Charged Leptons Measurements

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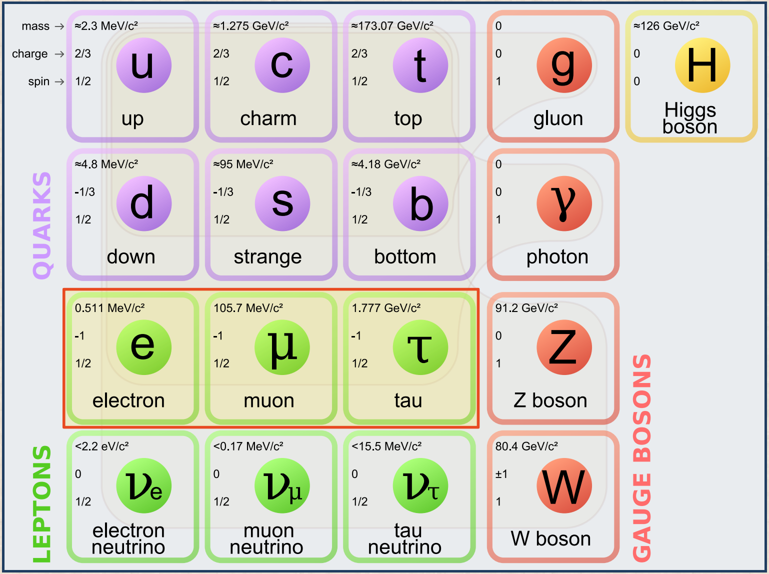
Charged Leptons in the Standard Model



Couplings of Charged Leptons in the Standard Model

Electroweak charges

fermions	$I_{W,L}$	$I_{W,R}$	Y_L	Y_R	$Q = g_{EM}\,[e]$ $= I_{W,L} + Y_L/2$ $= I_{W,R} + Y_R/2$	$g_{W,L}\,[e]$	$g_{Z,L}[e]$	$g_{Z,R}[e]$
$\nu_e,\ \nu_\mu,\ \nu_\tau$	$\frac{1}{2}$	0	-1	0	0	$+\frac{1}{2\sin\theta_W}$	$\frac{I_{W,L} - Q\sin^2\theta_W}{\sin\theta_w\cos\theta_w}$	$\frac{I_{W,R} - Q\sin^2\theta_W}{\sin\theta_w\cos\theta_w}$
$e^-,\ \mu^-,\ \tau^-$	$-\frac{1}{2}$	0	-1	-2	-1	$-\frac{1}{2\sin\theta_W}$	$\frac{I_{W,L} - Q\sin^2\theta_W}{\sin\theta_w\cos\theta_w}$	$\frac{I_{W,R} - Q\sin^2\theta_W}{\sin\theta_w\cos\theta_w}$
$u,\ c,\ t$	$\frac{1}{2}$	0	$+\frac{1}{3}$	$+\frac{4}{3}$	$+\frac{2}{3}$	$+\frac{1}{2\sin\theta_W}$	$\frac{I_{W,L} - Q\sin^2\theta_W}{\sin\theta_w\cos\theta_w}$	$\frac{I_{W,R} - Q\sin^2\theta_W}{\sin\theta_w\cos\theta_w}$
$d,\ s,\ b$	$-\frac{1}{2}$	0	$+\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2\sin\theta_W}$	$\frac{I_{W,L} - Q\sin^2\theta_W}{\sin\theta_w\cos\theta_w}$	$\frac{I_{W,R} - Q\sin^2\theta_W}{\sin\theta_w\cos\theta_w}$

QCD charges

- leptons have no color charge (no strong interation)

Higgs couplings

- charged leptons couplings proportional to mass

Some SM predictions on charged leptons

Lepton Flavour Universality (LFU)

- $g^e_X = g^\mu_X = g^\tau_X \quad [X = \gamma,\ W^\pm,\ Z]$

Very suppressed Lepton Flavour Violation (LFV)

- SM forbids LFV
- when SM is extended to accomodate neutrino mixing (SMν)
  - PNMS neutrino mixing matrix $U_{\ell i}, \quad \ell = e, \mu, \tau, \quad i = 1, 2, 3$
  - very suppressed LFV

e.g. $$\mathcal{B}(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\bar{\nu}_e \nu_\mu(\gamma))} = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{i1}}{m^2_w} \right|^2 \sim 10^{-54}$$

Decay widths

- $\Gamma(\mu \to e\bar{\nu}_e \nu_\mu(\gamma)) = \frac{\hbar}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^2} f(m_e^2/m_\mu^2)(1 + \delta^{\mu e}_{RC})$ (equiv. formulas for $\tau \to \mu/e\bar{\nu}_{\mu/e}\nu_\tau$)

Magnetic moments

- $\mu_\mu = g_\mu \frac{e}{2 m_e c}$ with $\frac{g_\mu - 2}{2} = a_\mu = \frac{\alpha}{2\pi} + \text{higher orders}$

Content of this lecture

- personal selection of important or interesting charged leptons measurements
- motivations (Standard Model tests, searches for New Physics)
- precision of measurements, limitations, prospects for improvements

Muon Lifetime

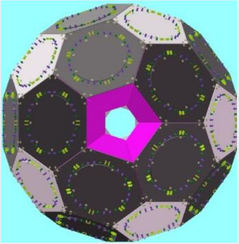
SM Electroweak predictions rely on experimental calibrations

α_{QED} [0.15 ppb]

CODATA 2018

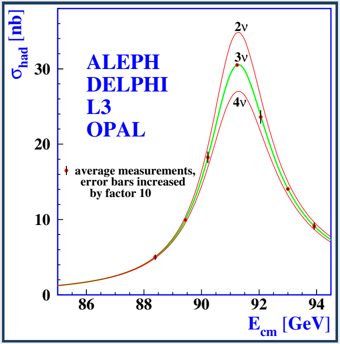
- spectroscopic measurements (Rb, Cs)
- electron magnetic anomaly $a_e = \frac{g_e - 2}{2}$

G_F [0.5 ppm]



- muon lifetime, MuLan 2011, PRL 106 (2011) 041803

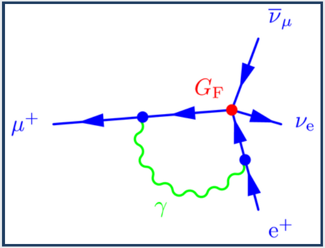
$m(Z^0)$ [23 ppm]



- LEP $e^+e^- \to \gamma, Z \to f\bar{f}$ Phys.Rept. 427 (2006) 257

Muon decay width and G_F

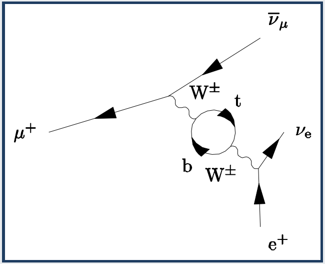
G_F defined as Fermi effective theory coupling



$$\Gamma[\mu \to \nu_\mu e \bar{\nu}_e(\gamma)] = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}(1 + \Delta q)$$

- Δq: radiative corrections to Fermi effective theory (phase space effects included)
  - 2nd order Nucl.Phys.B 564 (2000) 343 [170 ppb]
  - 3rd order Phys.Rev.D 104 (2021) 016003 [31 ppb]

Relation of G_F with EW SM theory



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}(1 + \Delta r)$$

- Δr: radiative corrections to W propagator
  - 2nd order Nucl.Phys.B 564 (2000) 343 [310 ppt]

Muon lifetime measurement, G_F determination

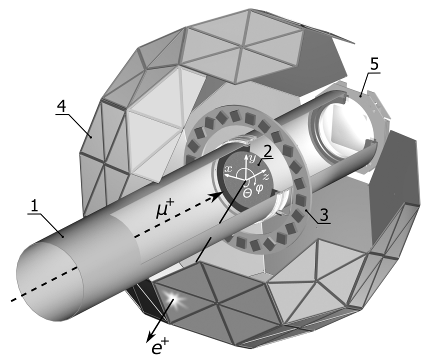
$$G_F = \sqrt{\frac{192\pi^3}{\tau_\mu m_\mu^5 (1+\Delta q)}} = 1.1663788(6)\cdot 10^{-5}\ \text{GeV}^{-2}$$ [500 ppb] PDG 2021

	uncertainty	uncertainty contribution
τ_μ	1000 ppb	500 ppb
m_μ	22 ppb	56 ppb
Δq	31 ppb	15 ppb

- τ_μ, MuLan 2011
- m_μ, CODATA 2018
  - primarily LAMPF 1999, muonium hyperfine splitting spectroscopy, Phys. Rev. Lett. 82 (1999) 711
- Δq, theory, Phys.Rev.D 104 (2021) 016003

Muon lifetime measurement – MuLan 2011

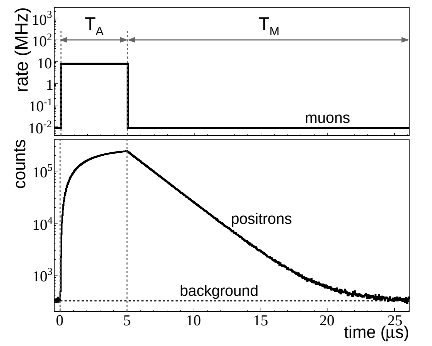
- measures lifetime of low-energy muons stopped on target
- most precise ever measurement of particle or atomic lifetime (1 ppm)
- using polarized μ^+ trapped in muonium with e^-, which have small $\Delta\tau_\mu \sim 1\,\text{ppb}$ (μ^- suffer capture in muonic atoms)
- large (2×10^{12}) sample of 29 MeV muons by PSI accelerator (p on target, π^+, $\pi^+ \to \mu^+\nu_\mu$)

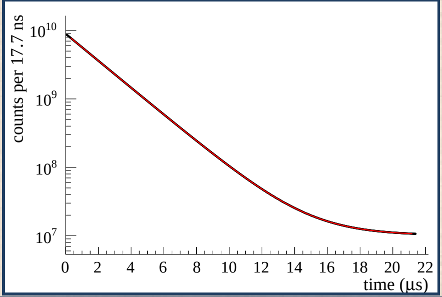


MuLan experiment

1. beam pipe
2. target
3. Halbach arrangement permanent magnet
4. spherical detector made of double plastic scintillator tiles
   - each tile has one inner & one outer identical scintillators
5. beam monitor

Muon lifetime measurement – MuLan 2011





accumulation of muons and observation of muon decays

- 5 μs bunch of muons on target
- 22 μs-long detection of muon-decay positrons
- repetition frequency 30 kHz

two targets to control muon polarization effects

- spin dephasing during accumulation reduces polarization
- permanent-magnet target, high B, 180 MHz precession
- second target, weak B, 1.8 MHz precession

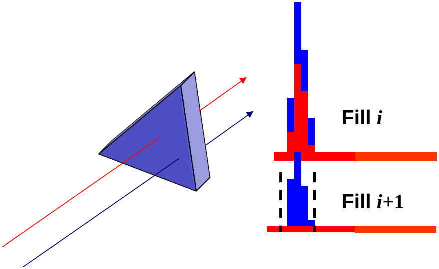
fit

- detect positrons as inner-outer coincidences in same tile
  - on average, 15 decays per cycle
- accumulate positrons in time bins
- correct for pileup precisely estimated using data
- fit time-bin counts with $N(t) = Ne^{-t/\tau_\mu} + C$
  - maximum likelihood fit, Poisson uncertainties

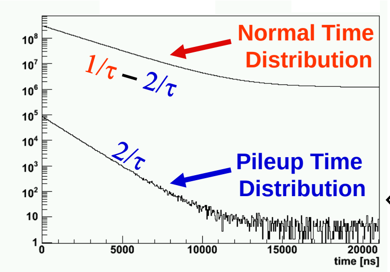
Mulan, pileup subtraction (from D.Hertzog 2014)

τ_{μ^+}

Leading order pileup is a ~5x10^{-4} effect, yet …



- Statistically reconstruct pileup time distribution
- Fit corrected distribution

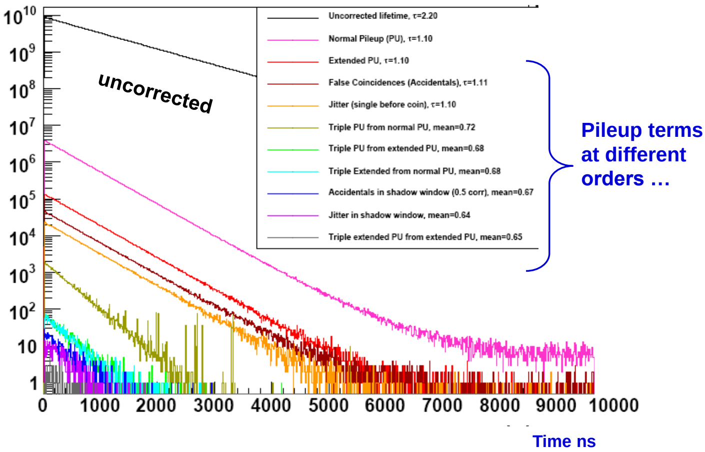


This is only the 1st order effect

Mulan, pileup subtraction (from D.Hertzog 2014)

τ_{μ^+}

Pileup to sub-ppm requires higher-order terms



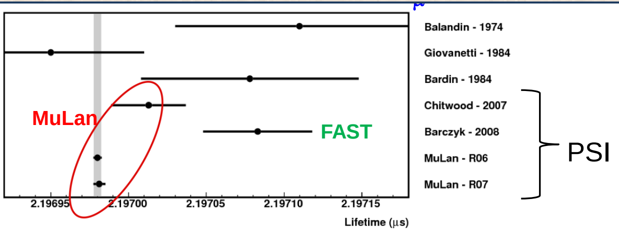
Mulan, result and systematics (from D.Hertzog 2014)

ppm units

Effect	2006	2007	Comment
Kicker extinction stability	0.20	0.07	Voltage measurements of plates
Upstream muon stops	0.10	0.10	Upper limit from measurements
Overall gain stability:	0.25	0.25	MPV vs time in fill; includes:
Timing stability	0.12	0.12	Laser with external reference ctr.
Pileup correction	0.20	0.20	Extrapolation to zero ADT
Residual polarization	0.10	0.20	Long relax; quartz spin cancelation
Clock stability	0.03	0.03	Calibration and measurement
Total Systematic	**0.42**	**0.42**	**Highly correlated for 2006/2007**
Total Statistical	**1.14**	**1.68**	

τ(R06) = 2 196 979.9 ± 2.5 ± 0.9 ps AK-3

τ(R07) = 2 196 981.2 ± 3.7 ± 0.9 ps Quartz



Muon lifetime measurement – MuLan 2011

$\tau_{\mu}(\text{Mulan 2011}) = 2\,196\,980.3 \pm 2.1\,(\text{stat.}) \pm 0.7\,(\text{syst.})\,\text{ps}$ [1.0 ppm]

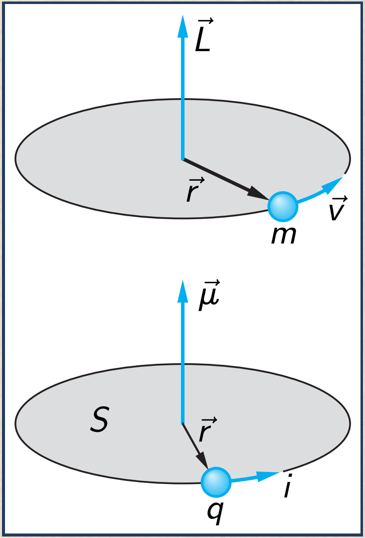
PRL 106 (2011) 041803, Phys.Rev.D 87 (2013) 052003

- not systematically limited
- main systematics
  - detector gain stability within measurement window - measured with data
  - pileup subtraction – measured with data
  - kicker stability in suppressing incoming muons after end of accumulation interval

primary measurement to determine G_F to 0.5 ppm

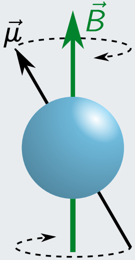
Muon $g-2$

Why "anomalous"? Magnetic momentum of rotating point-like charge



- $i = \dfrac{q}{T} = q\dfrac{v}{2\pi r}$
- $\mu = iS = q\dfrac{v}{2\pi r}\pi r^2 = \dfrac{q}{2}r\dfrac{mv}{m} = \dfrac{q}{2m}L$
- $\vec{\mu} = \dfrac{q}{2m}\vec{L}$

precession of magnetic momentum in magnetic field



Why "anomalous"? Some notation

particle x such as a muon, electron, proton, neutron

- magnetic moment $\vec{\mu}_x = g_x \frac{e}{2m_x} \vec{S}_x$,
  - e = absolute value of electron charge (used also for neutron)
  - $\vec{S}_x$ = spin (particle intrinsic angular momentum)
  - g_x = gyromagnetic ratio (defined also for neutral particles)
- classical charge distribution: $\rho_q / \rho_m = \text{constant} \quad \Rightarrow \quad g = 1$

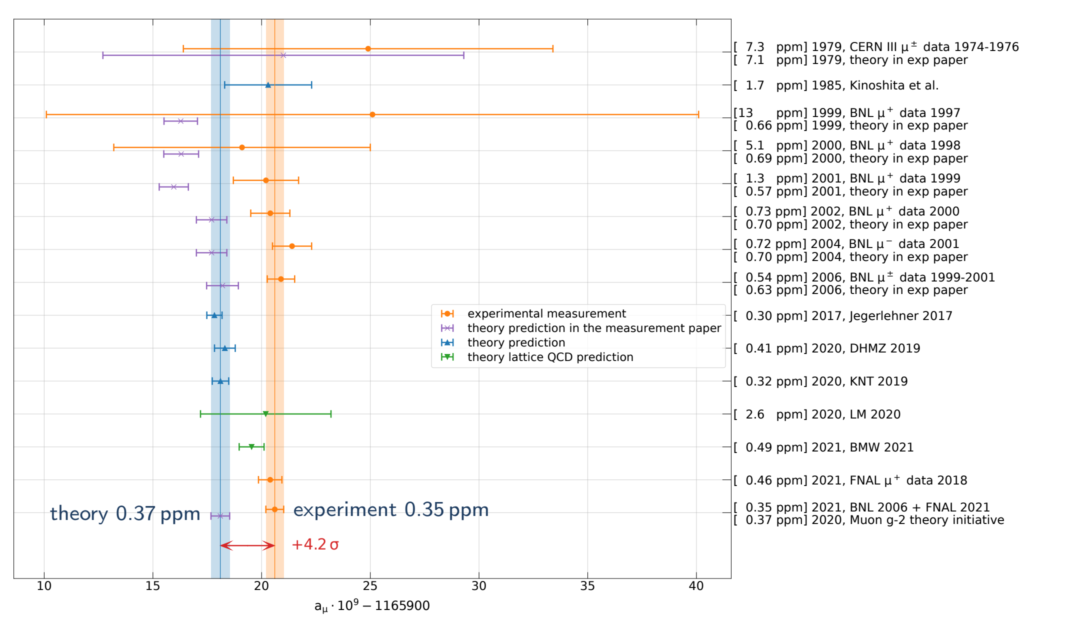
leptons (electron, muon, tau): spin 1/2 fundamental point-like particles

- $g_e,\ g_\mu,\ g_\tau = 2$ at first order and for Dirac equation (note: negative sign omitted for simplicity)
- $g_e,\ g_\mu,\ g_\tau = 2 + \epsilon$ $\quad \epsilon$ due to virtual particles exchanges
- $a_x = \frac{g_x - 2}{2}$ anomalous gyromagnetic ratio or magnetic anomaly
- Standard Model precisely predicts g_x, a_x

proton, neutron

- $g_p \simeq 5.6, \quad g_n \simeq -3.8$
- composite particles made of three quarks (mainly interacting with strong force)
- no precise Standard Model prediction of their magnetic moments
- strong force interactions are non-perturbative at low energy and difficult to compute

a_μ^{exp} vs. a_μ^{th} tests Standard Model to ~0.5 ppm



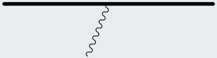
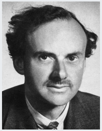
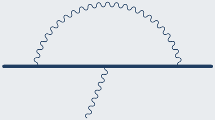
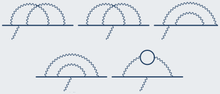
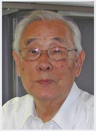
a_μ^{th}: remarkable successful sub-ppm SM perturbative calculation

$a_\mu^{\text{th}} = 116591810(43) \cdot 10^{-11}$ [0.37 ppm]

Muon $g-2$ theory initiative group, Dec 2020
Phys.Rept. 887 (2020) 1-166

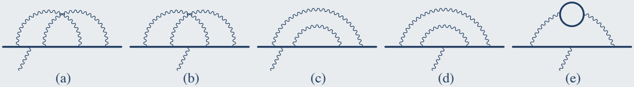
uncertainty contributions		[ppm]
QED	complete calculation to 5th order	0.001
EW	calculation to NLO	0.010
QCD	primarily non-perturbative	
- HVP	up to NNLO, primarily dispersive	0.340
- HLbL	up to NLO, dispersive + lattice QCD	0.150
total		0.370

Contributions to a_μ: QED

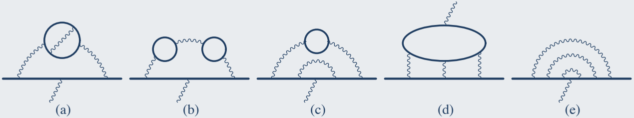
	a_μ contribution $[10^{-11}]$		order		main authors
	0	$\pm$ 0	0th	$(g_\mu{=}2)$	 P. Dirac, 1928
	116 140 973.233	$\pm$ 0.028	1th	$\frac{\alpha_{\text{QED}}}{2\pi}$	 
	413 217.6252	$\pm$ 0.0070	2nd		E. Remiddi *et al.*
	30 141.90226	$\pm$ 0.00033	3rd		 T. Kinoshita *et al.*
	381.004	$\pm$ 0.017	4th		
	5.0783	$\pm$ 0.0059	5th		

Contributions to a_μ: QED graphs

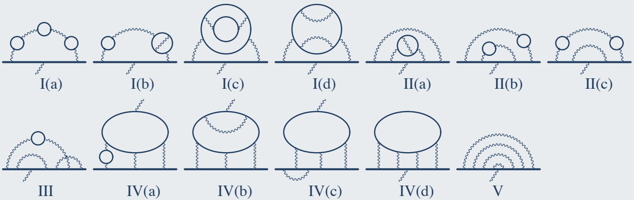
2nd order graphs



3th order graph

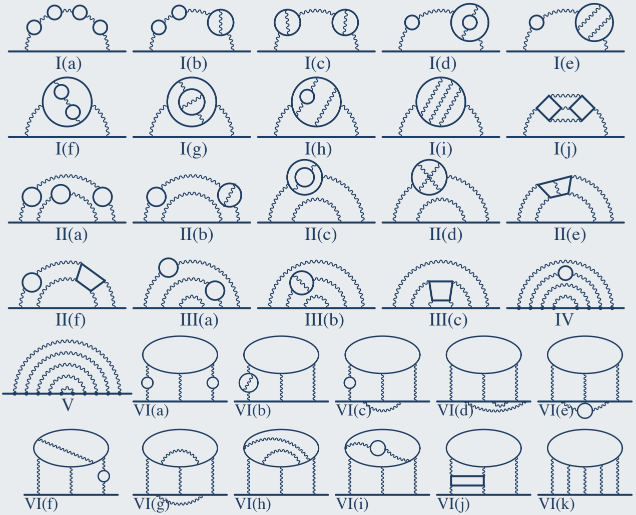


4th order graphs



Contributions to a_μ: QED graphs

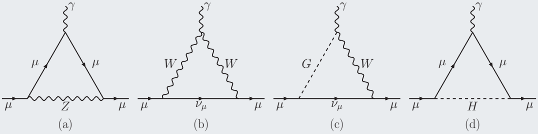
5th order graphs (12672)



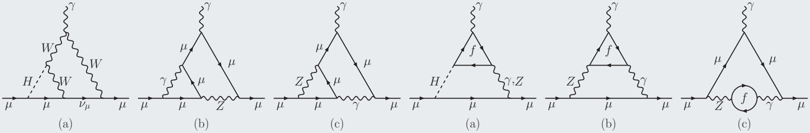
Contributions to a_μ: EW

a_μ contribution $\times \cdot 10^{11}$	order
153.6 ± 0.1	1st + 2nd

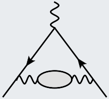
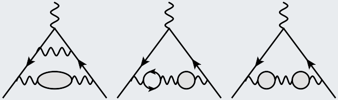
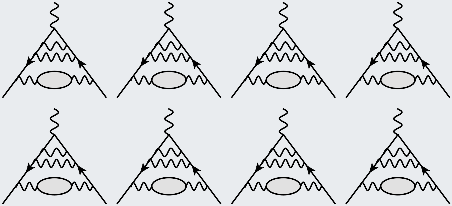
EW graphs, 1st order



EW graphs, 2nd order



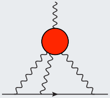
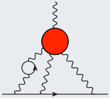
Contributions to a_μ: QCD HVP (hadronic vacuum polarization)

	a_μ contribution $[10^{-11}]$	order
	6931 $\pm$ 40	HLO HVP
	-98.3 $\pm$ 7	HNLO HVP
	12.4 $\pm$ 1	HNNLO HVP

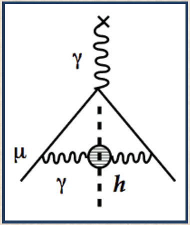
terminology

LO = "leading order", NLO = "next to leading order", NNLO = "next to next to leading order"

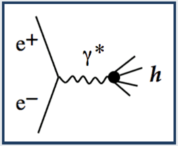
Contributions to a_μ: QCD HLbL (hadronic light-by-light)

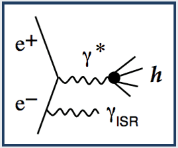
	a_μ contribution $[10^{-11}]$	order
	92 ± 19	HNLO HLbL
	2 ± 1	HNNLO HLbL

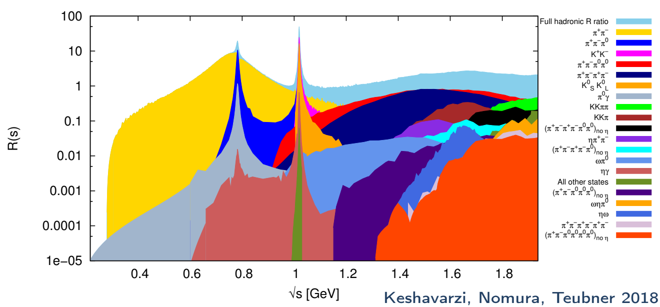
Hadronic contributions calculation



using measurements of $e^+e^- \to$ hadrons cross-section (or $\tau \to$ hadrons ν decay)

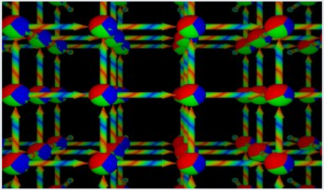




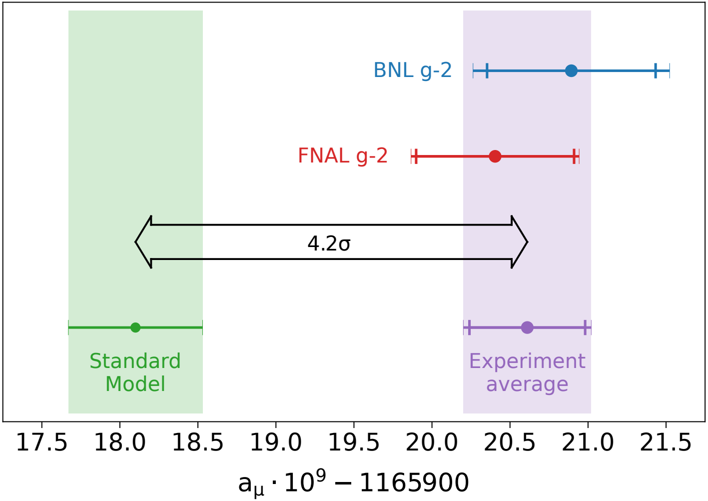


Keshavarzi, Nomura, Teubner 2018

using numeric calculations on discretized space-time (lattice QCD)



Experimental value of $a_\mu = (g_\mu - 2)/2$: BNL 2006, FNAL 2021



+3.66 σ, E821
$1165920.893(630) \cdot 10^{-9}$

+3.34 σ, E989 Run 1
$1165920.405(539) \cdot 10^{-9}$

+4.23 σ, E989 Run 1 + E821
$1165920.611(410) \cdot 10^{-9}$

Muon g-2 theory initiative
$1165918.100(430) \cdot 10^{-9}$

Motion and spin precession of muon in uniform magnetic field

muon spin precession relative to momentum

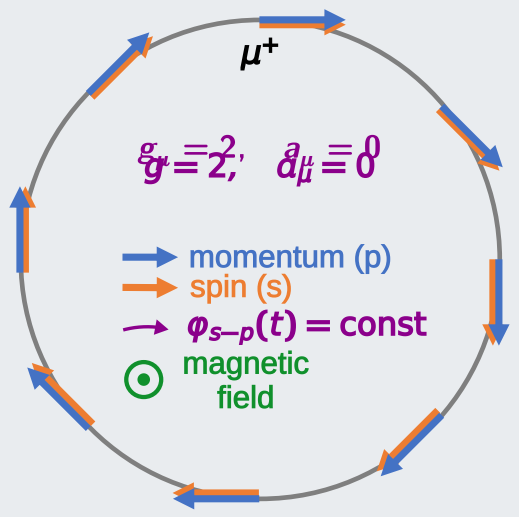
$$\omega_s \quad - \quad \omega_c \quad = \quad \omega_a$$

$$\boxed{-g_\mu \frac{eB}{2m_\mu} - (1-\gamma)\frac{eB}{m_\mu\gamma}} - \boxed{-\frac{eB}{m_\mu\gamma}} = \boxed{-a_\mu \frac{eB}{m_\mu}}$$

Larmor + Thomas precessions — cyclotron frequency — no γ!

- frequency measurements best for precision
- magnetic field NMR measurement also frequency
- angle between momentum and spin $\theta(t) = \omega_a t$

polarized muons in magnetic storage ring



Motion and spin precession of muon in uniform magnetic field

muon spin precession relative to momentum

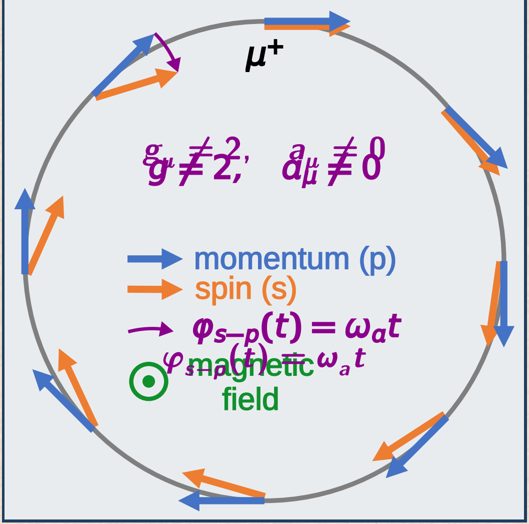
$$\omega_s - \omega_c = \omega_a$$

$$\boxed{-g_\mu \frac{eB}{2m_\mu} - (1-\gamma)\frac{eB}{m_\mu \gamma}} - \boxed{-\frac{eB}{m_\mu \gamma}} = \boxed{-a_\mu \frac{eB}{m_\mu}}$$

Larmor + Thomas precessions — cyclotron frequency — no γ!

- frequency measurements best for precision
- magnetic field NMR measurement also frequency
- angle between momentum and spin $\theta(t) = \omega_a t$

polarized muons in magnetic storage ring



Focusing electric field and magic energy

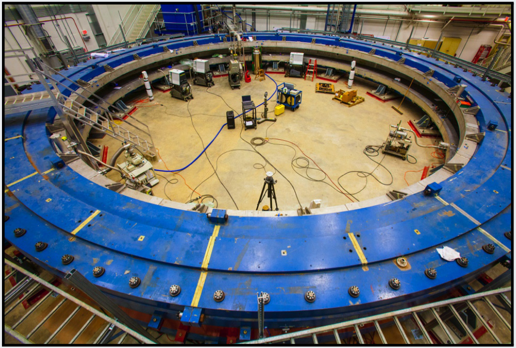
in presence of (focusing) electric field and motion not perfectly transverse to magnetic field

$$\vec{\omega}_a = -\frac{e}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)(\vec{\beta} \times \vec{E}) - a_\mu \frac{\gamma}{\gamma + 1}\left(\vec{\beta} \cdot \vec{B}\right)\vec{\beta}\right]$$

CERN 1975-, BNL, FNAL

$$p_\mu^{\text{magic}} = 3.094\ \text{GeV} \quad \Rightarrow \quad \gamma = 29.3$$

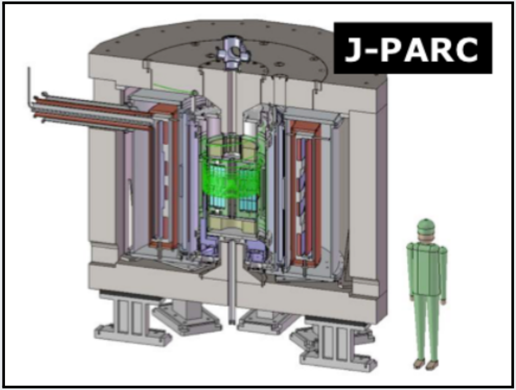
$$\Rightarrow \quad \left(a_\mu - \frac{1}{\gamma^2 - 1}\right) \simeq 0$$



J-PARC E34

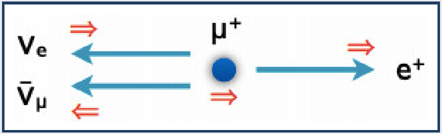
ultra-cold muons

$$E = 0 \quad \Rightarrow \quad \vec{\beta} \times \vec{E} = 0$$

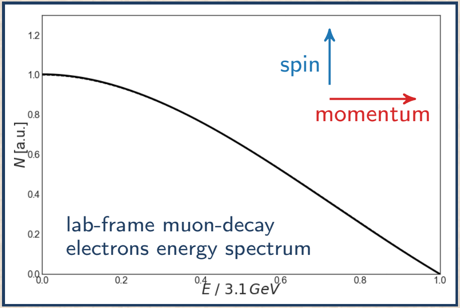


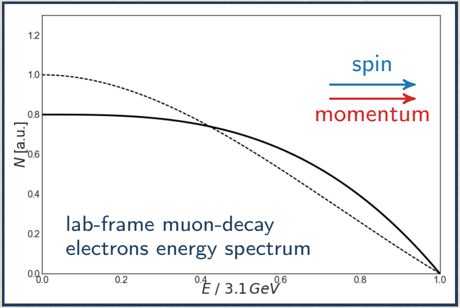
Rate of high-energy muon-decay electrons modulated with $\cos \omega_a t$

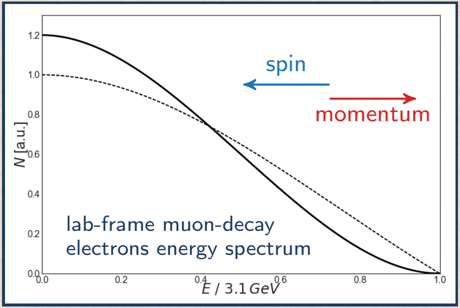
- because of parity violation in muon decay, decay electrons peak along muon spin



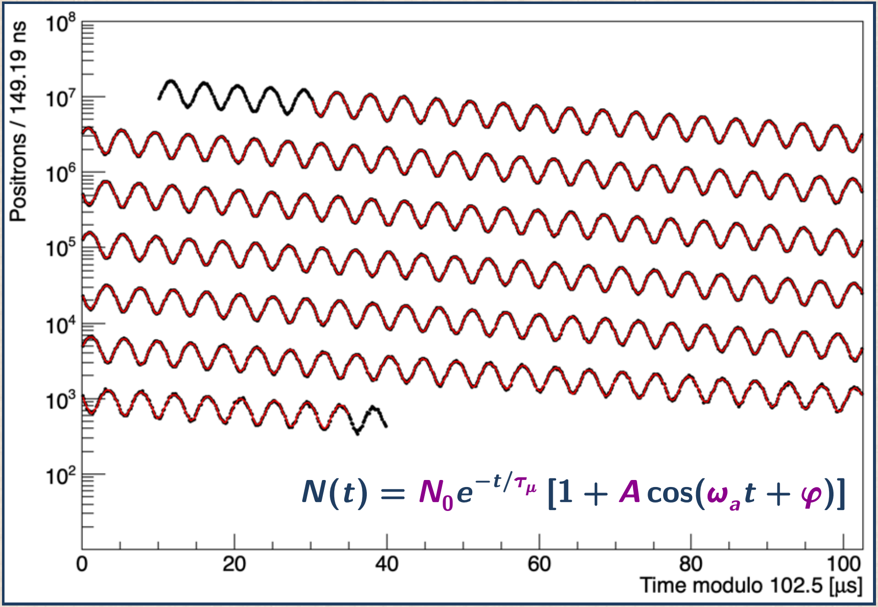
- electrons decaying along muon momentum have highest energy in laboratory frame
- $N_e(E_e > E_t) = N_{e0} e^{-t/\tau_\mu} (1 + A \cos \omega_a t)$







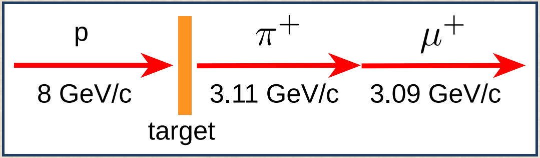
Count muon decays in bins of time ⇒ wiggle plot

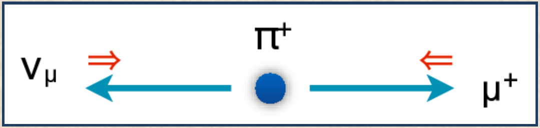


$$N(t) = N_0 e^{-t/\tau_\mu} [1 + A\cos(\omega_a t + \varphi)]$$

Production of polarized muons

- dump 8 GeV protons on target to produce pions
- select pions with momemtum $p \simeq 3.11$ GeV
- let them decay into muons
- in pion rest frame, because of parity violation in pion decay, μ^- spin is aligned with momentum (μ^+ spin is anti-aligned with momentum)
- in laboratory frame, highest energy muons are >90% polarized





- with 8 GeV protons on target, μ^+ are produced $\sim 4\times$ more frequently than μ^-

a_μ measurement: how sub-ppm precision can be obtained

measurement of magnetic field: ω_p

- proton spin precession frequency measures magnetic field (NMR): $\hbar\omega_p = 2\mu_p B$

measurements

- $\omega_a = a_\mu \frac{eB}{m_\mu}, \quad \hbar\omega_p = 2\mu_p B$

spin 1/2 particle x = proton, muon

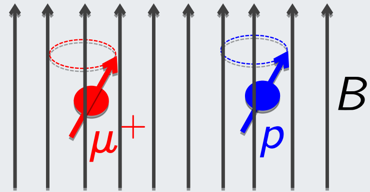
- $S_x = \frac{\hbar}{2}, \quad \mu_x = g_x \frac{e}{m_x} S, \quad a_x = \frac{g_x - 2}{2}$

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$$

muonium & hydrogen hyperfine transitions

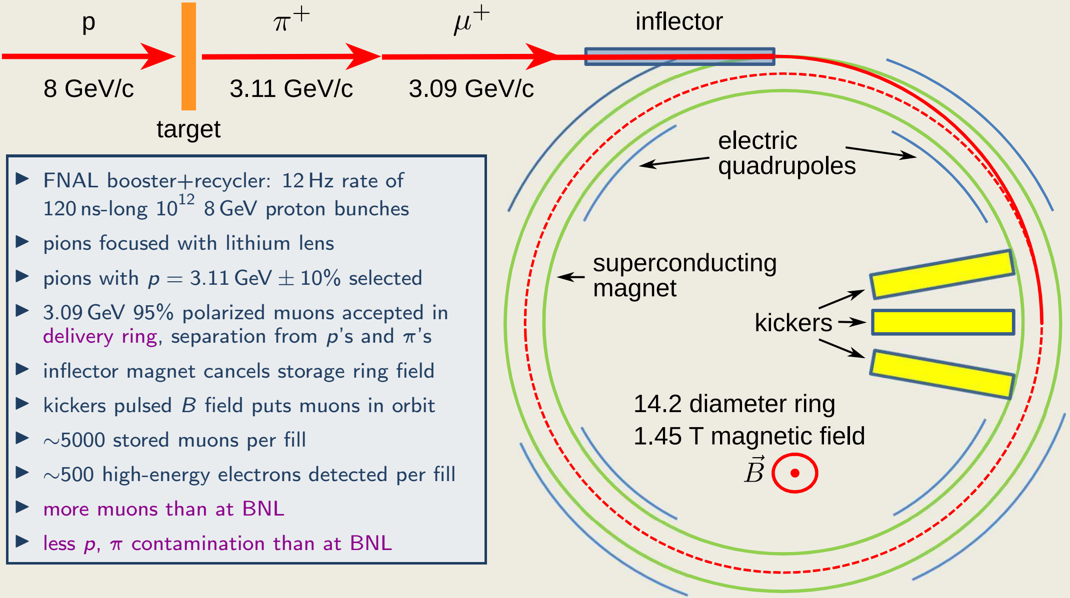
mainly LAMPF 1999 experiment
precision on CODATA 2018 fit: 22 ppb

ω_a & ω_p in same magnetic field



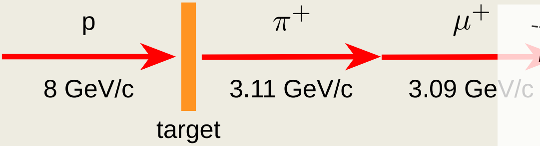
actually, best a_μ obtained by adding ω_a/ω_p measurement to Fundamental Physical Constants CODATA fit

Muon production, storage and decay at E989 FNAL experiment



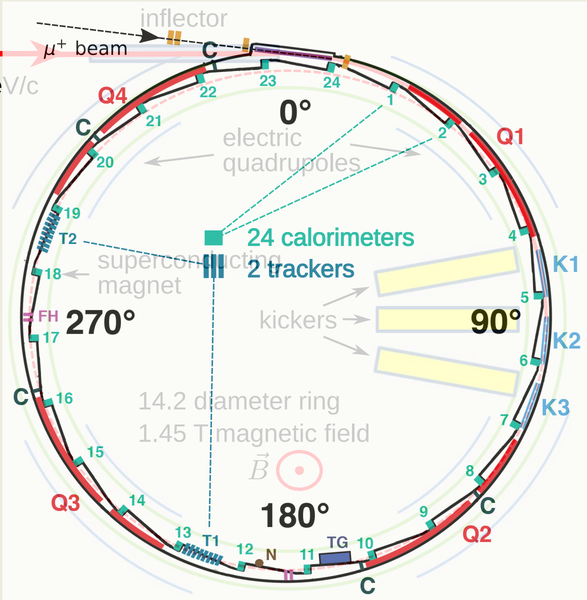
- FNAL booster+recycler: 12 Hz rate of 120 ns-long 10^{12} 8 GeV proton bunches
- pions focused with lithium lens
- pions with $p = 3.11$ GeV $\pm$ 10% selected
- 3.09 GeV 95% polarized muons accepted in delivery ring, separation from p's and π's
- inflector magnet cancels storage ring field
- kickers pulsed B field puts muons in orbit
- $\sim$5000 stored muons per fill
- $\sim$500 high-energy electrons detected per fill
- more muons than at BNL
- less p, π contamination than at BNL

Muon production, storage, decay and detection at FNAL

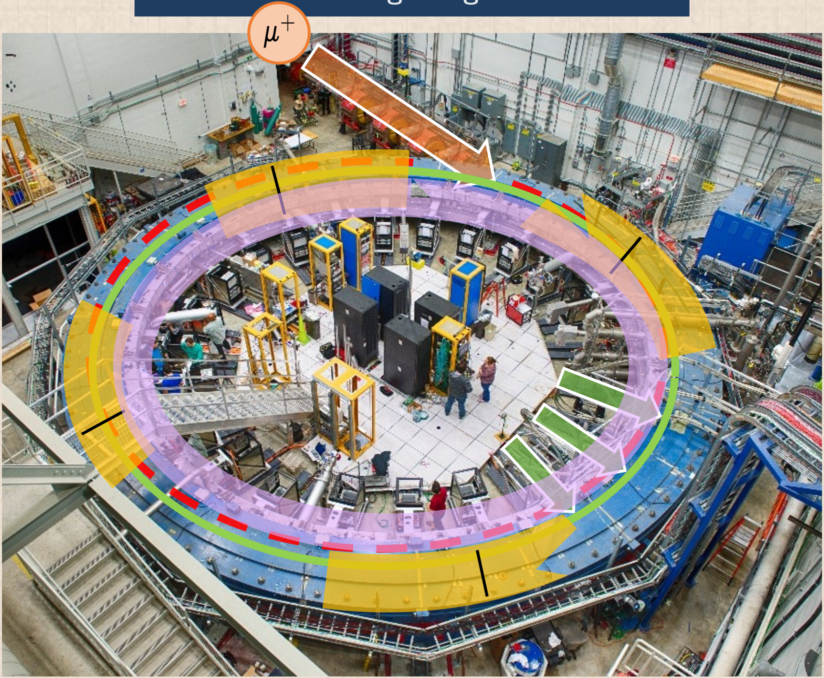


Muon decays detectors

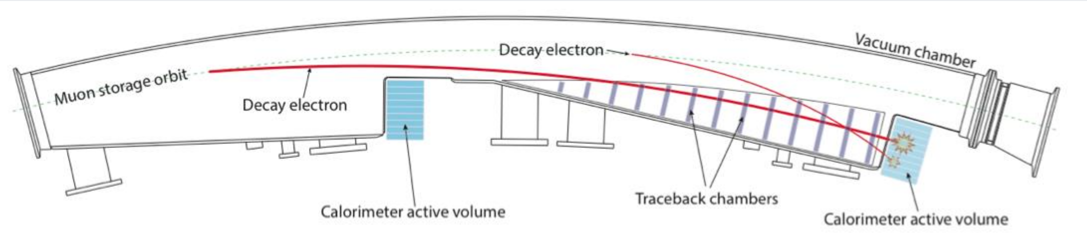
- 24 calorimeters
- 2 trackers



FNAL-E989 storage ring and detectors



Muon decays detectors



- 24 calorimeter modules of 6×9 PbF_2 crystals with 800 MHz-sampling SiPM readout
  - measure muon-decay electrons energy detecting Cherenkov light
  - accurate gain monitoring with laser calibration system
- 2 straw chamber trackers with total of about 1000 channels
  - reconstruct beam distribution inside storage ring from muon decay electrons

comparison with BNL E821 experiment

- more granular calorimeter, faster data acquisition
- improved calorimeter gain monitoring
- improved tracking

Conceptual formula for $R'_\mu(T) = \omega_a / \tilde{\omega}'_p(T)$

$$R'_\mu(T) = \frac{\omega_a}{\tilde{\omega}'_p(T)} \overset{\text{conceptually}}{=} \frac{f_{\text{blind}}\ \omega_a^{\text{m}}\ \left(1 + C_e + C_p + C_{\text{ml}} + C_{\text{pa}}\right)}{f_{\text{calib}}\ \langle \omega'_p(T)(x,y,\varphi) \times M(x,y,\varphi)\rangle\ \left(1 + B_k + B_q\right)}$$

ω_a measurement and corrections

- f_{blind} correction for blinding clock offset
- ω_a^m *measured* precession of muon spin relative to momentum rotation in magnetic field
- C_e ω_a electric field correction
- C_p ω_a pitch correction (vertical beam oscillations)
- C_{ml} ω_a muon loss correction
- C_{pa} ω_a phase acceptance correction

$\tilde{\omega}'_p(T)$ measurement and corrections

- f_{calib} magnetic field probes calibration
- $\omega'_p(T)(x,y,\varphi)$ measured shielded proton spin precession frequency map in storage ring
- $M(x,y,\varphi)$ muon beam distribution
- B_k $\tilde{\omega}'_p(T)$ kicker eddy fields correction
- B_q $\tilde{\omega}'_p(T)$ electric quadrupoles transient field correction

Data analysis

ω_a^m measurement

- 11 different measurements
- by 6 analysis groups
- using 4 different methods to fit rate of muon-decay positrons over time
  - variations of counting positrons exceeding a threshold energy
- blind analysis, different blinding offsets for different groups

$\tilde{\omega}'_p(T)$ measurement

- 2 different measurements
- by 2 analysis groups
- using 2 different methods

Reconstruction of positron energy deposits in calorimeters

readout

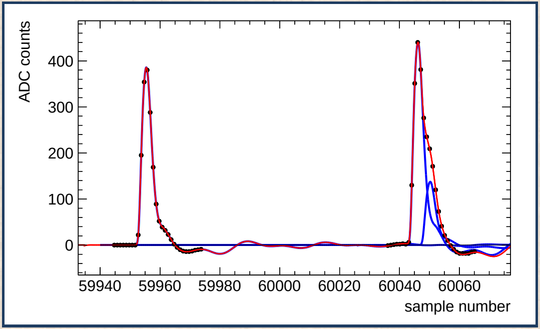
- record SiPM samples for all deposits $> 50\,\mathrm{MeV}$

fit using crystal pulses templates

- get template pulse for each crystal from data
- samples fit to one or more superposed templates

two reconstruction algorithms

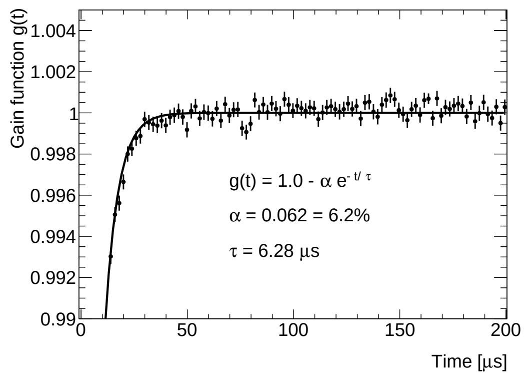
- local: fit individual crystals
- global: global fit over multiple crystals



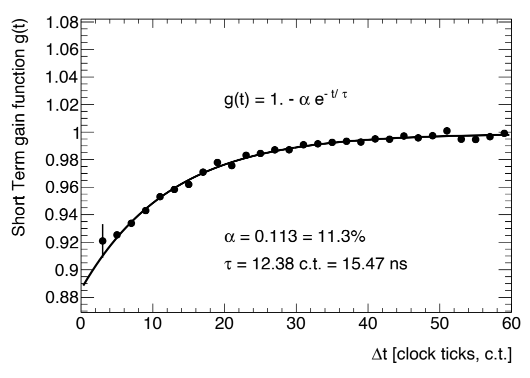
Calorimeter gain variation, measured with laser pulses and corrected

- SiPM gain is reduced by occurrence of preceding hits
- gain monitored by reading back reference laser light pulses injected in PbF_2 crystals
- positron energy measurement from SiPM readout corrected for average measured gain loss

μs time scale SiPM power supply recovery time

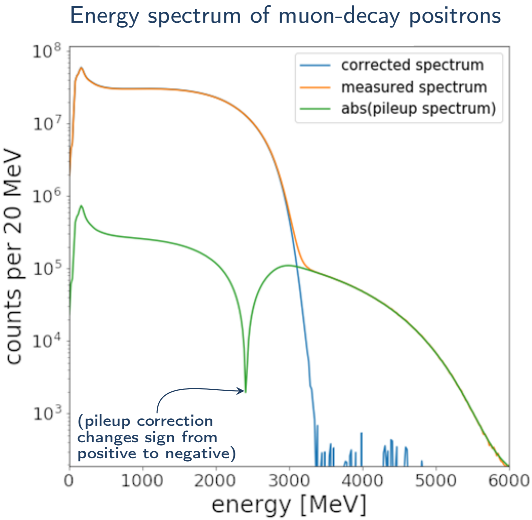


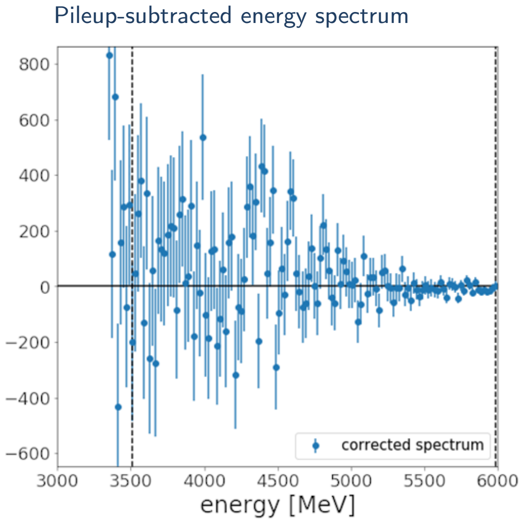
ns time scale SiPM pixel recovery time



Pileup

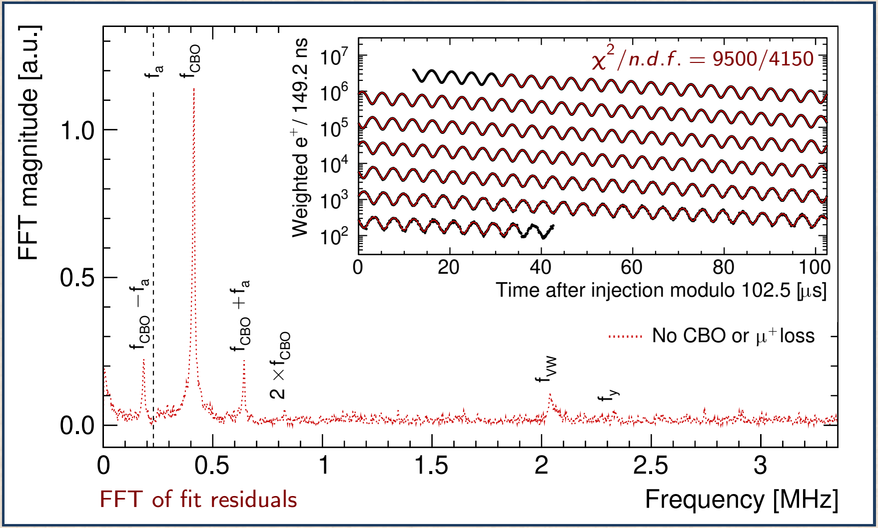
- 2+ decay positrons close in time and space can be reconstructed as a single positron with $E = E_1 + E_2$
- pileup probability in each time bin measured on data in displaced bin and statistically subtracted





Muon precession, 5 parameters ω_a fit model

- number of positron decays with $E > \sim 1.7\,\text{GeV}$, binned over time, from 30 to 650 μs
- fit with $N(t) = N_0 e^{-t/\tau_\mu}\,[1 + A\cos(\omega_a t + \varphi)]$



Muon precession, 22-parameters ω_a fit

- include beam dynamics oscillations of beam position and spread
- include effect of muon loss on collimators

$$N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A \cdot A_{BO}(t)\cos(\omega_a t + \varphi + \varphi_{BO}(t)))\cdot N_{\rm CBO}(t)\cdot N_{\rm VW}(t)\cdot N_y(t)\cdot N_{\rm 2CBO}(t)\cdot \Lambda(t)$$

$$A_{\rm BO}(t) = 1 + A_A\cos(\omega_{\rm CBO}(t)\cdot t + \varphi_A)e^{-\frac{t}{\tau_{\rm CBO}}}$$

$$\varphi_{\rm BO}(t) = A_\varphi\cos(\omega_{\rm CBO}(t)\cdot t + \varphi_\varphi)e^{-\frac{t}{\tau_{\rm CBO}}}$$

$$N_{\rm CBO}(t) = 1 + A_{\rm CBO}\cos(\omega_{\rm CBO}(t)\cdot t + \varphi_{\rm CBO})e^{-\frac{t}{\tau_{\rm CBO}}}$$

$$N_{\rm 2CBO}(t) = 1 + A_{\rm 2CBO}\cos(2\omega_{\rm CBO}(t)\cdot t + \varphi_{\rm 2CBO})e^{-\frac{t}{2\tau_{\rm CBO}}}$$

$$N_{\rm VW}(t) = 1 + A_{\rm VW}\cos(\omega_{\rm VW}(t)\cdot t + \varphi_{\rm VW})e^{-\frac{t}{\tau_{\rm VW}}}$$

$$N_y(t) = 1 + A_y\cos(\omega_y(t)\cdot t + \varphi_y)e^{-\frac{t}{\tau_y}}$$

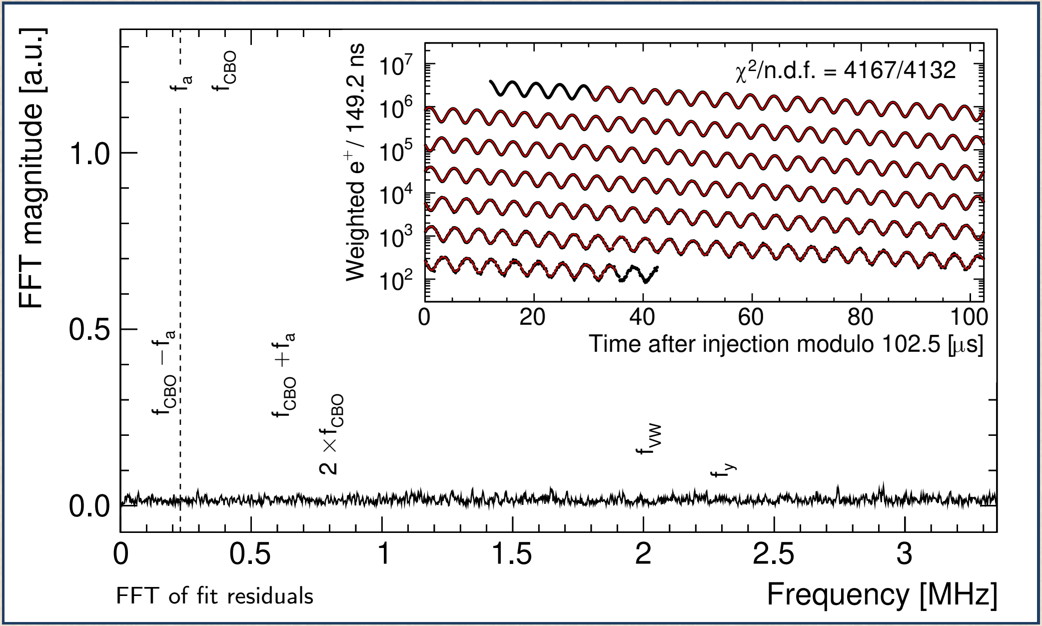
$$\Lambda(t) = 1 - k_{LM}\int_{t_0}^{t} L(t')e^{t'/\tau}\,dt'$$

$$\omega_{\rm CBO}(t) = \omega_0^{CBO} + \frac{A}{t}e^{-\frac{t}{\tau_A}} + \frac{B}{t}e^{-\frac{t}{\tau_B}}$$

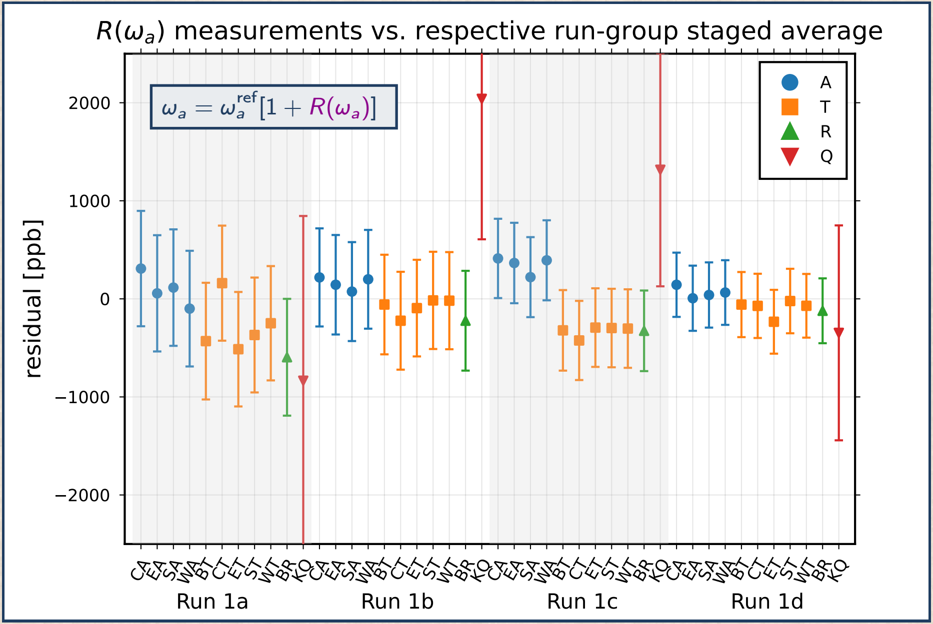
$$\omega_y(t) = F\omega_{\rm CBO}(t)\sqrt{2\omega_c/F\omega_{\rm CBO}(t) - 1}$$

$$\omega_{\rm VW}(t) = \omega_c - 2\omega_y(t)$$

22 parameters ω_a fit has χ^2/n.d.o.f. consistent with 1

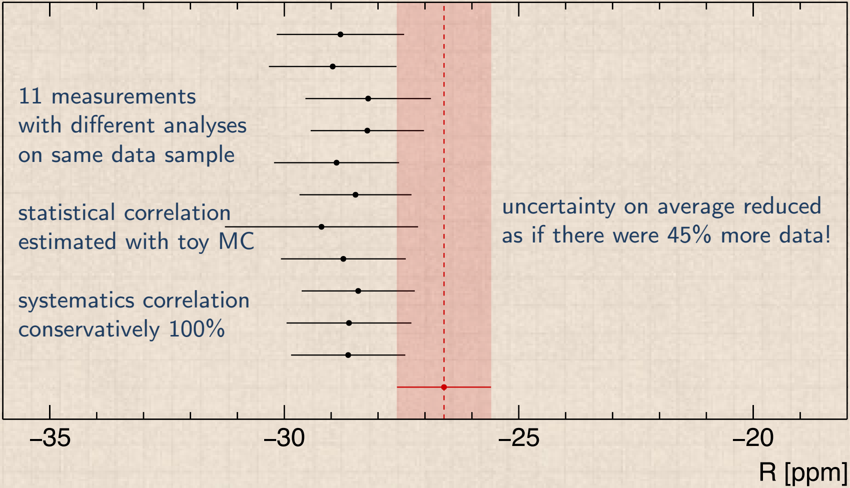


In each of 4 datasets, 11 ω_a measurements are consistent between each-other



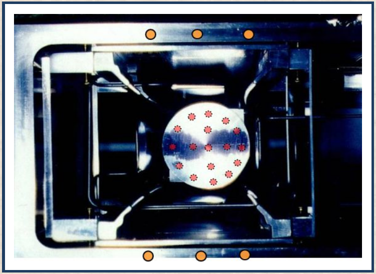
Averaging correlated measurements susceptible to correlations's estimation

- minimum χ^2 average of ∼critically correlated measurements very sensitive to imperfect correlations
- known statistical issue, can produce unrealistic fit result average and uncertainty

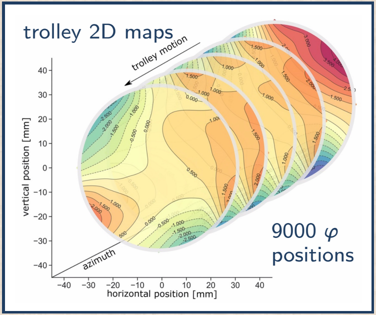


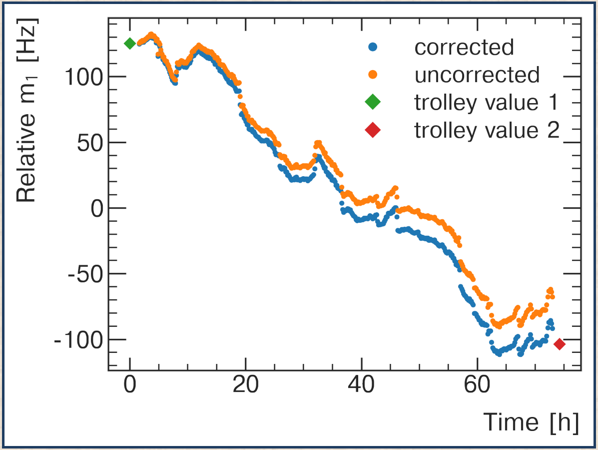
- solved using even weights, slightly suboptimal but stable

Measuring ω_p / magnetic field with fixed and trolley probes



- 378 fixed probes measure continuosly the magnetic field
- 17-probes trolley run along muons path every ~3 days
- fixed probes measurements corrected using trolley measurements





Measuring ω_p magnetic field: calibration of probes

calibration

- each trolley probe calibrated with absolute cylindrical probe placed in the same position inside the storage ring
- absolute cylindrical probe calibrated to reference absolute spherical probe in MRI magnet at Argonne National Laboratory
- absolute spherical probe consistent with novel absolute ^{3}He probe
- 17 probes calibration uncertainty 20 – 48 ppb

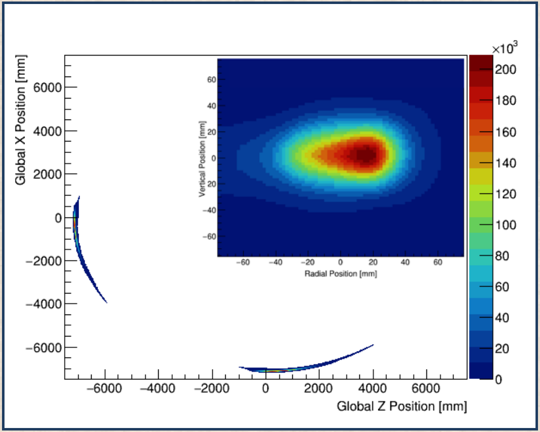
reference temperature

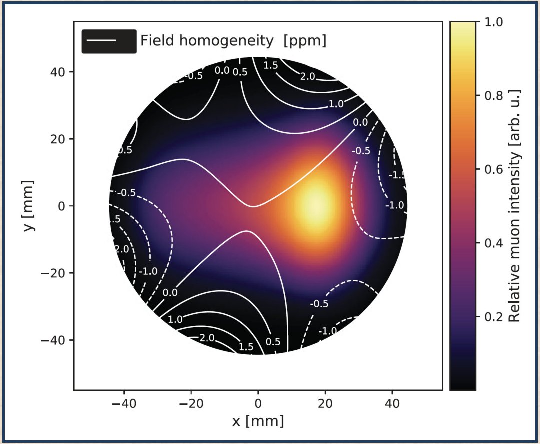
- magnetic field measurements corrected to be expressed as $\omega'_p(T)$, precession frequency of shielded proton spin in spherical water sample at reference temperature of 34.7 °C

absolute spherical probe

$\tilde{\omega}'_p(T)$ (magnetic field experienced by the muons) measured to 56 ppb

- tracker reconstructs muons decay vertices in parts of storage region
- bean dynamics simulation used to extrapolate to whole storage region
- magnetic field map averaged over muon distribution
- two independent groups did the measurement, one additional group the calibration





FNAL E989 a_μ measurement corrections and uncertainties

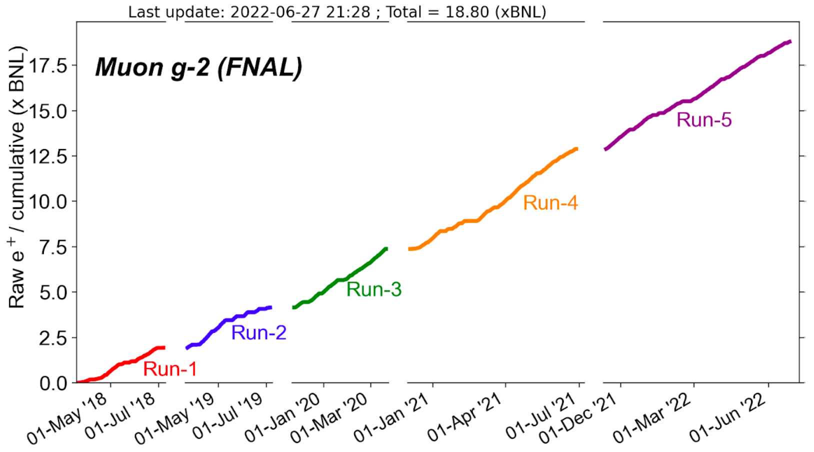
	Correction	Uncertainty	Design goal
ω_a^m (statistical)	–	434	100
ω_a^m (systematic)	–	56	
base clock	–	2	
C_e	489	53	
C_p	180	13	
C_{ml}	-11	5	
C_{pa}	-158	75	
ω_a beam dynamics corrections ($C_e + C_p + C_{ml} + C_{pa}$)	499	93	
ω_a total systematic	499	109	70
$\omega'_p(T)(x, y, \varphi)$	–	54	
$M(x, y, \varphi)$	–	17	
$\langle\omega'_p(T)(x, y, \varphi) \times M(x, y, \varphi)\rangle$	–	56	
B_q	-17	92	
B_k	-27	37	
$\tilde{\omega}'_p(T)$ transient fields corrections ($B_q + B_k$)	-44	99	
$\tilde{\omega}'_p(T)$ total [note: correction sign now for $\omega_a/\tilde{\omega}'_p(T)$]	44	114	70
$\omega_a/\tilde{\omega}'_p(T)$ total systematic	544	157	100
external measurements	–	25	
total [correction is for $\omega_a/\tilde{\omega}'_p(T)$]	544	462	140

FNAL E989 a_μ measurement main uncertainties

- statistically limited by number of observed muon decays
- phase-acceptance systematics limited by
  - understanding of beam transverse distribution and dependent measured muon spin phase
- measurement / estimate of transient magnetic fields due to
  - kickers operations
  - cycling of quadrupoles electric field
- several other non-negligible systematics contributions

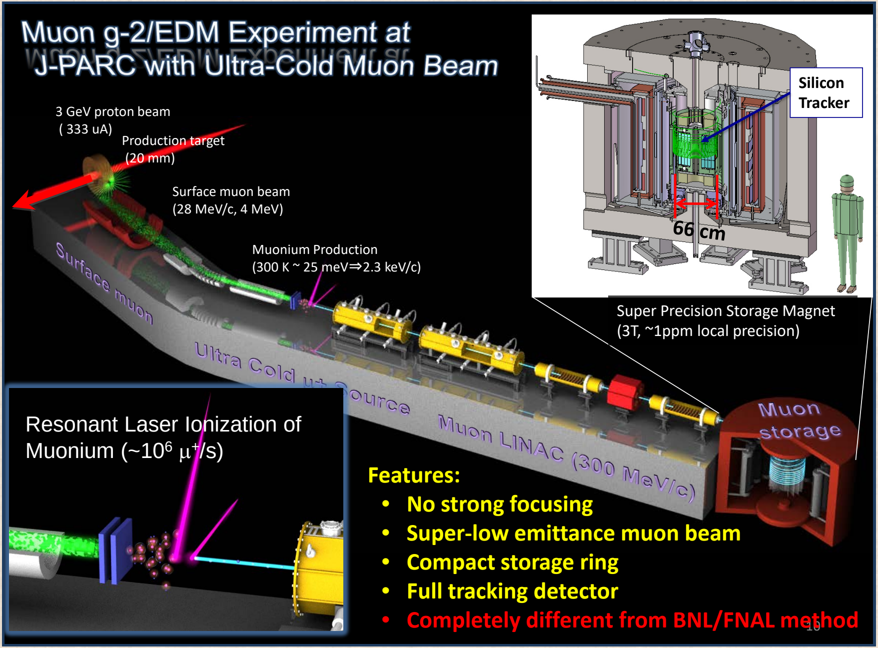
FNAL-E989 a_μ measurement prospects

FNAL-E989 data samples



- Run 1 result 0.46 ppm
- additional data may be collected if Run 6 is approved
- design goal 0.14 ppm may be exceeded in a few years time

J-PARC E34 Muon $g-2$ experiment [figure by K. Ishida, 2019]



J-PARC E34 Muon $g-2$ experiment

Summary of statistics and uncertainties

	Estimation
Total number of muons in the storage magnet	5.2×10^{12}
Total number of reconstructed e^+ in the energy window [200, 275 MeV]	5.7×10^{11}
Effective analyzing power	0.42
Statistical uncertainty on ω_a [ppb]	450
Uncertainties on a_μ [ppb]	450 (stat.)
	< 70 (syst.)
Uncertainties on EDM [10^{-21} $e\cdot$cm]	1.5 (stat.)
	0.36 (syst.)

Estimated systematic uncertainties on a_μ (first phase, approved)

Anomalous spin precession (ω_a)		Magnetic field (ω_p)	
Source	Estimation (ppb)	Source	Estimation (ppb)
Timing shift	< 36	Absolute calibration	25
Pitch effect	13	Calibration of mapping probe	20
Electric field	10	Position of mapping probe	45
Delayed positrons	0.8	Field decay	< 10
Diffential decay	1.5	Eddy current from kicker	0.1
Quadratic sum	< 40	Quadratic sum	56

PTEP 2019 (2019), 053C02

Muon Lepton Flavour Violation searches

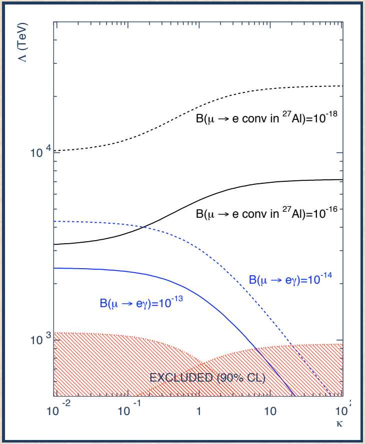
Muon Lepton Flavour Violation

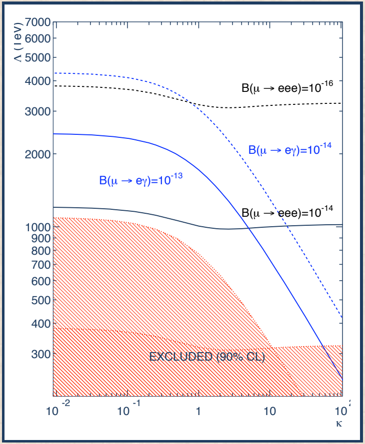
- effective theory for $\mu \to e\gamma$, $\mu \to eee$, $\mu N \to eN$ [Prog.Part.Nucl.Phys. 71 (2013) 75]

$$\mathcal{L}_{\text{CLFV}} = \begin{cases} \dfrac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R \sigma_{\mu\nu} e_L F^{\mu\nu} + h.c. + \\ \dfrac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L \gamma_\mu e_L \left(\bar{u}_L\gamma^\mu u_L + \bar{d}_L \gamma^\mu d_L\right) + h.c. \end{cases}$$

dipole term: SUSY, ...
$\mu \to e\gamma$, $\mu \to eee$

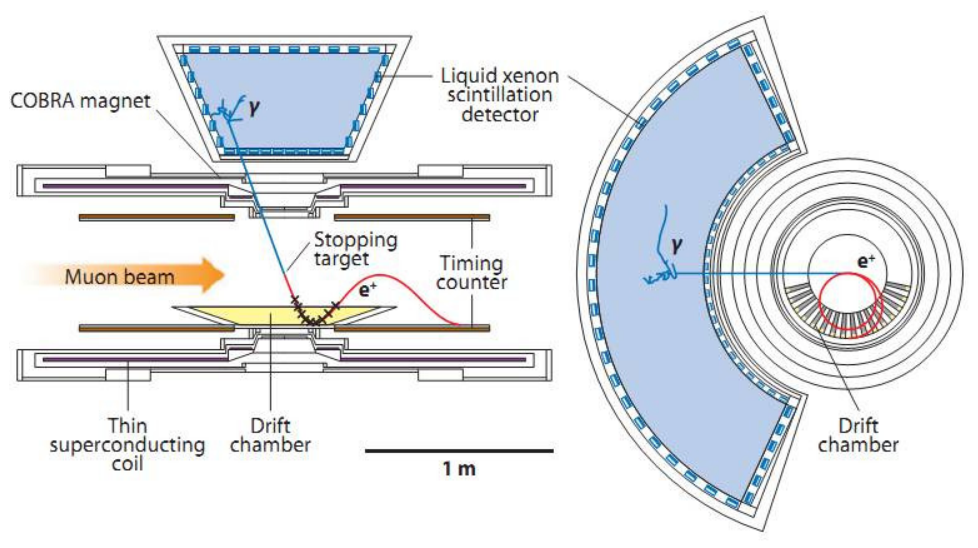
contact term, Z', leptoquark
$\mu N \to eN$, $\mu \to eee$
only via loops: $\mu \to e\gamma$





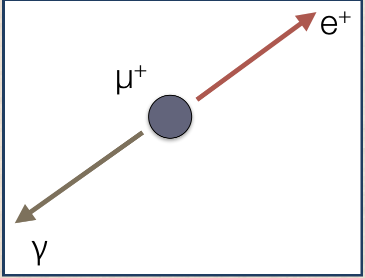
MEG experiment at PSI, search for $\mu \to e\gamma$

- PSI 28 MeV/c-momentum surface μ^+ beam, rate up to $R_\mu = 10^8$/s



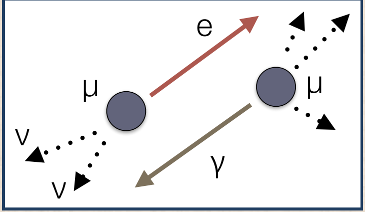
MEG experiment at PSI, search for $\mu \to e\gamma$

signal



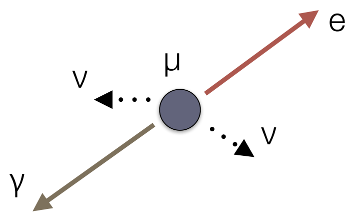
- $N \propto R_\mu$
- $E_\gamma = 52.8\,\text{MeV}$
- $E_{e+} = 52.8\,\text{MeV}$
- $\alpha_{e\gamma} = 180^\circ$
- $T_{e\gamma} = 0$

Accidental (Acc) background



- $N \propto R_\mu^2$
- $E_\gamma < 52.8\,\text{MeV}$
- $E_{e+} < 52.8\,\text{MeV}$
- $\alpha_{e\gamma} < 180^\circ$
- $T_{e\gamma} = \text{flat}$

Radiative Muon Decay (RMD)



- $N \propto R_\mu$
- $E_\gamma < 52.8\,\text{MeV}$
- $E_{e+} < 52.8\,\text{MeV}$
- $\alpha_{e\gamma} < 180^\circ$
- $T_{e\gamma} = 0$

note: in the data analysis $\theta_{e\gamma} = 180^\circ$ is simplified form of $\theta_{e\gamma} = 0$ and $\phi_{e\gamma} = 0$,
with $\theta_{e\gamma} = (\pi - \theta_e) - \theta_\gamma$ and $\phi_{e\gamma} = (\pi + \phi_e) - \phi_\gamma$

MEG experiment at PSI, search for $\mu \to e\gamma$

optimization

- optimal $R_\mu = 3{\cdot}10^7$ /s to suppress accidental background $\propto R_\mu^2$ [max $R_\mu = 1{\cdot}10^8$ /s]

analysis

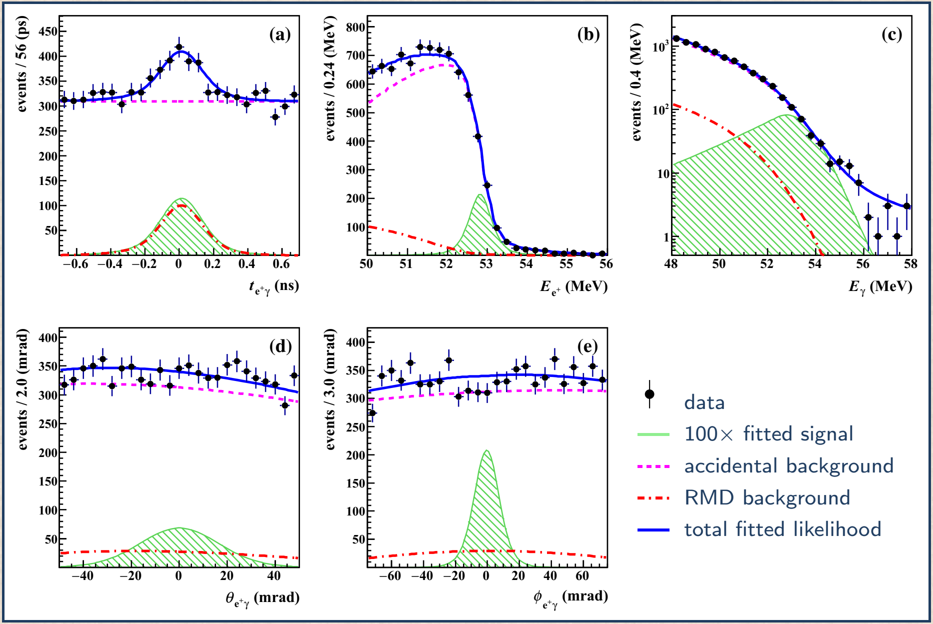
- loose selection of e^+, γ candidates with compatible detection times
- extended maximum likelihood fit with (simplified expression):

$$L(N_{\text{sig}}, N_{\text{RMD}}, N_{\text{acc}}) = \frac{e^{-(N_{\textbf{sig}}+N_{\textbf{RMD}}+N_{\textbf{acc}})}}{N_{\text{obs}}!} \prod_{1}^{N_{\textbf{obs}}} \left(N_{\text{sig}} F_{\text{sig}}(x_i) + N_{\text{RMD}} F_{\text{RMD}}(x_i) + N_{\text{acc}} F_{\text{acc}}(x_i)\right)$$

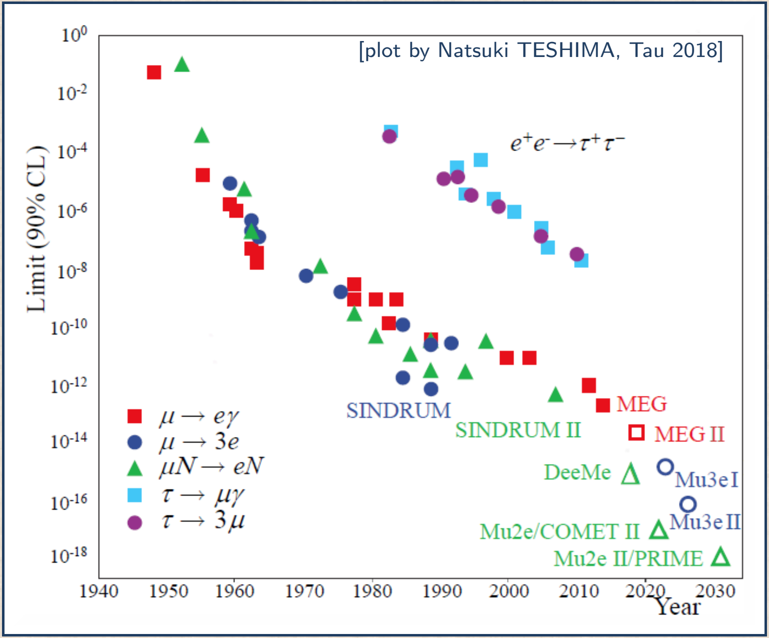
N_{obs} = number of events in signal window; $x_i = E_\gamma,\quad E_{e+},\quad \theta_{e\gamma},\quad \phi_{e\gamma},\quad T_{e\gamma}$

- N_{RMD} and N_{acc} constrained to observed events in sidebands with Gaussian terms (not shown above)
- 90% confidence interval of N_{sig} using Feldman-Cousins approach with profile-likelihood ratio ordering
- N_μ in signal acceptance computed by counting Michel and radiative muon decays
- $1/N_\mu$(signal acceptance) $= (5.84 \pm 0.21){\cdot}10^{-14}$ (single event sensitivity)
- no significant signal found $\mathcal{B}(\mu \to e\gamma) < 4.2{\cdot}10^{-13}$ at 90% CL Eur.Phys.J.C 76 (2016) 434

MEG experiment at PSI, likelihood projections



Muon LFV searches results and prospects



Multiple dedicated projects will search for muon LFV in near future

$\mu \to e\gamma$

- MEG-II at (PSI, Switzerland), $\sim 10\times$ more sensitive than MEG, SES$\sim < 6{\cdot}10^{-14}$

$\mu \to eee$

- Mu3e (PSI, Switzerland), $\mathcal{B} < 1{\cdot}10^{-16}$

$\mu N \to eN$

- DeeMe, C target, 1 year (JPARC, Japan), SES $\sim 1{\cdot}10^{-13}$
- DeeMe, SiC target, 4 years (JPARC, Japan), SES $\sim 5{\cdot}10^{-15}$
- Mu2e (FNAL, USA), SES $\sim 3{\cdot}10^{-17}$
- COMET-1 (JPARC, Japan), SES $\sim 3{\cdot}10^{-15}$
- COMET-2 (JPARC, Japan), SES $\sim 3{\cdot}10^{-17}$
- PRISM/PRIME (JPARC, Japan), SES $\sim 1{\cdot}10^{-18}$

note: $\mu N \to eN$ signature is 1 electron with $E_e \simeq m_\mu$ and does not suffer R_μ^2 accidental background

Lepton Universality tests with tau decays

Tau decay widths

decay width of τ decaying to μ and two neutrinos

$$\Gamma[\tau \to \nu_\tau \mu \bar{\nu}_\mu(\gamma)] = \Gamma_{\tau\mu} = \Gamma_\tau \mathcal{B}_{\tau\mu} = \frac{\mathcal{B}_{\tau\mu}}{\tau_\tau} = \frac{G_{F\tau} G_{F\mu} m_\tau^5}{192\pi^3} f\left(m_\mu^2/m_\tau^2\right) (1+\delta_W^{\tau\mu})(1+\delta_\gamma^\tau)$$

- expression for $\Gamma[\tau \to \nu_\tau e \bar{\nu}_e(\gamma)]$ replacing μ with e
- expression for $\Gamma[\mu \to \nu_\mu e \bar{\nu}_e(\gamma)]$ replacing τ with μ, $\mathcal{B}(\mu \to \nu_\mu e \bar{\nu}_e(\gamma)) = 1$
- $G_{F\tau} = \dfrac{g_\tau^2}{4\sqrt{2}M_W^2}$ Fermi effective theory constant, proportional to weak coupling squared [here without the weak corrections, different convention]
- experimental precision on $\mathcal{B}(\tau \to \ell \bar{\mu}_\ell \nu_\tau)$: [0.23%]
- $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ phase space [$\Delta \leq 3.7$ ppm, negligible]
- $\delta_W^{\tau\mu} = \dfrac{3}{5}\dfrac{m_\tau^2}{M_W^2} + \dfrac{9}{5}\dfrac{m_\mu^2}{M_W^2}$ weak radiative correction [$\Delta \leq 97$ ppb, negligible]
- $\delta_\gamma^\tau = \dfrac{\alpha(m_\tau)}{2\pi}\left(\dfrac{25}{4} - \pi^2\right)$ electro-magnetic radiative correction [Δ negligible]

all references and some more details in Heavy Flavour Averaging Group (HFLAV) reports, tau section
most recent preprint arXiv:1909.12524, web report

Lepton universality tests

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e}(1+\delta_\gamma^\mu)(1+\delta_W^{\mu e})}{\tau_\tau m_\tau^5 f_{\tau e}(1+\delta_\gamma^\tau)(1+\delta_W^{\tau e})}} = 1.0009 \pm 0.0014 = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\tau e}^{\text{SM}}}}$$ [HFLAV Winter 2022]

$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e}(1+\delta_\gamma^\mu)(1+\delta_W^{\mu e})}{\tau_\tau m_\tau^5 f_{\tau \mu}(1+\delta_\gamma^\tau)(1+\delta_W^{\tau \mu})}} = 1.0027 \pm 0.0014 = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau \mu}^{\text{SM}}}}$$ [HFLAV Winter 2022]

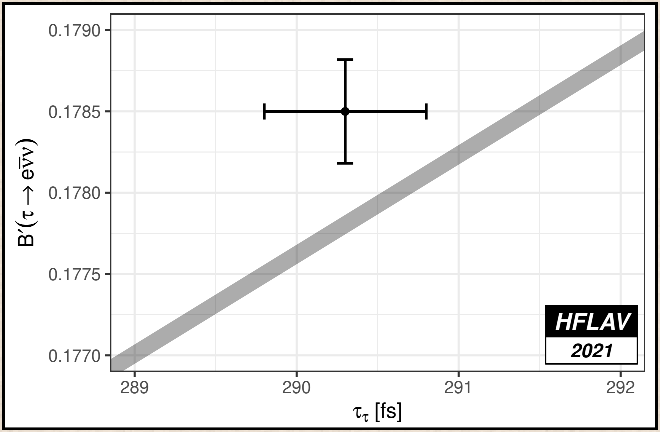
$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau e}} \frac{f_{\tau e}}{f_{\tau \mu}} \frac{(1+\delta_W^{\tau e})}{(1+\delta_W^{\tau \mu})}} = 1.0019 \pm 0.0014$$ [HFLAV Winter 2022]

- not independent! (3rd one is the ratio of the first two)
- 0.14% precision, uncertainties dominated by uncertainties on m_τ, τ_τ, $\mathcal{B}_{\tau e}$, $\mathcal{B}_{\tau \mu}$
- sizeable but precisely known EW radiative corrections

A.Crivellin notation

- $R_{\tau\mu}$ $2\,\sigma$ excess corresponds to $\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau\mu}}{\mathcal{B}_{\tau\mu}^{\text{SM}}}}$ $2\,\sigma$ excess

Canonical tau lepton universality test plot



$(g_\tau/g_{e\mu}) = 1.0018 \pm 0.0013$
$[g_{e\mu} = g_e = g_\mu$ assuming $g_e = g_\mu]$

$\Delta(g_\tau/g_{e\mu})$ contributions

input	Δinput	$\Delta(g_\tau/g_{e\mu})$
$\mathcal{B}'_{\tau\to e}$	0.180%	0.090%
τ_τ	0.172%	0.086%
m_τ	0.007%	0.017%
total		0.126%

best measurements

$\mathcal{B}'_{\tau\to e}$	ALEPH
τ_τ	Belle
m_τ	BES III

- $\mathcal{B}'(\tau\to e\bar\nu\nu)$ = average of $\begin{cases}\mathcal{B}(\tau\to e\bar\nu\nu)\\ \mathcal{B}(\tau\to \mu\bar\nu\nu)\frac{f_{\tau e}}{f_{\tau\mu}}\frac{(1+\delta_W^{\tau e})}{(1+\delta_W^{\tau\mu})}\end{cases}$
- $$\frac{\mathcal{B}'(\tau\to e\bar\nu\nu)\tau_\mu}{\mathcal{B}(\mu\to e\bar\nu\nu)\tau_\tau} = \frac{g_\tau^2}{g_{e\mu}^2}\frac{m_\tau^5 f_{\tau e}(1+\delta_\gamma^\tau)(1+\delta_W^{\tau e})}{m_\mu^5 f_{\mu e}(1+\delta_\gamma^\mu)(1+\delta_W^{\mu e})}$$
- $$\left(\frac{g_\tau}{g_{e\mu}}\right)^2 = \frac{\mathcal{B}'(\tau\to e\bar\nu\nu)}{\mathcal{B}(\mu\to e\bar\nu\nu)}\frac{\tau_\mu}{\tau_\tau}\frac{m_\mu^5}{m_\tau^5}\frac{f_{\mu e}(1+\delta_\gamma^\mu)(1+\delta_W^{\mu e})}{f_{\tau e}(1+\delta_\gamma^\tau)(1+\delta_W^{\tau e})}$$

Lepton universality tests with decays involving pseudoscalar mesons

$$\frac{\Gamma(\tau^- \to \nu_\tau P^-)}{\Gamma(P^- \to \ell^- \bar{\nu}_\ell)} = \frac{\mathcal{B}(\tau^- \to \nu_\tau P^-)\tau_P}{\mathcal{B}(P^- \to \ell^- \bar{\nu}_\ell)\tau_\tau} = \left|\frac{g_\tau}{g_\ell}\right|^2 \frac{m_\tau^3}{2 m_P m_\mu^2} \frac{(1 - m_P^2/m_\tau^2)^2}{(1 - m_\ell^2/m_P^2)^2}(1 + \delta_{\tau/P})$$

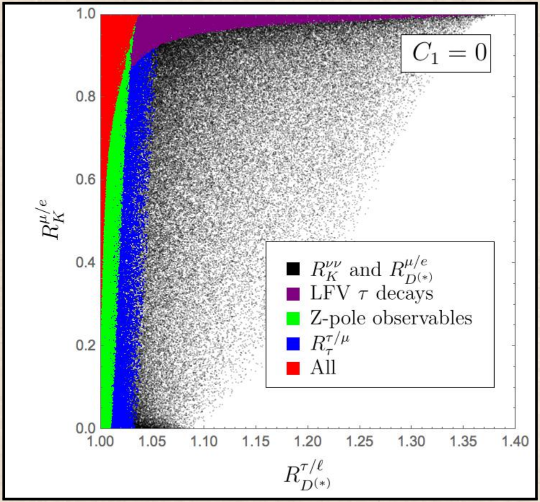
- meson decay constant and CKM matrix coefficient drop out in the ratio!
- part of radiative corrections also drop out
- because of hadronic uncertainties, $\Delta(1 + \delta_{\tau/P}) \sim 0.6\%$ [PRD 104 (2021) L091502]
- $P^- \to e^- \bar{\nu}_e$ decays helicity-suppressed, worse experimental precision

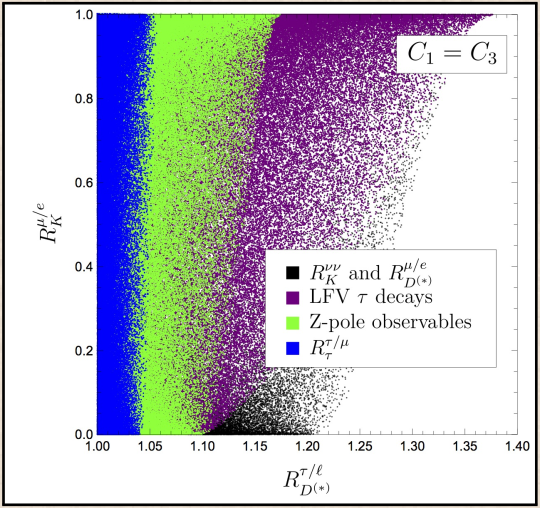
$$\left|\frac{g_\tau}{g_\mu}\right| = \sqrt{\frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\bar{\nu}_\mu)} \frac{2 m_h m_\mu^2 \tau_h}{m_\tau^3 \tau_\tau} \frac{(1 - m_\mu^2/m_P^2)^2}{(1 - m_P^2/m_\tau^2)^2} \frac{1}{(1 + \delta_{\tau/P})}} \qquad (P^- = \pi \text{ or } K)$$

- $\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9959 \pm 0.0038 \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9855 \pm 0.0075$
- precision limited by radiative corrections uncertainties

Tau Lepton universality constrains models for B $R^{\tau/\ell}_{D^{(*)}}$- $R^{\mu/e}_{K}$ anomalies

Feruglio, Paradisi, Pattori JHEP 09 (2017) 061
blue points correspond to parameter space region allowed by tau lepton universality





Tau Mass

Tau Mass

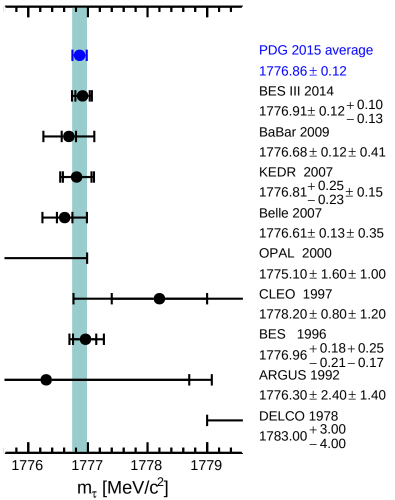
threshold scan technique

- measure onset of $\sigma(e^+e^- \to \tau^+\tau^-)(\sqrt{s})$
- DELCO, BES (92, 96), KEDR and BES III
- precise E_{beam} using
  - either resonant depolarization of polarized $e^\pm$
  - beam - laser Compton scattering
- statistically and sistematically most precise

pseudomass technique

- select $e^+e^- \to \tau^+\tau^-$ events
- fit pseudomass from visible tau-decay products
- ARGUS, OPAL, Belle, *BABAR*
- less precise

Tau Mass measurements



Threshold scan technique: tau pair cross-section vs. energy

$\sigma(e^+e^- \to \tau^+\tau^-)$

at lowest order

$$\sigma_{\tau\tau} = \frac{4\pi\alpha^2}{3s}\beta\left(\frac{3-\beta^2}{2}\right), \quad \beta = \frac{p_\tau}{E_\tau}, \quad s = E^2$$

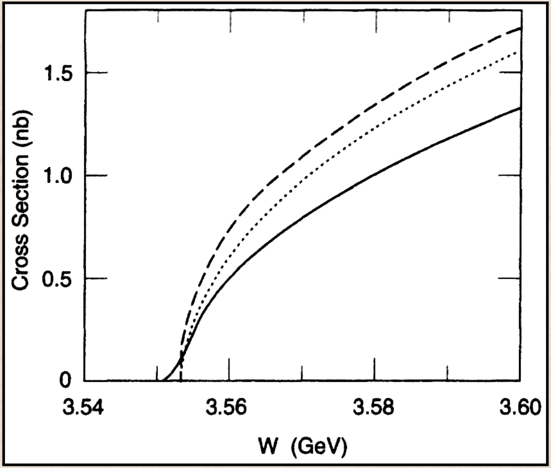
including radiative corrections, beam energy spread

$$\sigma(E, m_\tau, \sigma_E) = \frac{1}{\sqrt{2\pi}\sigma_E}\int_{2m_\tau}^{\infty} dE' e^{\frac{-(E-E')^2}{2(\sigma_E)^2}} \int_0^{1-\frac{4m^2}{E'^2}} dx F_I(x, E')\tilde{\sigma}(E'\sqrt{1-x}, m_\tau),$$

$$\tilde{\sigma}(E) = \frac{4\pi\alpha^2}{3E^2}\beta\left(\frac{3-\beta^2}{2}\right)\frac{F_C(\beta)F_\tau(\beta)}{[1-\prod(E)]^2}$$

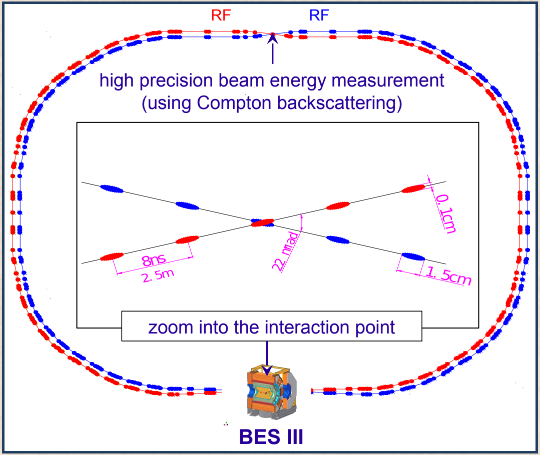
- $F_\tau(\beta)$ final state radiation from τ
- $F_C(\beta)$ Coulomb $\tau^+\tau^-$ interation in final state
- $\prod(E)$ QED corrections to photon propagator
- $F_I(x, E')$ initial state radiation probablity
- σ_E beam energy spread

Threshold scan technique: tau pair cross-section vs. energy



- dotted curve: lowest order prediction
- dashed curve: Coulomb correction and final state radiation
- solid curve: all effects

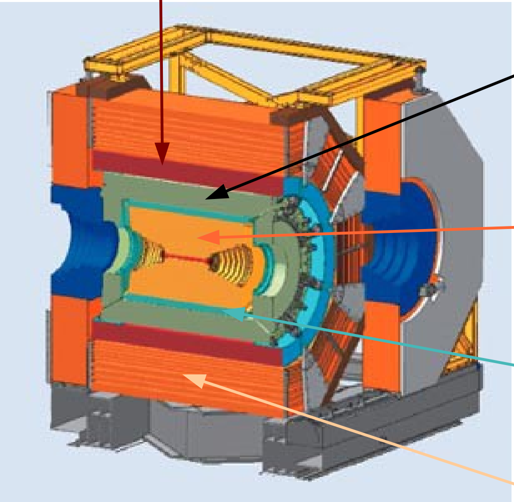
Tau Mass BES III measurement (2014), BEPC II storage ring (Beijing)



- double beam pipe for e^+, e^-
- large crossing angle
- beam energy: $1.0 - 2.3\,\text{GeV}$
- energy spread: $5.16{\cdot}10^{-4}$
- design luminosity: $1{\cdot}10^{33}\,/\text{cm}^2/\text{s}$ $\quad @\psi(3770)$
- beam energy measurement $5{\cdot}10^{-5}$

Tau Mass BES III measurement (2014), BES III detector

Super conducting magnet: 1 T



EMC: CsI cristal
- Energy resolution: **2.5% @1GeV**
- Spatial resolution: **6mm**

MDC:
- Spatial resolution: **σ_{xy}= 120μm**
- Momentum resolution:**0.5% @ 1GeV**
- **dE/dx** resolution: 6%

TOF:
Time resolution: **100ps** (barrel)
110ps (endcaps)

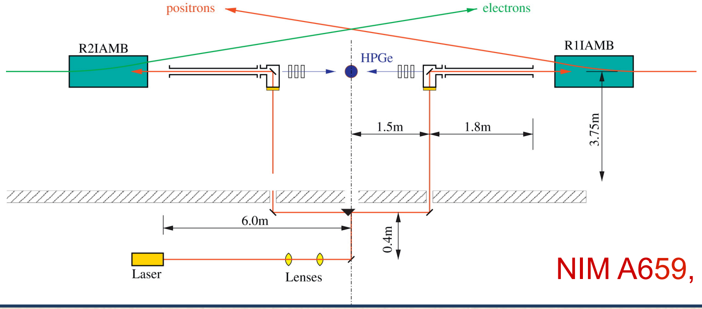
Muon ID:
9 layers RPC, 8 for endcaps

Tau Mass BES III measurement (2014), Beam Energy Measurement

- Determination of beam energy crucial for τ mass measurement
- The electron energy E_e is related to the maximal energy of the scattered photon E_γ by the kinematics of Compton scattering

$$E_e = \frac{E_\gamma}{2}\left[1+\sqrt{1+\frac{m_e^2}{\varepsilon_\gamma E_\gamma}}\right] \Rightarrow E_{CM} = 2\times\sqrt{\overline{E}_{e^+}\times\overline{E}_{e^-}}\times\cos\left(\frac{\theta_{e^+e^-}}{2}\right)$$

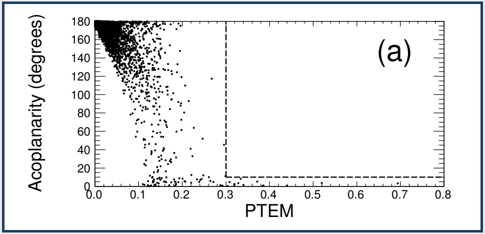
- σ(Energy) ~ 10^{-5}, Δ (Energy spread) / Energy spread ~ 6%



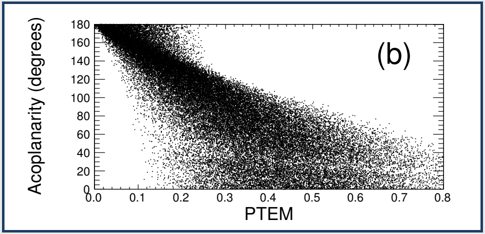
NIM A659, 21 (2011)

Selection of tau events

- select 1-prong vs. 1-prong with ee, $e\mu$, eh, $\mu\mu$, μh, hh, $e\rho$, $\mu\rho$, $\pi\rho$ $(h = \pi, K)$
- 2 good tracks, 0 or 2 photons
- require large acoplanarity angle $\phi_{\text{acopl}} = \pi - (\phi_2 - \phi_1)$
- require large $\dfrac{P_t}{\max(E_{\text{missing}})} = \dfrac{\left|(\vec{P}_1 + \vec{P}_2)_t\right|}{\sqrt{s} - P_1 - P_2}$
- EMC, TOF, Muon ID used for particle identification



ϕ_{acopl} vs. $\dfrac{P_t}{\max(E_{\text{missing}})}$, below tau threshold



ϕ_{acopl} vs. $\dfrac{P_t}{\max(E_{\text{missing}})}$, above tau threshold

Tau Mass BES III measurement (2014)

$$L(m_\tau, \mathcal{R}_{\text{data/MC}}, \sigma_B) = \prod_{i=1}^{4} \frac{\mu_i^{N_i} e^{-\mu_i}}{N_i!}$$

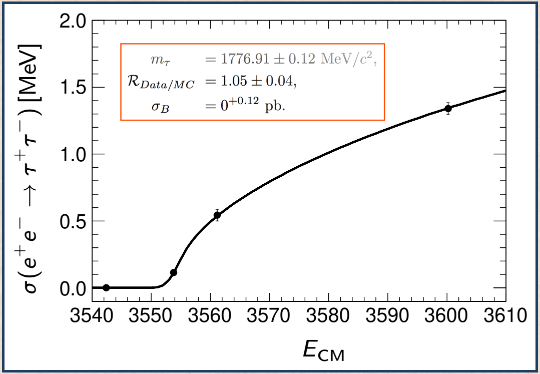
$\mathcal{R}_{\text{data/MC}} = \dfrac{\epsilon_{\text{data}}}{\epsilon_{\text{MC}}}$ *constant*

$\sigma_{\text{B}} =$ background cross-section, constant vs. E^i_{CM}

$\mu_i = [\mathcal{R}_{\text{data/MC}} \times \epsilon_i \cdot \sigma(E^i_{\text{CM}}, m_\tau) + \sigma_B] \cdot \mathcal{L}_i$

$\epsilon_i = \sum_j \epsilon_{ij} \mathcal{B}(\tau \to \text{final state j})$

$\mathcal{L}_i =$ luminosity at $E_{\text{CM}} = E^i_{\text{CM}}$



Tau Mass BES III measurement (2014), systematics and result

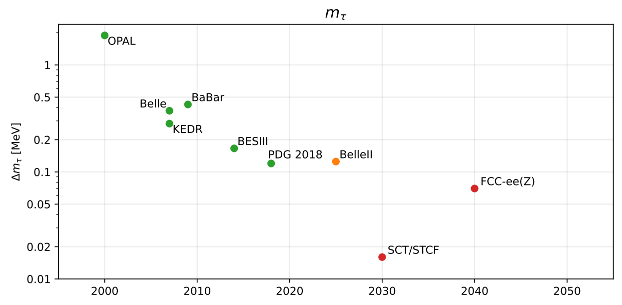
main systematics

- beam energy, modeling of selection efficiency, background shape

result

- $m_\tau = 1776.91 \pm 0.12\,(\text{stat.})^{+10}_{-13}\,(\text{syst.})$ MeV [PRD 90 (2014) 012001]
- $m(\tau)_{\text{WA}} = 1776.86 \pm 0.12$ [68 ppm]; $m(J/\psi)$ known more precisely [3.5 ppm]

tau mass measurement prospects



Tau Lifetime measurement by Belle (2014)

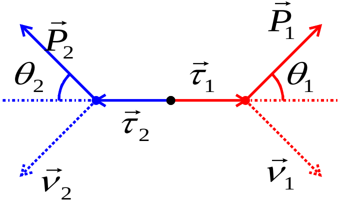
Measurement method

- use 3-prong vs. 3-prong events
- kinematically reconstruct the $\tau^+\tau^-$ direction in the center-of-mass frame
- measure correlation between sum of signed lifetimes and

Belle Tau Lifetime: method

Mikhail Shapkin, Tau2014

- In the CM frame for the reaction $e^+e^- \to \tau^+\tau^- \to 3\pi\nu\, 3\pi\nu$ flight directions of τ^+ and τ^- are back-to-back.
- Energy of each τ-lepton is $\sqrt{s}/2$.
- Each τ-lepton is decayed into $\tau \to 3\pi\nu$; mass of τ-lepton is taken from PDG; neutrino mass assumed to be zero.



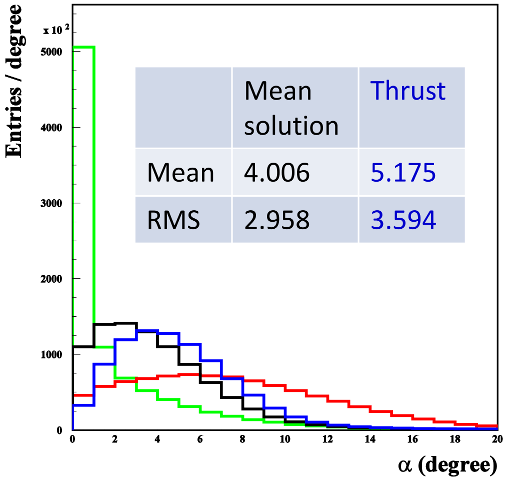
- $$\cos\theta = \frac{2E_\tau E_x - m_\tau^2 - m_x^2}{2P_\tau P_x} = \frac{2E_\tau E_x - m_\tau^2 - m_x^2}{2\sqrt{\left(E_\tau^2 - m_\tau^2\right)}P_x}$$
- $$\begin{cases} \left(\vec{P}_1 \cdot \vec{n}_+\right) = xP_{x1} + yP_{y1} + zP_{z1} = \left|P_1\right|\cos\theta_1 \\ \left(\vec{P}_2 \cdot \vec{n}_+\right) = xP_{x2} + yP_{y2} + zP_{z2} = -\left|P_2\right|\cos\theta_2 \\ \left(\vec{n}_+\right)^2 = x^2 + y^2 + z^2 = 1 \end{cases}$$

$\vec{n}_+$ is the unit vector in the direction of the positive τ-lepton

- Two solutions of quadratic equation are possible τ-lepton flight directions.

Belle Tau Lifetime: tau lepton direction resolution

Mikhail Shapkin, Tau2014



	Mean solution	Thrust
Mean	4.006	5.175
RMS	2.958	3.594

Angle between reconstructed and true τ-direction for $\tau\tau$ Monte Carlo events.

Monte Carlo samples:

- — Mean solution

$$\vec{n} = \frac{\vec{n}_1 + \vec{n}_2}{2}$$

- — n_1 - True solution
- — n_2 - Wrong solution
- — Thrust direction as τ-direction

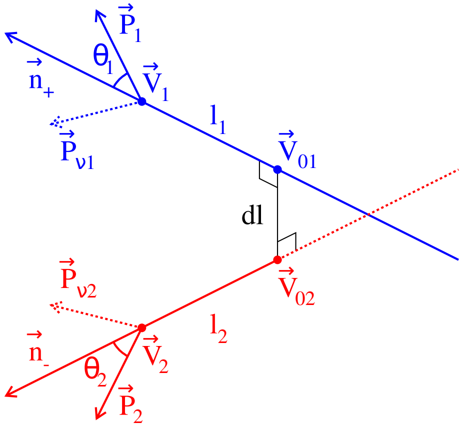
MC simulation predicts no bias from this choice

Belle Tau Lifetime: lifetime reconstruction

tau lifetime reconstruction

- convert $\tau^{\pm}$ direction in CM frame to 4-momentum using m_τ, E_τ^{CM}
- boost tau 4-momenta to lab frame (boost determined by asymmetric beam energies)
- each 4-momentum anchored to 3-prong vertex to compute tau paths (straight line approx.)
- origin = 3D closest approach of tau paths
- get 2 decay lengths from origin to 3-prong vertices
- $c\tau = \dfrac{l_\tau^{\text{lab}}}{(\beta\gamma)_\tau^{\text{lab}}}$

lab-frame tau momenta

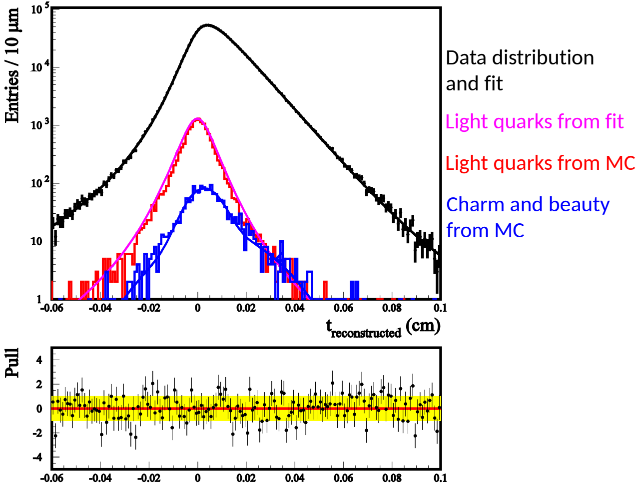


Belle Tau Lifetime: lifetime fit

tau lifetime fit model

- reconstructed lifetime components:
  - tau: resolution-convoluted $e^{-t/\tau}$
  - *uds*: resolution-convoluted $\delta(t)$
  - *bc*: estimated with simulation
- resolution from simulation with parameters fitted to data
- bkg normalizations from simulation

lifetime fit



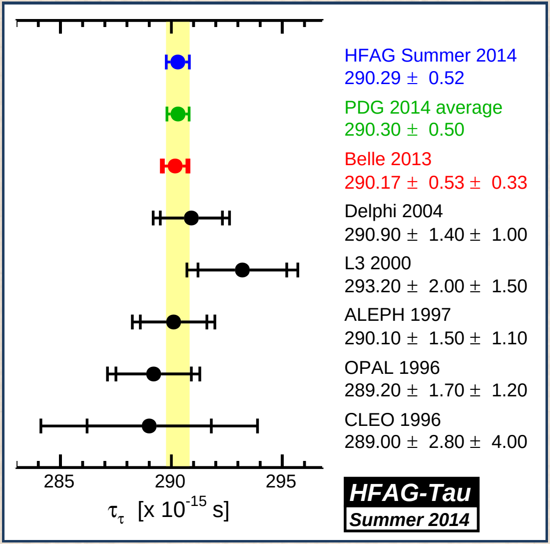
Belle Tau Lifetime: systematics

Mikhail Shapkin, Tau2014

Source of Systematics	$\Delta(c\tau)$ in μm
SVD alignment	0.090
Asymmetry of R-function	0.030
Fit range	0.020
ISR and FSR description	0.018
Beam energy calibration	0.016
Background contribution	0.010
Error of the τ-lepton mass	0.009
Total	**0.101**

Tau Lifetime

Tau lifetime

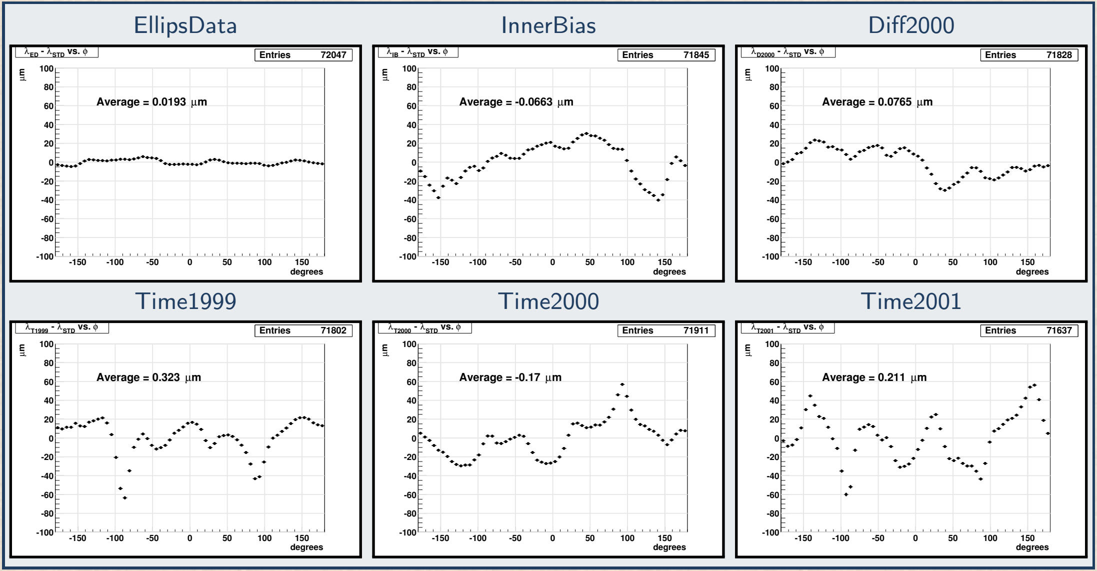


- LEP experiments, many methods
  - impact parameter sum (IPS)
  - momentum dependent impact parameter sum (MIPS
  - 3D impact parameter sum (3DIP)
  - impact parameter difference (IPD)
  - decay length (DL)
- Belle
  - 3-prong vs. 3-prong decay length
  - largest syst. error: alignment

Tau lifetime systematic effects related to detector misalignment

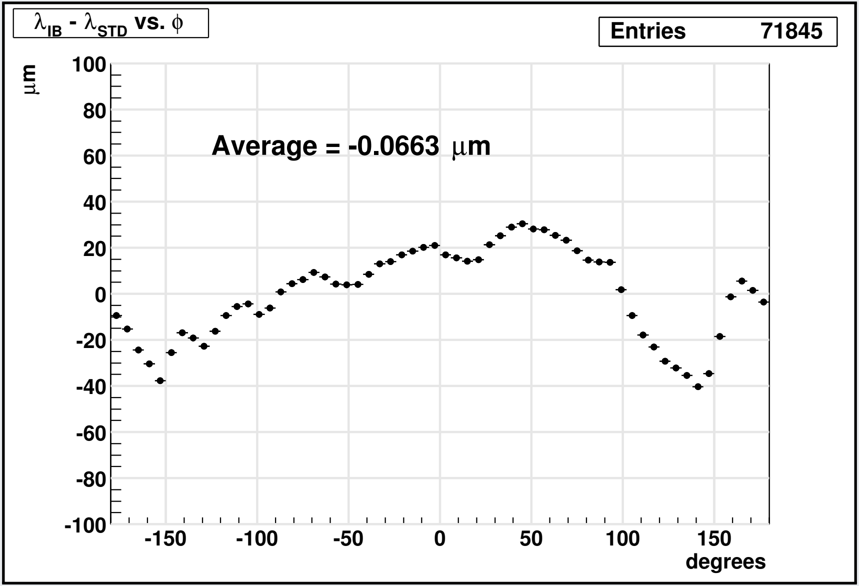
- detector misalignment causes huge effects on the reconstructed tau lifetime
  - in *BABAR*, up to $\pm 40\,\mu$m on a $200\,\mu$m mean transverse decay length
- however, under some conditions, these systematic offsets average to zero:
  - $e^+e^- \to \tau^+\tau^-$ experiments (B-factories, LEP)
  - get lifetime by reconstructing decay length projection onto transverse plane
  - signal sample with complete uniform distribution in azymuth

BABAR studies, tau lifetime with SVT misalignment files



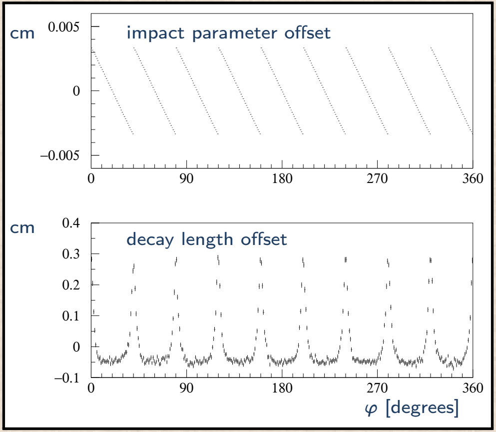
BaBar studies, tau lifetime with SVT misalignment files

InnerBias



Tau lifetime systematic effects related to detector misalignment

- crude simulation of ALEPH vertex detector with all wafers radially shifted out by 100 μm
- impact parameter offset $= 100\,\mu\text{m} \cdot \sin\alpha$
  - α = track angle w.r.t. normal of wafer
- decay length offset (measured using 3-tracks vertex)
  - negative when 3 tracks hit same wafer
  - positive when 3 tracks hit two wafers



- decay length offset ∼ derivative of impact parameter offset
  - can be understood with 1-st order model of most simple misalingments
  - confirmed empirically quite precisely in accurate simulations of real detectors
- averaging decay length over full range of azymuth angle is like integrating
  $\Rightarrow$ average offset = impact parameter offset variation from 0 to 2π i.e. zero

Tau lifetime systematic effects related to detector misalignment

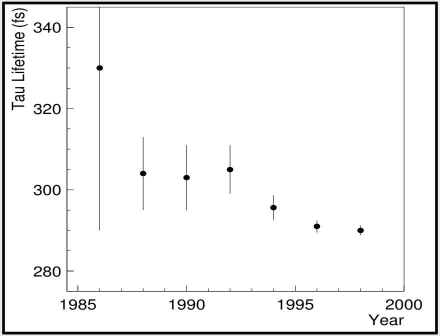
- large decay length offsets from detector misalignment, which average to zero when
  - transverse decay length measurements
  - events evenly distributed in complete asymuth angle range $0 - 2\pi$
- simulation of realistic misalignment can estimate the systematic effects
- when using measurements in space, out of the transverse plane, there are similar cancellations, however the coverage can never be complete as for azymuth
- decay length reconstruction absolute scale
  - not affected by e.g. wrong assumed radius of detector
  - rather, determined by assumed detector element sizes (e.g. silicon strip spacinv)

References

- S.Wasserbaech, Review of tau lifetime measurements, Nucl.Phys.Proc.Suppl. 76 (1999) 107
  S.Wasserbaech, Systematic biases in particle lifetime measurements, PRD 48, 4216 (1993)

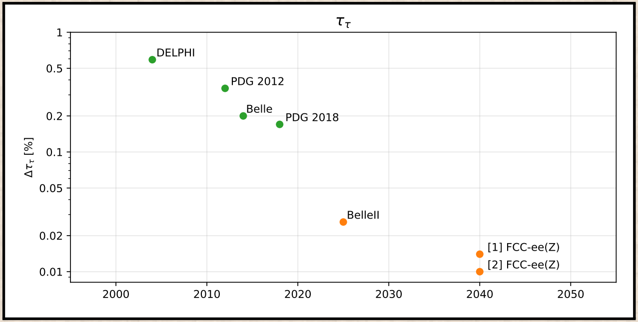
Tau lifetime reconstruction bias

- multiple scattering in detector induces correlated errors in d_0 & φ_0 helix parameters
  $\Rightarrow$ positive lifetime reconstruction bias
- 1985-1993 reconstruction bias not subtracted in 8 measurements of tau lifetime with DL method



- S.Wasserbaech, Systematic biases in particle lifetime measurements, PRD 48, 4216 (1993)
  - recommendation: always subtract estimated measurement biases

Tau lifetime measurement precision prospects



FCC estimate

[1] M. Dam and CDR

[2] A.L. FCC Workshop Jan 2020

Other estimates

- ESG 2019 docs

Tau leptonic branching ratios measurements

$\tau \to \mu\nu\bar{\nu}$

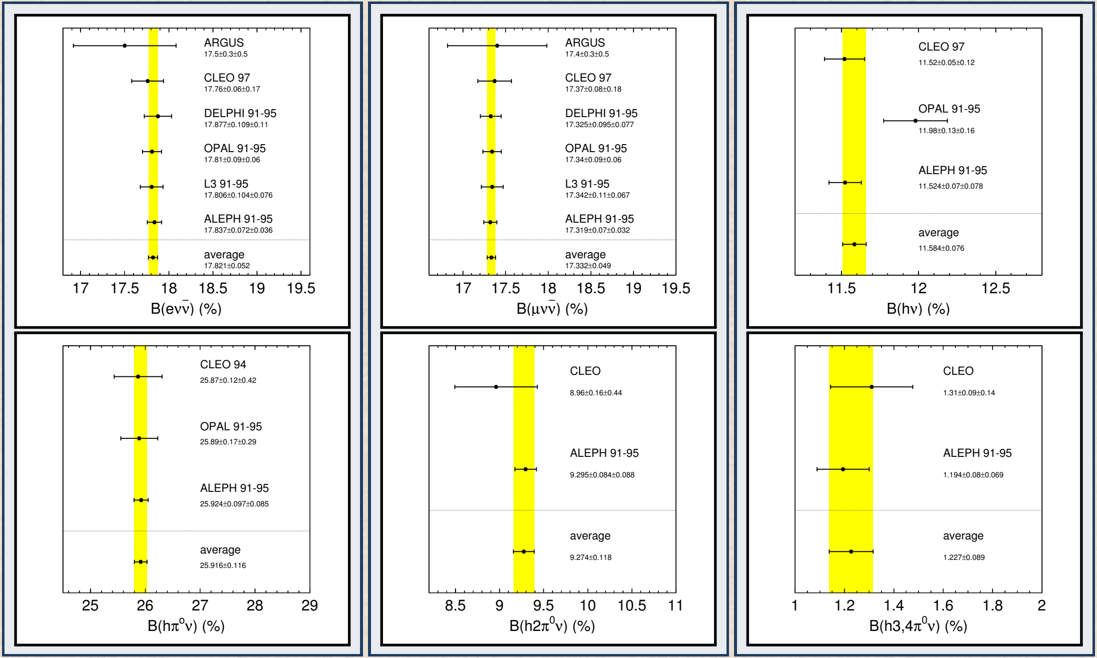
VALUE (%)	EVTS	DOCUMENT ID		TECN	COMMENT
17.39 ± 0.04	OUR FIT				
17.33 ± 0.05	OUR AVERAGE				
17.319 ±0.070 ±0.032	54k	[1] SCHAEL	2005C	ALEP	1991-1995 LEP runs
17.34 ±0.09 ±0.06	31.4k	ABBIENDI	2003	OPAL	1990-1995 LEP runs
17.342 ±0.110 ±0.067	21.5k	[2] ACCIARRI	2001F	L3	1991-1995 LEP runs
17.325 ±0.095 ±0.077	27.7k	ABREU	1999X	DLPH	1991-1995 LEP runs
• • • We use the following data for averages but not for fits. • • •					
17.37 ±0.08 ±0.18		[3] ANASTASSOV	1997	CLEO	E^{ee}_{cm} = 10.6 GeV

$\tau \to e\nu\bar{\nu}$

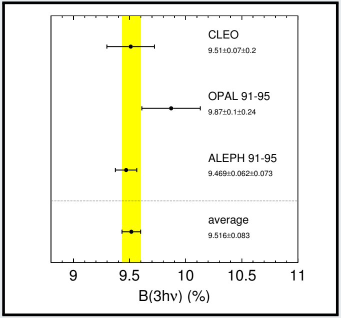
VALUE (%)	EVTS	DOCUMENT ID		TECN	COMMENT
17.82 ± 0.04	OUR FIT				
17.82 ± 0.05	OUR AVERAGE				
17.837 ±0.072 ±0.036	56k	[1] SCHAEL	2005C	ALEP	1991-1995 LEP runs
17.806 ±0.104 ±0.076	24.7k	[2] ACCIARRI	2001F	L3	1991--1995 LEP runs
17.81 ±0.09 ±0.06	33.1k	ABBIENDI	1999H	OPAL	1991--1995 LEP runs
17.877 ±0.109 ±0.110	23.3k	ABREU	1999X	DLPH	1991--1995 LEP runs
17.76 ±0.06 ±0.17		[3] ANASTASSOV	1997	CLEO	E^{ee}_{cm} = 10.6 GeV

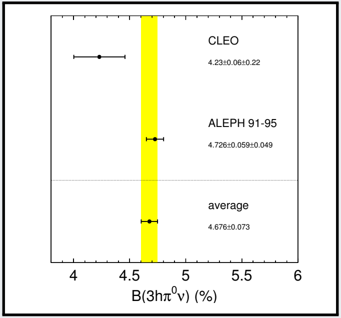
- no improvements from *B*-factories, despite ~500 M tau pairs w.r.t. 200 k
- ALEPH result significantly more precise that other 3 LEP experiments

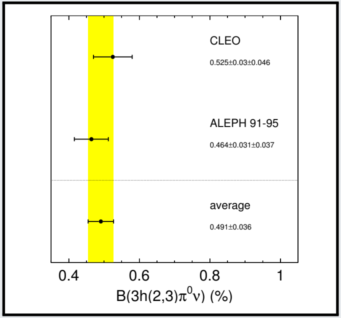
ALEPH final results on leptonic and other branching ratios (2005)

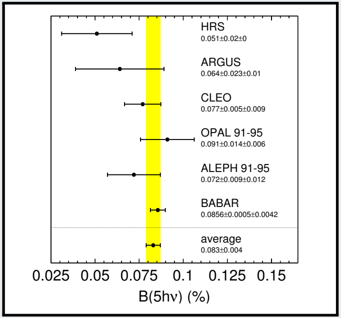


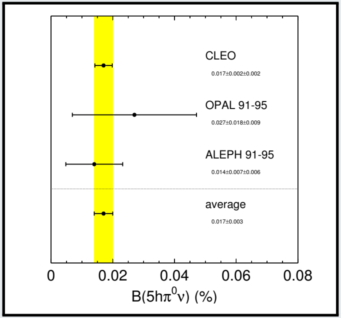
ALEPH final results on leptonic and other branching ratios (2005)



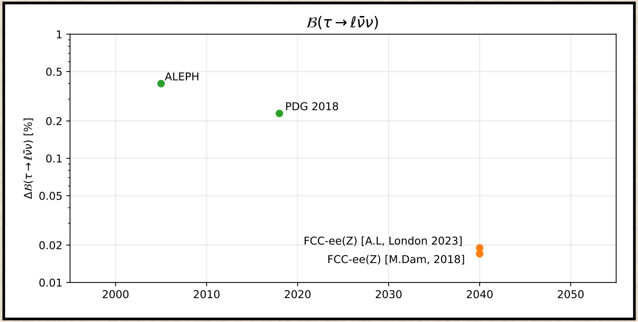








Tau leptonic branching ratio precision prospects



FCC estimate

- M. Dam Tau2018, Tau2021

- no improvements from B-factories, despite 500−1000 M tau pairs w.r.t. ∼100 k
- ALEPH result significantly more precise that other 3 LEP experiments

Tau Branching fractions measurements features

- branching fraction measurement: $\mathcal{B} = \dfrac{N(\tau \to X)}{N(\tau)}$

Z peak

- $\dfrac{N_{\text{obs}}(\tau \to X)/\epsilon(\tau \to X) - N_{\text{bkg}}(\tau \to X)}{N_{\text{obs}}(\tau)/\epsilon(\tau)}$
- $\epsilon(\tau \to X) \sim \epsilon(\tau \to X) \sim 100\%$
  MC modeling for $(100\% - \epsilon)$, small systematics
- small non-tau N_{bkg}
  - easier to suppress hadrons, e^+e^-, $\mu^+\mu^-$
- $\Delta\mathcal{L} \sim 0.1\%$

$\Upsilon(4s)$ peak

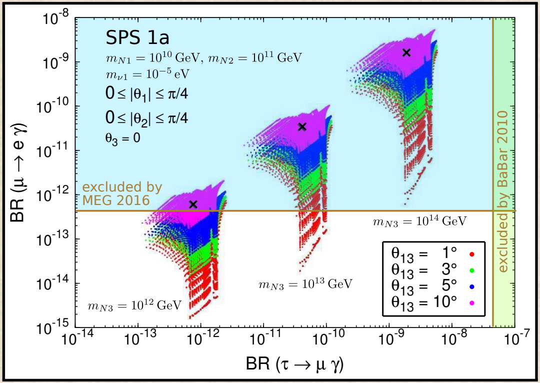
- $\dfrac{N_{\text{obs}}(\tau \to X)/\epsilon(\tau \to X) - N_{\text{bkg}}(\tau \to X)}{2\sigma(e^+e^- \to \tau^+\tau^-)\mathcal{L}}$
- $\epsilon(\tau \to X) \ll 100\%$
  MC modeling causes large systematics
- large non-tau N_{bkg}
  - more contamination of hadrons, e^+e^-, $\mu^+\mu^-$
- $\Delta\mathcal{L} \sim 1\%$

ALEPH at Z peak

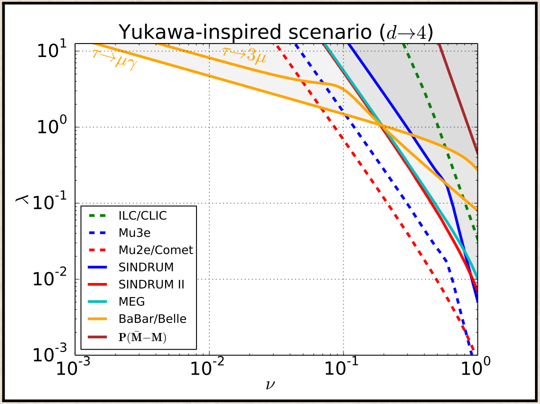
- single analysis for all topological $\mathcal{B}$s
  (by n. of tracks and n. of π^0)
- non-tau bkg from cross-feed of other channels
  all channels and almost all bkg fit on data

Tau LFV searches probe & constrain New Physics models

MSSM Seesaw
Antusch, Arganda, Herrero, Teixeira 2006



doubly charged scalar
Crivellin, Ghezzi, Panizzi, Pruna, Signer 2019



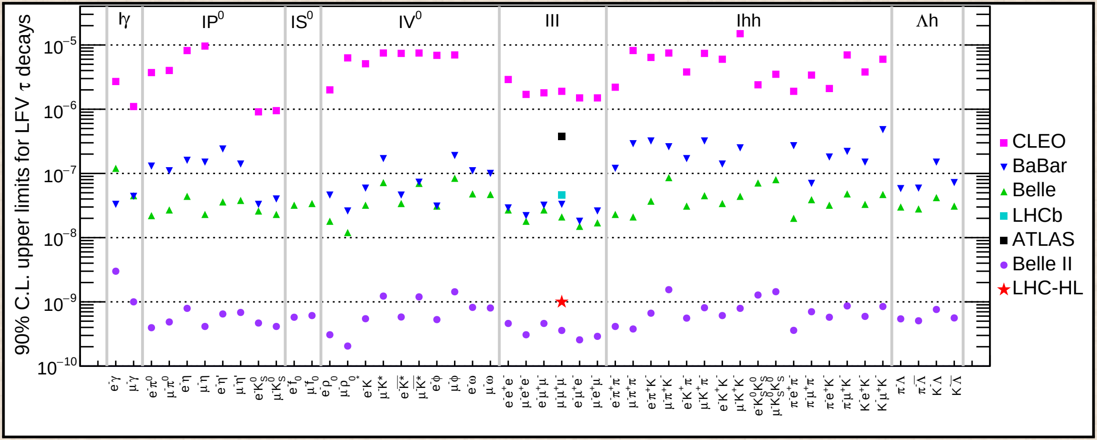
typical NP models

- $\mathcal{B}(\tau \to \mu\gamma) \sim 10\text{–}1000 \times \mathcal{B}(\mu \to e\gamma)$
- muon LFV searches more effective

specific models / parameter space regions

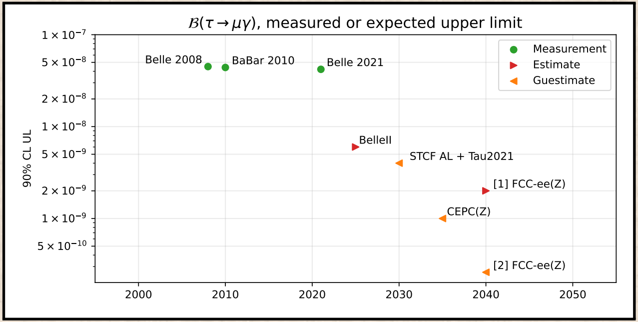
- part of plot only constrained by tau LFV limits

Tau LFV limits: present and future with Belle II and LHCb-HL



HL-LHC and HE-LHC opportunities, arXiv:1812.07638 [hep-ph]

FCC Tau LFV sensitivity for $\tau \to \mu\gamma$



FCC estimate for $\tau \to \mu\gamma$

[1] M. Dam simulation with 2% of full FCC statistics

[2] M. Dam 2021, guestimate with improved longitudinally segmented crystal EM calorimeter

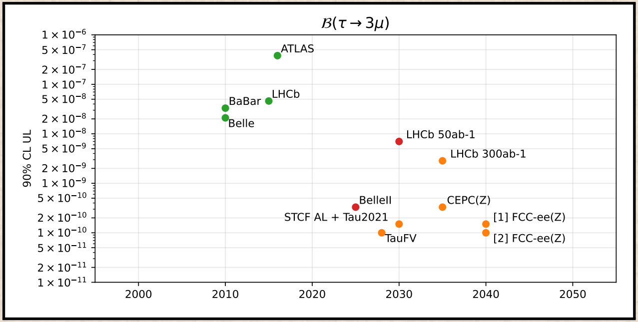
Other estimates

- ESG 2019 docs
- my extrapolation to 10y of SCTF limits presented at Tau2021

Plot notes

- Red more solid estimates
- Orange less solid estimates
- dates of future results are arbitrary, for plotting convenience

FCC Tau LFV sensitivity for $\tau \to 3\mu$



FCC estimate for $\tau \to \mu\mu\mu$

[1] my guestimate

[2] M. Dam, Tau2021

Other estimates

- ESG 2019 doc
- my extrapolation to 10y of SCTF limits presented at Tau2021

Conclusions

Conclusions

- muon measurements provide precise tests and far-reaching searches for New Physics
- tau measurements are most precise for testing LFU, hot topic today
- tau LFV searches are less poverful than muon LFV searches
  but if LFV is found then tau LFV searches required to distinguish the proposed New Physics models
- this presentation is largely incomplete
- for Muon Physics, see also
  - T.P.Gorringe, D.W.Hertzog, Precision muon physics, j.ppnp 06 (2015) 001
- for Tau Physics, see also
  - A. Pich, Precision Tau Physics, Prog.Part.Nucl.Phys. 75 (2014) 41
  - A. Pich, Historical overview of 30 years of tau lepton physics work,
    https://indico.cern.ch/event/848732/contributions/4506420/,
    https://inspirehep.net/literature/1993928

Backup Slides

$|V_{us}|$ determinations fron kaons

- $\Gamma(K \to \pi\ell\bar{\nu}_\ell[\gamma]) = \dfrac{G_F^2 m_K^5}{192\pi^3} C_K^2 \, S_{\text{EW}}^K \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\ell \left(1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU(2)}}^{K\pi}\right)^2$ $\quad K_{\ell 3}$
- $\dfrac{\Gamma(K^- \to \ell^- \bar{\nu}_\ell)}{\Gamma(\pi^- \to \ell^- \bar{\nu}_\ell)} = \dfrac{|V_{us}|^2}{|V_{ud}|^2} \left(\dfrac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \dfrac{m_K(1 - m_\ell^2/m_K^2)^2}{m_\pi(1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{\text{EM}})$ $\quad K_{\ell 2}$

$|V_{us}|$ from tau decays measurements

$|V_{us}|$ from tau inclusive

- $$\frac{R(\tau \to X_{\text{strange}}\,\nu)}{|V_{us}|^2} = \frac{R(\tau \to X_{\text{non-strange}}\,\nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}},$$ $\tau \to X_s \nu$

$|V_{us}|$ from tau exclusive

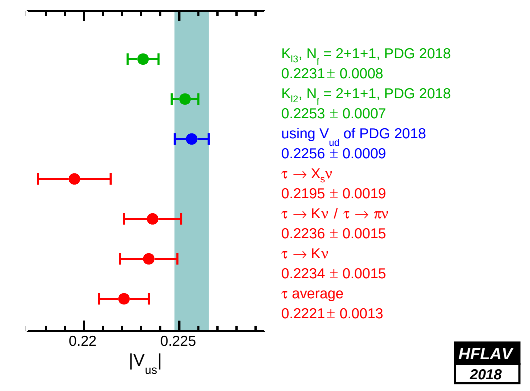
- $$\frac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 R_{\tau K/\tau\pi} \frac{\left(1 - m_K^2/m_\tau^2\right)^2}{\left(1 - m_\pi^2/m_\tau^2\right)^2}$$ $\tau \to K / \tau \to \pi$
- $$\Gamma(\tau^- \to K^- \nu_\tau) = \frac{G_F^2}{16\pi\hbar} |V_{us}|^2 f_{K\pm}^2 R_{\tau K} m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2$$ $\tau \to K$

CKM matrix first row unitarity test in $|V_{us}|$ perspective, PDG 2018

- PDG 2018 $|V_{ud}|$- $|V_{us}|$ review
  - $|V_{us}|$ from kaons $\sim$consistent with unitarity
- HFLAV 2018 report, PRL 93 (2004) 231803
  - $|V_{us}|$ from tau inclusive $>3\sigma$ discrepancy
  - $|V_{us}|$ from tau exclusive $\sim$consistent

note

- alternative $|V_{us}|$ determinations from $\tau \to X_s \nu$ exist, which are more consistent with kaons and CKM unitarity
  - Hudspith, Lewis, Maltman & Zanotti 2018
  - Boyle *et al.* 2018



- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ $\sim$consistent with unitarity except when using $|V_{us}|$ from tau inclusive

PDG 2020 $|V_{ud}|$ - $|V_{us}|$ review, HFLAV 2021 preliminary

$|V_{ud}|$

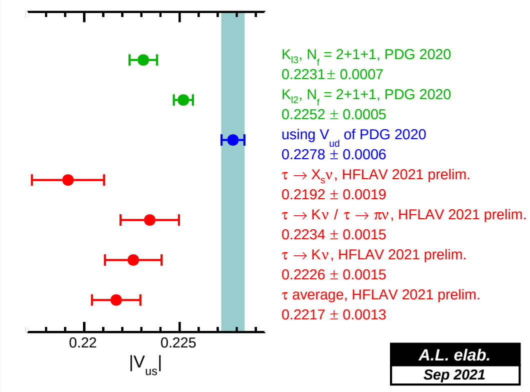
- new dispersive calculation of Δ_R^V inner or universal electroweak radiative corrections (RC) to superallowed nuclear beta decays
  Seng, Gorchtein & Ramsey-Musolf, Phys. Rev. D 100, 013001 (2019)
  - $\sim 2\times$ more precise
  - significant shift

$|V_{us}|$ from kaons

- updated more precise lattice QCD constants

$|V_{us}|$ from tau

- updated lattice QCD constants (minor)
- numerical typo fixed on $|V_{us}|$ from $\tau \to K\nu$



- $|V_{ud}|$ - $|V_{us}|_K$ anomaly $\sim 3\,\sigma$
  (scale factor = 2 on $|V_{us}|_K$ because of difference on $|V_{us}|_K$ from $K_{\ell 3}$ and $K_{\mu 2}$)

2021 update on $|V_{ud}|$ - $|V_{us}|$ anomaly, HFLAV 2021 preliminary

$|V_{ud}|$

- J.C.Hardy & Ii.S.Towner, PRC 102, 045501 (2020)
  - revised experimental inputs
  -

Marciano and Sirlin 2006	2.361 ± 0.038
Seng et al. 2018/2019	2.467 ± 0.022
Czarnecki, Marciano and Sirlin 2019	2.426 ± 0.032
Adopted value for Δ_R^V	2.454 ± 0.019

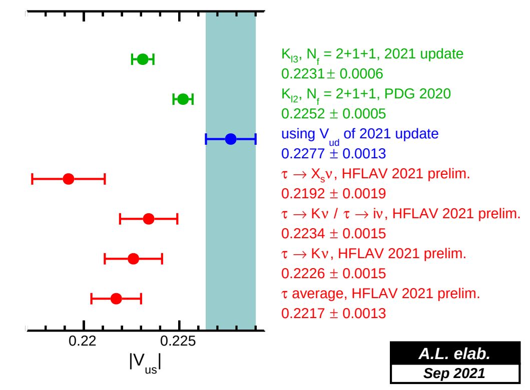
  - increased systematic uncertainty (new nuclear corrections)

$|V_{us}|$ from kaons

- improved K_{e3} radiative corrections, Seng, Gorchtein & Ramsey-Musolf, arXiv:2103.04843 [hep-ph]
- new calculation of $|V_{us}|_{K\ell 3}$ Seng, Galviz, Marciano, Meißner, arXiv:2107.14708 [hep-ph]

$|V_{us}|$ from tau

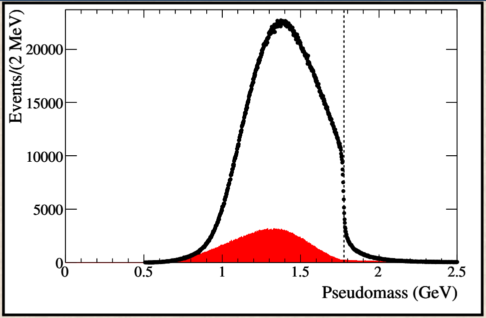
- using 2021 update $|V_{ud}|$ (minor)



- $|V_{ud}|$ - $|V_{us}|_K$ anomaly $\sim 3\,\sigma$
  - no scale factor on $|V_{us}|_K$
  - $\sim 5\,\sigma$ without increased $|V_{ud}|$ systematics

Digression on tau pseudo-mass

- $m_\tau = \sqrt{(P_h^\mu + P_\nu^\mu)^2} = \sqrt{P_h^{\mu 2} + P_\nu^{\mu 2} + 2(E_h E_\nu - \vec{p}_h \vec{p}_\nu)}$
- $E_\nu = \frac{E_{\text{CM}}}{2} - E_h, \quad P_\nu^{\mu 2} = m_\nu = 0, \quad P_h^{\mu 2} = m_h,$
- $m_\tau = \sqrt{m_h + 0 + 2[E_h(E_{\text{CM}}/2 - E_h) - p_h(E_{\text{CM}}/2 - E_h)\cos\theta_{h\nu}]}$
- $m_\tau = \sqrt{m_h^2 + 2(E_{\text{CM}}/2 - E_h^*)(E_h^* - p_h^* \cos\theta_{h\nu}^*)}$
- mass function of unknown $\theta_{h\nu}$ – get minimum possible mass by setting $\theta_{h\nu} = 0$
- **pseudo-mass** $m_\tau = \sqrt{m_h^2 + 2(E_{\text{CM}}/2 - E_h^*)(E_h^* - p_h^*)}$



- tau pseudo-mass distribution in *BaBar* tau mass measurement paper
- red component is estimated background

Tau Lifetime

τ MEAN LIFE

PDG 2019

VALUE (10^{-15} s)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
290.3 ± 0.5	**OUR AVERAGE**				
$290.17 \pm 0.53 \pm 0.33$	1.1M	BELOUS	2014	BELL	711 fb^{-1} E^{ee}_{cm}=10.6 GeV
$290.9 \pm 1.4 \pm 1.0$		ABDALLAH	2004T	DLPH	1991-1995 LEP runs
$293.2 \pm 2.0 \pm 1.5$		ACCIARRI	2000B	L3	1991--1995 LEP runs
$290.1 \pm 1.5 \pm 1.1$		BARATE	1997R	ALEP	1989--1994 LEP runs
$289.2 \pm 1.7 \pm 1.2$		ALEXANDER	1996E	OPAL	1990--1994 LEP runs
$289.0 \pm 2.8 \pm 4.0$	57.4k	BALEST	1996	CLEO	E^{ee}_{cm}= 10.6 GeV

tau lifetime precision

precision (ppm)	
1700	PDG 2019
2100	Belle
5900	DELPHI
6400	ALEPH
7200	OPAL
260	Belle II guestimate, extrapolating from 0.711 ab^{-1} to 50 ab^{-1}
5	FCC, stat. only extrapolation from ALEPH (1e5) to FCC (1.65e11) tau pairs

$\Rightarrow$ what are the limiting systematics?

Tau Lifetime systematics at LEP

DELPHI main systematics, Eur.Phys.J.C36:283-296,200

- IP impact parameter difference on 1-1-prong tau pairs
  - trimming, backgrounds, impact parameter resolution, alignment
- MD miss-distance on 1-1-prong tau pairs
  - resolution on MD, bias, selection
- DL transverse decay length on 3-1 and 3-3 prong tau pairs
  - alignment

ALEPH main systematics, Phys.Lett.B414:362-372,1997

- MIPS, momentum-weighted impact parameter sum
  - resolution on impact parameter sum, bias (from MC)
- 3DIP 3D impact parameter, Z. Phys. C 74, 387–398 (1997)
  - bias (from MC), vertex chisq cut
- IPD, impact parameter difference
  - resolution and trimming of outliers
- DL, decay length
  - vertex chisq cut

expect that all these systematics scale with $1/\sqrt{N_{\text{events}}}$
including alignment systematics
although questionable if up to a factor $1/{\sim}1300$

Tau Lifetime systematics at FCC

Alignment systematic

- alignment calibration precision improves with statistics
- misalignment effects zero at first order for uniform azymuthal acceptance
  S.R.Wasserbaech, Nucl.Phys.Proc.Suppl. 76 (1999) 107-116
  - still, questionable how far this holds
- related systematic that does not scale
  absolute length scale of vertex detector average elements spacing, realiable to 10^{-4} or 100 ppm

Systematics from kinematics of tau decay

- $\tau_\tau = \lambda_\tau / \beta\gamma = \lambda_\tau / \dfrac{\sqrt{E_\tau^2 - m_\tau^2}}{m_\tau} = \lambda_\tau / \dfrac{\sqrt{(E_{\text{beam}} - E_{\text{rad}}^{\text{MC}})^2 - m_\tau^2}}{m_\tau}$

systematic [ppm]	
1	E_{beam}
68	m_τ PDG 2019
7	m_τ possible measurement at Super Charm-Tau Factories
?	MC accuracy on average radiation energy loss (*) (estimated 100 ppm for BABAR)

(*) depends on

- accuracy of generator, can be checked measuring momentum distribution of di-muon events
- accuracy of simulation of efficiency of selection procedure vs. E_τ (scales with luminosity)