Neutrino Cosmology: Lecture I

Miguel Escudero Abenza
miguel.escudero@tum.de

XX LNF Summer School
Bruno Touschek
Frascati
14-07-2022

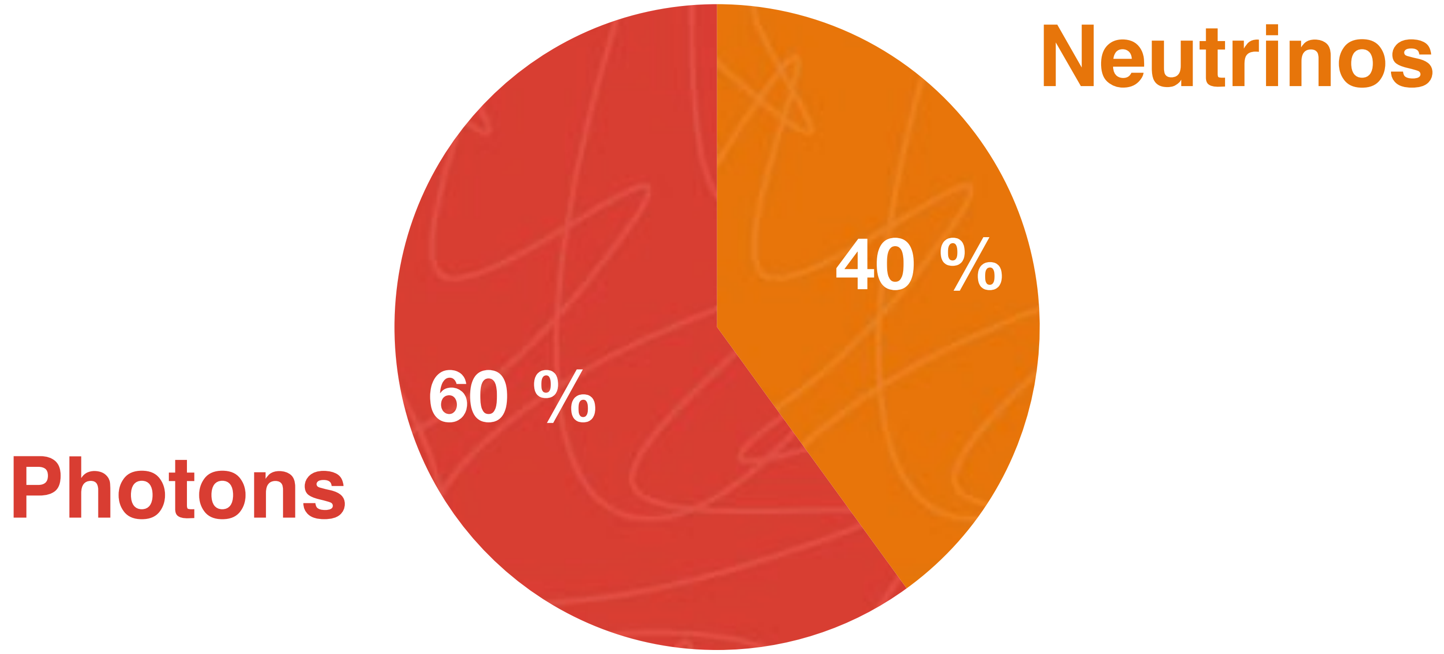
Unterstützt von / Supported by
Alexander von Humboldt
Stiftung / Foundation

Motivation

- **Neutrino masses are the only laboratory evidence of physics beyond the Standard Model**

Use neutrinos to understand open problems in Cosmology

- **Neutrinos are ubiquitous in Cosmology**



Use cosmological data to understand their properties

Goals

1) Understand what is the role played by neutrinos in Cosmology

2) Have a flavor of the types of BSM physics that can be tested with neutrinos in cosmology

3) Understand why one can derive neutrino mass bounds using cosmological data and what are the assumptions behind these constraints

4) See whether neutrinos may have anything to do with the Hubble tension

5) What are we going to learn in the upcoming years?

Set Up

Unlike neutrinos, I like to interact 😃

The plan is to learn and discuss. Therefore:

Questions and Comments are most welcome, at any time!!!!

Outline

Lecture I

Crash course on early Universe cosmology

Neutrino decoupling in the Standard Model

Evidence for the Cosmic Neutrino Background

Can we directly detect the Cosmic Neutrino Background?

Lecture II

Neutrino Masses in Cosmology

The Hubble tension and neutrinos

A Two-slides Cosmo Crash Course

Homogeneity and Isotropy implies Expansion!

— An homogeneous and isotropic Universe expands

—We have evidence that the early Universe was indeed fairly homogeneous and isotropic because $\Delta T_\gamma / T_\gamma \sim 10^{-4}$

— General Relativity relates the expansion rate of the Universe with the energy density in all the species contained on it

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

$$\downarrow$$

Friedmann Equation:

$$\boxed{H^2 = \frac{8\pi G}{3}\rho}$$

H **: Expansion rate**
(Hubble parameter)

ρ **: Energy density**

A Two-slides Cosmo Crash Course

Thermodynamics of the expanding Universe

— Particles were interacting efficiently in the early Universe. That means that they were for a long time in thermal equilibrium.

— The typical energy of a relativistic particle in thermal equilibrium is:

$$\boxed{\langle E \rangle \sim 3\,T}$$

— A species thus becomes non-relativistic when:

$$\boxed{T \lesssim m/3}$$

— In a Universe dominated by radiation (i.e. ultrarelativistic particles):

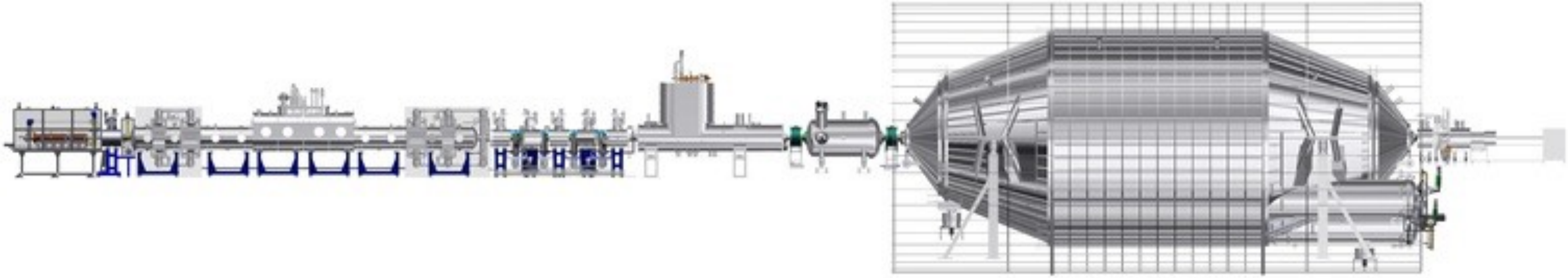
$$\boxed{H \sim \sqrt{\rho}/M_{\rm Pl} \sim T^2/M_{\rm Pl}}$$

On the Standard Model of Cosmology:

ΛCDM ≡ Universe currently dominated by a Cosmological Constant and with Cold Dark Matter

New Laboratory Neutrino Mass Bound

KATRIN experiment $^3\mathrm{H} \to {}^3\mathrm{He}^+ + e^- + \bar{\nu}_e$



Mainz and Troitsk (2004): $m_{\nu_e} < 2.2\,\mathrm{eV}$ **(95 % CL)**

Current laboratory bound: $m_{\nu_e} < 0.8\,\mathrm{eV}$ **(90 % CL, FC)**

2105.08533+1909.06048 (PRL)

$\sum m_\nu < 2.4\,\mathrm{eV}$ **(90 % CL, FC)**

KATRIN expected reach

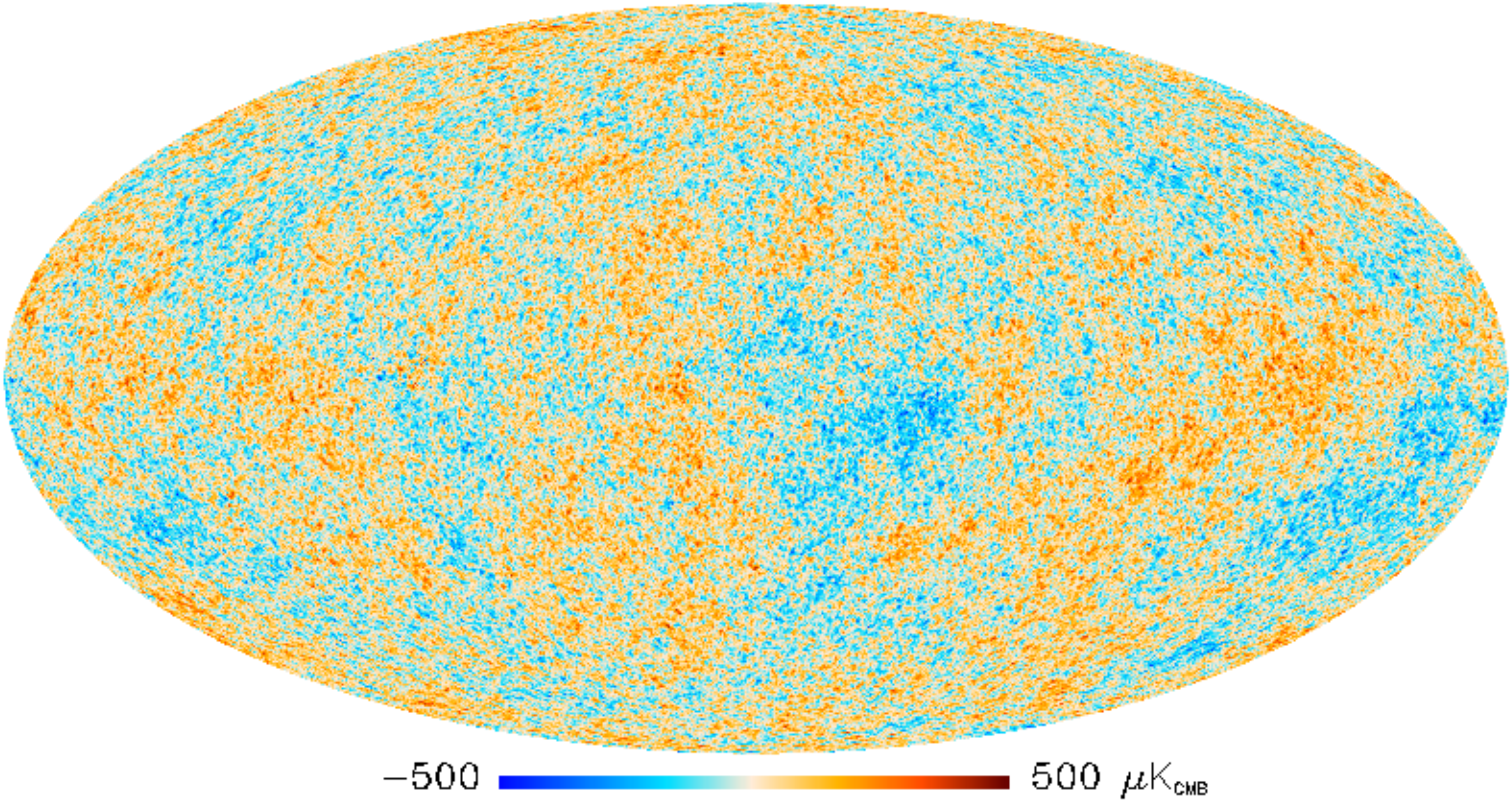
(in ~4 years)

$m_{\nu_e} < 0.2\,\mathrm{eV}$ **(90 % CL)**

$\sum m_\nu < 0.6\,\mathrm{eV}$

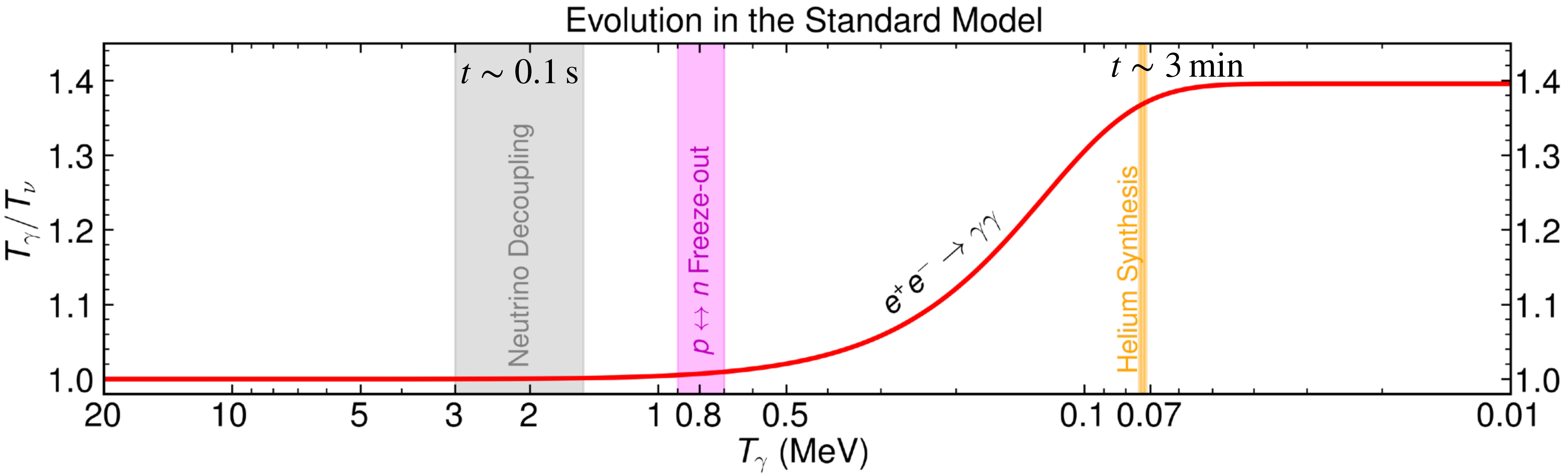
Planck Legacy Data is Public

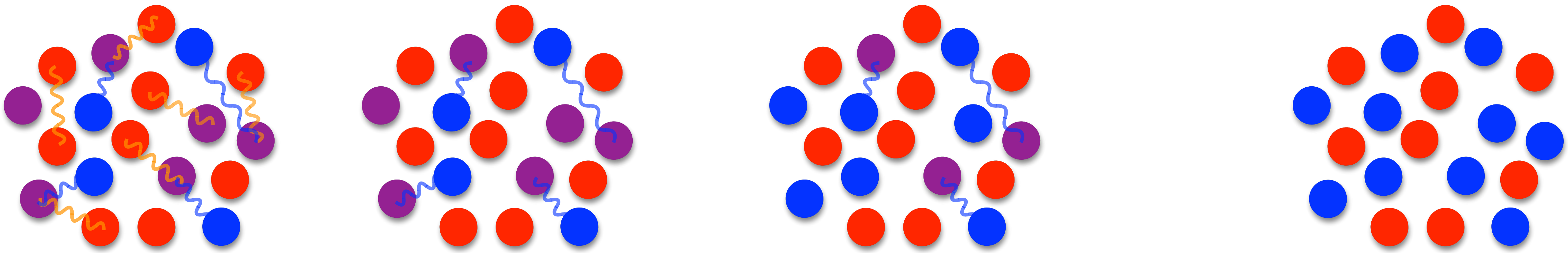
Planck Likelihoods 1907.12875



CLASS/CAMB MontePython/CosmoMC

Neutrino Decoupling





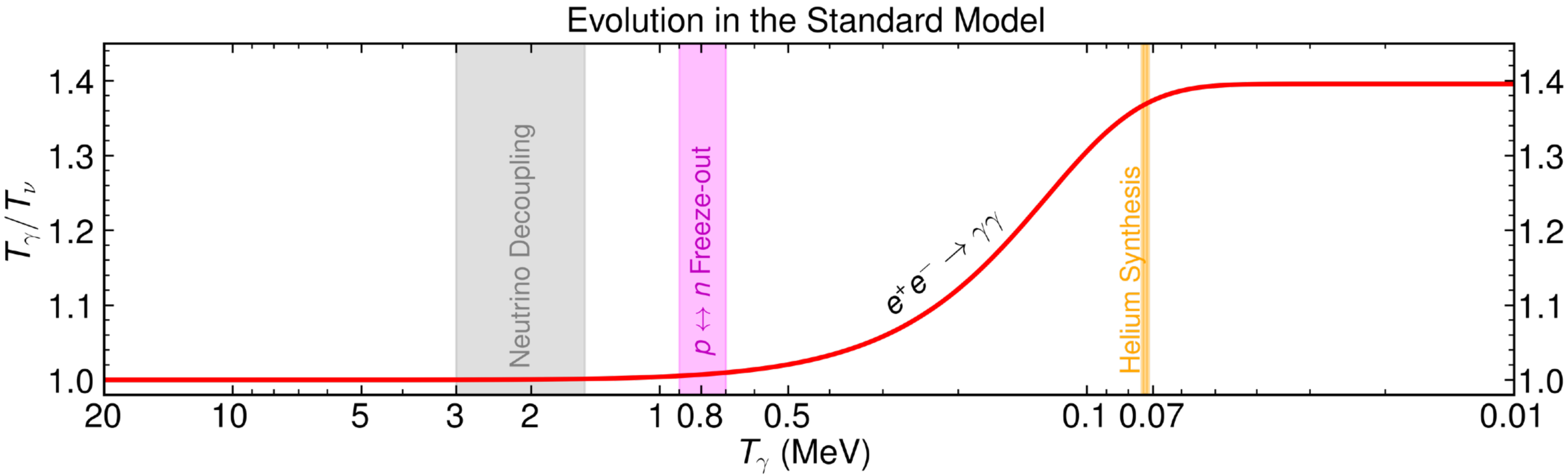
$$e^+ e^- \leftrightarrow \bar{\nu}_\alpha \nu_\alpha$$

$$e^\pm \nu_\alpha \leftrightarrow e^\pm \nu_\alpha$$

$$\nu_\alpha \nu_\beta \leftrightarrow \nu_\alpha \nu_\beta$$

🔴 **Neutrinos** 🟣 **Electrons** 🔵 **Photons** 〰 **Z-W (off-shell)**

Neutrino Decoupling

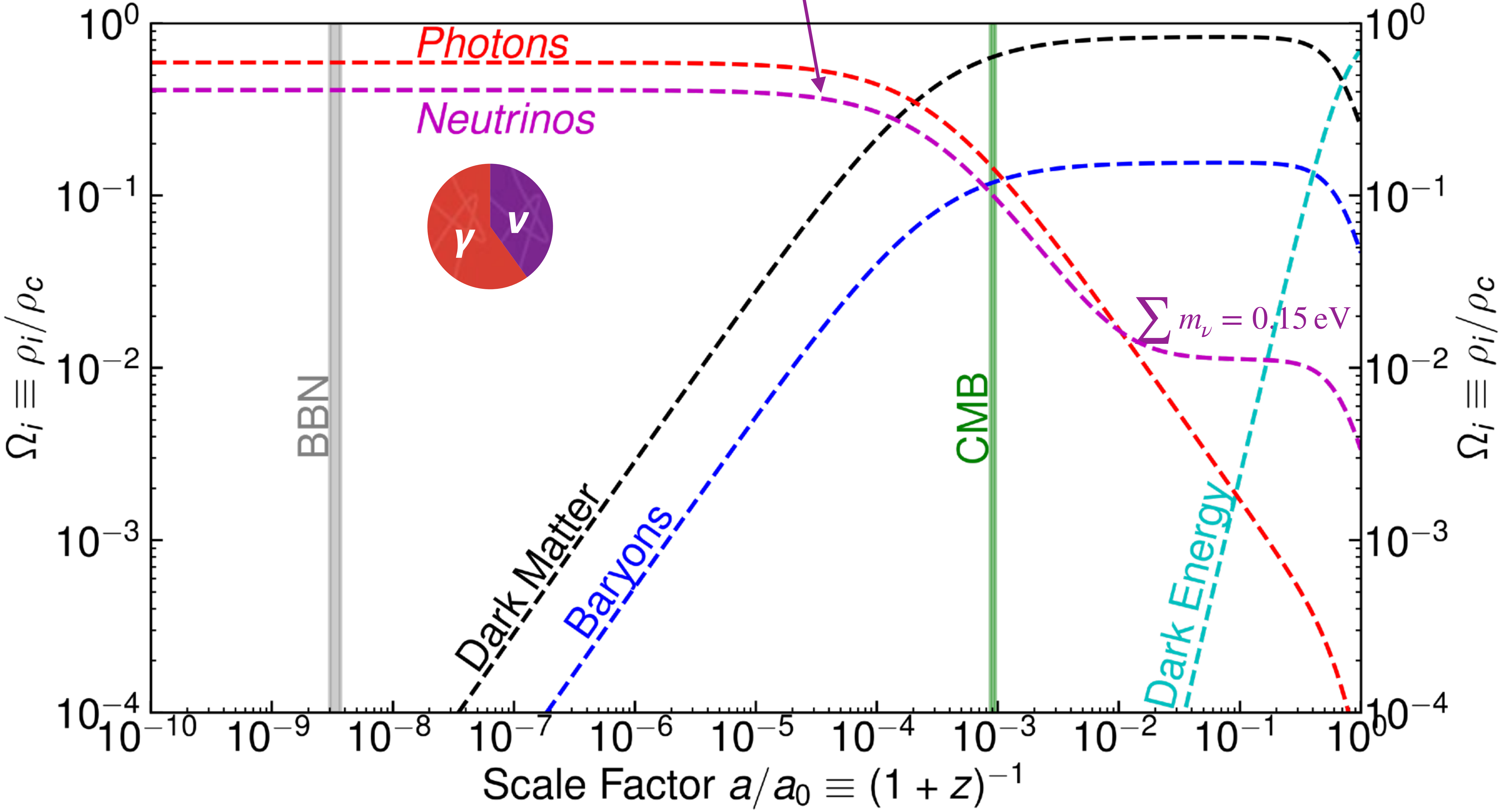


- **How do we measure the energy density in relativistic neutrino species?**
- **The key parameter is:** $N_{\rm eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{\rm rad}-\rho_\gamma}{\rho_\gamma}\right)$
- **when only neutrinos and photons are present:** $N_{\rm eff} = 3\left(\frac{1.4\,T_\nu}{T_\gamma}\right)^4$
- **The Standard Model value is:** $\boxed{N_{\rm eff}^{\rm SM} = 3.044(1)}$

Bennett, Buldgen, Drewes & Wong 1911.04504
Escudero Abenza 2001.04466
Akita & Yamaguchi 2005.07047
Froustey, Pitrou & Volpe 2008.01074
Gariazzo, de Salas, Pastor et al. 2012.02726
Hansen, Shalgar & Tamborra 2012.03948

Neutrino Evolution

Neutrinos are always a relevant species in the Universe's evolution



Non-Rel: $z_\nu^{\mathrm{non-rel}} \simeq 200 \dfrac{m_\nu}{0.1\,\mathrm{eV}}$

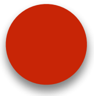
Hot DM: $\Omega_\nu h^2 = \sum m_\nu / (93.14\,\mathrm{eV})$

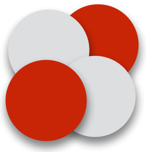
Evidence for Cosmic Neutrinos

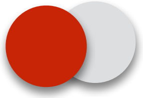
Big Bang Nucleosynthesis

See tomorrow's talk by Gianpiero!

Current measurements are broadly consistent with the SM picture


H ~ 75%


^{4}He ~ 25%


D ~ 0.005%

This implies that neutrinos should have been present:

1) **It is impossible to have successful BBN without neutrinos. They participate in $p \leftrightarrow n$ conversions up to $T \gtrsim 0.7\,\mathrm{MeV}$**

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$
$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$
$$n + \nu_e \leftrightarrow p + e^-$$

2) **Neutrinos contribute to the expansion rate $H \propto \sqrt{\rho}$**

By comparing predictions against observations, we know:

$$\boxed{N_{\rm eff}^{\rm BBN} = 2.86 \pm 0.28}$$

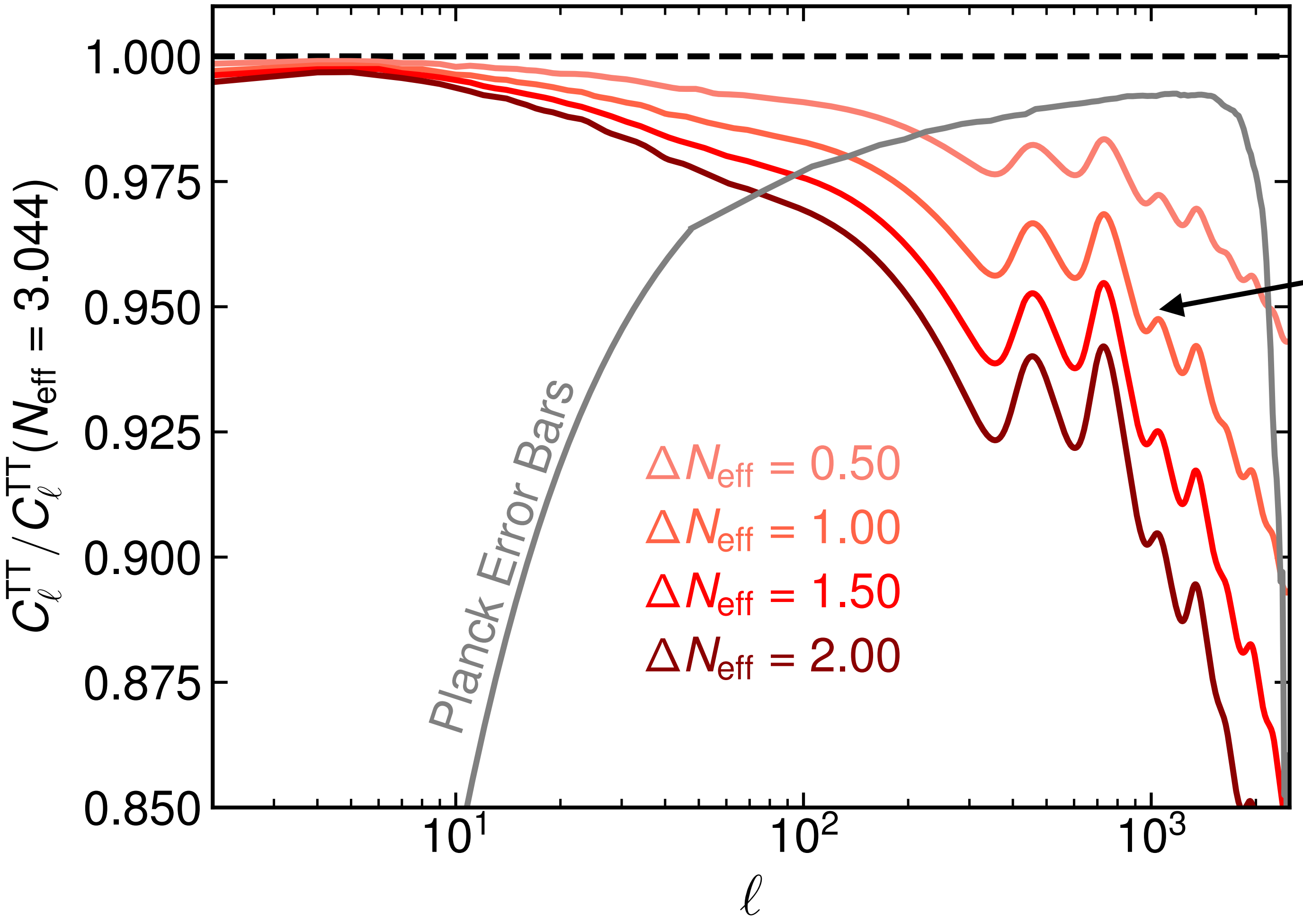
see e.g. Pisanti et al. 2011.11537

Evidence for Cosmic Neutrinos

Cosmic Microwave Background

See yesterday's talk by Paolo!

Why? **Ultra-relativistic neutrinos represent a large fraction of the energy density of the Universe, $H \propto \sqrt{\rho}$**



$N_{\rm eff}$ is constrained by the high-ℓ multipoles, i.e. Silk damping

$$\boxed{N_{\rm eff}^{\rm CMB+BAO} = 2.99 \pm 0.17}$$

Planck 2018 1807.06209

Evidence for Cosmic Neutrinos

- **Current constraints**

BBN $N_{\mathrm{eff}}^{\mathrm{BBN}} = 2.86 \pm 0.28$ **Pisanti et al. 2011.11537**

Planck+BAO $N_{\mathrm{eff}}^{\mathrm{CMB}} = 2.99 \pm 0.17$ **Planck 2018, 1807.06209**

- **Standard Model prediction:** $N_{\mathrm{eff}}^{\mathrm{SM}} = 3.044(1)$
- **Data is in excellent agreement with the Standard Model prediction**
- **This provides strong (albeit indirect) evidence for the Cosmic Neutrino Background.**

Today! **Tomorrow!**

Implications:

1) Stringent constraint on many BSM settings

2) We can use cosmological data to test neutrino properties

Neutrino Decoupling in the SM

Why it is worth investigating the process of neutrino decoupling?

1) The ultimate generation of CMB experiments are expected to measure Neff with a precision of 0.03!

That means that small effects cannot be neglected!

2) This will allow us to understand what can happen in scenarios beyond the Standard Model!

Why $N_{\rm eff}^{\rm SM}$ is not exactly 3?

1) Neutrino decoupling is not instantaneous

$$\sigma \sim G_F^2 E_\nu^2$$

2) Weak Interactions freeze out at T = 2-3 MeV. This is not too different from the electron mass and therefore there is some heating from e$^+$e$^-$ annihilation

$$n \langle \sigma v \rangle \simeq G_F^2 T^5 \simeq H$$

3) Finite temperature QED corrections

$$\delta m_e^2(T),\ \delta m_\gamma^2(T)$$

4) Neutrino oscillations are active at T < 5 MeV!

$$t_\nu^{\rm os} \sim \frac{12\,T}{\Delta m^2} \qquad t_{\rm exp} = \frac{1}{2H} \sim \frac{m_{Pl}}{3.44\sqrt{10.75}T^2} \qquad t_\nu^{\rm scat} \sim \frac{1}{G_F^2\,T^5}$$

Neff in the Standard Model

Methods to solve for neutrino decoupling:

The simplest method:

— Assume neutrinos decouple instantaneously and use entropy conservation to get the neutrino temperature today. Exercise!

Pros: Very easy to do 😃

Con: Does not include dynamics and is not too accurate 😐

The full method:

— Solve the actual Boltzmann equation describing $\nu - e$ and $\nu - \nu$ interactions

Pros: It gives the full result 😃

Con: It is considerably involved as it requires solving a system of *hundreds of stiff integrodifferential equations* ☹️

The intermediate method:

— Track the neutrino energy density of all the species assuming they follow thermal equilibrium distributions

Pros: It is fast, precise and allows one to easily include BSM species in the game! 😃

Solving for Neutrino Decoupling

A given particle species can be fully characterized by its distribution function:

Distribution function: $f \equiv$ **Number of particles in a phase space volume of** $\frac{1}{(2\pi\hbar)^3} d^3x d^3p$

In an homogeneous and isotropic Universe: $f(\vec{x}, \vec{p}) = f(|\vec{p}|) = f(p)$

From the distribution function we can extract all relevant properties of the system:

Number density:

$$n = g_i \int \frac{d^3p}{(2\pi)^3} f(p)$$

Energy density:

$$\rho = g_i \int \frac{d^3p}{(2\pi)^3} E\, f(p)$$

Pressure density:

$$p = g_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(p)$$

$g_i \equiv$ **Number of internal degrees of freedom for the given species**

$g_\gamma = 2$ $\quad$ $g_{e^+ + e^-} = 2 + 2 = 4$ $\quad$ $g_{\nu_e + \bar{\nu}_e} = ?$ $\quad$ $g_{\nu_e + \bar{\nu}_e} = 2$

(only the ν_L and $\bar{\nu}_R$ participate in the weak interactions!)

Equilibrium Thermodynamics

- **Particles that are efficiently interacting are said to be in thermal equilibrium.**
- **Species in thermal equilibrium follow the following distributions:**

Fermions:

$$f_{\rm FD}(E) = \frac{1}{1+e^{\frac{E-\mu}{T}}}$$

Bosons:

$$f_{\rm BE}(E) = \frac{1}{-1+e^{\frac{E-\mu}{T}}}$$

T is the temperature

μ is the chemical potential

In the majority of relevant physical cases $\mu \simeq 0$ because we have the same number of particles and anti particles. In that case:

for $T \gg m$	$n \sim T^3$	$\rho \sim T^4$	$p = \frac{1}{3}\rho$
for $T \ll m$	$n \sim (mT)^{3/2} e^{-m/T}$	$\rho \simeq mn$	$p = 0$

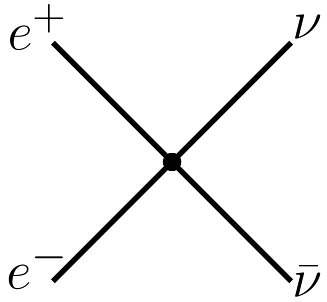
Take aways:

— The number density of non-relativistic particles decays exponentially if the species is in thermal equilibrium!

— That means that the energy density of the early Universe is dominated by relativistic particles!

Neutrino Decoupling

- Interactions between neutrinos and electrons were very efficient for T > 2 MeV. That means that we expect neutrinos to follow a distribution function that roughly resembles an equilibrium one



- The full description will be obtained by solving the full Boltzmann equation:

$$\frac{\partial f}{\partial t} - Hp\frac{\partial f}{\partial p} = C[f]$$

Here, H is the expansion rate of the Universe and $C[f]$ is the collision term that accounts for the interactions of neutrinos with any other species, e.g.: $e^+e^- \leftrightarrow \bar{\nu}\nu$

- The main issue is that:

$$C[f] \sim \int_{\mathrm{9D-PhaseSpace}} d\Pi[f_{\nu_\alpha} f_{\nu_\beta} - \ldots]$$

- The integral can be simplified to just 2D, but then this equation represents a system of stiff integrodifferential equation that can be rather difficult to solve

see Mangano et al. astro-ph/0111408 for early calculations and Bennet et al. 2012.02726 for the most recent one

see also the FortEPiaNO code: by Gariazzo, de Salas & Pastor

Neutrino Decoupling

- **A trick to solve it much more easily is to integrate it and make several approximations, see Escudero 1812.05605 and 2001.04466. This is what is typically done in the context of thermal Dark Matter or in Baryogenesis.**

$\frac{df}{dt} - pH\frac{df}{dp} = C[f]$ **integrating this equation by** $\frac{1}{(2\pi)^3}Ed^3p$ **yields:**

$$\boxed{\frac{d\rho}{dt} + 3(\rho + p)H = \int \frac{Ep^2}{2\pi^2}C[f]dp \equiv \frac{\delta\rho}{\delta t}}$$

To actually solve for this we need an ansatz for the distribution function of neutrinos. Lets assume they follow a Fermi-Dirac distribution with a temperature T_ν.

Once this is done one simply needs to solve two ordinary differential equations for T_ν and T_γ:

$$\frac{dT}{dt} = \frac{d\rho}{dt} \Big/ \frac{\partial\rho}{\partial T} = \left[-3H(\rho + p) + \frac{\delta\rho}{\delta t}\right] \Big/ \frac{\partial\rho}{\partial T}$$

The energy transfer rates are analytical expressions if one neglects the electron mass and assumes Maxwell-Boltzmann statistics for the distributions:

as a result of a 12D integral!:

$$\boxed{\left.\frac{\delta\rho_\nu}{\delta t}\right|_{\rm SM} = 4\frac{G_F^2}{\pi^5}\left(g_L^2 + g_R^2\right)\left[32\left(T_\gamma^9 - T_\nu^9\right) + 56\,T_\gamma^4 T_\nu^4\left(T_\gamma - T_\nu\right)\right]}$$

Neutrino Decoupling in the SM

Solutions after electron-positron annihilation:

Neutrino Decoupling in the SM	$T_{\nu_e} = T_{\nu_{\mu,\tau}}$		$T_{\nu_e} \neq T_{\nu_{\mu,\tau}}$		
Scenario	T_γ/T_ν	$N_{\rm eff}$	T_γ/T_{ν_e}	T_γ/T_{ν_μ}	$N_{\rm eff}$
Instantaneous decoupling	1.4010	3.000	1.4010	1.4010	3.000
Instantaneous decoupling + QED.	1.3998	3.010	1.3998	1.3998	3.010
FD+m_e collision term	1.3969	3.036	1.3957	1.3976	3.035
FD+m_e collision term + NLO-QED	1.39578	**3.045**	1.3946	1.3965	**3.044**

From these results we can draw some conclusions:

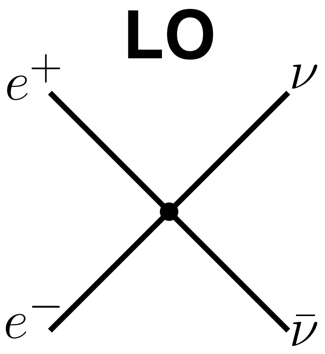
1) The main contributions to $\Delta N_{\rm eff}^{\rm SM}$ come from residual electron-positron annihilations into neutrinos $\Delta N_{\rm eff}^{\rm SM} \simeq 0.036$

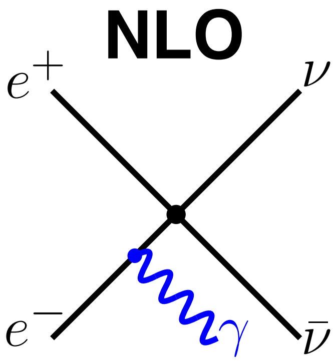
2) Finite temperature corrections contribute to $\Delta N_{\rm eff}^{\rm SM} \simeq 0.009$

3) Neutrino oscillations contribute to $\Delta N_{\rm eff}^{\rm SM} \simeq 0.001$

Outlook:

No one has so far included QED corrections to the neutrino-electron interactions.





It is expected that they will modify $\Delta N_{\rm eff}^{\rm SM} \simeq -0.001$ Escudero 20'

Constraints from $N_{\rm eff}$

$N_{\rm eff}$ measurements constrain very light particles that decoupled while relativistic after the Big Bang (very much like neutrinos). Their energy density is parametrized by

$$\boxed{\Delta N_{\rm eff} = N_{\rm eff} - 3.044}$$

We have thousands of BSM models where $\Delta N_{\rm eff} > 0$

Some examples:

- **Sterile Neutrino** $m_N \sim {\rm eV}$ $\Delta N_{\rm eff} = 1$ **(e.g. Gariazzo, de Salas & Pastor 1905.11290)**
- **Goldstone Bosons**

Editors' Suggestion

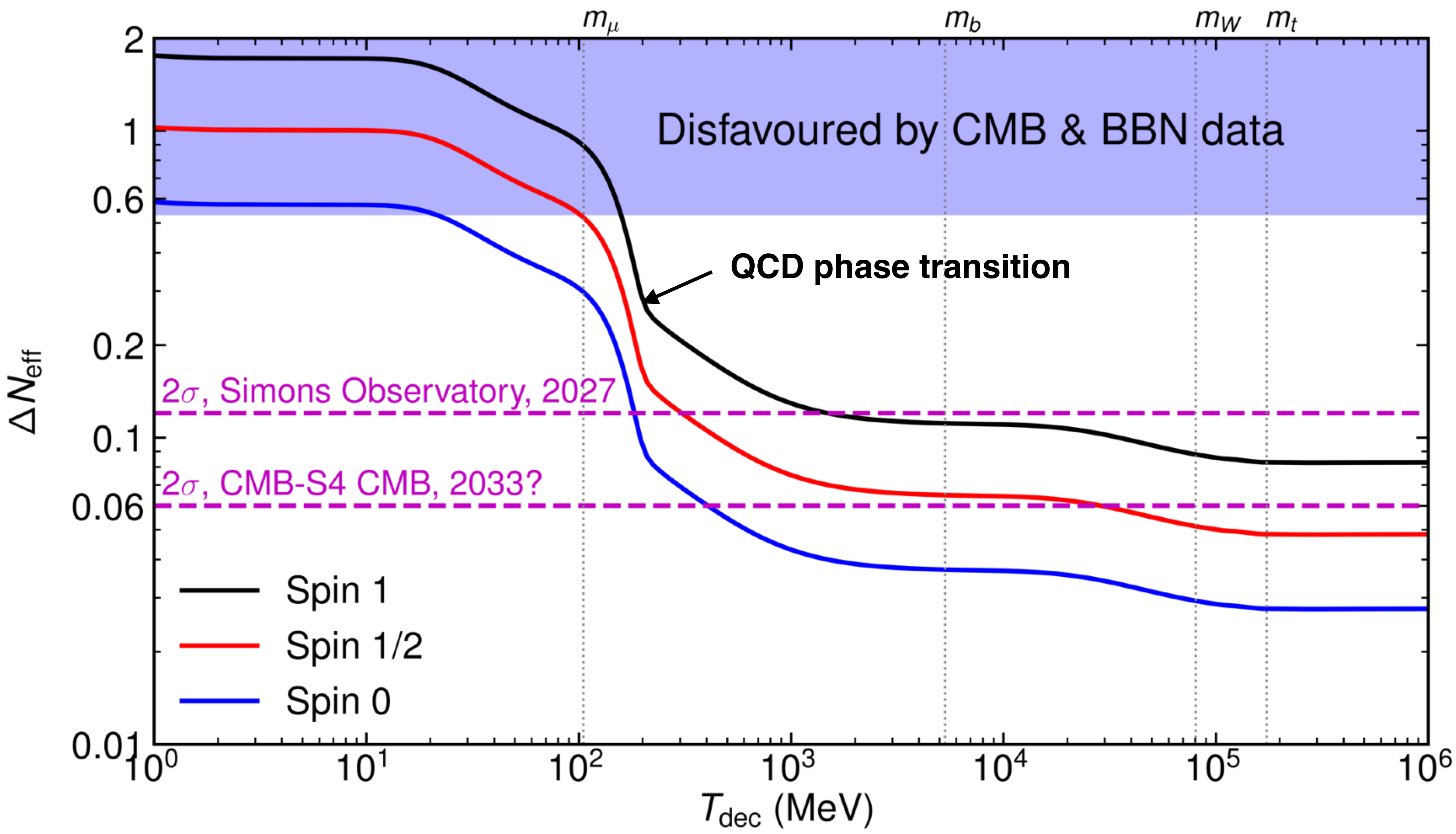
Goldstone Bosons as Fractional Cosmic Neutrinos

Steven Weinberg
Phys. Rev. Lett. **110**, 241301 – Published 10 June 2013

- **Other sterile long-lived particles** **Gravitino, hidden sector particles ...**

Constraints from N_{eff}

Contribution to $\Delta N_{\rm eff}$ from a massless particles that decoupled at $T_{\rm dec}$



Take Away:

Any new massless state in thermal equilibrium with the SM plasma should have decoupled at $T_{\rm dec} \gtrsim 100\,{\rm MeV}$

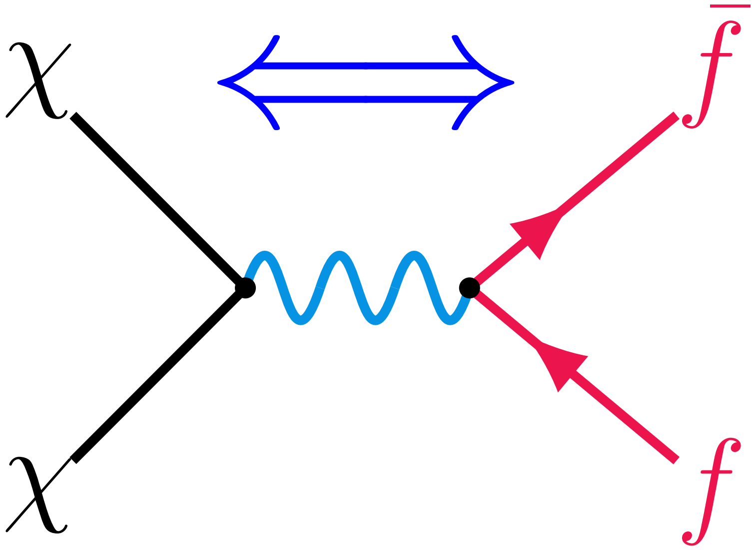
Constraints from $N_{\rm eff}$

Constraints are relevant in many other BSM settings:

WIMPs	$m_{\rm WIMP} > (4-10)\,{\rm MeV}$	**Sabti et al. 1910.01649** **Boehm et al. 1303.6270**
GeV-Sterile Neutrinos	$\tau_N \lesssim 0.05\,{\rm s}$	**Sabti et al. 2006.07387** **Dolgov et al. hep-ph/0008138**
Vector Bosons	$g \lesssim 10^{-10} \quad m \lesssim 10\,{\rm MeV}$	**Escudero et al. 1901.02010** **Kamada & Yu 1504.00711**
Axions		**Raffelt et al. 1011.3694** **Blum et al. 1401.6460**
Low Reheating	$T_{\rm RH} > (2-5)\,{\rm MeV}$	**de Salas et al. 1511.00672** **Hasegawa et al. 1908.10189**
Variations of G_N	$G_{\rm BBN}/G_0 = 0.98 \pm 0.03$	**Alvey et al. 1910.10730** **Copi et al. astro-ph/0311334**
PBHs	$6\times 10^8\,{\rm g} < M_{\rm PBH} < 2\times 10^{13}\,{\rm g}$	**Carr et al. 0912.5297** **Keith et al. 2006.03608**
Stochastic GW backgrounds	$\Omega_{\rm GW} h^2 < 3\times 10^{-6}$	**Caprini & Figueroa 1801.04268**

Constraints from N_{eff}: WIMPs

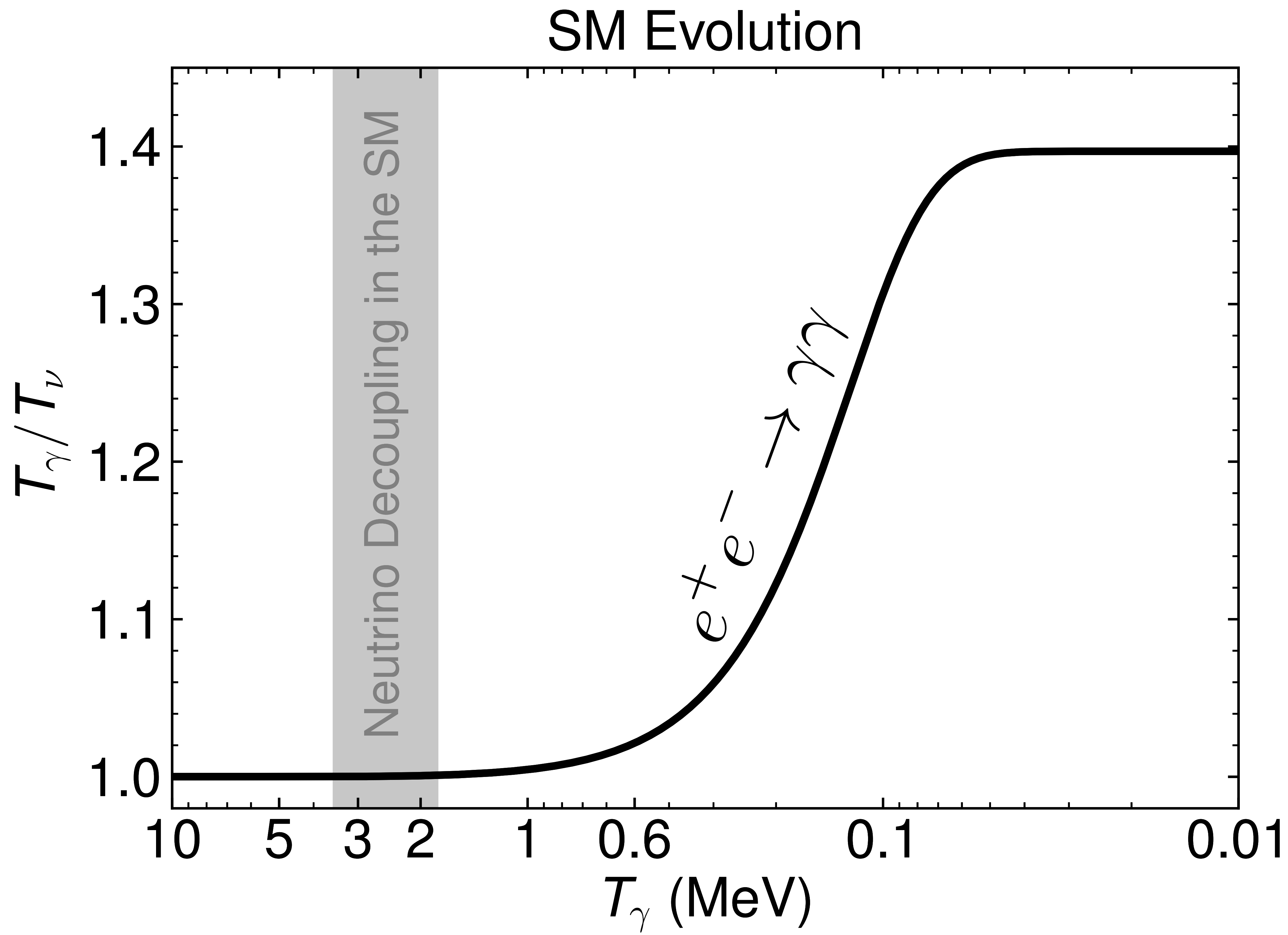
WIMPs are in thermal equilibrium until $T \sim m_\chi/20$



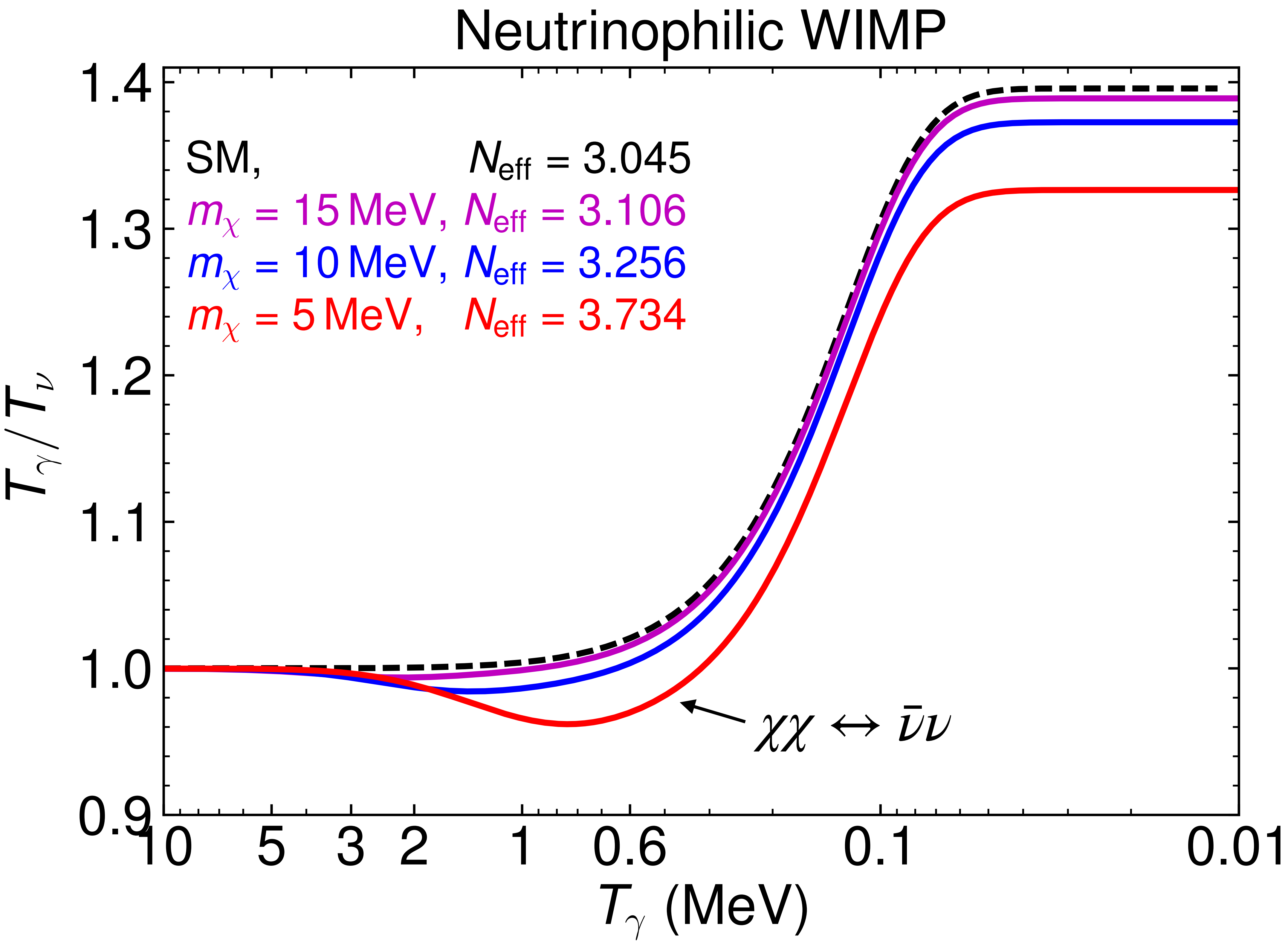
That means that WIMPs with $m_\chi \lesssim 20\,\mathrm{MeV}$ can affect neutrino decoupling, and therefore $N_{\rm eff}$

- **They can release entropy into the SM sectors**
- **Could delay the process of neutrino decoupling**

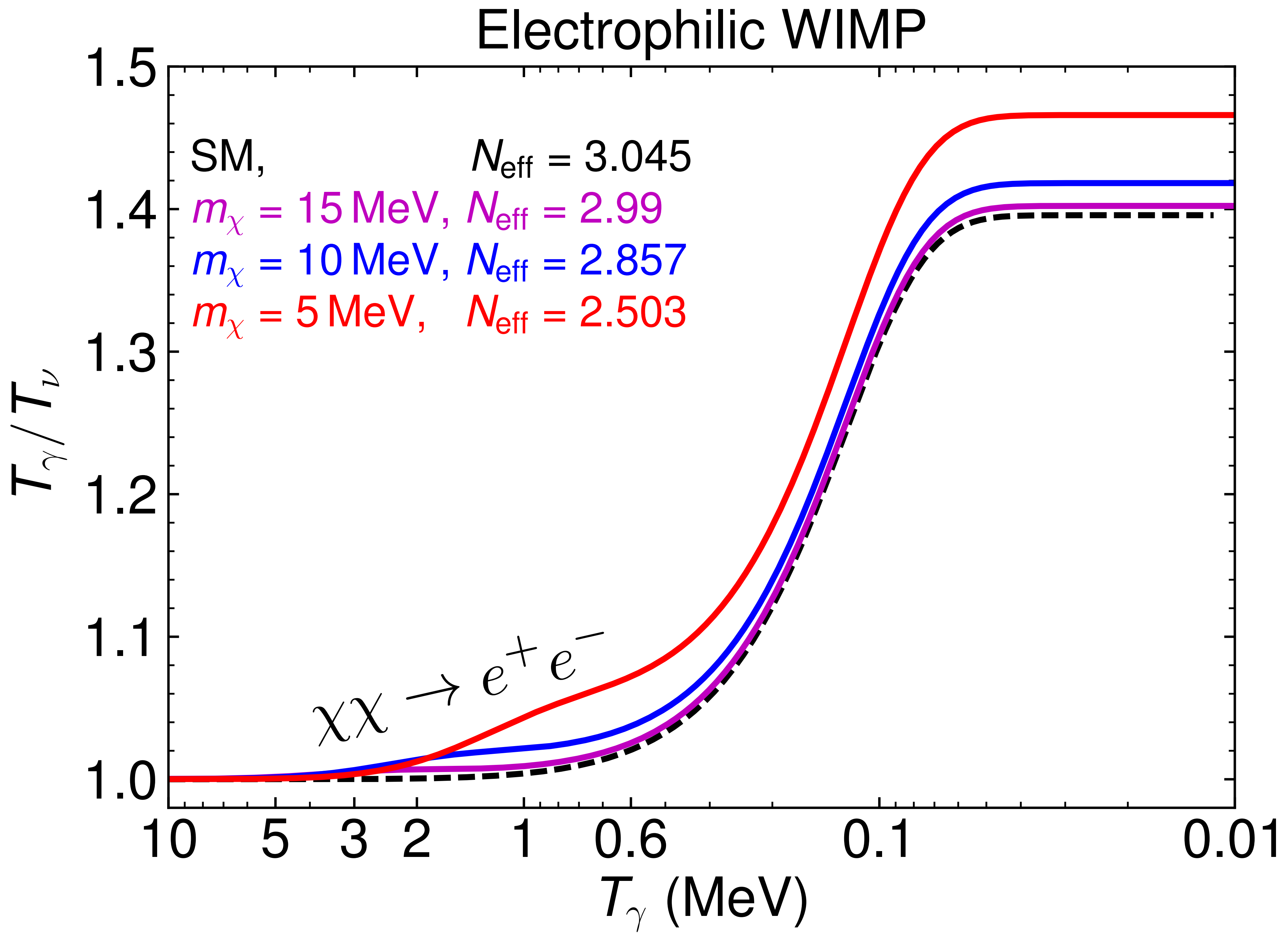
Impact of Thermal Dark Matter



Neutrinophilic WIMP: $N_{\mathrm{eff}} > 3.044$



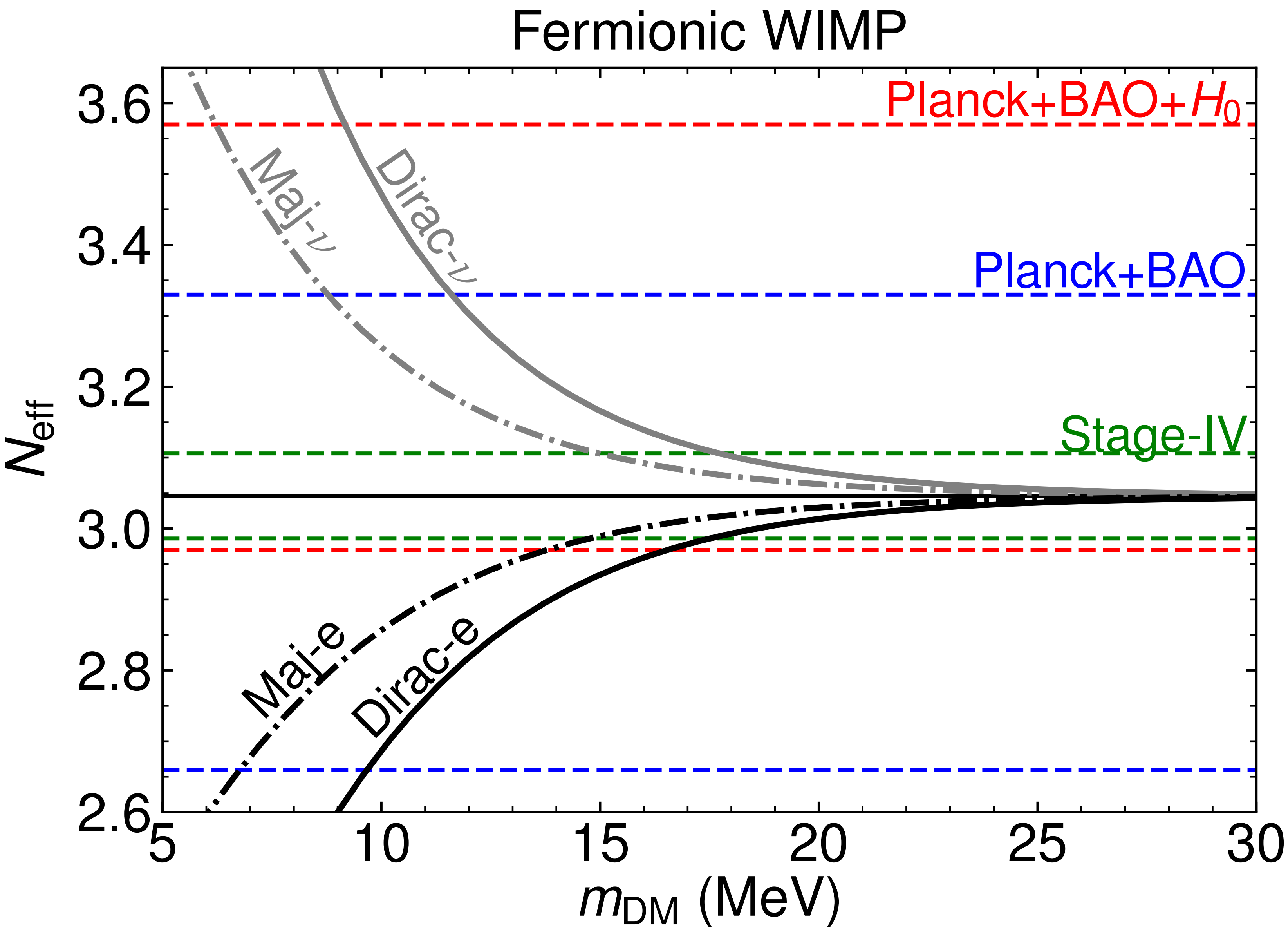
Electrophilic WIMP: $N_{\text{eff}} < 3.044$



This is one of the very few scenarios where $N_{\text{eff}} < 3.044$!

Lower bound on the DM mass

Comparing prediction vs. observations:



$$\boxed{m_{\rm DM} > 4\,{\rm MeV}}$$

at 95% CL

Sabti et al. 1910.01649
Boehm et al. 1303.6270

In addition, we could test WIMPs of $m_{\rm DM} \lesssim 15\,{\rm MeV}$ with CMB Stage-IV experiments

Summary

Number of effective neutrino species

$$N_{\rm eff}^{\rm BBN} = 2.86 \pm 0.28 \qquad N_{\rm eff}^{\rm CMB} = 2.99 \pm 0.17$$

$$N_{\rm eff}^{\rm SM} = 3.044(1)$$

Agreement between measurements of $N_{\rm eff}$ and the SM prediction implies:

Strong evidence that the CNB should be there as expected in the SM

This represents an important constraint on many BSM settings

e.g.: $m_{\rm WIMP} > 4\,{\rm MeV}$

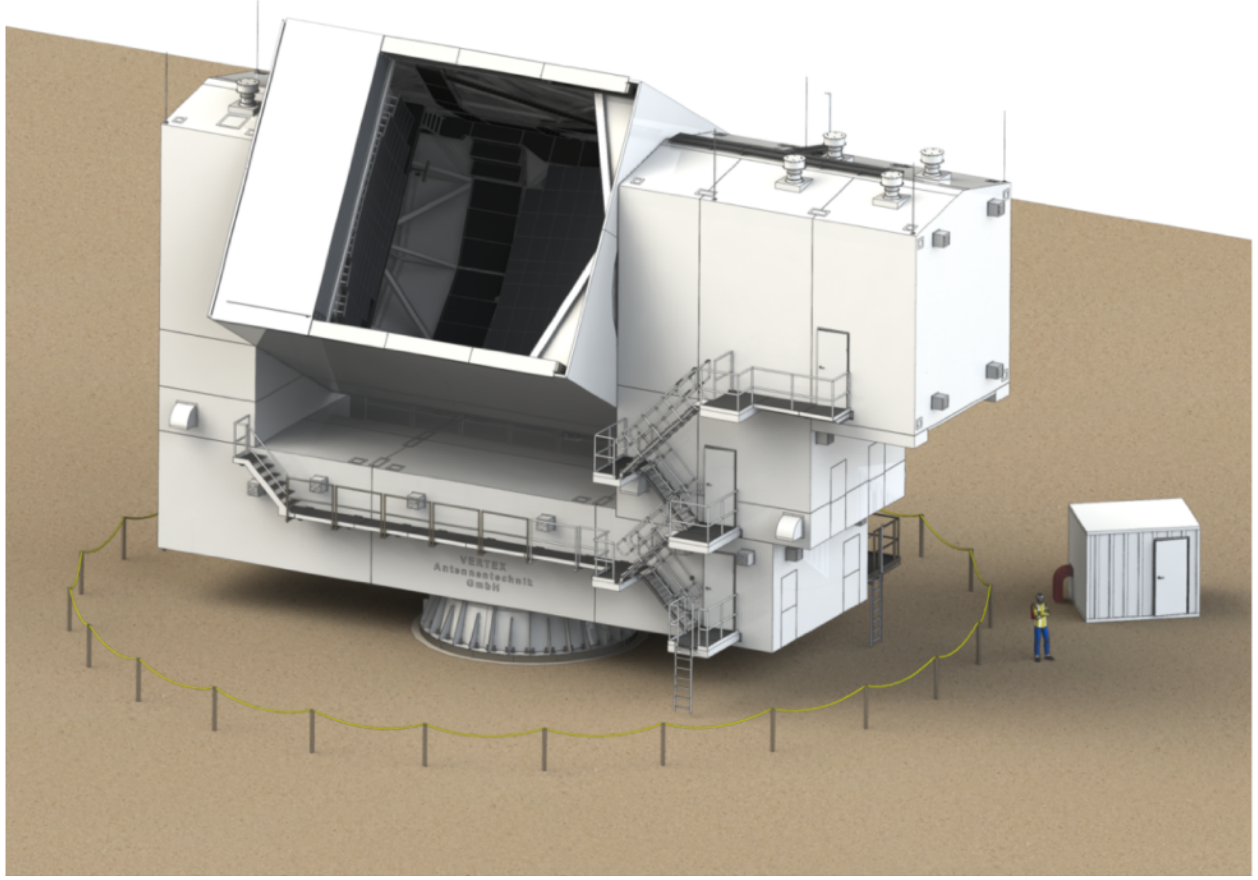
Outlook: Number of Neutrinos

The next generation of CMB experiments are expected to significantly improve the sensitivity on Neff.

Simons Observatory

$$\sigma(N_{\rm eff}) = 0.06$$ **~2028**

CMB-S4



$$\sigma(N_{\rm eff}) = 0.03$$ **~2035?**

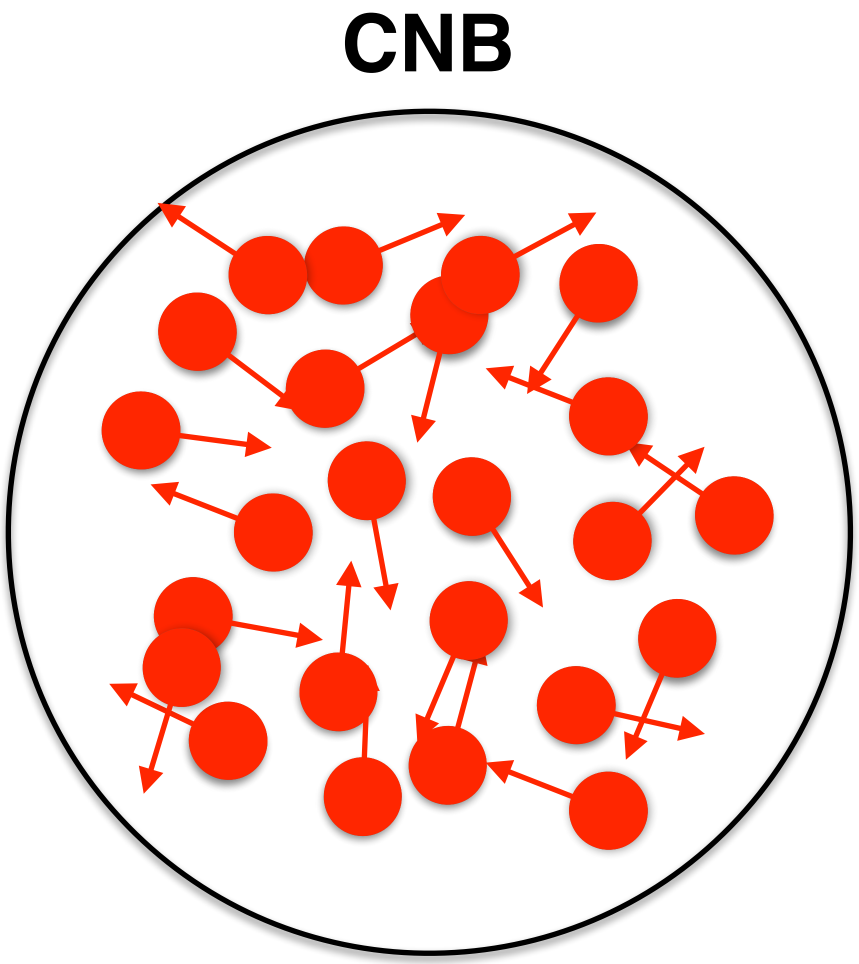
These measurements will represent an important test to BSM physics and perhaps may yield a BSM signal!

CNB Detection?

We have no direct evidence for the Cosmic Neutrino Background

Can we try to directly detect it?

The Cosmic Neutrino Background in the Standard Model:



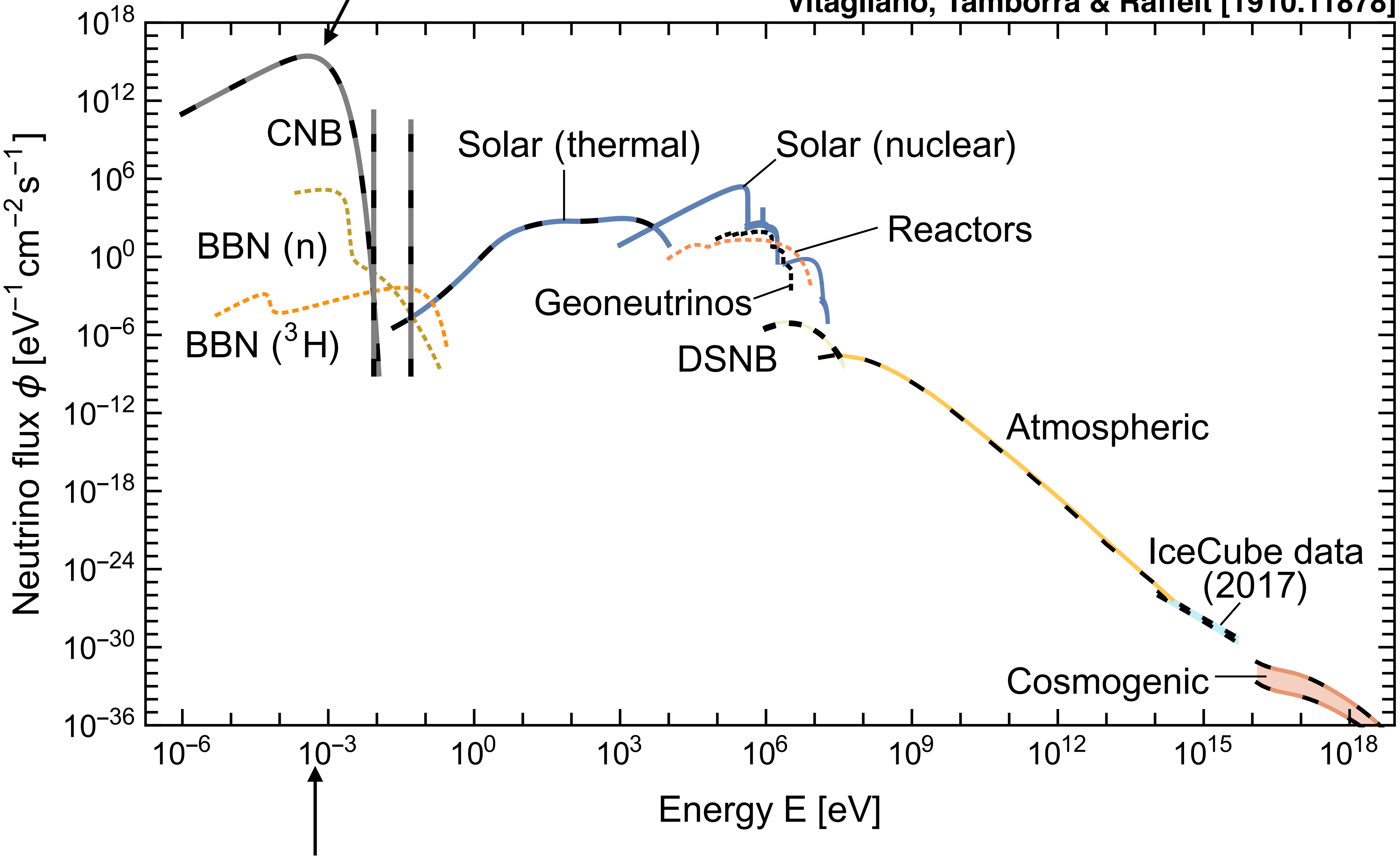
$$n_\nu^{\rm SM} \simeq 300\,{\rm cm}^{-3}$$ **Large number density**

$$T_\nu^{\rm SM} = T_\gamma/1.4 \simeq 1.95\,{\rm K}$$ **Very low energetic**

CNB Detection?

The CNB represents the largest flux of neutrinos on Earth! 😃

Vitagliano, Tamborra & Raffelt [1910.11878]



However, they are very low energetic ☹️

CNB Detection?

Directly detecting the CNB is VERY VERY challenging

Perhaps the best search strategy is via capture in beta decaying nuclei

$$\boxed{\nu_e + {}^3\mathrm{H} \rightarrow e^- + {}^3\mathrm{He}^+}$$

Weinberg [1962]

Indeed, a recent search at KATRIN was able to bound

$$\boxed{n_\nu < 10^{10}\, n_\nu^{\mathrm{SM}}}$$

[2202.04587]

The PTOLEMY collaboration has taken seriously the possibility of actually detecting it [1808.01892, 1902.05508, 2203.11228]

Perhaps in a couple of decades we have a measurement of the last background predicted by the Big Bang Theory!

CNB Detection?

Some experimental challenges:

Low number of events

$$\Gamma_{\nu_e + {}^3\mathrm{H} \to e^- + {}^3\mathrm{He}^+} \sim \frac{4(8)}{\mathrm{year}} \frac{M_T}{100\ \mathrm{g}} \frac{n_\nu}{56\,\mathrm{cm}^{-3}}$$

Huge beta background → **High energy resolution needed**

$${}^3\mathrm{H} \to e^- + {}^3\mathrm{He}^+ + \bar{\nu}_e \longrightarrow \Delta \lesssim m_\nu$$

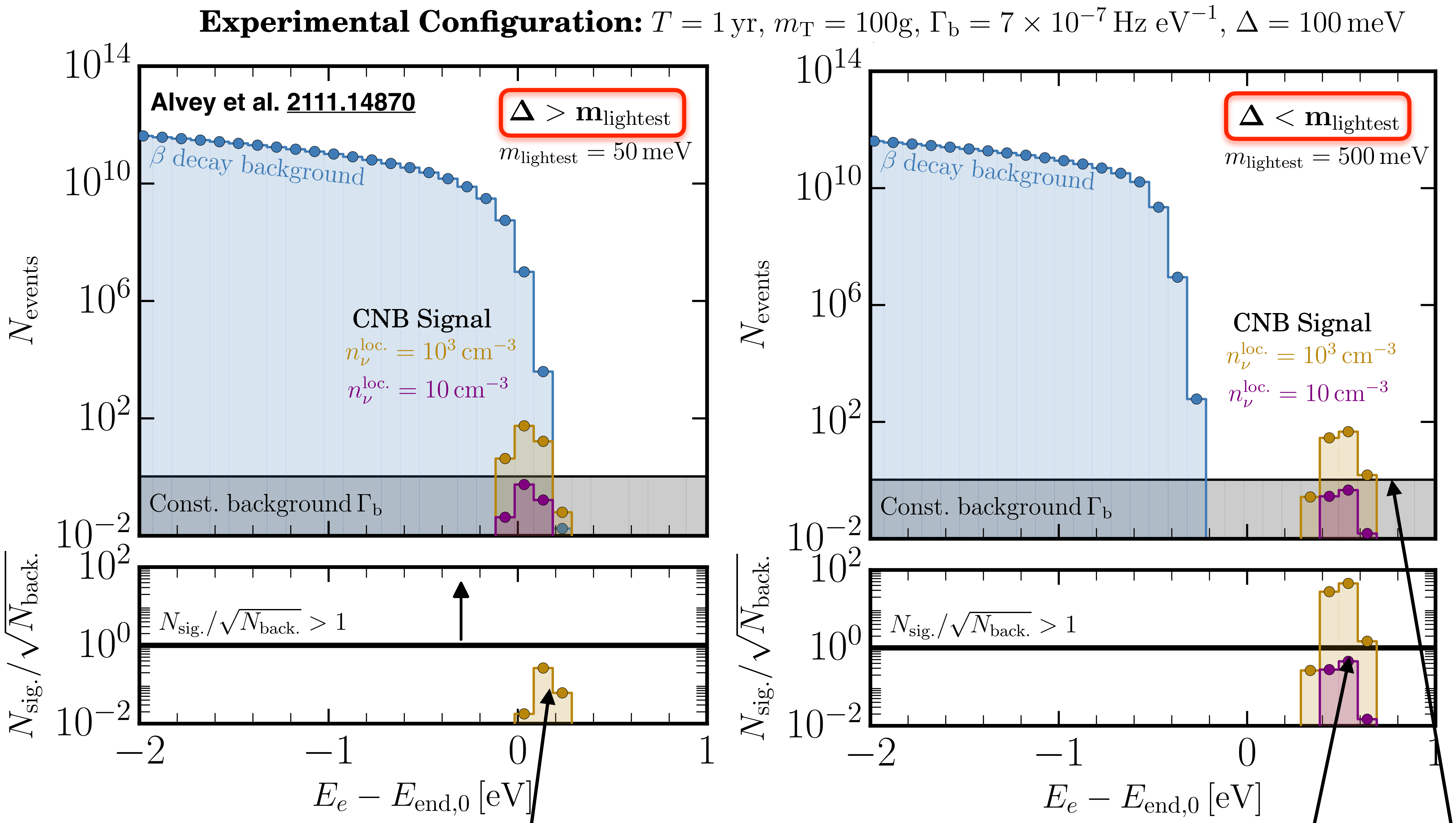
Physical challenges:

The deposition of ${}^3\mathrm{H}$ on graphene will in turn lead to a smearing of the spectrum with at least $\Delta E_e \sim 0.2\,\mathrm{eV}$ Cheipesh, Cheianov & Boyarsky [2101.10069], Nussinov & Nussinov [2108.03695]

The PTOLEMY collaboration is looking at potential remedies for this [2203.11228]. They suggest using perhaps tubularly shaped graphene sheets.

CNB Detection?

This makes rather complicated the CNB detection



No detection if $\Delta > m_\nu$

Detection possible if $\Delta < m_\nu$

But with a high exposure and low background

CNB Detection Status?

Several achievements needed:

Low background rate

Very high energy resolution

Large target mass

The PTOLEMY collaboration is planning to have a first prototype in ~1 year at the Gran Sasso Laboratory!

see talk by Marcello Messina in Neutrino 2022! Link

Take Home Messages

1) In the Standard Model, neutrinos are always a relevant component of the Universe across its entire history

2) When neutrinos are relativistic, their energy density is measured by $N_{\rm eff}$ which in the Standard Model is 3.044(1)

3) The agreement between measurements of $N_{\rm eff}$ and its prediction represents an important constraint for many BSM settings

4) We have indirect (albeit strong) evidence that the Cosmic Neutrino Background should be there. Its direct detection may happen in the next decades thanks to efforts such as PTOLEMY.

Key facts/numbers to remember

- **Neutrinos decouple at a temperature of $T \simeq 2\,\mathrm{MeV}$. From then onwards, they do not interact with anything.**

- **After e^+e^- have annihilated, neutrinos have a temperature of** $T_\nu \simeq T_\gamma/1.4$

- **There should be $n_\nu \simeq T_\nu^3 \simeq 300\,\mathrm{cm}^{-3}$ in every point in the Universe**

- **Neutrinos become non-relativistic when $T_\nu \lesssim m_\nu/3$.**

  This corresponds to $z_{\rm nr} \simeq 200\, m_\nu/(0.1\,\mathrm{eV})$

- **We have measured the mass squared differences between neutrinos which means that at least two of them should be non-relativistic today! Exercise: explicitly check when!**

Recommended References

Introductory:

Modern Cosmology
Scott Dodelson & Fabian Schmidt, Academic Press, 2020

General:

The Early Universe
Edward Kolb & Michael Turner, Front. Phys. 69, 1990

Introduction to the Theory of the Early Universe
Valery Rubakov & Dmitry Gorbunov, World Scientific, 2017

Advanced/Neutrinophilic:

Neutrino Cosmology
Lesgourgues, Mangano, Miele & Pastor, Cambridge University Press, 2013

Neutrinos in Cosmology
Alexander Dolgov, Physics Reports 370 (2002) 333–535

Kinetic Theory in the Expanding Universe
Jeremy Berstein, Cambridge University Press, 1988

Time for Questions and Comments

End of Lecture I

Thank you for your attention!

miguel.escudero@tum.de