Extraction of $|V_{ub}|$ from $B \to \pi l \nu$.

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The CKM matrix

- Precise determinations of the CKM elements necessary to probe the quark mixing mechanism of the Standard Model.
- Important ingredients in the theoretical predictions of several observables in the flavor sector.
- $V_{ub} \rightarrow$ Source of CP violation within the SM
  $\rightarrow$ Less precisely known.

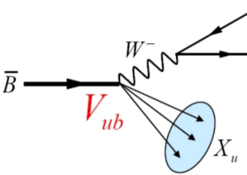
$$V_{\text{CKM}} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}$$

[PDG]

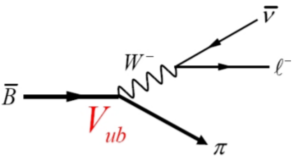
Measurements of $|V_{ub}|$

- The transition $b \to u l \bar{\nu}$ provides two avenues for determining $|V_{ub}|$ -

Inclusive



Exclusive



- Experimental and theoretical techniques for these two approaches different and largely independent → Important cross checks of our understanding.
- Mutual disagreement between exclusive and inclusive measurements.

$$|V_{ub}|^{exc} = (3.70 \pm 0.16) \times 10^{-3}, \quad |V_{ub}|^{inc} = (4.25 \pm 0.12^{+0.15}_{-0.14}) \times 10^{-3}, \quad (1)$$

differ by $\geq 2.2\ \sigma$ [PDG, 2020].

$|V_{ub}|$ from inclusive decays

- The theoretical description of inclusive $\bar{B} \to X_u l\bar{\nu}$ decays based on the Heavy Quark Expansion (an expansion in Λ_{QCD}/m_b).
- Total decay rate hard to measure due to the large background from $\bar{B} \to X_c l\bar{\nu}$ transitions → experimental cuts are necessary.
- In regions of phase space where $\bar{B} \to X_c l\bar{\nu}$ decays are suppressed, can't use HQE → introduce non-perturbative distribution functions(SF).
- Different approaches to model the shape function → extracted values of $|V_{ub}|$ model dependent.
- Recent analysis of the inclusive spectra with hadronic-tagging by Belle [arXiv:2102.00020] -

$$|V_{ub}|^{inc} = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}\,. \tag{2}$$

$|V_{ub}|$ from exclusive decays

- Exclusive determinations require knowledge of the form factors.

$$\langle \pi(p_\pi)|V_\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu \tag{3}$$

- $f_+(q^2 = 0) = f_0(q^2 = 0) \rightarrow$ cancel the divergence at $q^2 = 0$.

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$$\begin{aligned}\frac{d\Gamma}{dq^2}\left(\bar{B^0} \rightarrow \pi^+ l^- \bar{\nu}_l\right) =& \frac{G_F^2 |V_{ub}|^2}{24\pi^3 m_{B^0}^2 q^4}\left(q^2 - m_l^2\right)^2 \left|p_\pi(m_{B^0}, m_{\pi^+}, q^2)\right| \times \\ & \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B^0}^2 \left|p_\pi(m_{B^0}, m_{\pi^+}, q^2)\right|^2 \left|f_+\left(q^2\right)\right|^2\right. \\ & \left. + \frac{3m_l^2}{8q^2}\left(m_{B^0}^2 - m_{\pi^+}^2\right)^2 \left|f_0\left(q^2\right)\right|^2\right.\end{aligned} \tag{4}$$

- Model-independent parametrization based on general properties of analyticity, unitarity, and crossing symmetry.

Form factor parametrization

- For $\bar{B} \to \pi l \bar{\nu}_l$ decays, $m_l^2 \leq q^2 \leq (m_B - m_\pi)^2$.
- The z expansion → maps the kinematically allowed region within a disc of radius $|z| < 1$.

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_B + m_\pi)^2 \tag{5}$$

- Choosing $t_0 = (M_B + M_\pi)\,(\sqrt{M_B} - \sqrt{M_\pi})^2$ restricts z to $|z| < 0.28$ $\longrightarrow$ rapid convergence of the expansion.

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- BSZ parametrization -

$$f_i(q^2) = \frac{1}{1 - q^2/m_{R,i}^2} \sum_{k=0}^{N} a_k^i \left[z(q^2) - z(0)\right]^k \tag{6}$$

- Kinematic constraint → $a_0^0 = a_0^+$

Form factor parametrization

- BCL parametrization -

$$f_+(z) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{n=0}^{N_z-1} b_n^+ \left[z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z}\right], \tag{7}$$

$$f_0(z) = \sum_{n=0}^{N_z-1} b_n^0 z^n \,. \tag{8}$$

$$\Sigma(b^{0/+}, N_z) \equiv \sum_{m,n=0}^{N_z} B_{mn} \, b_m^{0/+} b_n^{0/+} \leq 1 \,, \tag{9}$$

where the element B_{mn} satisfies $B_{mn} = B_{nm} = B_{0|m-n|}$

- Kinematic constraint → replace one FF parameter in terms of the others.

$$b_3^0 = 45.70(b_0^+ - b_0^0) - 12.78 b_1^0 - 3.58 b_2^0 + 12.85 b_1^+ + 3.44 b_2^+ + 1.21 b_3^+ \tag{10}$$

Inputs for the extraction of $|V_{ub}|$

- $\bar{B} \to \pi l \bar{\nu}_l$ → the most promising decay mode for both experiment and theory.

  Four most precise measurements by BABAR and Belle -
- BABAR untagged $B^0 + B^+$ (6 q^2 bins) [arXiv:1005.3288v2] → BaBar(11)
- BABAR untagged $B^0 + B^+$ (12 q^2 bins) [arXiv:1201.1253] → BaBar(12)
- Belle untagged B^0 [arXiv:1012.0090] → Belle(11)
- Belle hadronic tagged B^0 and B^+ [arXiv:1306.2781] → Belle(13)

- Non-perturbative methods for the calculation of form factors:
  - Lattice QCD (LQCD)
    High q^2.
    (RBC-UKQCD/Fermilab-MILC)
  - Light-cone sum rules (LCSR)
    Low q^2.
    (arXiv: 1811.00983)

Methodology by HFLAV

HFLAV

Determine the average partial branching fraction in each q^2 bin, p-value $\sim 6\%$

Extract V_{ub} using above+ Lattice+LCSR(q^2=0), p-value $\sim 47\%$

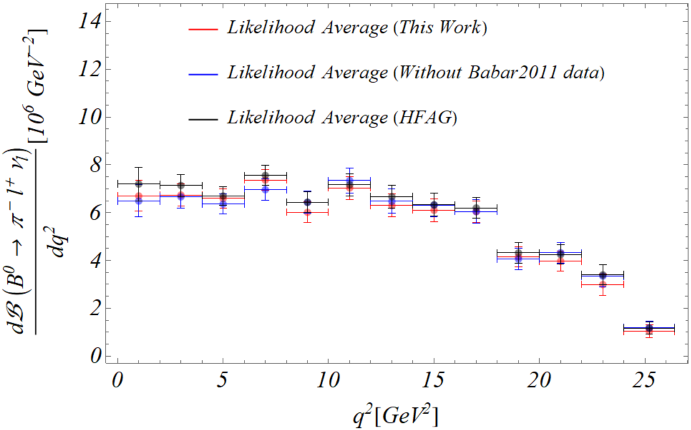
Observation : The analysis-method in BaBar(11) considerably different from that of BaBar(12) and Belle. BaBar 2012 significantly better than BaBar 2011.

Observation

- A closer look at the data shows that BaBar(11) untagged analysis of the $B^{0,+}$ modes have much lower statistics/yield (almost half) than the one published in the next year: BaBar(12).
- In 2011, the event selection has been optimized over the signal-enhanced region instead of the entire fit region, as was done in 2012.
- The analysis in 2011 uses only a subset of the full BaBar data-set.
- MILC also pointed out that BaBar (2011) is at odds with the rest.

Comparison of the average q^2 spectrum.

Likelihood Average (HFLAV) $\sim$ 6%, Average (This work) $\sim$ 1%, Average (Dropping BaBar(11) $\sim$ 24.8%.



Results with the new Lattice + LCSR inputs

22 data points (9 from LCSR, 13 from Lattice (3 for each of $f_{+,0}$ from UKQCD, 4 for f_+ and 3 for f_0 from MILC)

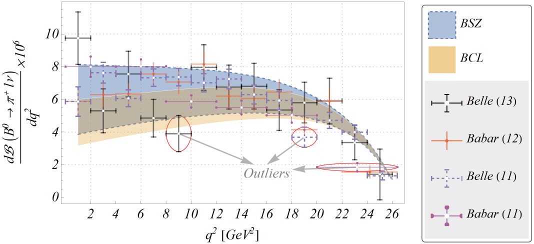
Parameters	Our Avg. q^2 spec. w/o BaBar(11) + New Lattice & LCSR + BaBar(11) re-introduced (p value = 0.75%)	Our Avg. q^2 spec. + New Lattice & LCSR (p value = 20.9%)	Our Avg. q^2 spec. w/o BaBar(11) + New Lattice & LCSR (p value = 31%)
$V_{ub} \times 10^3$	3.78(13)	3.78(13)	3.89(14)
b_0^+	0.410(12)	0.410(12)	0.408(12)
b_1^+	−0.526(44)	−0.526(44)	−0.561(46)
b_2^+	−0.39(13)	−0.39(13)	−0.40(13)
b_3^+	0.59(24)	0.59(24)	0.59(25)
b_0^0	0.540(16)	0.540(16)	0.536(16)
b_1^0	−1.617(66)	−1.617(66)	−1.647(66)
b_2^0	1.294(146)	1.294(146)	1.257(146)

Form-factors extracted only from the LCSR and lattice inputs

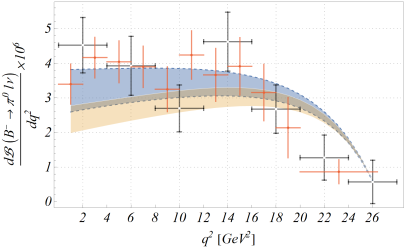
BSZ			
$\chi^2_{\min}/\mathrm{DOF}$	p-value(%)	Parameters	Values
4.48/15	99.6	a_0^+	0.213(22)
		a_1^+	-0.65(14)
		a_2^+	0.263(425)
		a_3^+	0.67(31)
		a_1^0	0.41(17)
		a_2^0	1.46(51)
		a_3^0	1.78(49)

BCL			
$\chi^2_{\min}/\mathrm{DOF}$	p-value(%)	Parameters	Values
12.88/15	61	b_0^+	0.396(13)
		b_1^+	-0.707(70)
		b_2^+	-0.36(18)
		b_3^+	0.77(32)
		b_0^0	0.521(17)
		b_1^0	-1.756(78)
		b_2^0	1.15(16)

Binned differential branching fraction plots



(a) (b)



(c)

Different scenarios

- ***Fit 1***: B^0 decays from Belle (2011) and Belle (2013); B^- decays from Belle (2013); the combined modes from BaBar (2011) and BaBar (2012).
    - *Fit 1A*: Experimental data (Fit 1) + synthetic Lattice data points,
    - *Fit 1B*: Experimental data (Fit 1) + synthetic Lattice data points + LCSR.

- ***Fit 2***: B^0 decays from Belle (2011), BaBar (2012), and Belle (2013); B^- decays from BaBar (2012) and Belle (2013).
    - *Fit 2A*: Experimental data (Fit 2) + synthetic Lattice data points,
    - *Fit 2B*: Experimental data (Fit 2) + synthetic Lattice data points + LCSR.

- ***Fit 3***: The combined modes from BaBar (2011) along with the *Fit 2* dataset.
    - *Fit 3A*: Experimental data (Fit 3) + synthetic Lattice data points,
    - *Fit 3B*: Experimental data (Fit 3) + synthetic Lattice data points + LCSR.

Different scenarios

BSZ Parametrization

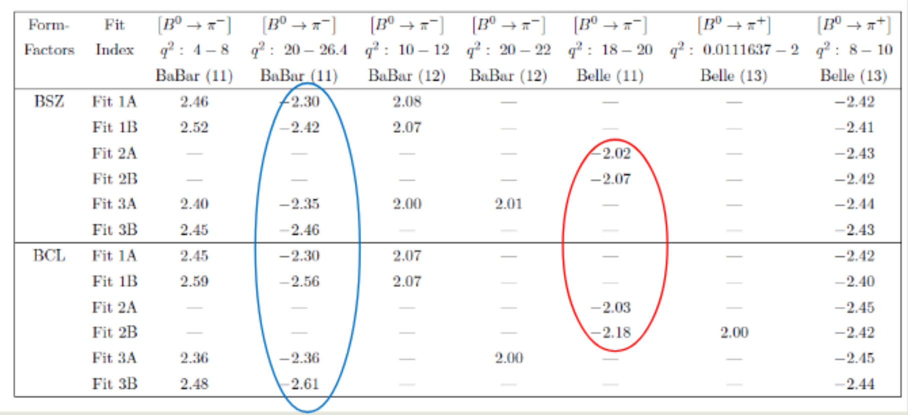
Run Name	Full				Dropped Pull > 2			
	χ^2_{min}/DOF	p-value(%)	$V_{ub} \times 10^3$		χ^2_{min}/DOF	p-value(%)	$V_{ub} \times 10^3$	
			Frequentist	Bayesian			Freq.	Bayes
Fit 1A	73.4/56	5.92	3.69(14)	3.67(14)	46.6/52	68.68	3.79(15)	$3.77\,\binom{15}{16}$
Fit 1B	77./65	14.57	3.74(13)	$3.73\,\binom{13}{14}$	49.3/61	85.77	3.83(14)	$3.82\,\binom{14}{16}$
Fit 2A	59.5/61	53.17	3.81(14)	3.79(15)	46./59	89.26	3.86(15)	$3.85\,\binom{15}{16}$
Fit 2B	62./70	74.23	3.85(14)	$3.83\,\binom{13}{15}$	48.3/68	96.63	3.91(14)	$3.89\,\binom{14}{15}$
Fit 3A	82.2/67	9.98	3.70(14)	3.69(14)	53.3/62	77.56	3.76(14)	$3.76\,\binom{15}{14}$
Fit 3B	85.9/76	20.54	3.75(13)	$3.74\,\binom{13}{14}$	62./73	81.79	3.84(14)	3.83(14)

BCL Parametrization

Run Name	Full				Dropped Pull > 2			
	χ^2_{min}/DOF	p-value(%)	$V_{ub} \times 10^3$		χ^2_{min}/DOF	p-value(%)	$V_{ub} \times 10^3$	
			Freq.	Bayes			Freq.	Bayes
Fit 1A	73.5/56	5.84	3.69(14)	$3.67\,\binom{13}{15}$	46.7/52	68.34	3.79(15)	3.78(15)
Fit 1B	92.1/65	1.51	3.79(13)	$3.78\,\binom{14}{13}$	63.2/61	39.84	3.89(14)	$3.87\,\binom{14}{15}$
Fit 2A	60.1/61	50.8	3.81(14)	3.81(15)	46.5/59	88.19	3.87(15)	$3.85\,\binom{14}{15}$
Fit 2B	75.9/70	29.42	3.91(14)	3.90(15)	58.3/67	76.64	3.96(14)	$3.96\,\binom{16}{14}$
Fit 3A	82.7/67	9.35	3.70(14)	$3.69\,\binom{13}{14}$	57.8./63	66.09	3.77(14)	3.76(15)
Fit 3B	101.4/76	2.73	3.80(13)	$3.79\,\binom{13}{15}$	76.3/73	37.27	3.90(14)	$3.89\,\binom{14}{15}$

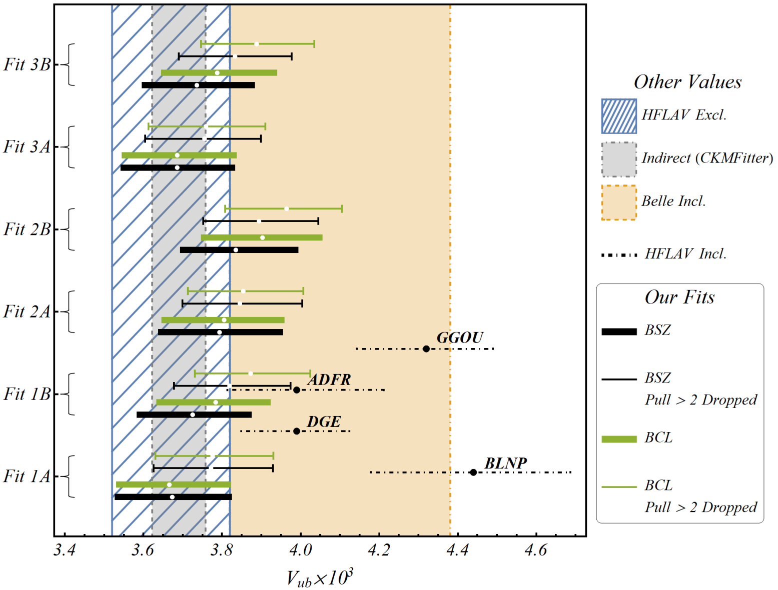
Observables with pull > 2 σ

$$pull_i = \frac{\mathcal{O}_i^{exp} - \mathcal{O}_i^{fit}}{\sigma_i^{exp}} \,. \tag{11}$$



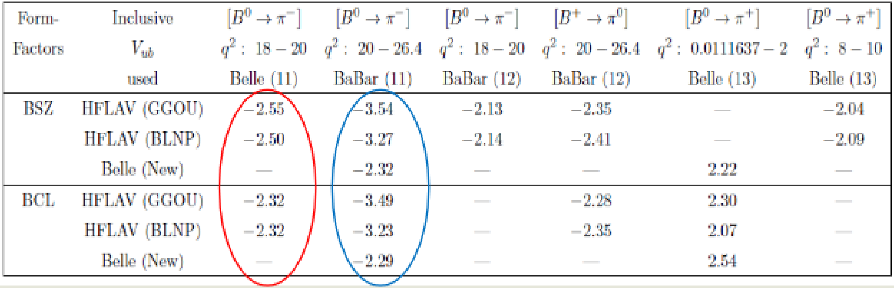
Form-Factors	Fit Index	$[B^0 \to \pi^-]$ q^2 : $4-8$ BaBar (11)	$[B^0 \to \pi^-]$ q^2 : $20-26.4$ BaBar (11)	$[B^0 \to \pi^-]$ q^2 : $10-12$ BaBar (12)	$[B^0 \to \pi^-]$ q^2 : $20-22$ BaBar (12)	$[B^0 \to \pi^-]$ q^2 : $18-20$ Belle (11)	$[B^0 \to \pi^+]$ q^2 : $0.0111637-2$ Belle (13)	$[B^0 \to \pi^+]$ q^2 : $8-10$ Belle (13)
BSZ	Fit 1A	2.46	−2.30	2.08	—	—	—	−2.42
	Fit 1B	2.52	−2.42	2.07	—	—	—	−2.41
	Fit 2A	—	—	—	—	−2.02	—	−2.43
	Fit 2B	—	—	—	—	−2.07	—	−2.42
	Fit 3A	2.40	−2.35	2.00	2.01	—	—	−2.44
	Fit 3B	2.45	−2.46	—	—	—	—	−2.43
BCL	Fit 1A	2.45	−2.30	2.07	—	—	—	−2.42
	Fit 1B	2.59	−2.56	2.07	—	—	—	−2.40
	Fit 2A	—	—	—	—	−2.03	—	−2.45
	Fit 2B	—	—	—	—	−2.18	2.00	−2.42
	Fit 3A	2.36	−2.36	—	2.00	—	—	−2.45
	Fit 3B	2.48	−2.61	—	—	—	—	−2.44

Comparison of $|V_{ub}|$ results



Deviations of theoretical predictions from data

$$dev_i = \frac{\mathcal{O}_i^{exp} - \mathcal{O}_i^{SM}}{\sqrt{(\sigma_i^{exp})^2 + (\sigma_i^{SM})^2}}, \quad (12)$$



Form-Factors	Inclusive V_{ub} used	$[B^0 \to \pi^-]$ q^2 : 18 – 20 Belle (11)	$[B^0 \to \pi^-]$ q^2 : 20 – 26.4 BaBar (11)	$[B^0 \to \pi^-]$ q^2 : 18 – 20 BaBar (12)	$[B^+ \to \pi^0]$ q^2 : 20 – 26.4 BaBar (12)	$[B^0 \to \pi^+]$ q^2 : 0.0111637 – 2 Belle (13)	$[B^0 \to \pi^+]$ q^2 : 8 – 10 Belle (13)
BSZ	HFLAV (GGOU)	−2.55	−3.54	−2.13	−2.35	—	−2.04
	HFLAV (BLNP)	−2.50	−3.27	−2.14	−2.41	—	−2.09
	Belle (New)	—	−2.32	—	—	2.22	—
BCL	HFLAV (GGOU)	−2.32	−3.49	—	−2.28	2.30	—
	HFLAV (BLNP)	−2.32	−3.23	—	−2.35	2.07	—
	Belle (New)	—	−2.29	—	—	2.54	—

Comparison of $|V_{ub}|^{exc.}$ obtained in this work

- ***Fit 2B-I***: Input used in *Fit 2B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}(Belle2011)$.
- ***Fit 3B-I***: Input used in *Fit 3B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}(BaBar2011)$.
- ***Fit 3B-II***: Input used in *Fit 3B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}(Belle2011)$ and $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}(BaBar2011)$.

Fit	BSZ				BCL			
Scenario	χ^2/DOF	p-value(%)	$V_{ub} \times 10^3$		χ^2/DOF	p-value(%)	$V_{ub} \times 10^3$	
			Frequentist	Bayesian			Frequentist	Bayesian
F2B-I	55.4/69	88.14	3.90(14)	$3.89^{+0.14}_{-0.15}$	68.85/69	48.25	3.96(14)	$3.95^{+0.14}_{-0.15}$
F3B-I	78.86/75	35.8	3.83(14)	3.83(13)	93.6/75	7.19	3.89(14)	3.89(14)
F3B-II	72.96/74	51.25	3.88(14)	$3.87^{+0.14}_{-0.15}$	87.2/74	13.99	3.94(14)	$3.93^{+0.14}_{-0.15}$

Conclusions

- We have extracted $|V_{ub}|$ analyzing all the available inputs on the exclusive $B \to \pi l \nu$ decays. This includes the data on the partial decay rates, inputs from lattice, and those from LCSR.
- We have identified BaBar(11) data (at least a part of it) as a probable source of bad quality fit. The fit scenarios (Fit 2A and 2B) without that data-set has an appreciable fit-probability.
- We found a very small number of data-points that compromise the fit-quality, and at the same time, influence the extraction of $|V_{ub}|$.
- From the full dataset after dropping $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}$ (Belle(11)) and $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}$ (BaBar(11)), the extracted $|V_{ub}| = (3.94(14)) \times 10^{-3}$.
  $\to$ Consistent with the recent one extracted from inclusive $B \to X_u \ell \nu_\ell$ decay by Belle within 1 σ.

Thank you!