Sifting through the SM for the hints of an ALP

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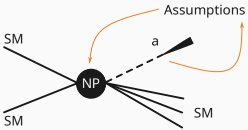
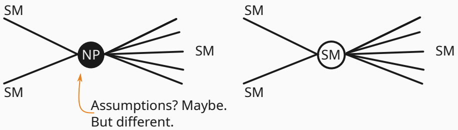
Direct vis à vis Indirect



Exotic Signatures

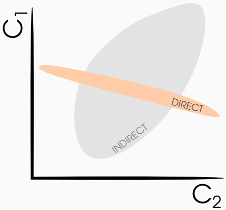
- Missing
- Displaced etc



Precision SM

- Branching Frac
- Shape etc

EFT approach:
Term by Term



Overview

Shift Symmetric $a \to a + c$

T$_1$: $aG\tilde{G}, \quad aW\tilde{W}, \quad aB\tilde{B}$

T$_2$: $\frac{\partial_\mu a}{f_a} \; [\; C_L^i \, \bar{q}_L^i t_{ij}^a q_L^j + C_R^i \, \bar{q}_R^i t_{ij}^a q_R^j \;]$

T$_3$: $\bar{q}_L \left[e^{i x_L^a T_a a/f_a} M_q \; e^{i x_R^b T_b a/f_a} \right] q_R$

- $q_{L/R} \to SU(3)_{L/R} \quad 3$
- Mass from QCD anomaly
- Rotate $aG\tilde{G}$ etc away
  - $q_R \to e^{i\frac{a}{f_a}} q_R$
- Redef Invariance

The 'bare' mass

- $\frac{1}{2} m_a^2 a^2$

Periodic Symmetry? $a \to a + \frac{2\pi}{n}$

- $aGG \quad aWW \quad aBB, \; \frac{1}{f_a} a \bar{q} \gamma^\mu q \; j_\mu$

! e.g., Leading terms of $\sin(a), \cos(a)$

- Portal couplings
- Maybe Later!

The Lagrangian we want

Theory of light mesons and an ALP: AχPT

- $\mathcal{L}_{A\chi PT} = \mathcal{L}^{SM}_{A\chi PT} + \mathcal{L}^{a}_{A\chi PT}$
  - In flavour basis
    - $a - \pi_0$ two-point functions $\rightarrow$ redefined π^0
    - $a\,\mathcal{O}(\pi_0, \pi_+, j^\mu_\pm, \cdots)$

$\Longrightarrow$ $\pi^0\ \mathcal{O}(\pi_0, \pi_+, j^\mu_\pm, \cdots)$ In mass basis

  - Goes on to mess up light-Meson mass spectrum
  - Modifies the $\langle K^+/\pi^+|\mathcal{O}|\pi^0\rangle$ form factors $\longleftarrow$
  - Modifies Meson decay Sum rules $\longleftarrow$

The same exercise for HQET
(!) Also check: Bauer et. al., 2102.13112, 2012.12272

To the chiral Lagrangian

- The symmetry breaking:

$$SU(3)_L \times SU(3)_R \xrightarrow{\langle 0|\bar{q}_L^j q_R^i|0\rangle \sim \Lambda^3 \delta^{ij}} SU(3)_V$$

- Non-linear representation of the Goldstones:

$$U_\pi \equiv e^{2t_i \frac{\pi^i}{f_\pi}} \xrightarrow{L\times R} L U_\pi R^\dagger$$

- The Lagrangian:

$$\mathcal{L} \supset \frac{f_\pi^2}{4}(D_\mu U_\pi)^\dagger (D^\mu U_\pi) + \Lambda \frac{f_\pi^2}{2} Tr[M U_\pi^\dagger] + h.c. + \cdots$$

$$D_\mu U_\pi = \partial_\mu U_\pi - i L_\mu U_\pi + i U_\pi R_\mu$$

$$J_\mu^{L\,a} = \frac{\delta \mathcal{L}}{\delta(\partial^\mu q_L)} \frac{\delta q_L}{\delta \alpha_L^a} + q_L \leftrightarrow \bar{q}_L \text{ (upstairs)}$$

$$J_\mu^{L\,a} = \frac{\delta \mathcal{L}}{\delta(\partial^\mu U_\pi)} \frac{\delta U_\pi}{\delta \alpha_L^a} + U_\pi \leftrightarrow \bar{U}_\pi = -i \frac{f_\pi^2}{2} \mathrm{Tr}\left[U_\pi^\dagger t^a \partial^\mu U_\pi \right] \text{ (downstairs)}$$

Let's focus: $K^+ \to \pi^0 \ell \nu$

$$\mathcal{L} = i G_F V_{\bar{s}u} \Big[f^+_{SM}(q^2) \{ K^+ \partial_\mu \pi_0 - \partial_\mu K^+ \pi_0 \} + f^-_{SM}(q^2) \partial_\mu (K^+ \pi_0) \Big] \, j^\mu_{-,\ell}$$

$f_-^{K^+\pi^0}(0) = 0$at LO in the SM

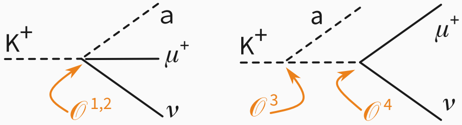
- $\langle \pi^0(p_\pi) | \bar{s}\gamma_\mu u | K^+(p_K) \rangle \equiv \frac{1}{\sqrt{2}} \Big[f^{K^+\pi^0}_{+,\mathrm{SM}}(q^2)\; Q_\mu + f^{K^+\pi^0}_{-,\mathrm{SM}}(q^2)\; q_\mu \Big]$
- $\langle \pi^+(p_\pi) | \bar{s}\gamma_\mu u | K^0(p_K) \rangle \equiv f^{K^0\pi^-}_{+,\mathrm{SM}}(q^2)\; Q_\mu + f^{K^0\pi^-}_{-,\mathrm{SM}}(q^2)\; q_\mu$
  - $Q^\mu = p_K^\mu + p_\pi^\mu; \quad q_\mu = p_K^\mu - p_\pi^\mu$

- $\dfrac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1 - \sqrt{3}\,\epsilon_{\eta-\pi_0} - \xi^2 \frac{C_3}{8} \left[C_A^3 + C_{LR}^3 + 2\sqrt{3} C_{LR}^8 \right]$

$$\xi = \frac{f_\pi}{f_a}$$

$K^+ \to \pi^0 \ell \nu$: AχPT

- $\mathcal{O}^1_{K^+_{\ell_3}}$: $(K^+\partial_\mu a - \partial_\mu K^+ a)\, j^\mu_{-,\ell}$ $\quad iG_F V_{\bar{s}u}\frac{\xi}{2}(\mathcal{C}_R - 2iC_W)$
- $\mathcal{O}^2_{K^+_{\ell_3}}$: $(K^+\partial_\mu a + \partial_\mu K^+ a)\, j^\mu_{-,\ell}$ $\quad iG_F V_{\bar{s}u}\frac{\xi}{2}(\mathcal{C}_R + 2iC_W)$
- $\mathcal{O}^3_{K^+_{\ell_3}}$: $\partial^\mu a\, (\partial_\mu K^+ K^- - K^+\partial_\mu K^-)$ $\quad \frac{i}{4}\frac{1}{f_\pi}\xi\,(\mathcal{C}_R + \sqrt{3}C^8_L)$
- $\mathcal{O}^4_{K^+_{\ell_3}}$: $\partial_\mu K^+ j^\mu_-$ $\quad -2f_\pi\, G_F V_{\bar{s}u}$



! $\mathcal{C}_\mathcal{R} = C^3_R + \sqrt{3}C^8_R$

Distortion of Shape of diff. distribution

SM+ALP χPT

$$\mathcal{L} = iG_F V_{\bar{s}u}\left[\tilde{f}^{+}(q^2)\{K^+\partial_\mu\pi_0 - \partial_\mu K^+\pi_0\} + \tilde{f}^{-}(q^2)\partial_\mu(K^+\pi_0)\right] j^{\mu}_{-,\ell}$$

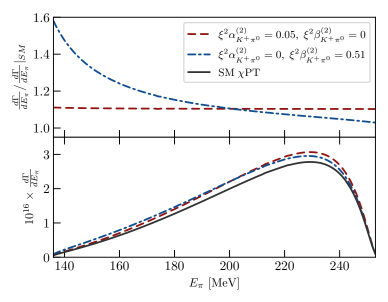
$\partial_\mu(K^+\pi^0) \rightarrow$ Brings out lepton mass

$$\tilde{f}_+(q^2) \rightarrow \left[1 + \frac{\xi^2\alpha^{(2)}_{K^+\pi^0}}{f^{+}(0)}\right] f^{+}_{\rm SM}(q^2)$$

$$\tilde{f}_-(q^2) \rightarrow \left[1 + \frac{\xi^2\beta^{(2)}_{K^+\pi^0}}{f^{-}(0)}\right] f^{-}_{\rm SM}(q^2)$$

$$\alpha^{(2)}_{K^+\pi^0} \equiv \alpha^{(2)}_{K^+\pi^0}(C_i)$$

$$\beta^{(2)}_{K^+\pi^0} \equiv \beta^{(2)}_{K^+\pi^0}(C_i)$$



Distortion of differential spectrum

$$\overline{|\mathcal{A}|}^2_{K_{l3}} = 2G_F^2|V_{\bar{s}u}|^2 C_{\rm cor}\left[1 + 2\,\xi^2 \frac{\alpha^{(2)}_{K^+\pi^0}}{f^+_{SM}(0)}\right](2H\cdot p_\ell\; H\cdot p_{\nu_\ell} - H^2 p_\ell \cdot p_{\nu_\ell}),$$

$$H_\mu \equiv f^+_{\rm SM}(q^2)\,Q_\mu + \left[1 + \xi^2\left(\frac{\beta^{(2)}_{K^+\pi^0}}{f^-_{SM}(0)} - \frac{\alpha^{(2)}_{K^+\pi^0}}{f^+_{SM}(0)}\right)\right] f^-_{\rm SM}(q^2)\,q_\mu$$

$$\simeq f^+_{\rm SM}(q^2)\,Q_\mu + \left[1 + \xi^2 \frac{\beta^{(2)}_{K^+\pi^0}}{f^-_{SM}(0)}\right] f^-_{\rm SM}(q^2)\,q_\mu, \;\because f^-_{SM}(0) \ll f^+_{SM}(0)$$

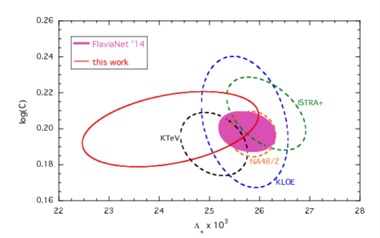
$$Q \equiv p_K + p_\pi; \quad q \equiv p_K - p_\pi; \quad C_{cor} \to \text{EW and EM corrections}$$

$$\therefore \alpha^{(2)} \to \text{Overall Scaling}; \qquad \beta^{(2)} \to \text{Mom. dependent Scaling.}$$

- This $\beta^{(2)}$ effect: previously unnoticed

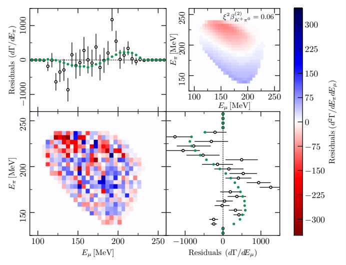
Bounds: NA48/2 differential distributions data

- SM expectations of FF params: Lattice[1]



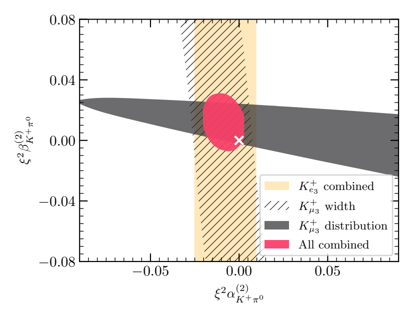
Lattice precision much lower than exp. precision

- True Value: NA48/2 → Dalitz Distr[2]



Marginalized distr.s on either side

- Theory Fitting Params : $\alpha^{(2)}_{K^+\pi^0}\xi^2$, $\beta^{(2)}_{K^+\pi^0}\xi^2$



- $\xi^2 \equiv \frac{f_\pi^2}{f_a^2}$

- Natural Next step:
  - $K \to a\mu\nu$ as bg for $K \to \mu\nu$ (.3%)
  - $\tau \to aK\nu$ as bg for $\tau \to K\nu$ (1%)

(2) 1808.09041

(1) 1602.04113

Flat Directions: "Phobias"

$$\mathcal{A}(K^+ \to a\ell\nu) \propto \alpha^{(1)}_{K^+a}$$
$$\mathcal{A}(K^+ \to \pi^0\ell\nu) \propto \alpha^{(2)}_{K^+a}$$

$$\alpha^{(2)}_{K^+\pi^0} = \frac{C_3}{2}\left(\alpha^{(1)}_{K^+a} + \frac{C_3}{4}\right)$$

- Strength of $K^+ \to \pi^0 e^+\nu$
- Strength of $K^+ \to a\, e^+\nu$

Sum Rules

- Add up amplitudes to get an idea of underlying theory
- In the SM:
  $\frac{1}{4}\left|f_{+,\,\mathrm{SM}}^{K^+\pi^0}(0)\right|^2 + \frac{3}{4}\left|f_{+,\,\mathrm{SM}}^{K^+\eta}(0)\right|^2 = 1$
  - Completeness of basis (Also think of Cabibbo angle etc)
- The same sum in the AχPT:
  - $\frac{1}{4}\left|\tilde{f}_{+}^{K^+\pi^0}(0)\right|^2 + \frac{3}{4}\left|\tilde{f}_{+}^{K^+\eta}(0)\right|^2 =$
    $1 - \frac{\xi^2}{16}\left(C_3 + \sqrt{3}C_8\right)^2 + \xi^2\frac{3}{16}(C_L^8)^2$
- The first term is expected
- These sums can tell us about the *structure* of the theory

Summary

- Low lying ALPs modify χPT in non-trivial ways
- These modifications can be observed and categorized
  - Meson mass spectrum
  - Differential widths (also sum rules)
- These modifications will complement direct searches
  - More precise computations of SM parameters needed
- Interplay between high-energy and flavour experiments

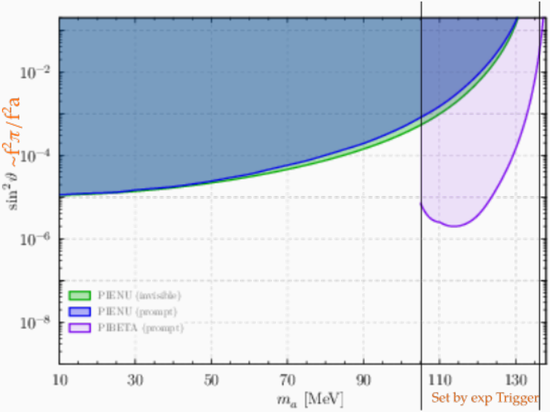
Thank You

Questions/Input/Critique?

2112.13147[hep-ph]
with S Ghosh and TS Roy

For Context

$\pi^+ \to ae^+\nu$



W. Altmannshofer, S. Gori, D. Robinson, 1909.00005

Form Factors etc

$$f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(t) = f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(0)\left[1+\lambda_{K^+\pi^0}^{+/0,\,(0)}\frac{t}{M_\pi^2}+\frac{1}{2}\lambda'^{+/0,\,(0)}_{K^+\pi^0}\frac{t^2}{M_\pi^4}\right]+\cdots\,,$$
$$f_{-,\,\mathrm{SM}}^{K^+\pi^0}(t) = \left[f_{0,\,\mathrm{SM}}^{K^+\pi^0}(t)-f_{+,\,\mathrm{SM}}^{K^+\pi^0}(t)\right]\frac{M_K^2-M_\pi^2}{t}\,. \tag{3.9}$$

We use the dispersive parametrization of the FFs, where the slope parameters are determined in terms of the slope of the vector, $f_+^{K^+\pi^0}(t)$, FF at $t=0$ and the slope of the scalar, $f_0^{K^+\pi^0}(t)$, FF at the Callan-Treiman point ($t_{CT}=M_K^2-M_\pi^2$), Λ_+ and C respectively [58]:

$$\lambda_{K^+\pi^0}^{+\,(0)} = \Lambda_+\,;$$
$$\lambda'^{+\,(0)}_{K^+\pi^0} = \left(\lambda_{K^+\pi^0}^{+\,(0)}\right)^2+5.79(97)\times10^{-4}\,;$$
$$\lambda_{K^+\pi^0}^{0\,(0)} = \frac{M_\pi^2}{t_{CT}}\left[\log(C)-0.0398(44)\right]\,;$$
$$\lambda'^{0\,(0)}_{K^+\pi^0} = \left(\lambda_{K^+\pi^0}^{0\,(0)}\right)^2+4.16(56)\times10^{-4}. \tag{3.10}$$

We tabulate the lattice determination [52] of Λ_+, C, and $f_{+,\,\mathrm{SM}}^{K^+\pi^0}(0)(=f_{0,\,\mathrm{SM}}^{K^+\pi^0}(0))$ in Table 3. We also note the correlations among these parameters, which we include in our computations.

Parameter	Correlation
$\Lambda_+ = 24.22(1.16)\times10^{-3}$	$\rho\,[\Lambda_+,\log(C)] = 0.376$
$\log(C) = 0.1998(138)$	$\rho\left[f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(0),\log(C)\right] = -0.719$
$f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(0) = 0.9709(46)$	$\rho\left[f_{+/0,\,\mathrm{SM}}^{K^+\pi^0}(0),\Lambda_+\right] = -0.228$

Table 3. Lattice determined values (left) and correlations (right) of the FF parameters in the dispersive formalism [52]. The quantities $\rho[i,j]$ are the correlation coefficients between the i-th and the j-th parameters.

$$\mathrm{Re}\left(\tilde f_+^{K^+\pi^0}(t)\right) = \left(\alpha_{K^+\pi^0}^{(0)}+\xi^2\alpha_{K^+\pi^0}^{(2)}+\delta\alpha_{K^+\pi^0}^{(0)}+\xi^2\delta\alpha_{K^+\pi^0}^{(2)}\right)$$
$$\times\left[1+\left(\lambda_{K^+\pi^0}^{+,\,(0)}+\xi^2\lambda_{K^+\pi^0}^{+,\,(2)}\right)\frac{t}{M_\pi^2}+\left(\lambda'^{+,\,(0)}_{K^+\pi^0}+\xi^2\lambda'^{+,\,(2)}_{K^+\pi^0}\right)\frac{t^2}{2M_\pi^4}\right]+\cdots$$
$$\simeq\left[1+\xi^2\frac{\alpha_{K^+\pi^0}^{(2)}}{\alpha_{K^+\pi^0}^{(0)}}\right]f_{+,\,\mathrm{SM}}^{K^+\pi^0}(t)\,,$$
$$\mathrm{Re}\left(\tilde f_-^{K^+\pi^0}(t)\right) = \left(\delta\beta_{K^+\pi^0}^{(0)}+\xi^2\beta_{K^+\pi^0}^{(2)}+\xi^2\delta\beta_{K^+\pi^0}^{(2)}\right)$$
$$\times\left[1+\left(\lambda_{K^+\pi^0}^{-,\,(0)}+\xi^2\lambda_{K^+\pi^0}^{-,\,(2)}\right)\frac{t}{M_\pi^2}+\left(\lambda'^{-,\,(0)}_{K^+\pi^0}+\xi^2\lambda'^{-,\,(2)}_{K^+\pi^0}\right)\frac{t^2}{2M_\pi^4}\right]+\cdots$$
$$\simeq\left[1+\xi^2\frac{\beta_{K^+\pi^0}^{(2)}}{\delta\beta_{K^+\pi^0}^{(0)}}\right]f_{-,\,\mathrm{SM}}^{K^+\pi^0}(t)\,,$$

The factor C_{cor} encapsulates effects that are not captured by the lattice computations of the FFs that we use in our numerical analyses. It is defined as:

$$C_{\mathrm{cor}} = S_{\mathrm{EW}}(1+\delta_{SU(2)}^K+\delta_{\mathrm{em}}^{k\ell})^2. \tag{3.7a}$$

$$\text{Here,}\quad S_{\mathrm{EW}} = 1+\frac{2\alpha_{\mathrm{EM}}}{\pi}\left(1-\frac{\alpha_s}{4\pi}\right)\log\frac{M_Z}{M_\rho}+\mathcal{O}\left(\frac{\alpha\alpha_s}{\pi^2}\right) = 0.0232\pm0.0003, \tag{3.7b}$$

encodes the short-distance contribution to the EM corrections [55, 56]. The other corrections, *viz.* $\delta_{SU(2)}^{K\ell}=(2.45\pm0.19)\%$ [57] and $\delta_{\mathrm{EM}}^{K\ell}=(0.016\pm0.25)\times10^{-2}$ [56] are the isospin breaking and long distance electromagnetic corrections respectively. In our calculations, We drop the $\delta_{\mathrm{EM}}^{K\ell}$ corrections as it is much smaller than the order we are working up to.

Lattice Form Factors: 1602.04113 (ETM collaboration)

Short Distance EW, EM, Isospin: 0801.1817 (FlaviaNet), NPB 196 (1982), 83 (Sirlin), 1704.04104 (Moulson)

$$\mathcal{L} \supset \frac{f_\pi^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle$$
$$= \frac{1}{2}(\partial_\mu\pi_a)^2+\frac{f_\pi^2}{8}(V_5^\mu)^2+\frac{f_\pi}{2}\partial_\mu\pi_a V_{5a}^\mu+\frac{1}{4}f_{abc}\pi_{[a}\partial_\mu\pi_{b]}V_c^\mu+\frac{1}{8f_\pi}f_{abc}\pi_a V^\mu_{5[b}V^\mu_{c]}$$

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