Quark Flavour Physics : Theory Aspects

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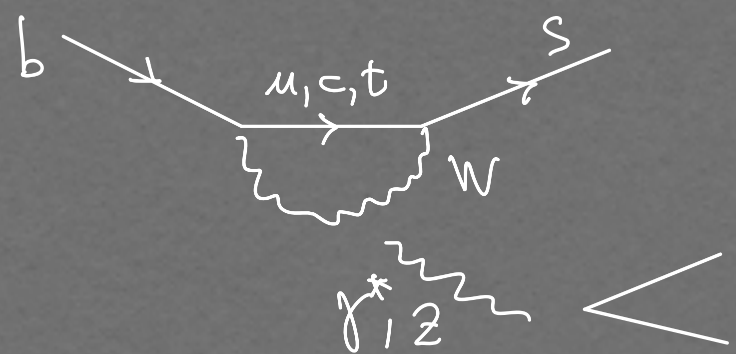
L2 :

- $b \to s$ transitions : basics
- The $b \to s\ell\ell$ effective Hamiltonian
- From $\mathcal{H}_{eff}$ to the amplitude
- Renormalization-scale (in)dependence
- "Matching & running"
- $b \to s$ phenomenology
  - errors on Wilson coefficients
  - form factors
  - $B \to K\ell\ell$ matrix-element structure ($\to R_K$)

b → s transitions : basics

- Why interesting

$b \to s$ transitions occur through diagrams of which the dominant one is often the "penguin":



① loop suppression

Integrals w/ 3-4 propagators

$$\hookrightarrow \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2-m^2)^n} = \frac{(\ldots)}{(4\pi)^{d/2}} (\ldots) \left(\frac{1}{m^2}\right)^{n-d/2}$$

$\mathcal{O}\left(\frac{1}{16\pi^2}\right)$

② GIM suppression

$$m_{u_i} = \{m_u, m_c, m_t\}$$

$$A \propto \sum_{i=1}^{3} V_{u_i b} V_{u_i s}^* f(m_{u_i}^2, m_W^2)$$

Now suppose $m_{u_i} = m$

$$\Rightarrow A \propto f(m^2, m_W^2) \sum_{i=1}^{3} V_{u_i b} V_{u_i s}^* = \emptyset$$

i.e. $A \to \emptyset$ if m_i degenerate (in particular if $m_i = 0$)

$$\to A \propto \sum_i V_{u_i b} V_{u_i s}^* m_i^2 =$$

$$V_{tb} V_{ts}^* (m_t^2 - m_u^2) + V_{cb} V_{cs}^* (m_c^2 - m_u^2)$$

i.e. $A \propto \dfrac{\text{differences of (quark mass)}^2}{m_W^2}$ ← "GIM suppression"

$\boxed{\text{But: amplitudes (in NP-sensitive regions) usually top-dominated}}$

③ CKM suppression: $A \propto$ off-diag terms of the CKM

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 0.97 & 0.23 & 1.3\cdot10^{-3} - i\,3\cdot10^{-3} \\ -0.23 & 0.97 & 0.04 \\ 8\cdot10^{-3} - i\,3\cdot10^{-3} & -0.04 & 0.9991 \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 & O(\lambda) & O(\lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

with $\lambda \hat{=}$ "Cabibbo angle" $\simeq 0.226$

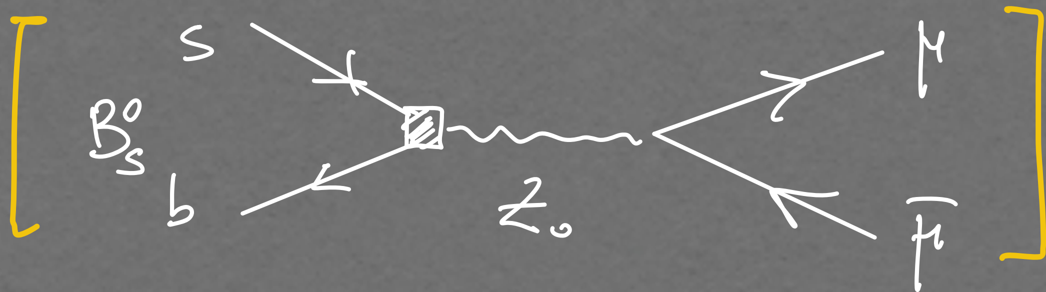
- hierarchical structure
- smaller entries the farther one goes from the diagonal

$$\underbrace{|V_{tb} V_{ts}| \simeq 0.04 \quad \times \frac{m_t^2}{m_W^2} \simeq 4}_{\text{typical CKM suppression in } b \to s} \; ; \; \underbrace{|V_{cb} V_{cs}| \simeq 0.04 \quad \times \frac{m_c^2}{m_W^2} \simeq 1.6\cdot10^{-4} \; ; \; |V_{ub} V_{us}| \simeq 8\cdot10^{-4} \quad \times \frac{m_u^2}{m_W^2} \simeq 6.2\cdot10^{-10}}_{\text{• typically subdominant in NP-sensitive regions}}$$

④ In some cases: chiral suppression

Example: $B_s \to \mu\mu$

(pseudo) scalar



must be in a spin-0 combination

↳ • longitudinal component of $Z_0 \Rightarrow$ Higgs interaction

$$\Rightarrow A \propto y_\mu \, \bar{\mu}_L \mu_R$$

• since $\mu_L \subset \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L = SU(2)_L$ doublet, whereas μ_R = singlet

$$\Rightarrow A \propto v$$

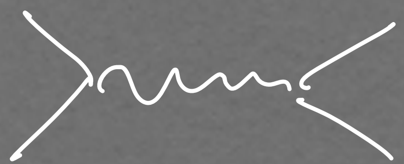
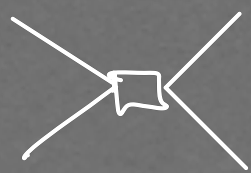
$$\longrightarrow A \propto y_\mu \cdot v = m_\mu \Rightarrow BR \propto \frac{m_\mu^2}{m_B^2}$$

with $m_\mu \sim 0.1$ GeV & $m_B \sim 5$ GeV this is a 4×10^{-4} suppression!

• The $b \to s\,\ell\ell$ effective Hamiltonian

• We already saw that internal propagators with a W or a top can be shrunk to a point with an error of $\mathcal{O}\left(\frac{p^2}{m^2_{W,t}}\right)$

external-particles' scale $\lesssim m_B^2 \simeq (5\ \text{GeV})^2$

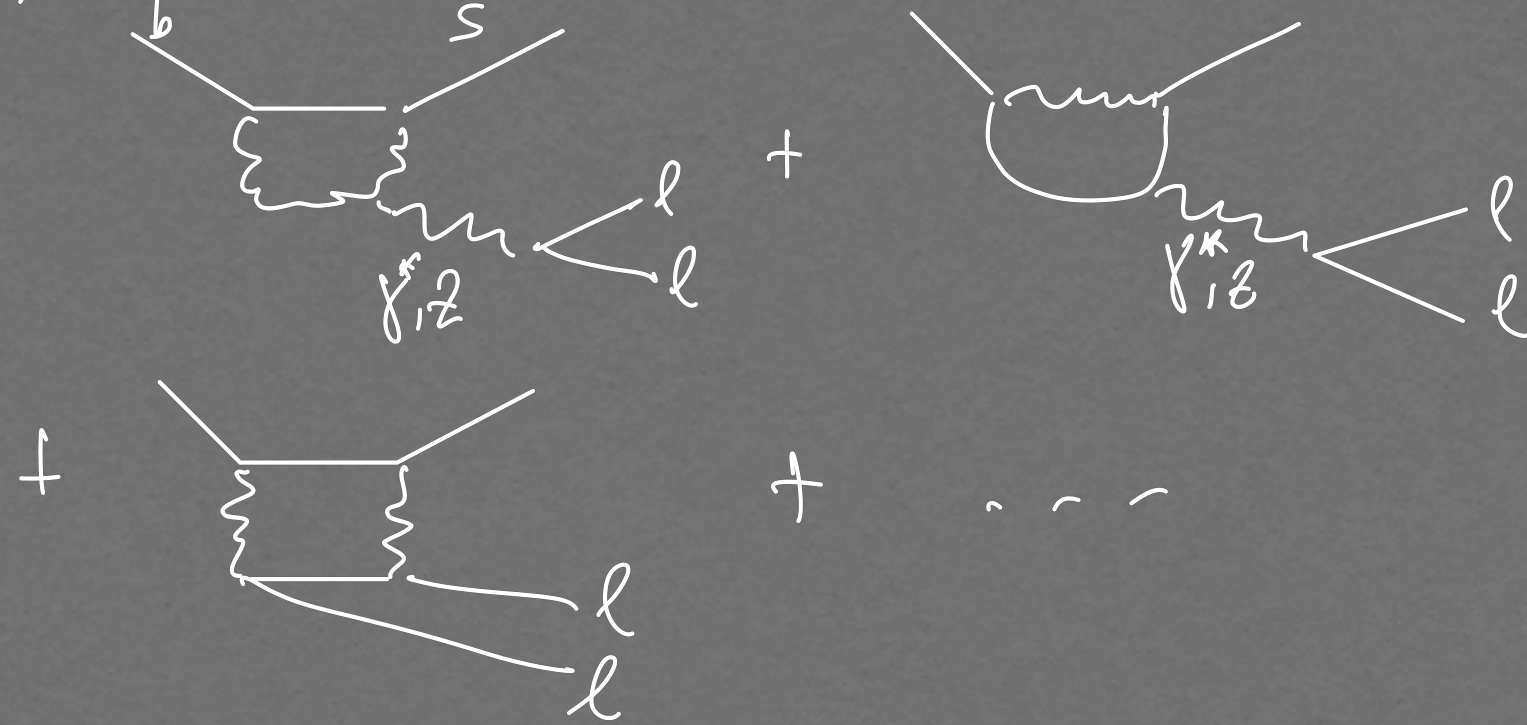
 $\simeq$  $+\ \mathcal{O}\left(p^2/m_W^2\right)$

this "effective amplitude" can be obtained with an "effective Hamiltonian"

Generalization of Fermi theory $\longrightarrow$ $\mathcal{H}_{\text{eff}} = \sum_i C_i \left(\bar{\psi}_1 \Gamma_A^{(i)} \psi_2\right)\left(\bar{\psi}_3 \Gamma_B^{(i)} \psi_4\right)$

• Similarly, the "renormalizable-theory amplitude"

$A =$



can be "matched" to an effective amplitude obtained from an Hamiltonian of the form

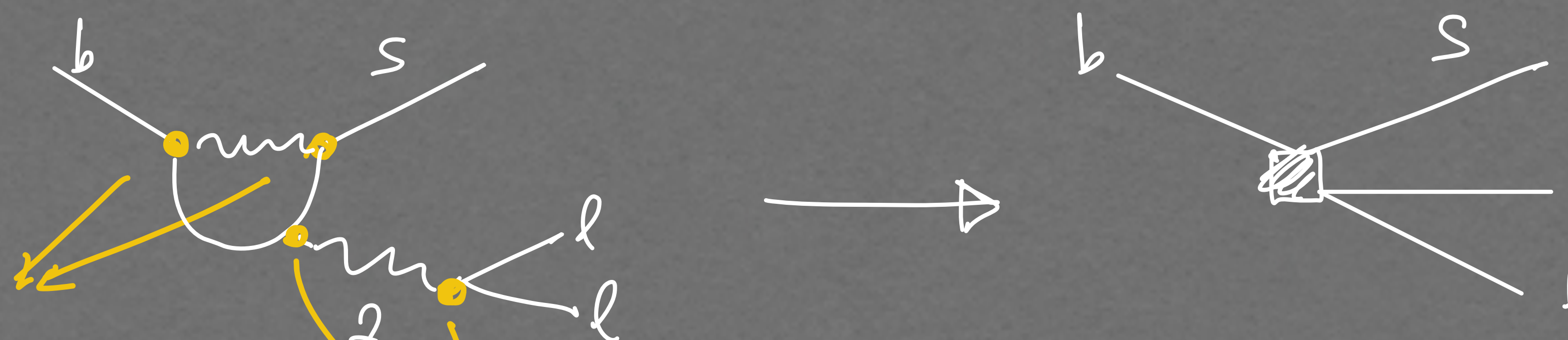
$$\mathcal{H}_{eff} = -\frac{4 G_F}{\sqrt{2}} V_{ts}^{*} V_{tb} \sum_i C_i(\mu)\, \mathcal{O}_i$$

where some of the $\mathcal{O}_i$ most relevant to us include:

- $$\mathcal{O}_{9} \equiv \frac{\alpha}{4\pi}\left(\bar{s}\gamma^{\mu}_{L} b\right)\left(\bar{\ell}\gamma_{\mu}\ell\right)$$

  $\mathcal{O}_{10}$: $\gamma_\mu \gamma_5$

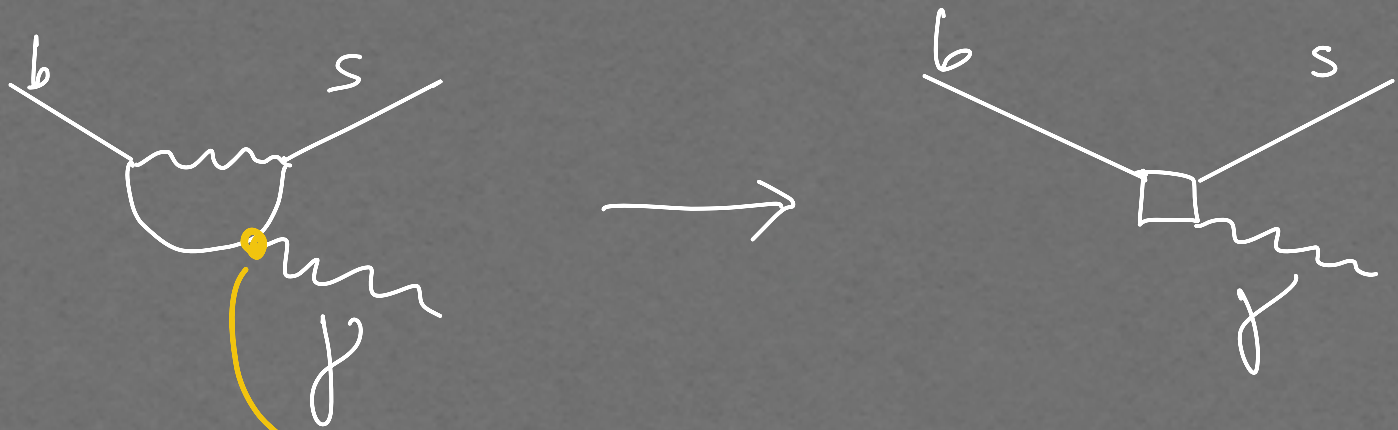
basically, "shrink the Z-penguin to a point"



$$\frac{g^2}{m_W^2} \to G_F$$

$(g^2 \sim e^2) \times$ loop factor $\to \frac{\alpha}{4\pi}$ factor in $\mathcal{O}_{9,10}$

- $\mathcal{O}_7 \equiv \frac{e}{16\pi^2} m_b \left(\bar{s}\,\sigma_{\mu\nu} P_R\, b\right) F^{\mu\nu}$



$(g \sim e) \times$ loop factor $\longrightarrow \frac{e}{16\pi^2}$ factor in $\mathcal{O}_7$

- Note: $\mathcal{O}_7$ has $\sigma_{\mu\nu} \propto [\gamma_\mu, \gamma_\nu]$ $\Leftarrow$ 2 gamma matrices

$\Rightarrow$ $\bar{s}_L \sigma^{\mu\nu} b_R$ or $\bar{s}_R \sigma^{\mu\nu} b_L$

- But the "building-block" weak interaction has one gamma matrix : $\bar{q}_L \gamma^\mu q'_L$

- So one needs to "flip the chirality" of either s or b

$\Rightarrow$ the "building-block" "interaction" for this are mass terms : $m_s \bar{s}_{L\,(R)} s_{R\,(L)}$ or $m_b \bar{b}_{L\,(R)} b_{R\,(L)}$

$\Rightarrow$ $\mathcal{O}_7$: $\bar{s}\sigma^{\mu\nu} P_R b \propto m_b$

- From $\mathcal{H}_{eff}$ to the amplitude

$$i\mathcal{M}(\bar{B}\to\bar{K}\ell\ell) = i\langle \bar{K}\ell\ell|\underbrace{\mathcal{L}_{eff}}_{=-\mathcal{H}_{eff}}|\bar{B}\rangle$$

w/ $$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb}\left(C_7\,\mathcal{O}_7 + C_9\,\mathcal{O}_9 + C_{10}\,\mathcal{O}_{10} + \ldots\right)$$

- One obtains: $$\mathcal{M}(\bar{B}\to\bar{K}\ell\ell) = \frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}V_{ts}^* V_{tb}$$

$$\cdot\Big\{ C_9(\mu)\cdot\langle\bar{K}|\bar{s}\gamma^\mu_L b|\bar{B}\rangle\,\bar{u}_\ell\gamma_\mu v_\ell$$

$$+ C_{10}(\mu)\cdot\langle\qquad\qquad\rangle\,\bar{u}_\ell\gamma_\mu\gamma_5 v_\ell$$

$$-\frac{2i\,m_b}{q^2}C_7(\mu)\cdot\langle\bar{K}|\bar{s}\sigma^{\mu\nu}q_\nu b_R|\bar{B}\rangle\,\bar{u}_\ell\gamma_\mu v_\ell\Big\}$$

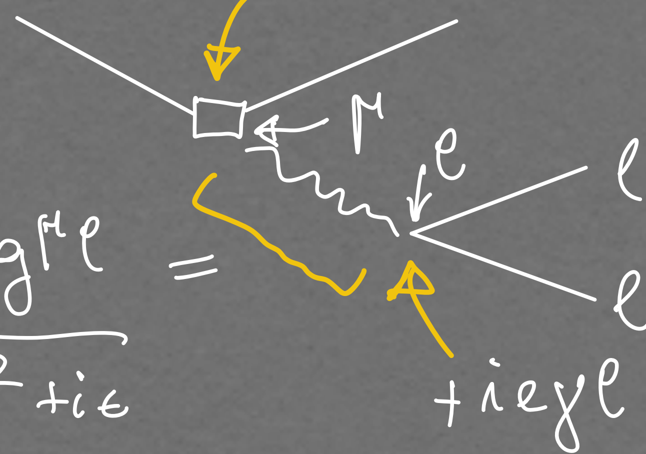
fermion fields   spinors

- Let us sketch the calculation of the C_7 piece.

$$O_7 = \frac{e}{16\pi^2} m_b \, \bar{s} \sigma^{\mu\nu} b_R \cdot F_{\mu\nu}$$

$$= \frac{e\, m_b}{16\pi^2} \bar{s} \sigma^{\mu\nu} b_R \big(\underbrace{-2 \partial_\nu A_\mu}_{-2i\epsilon^*_\mu q_\nu} \big)$$

$$\frac{-i g^{\mu\rho}}{q^2 + i\epsilon} =$$



$\Big\} \rightarrow$ outgoing momentum q

$$\hookrightarrow \mathcal{M}\Big|_{C_7} = +\frac{4 G_F}{\sqrt{2}} V_{ts}^* V_{tb} \, C_7 \cdot \frac{e\, m_b}{16\pi^2} \langle \bar{K} | \bar{s} \sigma^{\mu\nu} b_R | \bar{B} \rangle$$

$$\cdot (-2i q_\nu) \frac{(-i g_{\mu\rho})}{q^2 + i\epsilon} \cdot (+ie \, \bar{u}_\ell \gamma^\rho v_\ell)$$

$$= \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{\alpha}{\pi} \cdot C_7 \cdot \left(- \frac{2 i m_b}{q^2} \right) \cdot \langle \bar{K} | \bar{s} \sigma^{\mu\nu} q_\nu b_R | \bar{B} \rangle$$

$$\cdot \bar{u}_\ell \gamma_\mu v_\ell$$

- Quick comparison w/ $C_{9,10}$ contributions

$$\mathcal{M}_{C_{9,10}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi} V_{ts}^* V_{tb} \cdot$$

$$\cdot \left\{ C_9(\mu) \cdot \langle \bar{K} | \bar{s} \gamma^\mu_L b | \bar{B} \rangle \, \bar{u}_\ell \gamma_\mu v_\ell \right.$$

$$\left. + C_{10}(\mu) \cdot \langle \qquad \rangle \, \bar{u}_\ell \gamma_\mu \gamma_5 v_\ell \right\}$$

$C_9 \simeq 4.3$; $C_{10} \simeq -4.3$; $C_7 \simeq -0.3$

numerically smaller, but contrib. is enhanced for small q^2

$$\boxed{\text{Which (combination of) effective interactions one given observable probes depends on the kinematic region considered}}$$

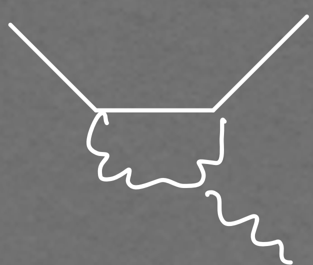
Renormalization-scale (in)dependence

- The C_i are functions of μ: renormalization scale
- And so are $\langle$ final states $| \mathcal{O}_i |$ initial states $\rangle$.

$$= \langle \qquad \rangle(\mu)$$

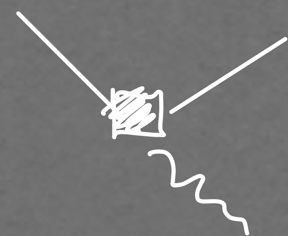
- Why $C_i = C_i(\mu)$

They are obtained through



full SM amplitude

$$\xrightarrow[\text{heavy intermediate particles}]{\text{integrate out}}$$



$$\underbrace{G_F \cdot C}_{\text{effective coupling}} \cdot \underbrace{\mathcal{O}}_{\text{effective operator}}$$

- Since the diags to the left diverge, one needs to "renormalize" the amplitude
  - ↳ stipulate its physical value at a given scale
  - ↳ renormalization introduces dependence on an arbitrary scale

- We're familiar with SM "scale-dependent" couplings
  i.e. we know that $\alpha_s(M_Z) = 0.12$
  whereas $\alpha_s(\lesssim 1\,\mathrm{GeV}) \gtrsim O(1)$.

The C_i are also couplings, so $C_i = C_i(\mu)$

- But also $\langle \mathcal{O}_i \rangle = \langle \mathcal{O}_i \rangle(\mu)$

  - These $\langle\ \rangle$ are calculated with non-perturbative methods, e.g. a space-time lattice (LQCD)

  - The lattice spacing works as an UV-regulator

  - As the lattice spacing $\to 0$, one should reobtain continuum QCD

    $\Longrightarrow$ LQCD requires renormalization as well

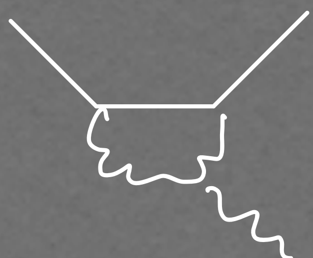
- $C_i(\mu) \langle \mathcal{O}_i \rangle(\mu) = \mu\text{-independent}$

  (at any order in perturbation theory)

- Recap of steps for calculating an observable:

"matching & running"

① Calculate $C_i(\mu_h)$ by identifying (= "matching")



$$+ \ldots = C_i(\mu_h)\, \mathcal{O}_i$$

A_{full} = sum of diagrams in the "full" theory: SM or beyond

- Fixes $C_i(\mu_h)$
- The heavy scales in the logs will appear in logs:

$$C_i(\mu_h) \propto \begin{pmatrix} \text{full-theory} \\ \text{couplings} \end{pmatrix} \times \text{logs}\left(\frac{m_W, m_t, \ldots}{\mu_h}\right)$$

- What's the "natural" value for μ_h ?

Any value that does NOT give rise to large logs

$\Rightarrow \mu_h \sim \mathcal{O}(m_W)$ (= "matching scale")

(2) Calculate $\langle \text{final states} | \mathcal{O}_i | \text{initial states} \rangle (\mu_s)$ using non-perturbative methods

- What's the "natural" value for μ_s ?

It depends on the process : $\mu_s \sim \mathcal{O}(\text{mass of external states})$

$\hookrightarrow \mu_h \gtrsim 100$ GeV whereas $\mu_s \lesssim 5$ GeV

How to connect the two ?

③ RGE – evolve the C_i from μ_h to μ_s
"
Renormalization-group equations

- RGE equations for C_i = eqs. dictating the $\log\mu$ dependence of the C_i

$$\frac{d}{d\log\mu}\vec{C} = -\hat{\gamma}\vec{C}$$

$$\begin{pmatrix} C_1 \\ C_2 \\ \vdots \end{pmatrix}$$

"anomalous dimension matrix": would be zero if the C_i were scale-indep

- Procedure is entirely analogous to e.g. running of $\alpha_s = \frac{g_s^2}{4\pi}$

$$dg_s / d\log\mu = \beta(g_s)$$ "β-function of QCD"

$b \to s$ phenomenology

- We can now apply all previous concepts to $\mathcal{H}_{eff}^{b\to s\ell\ell}$

$$\mathcal{H}_{eff}^{b\to s\ell\ell} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=7,9,10,\dots} C_i(\mu)\, \mathcal{O}_i$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b\, \bar{s}\sigma_{\mu\nu} P_R b\, F^{\mu\nu}$$

$$\mathcal{O}_9 = \frac{\alpha}{4\pi} \bar{s}\gamma_L^\mu b\, \bar{\ell}\gamma_\mu \ell$$

$$\mathcal{O}_{10} = \quad '' \quad \bar{\ell}\gamma_\mu\gamma_5 \ell$$

- We can be more precise on the $C_i(\mu)$ definition.
  - $C_i(\mu = \mu_h \sim m_W)$: "matching"

$$C_7(m_W) = -0.19 \quad ; \quad C_9(m_W) \simeq +2 \quad ; \quad C_{10}(m_W) \simeq -4.1$$

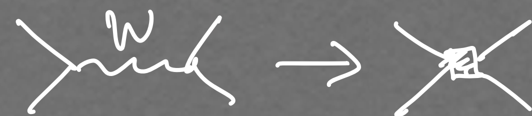
  - $C_i(\mu = \mu_s \sim m_b)$: "running"

$$C_7(m_b) = -0.33 \quad (73\%\ \text{effect})$$

$$C_9(m_b) \cong +4 \quad (100\%\ \text{effect})$$

overlooking this effect, a BR would be off by a factor 4 !

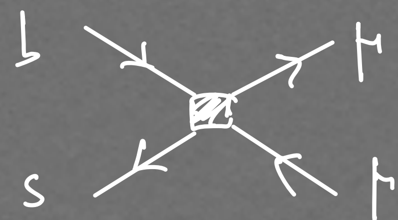
- Why such large effects?

  - While RGE-running, C_7 & C_9 get contributions from other, 4-quark operators
  - these operators are generated at tree level: 

    4-q operator

    so they have large couplings

  - Example

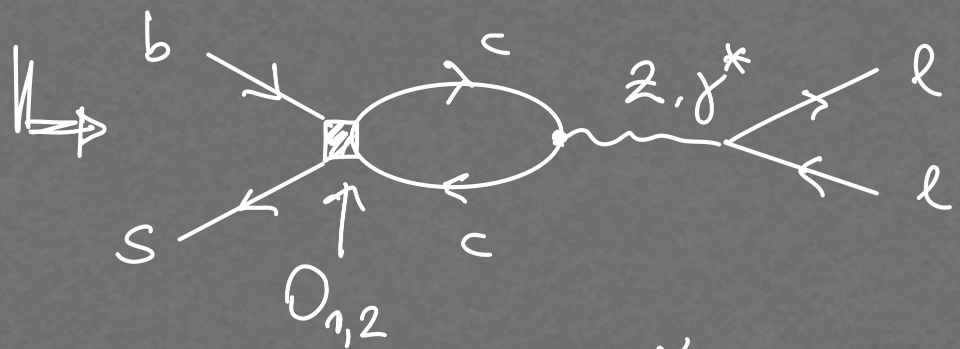
    - Start from $\mathcal{O}_9$: 

    $$(\bar{s}\gamma^{\mu}_{L} b)(\bar{\ell}\gamma_{\mu}\ell)$$

    - Then consider the operators

$$\mathcal{O}_1 = (\bar{s}\gamma^{\mu}_{L} b)(\bar{c}\gamma_{\mu L} c) \qquad \mathcal{O}_2 = (\bar{s}\gamma^{\mu}_{L} c)(\bar{c}\gamma_{\mu L} b)$$

$$C_2(m_W) = -1 \,!$$



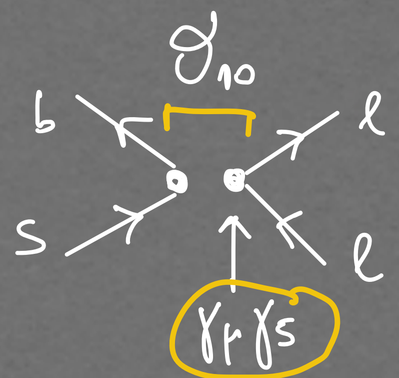
contribution to O_9

contribution to O_7

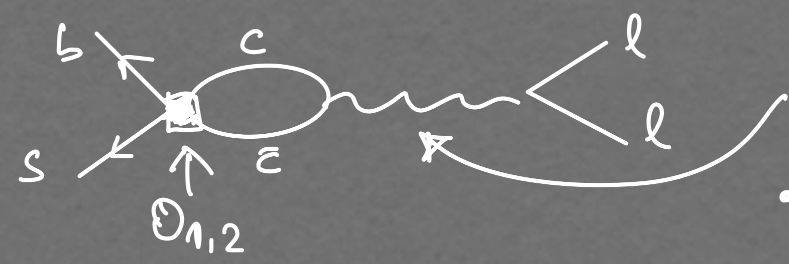
– Such contributions effectively included through RGE-running

$\boxed{\text{The actual "minimal" basis of } O_i \text{ for precision calculations includes } O_{7,9,10,1,2} \text{ (Grinstein-Savage-Wise, NPB 1989)}}$

- What about O_{10}? It does not mix w/ $O_{1,2}$



At dim 6, this structure cannot be obtained from



- either vector current $\Rightarrow \gamma_\mu$, not $\gamma_\mu \gamma_5$
- or Z propagator $\longrightarrow$ dim 8

- **Recap about $C_{7,9,10}$ matching & running**

matching
$$C_7(m_W) \cong -0.19$$
$$C_9(m_W) \cong +2$$
$$C_{10}(m_W) = -4.3$$

$\xrightarrow{\text{RGEs}}$

$$C_7(m_b) \simeq -0.33$$
$$C_9(m_b) \simeq +4.3$$
$$C_{10}(m_b) \simeq -4.3$$

Note: $C_9 \simeq -C_{10}$ at m_b

Remember: $\mathcal{O}_9 \propto (\bar{s}\gamma^\mu_L b)(\bar{\ell}\gamma_\mu \ell)$
$\mathcal{O}_{10} \propto (\bar{s}\gamma^\mu_L b)(\bar{\ell}\gamma_\mu\gamma_5 \ell)$ $\Rightarrow$ $C_9 \simeq -C_{10}$ (at m_b) implies approximate $(V-A)\times(V-A)$ structure

- Why interesting.

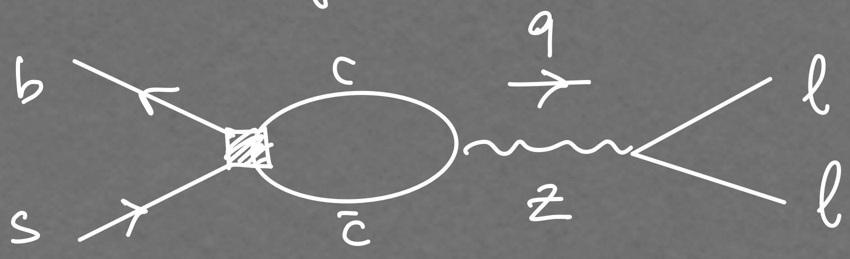
- New-physics contributions arise at $\Lambda_{NP} \gg m_W$ (= EW scale)
- At Λ_{NP}, EW symmetry restored
- I.e. effective operators written with $SU(2)_L \times U(1)_Y$ - symmetric SM matter multiplets
- E.g. $\bar{Q}_L \gamma^\mu Q_L \bar{L}_L \gamma_\mu L_L \longrightarrow (V-A)\times(V-A)$ structure

• Errors on Wilson coefficients

• In "non-resonant" kinematic regions, it's the perturbation theory error, $\mathcal{O}(1\%)$

↙ ?

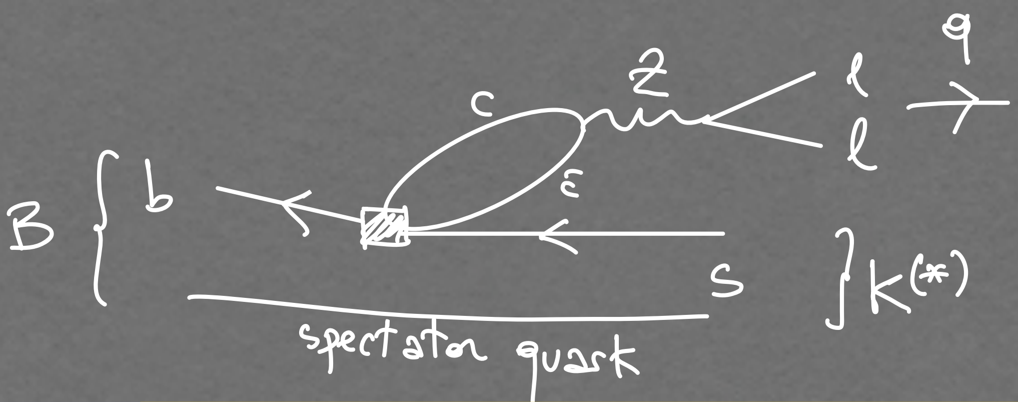
Take again



• For $B_s \to \ell\ell$, the $\ell\ell$ momentum q is fixed : $q^2 = m_{B_s}^2$

• For $B \to K\ell\ell$ it is not

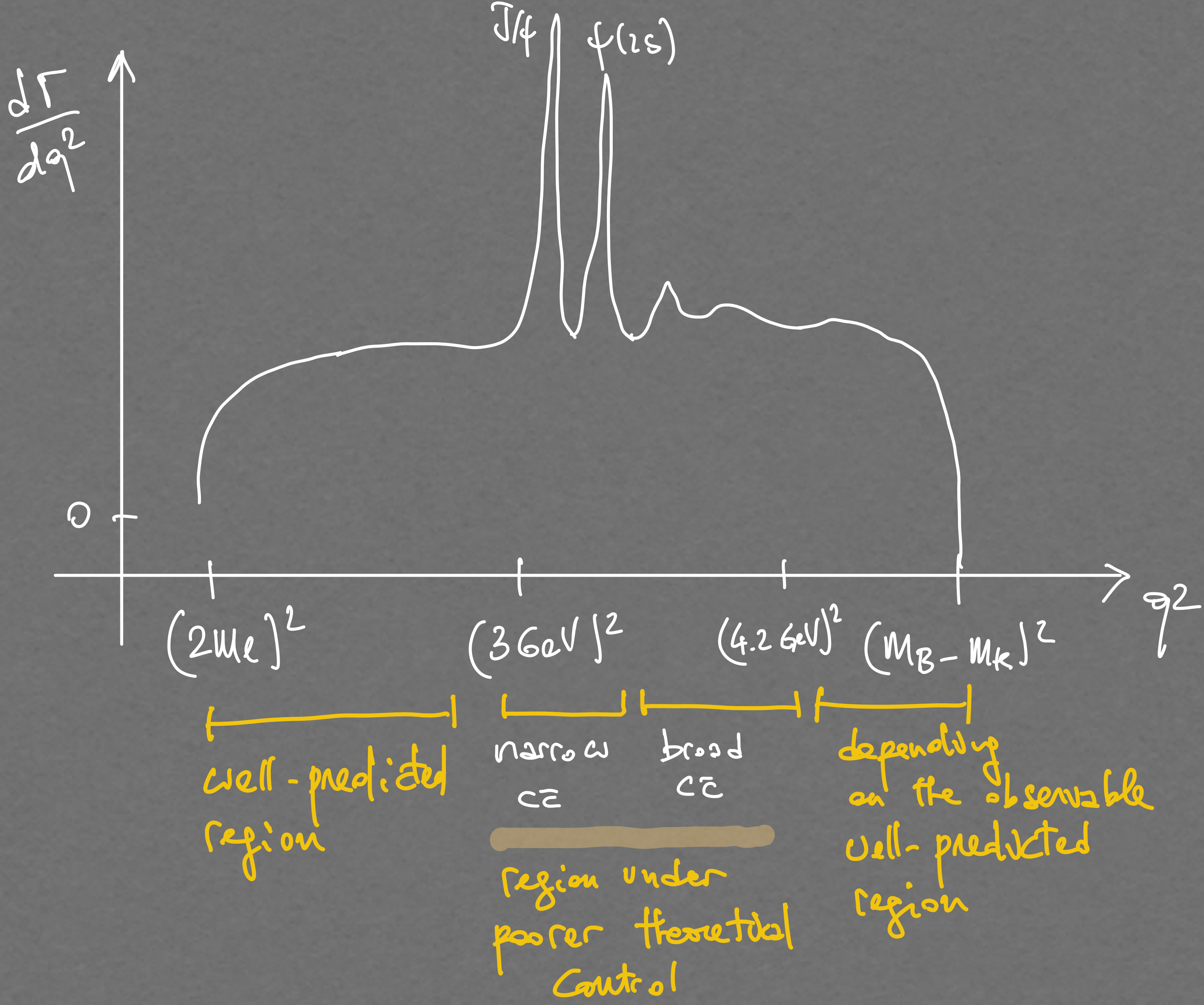
• $B \to K^{(*)}\ell\ell$ case :



• $q^2 \in [(2m_\ell)^2, (m_B - m_{K^{(*)}})^2]$

$\gtrsim (4.4\ \text{GeV})^2$

The q^2 range includes regions where intermediate resonances can be produced : $m_{J/\psi} \simeq 3.1$ GeV ; $m_{\psi(2S)} \simeq 3.7$ GeV ; etc.



$\frac{d\Gamma}{dq^2}$
0
J/ψ
$\psi(2S)$
q^2
$(2m_\ell)^2$
$(3\,GeV)^2$
$(4.2\,GeV)^2$
$(m_B - m_K)^2$
well-predicted region
narrow $c\bar{c}$
broad $c\bar{c}$
region under poorer theoretical control
depending on the observable well-predicted region

Form factors

- Our amplitudes $= \langle \text{final states} | -\mathcal{H}_{eff} | \text{initial states} \rangle$

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu)\, \mathcal{O}_i$$

$\hookrightarrow$ We still need to calculate $\langle fin | \mathcal{O}_i | in \rangle \equiv \langle \mathcal{O}_i \rangle$

- Remember the structure we found for $\mathcal{O}_{7,9,10}$

$$\mathcal{O}_7 \propto (\bar{s}\sigma_{\mu\nu} P_R b)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_9 \propto (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_{10} \propto (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\sim \quad J_i^{(q)} \times J_i^{(\ell)}$$

$\hookrightarrow$ The difficult part of the calculation is

$$\langle fin | J_i^{(q)} | in \rangle$$

- Vector matrix elements (example of $B(p_B) \to K(p_K)\ell\bar{\ell}(q)$)

$$\langle K(p_K)|\,\bar{s}\gamma_\mu b\,|B(p_B)\rangle = f_+(q^2)(p_B+p_K)_\mu + f_-(q^2)(p_B-p_K)_\mu$$

"form factors": functions of the kinematic invariants of the problem

$$(p_B+p_K)^2 = m_B^2 + m_K^2 + 2\,p_B\cdot p_K$$

$$q^2 = (p_B-p_K)^2 = \quad '' \quad -2\,p_B\cdot p_K$$

So we can choose m_B^2, m_K^2, q^2

$$\langle K(q_K)|\,\bar{s}\gamma_\mu\gamma_5 b\,|B(p_B)\rangle = \emptyset \quad \text{by parity}$$

- Tensor matrix elements

$$\langle K(p_K)|\,\bar{s}\,\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,|B(p_B)\rangle = i\left\{(p_B+p_K)_\mu q^2 - (m_B^2-m_K^2)q_\mu\right\}$$

γ_5 piece gives $\emptyset$

$$\cdot\, f_T(q^2)\,\cancel{(m_B+m_K)}$$

- FFs & Lattice-QCD (LQCD)

- For suitable q^2 values, these FFs can be estimated <u>non-perturbatively</u> using QCD in discretized space-time

- LQCD "prefers" large q^2

  In fact, external states are isolated through exponentials:

  $e^{-m_{B,K}t}$

  $\longrightarrow$ The natural $q^2 = (p_B - p_K)^2 = (m_B - m_K)^2 = q^2_{max}$

- To calculate FFs at $q^2 < q^2_{max}$, <u>need Fourier transform</u>

  $\downarrow$

  (noisy) sum over lattice sites

- $B \to K\ell\ell$: matrix-element structure [understanding why $R_K \simeq 1$]

$$\mathcal{M}(B \to K\ell\ell) = \langle \ell^+(p_1)\, \ell^-(p_2)\, K(p_K) | -\mathcal{H}_{\rm eff}^{b\to s\ell\ell} | B(p_B)\rangle$$

$$= +\frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi} V_{ts}^* V_{tb} \Big\{ C_9 \Big[\langle K | \bar{s} \gamma^\mu P_L b | B\rangle\, \bar{u}_\ell(p_2) \gamma_\mu v_\ell(p_1) \Big]_{1}$$

$$+ C_{10} \Big[\langle K | \quad '' \quad | B \rangle\, \bar{u}_\ell(p_2) \gamma_\mu \gamma_5 v_\ell(p_1) \Big]_{2}$$

$$- \frac{2 i m_b}{q^2} C_7 \Big[\langle K | \bar{s} \sigma^{\mu\nu} q_\nu P_L b | B \rangle\, \bar{u}_\ell(p_2) \gamma_\mu v_\ell(p_1) \Big]_{3} \Big\}$$

- Note: this is of the form

$$\mathcal{M} = \# \Big\{ \underbrace{T_\mu^{(1)}}_{\propto C_9 \text{ or } C_7} \bar{\ell} \gamma^\mu \ell + \underbrace{T_\mu^{(2)}}_{\propto C_{10}} \bar{\ell} \gamma^\mu \gamma_5 \ell \Big\}$$

• Let's look closer

$$[\quad]_{1} = \frac{1}{2}\underbrace{\Big(f_+ \overset{p_B+p_K}{p_\mu} + f_- q_\mu\Big)}\bar{u}_\ell(p_2)\gamma^\mu v_\ell(p_1) = \frac{f_+}{2}\, p_\mu\, \bar{u}_\ell(p_2)\gamma^\mu v_\ell(p_1)$$

$$\left(q_\mu \bar{u}_\ell \gamma^\mu v_\ell = \varnothing\right)$$

$$[\quad]_{2} = \frac{1}{2}\Big(\qquad '' \qquad\Big)\bar{u}_\ell \gamma^\mu \gamma_5 v_\ell = \frac{f_+}{2}\, p_\mu\, \bar{u}\gamma^\mu\gamma_5 v$$

$$+ f_- m_\ell \bar{u}\gamma_5 v$$

$$[\quad]_{3} = \frac{i}{2q^2}\Big(p_\mu q^2 - (m_B^2 - m_K^2)\, q_\mu\Big)\frac{f_T}{m_B + m_K}\, \bar{u}\gamma^\mu v$$

$$= \frac{i}{2}\, p_\mu \frac{f_T}{m_B + m_K}\, \bar{u}\gamma^\mu v$$

- So $\mathcal{M}(B \to K\ell\ell)$ simplifies considerably:

$$\mathcal{M}(B \to K\ell\ell) =$$

$$+\frac{G_F}{2\sqrt{2}} \frac{\alpha}{\pi} V_{ts}^* V_{tb} \left\{ \overbrace{\left(C_9 f_+ + C_7 \frac{2 m_b f_T}{m_B + m_K} \right)}^{\equiv V_p} p_\mu \bar{u} \gamma^\mu v \right.$$

$$\left. + \underbrace{C_{10} \Big(f_+}_{\equiv A_p} p_\mu \bar{u} \gamma^\mu \gamma_5 v + 2 m_\ell \bar{u} \gamma_5 v \Big) \right\}$$

- We can now calculate the width

$$\frac{d\Gamma}{dq^2}(B \to K\ell\ell) \propto G_F^2 \frac{\alpha^2}{\pi^2} \left| V_{ts}^* V_{tb} \right|^2 \left\{ |V_p|^2 + |A_p|^2 + \left(\frac{m_\ell}{m_B} \right)^2 \text{- suppressed} \right\}$$

- Now let's consider R_K:

$$R_K[q_1^2, q_2^2] \equiv \frac{\int_{q_1^2}^{q_2^2} \frac{d\Gamma}{dq^2}(B\to K\mu\mu)\, dq^2}{\left(\quad '' \quad ee \quad \right)}$$

$$= 1 + \mathcal{O}\left(\frac{m_\mu^2}{m_B^2}\right) \sim 10^{-4}$$

– Actual error $\mathcal{O}(1\%)$ due to QED corrections

– 3-body decay implies

$s = (p_B - p_K)^2 = q^2$; $t = (p_B - p_1)^2$; $u = (p_B - p_2)^2$

with $s + t + u = m_B^2 + m_K^2 + 2m_\ell^2 \rightarrow$ only 2 indep. invariants

$\Longrightarrow$ width is, at most, doubly differential.

Lepton-flavour-universality (LFU) tests

- The R_K ratio mentioned above is the best-known example of a whole suite of LFU tests today available.

$$R_K[q_1^2, q_2^2] \equiv \frac{\int_{q_1^2}^{q_2^2} \frac{d\Gamma}{dq^2}(B \to K \mu\mu)\, dq^2}{ee}$$

$R_K[1, 6\ \text{GeV}^2] < 1$

$R_{K^*}[1.1, 6\ \text{GeV}^2] < 1$

$R_{K^*}[0.045, 1.1\ \text{GeV}^2] < 1$

+ many $BR(b \to s \mu\mu)$ below SM (both in mesonic & baryonic channels) whereas " ee channels are SM-like within (large) errors

+ angular data in $B \to K^* \mu\mu$ showing departures consistent w/ same picture

$$R_{D^{(*)}} \equiv \frac{BR(B \to D^{(*)} \tau \nu)}{BR(B \to D^{(*)} \ell \nu)\big|_{\substack{\ell = e + \mu \\ \text{average}}}}$$

$\hookrightarrow$ $(R_D)_{exp} > (R_D)_{SM}$ & $(R_{D^*})_{exp} > (R_D)_{SM}$

• New effects : EFT picture

- We saw that $b \to s \ell\ell$ ampl. described by

$$\mathcal{H}_{eff} = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \left(C_i \mathcal{O}_i + \text{"primed"} \right)$$

where, in the SM, the $\mu \sim m_b$ dynamics is dominated by

$$\mathcal{O}_7 \sim (\bar{s}\, \sigma_{\mu\nu} P_R\, b) F^{\mu\nu} \quad ; \quad \mathcal{O}_9 \sim (\bar{s} \gamma^\mu_L b)(\bar{\ell} \gamma_\mu \ell) \quad ; \quad \mathcal{O}_{10} \sim (\bar{s} \gamma^\mu_L b)(\bar{\ell} \gamma_\mu \gamma_5 \ell)$$

- New dynamics best appreciated in the chiral basis for $\mathcal{O}_9, \mathcal{O}_{10}$:

$$\mathcal{O}_{XY} \equiv (\bar{s} \gamma^\mu P_X b)(\bar{\ell} \gamma_\mu P_Y \ell)$$

↳ Vectorial currents can be promoted to full $SU(2)_L$ invariants

i.e. written in terms of Q_L, L_L, u_R, d_R, e_R

- The SM (accidentally) helps:

  - at $\mu \sim m_b$: $C_9 \simeq +4$; $C_{10} \simeq -4$

$$\Rightarrow C_9 \mathcal{O}_9 + C_{10} \mathcal{O}_{10} \simeq C_9 (\mathcal{O}_9 - \mathcal{O}_{10})$$

$$= 2 C_9 \underbrace{(\bar{s} \gamma^\mu_L b)(\bar{\ell} \gamma_{\mu L} \ell)}_{(V-A)\times(V-A) \text{ structure}}$$

we saw that this is accidental in the SM

- because $C_9(M_W) \approx +2$ whereas $C_{10}(M_W) \simeq -4$
- & large RGE through $\mathcal{O}_1, \mathcal{O}_2$

So, $\delta C_9(\mu \sim \Lambda_{NP}) = -\delta C_{10}(\mu \sim \Lambda_{NP})$ is well-defined also at $\mu \sim m_b$

- So we can rewrite

$$\mathcal{H}_{\text{eff}} = -\underbrace{\frac{4G_F}{\sqrt{2}}}_{2/v^2} V_{tb}V_{ts}^* \left\{ \underbrace{C_9}_{\frac{C_9+C_{10}}{2} + \frac{C_9-C_{10}}{2}} \frac{\alpha}{4\pi}(\bar{s}\gamma^\mu_L b)(\bar{\ell}\gamma_\mu \ell) + \underbrace{C_{10}}_{\frac{C_9+C_{10}}{2} - \frac{C_9-C_{10}}{2}} \frac{\alpha}{4\pi}(\bar{s}\gamma^\mu_L b)(\bar{\ell}\gamma_\mu\gamma_5 \ell) + \ldots \right\}$$

$$\frac{C_9+C_{10}}{2} = C_+ \qquad \frac{C_9-C_{10}}{2} = C_-$$

$$= -\underbrace{V_{tb}V_{ts}^*}_{0.04} \underbrace{\frac{\alpha}{4\pi}}_{5\cdot 10^{-4}} \underbrace{\frac{2}{v^2}}_{\frac{1}{(174\,\text{GeV})^2}} \left\{ \underbrace{2C_+}_{\sim -0.2} \underbrace{(\bar{s}\gamma^\mu_L b)(\bar{\ell}\gamma_{\mu R} \ell)}_{\mathcal{O}_{LR}} + \underbrace{2C_-}_{\sim +8.4} \underbrace{(\bar{s}\gamma^\mu_L b)(\bar{\ell}\gamma_{\mu L} \ell)}_{\mathcal{O}_{LL}} + \ldots \right\}$$

$$= \frac{1}{(\sim 36\,\text{TeV})^2} \sum_{X,Y} C_{XY}\,\mathcal{O}_{XY} \qquad \text{w/ e.g. } C_{LL} = 2C_-$$

$$\boxed{\text{a similar exercise for } b \to c \ell \nu \text{ (relevant for } R_{D^{(*)}}) \text{ yields much lower scale} \sim \text{TeV}}$$

∘ R_K dependence on δC_i

$$R_K = \frac{\left(\int \frac{d\Gamma}{ds}\, ds\right)\Big|_{\mu\mu}}{\left(\quad '' \quad\right)\Big|_{ee}}$$

with
$$\frac{d\Gamma}{ds} \propto G_F^2\, \alpha^2\, |V V^*|^2 \left\{ \underbrace{|V_\ell|^2}_{} + \overbrace{|A_\ell|^2}^{(C_{10}+C_{10}')f_+} \right\} \left(1 + \mathcal{O}(m_\ell^2/m_B^2)\right)$$

$$V_\ell = (C_9^{eff} + C_9')f_+ + \underbrace{\frac{2 m_b}{m_B + m_K}}_{\text{small}} (C_7^{eff} + C_7')$$

$$\Rightarrow R_K = \frac{|C_9^{SM} + \delta C_9^{(\mu)} + \delta C_9'^{(\mu)}|^2 + |C_{10}^{SM} + \delta C_{10}^{(\mu)} + \delta C_{10}'^{(\mu)}|^2}{(\mu) \to (e)}$$

$$\simeq 1 + \frac{\left\{\mathrm{Re}\left(\delta C_9^{(\mu)} + \delta C_9'^{(\mu)} - \delta C_{10}^{(\mu)} - \delta C_{10}'^{(\mu)}\right)\right\} - \left\{(\mu) \to (e)\right\}}{C_9^{SM}}$$

$$\Rightarrow \boxed{R_K - 1 \Longleftrightarrow \delta C_i / C_9^{SM}}$$

- Simultaneous explanations of $R_{K^{(*)}}$ & $R(D^{(*)})$
  - Common explanation very appealing
    - both are signs of LUV
    - Left-handed interactions (among the) favorite in an EFT description of both
      - $\rightarrow$ Consistent with $SU(2)_L \times U(1)_Y$ - gauge - symmetric new interactions above the EW scale

- E.g. $\mathcal{O}^{(1)} = G_1 \, \bar{Q}'_L \gamma^\mu Q'_L \; \bar{L}'_L \gamma_\mu L'_L$

$\mathcal{O}^{(3)} = G_3 \, \bar{Q}'_L \gamma^\mu \sigma^a Q'_L \; \bar{L}'_L \gamma_\mu \sigma^a L'_L$

(primed = to be rotated to mass eigenbasis)

Since $(\sigma^a)_{ij} (\sigma^a)_{kl} = \underbrace{2 \delta_{il} \delta_{kj}} - \delta_{ij} \delta_{kl}$

$$\mathcal{O}^{(3)} \propto (\bar{Q}'_L)^i \gamma^\mu (Q'_L)^j \, (\bar{L}'_L)^j \gamma_\mu (L'_L)^i$$

$i \neq j$: charged currents, e.g.

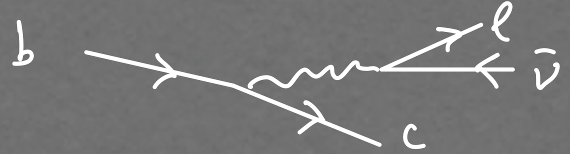
$i=1, j=2 \Rightarrow (\bar{q}'_u \gamma^\mu_L q'_d)(\bar{e}' \gamma_{\mu L} \nu')$

$\Rightarrow$ after rotation to mass eigenbasis, this will generate $b \to c \ell \nu$

- Common explanation challenging

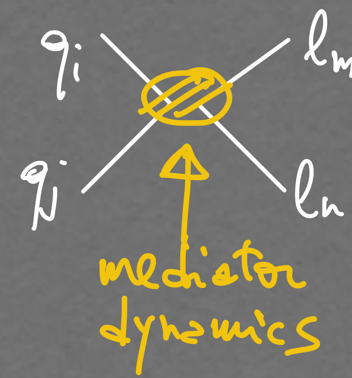
① $R_{K^{(*)}} \Rightarrow 10 \div 20\%$ effect in $b \to s$ (SM loop) amplitude

$R(D^{(*)}) \Rightarrow$ " " $b \to c$ (SM tree) amplitude



② Both $R_{K^{(*)}}$ & $R(D^{(*)})$ are semi-leptonic obs.

semi-lep. operators : quark current $J^{(q)}_{ij}$

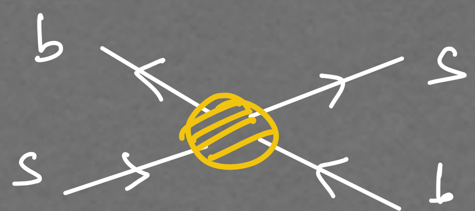


lepton current $J^{(\ell)}_{mn}$

↳ The new dynamics must produce:

- large, $\mathcal{O}(10\%)$ effects in $J^{(q)} \times J^{(\ell)}$ processes
- but small enough effects in $J^{(q)} \times J^{(q)}$ processes

e.g. B_s

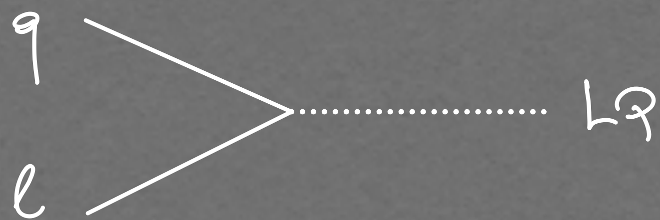


$\overline{B}_s$ which is SM-like

and in $J^{(\ell)} \times J^{(\ell)}$ processes, like $\tau \to 3\mu$, ...

- Leptoquarks (LQs)

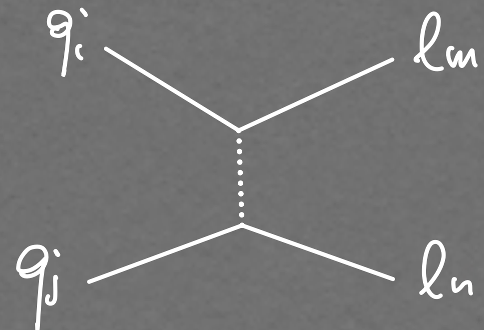
LQs are massive bosons coupled to a quark & a lepton



(... "genuine" LQs at least)

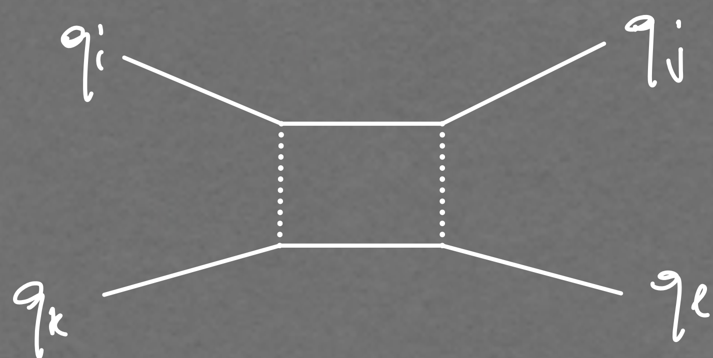
Issue n. 2:

By construction



is tree

whereas



is loop (and same for $J^{(\ell)} \times J^{(\ell)}$)

Issue n. ①: $b \to s \ell\ell$ NP $\ll$ $b \to c \ell \nu$ NP

Take a LQ $S_1 \sim (\underline{3}, \underline{1}, 1/3)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$

Possible couplings:

$$\underbrace{\overline{Q}^c_L \, L_L}\, S_1 \qquad \overline{u}^c_R \, e_R \, S_1$$

	$\overline{Q}^c_L L_L S_1$	$\overline{u}^c_R e_R S_1$
$SU(2)_L$	singlet	singlet
$U(1)_Y$	$+\frac{1}{6} - \frac{1}{2} + \frac{1}{3} = 0$	$\frac{2}{3} - 1 + \frac{1}{3} = 0$

Remember

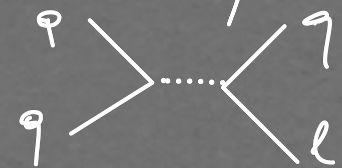
	Q	$= Y$	$+ T_3$
Q_L	$2/3$	$1/6$	$1/2$
	$-1/3$	$1/6$	$-1/2$
u_R	$2/3$		0
d_R	$-1/3$		0
L_L	0	$-1/2$	$1/2$
	-1	$-1/2$	$-1/2$
e_R	-1		0

Other couplings?

	$\bar{d}_R^c$	ν_R	S_1	$\bar{Q}_L^c$	Q_L	S_1^*	$\bar{u}_R^c$	d_R	S_1^*
$U(1)_Y$	$-1/3$	0	$+1/3$	$1/6$	$1/6$	$-1/3$	$\frac{2}{3}$	$-1/3$	$-1/3$

can be set to 0 if ν_R does not exist

"di-quark" couplings: dangerous for proton decay



– <u>ONLY</u> in absence of such couplings LQs have well-defined B & L qn's

Back to "main" S_1 couplings

$\bar{Q}_L^c L_L S_1$; $\bar{u}_R^c e_R S_1$

$\underbrace{\bar{Q}_L^c L_L S_1}_{\downarrow}$

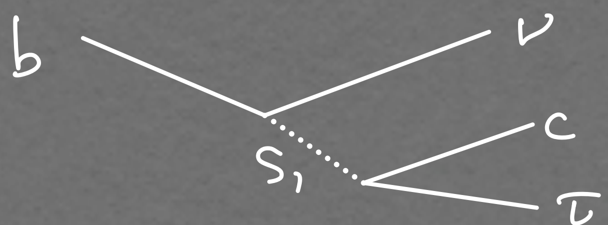
$\bar{u}_L^c e_L S_1$ & $\bar{d}_L^c \nu_L S_1$

$\Rightarrow$ up-quark - down-lepton

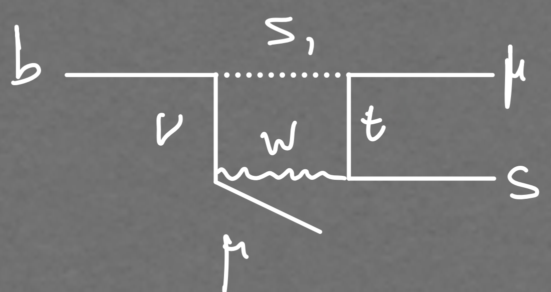
or

down-quark - up-lepton

↳ Generated amplitudes



$b \to c \tau \nu$ at tree level



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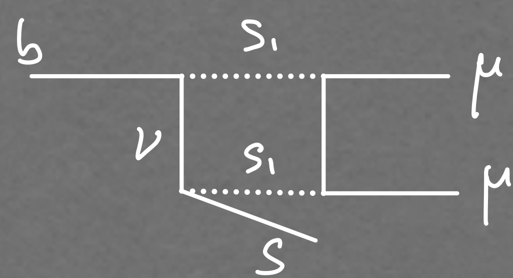

$\Rightarrow$ $b \to s \mu\mu$ at loop level

+ constraints...