

Big Bang Nucleosynthesis as a cosmological probe

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SUMMARY

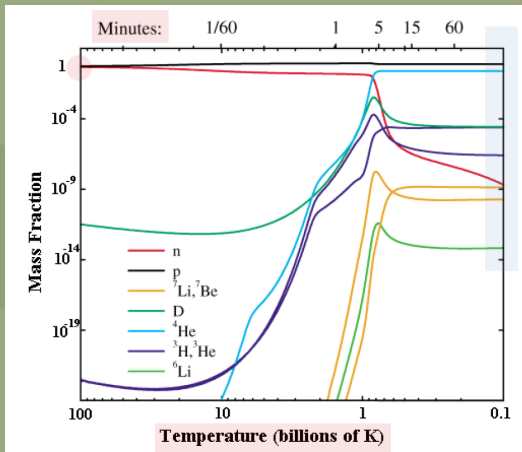
- Preliminaries
- BBN: overview and simple results
- Improving precision: weak rates, neutrino decoupling and nuclear chain rates
- Observations
- Standard BBN
- Non standard scenarios

Preliminaries: BBN in few words

1. Less than 1 second after the bang, the plasma of γ , e^- , ν , n , p (and their antiparticles) is in equilibrium.
2. At $T \sim 1$ MeV (1 second) neutrinos decouple because their weak interactions go out of equilibrium with respect to expansion.
3. n/p ratio (fortunately) freezes out just soon after neutrinos, at $T_D \sim 800$ keV; then, when a sufficient abundance of deuterium forms at $T_{\text{BBN}} \sim 100$ keV, the nuclear chain starts: (almost) all neutrons present at this moment go into ${}^4\text{He}$.

The final result is a universe made by 75% of hydrogen, 25% of ${}^4\text{He}$ (and negligible yields of the other elements up to ${}^7\text{Li}$).

mass fraction of ${}^4\text{He}$ in a simple equation



$$Y_p = \frac{4 n_{{}^4\text{He}}}{n_n + n_p} = \frac{4 n_n / 2}{n_n + n_p} = \frac{2}{1 + n_p / n_n} = 0.25$$

Key pillar of the Hot Big Bang Model
BBN is an overconstrained scenario

Theoretical predictions depends on two parameters:

$$\Omega_b$$

$$N_\nu$$

Preliminaries

History

- 1946 Gamow: nuclear reactions in the early universe might explain the abundances of elements.
- Fermi and Turkevich: lack of stable nuclei with mass 5 and 8 prevents significant production of nuclei more massive than ${}^7\text{Li}$.
- 1964 Peebles, Hoyle and T aylor: $Y_p \approx 0.25$.
- 1967 Wagoner, Fowler and Hoyle: first detailed calculation of light nuclei abundances.
-Schramm, Turner, Steigman,....and many others

Preliminaries

- FRW universe: spatial homogeneity and isotropy

Einstein equations:
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

Perfect fluid:
$$T_{\mu\nu} = \text{diag}(\rho, -p, -p, -p)$$

Equation of state:
$$p = p(\rho)$$

Friedmann equation:
$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

Equilibrium versus non equilibrium: may we describe particle distribution in phase space via equilibrium Fermi Dirac or Bose Einstein distribution functions?

Preliminaries

Formally not (but for massless particles)
Practically yes, to a very good precision.

- As long as particle interactions are fast with respect to the expansion rate:

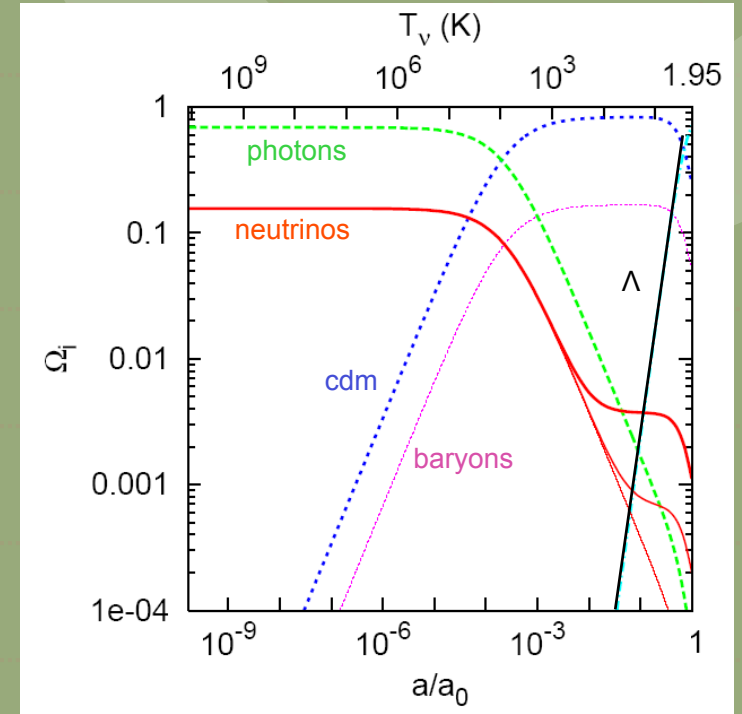
$$f_i = \left(\exp\left(\frac{E - \mu_i}{T(t)}\right) \pm 1 \right)^{-1}$$

When radiation ($p=\rho/3$) dominates the energy density (g_i are the number of relativistic d.o.f.)

$$T(t) \approx \left(\frac{1}{\sqrt{\sum g_i}} \frac{1}{t(\text{sec})} \right)^{1/2} \text{ MeV}$$

!! Out of equilibrium phases are crucial in the history of the universe

Tools: a simple criterium and the Boltzmann transport equations



Preliminaries

Fiducial criterium: interaction timescale versus expansion rate

$$\Gamma \equiv \langle \sigma v \rangle n > H(T) \quad \text{Equilibrium}$$

$$\Gamma \equiv \langle \sigma v \rangle n \leq H(T) \quad \text{Freeze out}$$

Kinetic equation for the phase space distribution of a specie i (in FRW $f(p)$):

$$\frac{1}{E} \mathcal{L}(f) = \frac{\partial f(t, p)}{\partial t} - H p \frac{\partial f(t, p)}{\partial p} = \mathbf{C}(f(p, t); f_i)$$

Liouville operator

Collisional integral

$$\begin{aligned} \mathbf{C}(f_a; f_b, f_c, f_d) &= \frac{1}{E_a} \int d\pi(p_b) d\pi(p_c) d\pi(p_d) (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d) \\ &\times \left[|\mathcal{M}_{cd,ab}|^2 f_c(p_c, t) f_d(p_d, t) (1 \pm f_a(p_a, t)) (1 \pm f_b(p_b, t)) \right. \\ &\left. - |\mathcal{M}_{ab,cd}|^2 f_a(p_a, t) f_b(p_b, t) (1 \pm f_c(p_c, t)) (1 \pm f_d(p_d, t)) \right] \end{aligned} \quad (2.94)$$

Preliminaries

Evolution of number density

$$n_\mu(t) = g \int \frac{d^3p}{(2\pi)^3} \frac{P^\mu}{E} f(p, t)$$



assuming isotropy only $n=n_0$ is non zero

$$n(t) = g \int \frac{d^3p}{(2\pi)^3} f(p, t)$$

$$\int \frac{d^3p_a}{(2\pi)^3} \left(\frac{\partial f(t, p_a)}{\partial t} - H p_a \frac{\partial f(t, p_a)}{\partial p_a} \right) = \dot{n}_a + 3H n_a = \int \frac{d^3p_a}{(2\pi)^3} \mathbf{C}$$

Exercise: show that for binary collisions $a + b \leftrightarrow c + d$ and equilibrium distributions the collisional integral vanishes! (indeed it vanishes for arbitrary interactions!!)

1. Species share the same temperature T (kinetic equilibrium)
2. $\mu_a + \mu_b = \mu_c + \mu_d$ (chemical equilibrium)

Preliminaries

Saha equation:

Assuming kinetic equilibrium (for simplicity I assume T reversal invariance):

$$Ha \frac{dn_a}{da} + 3Hn_a = \int d\pi(p_a) d\pi(p_b) d\pi(p_c) d\pi(p_d) (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d) \times |\mathcal{M}_{ab,cd}|^2 \left[e^{-(E_c+E_d)/T} e^{\mu_c/T} e^{\mu_d/T} - e^{-(E_a+E_b)/T} e^{\mu_a/T} e^{\mu_b/T} \right] \quad (2.119)$$

and using energy conservation

$$a^{-2} \frac{d}{da} (n_a a^3) = \frac{\langle \sigma |v| \rangle n_b}{H} n_a \left(\exp \left(\frac{\mu_c + \mu_d - \mu_a - \mu_b}{T} \right) - 1 \right)$$

If the scattering rate is large compared to H chemical equilibrium holds, which can be recast as

$$n_i \sim e^{\mu_i/T} \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T}$$

$$\frac{n_c n_d}{n_a n_b} = \frac{\int d^3p e^{-E_c/T} \int d^3p e^{-E_d/T}}{\int d^3p e^{-E_a/T} \int d^3p e^{-E_b/T}}$$

Saha equation

Preliminaries

When equilibrium holds

$$\rho_i = 3P_i = \begin{cases} \frac{\pi^2}{30} g T_i^4, & \text{boson} \\ \frac{7}{8} \frac{\pi^2}{30} g T_i^4, & \text{fermion} \end{cases}$$

For fermions (protons, neutrons, electrons, positrons, neutrinos) in presence of a chemical potential

$$\rho_i = 3P_i = \frac{7}{8} \frac{\pi^2}{30} g T_i^4 \left(1 + \frac{30\xi_i^2}{7\pi^2} + \frac{15\xi_i^4}{7\pi^4} \right) \quad \xi = \mu/T$$

BBN: overview and simple results

Free parameters, nuclear rates, weak rates
cosmological model



Nuclide
abundances

In the standard, minimal model the only free
parameter is the baryon to photon number
density

$$\eta = n_b/n_\gamma = 274 \cdot 10^{-10} \Omega_b h^2 \approx 10^{-9}$$

Non standard models: extra species, chemical
potentials, low energy inflation models, extra
dimensions...

BBN in four steps

- | | | |
|------|----------------------|--|
| i) | initial conditions | $T > 1 \text{ MeV}$ |
| ii) | n/p ratio freeze out | $T \approx 1 \text{ MeV}$ |
| iii) | D bottleneck | $T \approx 0.1 \text{ MeV}$ |
| iv) | nuclear chain | $0.1 \text{ MeV} > T > 0.01 \text{ MeV}$ |

BBN overview and simple results

BBN codes

R.V. Wagoner, *Astrophys. J. Suppl.* 18 (1969) 247; R.V. Wagoner, *Astrophys. J.* 179 (1973) 343.

L.H. Kawano, 1988. Preprint FERMILAB-Pub-88=34-A; L.H. Kawano, 1992. Preprint FERMILAB-Pub-92=04-A.

R.E. Lopez, M.S. Turner, *Phys. Rev. D* 59 (1999) 103502.

E. Lisi, S. Sarkar, F.L. Villante, *Phys. Rev. D* 59 (1999) 123520.

K.A. Olive, G. Steigman, T.P. Walker, *Phys. Rep.* 333334 (2000) 389.

S. Esposito, G. Mangano, G. Miele, O. P., *JHEP* 0009 (2000) 038; P.D. Serpico, et al., *JCAP* 0412 (2004) 010

...

PARthENoPE: O. Pisanti et al., *Comp. Phys. Comm.* 178 (2008) 956; *Comp. Phys. Comm.* 233 (2018) 237

AlterBBN: A. Arbey, *Comp. Phys. Comm.* 183 (2012) 1822

PRIMAT: C. Pitrou, A. Coc, J.-P. Uzan, E. Vangioni, *Phys. Rep.* 754 (2018) 1

S. Gariazzo, P.F. de Salas, O. Pisanti, R. Consiglio,
Comput.Phys.Commun. (2022) 108205, 271

Three public codes, all of them essentially equivalent from the numerical point of view.

BBN: overview and simple results

i) Initial conditions

Notations: $n_A = N_A/V$; $\eta = n_b/n_\gamma = 274 \cdot 10^{-10} \Omega_b h^2$
 $X_A = n_A/n_H$ $Y_P = 4 n_{4\text{He}}/n_b$

- For large temperatures all nuclear species are kept in chemical equilibrium (Nuclear Statistical Equilibrium NSE)

$$\mu_A = Z\mu_p + (A-Z)\mu_n$$

B_A = binding energy

$$n_A = g_A \frac{A^{3/2}}{2^A} \left(\frac{2\pi}{T} \right)^{3(A-1)/2} n_p^Z n_n^{A-Z} e^{B_A/T}$$

$$\frac{n_A}{n_b} = \left(\frac{2\zeta(3)}{\sqrt{\pi}} \right)^{A-1} \frac{g_A}{2} A^{3/2} \left(\frac{T}{m_N} \right)^{3(A-1)/2} \eta^{A-1} \left(\frac{n_p}{n_b} \right)^Z \left(\frac{n_n}{n_b} \right)^{A-Z} e^{B_A/T}$$

BBN overview and simple results

nucleus	B_A (MeV)	B_A/A (MeV)
D	2.23	1.1
^3H	6.92	2.3
^3He	7.72	2.6
^4He	28.30	7.1
^6Li	31.99	5.3
^7Li	39.25	5.6
^7Be	37.60	5.4
^{12}C	92.2	7.7

BBN overview and simple results

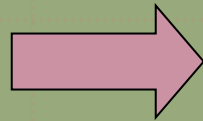
ii) n/p ratio freeze out

- The density ratio of n and p is kept in chemical equilibrium by weak processes:

$$\nu_e + n \leftrightarrow e^- + p$$

$$\bar{\nu}_e + p \leftrightarrow n + e^+$$

$$\bar{\nu}_e + e^- + p \leftrightarrow n$$



$$\frac{n_n}{n_p} = \exp\left(-\frac{m_n - m_p}{T}\right)$$

All other nuclei abundances are negligible for $T > 1$ MeV

BBN overview and simple results

As for purely leptonic weak interactions, also $n \leftrightarrow p$ processes freeze out at T of order of 1 MeV

- Standard calculation:
- Thermal averaged weak rates are evaluated at tree level with V-A theory and in the infinite nucleon mass limit (Born approximation).
- Example:

$$\Gamma(n \rightarrow p + e^- + \bar{\nu}_e) = \frac{G_F^2 (c_V^2 + 3c_A^2)}{2\pi^3} \int dp_e p_e^2 E_\nu^2 \Theta(E_\nu) f(E_\nu) f(E_e)$$

G_F , c_V and c_A are well known from muon decay, beta decays and neutron beta decay angular distribution.

$$G_F = 1.166378 8 (6) 10^{-5} \text{ GeV}^{-2}; \quad c_V = 0.97373 (31)$$

$$c_A/c_V = 1.2754 (13)$$

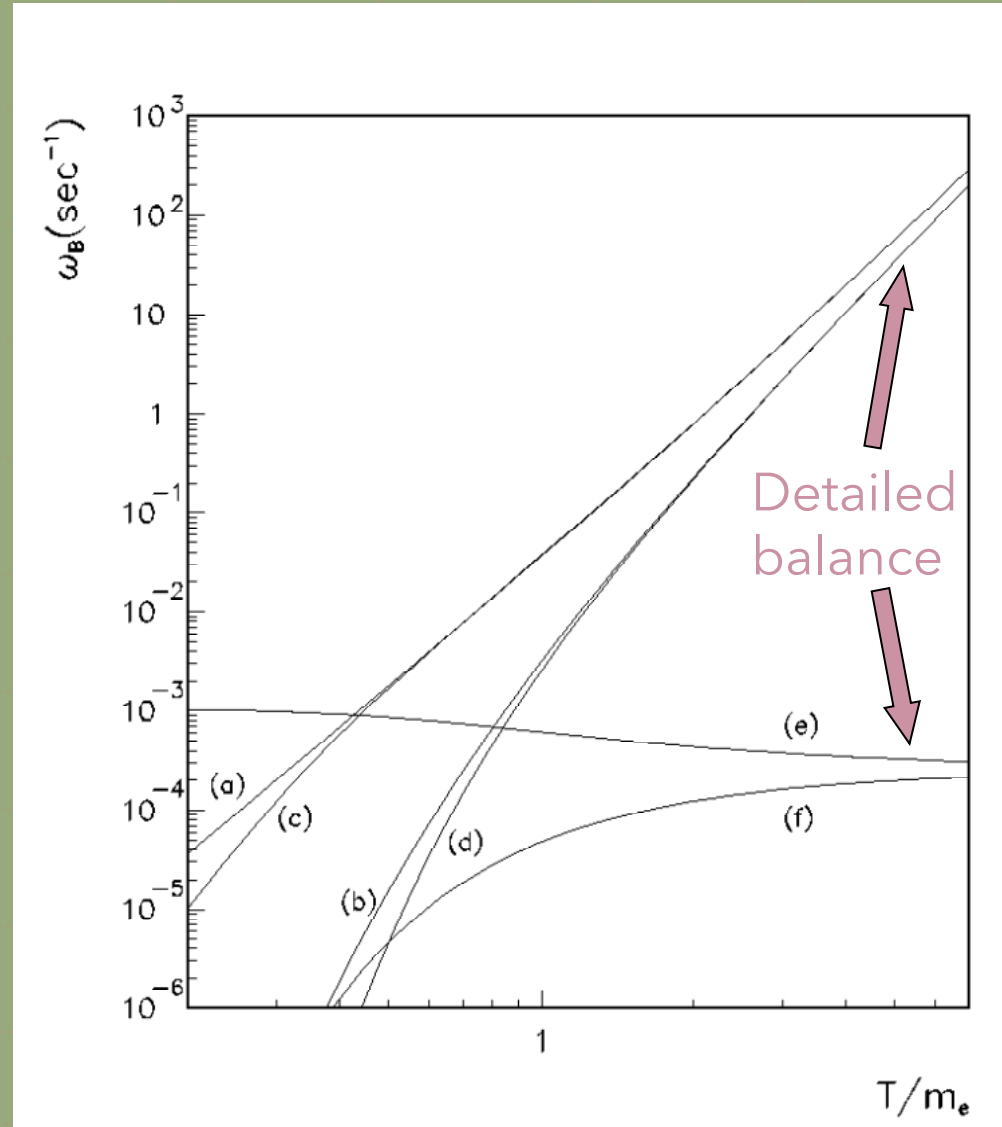
BBN overview and simple results

$$\Gamma_{np} = H \quad \text{freeze out}$$

Assuming that radiation energy density is dominated by radiation (photons, neutrinos...see later for extra species) i.e. H goes like T^2

$$G_F^2 T^2 f\left(\frac{Q}{T}, \frac{m_e}{T}\right) \approx G_N T^2$$

Freezing temperature 0.8 MeV
Exercise: what if the universe is matter dominated?



- (a) $\nu_e + n \rightarrow e^- + p$
- (b) $e^- + p \rightarrow \nu_e + n$
- (c) $e^+ + n \rightarrow \bar{\nu}_e + p$
- (d) $\bar{\nu}_e + p \rightarrow e^+ + n$
- (e) $n \rightarrow e^- + \bar{\nu}_e + p$
- (f) $e^- + \bar{\nu}_e + p \rightarrow n$

BBN overview and simple results

Using weak rates $X_n = 0.150 = (1 + \exp((m_n - m_p)/T))^{-1}$

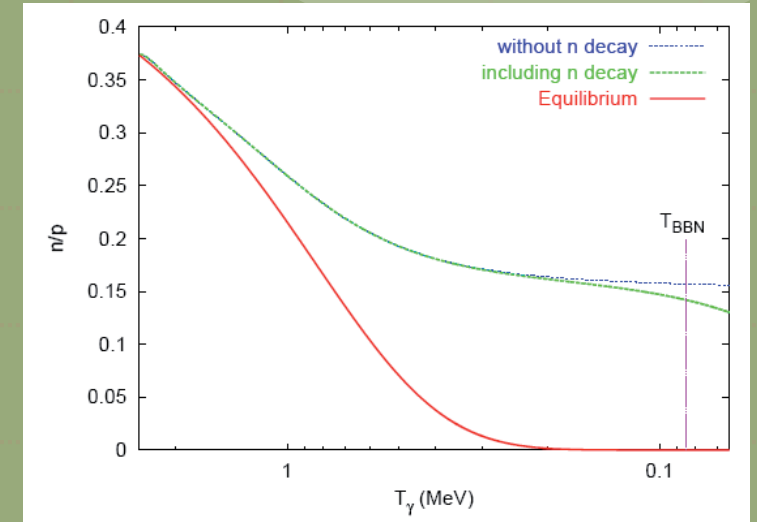
This ratio slightly decreases, due to neutron decay:

$$X_n = X_n \exp(-t/\tau_n) = 0.122$$

- A first rough estimate:
- Since basically all neutrons are eventually captured in ^4He nuclei (largest gain in energy), neglecting all other nuclei:

$$Y_p \approx \frac{4 n_n}{n_p + n_n} \approx 2 \cdot 0.122 = 0.244$$

This rough estimate turns out to be rather accurate indeed!!



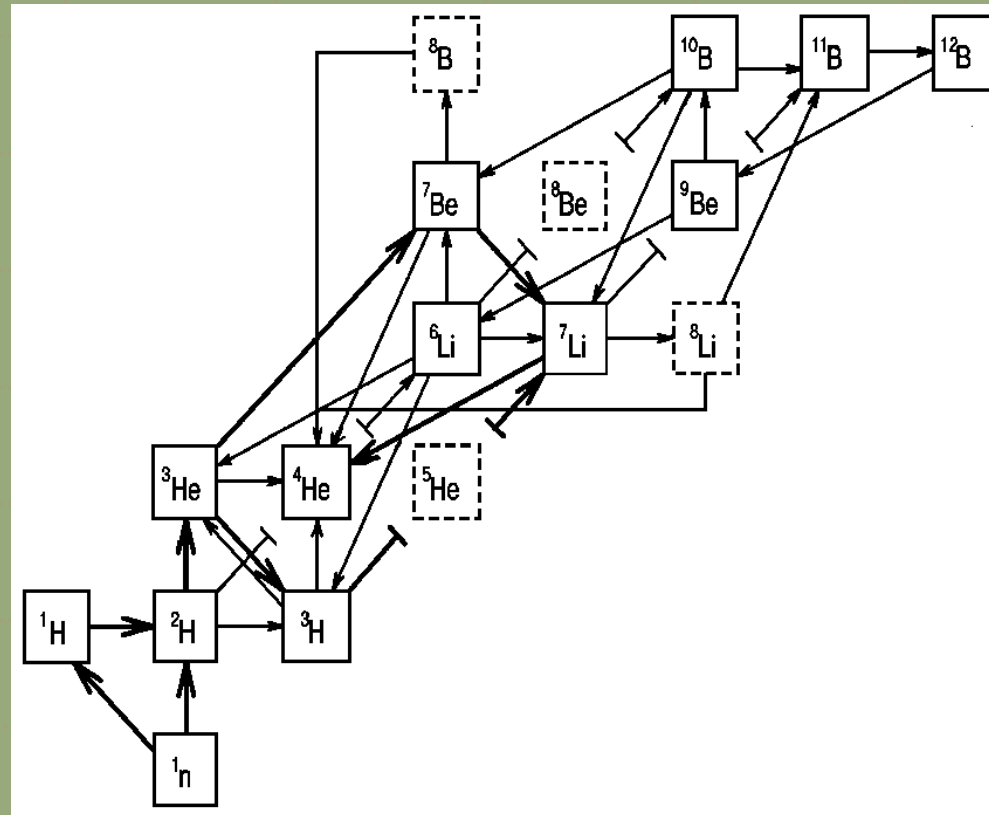
BBN overview and simple results

iii) D bottleneck

Deuterium formation is crucial for triggering the complicated nuclear reaction chain:



disfavoured (low density)



BBN overview and simple results

Two competing processes:

fusion:



photodissociation:



One would expect that when T just drops below $B_D = 2.23$ MeV, photodissociation processes become ineffective. However: too many photons!!

$$\frac{X_D}{X_n X_p} = \frac{12\zeta(3)}{\sqrt{\pi}} \left(\frac{T}{m_p} \right)^{3/2} \eta \exp(B_D / T)$$

Saha equation

- Deuterium formation starts (rapidly leading to ^4He) only when $\eta \exp(B_D/T^*) = 1$.
- As we will see η is of order 10^{-9} , so $T^* = 0.08$ MeV.

BBN overview and simple results

iv) Nuclear chain

Once D is produced, ${}^4\text{He}$ is rapidly formed, along with small fractions of ${}^3\text{H}$, ${}^3\text{He}$, ${}^6\text{Li}$, ${}^7\text{Li}$ and ${}^7\text{Be}$.

${}^7\text{Be}$ eventually gives ${}^7\text{Li}$ by electron capture:



Though both ${}^{12}\text{C}$ and ${}^{16}\text{O}$ have larger binding energy than ${}^4\text{He}$, they are not produced in sensible amounts since:

- i) No tightly bound isotopes with $A=5, 8$
- ii) Coulomb barrier start to be significant
- iii) Low baryon density suppress triple α processes
(@ 0.1 MeV baryon density is earth atmosphere density at ground level)

BBN overview and simple results

How to evaluate nuclei yields? BBN code: solving a set of coupled differential equations:

Z \ N	0	1	2	3	4	5	6	7	8
0		n							
1	H	² H	³ H						
2		³ He	⁴ He						
3				⁶ Li	⁷ Li	⁸ Li			
4				⁷ Be		⁹ Be			
5				⁸ B		¹⁰ B	¹¹ B	¹² B	
6						¹¹ C	¹² C	¹³ C	¹⁴ C
7						¹² N	¹³ N	¹⁴ N	¹⁵ N
8							¹⁴ O	¹⁵ O	¹⁶ O

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi}{3m_{Pl}^2} (\rho_\gamma + \rho_{e^\pm} + \rho_b + \rho_v)}$$

$$\frac{\dot{n}_b}{n_b} = -3 \frac{\dot{a}}{a}$$

$$\dot{T} = \Phi(t, X_a)$$

$$Q_{lepton}(\mu_e, T) = -Q_{baryon}(X_a)$$

$$\dot{X}_a = \sum_{b,c,d} N_a \left(\Gamma(c+d \rightarrow a+b) \frac{(X_c)^{N_c}}{N_c!} \frac{(X_d)^{N_d}}{N_d!} - \Gamma(a+b \rightarrow c+d) \frac{(X_a)^{N_a}}{N_a!} \frac{(X_b)^{N_b}}{N_b!} \right)$$

a : scale factor

ρ_v : energy density

of relativistic

species ($m < 1$ MeV)

μ_e : electron chemical potential

Improving precision: weak rates, neutrino decoupling and nuclear chain rates

Inputs:

nuclear rates (experimental values extrapolated in the relevant energy range)

baryon density (η)

energy density in relativistic degrees of freedom:

$$N_\nu \equiv \frac{\rho_\nu}{\frac{7\pi^2}{120} T^4}$$

$$\rho_R = \rho_\gamma \left[1 + \frac{7}{8} 3.045 \left(\frac{4}{11} \right)^{4/3} \right] + \Delta N_{eff} \cdot \rho_\gamma \left[\frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \right]$$

The present (and future) precision of astrophysical observations of primordial nuclide abundances led to a large effort in improving precision of theoretical predictions for ^4He and deuterium (mainly), i.e.

1. Weak rates now computed including radiative corrections
2. More precise data on nuclear cross sections and «ab initio» nuclear theoretical calculations
3. Neutrino evolution including oscillations and obtained solving the full kinetic equations

Improving precision...

- Accuracy of the BBN codes. Standard physics, theoretical framework well established, but outputs of the nuclear network depend on the determination of several critical reactions. In the past mainly experimental measures (not always in the relevant energy range for BBN, 10÷400 keV in the center of mass), now also theoretical calculations.



experimental reaction data and analysis methods

- Accuracy of primordial elements abundances measurement. Indirect observations, since stars have changed the chemical composition of the universe. Strategies are observation in “primordial” systems or careful account for chemical evolution: increasingly precise astrophysical data on D (1%), He measured by different groups with less than 1.5% accuracy but one determination is at 4% distance, the situation is not clear for Li (the value is a factor 2-3 below the BBN prediction, lithium depletion problem).



systematics and astrophysical evolution

Improving precision...

Example of the issue: neutron decay. In the Born approximation the thermal averaged rate in the limit of vanishing densities is

$$\tau_n^{-1} = \frac{G_F^2 (c_V^2 + 3c_A^2)}{2\pi^3} m_e^5 \int_1^{\Delta/m_e} d\varepsilon \varepsilon \left(\varepsilon - \frac{\Delta}{m_e} \right)^2 (\varepsilon^2 - 1)^{1/2}$$

$$\tau_n(\text{th}) = 961 \text{ s}$$

S. Esposito, G. M., G. Miele, O. Pisanti, Nucl. Phys. B 540 (1999) 3

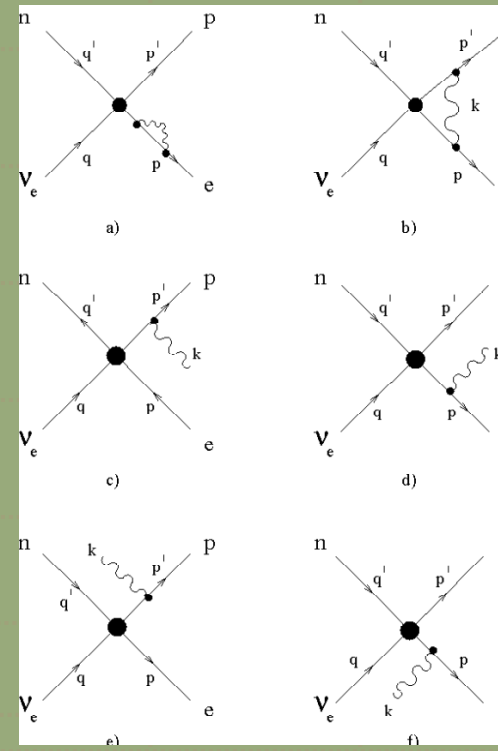
Corrections to the weak rates:

- radiative corrections $O(\alpha)$
- finite nucleon mass corrections $O(T/m_N)$
- plasma effects ($\propto T/m_e$)

$$\tau_n(\text{th}) = 893.9 \text{ s}$$

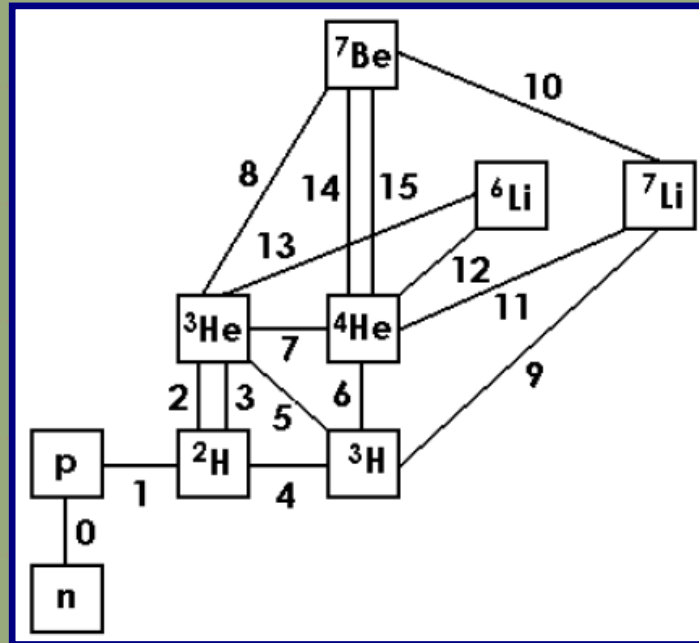
$$\tau_n(\text{exp}) = 878.4 \pm 0.5 \text{ s}$$

Weak rates are the main issue for calculating Y_p , and the main uncertainty is the experimental error in the neutron lifetime.



Improving precision...

In the last decade more precise data have been obtained on nuclear cross sections in the CM energy range relevant for BBN. Ab initio calculations and LUNA result on dpgamma!



Deuterium synthesis

Symbol	Reaction	Symbol	Reaction
R_0	τ_n	R_8	${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$
R_1	$p(n, \gamma)d$	R_9	${}^3\text{H}(\alpha, \gamma){}^7\text{Li}$
R_2	${}^2\text{H}(p, \gamma){}^3\text{He}$	R_{10}	${}^7\text{Be}(n, p){}^7\text{Li}$
R_3	${}^2\text{H}(d, n){}^3\text{He}$	R_{11}	${}^7\text{Li}(p, \alpha){}^4\text{He}$
R_4	${}^2\text{H}(d, p){}^3\text{H}$	R_{12}	${}^4\text{He}(d, \gamma){}^6\text{Li}$
R_5	${}^3\text{He}(n, p){}^3\text{H}$	R_{13}	${}^6\text{Li}(p, \alpha){}^3\text{He}$
R_6	${}^3\text{H}(d, n){}^4\text{He}$	R_{14}	${}^7\text{Be}(n, \alpha){}^4\text{He}$
R_7	${}^3\text{He}(d, p){}^4\text{He}$	R_{15}	${}^7\text{Be}(d, p)2\,{}^4\text{He}$

before
LUNA

Reaction	Rate symbol	$\sigma_{2\text{H}/\text{H}} \times 10^5$
$p(n, \gamma){}^2\text{H}$	R_1	± 0.002
$d(p, \gamma){}^3\text{He}$	R_2	± 0.062
$d(d, n){}^3\text{He}$	R_3	± 0.020
$d(d, p){}^3\text{H}$	R_4	± 0.013

0.1
%
87%
9%
3.8
%

Di Valentino et al., Phys. Rev. D90 (2014) no. 2, 023543

V. Mossa et al, Nature 587 (2020) 7833, 210
L.E. Marcucci et al, Phys.Rev.Lett. 116 (2016) 10, 102501

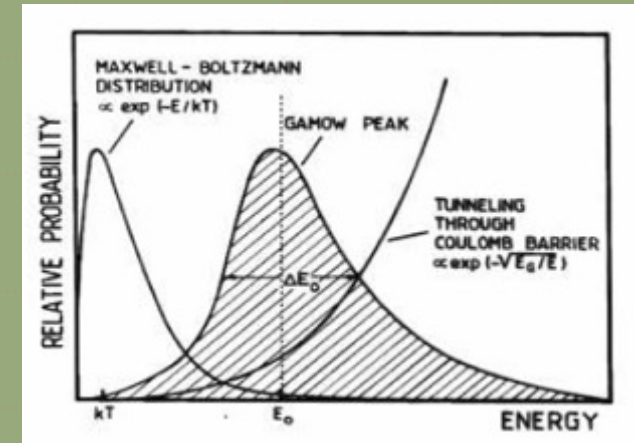
Improving precision

Nuclear cross sections

The S-factor is the intrinsic nuclear part of the reaction probability for charged particle induced reactions and is fitted from data (problem: datasets cover limited energy ranges and have different normalization errors, in some cases not even estimated).

$$\langle \sigma v \rangle = \sqrt{\frac{8}{\pi \mu_{ab}}} T^{-3/2} \int_0^{\infty} dE E \sigma(E) e^{-E/T}$$

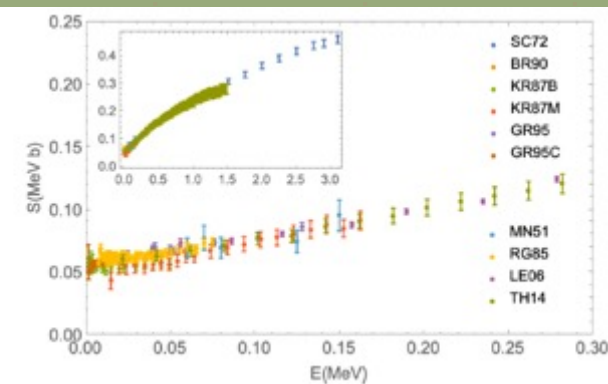
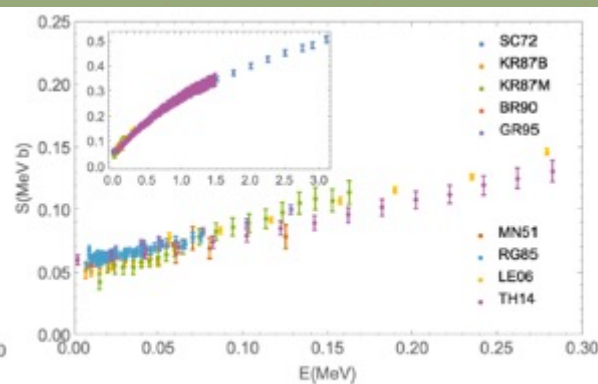
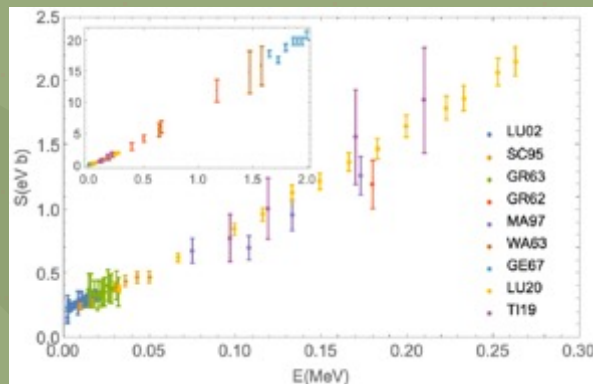
$$S(E) = \sigma(E) E e^{\sqrt{E_G/E}}$$



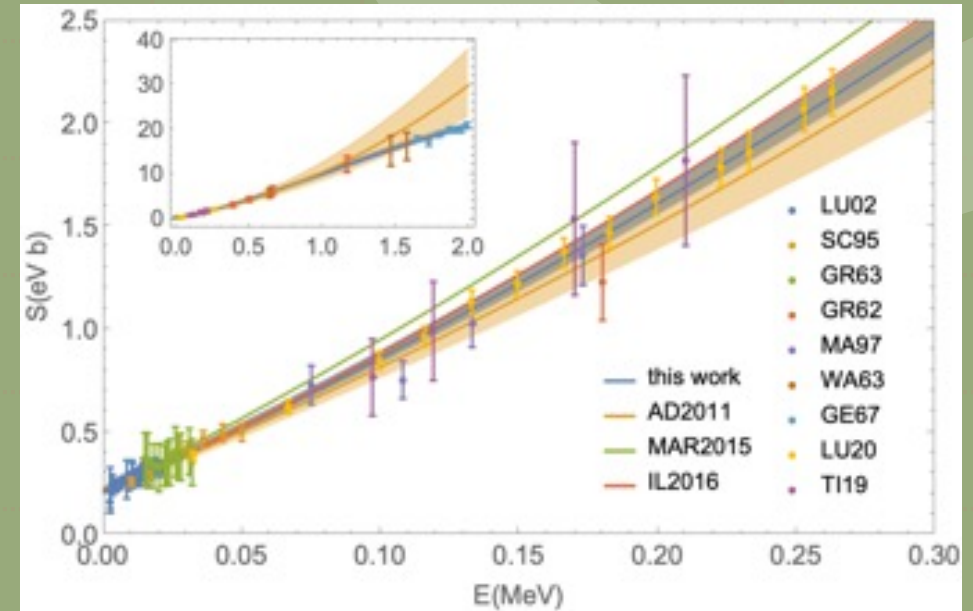
dp γ

ddn

ddp



Improving precision



Before LUNA

- previous data were scarce in the BBN range with $\sim 9\%$ uncertainty
- phenomenological fit by Adelberger et al. (AD2011, orange line and band)
- *ab initio* theoretical prediction by Marcucci et al. (2005) updated in 2016 (green line), 15% higher than AD2011
- Bayesian analysis by Iliadis et al. (2016, red line)

V. Mossa et al., Nature 587 (2020) 7833,
210

After LUNA

- very precise data (yellow points), $\Delta S/S \leq 2.6\%$, in $[30,300]$ keV E_{cm}
- S-factor global fit (dominated by LUNA data) with 3rd order polynomial, $\chi_{\text{red}}^2 = 1.02$ (Nature 2020, blue line and band)

Improving precision

Rate uncertainties

O.Pisanti et al, JCAP 04
(2021) 020

	$10^5 \sigma_i$	$\sigma_i(\%)$
$d(p, \gamma) {}^3\text{He}$	0.014	11
$d(d, n) {}^3\text{He}$	0.035	69
$d(d, p) {}^3\text{H}$	0.019	20

Yeh et al., JCAP 03 (2021)
046

Reaction i	$10^5 \sigma_i(\text{D/H})$	$10^5 \sigma_{\text{omit } i}(\text{D/H})$
$d(p, \gamma) {}^3\text{He}$	0.036	0.097
$d(d, n) {}^3\text{He}$	0.081	0.065
$d(d, p)t$	0.054	0.089
${}^3\text{He}(d, p) {}^4\text{He}$	0.002	0.103
$p(n, \gamma)d$	0.002	0.103
${}^3\text{He}(n, p)t$	0.002	0.103
all	0.103	–

12%
61%
27%

Improving precision...

Neutrino properties and evolution influences BBN predictions:

- They enter weak rates (ν_e)
- They contribute to the total energy density in the universe i.e. the expansion rate H

$$f_\alpha(k, t) = e^{-\Gamma_\alpha(t)t} \frac{\sum_\beta P_{\alpha\beta}(\mu, B, \theta, m, t)}{\exp(\sqrt{k^2 + m_\alpha^2} / T(t) - \xi_\alpha) + 1} + \delta_\alpha(k, t)$$

unstable ν 's: Ω_{tot} ,
LSS, CMB, e^+ flux

ν oscillations and
magnetic
moments: BBN

non-thermal
effects: CMB,
LSS, ...

of
species:
BBN,
CMB, ...

ν mass: Ω_{tot} ,
LSS, UHECR

ν chemical
potential:
BBN, LSS

Improving precision...

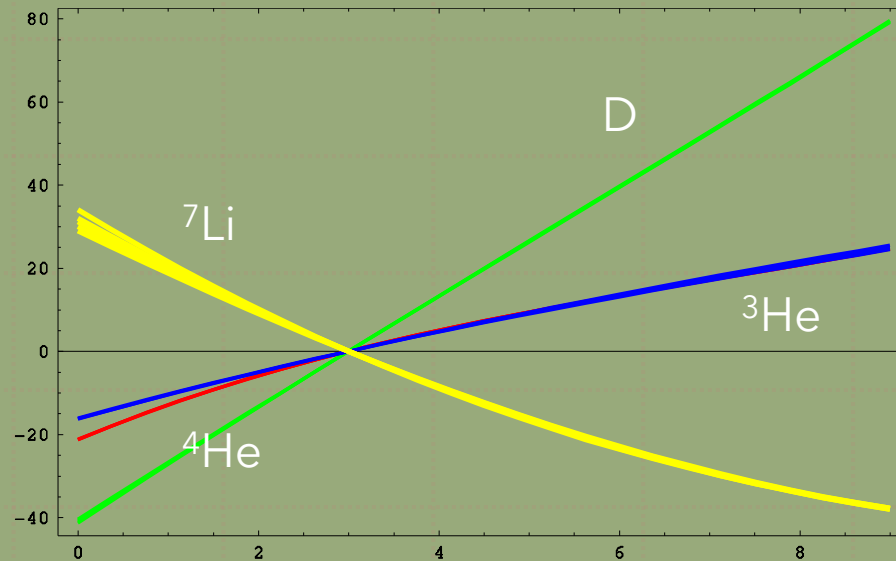
BBN and neutrinos

$$\rho_R = \rho_\gamma + \rho_\nu + \rho_x = \left(1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\nu}^{eff} \right) \rho_\gamma$$

They couple to gravity and contribute to expansion

Faster expansion \longrightarrow earlier weak process freeze-out

\longrightarrow more neutrons per protons



Bounds on extra light particles or exotic neutrino features

Improving precision...

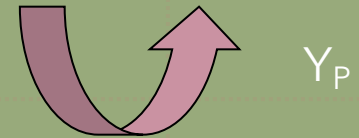
BBN and Neutrino Asymmetry: a leptometer

Large neutrino chemical potentials are not forbidden. They affect BBN!

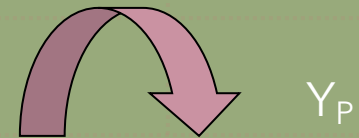
1) chemical potentials contribute to N_ν (if no extra d.o.f.)

$$N_\nu = 3 + \sum_i \left(\frac{30\xi_i^2}{7\pi^2} + \frac{15\xi_i^4}{7\pi^4} \right) + \dots$$

$$\xi_i \equiv \frac{\mu_{\nu_i}}{T}$$



2) a positive electron neutrino chemical potential μ_e (more neutrinos than antineutrinos) favour $n \rightarrow p$ with respect to $p \rightarrow n$ processes.



3) Neutrino oscillations mix the three standard active neutrino flavors. We can take all of them equal.

Improving precision...

As the Universe expands, particle densities are diluted and temperature falls. Weak interactions become ineffective to keep neutrinos in good thermal contact with the e.m. plasma

Rough, but quite accurate estimate of the decoupling temperature

Rate of weak processes \sim Hubble expansion rate

$$\Gamma_w \approx \sigma_w |v| n, H^2 = \frac{8\pi\rho_R}{3M_p^2} \rightarrow G_F^2 T^5 \approx \sqrt{\frac{8\pi\rho_R}{3M_p^2}} \rightarrow T_{dec}^v \approx 1 \text{ MeV}$$

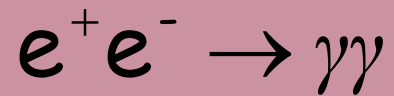
Since ν_e have both CC and NC interactions with e^\pm

$$\mathbf{T_{dec}(\nu_e) \sim 2 \text{ MeV}}$$

$$\mathbf{T_{dec}(\nu_{\mu,\tau}) \sim 3 \text{ MeV}}$$

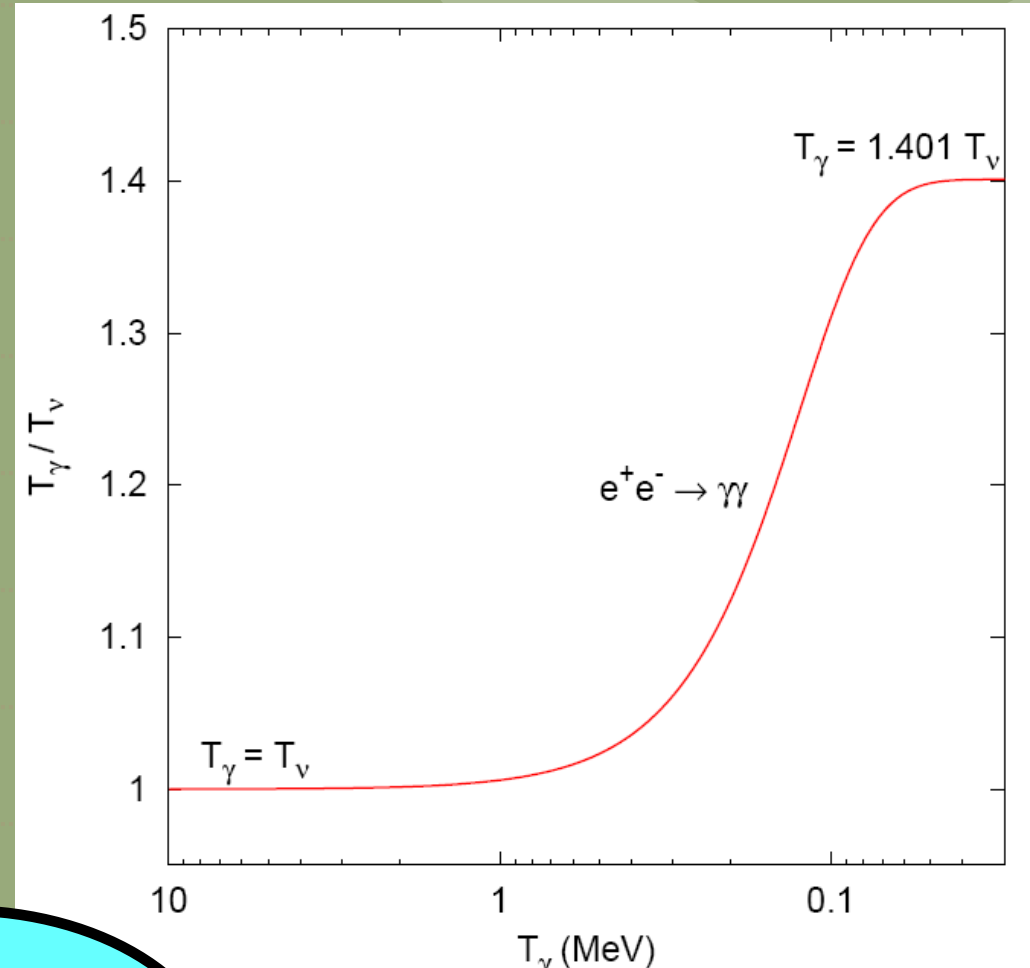
Improving precision...

At $T \sim m_e$, electron-positron pairs annihilate



heating photons but not the decoupled neutrinos (entropy conservation)

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$

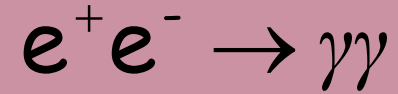


$$f_\nu(p, T) = \frac{1}{e^{p/T_\nu} + 1}$$

Improving precision...

Non-instantaneous neutrino decoupling

At $T \sim m_e$, e^+e^- pairs annihilate heating photons



But, since $T_{\text{dec}}(\nu)$ is close to m_e , neutrinos share a small part of the entropy release


$$\mathbf{f}_\nu = \mathbf{f}_{\text{FD}}(\mathbf{p}, T_\nu) [1 + \delta f(\mathbf{p})]$$

Improving precision..

Momentum-dependent Boltzmann equation

$$\left(\frac{d}{dt} - Hp \frac{d}{dp_1} \right) f_\nu(p_1, t) = I_{coll}(p_1, t)$$



Statistical Factor

$\frac{1}{2E_1} \int \prod_{i=2}^4 \left(\frac{d^3 p_i}{(2\pi)^3 2E_i} \right)$	$(2\pi)^4 \delta^4(P_1 + P_2 - P_3 - P_4)$	$ M ^2$	F
--	--	---------	-----

9-dim Phase Space

ΣP_i conservation

Process

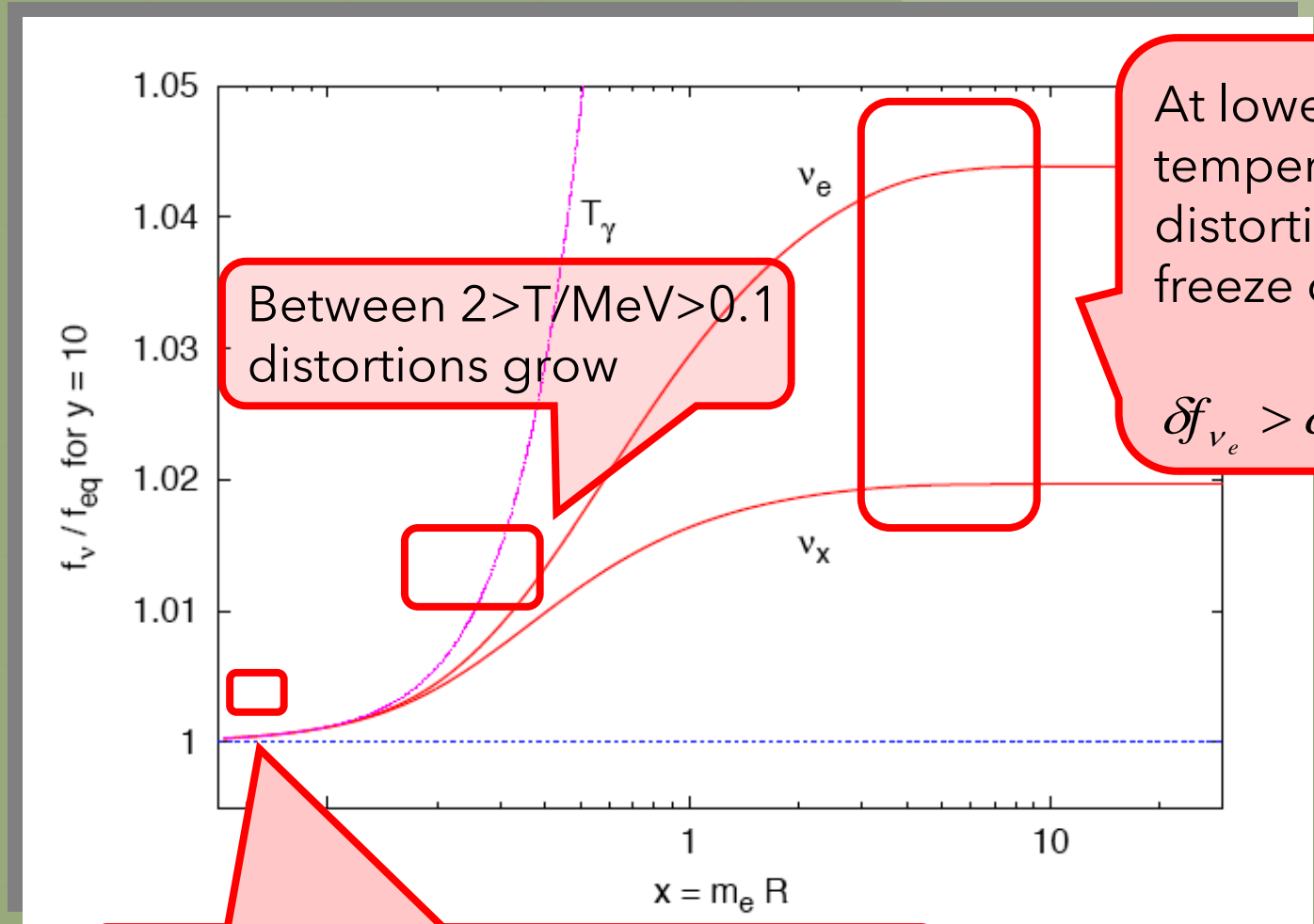
$$F = f_3 f_4 [1 + f_1] [1 - f_2] - f_1 f_2 [1 + f_3] [1 - f_4]$$

+ evolution of total energy density:

$$\frac{d\rho_R}{dt} = -3H(\rho_R + P_R)$$

Improving precision...

Evolution of f_ν for a particular momentum $p=10T$



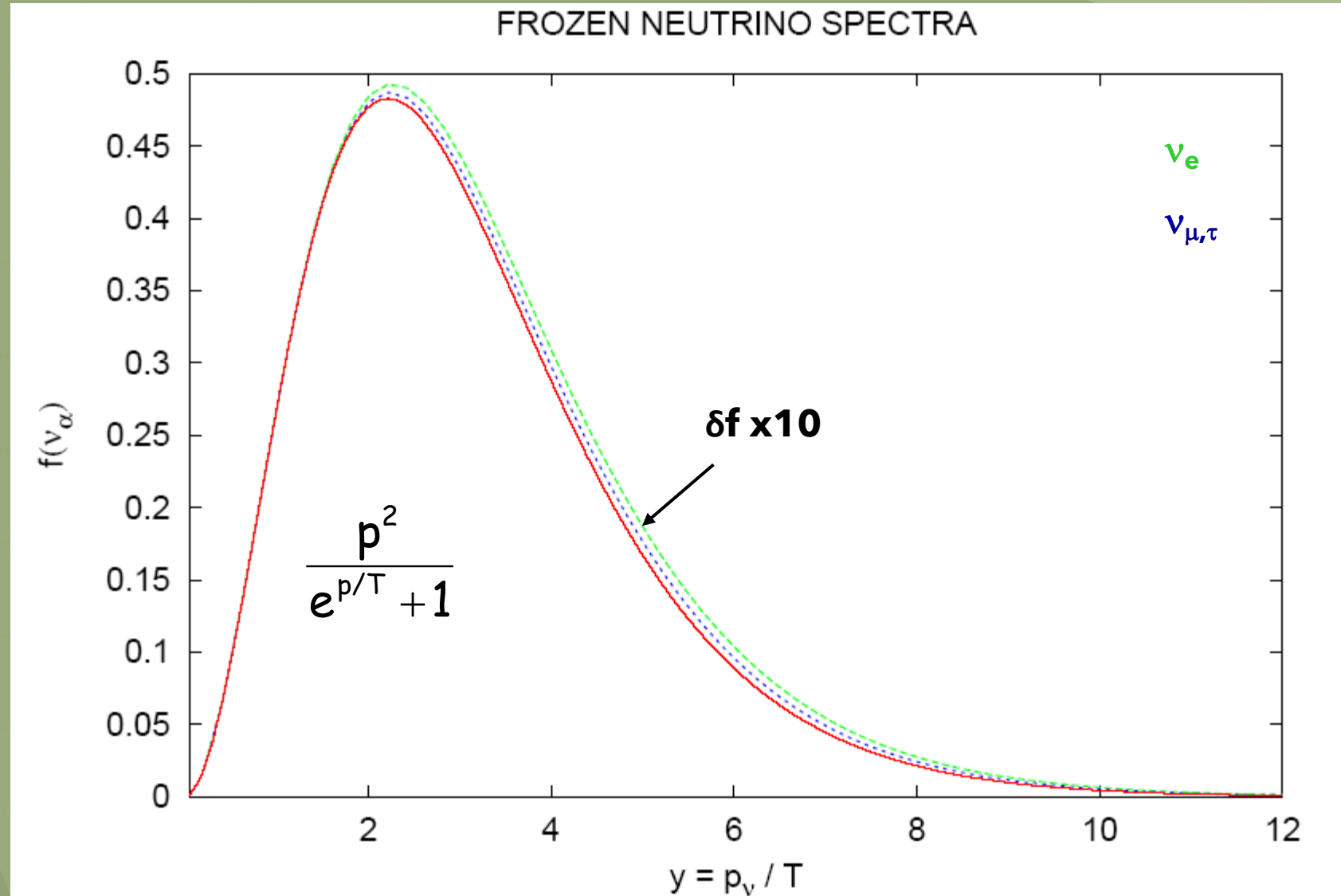
Between $2 > T/\text{MeV} > 0.1$
distortions grow

At lower
temperatures
distortions
freeze out

$$\delta f_{\nu_e} > \delta f_{\nu_\mu}$$

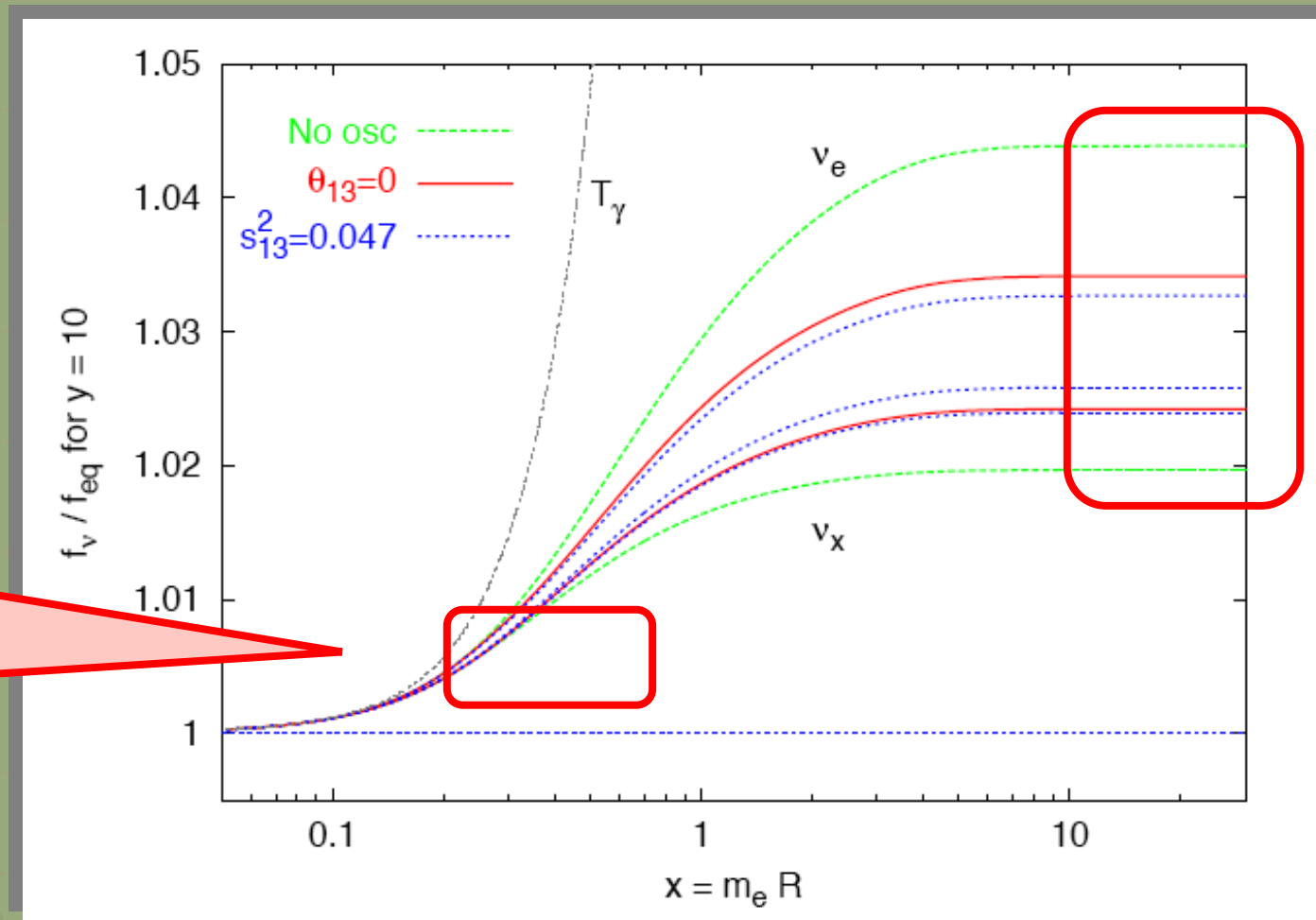
For $T > 2$ MeV neutrinos are coupled

Improving precision



Improving precision...

Effects of flavour neutrino oscillations on the spectral distortions



Around $T \sim 1$ MeV the oscillations start to modify the distortion

The variation is larger for ν_e

Improving precision...

Results

	$T_{fin}^\gamma / T_0^\gamma$	$\delta\rho_{\nu e}(\%)$	$\delta\rho_{\nu\mu}(\%)$	$\delta\rho_{\nu\tau}(\%)$	N_{eff}
Instantaneous decoupling	1.40102	0	0	0	3
SM	1.3978	0.94	0.43	0.43	3.046
+3v mixing ($\theta_{13}=0$)	1.3978	0.73	0.52	0.52	3.046
+3v mixing ($\sin^2\theta_{13}=0.047$)	1.3978	0.70	0.56	0.52	3.046

Dolgov, Hansen & Semikoz, NPB 503 (1997) 426
G.M. et al, PLB 534 (2002) 8
G.M. et al, NPB 729 (2005) 221

Observations

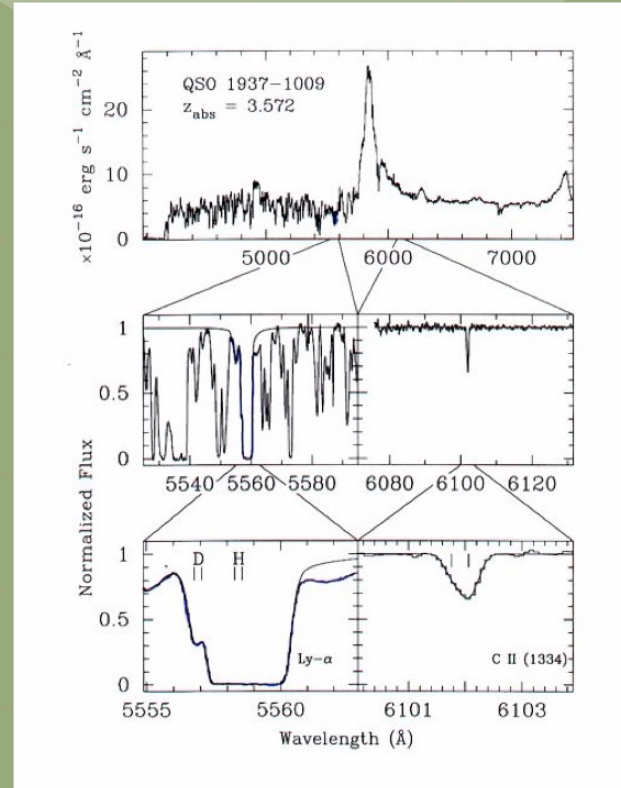
Data (the quest for “primordiality”)

- ^2H : it is only destroyed. Observation of Lyman absorption lines by neutral H and D (HI, DI) gas clouds (Damped Lyman- α , DLAs) at red-shift $z \approx 2 - 3$ placed along the line of sight of distant quasar. Few systems, but next generation 30-m class telescopes will increase the number.
- ^3He : in stellar interior can be either produced by ^2H -burning or destroyed in the hotter regions. It was observed only within Milky Way. Next generation 30-m class telescopes may measure $^3\text{He}/^4\text{He}$.
- ^4He : it is produced inside stars. Observation in ionized gas regions (HeII \rightarrow HeI recombination lines) in low metallicity environments (BCG or dwarf irregular), with O abundances 0.02 - 0.2 times those in the sun. Then, regression to zero metallicity. Large systematics (1% accuracy at best), but CMB allows interesting measure via ^4He effect on acoustic peak tail.
- ^7Li : it is produced (BBN and spallation) and destroyed. Observation of absorption lines in spectra of halo stars of POP II. Spite plateau at medium metallicity, but scattered points at low metallicity. The experimental value is a factor 2-3 below the BBN prediction. Attempts at solutions: nuclear rates, stellar depletion, new particles decaying at BBN, axion cooling, variation of fundamental constants. However, a measure from the Small Magellanic Cloud is at BBN level.

Observations

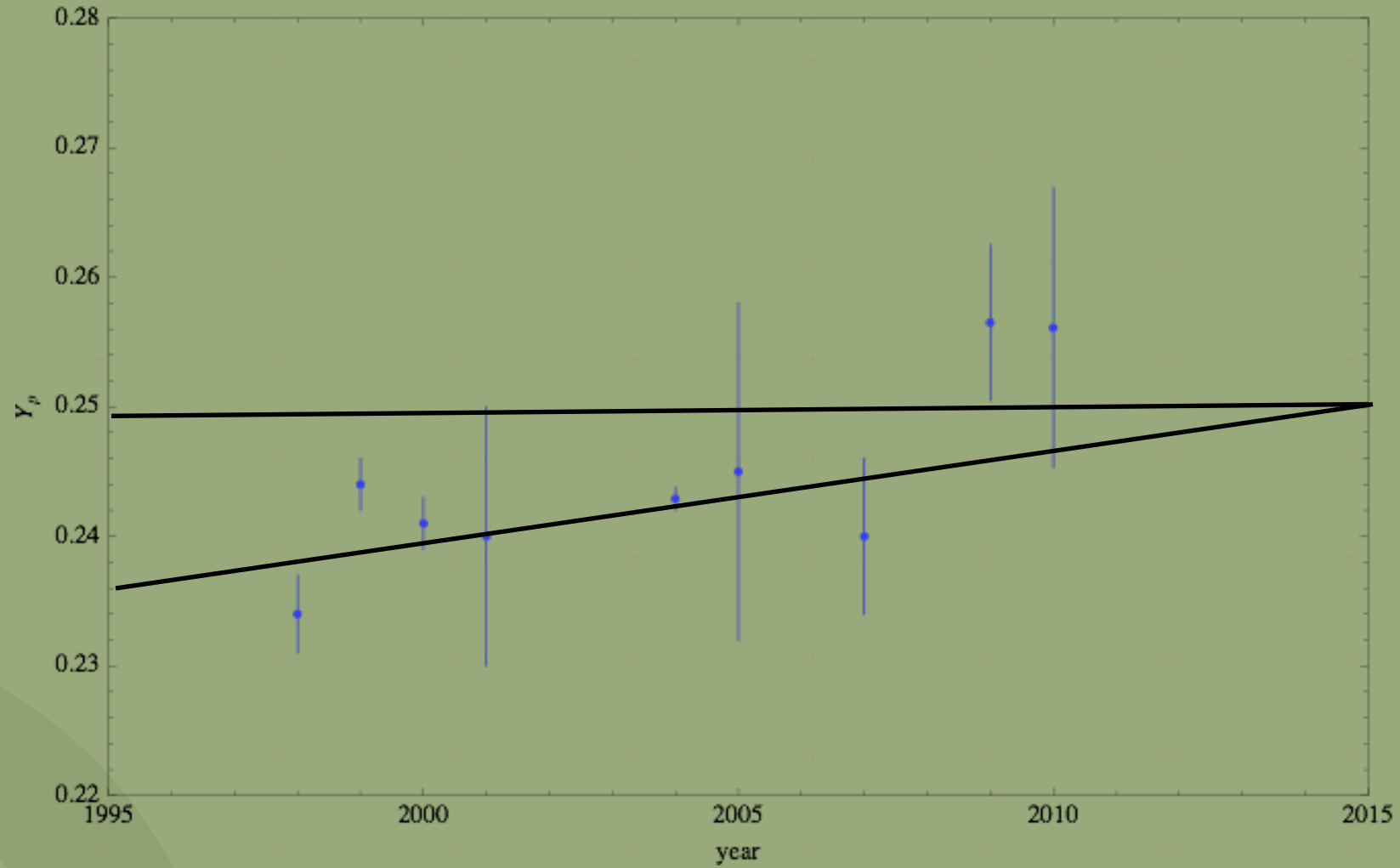
^2H

- Determination of D/H at high redshift help ensure that the observed abundance is close to primordial one.
- From a set of five high quality absorbers it was determined $^2\text{H}/\text{H}=(2.53\pm 0.04)\cdot 10^{-5}$ (R. Cooke et al., *Astrophys.J.* 781 (2014) 31).
- A measure $^2\text{H}/\text{H}=(2.45\pm 0.28)\cdot 10^{-5}$ at $z=3.256$ remains debated (S. Reimer-Sorensen et al., *MNRAS* 447 (2015) 2925).
- After recent new observations or reanalyses of existing data the new value, with 1.2% uncertainty, is $^2\text{H}/\text{H}=(2.527\pm 0.030)\cdot 10^{-5}$ (R. Cooke et al., *Astrophys.J.* 855 (2018) 102).
- The weighted mean of the latest 11 measures gives $^2\text{H}/\text{H}=(2.55\pm 0.03)\cdot 10^{-5}$ (B.D. Fields et al., *JCAP* 03 (2020) 010).
- Very promising improvement foreseen in the measure by 30 m class telescopes.



Observations

${}^4\text{He}$ "evolution"

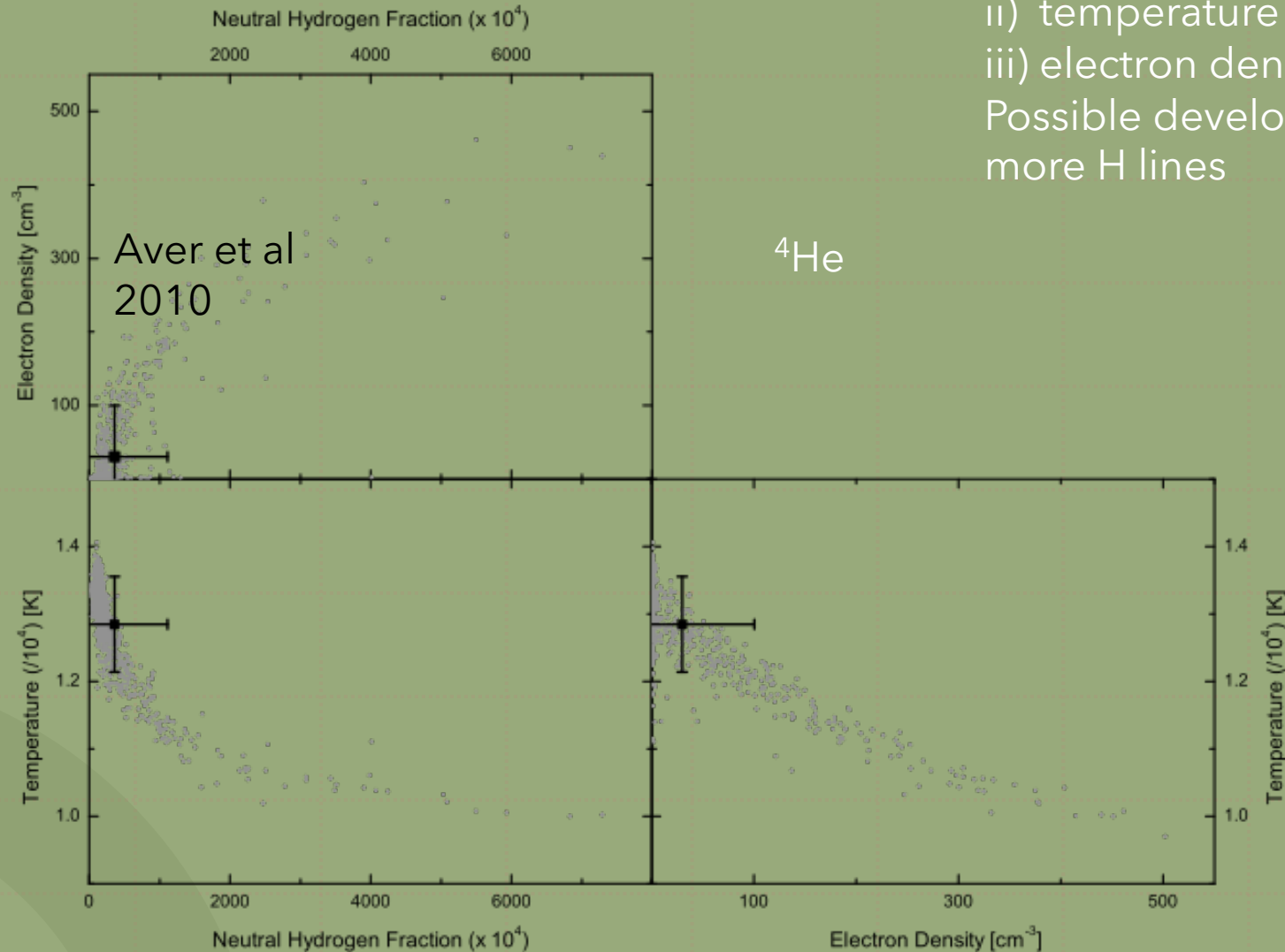


Observations

Main sources of systematics:

- i) interstellar reddening
- ii) temperature of clouds
- iii) electron density

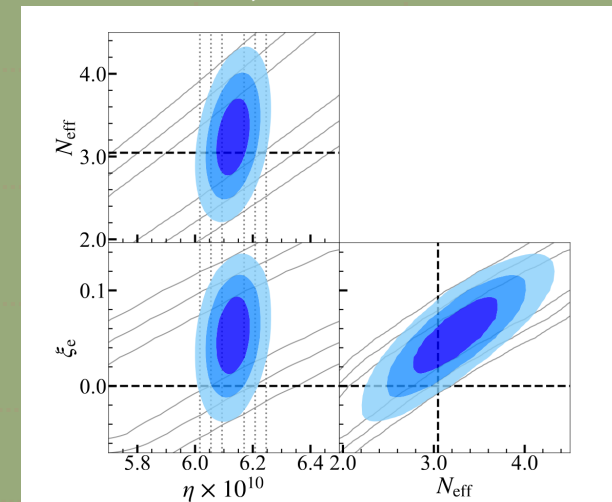
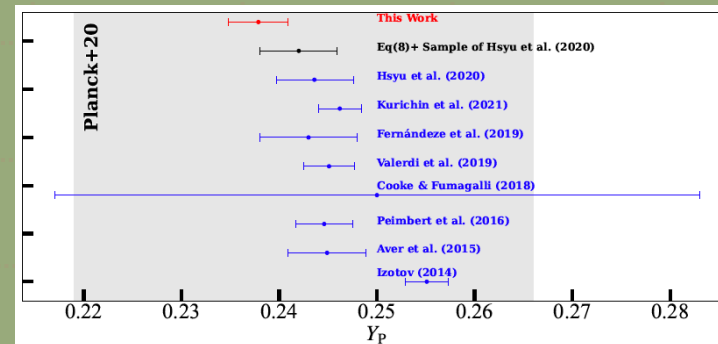
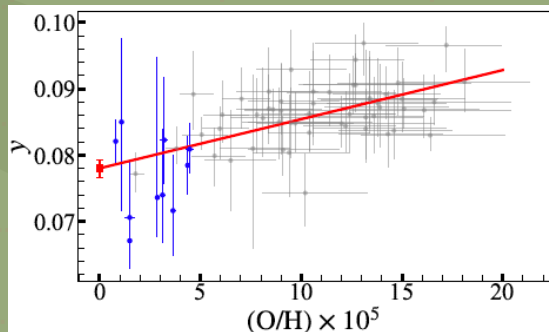
Possible developments: using more H lines



Observations

^4He

- The theoretical model used for extracting the abundance contains several physical parameters (among which ^4He abundance, electron density, optical depth, temperature, neutral H fraction). However, there was a degeneracy between the electron density and the temperature of the gas.
- More recently, the near-infrared (NIR) line $\text{HeI}\lambda 10830$ was included in the analysis, which is key to removing such a degeneracy.
- From the study of 54 galaxies (three of which are Extremely Metal Poor Galaxies, EMPGs, less than 10% of solar metallicity), it results $Y_p = 0.2436 \pm 0.0040$ (T. Hsyu et al, *Astrophys.J.* 896 (2020) 77).
- An alternative method consists in studying intergalactic absorption lines in almost primordial clouds between us and a background quasar, from which $Y_p = 0.250 \pm 0.033$ (C. Sykes et al, *MNRAS* 492 (2020) 2151). Same authors give $Y_p = 0.248 \pm 0.001$ as a weighted average of all recent determinations.
- Adding to the sample 10 EMPGs, a new results was released recently, $Y_p = 0.2379 \pm 0.0030$ (A. Matsumoto et al, e-Print: 2203.09617).

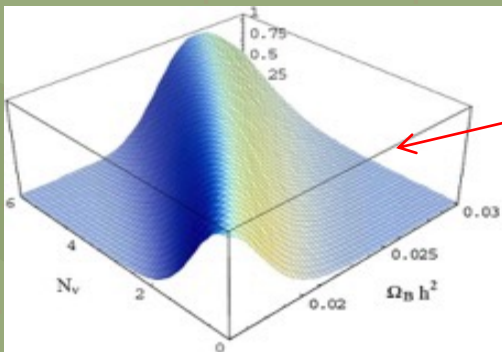


Standard BBN

- Choose the scenario, that is the parameters of your model: A, B,
- Run your favourite BBN code and determine the theoretical abundances $X_i(A,B,...)$ with corresponding uncertainties $\sigma_i(A,B,...)$.
- Construct likelihood functions for your abundances:

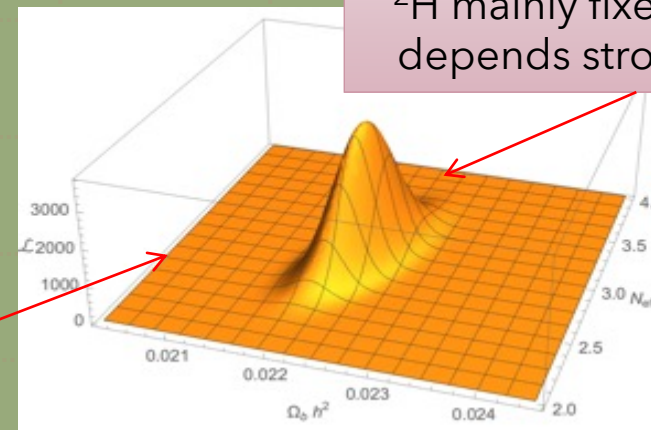
$$L_i(N_{eff}, \eta) = \frac{1}{2\pi\sigma_i^{th}(N_{eff}, \eta)\sigma_i^{ex}} \int dx \exp\left(-\frac{(x - Y_i^{th}(N_{eff}, \eta))^2}{2\sigma_i^{th}(N_{eff}, \eta)^2}\right) \exp\left(-\frac{(x - Y_i^{ex})^2}{2\sigma_i^{ex}}\right)$$

- Determine confidence level contours from the comparison of theoretical and experimental quantities.



For free N_{eff} , ^2H alone is not sufficient in breaking the degeneracy...

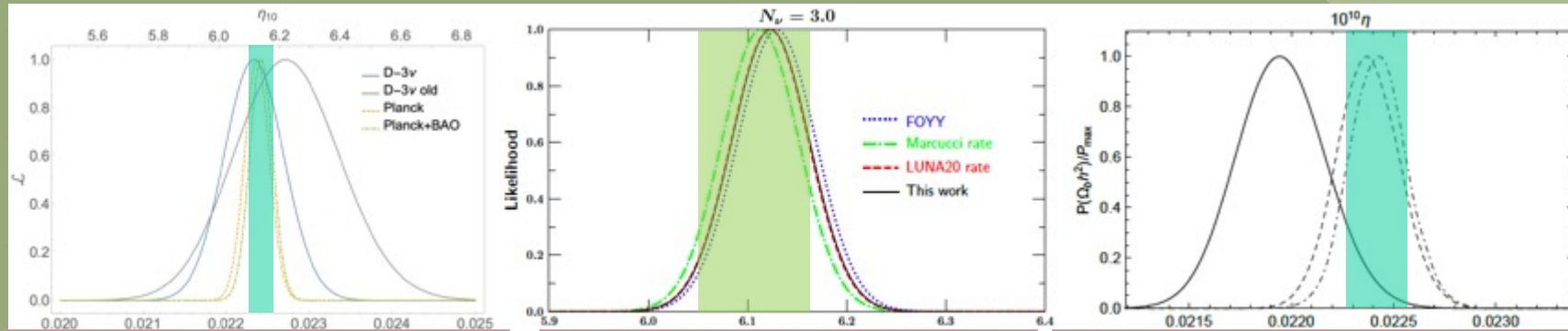
... and you need to add another observable (e.g. ^4He) or a prior (e.g. Ω_b Planck)



^2H mainly fixes $\Omega_B h^2$, ^4He depends strongly on N_{eff}

Standard BBN

BBN/CMB concordance. Only free parameter is the baryon density (baryon to photon ratio)



O.P. et al, JCAP 04 (2021) 020

Yeh et al., JCAP 03 (2021) 046

Pitrou et al., M.N.R.Astr.Soc. 502 (2021) 2, 2474

- A(blue) and B(black) in fair agreement with each other and with Planck (1σ green bands)
- C(solid) shows 1.84σ tension with Planck
- Likelihoods come from:
 - A: only D_{BBN} , $D/H=2.527 \pm 0.030$
 - B: $D_{\text{BBN}} + Y_{\text{pBBN}} + \text{CMB}$, $D/H=2.55 \pm 0.03$, $Y_{\text{p}}=0.2453 \pm 0.0034$
 - C: $D_{\text{BBN}} + Y_{\text{pBBN}}$, $D/H=2.527 \pm 0.030$, $Y_{\text{p}}=0.2453 \pm 0.0034$
- Planck green bands correspond to:
 - A: Planck + $Y_{\text{p}}(\omega_{\text{b}})$ + lensing + BAO
 - B: Planck + lensing
 - C: Planck + $Y_{\text{p}}(\omega_{\text{b}})$ + lensing + BAO

Non standard scenarios

BBN is a powerful «cosmological probe» and can test more exotic scenarios for either the cosmological model or fundamental interactions. In particular when combined with CMB data (Planck)

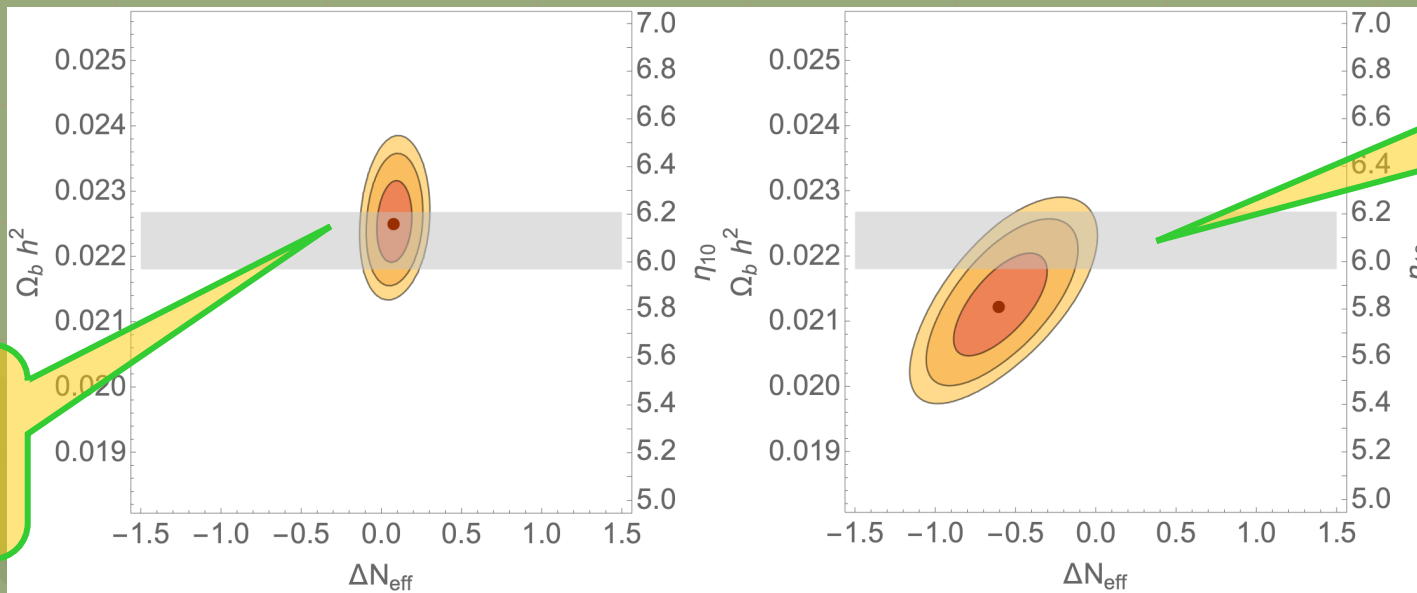
Few examples:

- Non standard neutrino distribution in phase space
- **Neutrino chemical potentials, i.e. neutrino-antineutrino (helicity) asymmetry**
- Non standard lepton interactions
- **Sterile neutrinos, dark radiation**
- **Low reheating at the MeV scale**
- Massive particles in the MeV range or heavier
- Varying coupling constant
- Extra-dimensions
- ...

Non standard scenarios

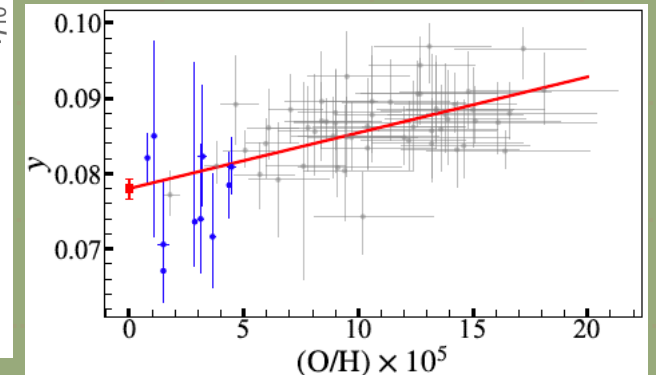
BBN and CMB indirect probes of non-standard cosmological models. In particular, BBN is strongly sensitive to the Hubble parameter. Since at BBN epoch $\rho \simeq \rho_R$ a possible departure from the standard scenario can show up in N_{eff} .

To break the degeneracy the ^4He abundance is employed with two different Y_p astrophysical measures, resulting in compatibility or tension of BBN with the Planck measure of the baryon density (the grey band is the $2\text{-}\sigma$ marginalized region from the Planck analysis with free N_{eff}).



2- σ Planck band for free N_{eff}

Matsumoto et al,
arXiv:2203.09617
(2022)



Non standard scenarios

Degenerate neutrinos?

Until neutrinos are coupled (and after their decoupling, till electron-positron annihilation) they are described by an equilibrium FD distribution, which depends on their chemical potential, μ_ν .

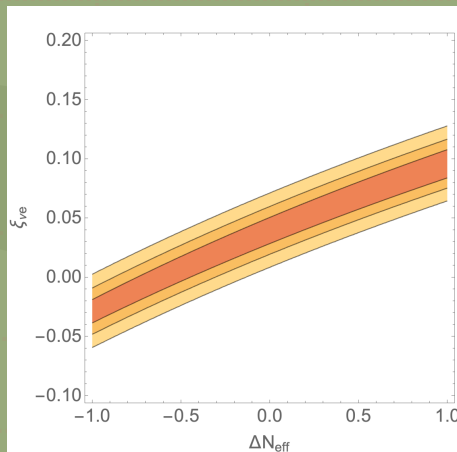
$$f_{eq}(p, T) = \frac{1}{e^{\frac{p \cdot \mu_\nu}{T}} + 1}$$

degeneracy parameter, invariant under cosmic expansion

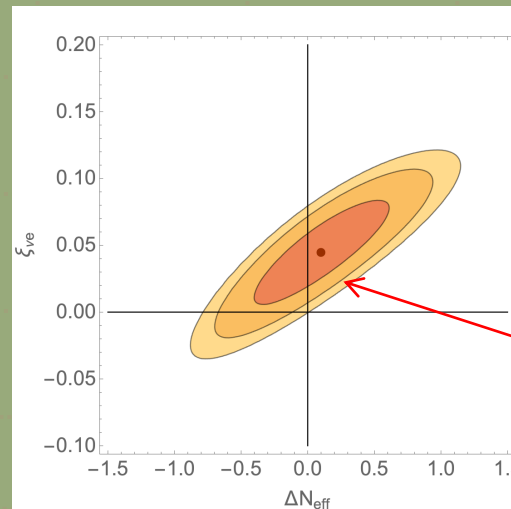
$$\xi_i \equiv \frac{\mu_{\nu i}}{T}$$

Chemical potentials contribute in increasing the energy density, so increasing the effective number of neutrinos. All flavours contribute to N_{eff} , giving a faster expansion \rightarrow more ${}^4\text{He}$; only $\xi_{\nu e}$ contribute to weak rates (a positive value \rightarrow more neutrinos \rightarrow less neutrons \rightarrow less ${}^4\text{He}$): degeneracy in the $\xi_{\nu e} - \Delta N_{eff}$ plane.

$$\Delta N_{eff}^{(\xi)} = \sum_i \left(\frac{30\xi_i^2}{7\pi^2} + \frac{15\xi_i^4}{7\pi^4} \right)$$



Planck prior



$$\xi_{\nu e} = 0.046 \pm 0.025$$

$$N_{eff} = 3.14 \pm 0.33$$

1- σ

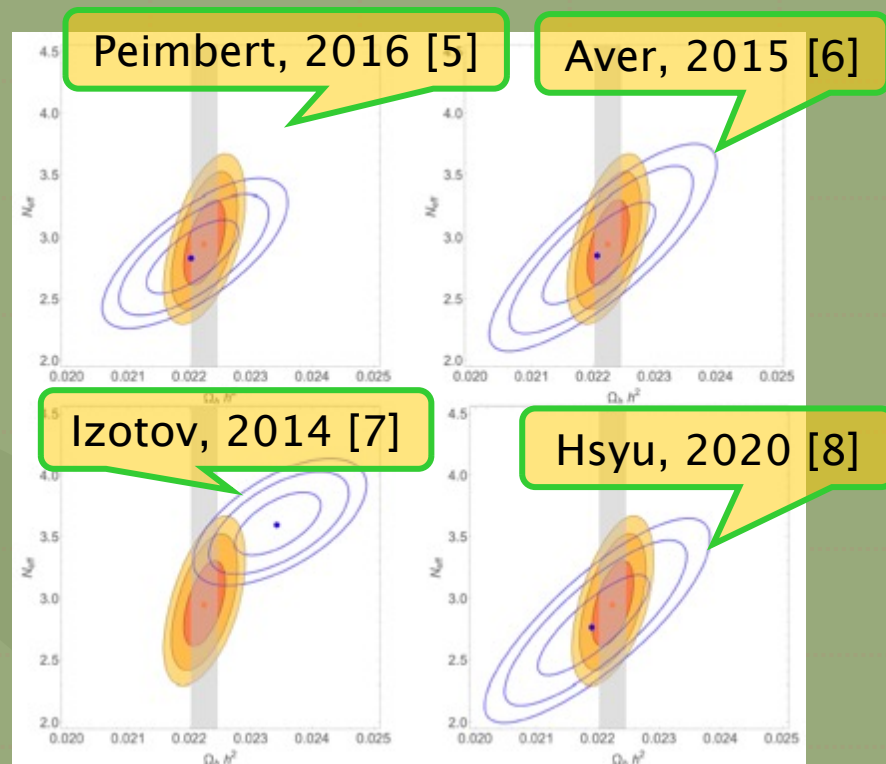
tension with standard scenario using Matsumoto et al Y_p determination

Non standard scenarios

Dark radiation

BBN and CMB indirect probes of non-standard cosmological models. In particular, BBN is strongly sensitive to the Hubble parameter. Since at BBN epoch $\rho \approx \rho_R$ a possible departure from the standard scenario can show up in N_{eff} .

To break the degeneracy an abundance orthogonal to D (^4He , blue contours) or an independent constraining information (CMB, orange contours).



- Different Y_p estimates result in compatibility or tension of BBN with the Planck measure of the baryon density and amount of radiation -> systematics in the astrophysical measurement of Y_p can play a major role.

	ω_b	N_{eff}
Planck	0.02237 ± 0.00015	3.045
Planck+BAO	0.02242 ± 0.00014	3.045
D-3ν	0.02233 ± 0.00036	3.045
D+Planck	0.02224 ± 0.00022	2.95 ± 0.22
BBN [5]	0.0220 ± 0.0005	2.84 ± 0.20
BBN [6]	0.0221 ± 0.0006	2.86 ± 0.28
BBN [7]	0.0234 ± 0.0005	3.60 ± 0.17
BBN [8]	0.0219 ± 0.0006	2.78 ± 0.28

Non standard scenarios

Sterile neutrinos

Hints for sterile neutrino states from
long(short) standing anomalies

LSND, MiniBoone

Reactor anomaly

Gallium anomaly

$$m_\nu \approx \text{eV}, \quad \sin^2 \theta_{as} \approx 10^{-2}$$

With standard assumptions too many sterile neutrinos in the early universe, produced via oscillations, i.e. a larger N_{eff} if oscillations are effective before neutrino decoupling, and distortion of standard neutrino (ν_e) distribution in phase space

Non standard scenarios

The standard case, after Planck 2013

$$N_{\text{eff}} < 3.30 \pm 0.27$$

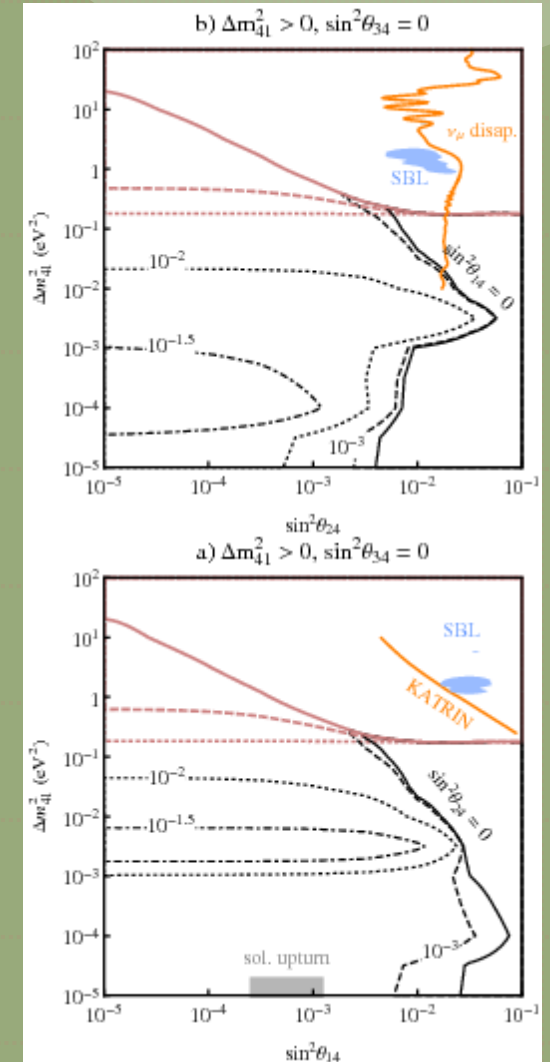
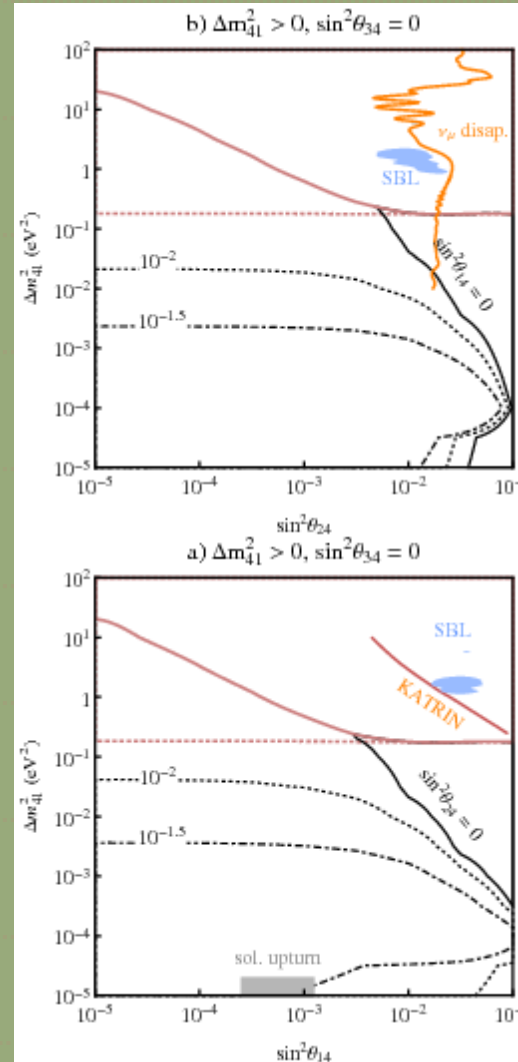
$$m_s < 0.38 \text{ eV}$$

New Planck analysis
even stronger!

(Planck XIII 2015-2018)

$$N_{\text{eff}} = 3.04 \pm 0.22$$

$$m_s < 0.38 \text{ eV}$$



Non standard scenarios

Lepton asymmetry suppresses sterile production (or might enhance it through a MSW resonance) via a matter potential term

$$H_\nu = \sqrt{2} G_F \eta_\nu$$

This renders the equation of motion non linear

Usual approximation: mean momentum $\langle p \rangle = 3.15 T$ and

1+1 neutrinos

Unsatisfactory, for several reasons:

- Oscillation is a mode dependent effect, and thus sterile production can start at different times and results into a different yield
- Oscillations may deform electron neutrino spectrum, and this in turn can change BBN prediction
- In 1+1 scenarios no "repopulation" and interplay of the active neutrinos via standard mixing



Non standard scenarios

Evolution of the neutrino density matrix

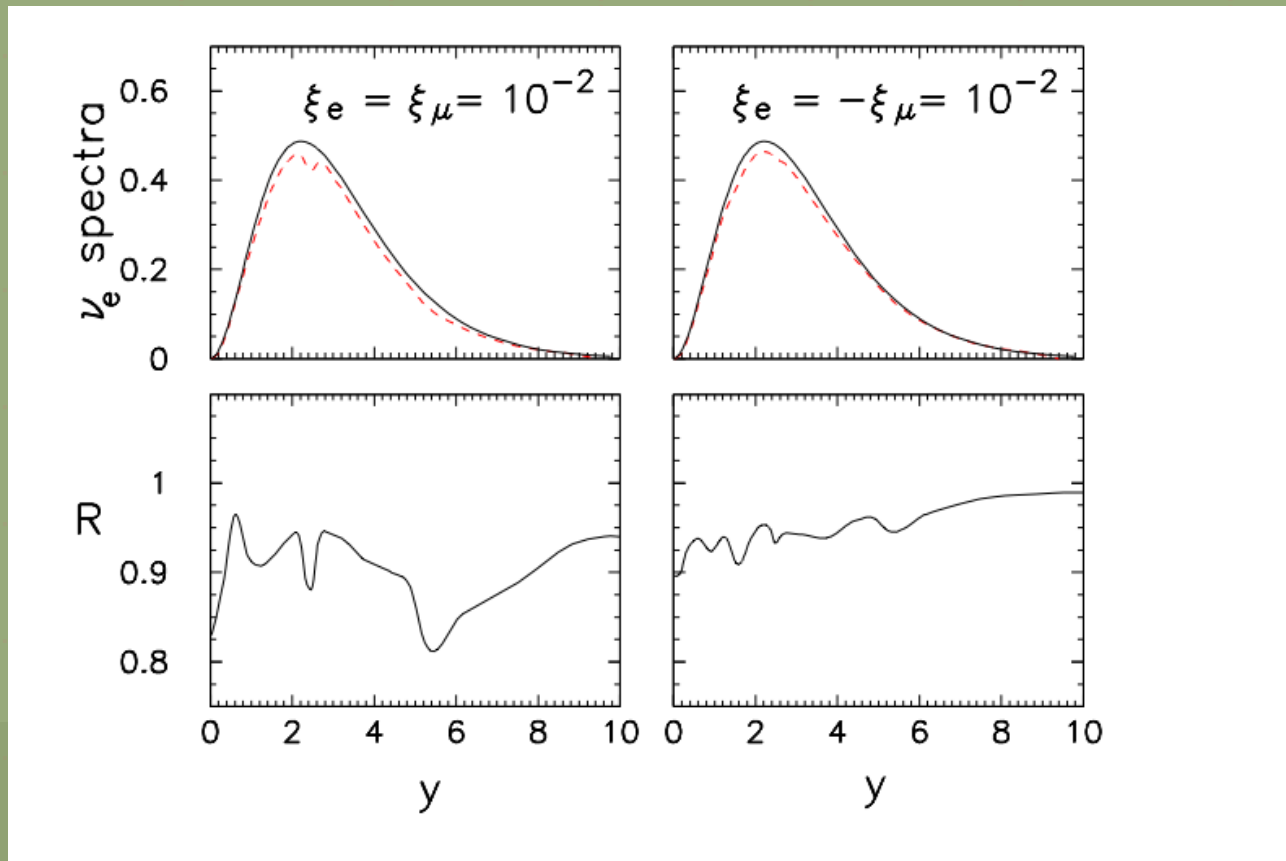
$$\rho(x, y) = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} & \rho_{es} \\ \rho_{\mu e} & \rho_{\mu\mu} & \rho_{\mu s} \\ \rho_{se} & \rho_{s\mu} & \rho_{ss} \end{pmatrix}$$

$$\begin{aligned} i \frac{d\rho}{dx} &= + \frac{x^2}{2m^2 y \overline{H}} [M^2, \rho] + \frac{\sqrt{2}G_F m^2}{x^2 \overline{H}} \left[\left(-\frac{8 y m^2}{3 x^2 m_W^2} E_\ell - \frac{8 y m^2}{3 x^2 m_Z^2} E_\nu + N_\nu \right), \rho \right] \\ &+ \frac{x \widehat{C}[\rho]}{m \overline{H}}, \\ i \frac{d\bar{\rho}}{dx} &= - \frac{x^2}{2m^2 y \overline{H}} [M^2, \bar{\rho}] + \frac{\sqrt{2}G_F m^2}{x^2 \overline{H}} \left[\left(+\frac{8 y m^2}{3 x^2 m_W^2} E_\ell + \frac{8 y m^2}{3 x^2 m_Z^2} E_\nu + N_\nu \right), \bar{\rho} \right] \\ &+ \frac{x \widehat{C}[\bar{\rho}]}{m \overline{H}}, \end{aligned}$$

x=m a

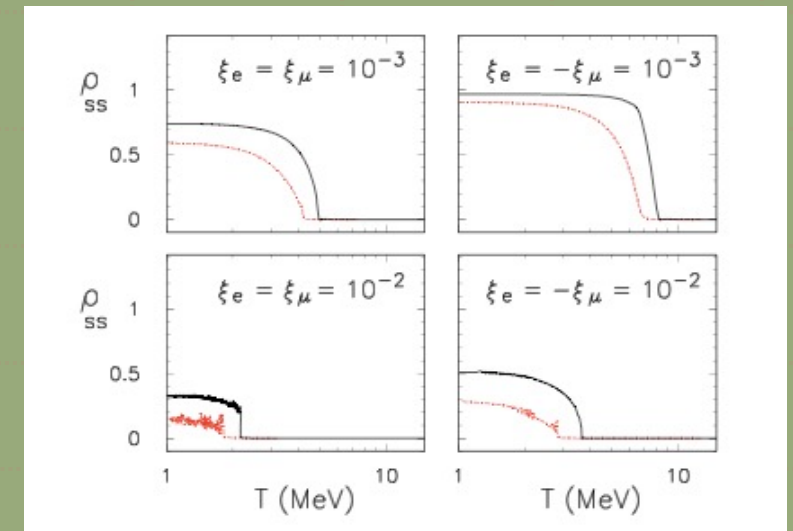
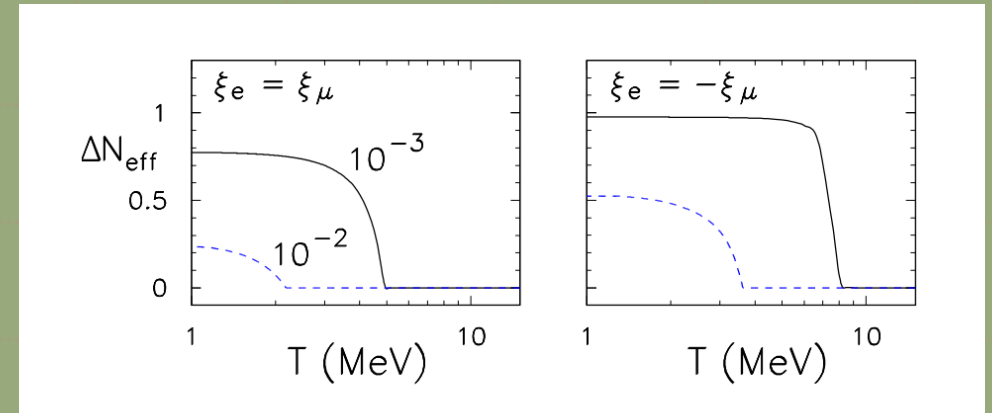
Non standard scenarios

N.Saviano et al, 2013



$y=p/a$

multi-momentum
average momentum



Non standard scenarios

Low reheating scenarios: universe energy density is dominated by a scalar field decaying into standard particles in the MeV energy range (E is the $e^+ - e^-$ energy density)

$$\frac{d\rho_\phi}{dt} = -\Gamma_\phi \rho_\phi - 3H\rho_\phi$$

$$\frac{d\varrho_{\mathbf{p}}}{dt} = -i[\Omega_{\mathbf{p}}, \varrho_{\mathbf{p}}] + C(\varrho_{\mathbf{p}})$$

$$\Omega_{\mathbf{p}} = \frac{M^2}{2p} - \frac{8\sqrt{2}G_{\text{FP}}}{3m_{\text{W}}^2} E.$$

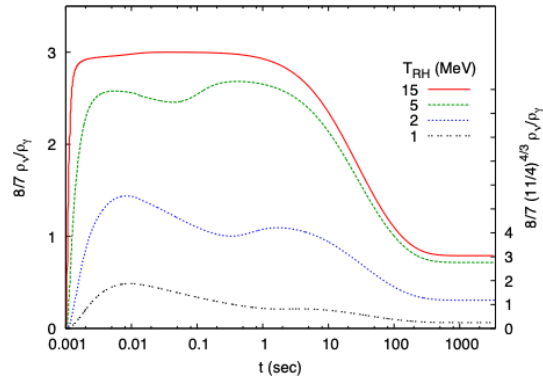


FIG. 1: Time evolution of the ratio of energy densities of neutrinos and photons, normalized in such a way that it corresponds to N_{eff} before (left) and after (right) e^\pm annihilations. Four cases with different values of the reheating temperature are shown.

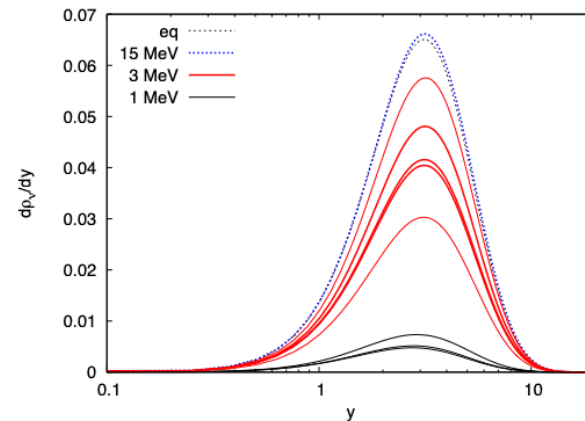


FIG. 2: Final differential spectra of neutrino energies as a function of the comoving momentum for three values of the reheating temperature, compared to an equilibrium spectrum (thin dotted black line). The three thick solid lines for $T_{\text{RH}} = 3$ (middle red lines) and 1 MeV (lower black lines) correspond, from larger to smaller values, to ν_e, ν_μ and ν_τ , respectively. For $T_{\text{RH}} = 3$ MeV we also include the case without flavor oscillations (thin red lines, upper for ν_e and lower for $\nu_{\mu,\tau}$).

Non standard scenarios

Depending on the reheating temperature (roughly the time of decay of the scalar field) there is a distortion of neutrino distribution and their abundance

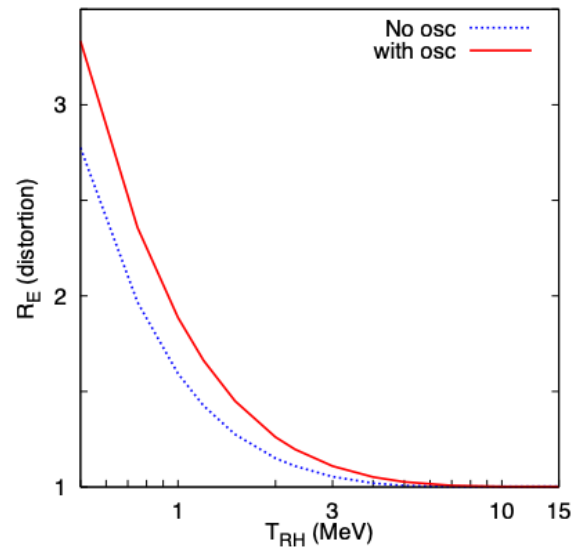


FIG. 3: Distortion of the electron neutrino spectrum parameterized with R_E (defined in the text) as a function of the reheating temperature. A value $R_E > 1$ indicates a significant spectral distortion with respect to equilibrium. Solid curve is for oscillating neutrinos, while dotted is for the no oscillation case and is reported for comparison.

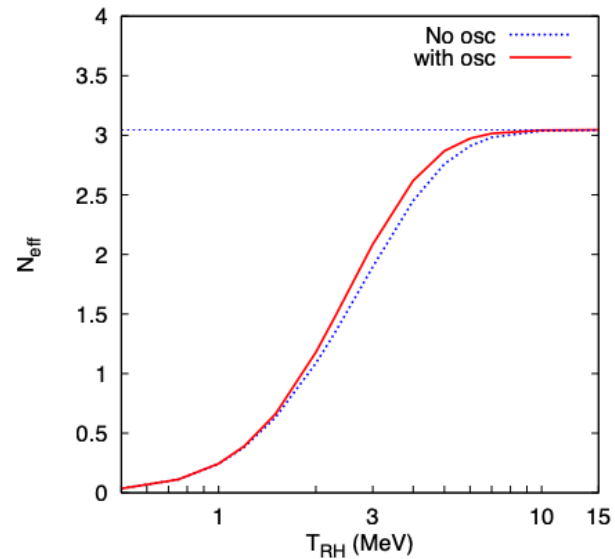


FIG. 4: Final contribution of neutrinos to the radiation energy density in terms of N_{eff} , as a function of the reheating temperature. The horizontal line indicates the standard value, $N_{\text{eff}} = 3.046$.

Non standard scenarios

...which leads to potentially large changes in both ^4He and deuterium abundances

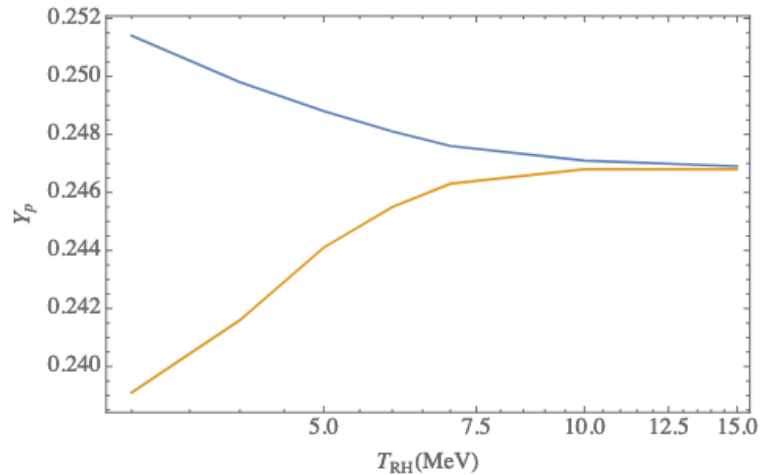


FIG. 6: Values of the primordial helium yield, Y_p , for different values of T_{RH} , taking into account neutrino oscillations (upper blue line) and in absence of the oscillations (lower yellow line).

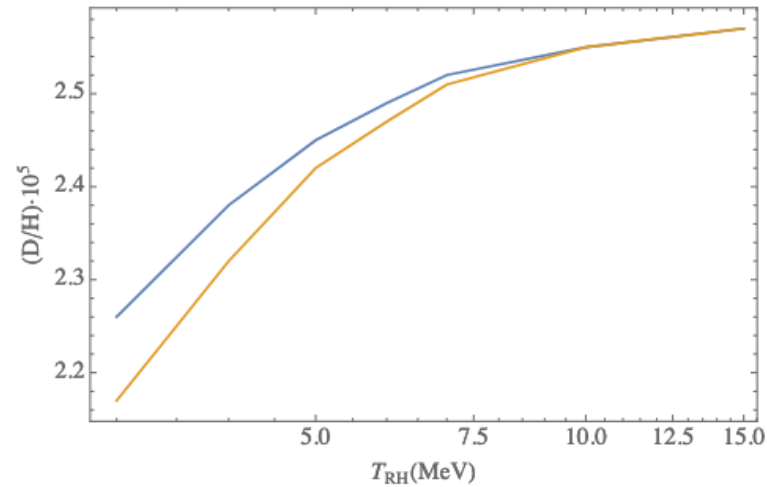


FIG. 7: Values of the deuterium to hydrogen ratio D/H , as a function of T_{RH} , with and without neutrino oscillations (upper blue and lower yellow lines, respectively).

Few conclusions

- BBN, alone or combined with other cosmological probes (CMB, LSS,...) can constrain exotic physics beyond the Standard Model
- Presently, up to some claims of a 2 sigma level tension, the standard picture is consistent
- New astrophysical precise data are expected in the next years or so, maybe urging theorists to further improve the precision of the BBN prediction as well as nuclear rate determinations