Slow-roll inflation in Palatini F(R) gravity

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Metric vs Palatini [2]

Metric formulation

Only DOF is the metric $g_{\mu\nu}$, connection Γ is algebrically related to the metric and is assumed to be the Levi-Civita one

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \mathcal{L}_m(g_{\mu\nu}, \phi, \partial\phi) \right]$$

Variation with respect to $g_{\mu\nu}$ yields EoM i.e. Einstein equations

Ricci scalar $R = g^{\alpha\beta} R_{\alpha\beta}$ depends on the metric through the

Levi-Civita connection $\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\lambda} (g_{\beta\lambda,\gamma} + g_{\lambda\gamma,\beta} - g_{\beta\gamma,\lambda})$

[2] T. Koivisto and H. Kurki-Suonio, Cosmological perturbations in the Palatini formulation of modified gravity, Class. Quant. Grav. 23 (2006)

Metric vs Palatini

Palatini formulation

In Palatini we have both metric $g_{\mu\nu}$ and the connection Γ as DOF's

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} f(\phi) R(\Gamma) + \mathcal{L}_m(g_{\mu\nu}, \Gamma, \phi, \partial\phi) \right]$$

Variation with respect to Γ yields EoM $\Gamma = \Gamma(g_{\mu\nu}, \phi)$

Variation with respect to $g_{\mu\nu}$ yields analogue of Einstein equations

Ricci tensor $R_{\alpha\beta}(\Gamma)$ is built from connection only

What if we have F(R) in the action?

Metric vs Palatini

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(R) + \frac{1}{2}f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

F(R) term produces drastically different theories

In **metric formulation** there is one more scalar degree of freedom

Bi-field inflationary configuration

In **Palatini formulation** this scalar DOF is not present

Single field inflationary configuration

Different inflationary predictions

Inflation basics

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3P)$$

Problem of initial conditions: current universe is homogeneous and flat, but time evolution drives it away from homogeneity flatness

Inflation is currently the most accepted and elegant solution for solving the horizon and flatness problem

Period of accelerated expansion for which the scale factor $a(t)$ grows exponentially

Inflation basics

Inflation also allows to predict the spectrum of primordial fluctuations in CMB

Information about inflation is encoded in the power spectrum of scalar and tensor fluctuations

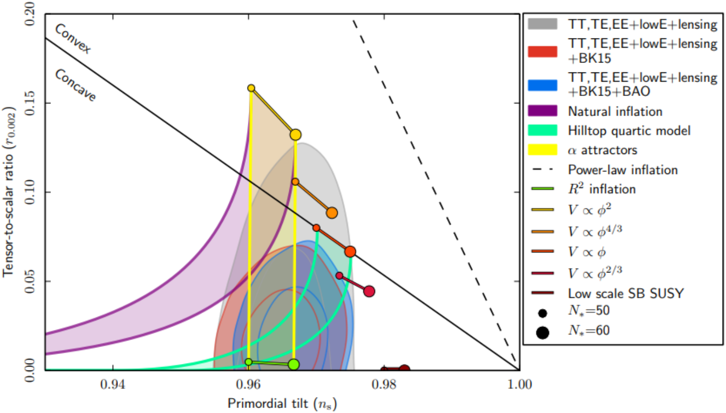
CMB observables are easily related to so called slow-roll parameters $\epsilon = \frac{1}{2}(\frac{V'}{V})^2$, $\eta = \frac{V''}{V}$

When $\epsilon \ll 1$ inflation takes place, while when $\eta \ll 1$ we can safely use slow-roll approximation

Slow-roll and CMB observables

We can relate ϵ and η to the CMB observables

Scalar to tensor ratio $r = 16\epsilon$, spectral index $n_s = 1 + 2\eta - 6\epsilon$, amplitude of the power spectrum $A_s = \frac{V}{24\pi^2\epsilon}$



Palatini inflation in presence of an R^2 term

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(R) + \frac{1}{2}f(\phi)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

We introduce auxiliary field ζ and transform to Einstein frame

If $F(R) = R + \alpha R^2$ we can explicitly solve the EoM for the auxilary field ζ and put the solution back in the action to obtain

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \textit{kinetic terms} - \frac{U_0}{1 + 8\alpha U_0}\right]$$

with $U_0 = \frac{V(\phi)}{[1+f(\phi)]^2}$ conformally transformed potential for $\alpha = 0$

Palatini inflation in presence of an R^2 term [3]

For large α we obtain a flat potential independently of $V(\phi)$ choice

Slow-roll $\epsilon = \frac{1}{2}(\frac{U'}{U})^2 \ll 1, \eta = \frac{U''}{U} \ll 1$

$r = 16\epsilon = \frac{r_0}{1+8\alpha U_0}$
$n_s = n_s^0$
$A_s = A_s^0$
$N_e = N_e^0$

Suppression of scalar to tensor ratio, other parameters unchanged

[3] Enckell V.M., Enqvist K., Räsänen S. Wahlman L.P., Inflation with R^2 term in the Palatini formulation, Journal of Cosmology and Astroparticle Physics, 2019

Slow-roll inflation in F(R) Palatini gravity

New method which allows to derive inflationary predictions in presence of higher order terms in the action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} F(R) - \frac{1}{2} k(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

By means of a conformal transformation we obtain the Einstein frame action

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} - \frac{1}{2} g_E^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi, \zeta) \right]$$

with $\frac{\partial \chi}{\partial \phi} = \sqrt{\frac{k(\phi)}{F'(\zeta)}}$; $U(\chi, \zeta) = \frac{V(\phi(\chi))}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)}$

Slow-roll inflation in F(R) Palatini gravity

EoM obtained by varying ζ gives

$$2F(\zeta) - \zeta F'(\zeta) - k(\phi)\partial^\mu\phi\partial_\mu\phi F'(\zeta) - 4V(\phi) = 0$$

In **slow-roll** limit the equation reduces to

$$G(\zeta) \equiv \frac{1}{4}[2F(\zeta) - \zeta F'(\zeta)] = V(\phi)$$

Cannot be solved for general $F(R)$

Trick: use ζ as computational variable using above equation as a constraint

Slow-roll inflation in F(R) Palatini gravity

Then $U(\chi,\zeta) = \frac{G(\zeta)}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} = \frac{1}{4}\frac{\zeta}{F'(\zeta)} = U(\zeta)$

Notice that:

1) $U(\zeta) = \frac{\zeta}{4F'(\zeta)}$ implies $\zeta > 0$ since $F'(\zeta)$ must be positive to allow for correct sign of kinetic term (and Weyl transformation)

2) Only the asymptotic form $F(R) \sim R^2$ can give asymptotically flat potential regardless of Jordan frame $V(\phi)$

In order to compute the slow roll parameters we need the derivatives of $U(\zeta)$ with respect to χ

Slow-roll inflation in F(R) Palatini gravity

Slow-roll parameters:

$$\epsilon(\zeta) = \frac{1}{2} g^2 (\frac{U'}{U})^2$$

$$\eta(\zeta) = \frac{g g' U' + g^2 U''}{U}$$

$$r(\zeta) = 8 g^2 \left(\frac{U'}{U}\right)^2$$

$$n_s(\zeta) = 1 + \frac{2g}{U^2}\left(g' U' U + g U'' U - 24 g U'^2\right)$$

$$A_s(\zeta) = \frac{U^3}{12\pi^2 g^2 U'^2}$$

$$N_e = \int_{\zeta_f}^{\zeta_N} \frac{U}{g^2 \partial U / \partial \zeta} d\zeta$$

Chain rule:

$$\frac{\partial}{\partial \chi} f(\zeta) = \frac{\partial \phi}{\partial \chi} \frac{\partial \zeta}{\partial \phi} \frac{\partial}{\partial \zeta} f(\zeta) \equiv g(\zeta) f'(\zeta)$$

with $g(\zeta) = \sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}} (\frac{\partial G}{\partial \zeta} \frac{\partial V^{-1}}{\partial G})^{-1}$

Slow-roll inflation in F(R) Palatini gravity

Necessary conditions for inflation scenario:

1) $F(R) \sim R$ for $R \sim 0$ to recover GR in the low-energy limit

2) $F'(R) > 0$ to have correct sign of kinetic term

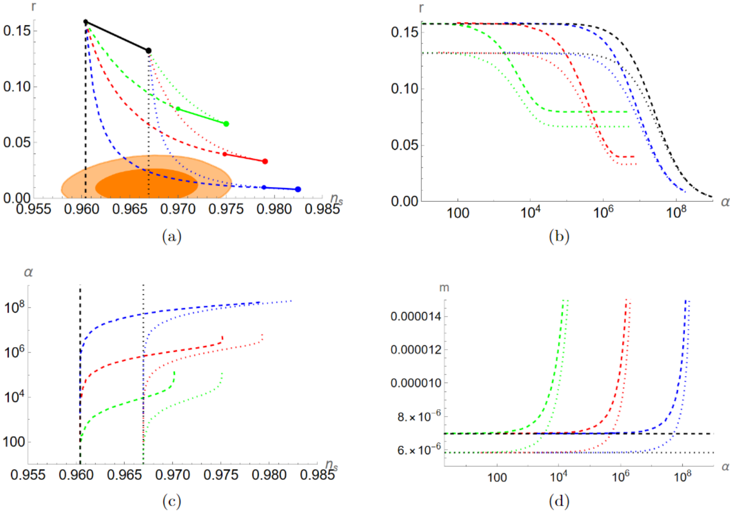
3) $V(\phi), G(\zeta)$ bijective at least in the subdomain where inflation happens

Test scenarios:

$$F(R) = R + \alpha R^n,\ k(\phi) = 1,\ V(\phi) = \frac{m^2}{2}\phi^2$$

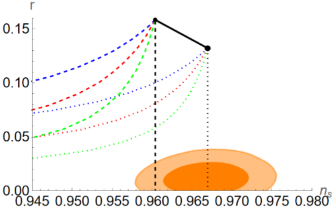
Test scenarios

Green n=3/2, red n=7/4, blue n=31/16

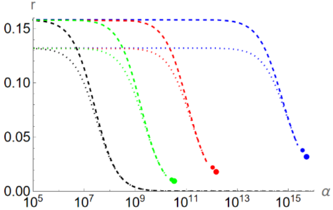


Test scenarios

Green n=5/2, red n=9/4, blue n=3



(a)



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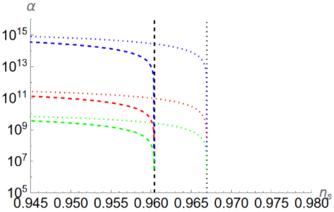

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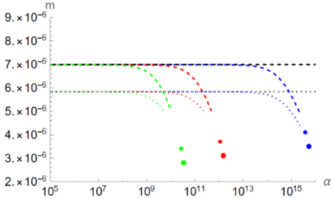

(d)

Conclusions

1) We developed a method that allows computation of inflationary parameters for $F(R)$ Palatini gravity

2) We tested our method for models $F(R) = R + \alpha R^n$ with a canonical kinetic term and a quadratic potential and analyzed their behavior in the slow-roll regime

3) We found out that $n = 2$ is an unstable solution: if the effective n slightly deviates from $n = 2$ the asymptotic configurations drastically differ

4) We went beyond slow-roll and found that for $n < 2$ all trajectories with large enough field value enter inflation and slow roll, while for $n > 2$ slow-roll regime is not on attractor on global scale

Thank you for the attention