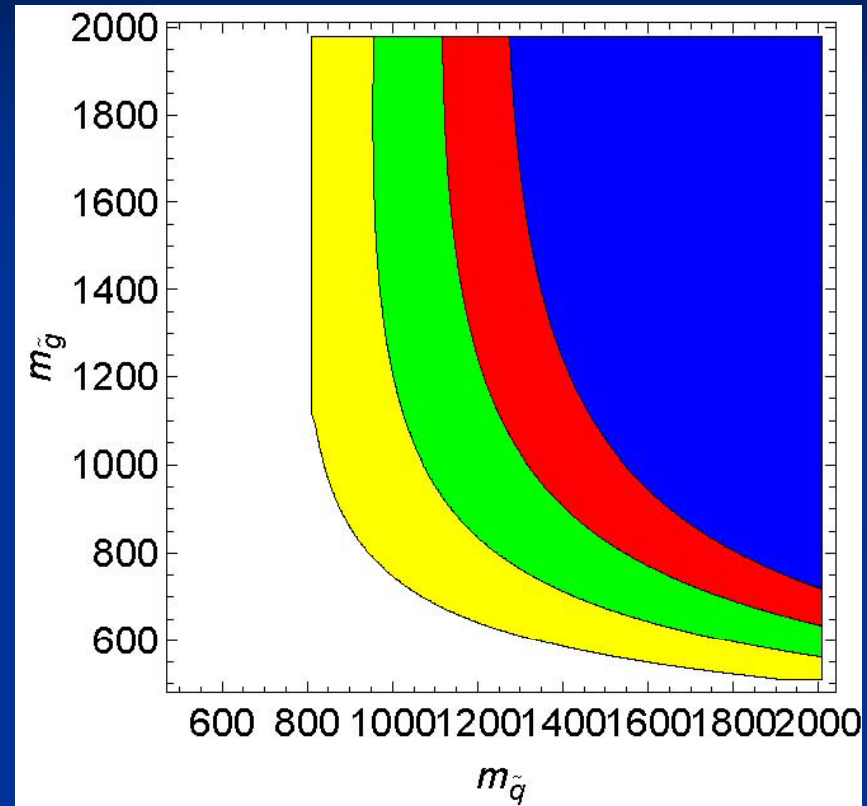
# CKM generation in the up-sector:

$$\Sigma_{13}^{uLR} = m_t V_{td}^*$$

$$\Sigma_{23}^{uLR} = m_t V_{cb}^*$$

- Constraints from Kaon mixing.
- $\delta_{31}^{uLR}, \delta_{32}^{uLR}$  unconstrained from FCNC processes.
- $\delta_{31}^{uLR}$  can induce a sizable right-handed W coupling.

A.C. 2009



■  $M_2 = 200\text{GeV}$

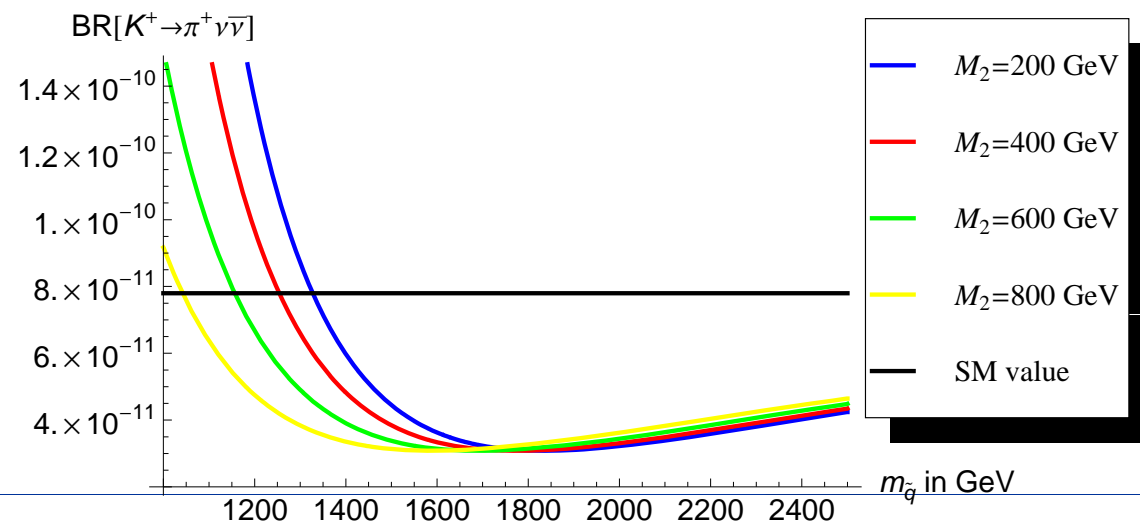
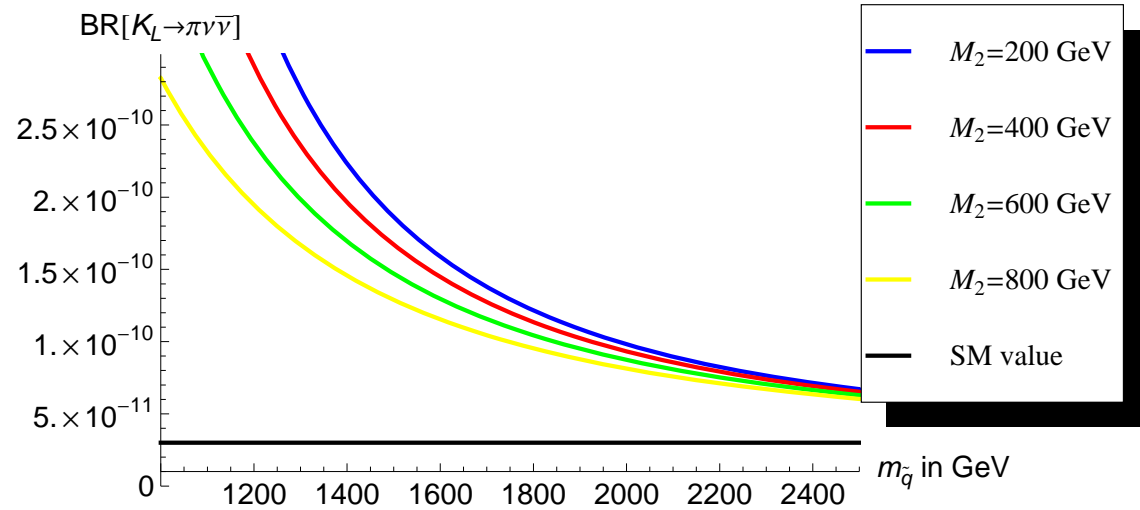
■  $M_2 = 400\text{GeV}$

■  $M_2 = 400\text{GeV}$

■  $M_2 = 800\text{GeV}$

- Effects in  $K \rightarrow \pi \nu \bar{\nu}$

- Verifiable predictions for NA62





# Conclusions

- The MSSM possesses many new sources of flavor and CP violation
- Self-energies can be chirally enhanced and of order one.
- Flavour-conserving non-holomorphic corrections induce flavour-changing neutral Higgs couplings proportional to  $A$ -terms.
  - ➔ Self-energies have physical effects.
- Radiative flavor-violation in the MSSM is an interesting solution to the SUSY CP and SUSY flavor problem.
- Constraints from  $b \rightarrow s\gamma$  and Kaon mixing are satisfied for SUSY masses  $O(1\text{TeV})$ .
- Large effects in  $K \rightarrow \pi\nu\nu$  are possible.
- $B_s$  mixing phase can be explained.