# Quark Flavour Physics : Theory Aspects

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L1 :

- How "flavour" interactions (and FCNCs) arise within the SM
- Parenthesis : the Fermi coupling
- How FCNCs led to the charm discovery : the GIM mechanism
- Estimating $m_c$ : $K-\bar{K}$ mixing ( Gaillard - Lee )
- CP violation : basics
- $K^0 - \bar{K}^0$ mixing
- ~~CP~~ & # of matter generations

## How "flavour" interactions arise within the SM.

- SM: unique Lagrangian arising from requiring:

(• dim 4 + Lorentz invariance + locality)

- $\underbrace{SU(3)_c \times SU(2)_L \times U(1)_Y}$ gauge invariance

"spontaneously" broken, but leaving:

- $Q_{em} = \underset{\substack{\uparrow \\ U(1)_Y \\ \text{q.n.}}}{Y} + \underset{\substack{\uparrow \\ \left(\frac{\sigma_3}{2}\right)_L \text{ q.n.}}}{T_3} = \text{conserved}$

- matter content

| | | | Comments |
|---|---|---|---|
| fermions | $Q_L$ $\quad$ $L_L$ | $u_R$ $\quad$ $d_R$ $\quad$ $e_R$ | (and no $\nu_R$ !) |
| scalars | $\Phi$ | | (just one Higgs doublet) |
| | $SU(2)_L$ doublets | $SU(2)_L$ singlets | |

- $Q_{em} = Y + T_3$ allows to fix all q.n.'s

$$
\begin{array}{llccc}
 & & Q_{em} = & T_3 & + \; Y \\
\text{e.g.} & Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} & 2/3 & 1/2 & 1/6 \\
 & & -1/3 & -1/2 & 1/6 \\
 & u_R & 2/3 & \emptyset & 2/3
\end{array}
$$

$$\Phi = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \frac{1}{\sqrt{2}} \qquad \begin{matrix} 1 & 1/2 & \boxed{1/2} \\ 0 & -1/2 & \boxed{1/2} \end{matrix}$$

- Since one of $\varphi_i$ must break the $SU(2)_L \times U(1)_Y$ symmetry "spontaneously", this component must be em.-neutral.

  This restricts $Y_\Phi = \pm 1/2$ and one can choose $+\frac{1}{2}$

- No loss of generality with choice $Y_\Phi = +1/2$, in fact

$$\tilde{\Phi} \equiv i\sigma_2 \Phi^* \qquad \begin{matrix} 0 & 1/2 & -1/2 \\ -1 & -1/2 & -1/2 \end{matrix}$$

- With all q.n.'s fixed, all interactions are fixed
  - the "gauge sector", through the matter covariant derivatives:

$$\sum_{f = Q_L, u_R, \ldots} \bar{f}\, i \not{D} f + (D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$$+ \text{ Yang-Mills Lagrangian terms}$$

  - the "Yukawa sector": fermion-antifermion-scalar interactions allowed by gauge inv.

$$\bar{Q}_L \Phi d_R \; ; \; \bar{L}_L \Phi e_R \; ; \; \bar{Q}_L \tilde{\Phi} u_R$$

- But we know there are (at least) 3 replicas of fermionic matter

$$\hookrightarrow \bar{Q}_L^i\, \Phi\, (Y_d)^{ij}\, d_R^j + \bar{Q}_L^i\, \tilde{\Phi}\, (Y_u)^{ij}\, u_R^j + \text{h.c.}$$

$$= -\mathcal{L}_Y^{(q)} \qquad \left(\text{with } i,j = 1,\dots,3\right)$$

- And we know the e.m.-neutral component of $\Phi$ takes a v.e.v.

$$\Phi = \frac{1}{\sqrt{2}} \underbrace{\begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}}_{\varphi_{em} = \emptyset} \xrightarrow[\text{v.e.v.}]{} \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \& \quad \langle \tilde{\Phi} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\hookrightarrow -\mathcal{L}_Y^{(q)}\Big|_{\langle \phi \rangle} = \bar{Q}_L^i \underbrace{(Y_d)^{ij} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}}_{(M_d)_{ij}} d_R^j + \bar{Q}_L^i \underbrace{(Y_u)^{ij} \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}}_{(M_u)_{ij}} u_R^j + \text{h.c.}$$

- Diagonalizing mass matrices

$M_u, M_d$ : $3\times 3 \in \mathbb{C}$ ; no symmetry imposing that they be diag. or $\in \mathbb{R}$

- A general $M \in \mathbb{C}$ can be made diag & real by

$$U_L^\dagger M_u U_R = \hat{M}_u \qquad D_L^\dagger M_d D_R = \hat{M}_d$$

- So we can redefine ( $\wedge \equiv$ mass eigen basis )

$$\underbrace{u_{\substack{L\\R}} = \boxed{U_{\substack{L\\R}}}\, \hat{u}_{\substack{L\\R}} \quad \& \quad d_{\substack{L\\R}} = \boxed{D_{\substack{L\\R}}}}_{\text{"chiral" unitary transformations}} \hat{d}_{\substack{L\\R}}$$

Note: $Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ ← have different chiral transformations

- Consequences in gauge interactions?

gauge int's: $\bar{q}\, i\gamma^\mu (\partial_\mu + i g V_\mu + \dots) q$

- e.m.-neutral gauge int's are unaffected

e.g. $Z^0_\mu \big( \underbrace{\bar{u}_L \gamma^\mu u_L}_{\bar{\hat{u}}_L V_L^\dagger \gamma^\mu V_L \hat{u}_L} + \dots$

$V_L^\dagger \gamma^\mu V_L = \mathbb{1}$

$\Rightarrow$ tree-level neutral currents conserve flavour

• e.m.-charged gauge interactions :

$$\mathcal{L}_{qqW} = +\frac{g}{\sqrt{2}}\left(\underbrace{\bar{u}_L \gamma^\mu d_L}_{\bar{\hat{u}}_L \underbrace{U_L^\dagger \gamma^\mu D_L}_{= V_{CKM} \neq \mathbb{1}} \hat{d}_L} W_\mu + h.c.\right)$$

$$= + \frac{g}{\sqrt{2}} \sum_{i,j} \bar{\hat{u}}_L^i \left(V_{CKM}\right)_{ij} \gamma^\mu \hat{d}_L^j \, W_\mu + h.c.$$

• since $U_L \neq D_L$ , $V_{CKM} \equiv U_L^\dagger D_L \neq \mathbb{1}$

Cabibbo-Kobayashi-Maskawa

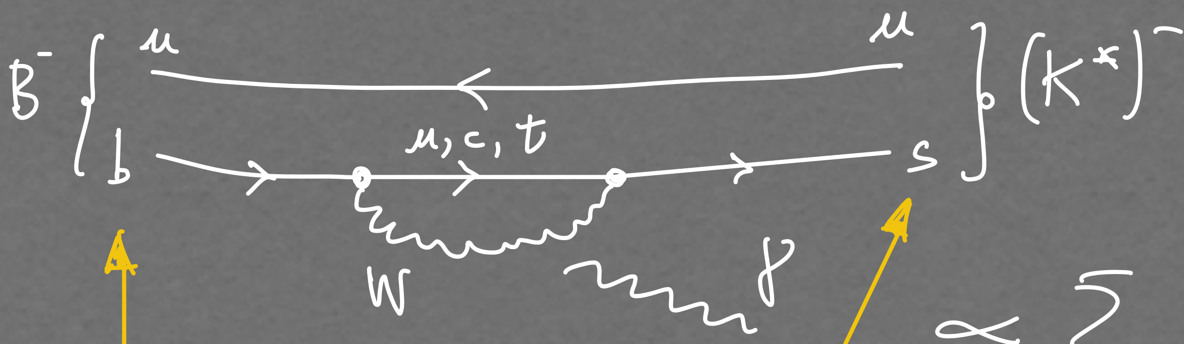
• $V_{CKM}^\dagger V_{CKM} = \mathbb{1}$

- Feynman rule


FR : $+i\frac{g}{\sqrt{2}}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)(V_{CKM})_{ij}$

(and conversely, $u^i \to d^j\, W^{(+)}$ will be $\propto (V^*_{CKM})_{ij}$ )

- So, we can build:




i.e. $B^- \to (K^*)^- \gamma$

$$\propto \sum_{q_u = u,c,t} V_{q_u b} V^*_{q_u s} \cdot f(m^2_{q_u})$$

- external quarks
  - of different generations
  - but same charge

$\Rightarrow$ Flavour-changing neutral currents (FCNCs)

- the couplings $V_{q_u b} V^*_{q_u s}$ are $\ll 1$
- loop suppression factor

$\Rightarrow$ FCNCs small in the SM

## Parenthesis : the Fermi coupling

- The hierarchy $m_q \ll m_W$ (exception : $m_t$) allows to introduce the "Fermi theory"

an "effective field theory" (EFT) that permitted to make weak-decay predictions well before the EW theory was formulated

- We saw that $\mathcal{L}_{qqW} = \frac{g}{\sqrt{2}} \bar{\hat{u}}_L \gamma^\mu V \hat{d}_L W_\mu + h.c.$

flavour eigenstates = mass eigenstates

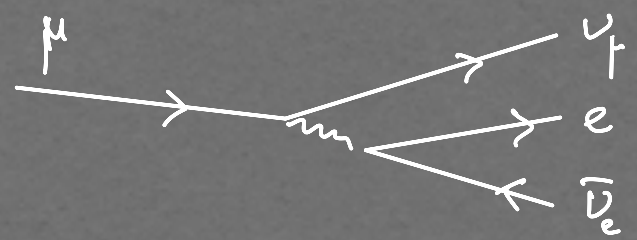
- in the lepton sector the analog is

$$\mathcal{L}_{\ell\ell W} = \frac{g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu \ell_L W_\mu + h.c.$$

flavour eigenstates = mass eigenstates

flavour eigenstates ≠ mass eigenstates $\left(\nu_L = U \nu_L^{mass}\right)$

- So, consider e.g. $\mu$ decay:




in Feynman gauge

small

$$i\mathcal{M} = \left(\frac{ig}{\sqrt{2}}\right)^2 \left(\bar{u}_\nu \gamma_L^\alpha u_\mu\right) \frac{-i\, g_{\alpha\beta}}{p^2 - m_W^2} \left(\bar{u}_e \gamma_L^\beta v_\nu\right)$$

$$\frac{1}{p^2 - m_W^2} \simeq -\frac{1}{m_W^2}\left(1 + \frac{p^2}{m_W^2} + \dots\right)$$

$$\hookrightarrow \simeq -\frac{i g^2}{2 m_W^2} \left(\bar{u}_\nu \gamma_L^\alpha u_\mu\right)\left(\bar{u}_e \gamma_{L\alpha} v_\nu\right) = i \langle e \nu \bar{\nu} | \underbrace{\mathcal{L}_{\text{eff}}}_{= -\mathcal{H}_{\text{eff}}} | \mu \rangle$$

- I.e. we can get the same amplitude using a local 4-fermion interaction w/ "effective" coupling $\mathcal{O}(g^2/m_W^2)$

$$\mathcal{H}_{\text{eff}} = + \frac{4 G_F}{\sqrt{2}} \left(\bar{\psi}_\nu \gamma_L^\alpha \psi_\mu\right)\left(\bar{\psi}_e \gamma_{\alpha L} \psi_\nu\right)$$

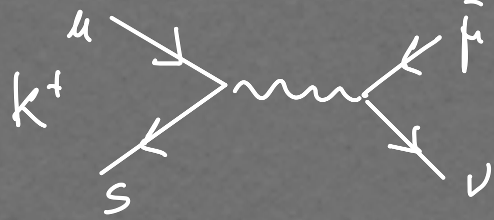
$$G_F \simeq 10^{-5}\ \text{GeV}^{-2} \simeq \frac{1}{(300\ \text{GeV})^2}$$

w/ $\boxed{\frac{4 G_F}{\sqrt{2}} \equiv \frac{g^2}{2 m_W^2} = \text{measured in one reference process}}$

## How FCNCs led to the charm discovery: the GIM mechanism

- With the above interaction, we can understand quantitatively processes such as $K_L \to \mu\mu$

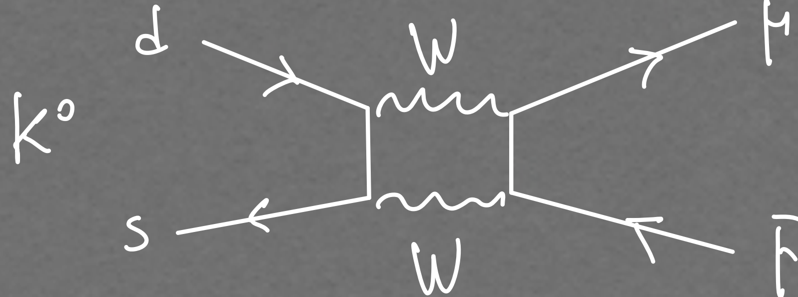
- Reference: $\Gamma(K^+ \to \mu^+ \nu)$




$1 \overset{W}{\sim\sim\sim} \to 1$ power of $G_F$

$\Rightarrow \Gamma(K^+ \to \nu\bar{\mu}) \sim G_F \cdot \#$

- $\Gamma(K_L \to \mu\bar{\mu})$ ?




$\overset{?}{\Longrightarrow} \Gamma(K_L \to \mu\bar{\mu})$

$\sim G_F^2 \cdot m_W^2 \cdot \underbrace{\text{loop}}_{1/16\pi^2} \cdot \#$

?

- This would yield

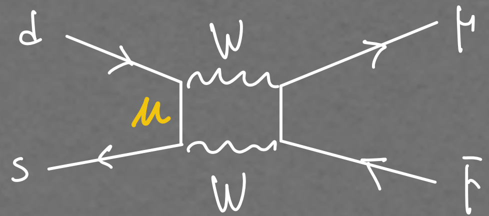
$$\frac{\Gamma(K_L \to \mu\bar{\mu})}{\Gamma(K^+ \to \nu\bar{\mu})} \sim 6\% \qquad \text{whereas exp gave } 4\cdot 10^{-9}$$

$\Rightarrow$ It looked like $\Gamma(K_L \to \mu\bar{\mu}) \sim G_F^2 \cdot \Lambda^2 \cdot \text{loop} \cdot \#$

$$\boxed{\text{with } \Lambda \ll m_W}$$

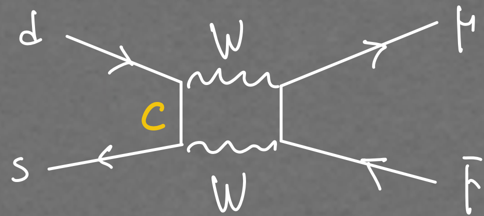
- A radical solution: another quark besides the known $u, d, s$ : the charm

Then:




$\sim V_{ud} \cdot V_{us}^* \cdot m_u^2$

$= c_\theta \, s_\theta \, m_u^2$




$\sim V_{cd} V_{cs}^* \cdot m_c^2$

$= - c_\theta s_\theta \, m_c^2$

$$V_{2\times 2} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$$

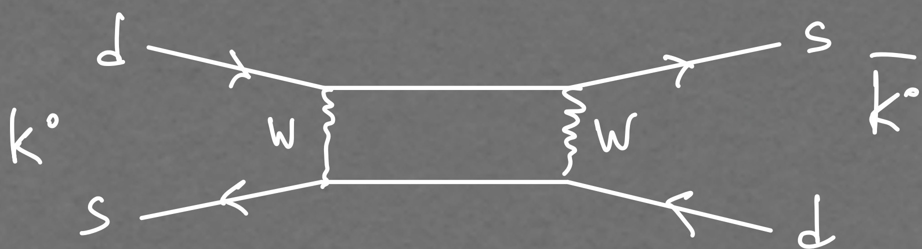
$$\Rightarrow A(K_L \to \mu\bar{\mu}) \sim \frac{m_u^2 - m_c^2}{m_W^2}$$

with

- $m_c^2 \ll m_W^2$
- but $m_c^2 \gg m_u^2$
  so as to introduce
  a new scale

## Estimating $m_c$ : $K-\bar{K}$ mixing ( Gaillard - Lee )

- Consider the process




GIM factor

$$A \approx \left(\frac{g}{\sqrt{2}}\right)^4 (s_\theta c_\theta)^2 \cdot \frac{1}{16\pi^2} \cdot \frac{m_c^2 - m_u^2}{(-m_W^2)^2} \cdot \langle \bar{K}^0 | (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d) | K^0 \rangle$$

gauge coupling

CKM structure

loop factor

2 W propagators

Fermi-theory operator evaluated between external states

$$= \langle \bar{K}^0 | \mathcal{H}_{eff} | K^0 \rangle \equiv M_{12} = \Delta m_K \cdot m_K$$

mass difference between $K_L$ & $K_S$

average mass

- The matrix elem. is calculated using non-perturbative methods.

$$\langle \bar{K}^0 | (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu_L d) | K^0 \rangle \sim f_K^2 \cdot m_K^2$$

- Let's put numbers (use of $g = \frac{e}{s_W}$ ; $\alpha = \frac{e^2}{4\pi}$)

$$\Delta m_K \cdot m_K = M_{12} = \left(\frac{g}{\sqrt{2}}\right)^4 (s_\theta c_\theta)^2 \cdot \frac{1}{16\pi^2} \cdot \frac{m_c^2 - m_u^2}{(-m_W^2)^2} \cdot \langle \bar{K}^0 | (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu_L d) | K^0 \rangle$$

$$\simeq \frac{G_F}{s_W^2} \cdot \frac{\alpha}{4\pi} \cdot s_\theta^2 c_\theta^2 \cdot \frac{m_c^2}{m_W^2} \cdot f_K^2 \cdot m_K^2$$

with $G_F = 1.2 \cdot 10^{-5}\ \text{GeV}^{-2}$ $\quad s_W^2 = 0.23$

$\alpha = 1/137$ $\quad \Delta m_K = 3.5 \cdot 10^{-15}\ \text{GeV}$

$f_K = 0.16\ \text{GeV}$

$m_K = 0.49\ \text{GeV}$

$\{c_\theta, s_\theta\} = \{0.974, 0.225\}$

$m_W = 80.38\ \text{GeV}$

$\Rightarrow \Delta m_K \cdot m_K = M_{12}(m_c)$

$\hookrightarrow m_c \sim 0.6\ \text{GeV}$

vs. $\bar{m}_c \simeq 1.27\ \text{GeV}$

- Not bad, given the ROUGH estimate

In the SM, $\Delta m_K$ still difficult to estimate properly:

- 70% short-distance boxes w/ charm & top & interference
  by far dominant
- 30% long-distance contributions, difficult to control

## CP violation : basics

| | | |
|---|---|---|
| | | prediction |
| ∘ Consistent $qqW$ interactions w/ 2 generations | $\Rightarrow$ | charm |
| " " " " 3 " | $\Rightarrow$ | CP violation |

∘ What is CP ?

C: particle (of given charge) $\xleftrightarrow{C}$ antiparticle (= "particle", with e.m. charge sign-flipped)

P: $\vec{x} \xleftrightarrow{P} -\vec{x}$, if $\vec{x}$ is a vector (as opposed to an axial vector)

$\Rightarrow$ Left-handed particle $\xleftrightarrow{P}$ Right-handed particle

CP : performs the two operations together

Ex. : $\underset{L}{e^-_R} \overset{CP}{\longleftrightarrow} \underset{R}{e^+_L}$

- Why CP important?

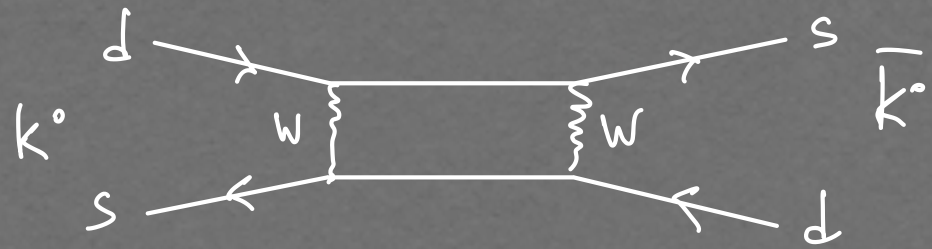
If CP exact, LH protons $\longleftrightarrow$ RH antiprotons

$\Rightarrow$ $\cancel{CP}$ required at some point in the history of the Universe to generate BAU (along with
- $\cancel{B}$
- $\cancel{C}$
- out of equilibrium )

- Realization of $\cancel{CP}$ importance was inspired by measured $\cancel{CP}$ in $K^0 - \bar{K}^0$.

## $K^0 - \bar{K}^0$ mixing

- We have already seen this process




- $K^0$ can, as $t$ evolves, become its antiparticle, and viceversa
- Neither of $K^0$ or $\bar{K}^0$ are well-defined particles
- A good particle: a state with definite mass & certain q.n.

(= properties under conservation laws) e.g. charge, spin, ...

mass: $$\mathcal{H} \, |e(\vec{p}=0)\rangle = m_e \, |e(\vec{p}=0)\rangle$$

Hamiltonian — eigenvalue

$\rightarrow$ a good particle is an eigenstate of the relevant $\mathcal{H}$

- How to define good physical states for the $K^0 - \bar{K}^0$ system.

  - $$\left.\begin{aligned} C|K^0\rangle &= |\bar{K}^0\rangle \\ C|\bar{K}^0\rangle &= |K^0\rangle \end{aligned}\right\} \Rightarrow |K_\pm\rangle \equiv \frac{|K^0\rangle \pm |\bar{K}^0\rangle}{\sqrt{2}}$$ are maybe a good basis?

  - The relevant $\mathcal{H}$ is $\mathcal{H}_{weak}$, and $C$ is not a good symmetry $\left([\mathcal{H}, C] \neq \emptyset\right)$

  - $CP$ is (almost) conserved by $\mathcal{H}_{weak}$, and $CP$ acts basically like $C$

    One can suitably define (quark) phases so that

    $$\left.\begin{aligned} CP|K^0\rangle &= |\bar{K}^0\rangle \\ CP|\bar{K}^0\rangle &= |K^0\rangle \end{aligned}\right\} \Rightarrow CP|K_\pm\rangle = \pm|K_\pm\rangle$$ ← CP-even ← CP-odd

  - $CP$ is almost conserved $\Rightarrow |K_\pm\rangle$ are approximate, but imperfect eigenstates

We can represent the imperfection as

$$|K_S\rangle \sim |K_+\rangle + \bar{\epsilon}\,|K_-\rangle \quad \leftarrow \text{mostly CP-even}$$

$$|K_L\rangle \sim |K_-\rangle + \bar{\epsilon}\,|K_+\rangle \quad \leftarrow \text{mostly CP-odd}$$

$\bar{\epsilon}$ : quantifies the amount of $\cancel{CP}$

- How will the $|K_{S,L}\rangle$ decay? look at decays to CP eigenstates

  E.g. $|2\pi\rangle$ = CP even
  
  $|3\pi\rangle$ = CP odd

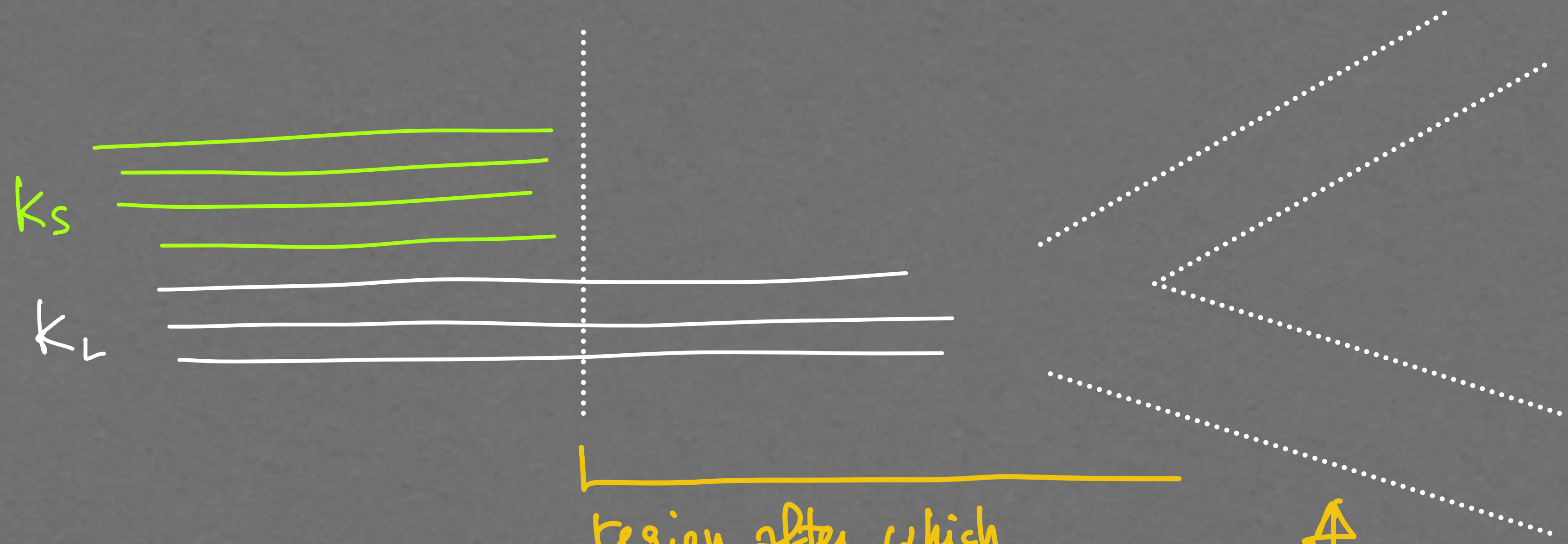
  ↳ $|K_+\rangle \rightarrow |2\pi\rangle$ & $\not\rightarrow |3\pi\rangle$
  
  $|K_-\rangle \rightarrow |3\pi\rangle$ & $\not\rightarrow |2\pi\rangle$
  
  (overlooking direct $\cancel{CP}$)

  ↳ $|K_S\rangle$ decays most of the time to $|2\pi\rangle$, and sometimes to $|3\pi\rangle$
  
  $|K_L\rangle$ " $|3\pi\rangle$ " $|2\pi\rangle$
  
  CP violation
  
  (that's why it's L, for "long")

- How to translate this into a CP observable

$$\frac{N_{\text{events}}(K_L \to 2\pi)}{N_{\text{events}}(K_S \to 2\pi)}$$

Experimental setup (Cronin-Fitch, 1964; Nobel 1980)




region after which most of the $K_S$ have decayed

instrument this region to look for $2\pi$ with $m_{2\pi} = m_{K_L}$

## $\cancel{CP}$ & # of matter generations

- $y_{u,d}$ off-diagonal $\Longleftarrow$ $V_{CKM} \neq \mathbb{1}$ (i.e. flavor violation)

- $y_{u,d} \in \mathbb{C}$ $\Longleftarrow$ $\cancel{CP}$

- $\cancel{CP}$ requires complex $y_{u,d}$. But not any complex $y_{u,d}$ will give physical $\cancel{CP}$

- Let's consider the case of 2 generations

$$V_{CKM} = V_{2\times2} = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$$

acting as; $(\bar{u} \quad \bar{c}) \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$

- Even if $V_{ud}, V_{us} \in \mathbb{C}$, we can move their phases into the def. of $d, s$, respectively

$\Rightarrow V_{ud}, V_{us} \in \mathbb{R}$ in all generality

- $V_{2\times 2}$ is unitary

$$V^\dagger \cdot V = \begin{pmatrix} V_{ud} & V_{cd}^* \\ V_{us} & V_{cs}^* \end{pmatrix} \cdot \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \mathbb{1}$$

$$\Rightarrow (V^\dagger \cdot V)_{12} = V_{ud} V_{us} + \underbrace{V_{cd}^* V_{cs}}_{\text{must be real}} = \emptyset$$

i.e. $\arg(V_{cd}) = \arg(V_{cs})$

$\hookrightarrow$ Since $V_{cd}$ & $V_{cs}$ always multiply $c$ their common phase can be absorbed into the $c$ def.

$\hookrightarrow$ So for 2 generations there's no physical $\cancel{CP}$

- Conditions for physical $\cancel{CP}$
  - Take $N$ generations $\Longleftrightarrow$ $2N$ quarks
  - $V_{CKM}$ has $N^2$ parameters

$$N^2 = \underbrace{\frac{N(N-1)}{2}}_{\text{Euler angles}} + \underbrace{\left(N^2 - \frac{N(N-1)}{2}\right)}_{\text{phases}} \qquad \left(N^2 - \frac{N(N-1)}{2}\right) = \frac{N(N+1)}{2}$$

  - physical phases

$$\frac{N(N+1)}{2} - \underbrace{(2N-1)}_{\substack{\text{redefinitions} \\ \text{of quark fields}}} \longrightarrow \quad \begin{array}{cc} N=2 & N=3 \\ \emptyset & 1 \end{array}$$

$\hookrightarrow$ $\cancel{CP}$ only possible for $N \geq 3$

Within the SM, $\cancel{CP}$ is due to 1 single weak phase