XX Frascati Summer School "Bruno Touschek"
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QCD and Jets at Colliders

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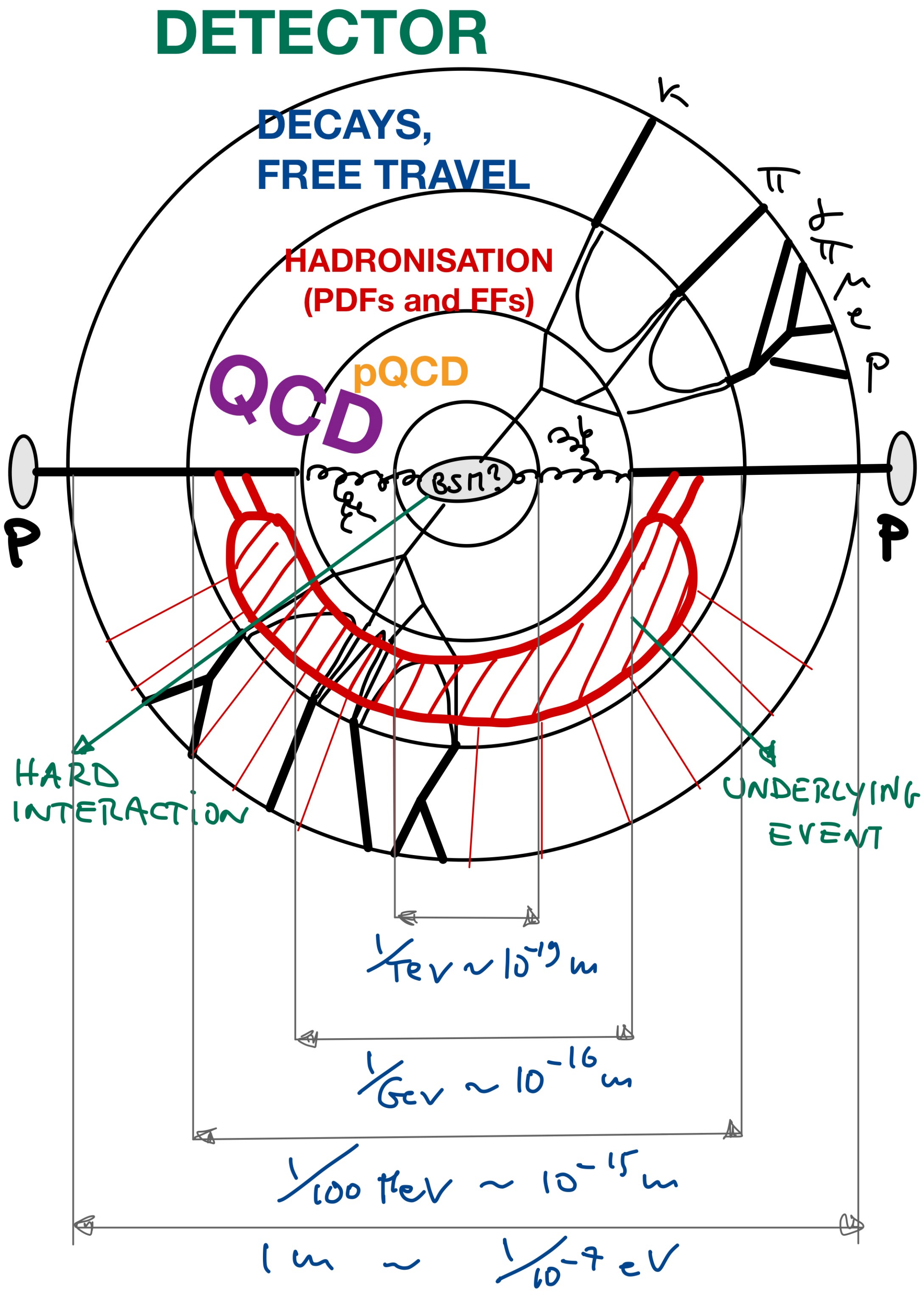
Lecture 1: QCD and perturbative calculations and tools

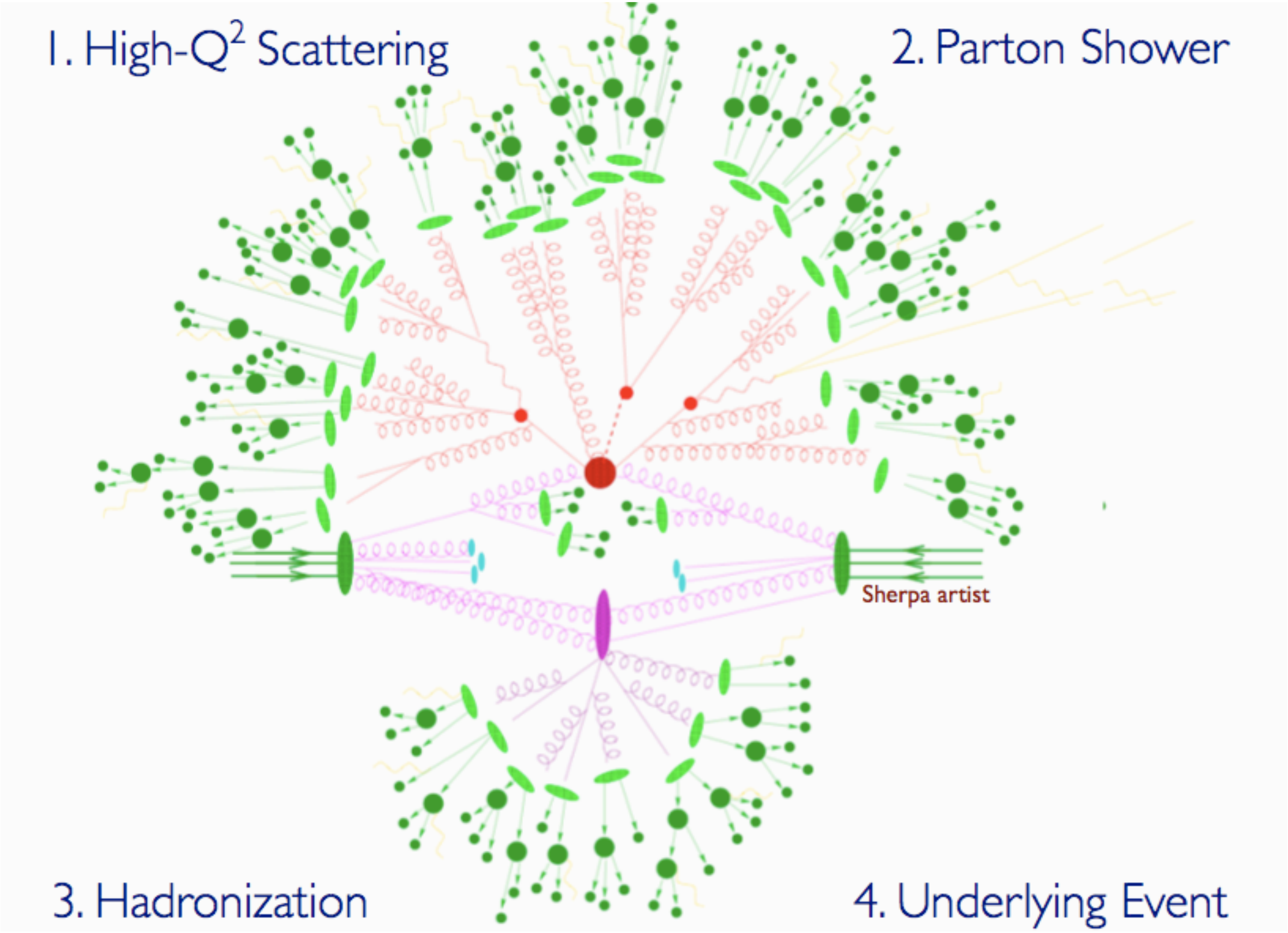
Lecture 2: Jet algorithms and substructure

A hadron collider event

[NB. NOT to scale!]



Strong interactions are complicated



Predictions

"We are driven to the conclusion that the Hamiltonian method for strong interactions is dead and must be buried, although of course with deserved honor"

Lev Landau

"The correct theory [of strong interactions] will not be found in the next hundred years"

Freeman Dyson

We have come a long way towards disproving these predictions

Books and "classics"...

- T. Muta, *Foundations of Quantum Chromodynamics*, World Scientific (1987)
- R.D. Field, *Applications of perturbative QCD*, Addison Wesley (1989)
  Great for specific examples of detailed calculations
- R.K. Ellis, W.J. Stirling and B.R. Webber, *QCD and Collider Physics*, Cambridge University Press (1996)
  Phenomenology-oriented
- G. Sterman, *An Introduction to Quantum Field Theory*, Cambridge University Press (1993)
  A QFT book, but applications tilted towards QCD
- Dokshitzer, Khoze, Muller, Troyan, *Basics of perturbative QCD*, http://www.lpthe.jussieu.fr/~yuri
  For the brave ones
- Dissertori, Knowles, Schmelling, *Quantum Chromodynamics: High Energy Experiments and Theory*, Oxford Science Publications
- Campbell, Huston, Krauss, *The Black Book of Quantum Chromodynamics*, Oxford University Press
  Perhaps the most recent QCD book
- M.L. Mangano, *Introduction to QCD*, http://doc.cern.ch//archive/cernrep//1999/99-04/p53.pdf
- S. Catani, *Introduction to QCD*, CERN Summer School Lectures 1999

...and more recent lectures, slides and...videos

- Gavin Salam,
  - "Elements of QCD for Hadron Colliders", http://arxiv.org/abs/arXiv:1011.5131
  - http://gsalam.web.cern.ch/gsalam/teaching/PhD-courses.html

- Peter Skands
  - 2015 CERN-Fermilab School lectures, http://skands.physics.monash.edu/slides/
  - "Introduction to QCD", http://arxiv.org/abs/arXiv:1207.2389

- Fabio Maltoni
  - "QCD and collider physics", GGI lectures,
    https://www.youtube.com/playlist?list=PL1CFLtxeIrQqvt-e8C5pwBKG4PljSyouP

- Search YouTube for "GGI Thaler", "GGI Soyez", "GGI Catani" "GGI Peskin"

- Search You Tube/web for "ICTP particle physics summer school"

QED v. QCD

QED has a wonderfully simple lagrangian, determined by local gauge invariance

$$\mathcal{L} = \bar\psi(i\partial\!\!\!/ - m)\psi - e\bar\psi A\!\!\!/\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

In the same spirit, we build **QCD**:

a non abelian local gauge theory, based on SU(3)$_{\text{colour}}$,

with 3 quarks (for each flavour) in the fundamental representation of the group and 8 gluons in the adjoint

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + \sum_f \bar\psi_i^{(f)}(i\partial\!\!\!/ - m_f)\psi_i^{(f)} - \bar\psi_i^{(f)}(g_s t^a_{ij} A\!\!\!/_a)\psi_j^{(f)}$$

Gauge Fields and their interact.

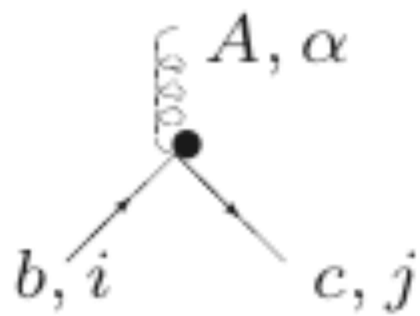
Matter

Interaction

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

1. Colour

quark-gluon interaction



$$-ig\,(t^A)_{cb}\,(\gamma^\alpha)_{ji}$$

colour matrix (generator of $SU(3)_{colour}$)

$$t^A{}_{cb}$$

Index of the **adjoint** representation

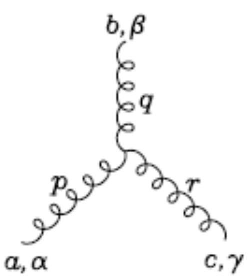
Indices of the **fundamental** representation

2. Gauge bosons self couplings

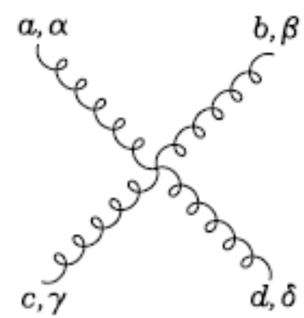
In QCD the gluons interact among themselves:

$$\mathcal{L}_{YM} = -\frac{1}{4}\sum_a F^a_{\mu\nu}F^{a\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$



$$= g f^{abc}\left[g^{\alpha\beta}(p-q)^\gamma + g^{\beta\gamma}(q-r)^\alpha + g^{\gamma\alpha}(r-p)^\beta\right]$$



$$\begin{aligned} = &-ig^2 f^{xac} f^{xbd}\left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) \\ &-ig^2 f^{xad} f^{xbc}\left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta}\right) \\ &-ig^2 f^{xab} f^{xcd}\left(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) \end{aligned}$$

New Feynman diagrams, in addition to the 'standard' QED-like ones

Direct consequence of non-abelianity of theory

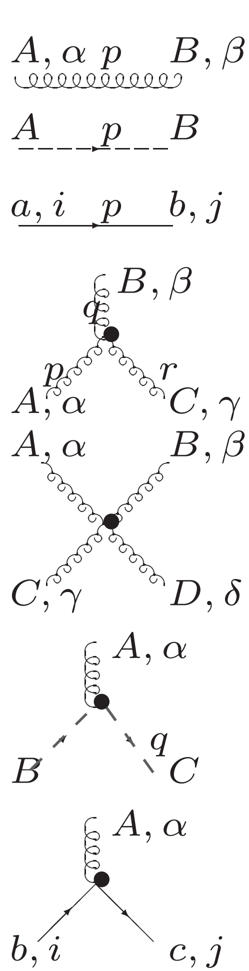
3. Need for ghosts

Cancel unphysical degrees of freedom that would otherwise propagate in covariant gauges

ghost propagator

gluon-ghost vertex

gauge parameter



gluon propagator	$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^\alpha p^\beta}{p^2+i\varepsilon}\right]\frac{i}{p^2+i\varepsilon}$
ghost propagator	$\delta^{AB}\frac{i}{p^2+i\varepsilon}$
quark propagator	$\delta^{ab}\frac{i}{(\not p-m+i\varepsilon)_{ji}}$
three-gluon vertex	$-gf^{ABC}\left[g^{\alpha\beta}(p-q)^\gamma + g^{\beta\gamma}(q-r)^\alpha + g^{\gamma\alpha}(r-p)^\beta\right]$ (all momenta incoming)
four-gluon vertex	$-ig^2 f^{XAC}f^{XBD}(g_{\alpha\beta}g_{\gamma\delta}-g_{\alpha\delta}g_{\beta\gamma})$ $-ig^2 f^{XAD}f^{XBC}(g_{\alpha\beta}g_{\gamma\delta}-g_{\alpha\gamma}g_{\beta\delta})$ $-ig^2 f^{XAB}f^{XCD}(g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma})$
gluon-ghost vertex	$gf^{ABC}q^\alpha$
quark-gluon vertex	$-ig\left(t^A\right)_{cb}\left(\gamma^\alpha\right)_{ji}$

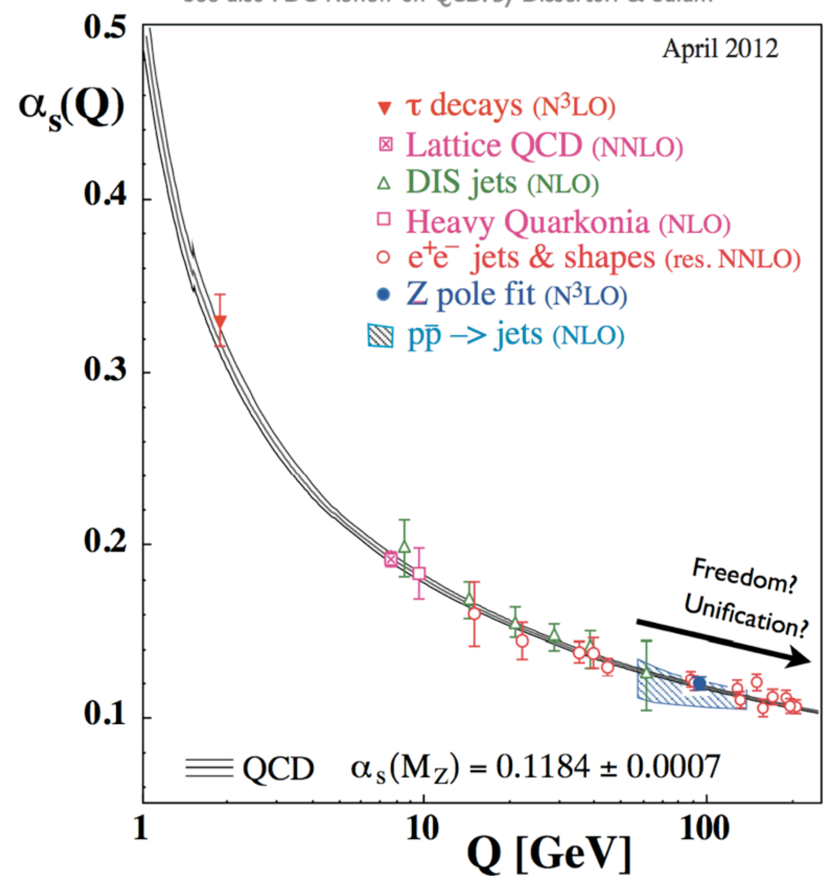
Table 1: Feynman rules for QCD in a covariant gauge.

Macroscopic differences

1. Confinement (probably -- no proof in QCD)

We never observe the fundamental degrees of freedom (quarks and gluons). They are always confined into hadrons.

From S. Bethke, Nucl.Phys.Proc.Suppl. 234 (2013) 229
See also PDG Review on QCD. by Dissertori & Salam

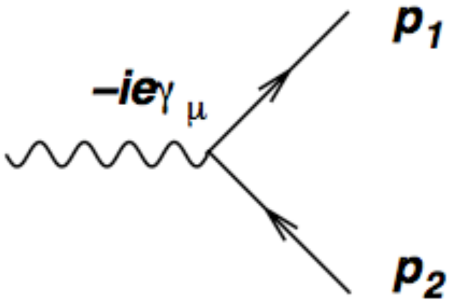


2. Asymptotic Freedom

The running coupling of the theory, α_s, **decreases** at large energies

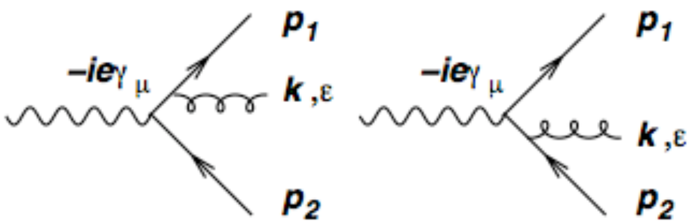
Start with $\gamma^* \to q\bar{q}$:

$$\mathcal{M}_{q\bar{q}} = -\bar{u}(p_1) i e_q \gamma_\mu v(p_2)$$



Emit a gluon:

$$\mathcal{M}_{q\bar{q}g} = \bar{u}(p_1) i g_s \not{\epsilon} t^A \frac{i}{\not{p}_1 + \not{k}} i e_q \gamma_\mu v(p_2) - \bar{u}(p_1) i e_q \gamma_\mu \frac{i}{\not{p}_2 + \not{k}} i g_s \not{\epsilon} t^A v(p_2)$$



In the **soft** limit , k << $p_{1,2}$

$$\mathcal{M}_{q\bar{q}g} \simeq \bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} - \frac{p_2.\epsilon}{p_2.k} \right)$$

Squared amplitude, including phase space

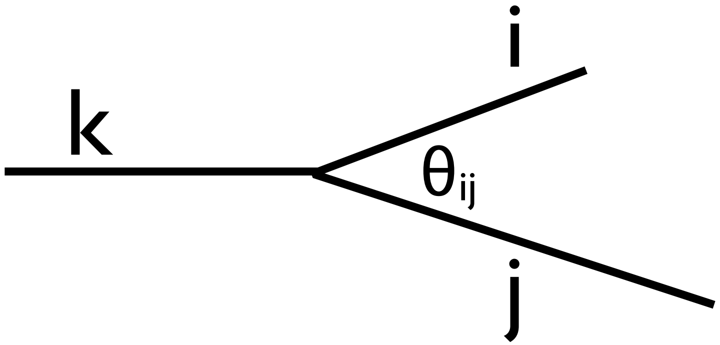
$$d\Phi_{q\bar{q}g}|M^2_{q\bar{q}g}| \simeq (d\Phi_{q\bar{q}}|M^2_{q\bar{q}}|)\,\frac{d^3\vec{k}}{2E(2\pi)^3}C_F g_s^2\frac{2p_1.p_2}{(p_1.k)(p_2.k)}$$

Factorisation: Born × radiation

Changing variables (use energy of gluon E and emission angle θ) we get for the radiation part

$$dS = \frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\sin\theta}\frac{d\phi}{2\pi}$$

QCD emission probability



$$\frac{dP_{k\to ij}}{dE_i d\theta_{ij}} \sim \frac{\alpha_s}{\min(E_i, E_j)\theta_{ij}}$$

Singular in the soft ($E_{i,j} \to 0$) and in the collinear ($\theta_{ij} \to 0$) limits.
Logarithmically divergent upon integration

The divergences can be cured by the addition of virtual corrections and/or **if** the definition of an observable is appropriate

Altarelli-Parisi kernel

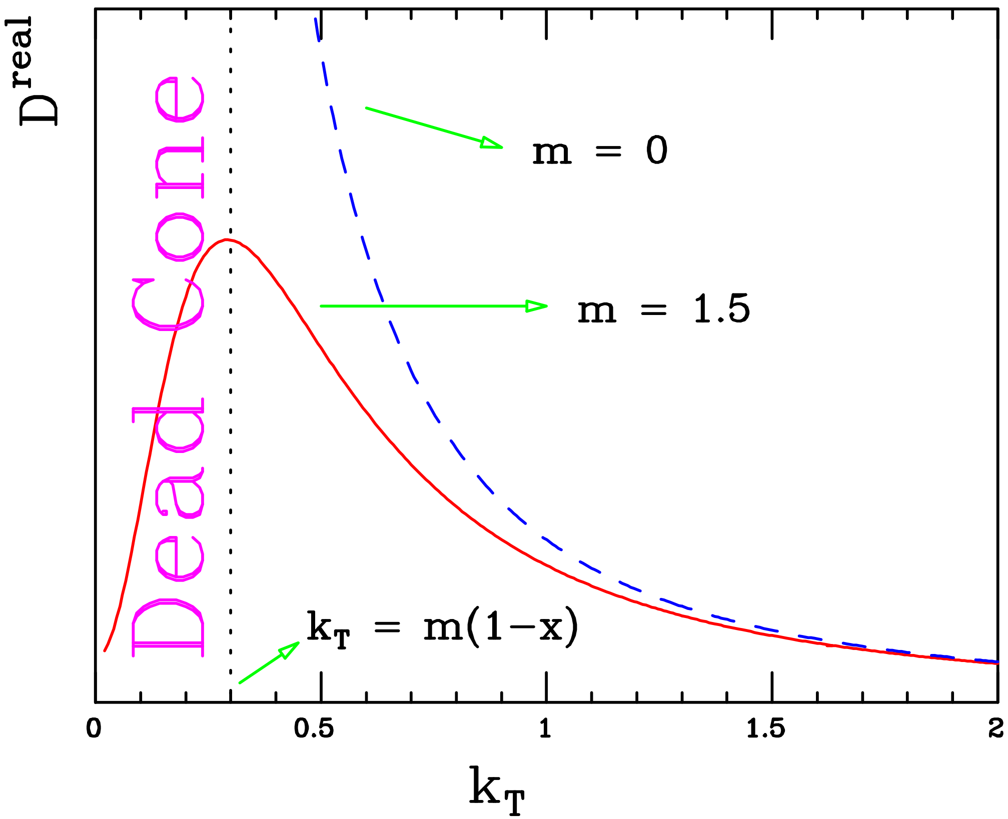
Using the variables $E=(1-z)p$ and $k_t = E\theta$ we can rewrite

$$dS = \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi} \quad \longrightarrow \quad \frac{\alpha_s C_F}{\pi} \frac{1}{1-z} dz \frac{dk_t^2}{k_t^2} \frac{d\phi}{2\pi}$$

'almost' the Altarelli-Parisi splitting function P_{qq}

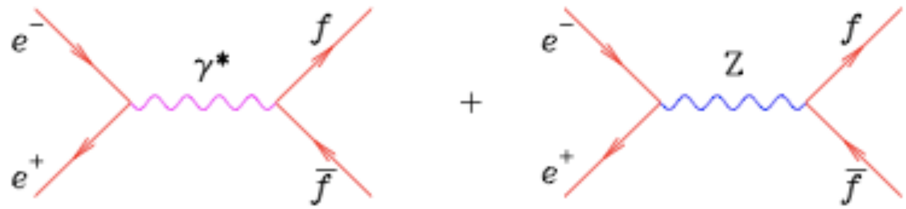
If the quark is massive the collinear singularity is **screened**

$$\frac{\alpha_s C_F}{\pi}\frac{1}{1-z}dz\frac{dk_t^2}{k_t^2}\frac{d\phi}{2\pi} \to \frac{\alpha_s C_F}{\pi}\frac{1}{1-z}dz\frac{dk_t^2}{k_t^2+(1-z)^2m^2}\frac{d\phi}{2\pi}+\cdots$$



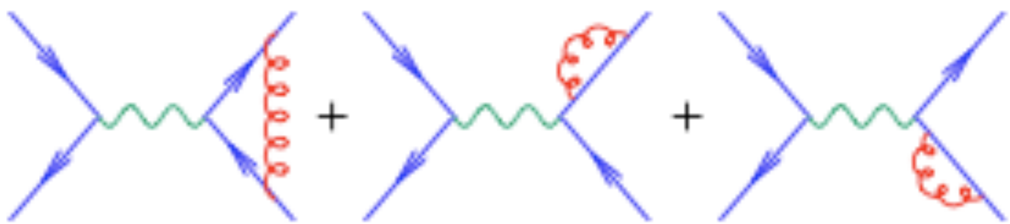
Easiest higher order calculation in QCD. Calculate $e^+e^- \to$ qqbar+X in pQCD

Born

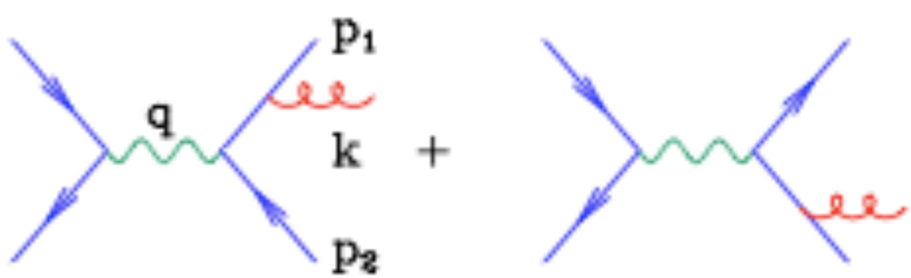


α_s^0

Virtual



Real



α_s^1

$e^+e^- \to$ hadrons

Regularize with dimensional regularization, expand in powers of ε

$$\sigma^{q\bar{q}g} = 2\sigma_0 \frac{\alpha_S}{\pi} H(\epsilon) \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon)\right]$$

Real

$$\sigma^{q\bar{q}} = 3\sigma_0 \left\{1 + \frac{2\alpha_S}{3\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right]\right\}$$

Virtual

$$R = 3 \sum_q Q_q^2 \left\{1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2)\right\}$$

Sum

Real and virtual, separately divergent, 'conspire' to make total cross section finite

Cancellation of singularities

The cancellation of real and virtual (infrared) singularities is not accidental

Block-Nordsieck theorem

IR singularities cancel in sum over soft unobserved photons in final state

(formulated for massive fermions ⇒ no collinear divergences)

Kinoshita-Lee-Nauenberg theorem

IR and collinear divergences cancel in sum over degenerate initial and final states

These theorems suggest that the observable must be crafted in a proper way for the cancellation to take place

A mine of QCD information

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum Q_q^2 \left(1 + \frac{3}{4} C_F \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

Number of colours

Colour factor (Casimir of fundamental representation of SU(3))

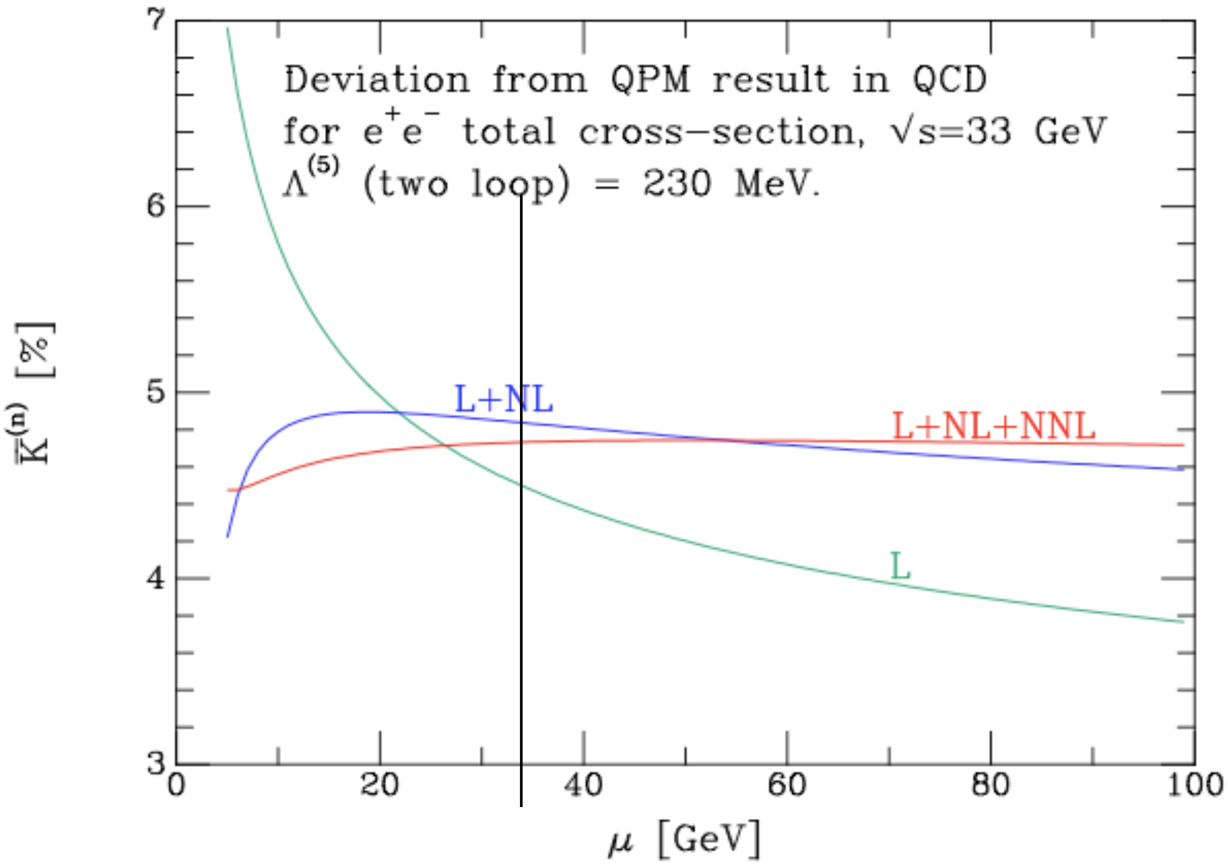
Electric charge of quarks (and their number, as you go through thresholds in energy)

Running of α_S, and its renormalisation group equation

Scale dependence

When renormalisation (or factorisation) becomes necessary, perturbative calculations end up depending on artificial scales

$$K_{QCD} = 1 + \frac{\alpha_S(\mu^2)}{\pi} + \sum_{n\geq 2} C_n\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_S(\mu^2)}{\pi}\right)^n$$



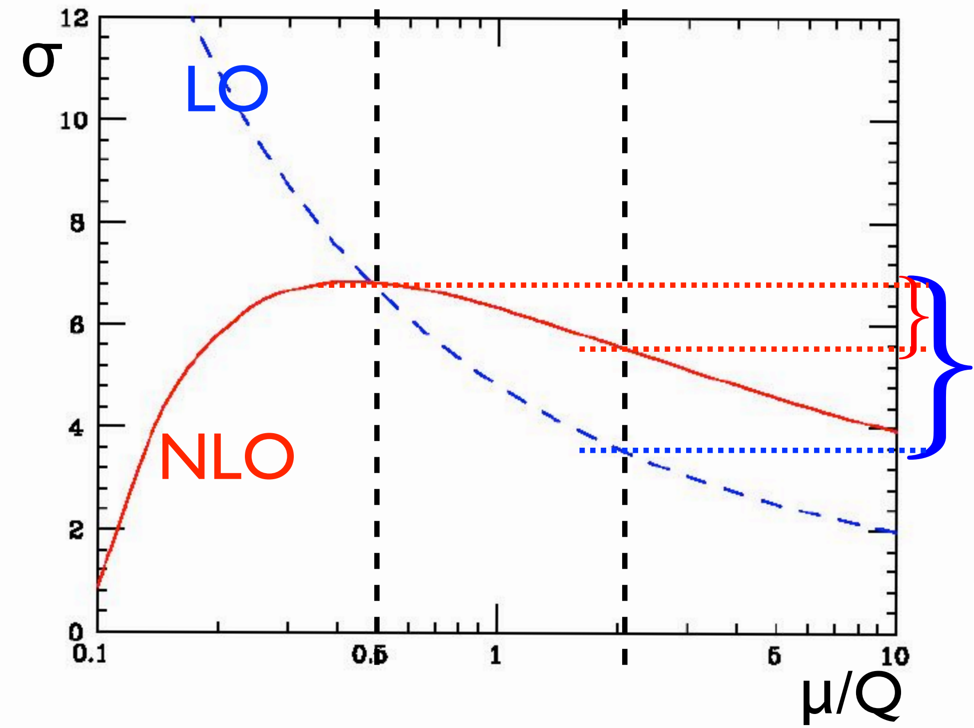
The cross section prediction varies with the renormalisation scale choice.

Which value do we pick for μ?

Theoretical uncertainties

μ cannot be fixed exactly, and only a physical cross section (and not its perturbative approximation) would be completely independent of μ

$$\frac{d}{d\ln\mu^2}\sigma^{phys} = 0$$



In real life: residual dependence at one order higher than the calculation

$$\frac{d}{d\ln\mu^2}\sum_{p=1}^{N} c_p(\mu)\alpha_s^p(\mu) \sim \mathcal{O}(\alpha_s^{N+1})$$

→ Vary scales (around a physical one) to **ESTIMATE** the uncalculated higher orders

Nota bene

In a perturbative calculation $\sum_{p=1}^{N} c_p(\mu)\alpha_s^p(\mu)$ the coefficients $c_p(\mu)$ have the form

$$c_p(\mu) = c_{p0} + \sum_{k=1}^{p-1} c_{pk} \ln^k \mu$$

The coefficients c_{pk}, with $k \geq 1$, can be obtained from the lower order coefficients $c_n(\mu)$ with $n < p$.

E.g. $$R(\alpha_s(\mu^2), \mu^2/s) = R_0 \left[1 + \frac{\alpha_s(\mu^2)}{\pi} + \left(c_{20} + \pi b_0 \ln \frac{\mu^2}{s} \right) \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^2 + \mathcal{O}(\alpha_s^3) \right]$$

The only genuine higher order prediction is c_{p0}. The other coefficients ensure cancellation of the scale dependence at order N

If you get the c_{p0} wrong, but obtain the other coefficients from lower order calculations, you still get a reduction in scale dependence even if your calculation is incorrect

Non-perturbative contributions

We have calculated $\sum_q \sigma(e^+e^- \to q\bar{q})$ in **perturbative** QCD

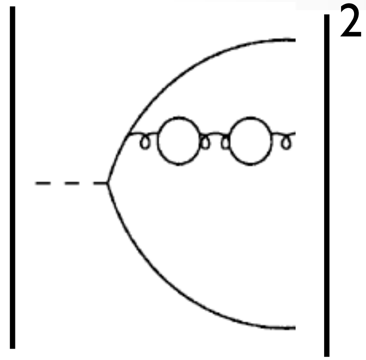
However

$$\sum_q \sigma(e^+e^- \to q\bar{q}) \neq \sigma(e^+e^- \to \mathrm{hadrons})$$

The (small) difference is due to hadronisation corrections, and is of non-perturbative origin

We cannot calculate it in pQCD, but in some cases we can get an idea of its behaviour from the incompleteness of pQCD itself

Suppose we keep calculating to higher and higher orders:



$$\rightarrow \alpha_s^{n+1}\, \beta_{0f}^{n}\, n!$$

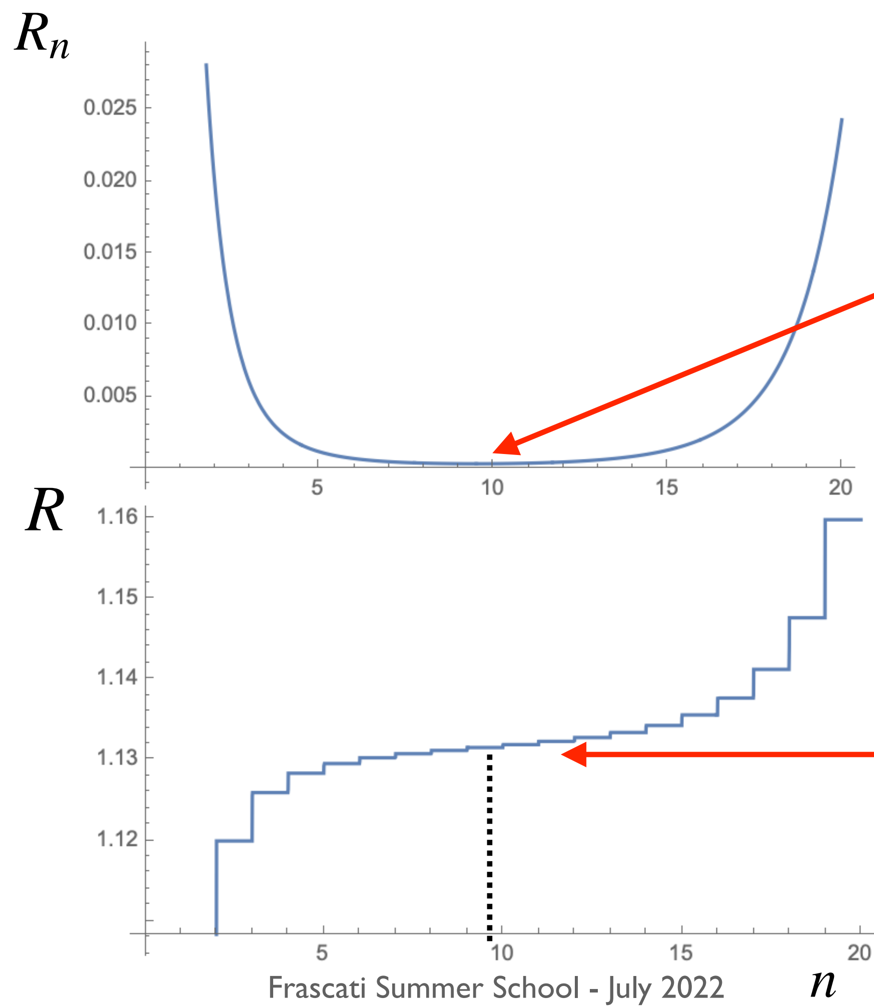
Factorial growth

This is big trouble: the series is not convergent, but only asymptotic

Evidence: try summing

$$R = \sum_{n=0}^{\infty} \alpha^n\, n!$$

$$(\alpha = 0.1)$$



minimal term

$n_{min} \simeq 1/\alpha$

Asymptotic value of the sum:

$$R^{asymp} \equiv \sum_{n=0}^{n_{min}} R_n$$

Power corrections

The renormalons signal the incompleteness of perturbative QCD

One can only define what the sum of a perturbative series is (like truncation at the minimal term)

The rest is a genuine ambiguity, to be eventually lifted by non-perturbative corrections:

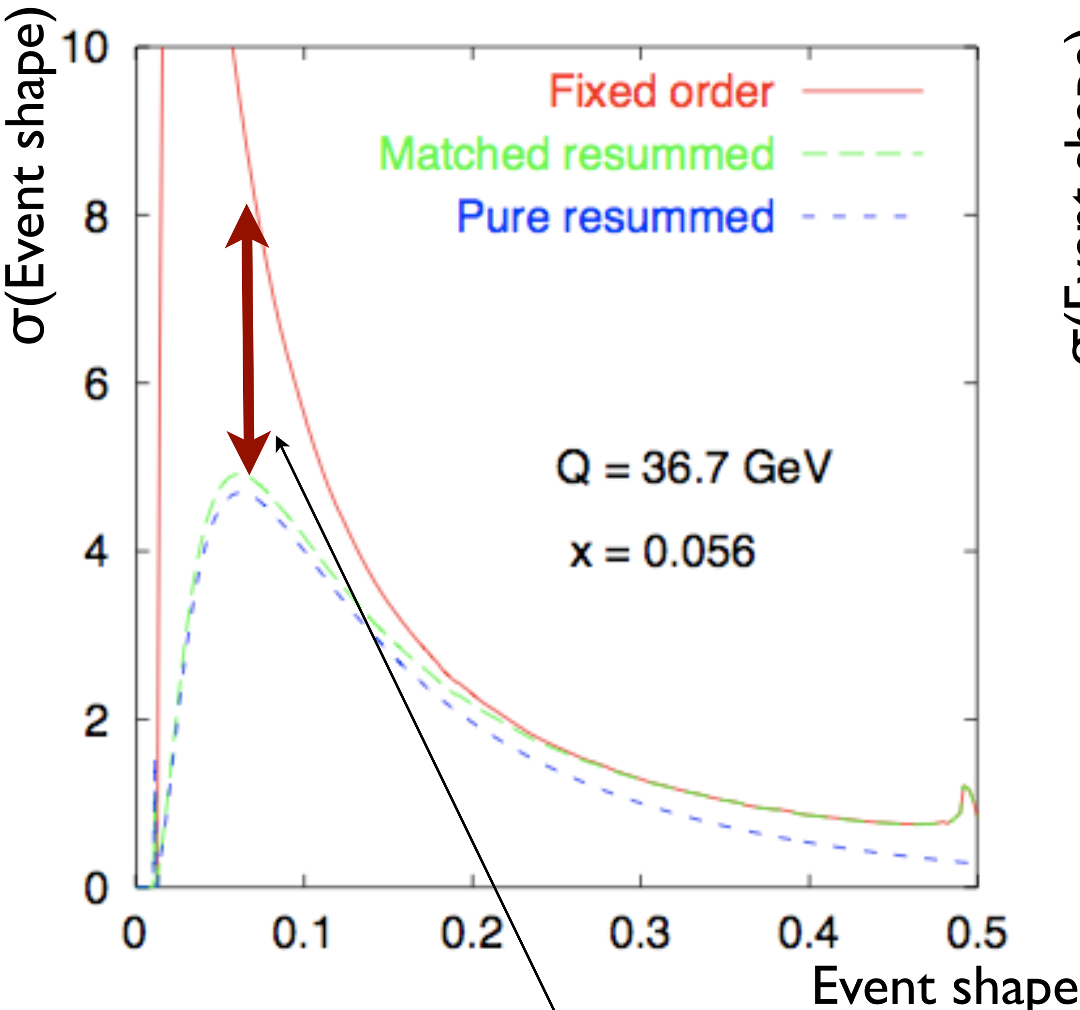
$$R^{true} = R^{pQCD} + R^{NP}$$

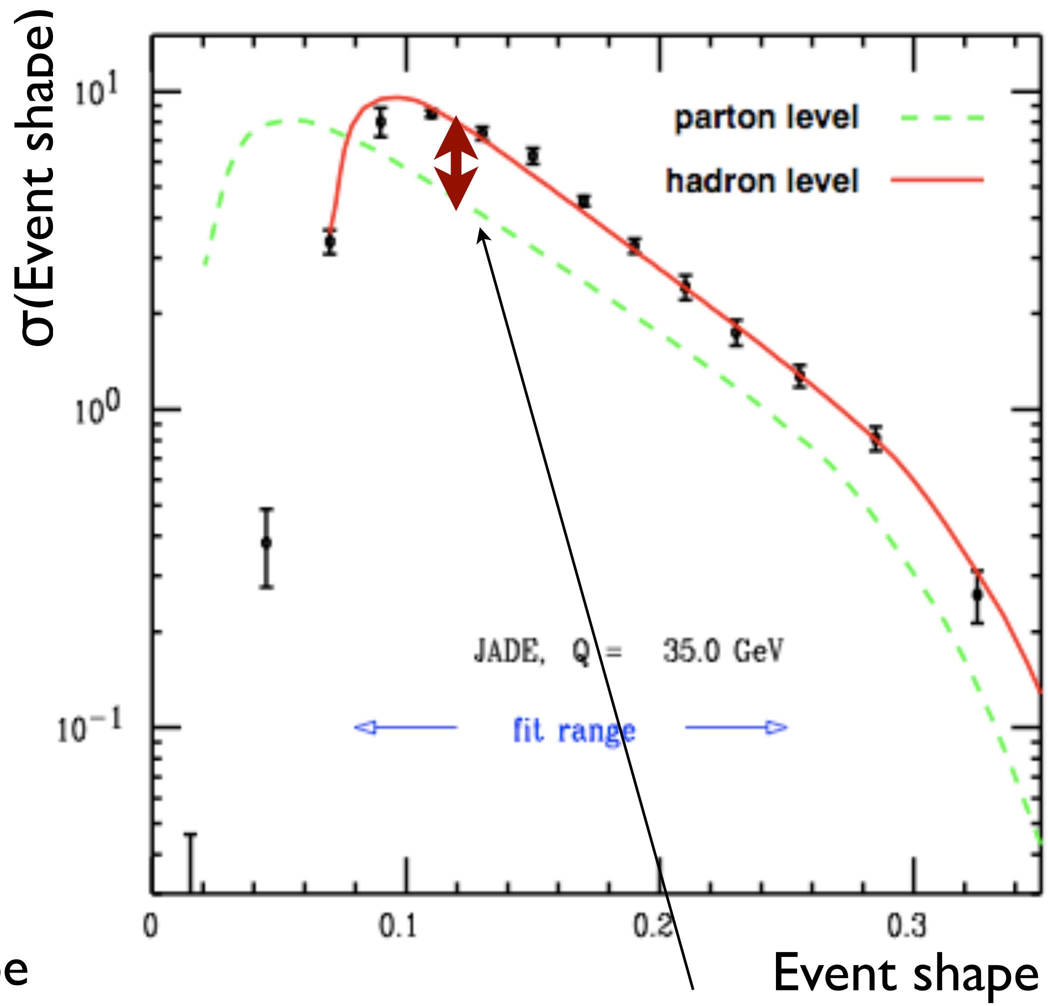
In QCD these non-perturbative corrections take the form of power suppressed terms:

$$R^{NP} \sim \exp\left(-\frac{p}{\beta_0 \alpha_s}\right) = \exp\left(-p \ln\frac{Q^2}{\Lambda^2}\right) = \left(\frac{\Lambda^2}{Q^2}\right)^p$$

The value of p depends on the process, and can sometimes be predicted by studying the perturbative series: pQCD - NP physics bridge

Perturbative (and NP) QCD predictions





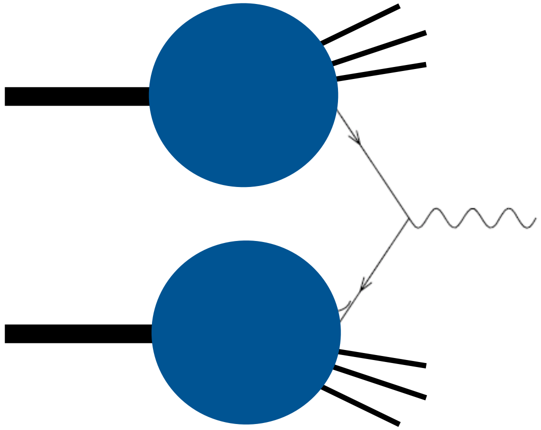
Effect of higher order logarithms (resummation)

Effect of non-perturbative power corrections

Power corrections models (i.e. Monte Carlo hadronisation) can be built and tested on data

pQCD calculations: hadrons

Turn hadron production in e+e- collisions around: Drell-Yan.



Still easy in **Parton Model**: just a convolution of probabilities

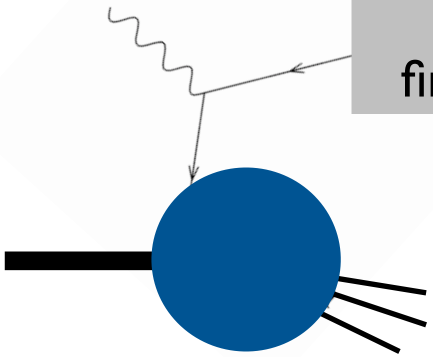
$$\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim \int d\xi_1 d\xi_2 \sum_{a=q\bar{q}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{a\bar{a}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}$$

$$\times(\text{probability to find parton a}(\xi_1)\text{ in }N)$$

$$\times(\text{probability to find parton }\bar{\text{a}}(\xi_2)\text{ in }N)$$

This isn't anymore an **inclusive process** as far as hadrons are concerned:
I find them in the initial state, **I can't 'sum over all of them'**

Still, the picture holds at tree level (parton model)
The parton distribution functions can be roughly equated to those extracted from DIS



Sum over all final state hadrons

Challenges in QCD

The non-inclusiveness of a general strong interaction process is a threat to calculability.

What do we do if we can't count on Bloch-Nordsieck and Kinoshita-Lee-Nauenberg?

QCD calculations adopt two strategies:

- Infrared and collinear safe observables
    - less inclusive but still calculable in pQCD
- Factorisation
    - trade divergencies for universal measurable quantities

A generic (not fully inclusive) observable *O* is **infrared and collinear safe** if

$$O(X; p_1, \ldots, p_n, p_{n+1} \to 0) \to O(X; p_1, \ldots, p_n)$$

$$O(X; p_1, \ldots, p_n \parallel p_{n+1}) \to O(X; p_1, \ldots, p_n + p_{n+1})$$

Infrared and collinear safety demands that, in the limit of a collinear splitting, or the emission of an infinitely soft particle, the observable remain **unchanged**

IRC safety: proof

Cancellation of singularities in **total cross section** (KLN)

$$\sigma_{tot} = \int_n |M_n^B|^2 d\Phi_n + \int_n |M_n^V|^2 d\Phi_n + \int_{n+1} |M_{n+1}^R|^2 d\Phi_{n+1}$$

A generic observable

$$\begin{aligned}\frac{dO}{dX} &= \int_n |M_n^B|^2 O(X;p_1,\ldots,p_n) d\Phi_n \\ &+ \int_n |M_n^V|^2 O(X;p_1,\ldots,p_n) d\Phi_n + \int_{n+1} |M_{n+1}^R|^2 O(X;p_1,\ldots,p_n,p_{n+1}) d\Phi_{n+1}\end{aligned}$$

In order to ensure the same cancellation existing in σ_{tot}, the definition of the observable must not affect the soft/collinear limit of the real emission term, because it is there that the real/virtual cancellation takes place

Example of IRC-safe observable

Thrust $$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}$$

- If a $\mathbf{p}_i \to 0$ (i.e. **soft** limit) it does not contribute to the sum
  $\Rightarrow$ *T* is **unchanged** $\Rightarrow$ OK

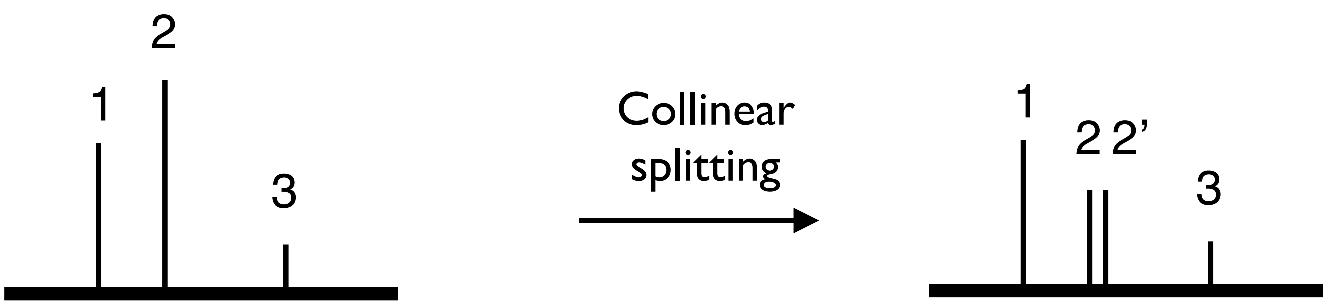
- If a $\mathbf{p}_i \to (1-\lambda)\mathbf{p}_i + \lambda\mathbf{p}_i$ (i.e. **collinear** splitting) then
  - In the numerator
    $|(1-\lambda)\mathbf{p}_i \cdot \mathbf{n}| + |\lambda\mathbf{p}_i \cdot \mathbf{n}| = (1-\lambda)|\mathbf{p}_i \cdot \mathbf{n}| + \lambda|\mathbf{p}_i \cdot \mathbf{n}| = |\mathbf{p}_i \cdot \mathbf{n}|$
  - In the denominator
    $|(1-\lambda)\mathbf{p}_i| + |\lambda\mathbf{p}_i| = (1-\lambda)|\mathbf{p}_i| + \lambda|\mathbf{p}_i| = |\mathbf{p}_i|$

  $\Rightarrow$ *T* is **unchanged** $\Rightarrow$ OK

Examples of IRC-unsafety

- Multiplicity (e.g. of gluons) is **soft unsafe**
  - One can always emit an additional infinitely soft particle, or split collinearly another, while conserving energy and momentum

- The hardest particle in an event is **collinear unsafe**



Hardest is 2 Hardest is 1

Note that IRC safety is a requirement for **perturbative calculability**. We can observe the hardest particle in an event, as we can count the number of pions. But we can't calculate their cross sections perturbatively

Drell-Yan: factorisation

Non fully inclusive process (hadrons in initial state):
non cancellation of collinear singularities in pQCD

Same procedure used for renormalising the coupling:
reabsorb the divergence into bare non-perturbative quantities, the parton probabilities (collinear factorisation)

The factorisation theorem

$$\sigma^{phys} = F^{bare} F^{bare} \sigma^{divergent}(\varepsilon) = F(\mu) F(\mu) \hat{\sigma}(\mu)$$

- infrared regulator → (ε)
- Parton Distribution Function → F
- factorisation scale → μ
- short-distance cross section → $\hat{\sigma}$

and (schematically) $F(\mu) = F^{bare}\left(1 + \alpha_s P \log\frac{\mu^2}{\mu_0^2}\right)$ → This factor universal

Drell-Yan: NLO result

$$\frac{d^2\hat{\sigma}^{(1)}_{q\bar{q}\to\gamma^* g}(z,Q^2,\mu^2)}{dQ^2} = \sigma_0(Q^2)\left(\frac{\alpha_s(\mu)}{\pi}\right)\left\{2(1+z^2)\left[\frac{\ln(1+z^2)}{1-z}\right]_+ - \frac{[(1+z^2)\ln z\,]}{(1-z)} + \left(\frac{\pi^2}{3}-4\right)\delta(1-z)\right\}$$

$$+\,\sigma_0(Q^2)\,C_F\,\frac{\alpha_s}{\pi}\left[\frac{1+z^2}{1-z}\right]_+ \ln\left(\frac{Q^2}{\mu^2}\right)$$

→ soft and collinear large log

→ residual of collinear factorisation

A prototype of QCD calculations: many finite terms but, more importantly, a few characteristic large logarithms

In many circumstances and kinematical situations the logs are much more important than the finite terms: hence in pQCD resummations of these terms are often phenomenologically more relevant than a full higher order calculation

Cascade

Factorisation

$$\sigma^{phys} = F(\mu)\hat{\sigma}(\mu)$$

Evolution

$$F(\mu) = F^{bare}\left(1 + \alpha_s P \log\frac{\mu^2}{\mu_0^2}\right)$$

$$\frac{d}{d\ln\mu^2}\ln\sigma^{phys} = 0 \quad \Rightarrow \quad \frac{d\ln\hat{\sigma}(\mu)}{\ln\mu^2} = -\frac{d\ln F(\mu)}{\ln\mu^2} = -\alpha_s P$$

DGLAP evolution equations for PDF's

Resummation

Solution of evolution equations resums higher order terms

Responsible for scaling violations

(for instance in DIS structure functions)

DGLAP equations

[Dokshitzer, Gribov, Lipatov, Altarelli, Parisi]

$$\frac{df_q(x,t)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{qq}(z)\, f_q(\frac{x}{z},t) + P_{qg}(z) f_g(\frac{x}{z},t)\right]$$

$$\frac{df_g(x,t)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[P_{gq}(z)\sum_{i=q,\bar{q}} f_i\left(\frac{x}{z},t\right) + P_{gg}(z) f_g\left(\frac{x}{z},t\right)\right]$$

The Altarelli-Parisi kernels control the evolution of the Parton Distribution Functions

Altarelli-Parisi kernels

[Altarelli-Parisi, 1977, Dokshitzer, 1977]

$$P_{gg} \to 2C_A \left\{ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right\} + \delta(1-x) \left[\frac{11C_A - 2n_f}{6} \right]$$

$$P_{qq}(z) \to \left(\frac{1+z^2}{1-z} \right)_+ \equiv \frac{1+z^2}{1-z} - \delta(1-z) \int_0^1 dy \left(\frac{1+y^2}{1-y} \right)$$

$$P_{qg} = \frac{1}{2} \left[z^2 + (1-z)^2 \right]$$

$$P_{gq}(z) = \quad C_F \frac{1+(1-z)^2}{z}$$

Higher orders: Curci-Furmansky-Petronzio (1980), Moch, Vermaseren, Vogt (2004)

Altarelli-Parisi kernels: NLO

$$P^{(1)}_{\rm ps}(x) = 4\,C_F n_f\left(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4{\rm H}_0 + x^2\left[\frac{8}{3}{\rm H}_0 - \frac{56}{9}\right] + (1+x)\left[5{\rm H}_0 - 2{\rm H}_{0,0}\right]\right)$$

$$P^{(1)}_{\rm qg}(x) = 4\,C_A n_f\left(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x){\rm H}_{-1,0} - 2p_{\rm qg}(x){\rm H}_{1,1} + x^2\left[\frac{44}{3}{\rm H}_0 - \frac{218}{9}\right]\right.$$
$$\left. + 4(1-x)\left[{\rm H}_{0,0} - 2{\rm H}_0 + x{\rm H}_1\right] - 4\zeta_2 x - 6{\rm H}_{0,0} + 9{\rm H}_0\right) + 4\,C_F n_f\left(2p_{\rm qg}(x)\left[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2\right.\right.$$
$$\left.\left. - \zeta_2\right] + 4x^2\left[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2}\right] + 2(1-x)\left[{\rm H}_0 + {\rm H}_{0,0} - 2x{\rm H}_1 + \frac{29}{4}\right] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2}{\rm H}_0\right)$$

$$P^{(1)}_{\rm gq}(x) = 4\,C_A C_F\left(\frac{1}{x} + 2p_{\rm gq}(x)\left[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6}{\rm H}_1\right] - x^2\left[\frac{8}{3}{\rm H}_0 - \frac{44}{9}\right] + 4\zeta_2 - 2\right.$$
$$\left. - 7{\rm H}_0 + 2{\rm H}_{0,0} - 2{\rm H}_1 x + (1+x)\left[2{\rm H}_{0,0} - 5{\rm H}_0 + \frac{37}{9}\right] - 2p_{\rm gq}(-x){\rm H}_{-1,0}\right) - 4\,C_F n_f\left(\frac{2}{3}x\right.$$
$$\left. - p_{\rm gq}(x)\left[\frac{2}{3}{\rm H}_1 - \frac{10}{9}\right]\right) + 4\,C_F^2\left(p_{\rm gq}(x)\left[3{\rm H}_1 - 2{\rm H}_{1,1}\right] + (1+x)\left[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2}{\rm H}_0\right] - 3{\rm H}_{0,0}\right.$$
$$\left. + 1 - \frac{3}{2}{\rm H}_0 + 2{\rm H}_1 x\right)$$

$$P^{(1)}_{\rm gg}(x) = 4\,C_A n_f\left(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\left(\frac{1}{x} - x^2\right) - \frac{2}{3}(1+x){\rm H}_0 - \frac{2}{3}\delta(1-x)\right) + 4\,C_A^2\left(27\right.$$
$$ + (1+x)\left[\frac{11}{3}{\rm H}_0 + 8{\rm H}_{0,0} - \frac{27}{2}\right] + 2p_{\rm gg}(-x)\left[{\rm H}_{0,0} - 2{\rm H}_{-1,0} - \zeta_2\right] - \frac{67}{9}\left(\frac{1}{x} - x^2\right) - 12{\rm H}_0$$
$$\left. - \frac{44}{3}x^2{\rm H}_0 + 2p_{\rm gg}(x)\left[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2{\rm H}_{1,0} + 2{\rm H}_2\right] + \delta(1-x)\left[\frac{8}{3} + 3\zeta_3\right]\right) + 4\,C_F n_f\left(2{\rm H}_0\right.$$
$$\left. + \frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\left[4 - 5{\rm H}_0 - 2{\rm H}_{0,0}\right] - \frac{1}{2}\delta(1-x)\right)\,.$$

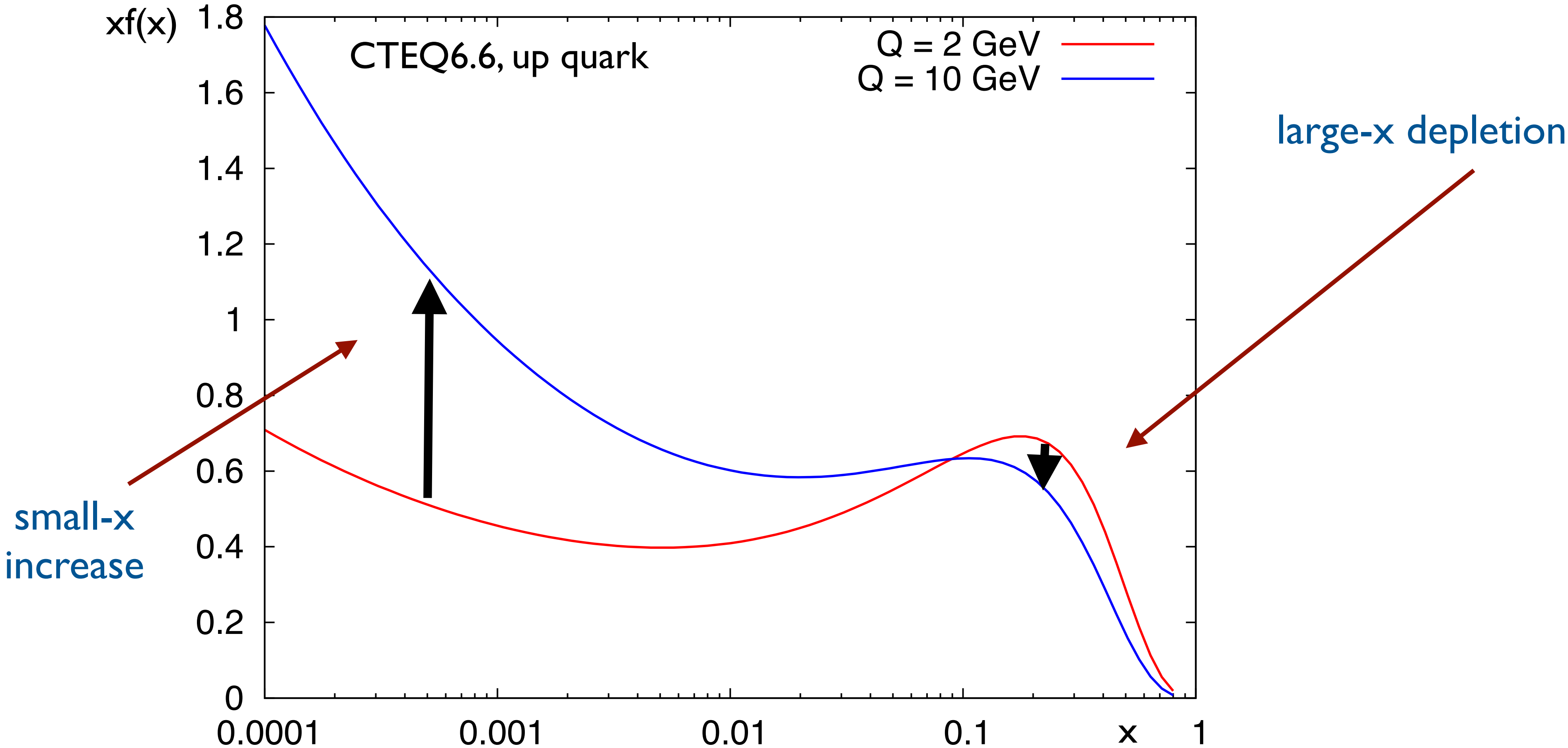
$$P_{ab} = \frac{\alpha_s}{2\pi}P^{(0)} + \frac{\alpha_s^2}{16\pi^2}P^{(1)}$$

Curci, Furmanski & Petronzio '80

Altarelli-Parisi kernels: NNLO

NNLO, $P_{ab}^{(2)}$: Moch, Vermaseren & Vogt '04

DGLAP evolution of PDFs



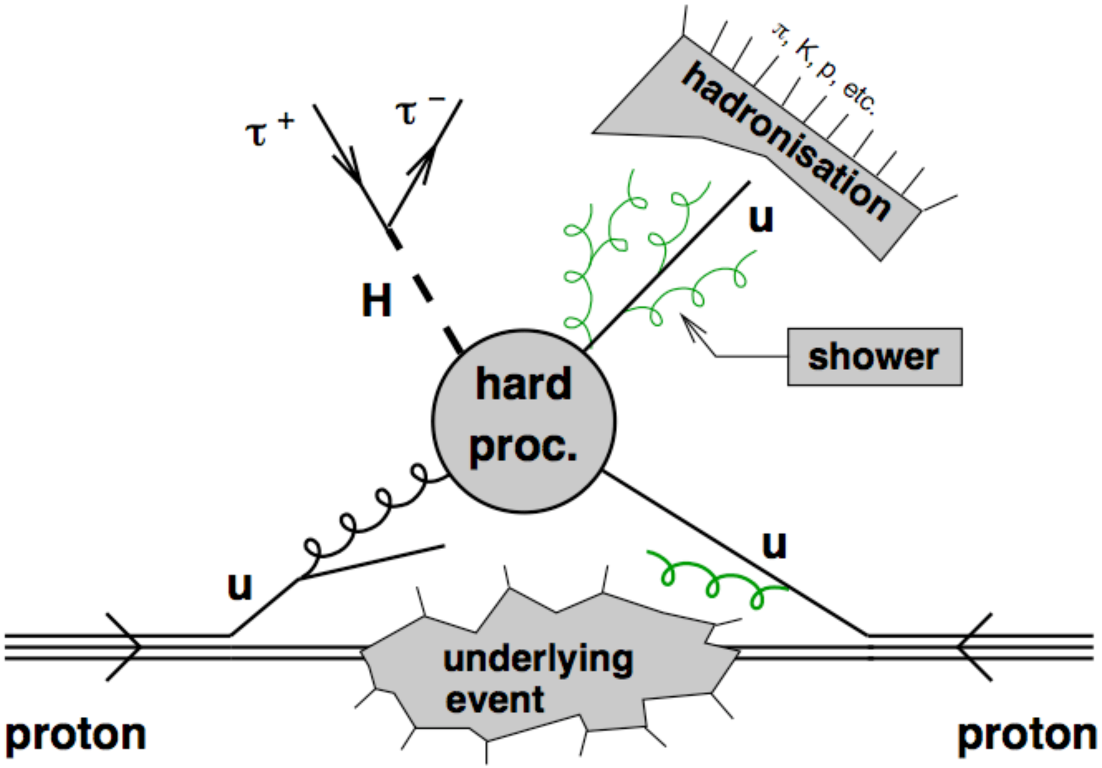
Evolution (i.e. higher momentum scale) produces more partons at small momentum fraction (because they lose energy by radiating)

As for the coupling, one can't predict PDF's values in pQCD, but only their evolution

Take-home points

- universal character of soft/collinear emission
- both real and virtual diagrams usually contribute to an observable (and are both needed to cancel divergencies)
- not everything is perturbatively calculable. Restrict to IRC-safe observables and/or employ factorisation
- Factorisation leads to resummation

Ingredients and tools



- PDFs
- Hard scattering and shower
- Final state tools

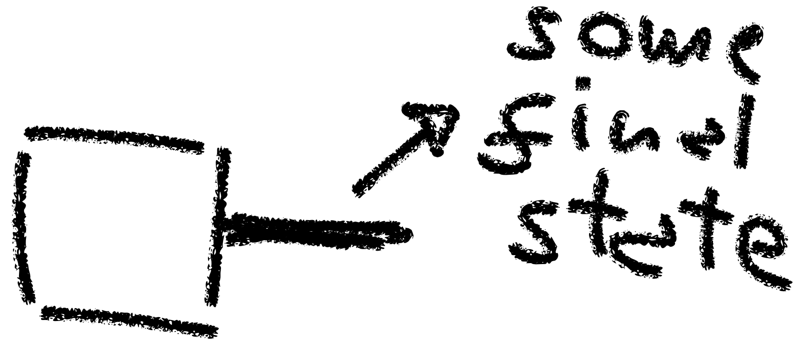
(Higher order) calculations

What goes into them ?

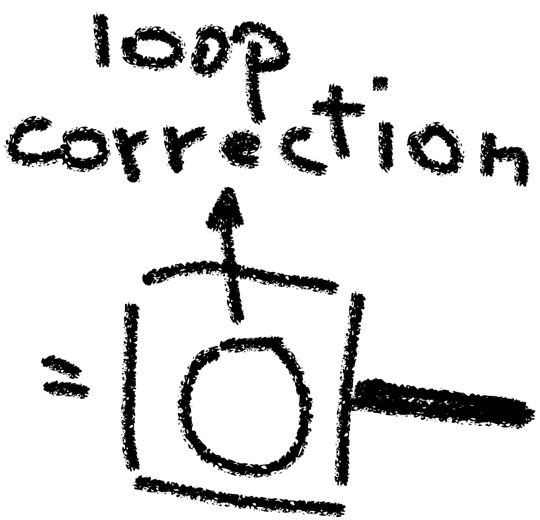
(Very superficial and schematic presentation of their structure and of some of the tools, and no pretence of actually explaining how to do things)

Nomenclature

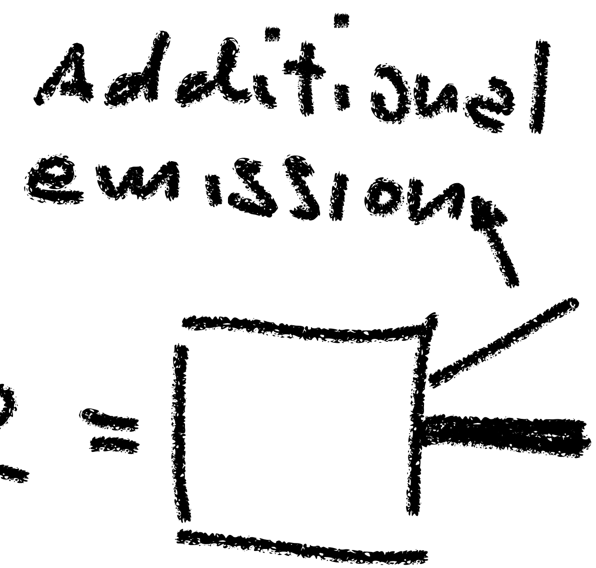
$P =$



$P_{1L} =$



$P_{+1e} =$



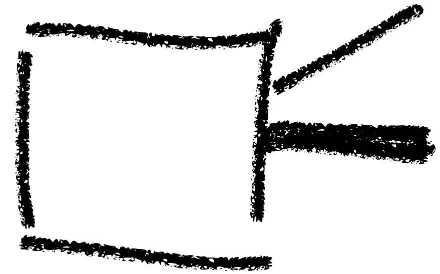
N.B.

$$P + \underline{1}\,e \neq P + \underline{1}\,\text{jet}$$

e = emission

or

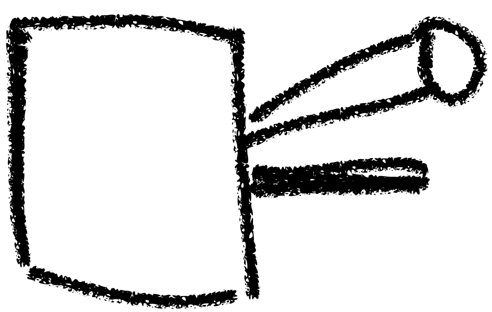
e = eeg

P + 1e = 

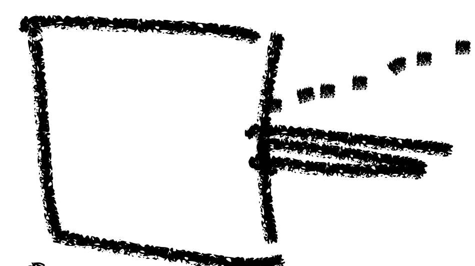
Contributes to:

P + 1 jet

P + X

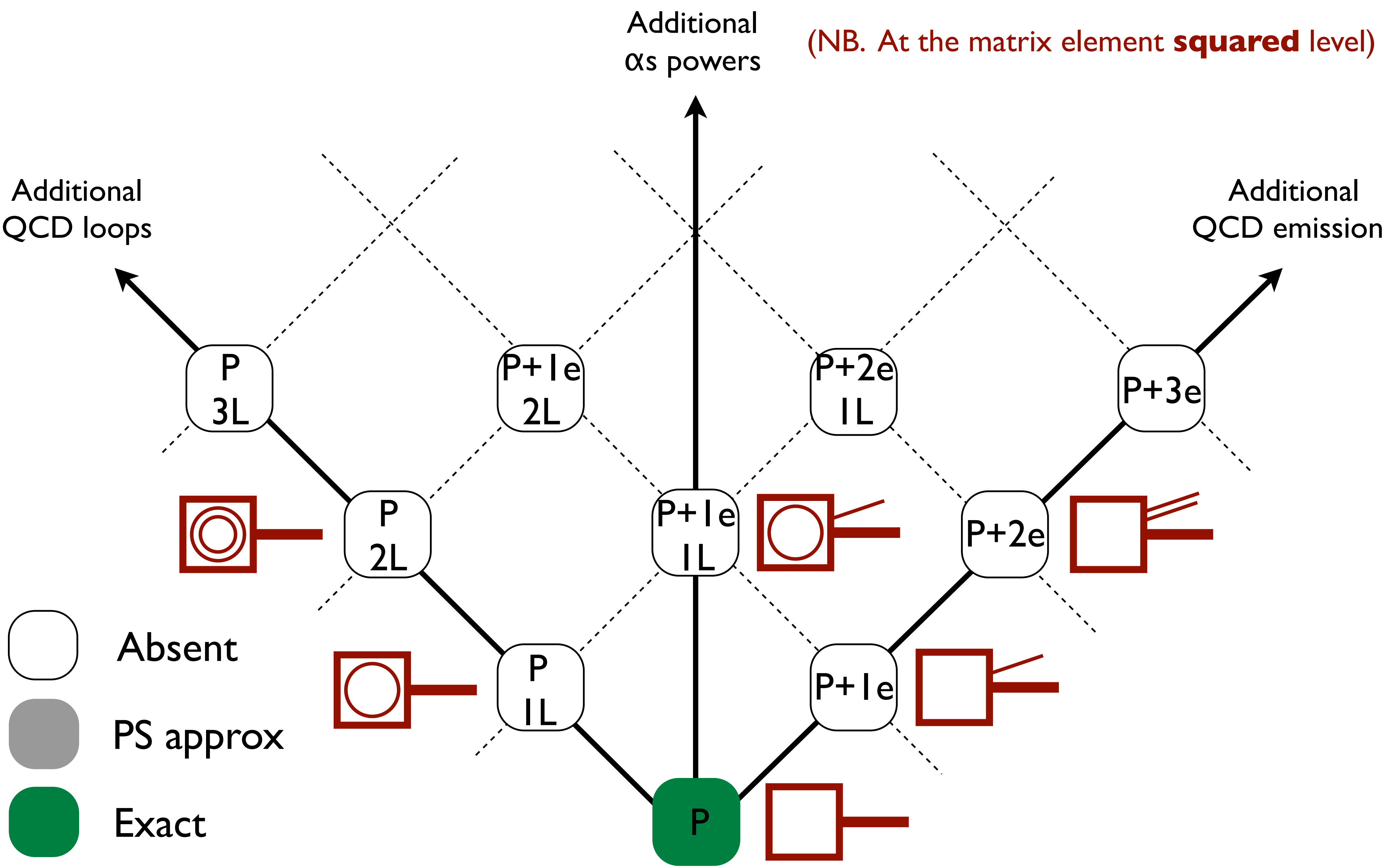


or

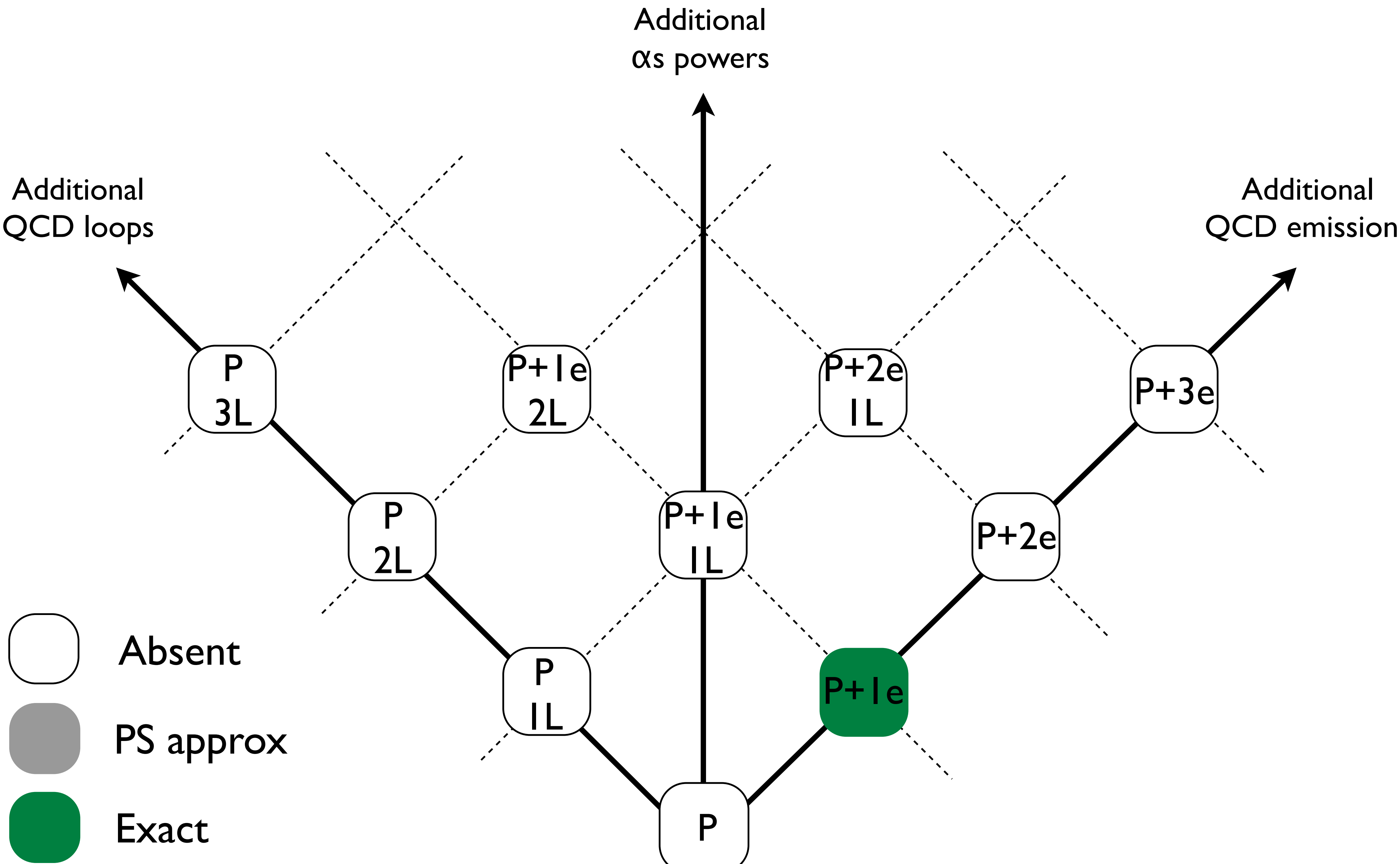


(if e is integrated over)

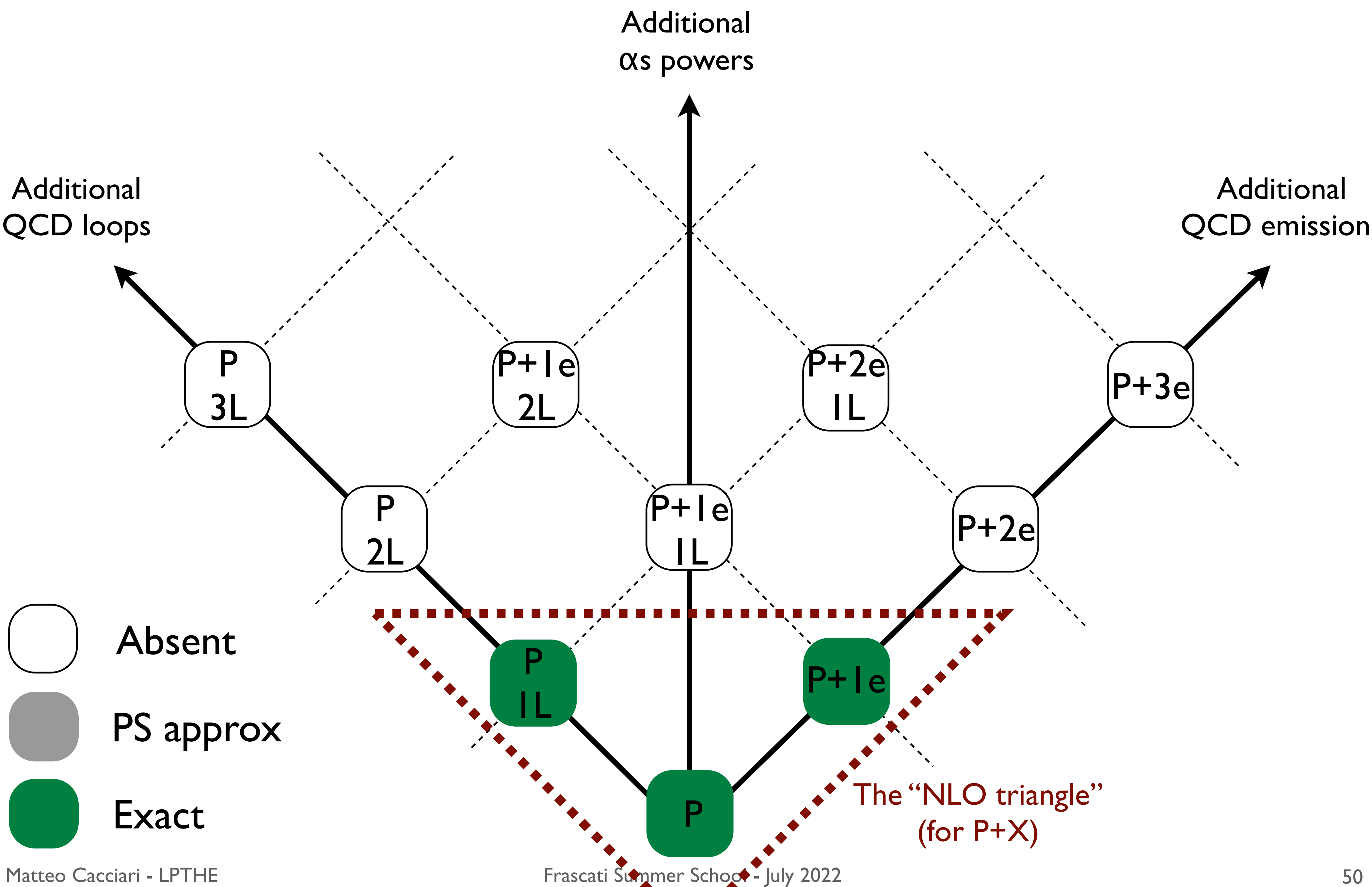
Process P exact at LO, nothing else



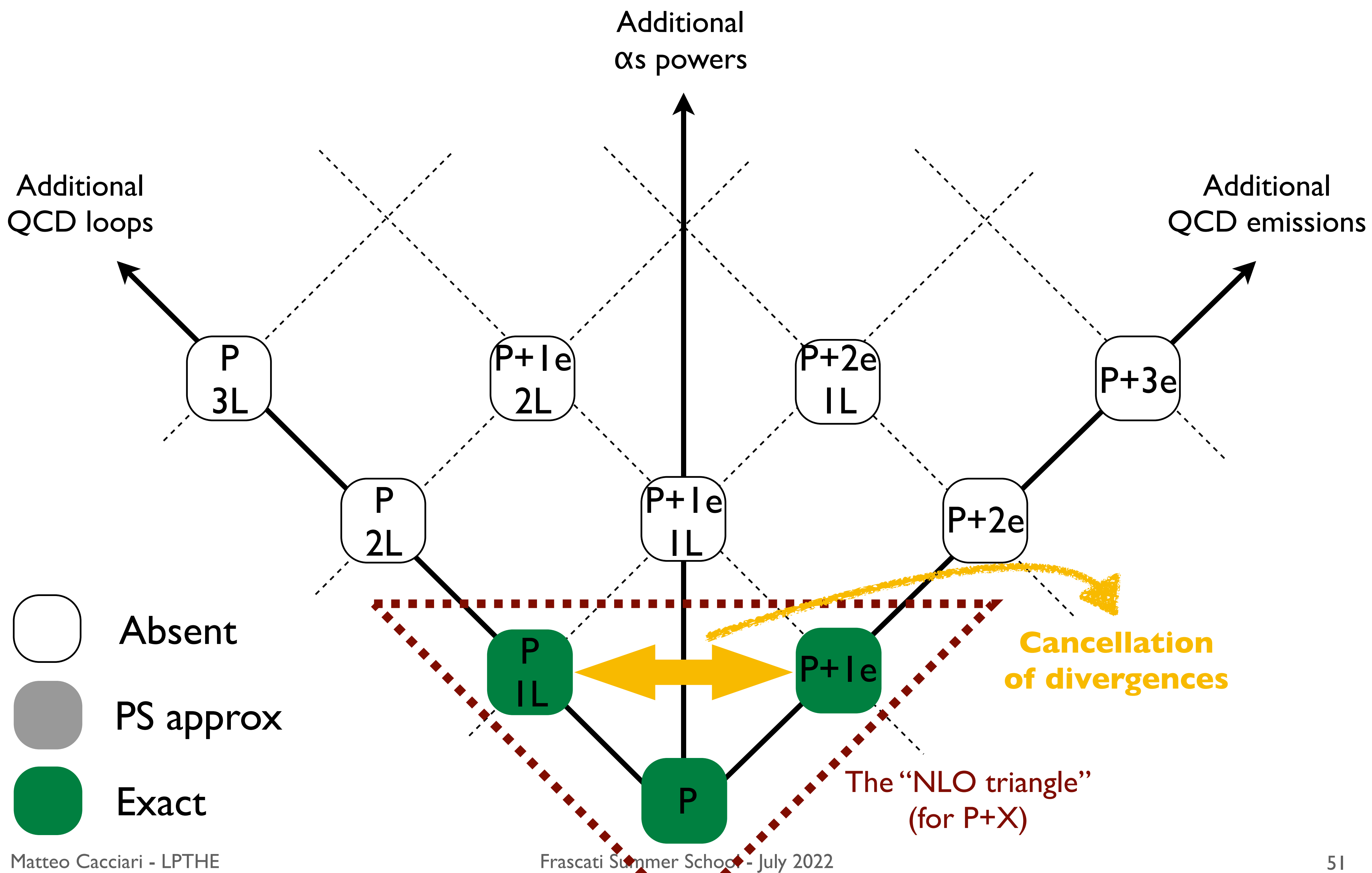
Process P+1j exact at LO, nothing else



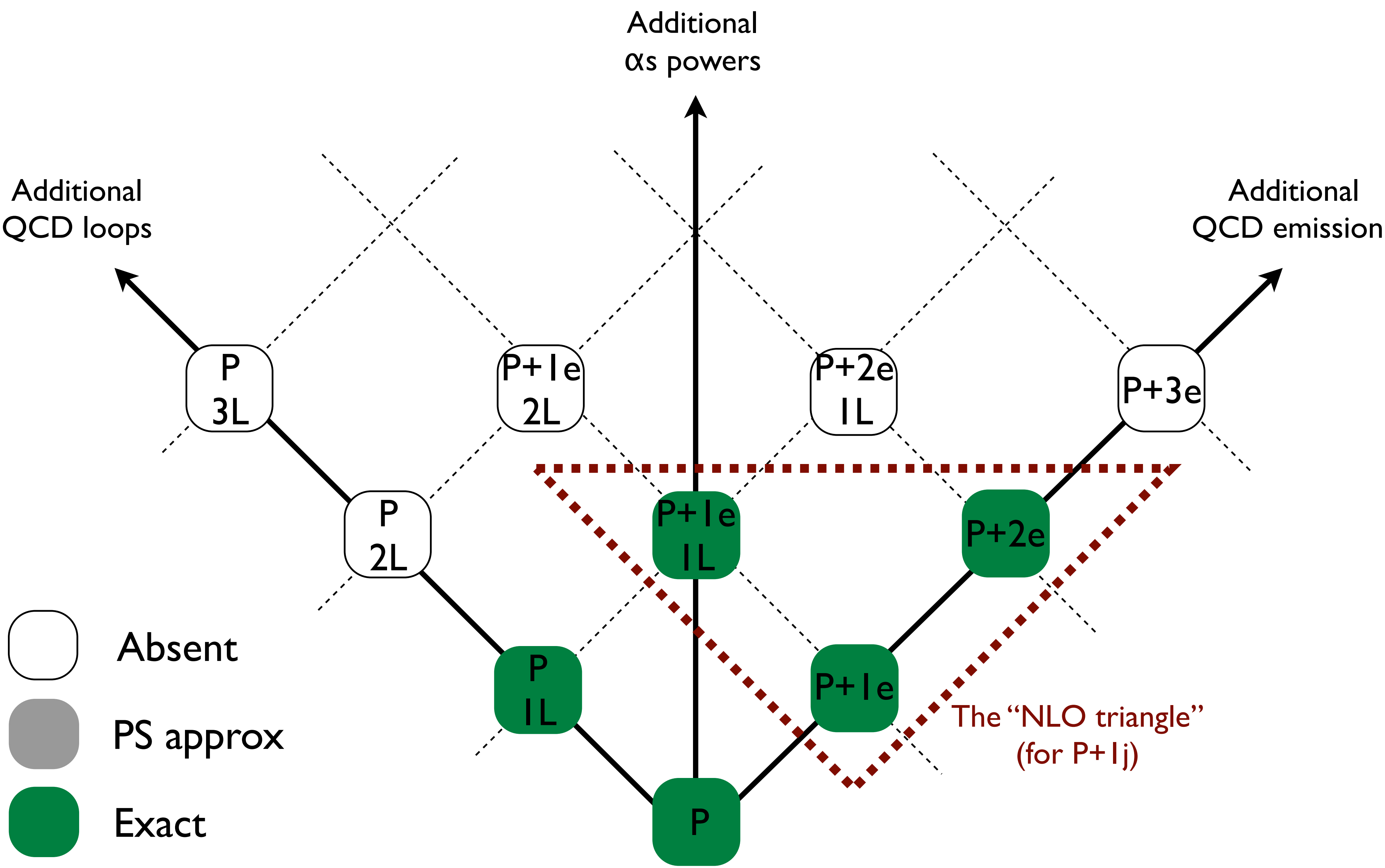
Process P exact at NLO, P+1j exact at LO, nothing else



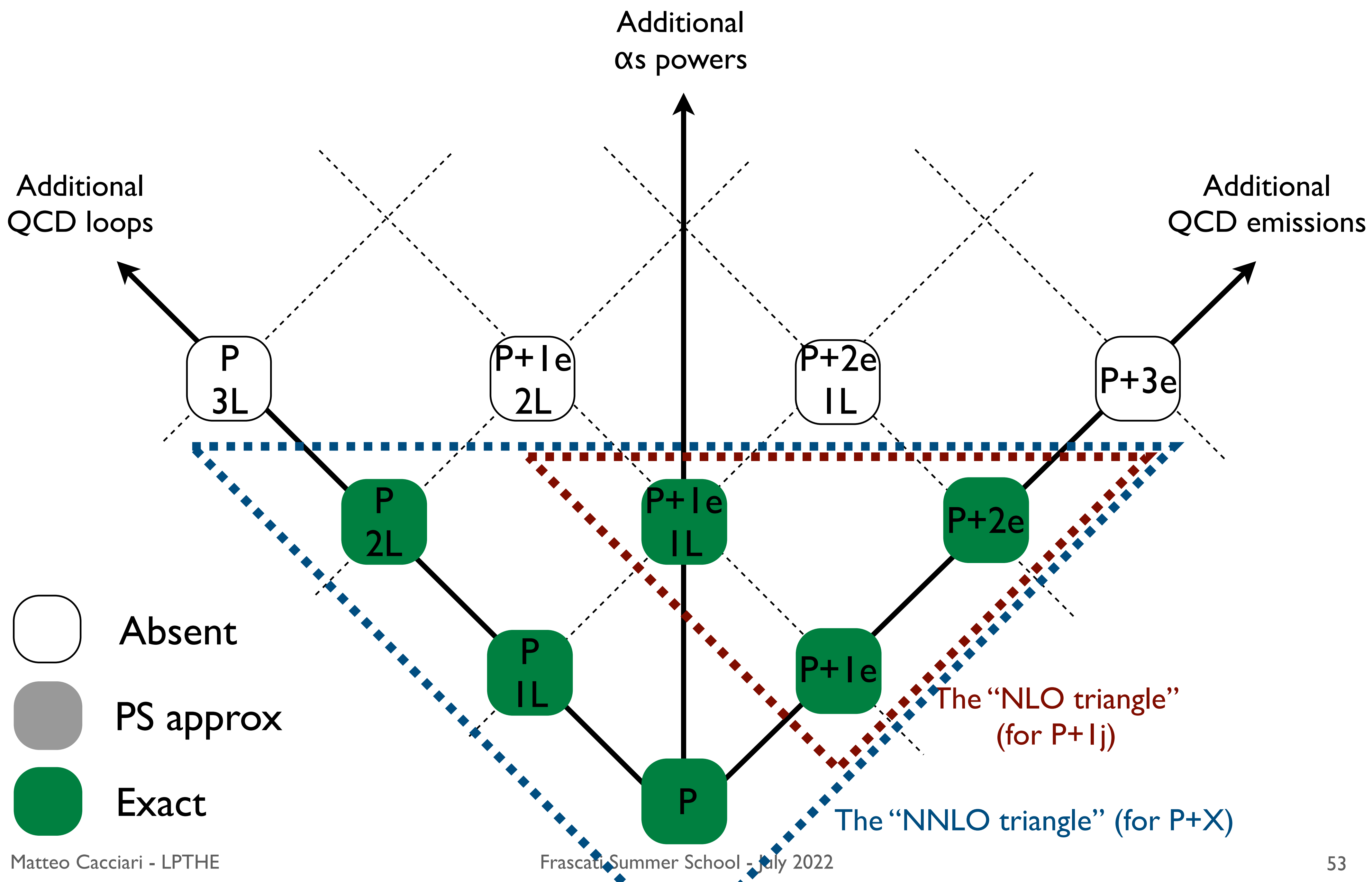
Process P exact at NLO, P+1j exact at LO, nothing else



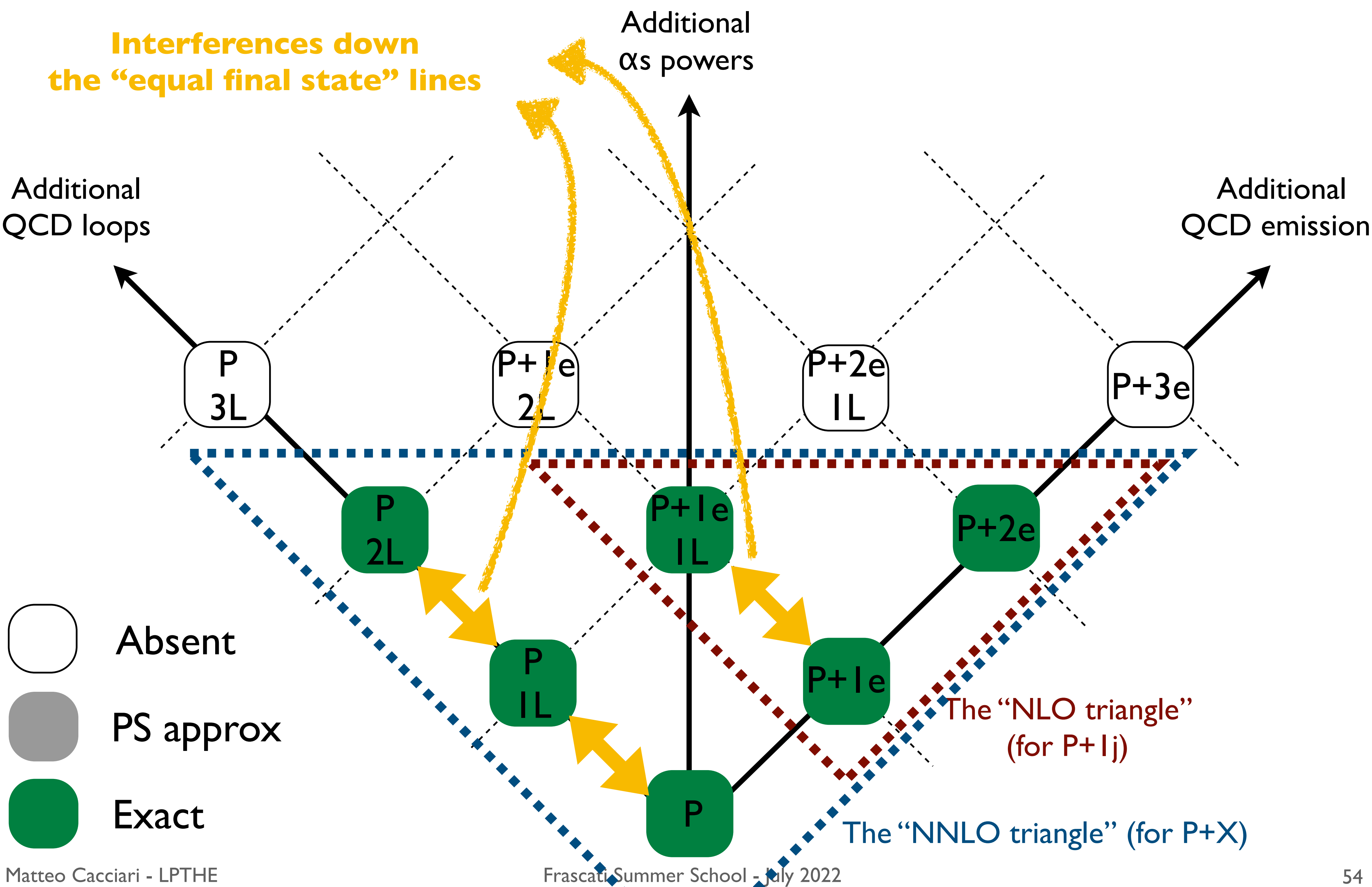
Process P and P+1j exact at NLO, P+2j at LO



Process P exact at NNLO, P+1j exact at NLO, P+2j at LO



Process P exact at NNLO, P+1j exact at NLO, P+2j at LO



Tools for the hard scattering

Can be divided in

- **Integrators**
  - evaluate the (differential) cross section by integrating the calculation over the phase space, yielding (partly) inclusive quantities
  - Produce weighted events (the weight being the value of the cross section)
  - Calculations exist at LO, NLO, NNLO, NNNLO
- **Generators**
  - generate fully exclusive configurations
  - Events are unweighted (i.e. produced with the frequency nature would produce them)
  - Easy at LO, get complicated when dealing with higher orders

Fixed order calculation

Born

$$d\sigma^{Born} = B(\Phi_B) d\Phi_B$$

NLO

$$d\sigma^{NLO} = [B(\Phi_B) + V(\Phi_B)]\, d\Phi_B + R(\Phi_R) d\Phi_R$$

$$d\Phi_R = d\Phi_B \, d\Phi_{rad}$$

$$d\Phi_{rad} = d\cos\theta \, dE \, d\phi$$

Problem:
$V(\Phi_B)$ and $\int R d\Phi_R$ are divergent

Subtraction terms

An observable *O* is **infrared and collinear safe** if

$$O(\Phi_\mathrm{R}(\Phi_\mathrm{B}, \Phi_\mathrm{rad})) \to O(\Phi_\mathrm{B})$$

Soft or collinear limit

One can then write, with R→C in the soft/coll limit,

This integration performed analytically

$$\langle O\rangle = \int \left[B(\Phi_B) + V(\Phi_B) + \int C(\Phi_R) d\Phi_{rad}\right] O(\Phi_B) d\Phi_B$$

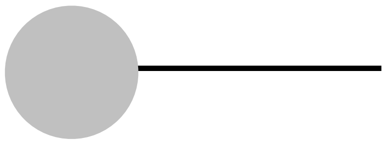
$$+ \left[R(\Phi_R)O(\phi_R) - C(\Phi_R)O(\Phi_B)\right] d\Phi_R$$

Separately finite

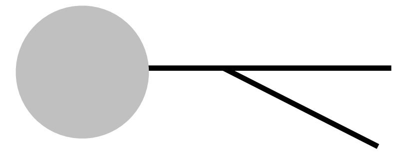
This (or a similar) cancellation will always be implicit in all subsequent equations

Parton Shower Monte Carlo

Exploit factorisation property of soft and collinear radiation



σ_n



σ_{n+1}

Factorisation

$$d\sigma_{n+1}(\Phi_{n+1}) = \mathcal{P}(\Phi_{\rm rad})\, d\sigma_n(\Phi_n)\, d\Phi_{\rm rad}$$

Emission probability

$$\mathcal{P}(\Phi_{\rm rad})\, d\Phi_{\rm rad} \approx \frac{\alpha_S(q)}{\pi}\frac{dq}{q}P(z,\phi)\, dz\frac{d\phi}{2\pi}$$

Iterate emissions to generate higher orders (in the soft/collinear approximation)

Parton Shower MC

Based on the **iterative emission of radiation** described in the **soft-collinear limit**

$$d\sigma^{(MC)}(\Phi_R)d\Phi_R = B(\Phi_B)d\Phi_B\mathcal{P}(\Phi_{rad})d\Phi_{rad}$$

Pros: soft-collinear radiation is resummed to all orders in pQCD

Cons: hard large-angle radiation is missing

Overall accuracy will be leading log (LL) for the radiation, and leading order (i.e. Born) for the integrated cross sections

A key ingredient of a parton shower Monte Carlo:

Sudakov form factor

$\Delta(t_1,t_2)$

Probability of **no emission** between the scales t_1 and t_2

Example:

- decay probability per unit time of a nucleus = c_N
- Sudakov form factor $\Delta(t_0,t) = \exp(-c_N(t-t_0))$

Probability that nucleus does **not** decay between t_0 and t

Sudakov form factor: derivation

Decay probability per unit time = $\frac{dP}{dt} = c_N$

Probability of **no** decay between t_0 and t = $\Delta(t_0,t)$ [with $\Delta(t_0,t_0)$ = 1]

⇒ Probability of decay between t_0 and t = 1- $\Delta(t_0,t)$

[unitarity: either you decay or you don't]

Decay probability per unit time **at time t** can be written in two ways:

1. $$P^{\rm dec}(t) = \frac{d}{dt}\Big(1 - \Delta(t_0,t)\Big) = -\frac{d\Delta(t_0,t)}{dt}$$

2. $$P^{\rm dec}(t) = \Delta(t_0,t)\frac{dP}{dt}$$

No decay until t, probability per unit time to decay at t

Sudakov form factor: derivation

Equating the two expressions for $P^{dec}(t)$ we get

$$-\frac{d\Delta(t_0,t)}{dt} = \Delta(t_0,t)\frac{dP}{dt}$$

We can solve the differential equation using $dP/dt = c_N$ and we get

$$\Delta(t_0,t) = \exp(-c_N(t-t_0))$$

If the decay probability depends on t (and possibly other variables, call them z) this generalises to

$$\Delta(t_0,t) = \exp\left(-\int_{t_0}^{t} dt' \int dz\, c_N(t',z)\right)$$

Sudakov form factor in QCD

Emission probability

$$\mathcal{P}(\Phi_{\rm rad})\,\mathrm{d}\Phi_{\rm rad} \approx \frac{\alpha_S(q)}{\pi}\frac{\mathrm{d}q}{q}P(z,\phi)\,\mathrm{d}z\frac{\mathrm{d}\phi}{2\pi}$$

Sudakov form factor = probability of **no emission**

from large scale q_1 to smaller scale q_2

$$\Delta_S(q_1,q_2) = \exp\left[-\int_{q_2}^{q_1}\frac{\alpha_S(q)}{\pi}\frac{\mathrm{d}q}{q}\int_{z_0}^{1}P(z)\,\mathrm{d}z\right]$$

Conventions for Sudakov form factor

$$\Delta_S(q_1, q_2) = \exp\left[-\int_{q_2}^{q_1} \frac{\alpha_S(q)}{\pi}\frac{dq}{q}\int_{z_0}^{1} P(z)\,dz\right]$$

Full expression, with details of soft-collinear radiation probability

$$\Delta(p_T) = \exp\left[-\int_{p_T}^{Q} \frac{\frac{d\sigma^{(MC)}}{dy\,dp'_T}}{\frac{d\sigma^{(B)}}{dy}} dp'_T\right]$$

Dropped upper limit, taken implicitly to be the hard scale Q

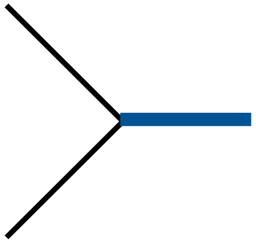
$$\Delta_R(p_T) = \exp\left[-\int \frac{R}{B}\Theta(k_T(\Phi_R) - p_T)d\Phi_{rad}\right]$$

Introduced suffix (R in this case) to indicate expression used to described radiation

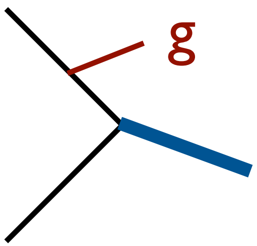
$$\Delta_R(p_T) = \exp\left[-\int_{p_T} \frac{R}{B} d\Phi_{rad}\right]$$

Integration boundaries only implicitly indicated

PS example: Higgs plus radiation



Leading order.
No radiation, Higgs $p_T = 0$



With emission of radiation
Higgs $p_T \neq 0$

Description of hardest emission in PS MC (either event is generated)

$$\frac{d\sigma^{(MC)}}{dy\, dp_T} = \frac{d\sigma^{(B)}}{dy}\delta(p_T)\Delta(Q_0) + \Delta(p_T)\frac{d\sigma^{(MC)}}{dy\, dp_T}$$

$$\Delta(p_T) = \exp\left[-\int_{p_T}^{Q} \frac{\frac{d\sigma^{(MC)}}{dy\, dp'_T}}{\frac{d\sigma^{(B)}}{dy}} dp'_T\right]$$

Sudakov form factor

- $\frac{d\sigma^{(B)}}{dy}$: x-sect for no emission
- $\delta(p_T)\Delta(Q_0)$: prob. of **no emission** (down to the PS cutoff)
- $\Delta(p_T)$: prob. of no emission down to p_T
- $\frac{d\sigma^{(MC)}}{dy\, dp_T}$: x-sect for **emission at p_T**, as described by the MC

Toy shower for the Higgs p_T

Gavin Salam has made public a ‘toy shower’ that generates the Higgs transverse momentum via successive emissions controlled by the Sudakov form factor

$$\Delta(p_T) = \exp\left[-\frac{2\alpha_s C_A}{\pi}\ln^2\frac{p_{T,\max}^2}{p_T^2}\right]$$

You can get the code at
https://github.com/gavinsalam/zuoz2016-toy-shower

NB. In order to get more realistic results you need at least at the code in v2

Shower unitarity

It holds

$$\int_0^Q \left[\delta(p_T)\Delta(Q_0) + \frac{\Delta(p_T)\frac{d\sigma^{(MC)}}{dy\,dp_T}}{\frac{d\sigma^{(B)}}{dy}} \right] dp_T = \Delta(Q_0) + \int_{Q_0}^Q \frac{d\Delta(p_T)}{dp_T} dp_T = \Delta(Q) = 1$$

Shower unitarity

so that

$$\int_0^Q dp_T \frac{d\sigma^{(MC)}}{dy\,dp_T} = \frac{d\sigma^{(B)}}{dy} \int_0^Q \left[\delta(p_T)\Delta(Q_0) + \frac{\Delta(p_T)\frac{d\sigma^{(MC)}}{dy\,dp_T}}{\frac{d\sigma^{(B)}}{dy}} \right] dp_T = \frac{d\sigma^{(B)}}{dy}$$

A parton shower MC correctly reproduces the Born cross section for integrated quantities

This amounts to introducing approximate virtual corrections, whose job is simply cancelling divergencies from real emission and nothing more (nor less)

PS MC in different notation

Writing the real cross section as described by the Monte Carlo (i.e. with the parton shower) simply as R^{MC}, we can rewrite

$$d\sigma^{MC} = Bd\Phi_B \left[\Delta_{MC}(Q_0) + \Delta_{MC}(p_T)\frac{R^{MC}}{B}d\Phi_{rad}\right]$$

with $$\Delta_{MC}(p_T) = \exp\left[-\int_{p_T}\frac{R^{MC}}{B}d\Phi_{rad}\right]$$

as our Master Formula for a Parton Shower Monte Carlo.

Thanks to the shower unitarity, it holds

$$\Rightarrow \int d\sigma^{MC} = \int Bd\Phi_B = \sigma^{LO}$$

Matrix Element corrections

In a PS Monte Carlo $R^{(MC)}(\Phi_R) = B(\Phi_B)\mathcal{P}(\Phi_{rad})$ soft-collinear approximation

Replace the MC description of radiation with the **correct** one:

$$\mathcal{P}(\Phi_{rad}) \to \frac{R}{B}$$

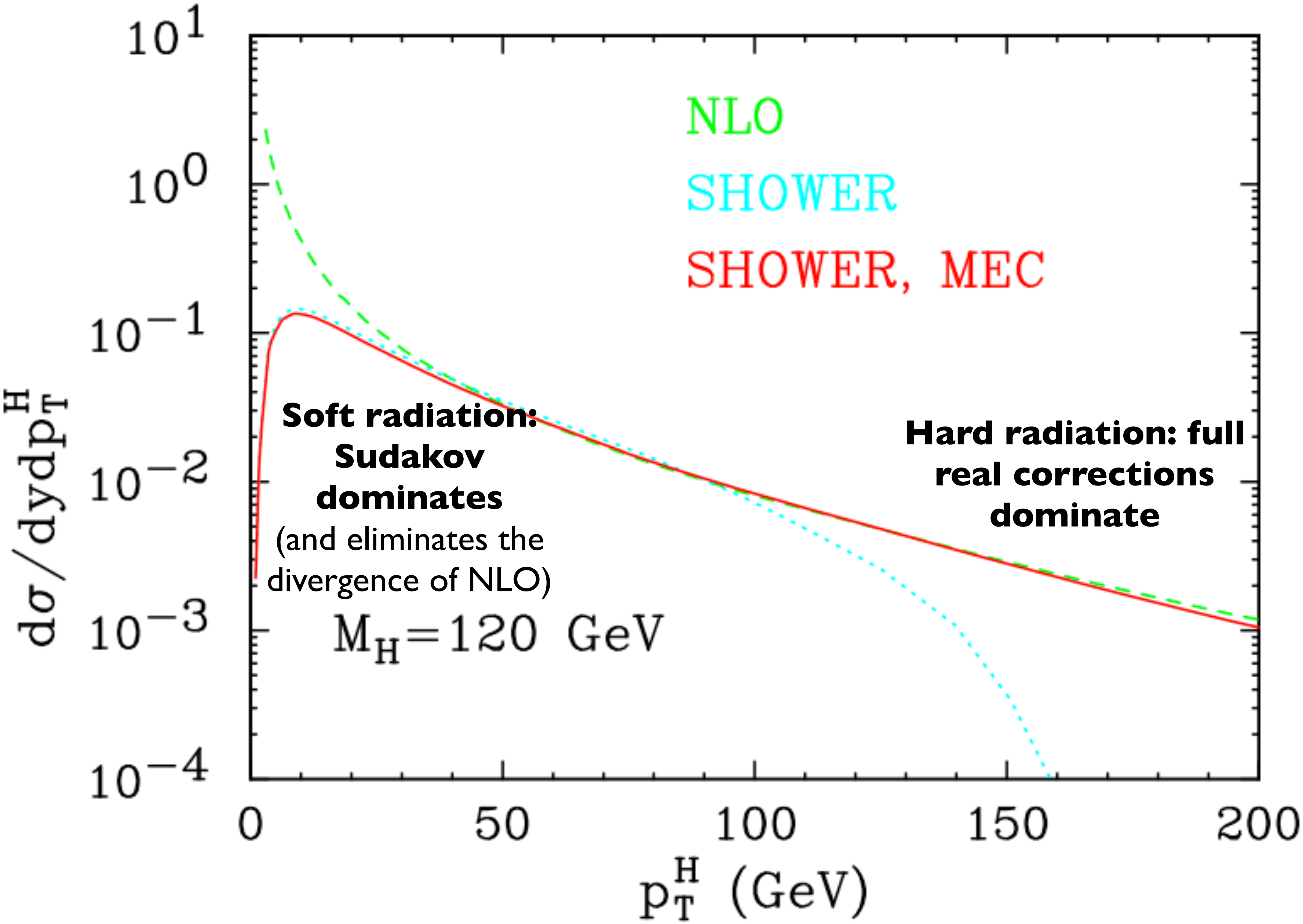
The Sudakov becomes

$$\Delta(p_{\mathrm{T}}) = \exp\left[-\int_{p_{\mathrm{T}}}^{Q} \frac{\frac{\mathrm{d}\sigma^{(\mathrm{MC})}}{\mathrm{d}y\,\mathrm{d}p'_{\mathrm{T}}}}{\frac{\mathrm{d}\sigma^{(\mathrm{B})}}{\mathrm{d}y}} \mathrm{d}p'_{\mathrm{T}}\right] \longrightarrow \Delta_R(p_T) = \exp\left[-\int \frac{R}{B}\Theta(k_T(\Phi_R) - p_T)d\Phi_{rad}\right]$$

and the x-sect formula for the hardest emission

$$d\sigma^{MEC} = Bd\Phi_B\left[\Delta_R(Q_0) + \Delta_R(p_T)\frac{R}{B}d\Phi_{rad}\right]$$

Matrix Element corrections

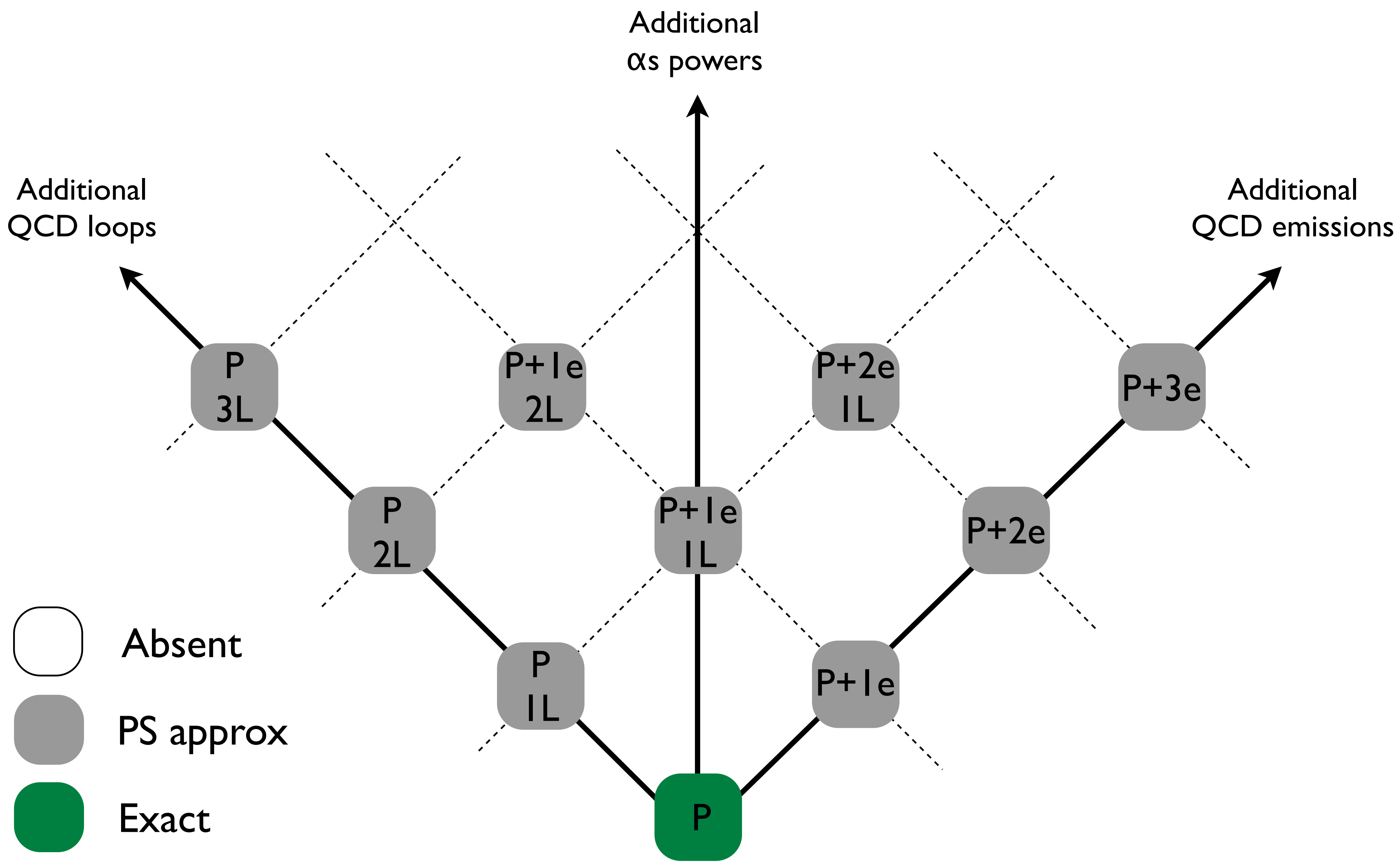


We wish to go beyond a Parton Shower (+MEC) Monte Carlo, so that

- we can successfully interface **matrix elements for multi-parton production with a parton shower**
- we can successfully interface a **parton shower with a NLO calculation**

It's a **quest for exactness** of ever more complex processes

Process P exact at LO, the rest PS approximation

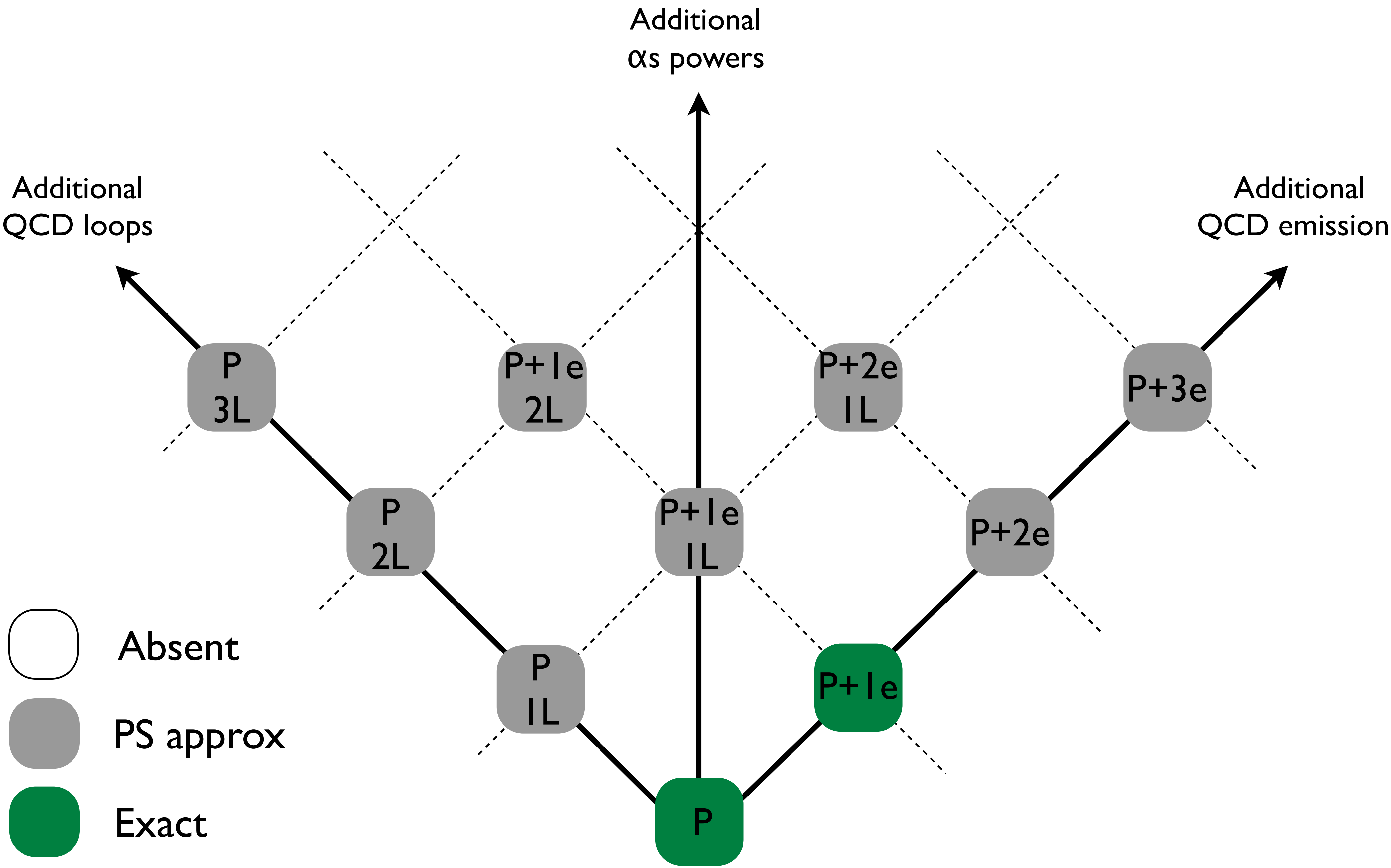


Absent

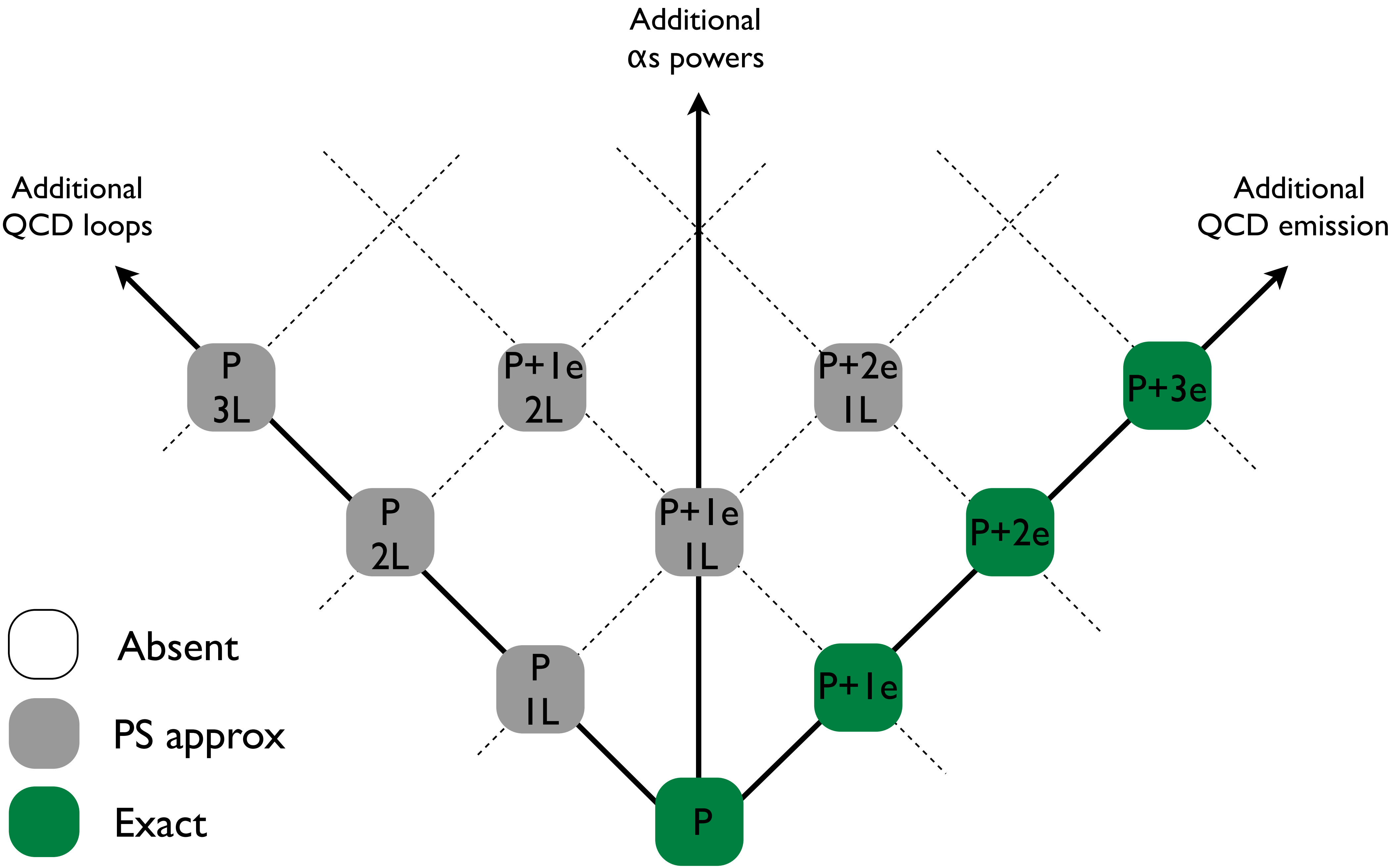
PS approx

Exact

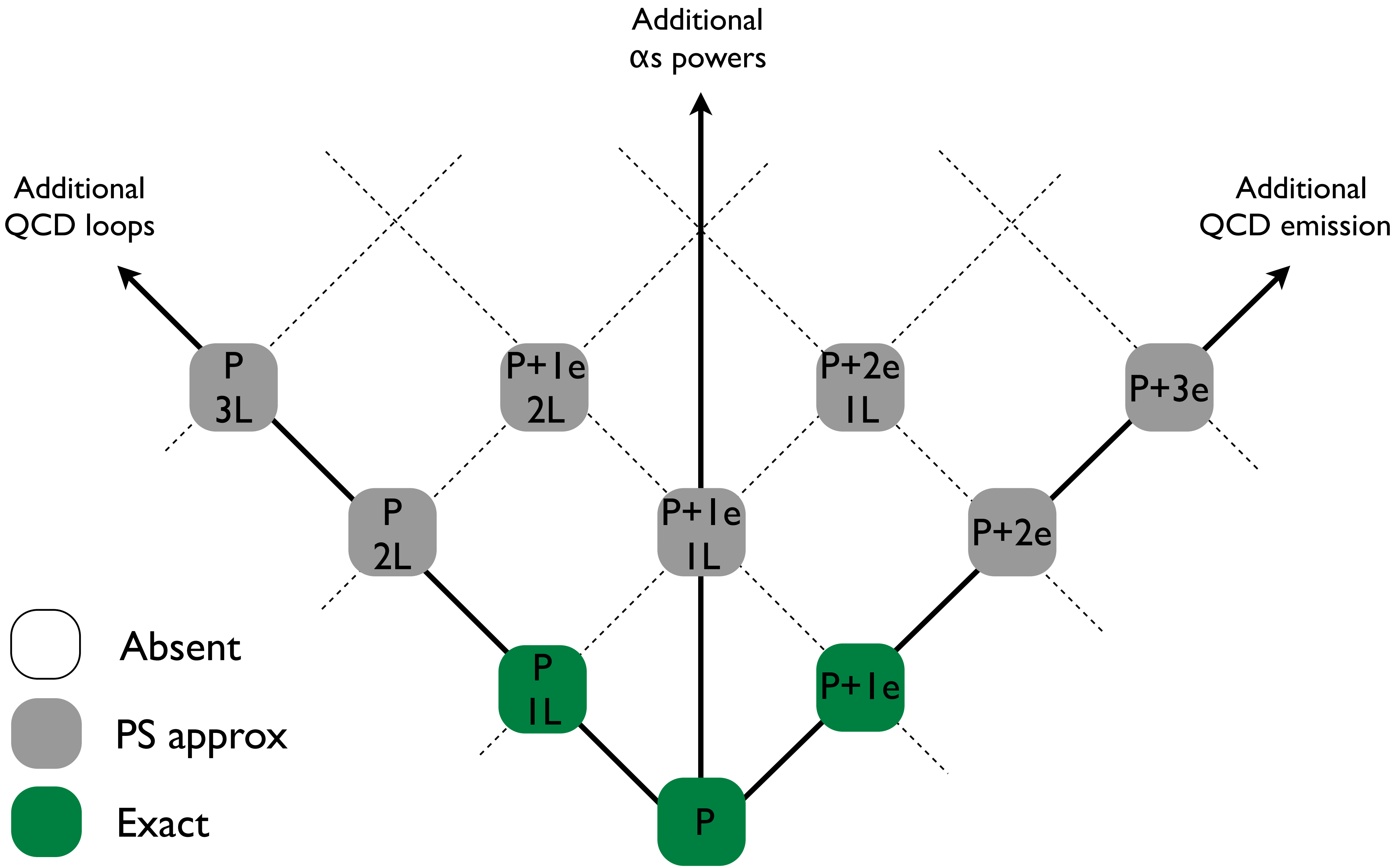
Process P and P+1j exact at LO, the rest PS approximation [PS+MEC or PS from ME for P+1e]



Process P, P+1j, P+2j, ... exact at LO, the rest PS approx. [PS+Matrix Element (CKKW, MLM,....)]



Process P exact at NLO, the rest PS approximation [PS+NLO (MC@NLO, POWHEG,...)]



‘MonteCarlos at NLO’:

- MC@NLO [Frixione and Webber, 2002]
- POWHEG [Nason, 2004]

NB. MC@NLO is a code, POWHEG is a method

Having evolved into (semi)automated forms:

- The POWHEG BOX [powhegbox.mib.infn.it 2010]
- MadGraph5_aMC@NLO [amcatnlo.cern.ch 2011]

MCs at NLO

Matrix-element corrected shower Monte Carlos still have leading order accuracy for the total rates

$$d\sigma^{MEC} = Bd\Phi_B \left[\Delta_R(Q_0) + \Delta_R(p_T)\frac{R}{B}d\Phi_{rad}\right] \quad \text{and} \quad \Delta_R(Q_0) + \int \Delta_R(p_T)\frac{R}{B}d\Phi_{rad} = 1$$

$$\Rightarrow \int d\sigma^{MEC} = \int Bd\Phi_B = \sigma^{LO}$$

We want to do better, and **merge** PS and NLO, so that

$$\int d\sigma^{PS+NLO} = \int (B+V)d\Phi_B + \int Rd\Phi_R = \sigma^{NLO}$$

Idea: remove from the NLO the terms that are already generated by the parton shower (NB. MC-specific)

$$d\sigma^{MC@NLO} = \bar{B}_{MC} d\Phi_B \left[\Delta_{MC}(Q_0) + \Delta_{MC}(p_T)\frac{R^{MC}}{B} d\Phi_{rad}\right] + [R - R^{MC}] d\Phi_R$$

$$\bar{B}_{MC} = B + \left[V + \int R^{MC} d\Phi_{rad}\right]$$

'soft' event — MC shower — 'hard' event

It is easy to see that, as desired,

$$\int d\sigma^{MC@NLO} = \int d\sigma^{NLO}$$

Idea: generate hardest radiation first, then pass event to MC for generation of subsequent, softer radiation

$$d\sigma^{POWHEG} = \bar{B} d\Phi_B \left[\Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right]$$

$$\bar{B} = B + \left[V + \int R \, d\Phi_{rad} \right]$$

NLO x-sect

MC shower

It is easy to see that, as desired,

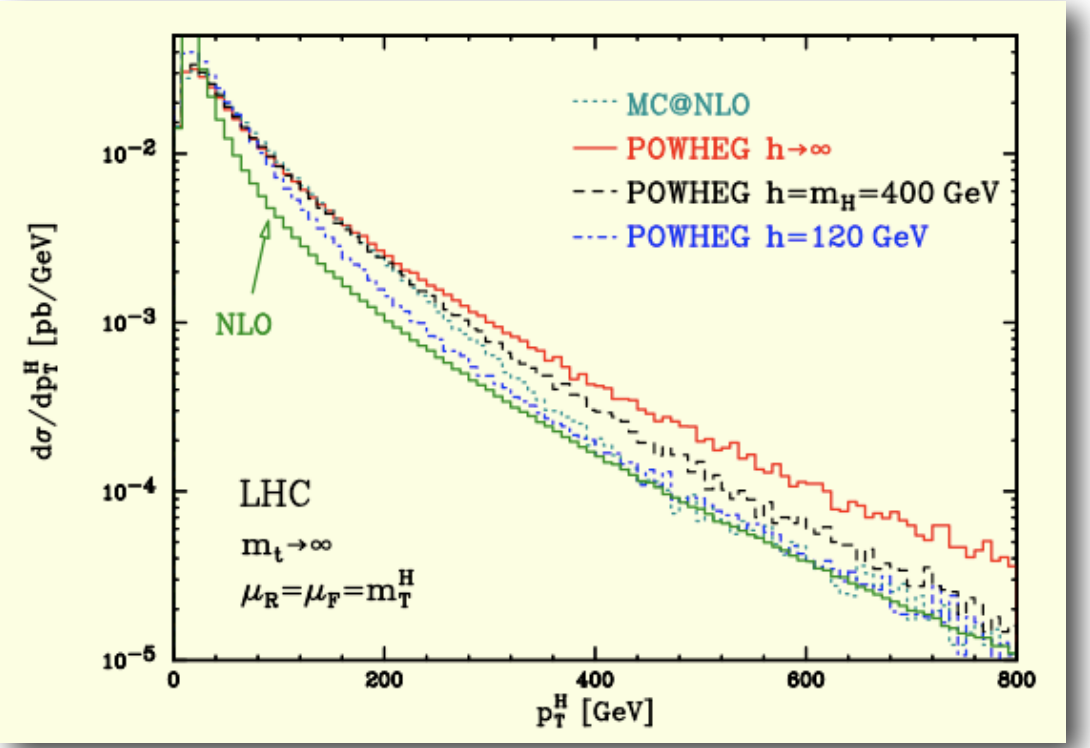
$$\int d\sigma^{POWHEG} = \int d\sigma^{NLO}$$

Large p_T enhancement in POWHEG

The 'naive' formulation for POWHEG is

$$d\sigma^{POWHEG} = \bar{B} d\Phi_B \left[\Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right]$$

In this form $\bar{B} d\Phi_B$ provides the NLO K-factor (order 1+ O(α_s)) , but also associates it to large p_T radiation, where the calculation is already O(α_s) (but only LO accuracy).



This generates an effective (but not necessarily correct) O(α_s^2) term (i.e. NNLO for the total cross section)

OK because beyond nominal accuracy, but one may feel uncomfortable with such large numerical factors

Modified POWHEG

The 'problem' with the naive POWHEG comes from the hard radiation being enhanced by spurious higher orders. In order to suppress this effect, we split

$$R = R^S + R^F \qquad R^S \equiv \frac{h^2}{h^2 + p_T^2} R \qquad R^F \equiv \frac{p_T^2}{h^2 + p_T^2} R$$

R^S: Contains singularities

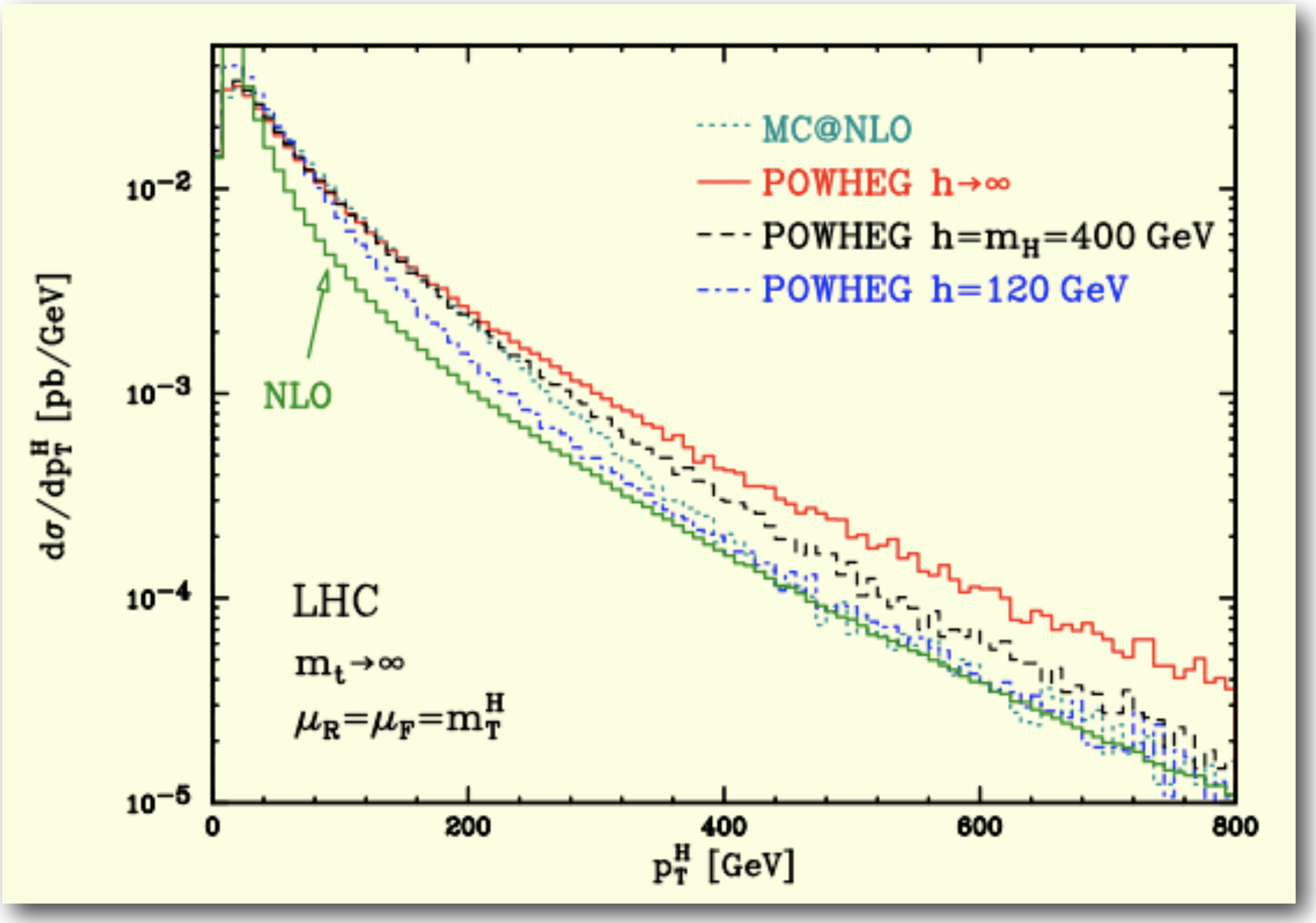
R^F: Regular in small p_T region

$$d\sigma^{POWHEG} = \bar{B}^S d\Phi_B \left[\Delta_S(Q_0) + \Delta_S(p_T) \frac{R^S}{B} d\Phi_{rad} \right] + R^F d\Phi_R$$

$$\bar{B}^S = B + \left[V + \int R^S \, d\Phi_{rad} \right] \qquad \Delta_S(p_T) = \exp\left[- \int_{p_T} \frac{R^S}{B} d\Phi_{rad} \right]$$

Modified POWHEG

In the $h \to \infty$ limit the exact NLO result is recovered



Comparisons

$$d\sigma^{MC} = Bd\Phi_B \left[\Delta(Q_0) + \Delta(p_T)\frac{R^{MC}}{B}d\Phi_{rad}\right]$$

$$d\sigma^{MEC} = Bd\Phi_B \left[\Delta_R(Q_0) + \Delta_R(p_T)\frac{R}{B}d\Phi_{rad}\right]$$

$$d\sigma^{NLO} = [B+V]\, d\Phi_B + Rd\Phi_R$$

$$d\sigma^{MC@NLO} = \bar{B}_{MC}d\Phi_B \left[\Delta_{MC}(Q_0) + \Delta_{MC}(p_T)\frac{R^{MC}}{B}d\Phi_{rad}\right] + [R - R^{MC}]d\Phi_R$$

$$d\sigma^{POWHEG} = \bar{B}^S d\Phi_B \left[\Delta_S(Q_0) + \Delta_S(p_T)\frac{R^S}{B}d\Phi_{rad}\right] + R^F d\Phi_R$$

$$R = R^S + R^F$$

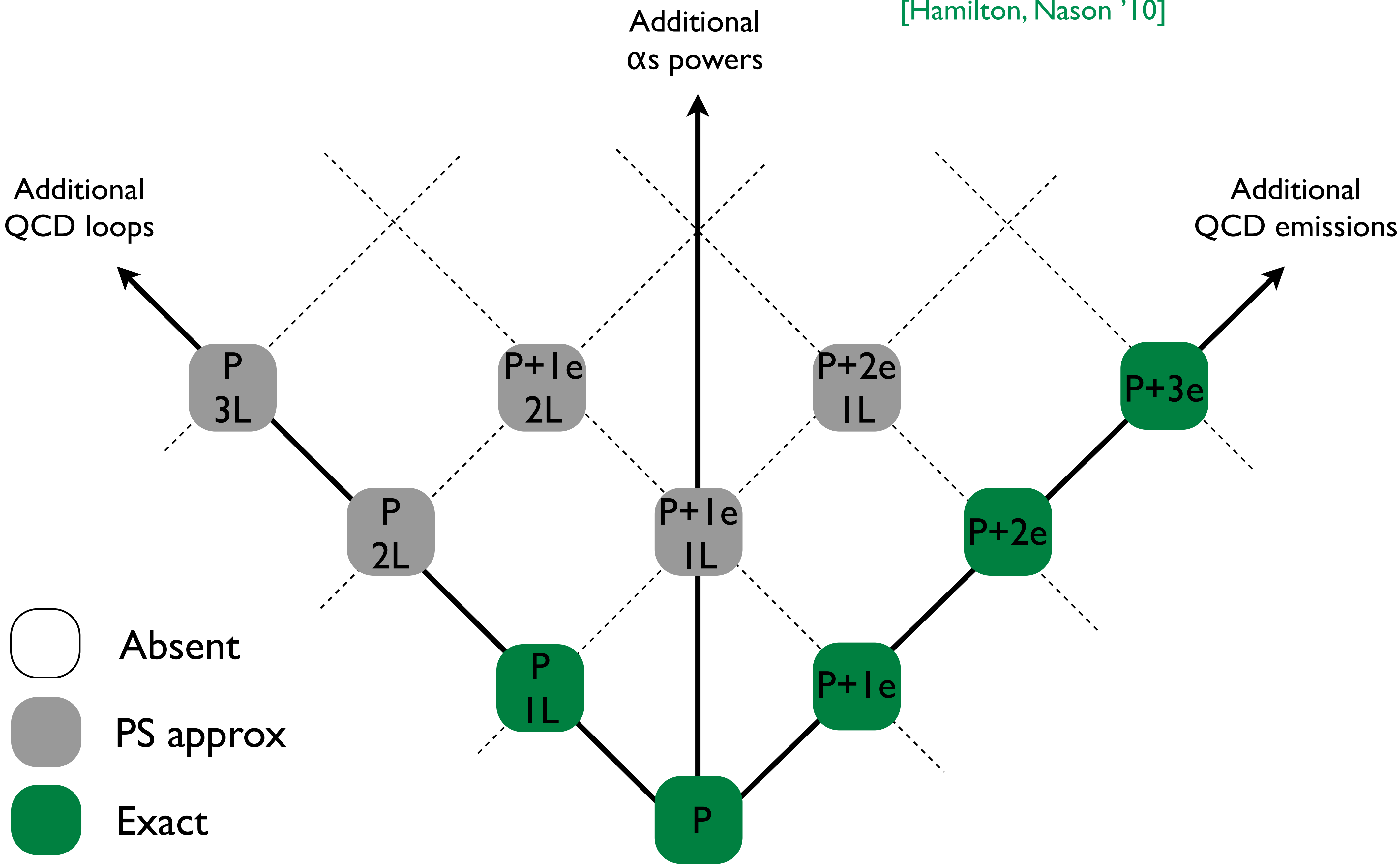
$$R^S \equiv \frac{h^2}{h^2 + p_T^2}R$$

$$R^F \equiv \frac{p_T^2}{h^2 + p_T^2}R$$

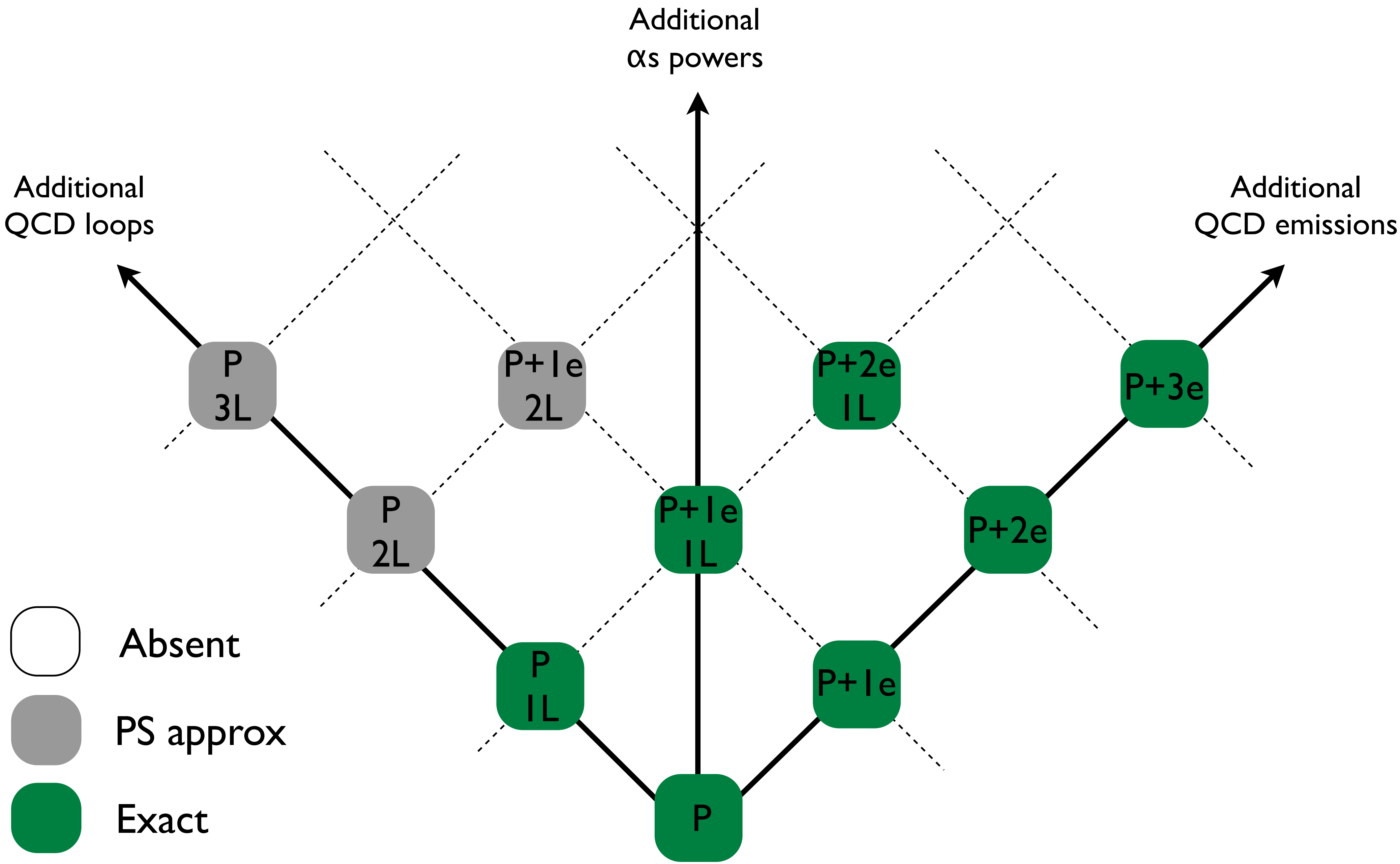
POWHEG approaches MC@NLO if $R^S \to R^{MC}$

Process P exact at NLO, P+1j, P+2j,... at LO, the rest PS [PS+NLO+ME (MENLOPS,...)]

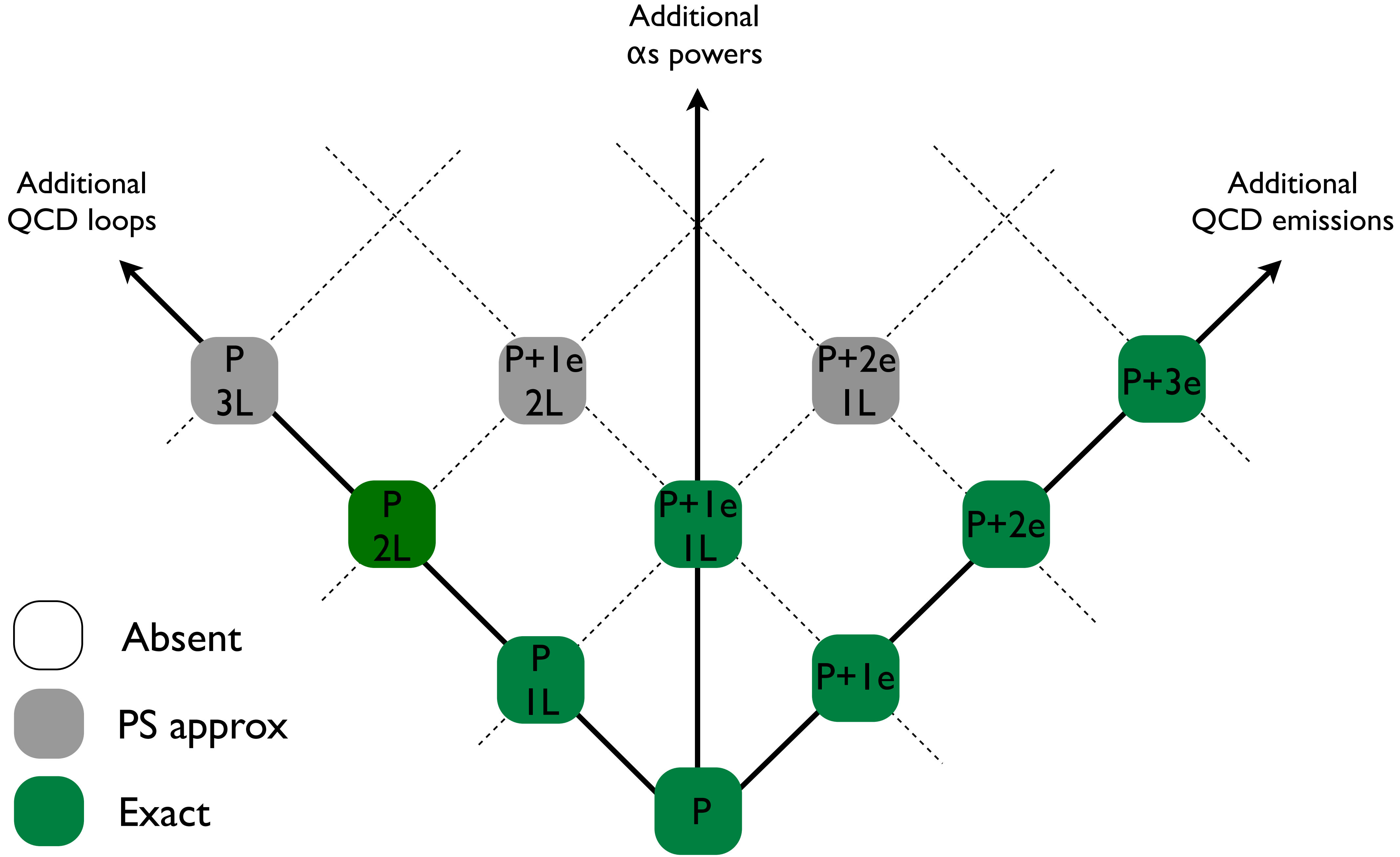
[Hamilton, Nason '10]



Process P, P+1j, P+2j,... exact at NLO, the rest PS [PS+NLO+ME$_{NLO}$ (MEPS@NLO, MiNLO', ...)]



Process P exact at NNLO, P+1j, at NLO, the rest PS [PS+NNLO+ME$_{NLO}$ (MINNLO$_{PS}$, …)]



Take home points

Monte Carlos in QCD are complicated. I only scratched the surface here and gave almost no details. If interested, check lectures of real MC people (Sjostrand, Skands, Nason, Maltoni, Frixione, Krauss, Hoeche, Richardson, Webber and many others)

Monte Carlos exploit property of universality of soft/collinear radiation to resum its effects to all orders (within some approximations)

Effects of multi-parton, hard, large-angle radiation can be included via exact calculations and proper (and delicate) mergings. State of the art accuracy is NLO and, in some cases, NNLO

The result is a detailed description of the final state, covering as much phase space as possible. Accurate descriptions of data are usually achieved