ALL-ORDER RESULTS IN GAUGE THEORIES

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OUTLINE

- **Precision in particle physics**

- **The high-energy limit**

  JHEP 08 (2020), 116, [arXiv:2006.01267 [hep-ph]],

  Phys. Rev. D 103 (2021), L111501, [arXiv:2012.00613 [hep-ph]],

  JHEP 03 (2022), 053, [arXiv:2111.10664 [hep-ph]],

  Phys. Rev. Lett. 128 (2022) no.13, 132001, [arXiv:2112.11098 [hep-ph]].

- **The threshold limit**

  JHEP 1903 (2019) 043, [[arXiv:1809.10631[hep-ph]],

  JHEP 1911 (2019) 002, [arXiv:1905.13710 [hep-ph]],

  JHEP 01 (2020), 094, [arXiv:1910.12685 [hep-ph]],

  JHEP 20 (2020), 078, [arXiv:1912.01585 [hep-ph]],

  Phys.Rev.D 103 (2021) 3, 034022, [arXiv:2008.01736 [hep-ph]],

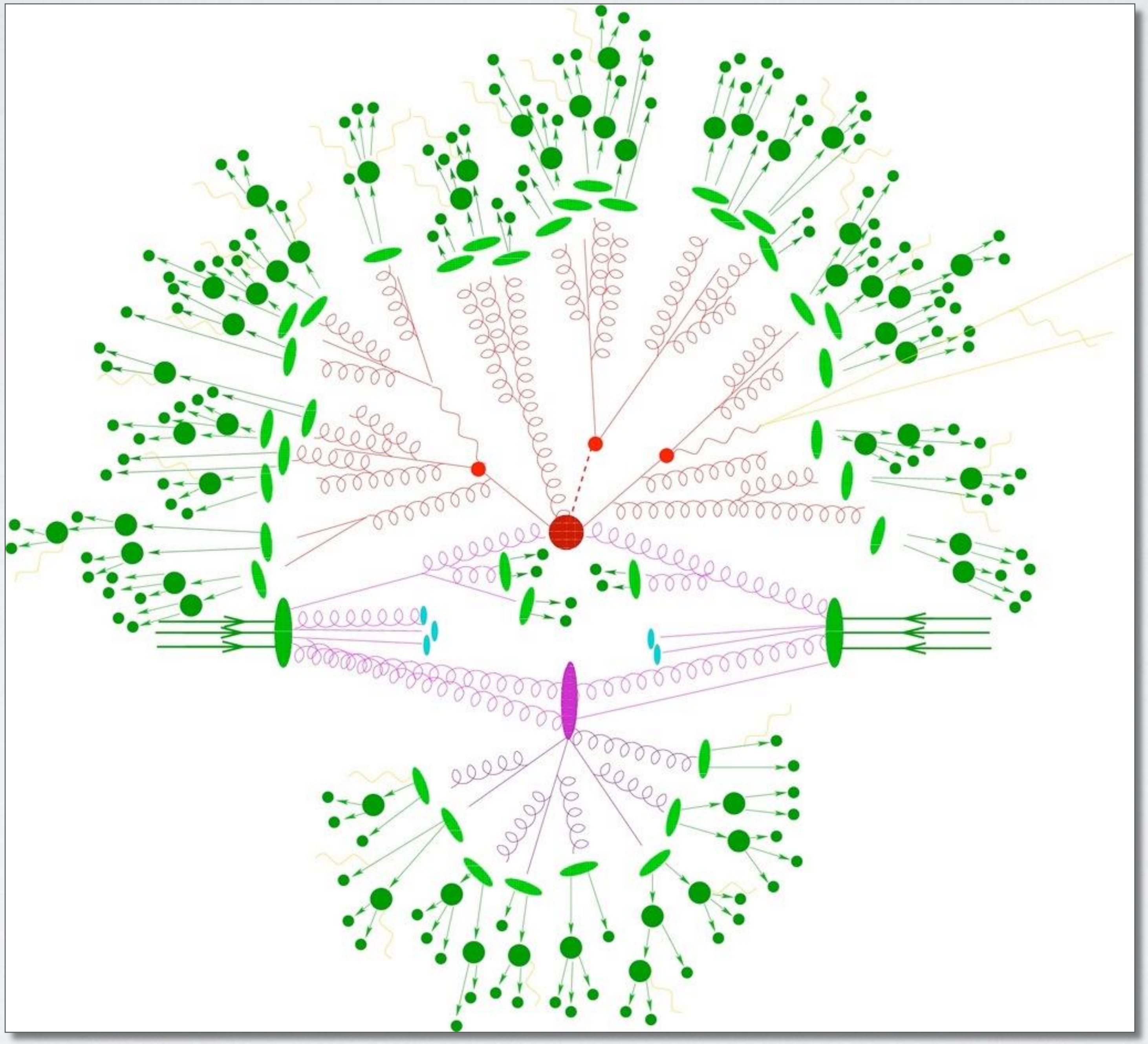
  JHEP 10 (2020), 196, [arXiv:2008.04943 [hep-ph]],

  JHEP 05 (2021), 114, [arXiv:2101.07270 [hep-ph]],

  JHEP 10 (2021), 061, [arXiv:2107.07353 [hep-ph]],

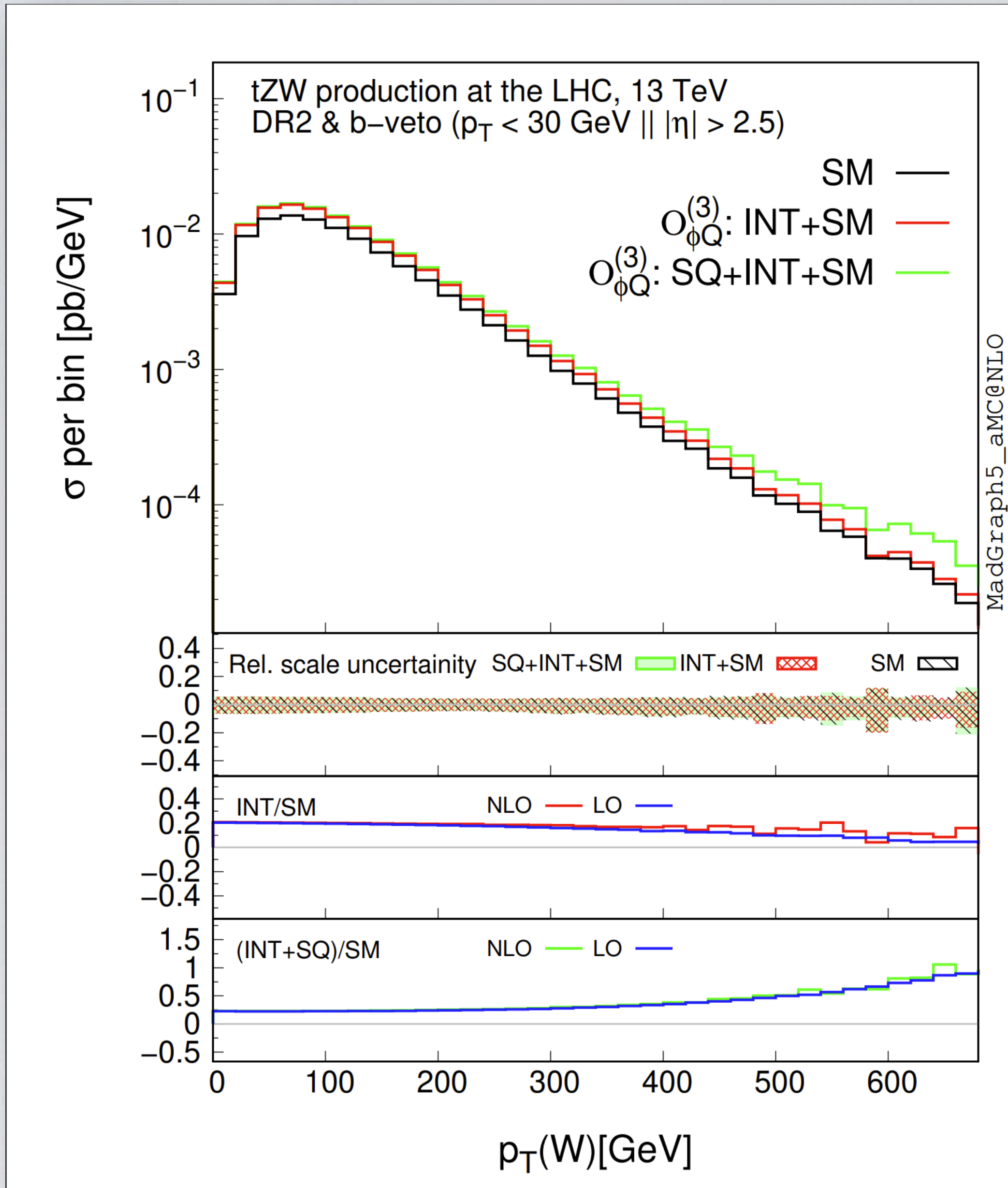
  JHEP 12 (2021), 087, [arXiv:2109.09752 [hep-ph]].

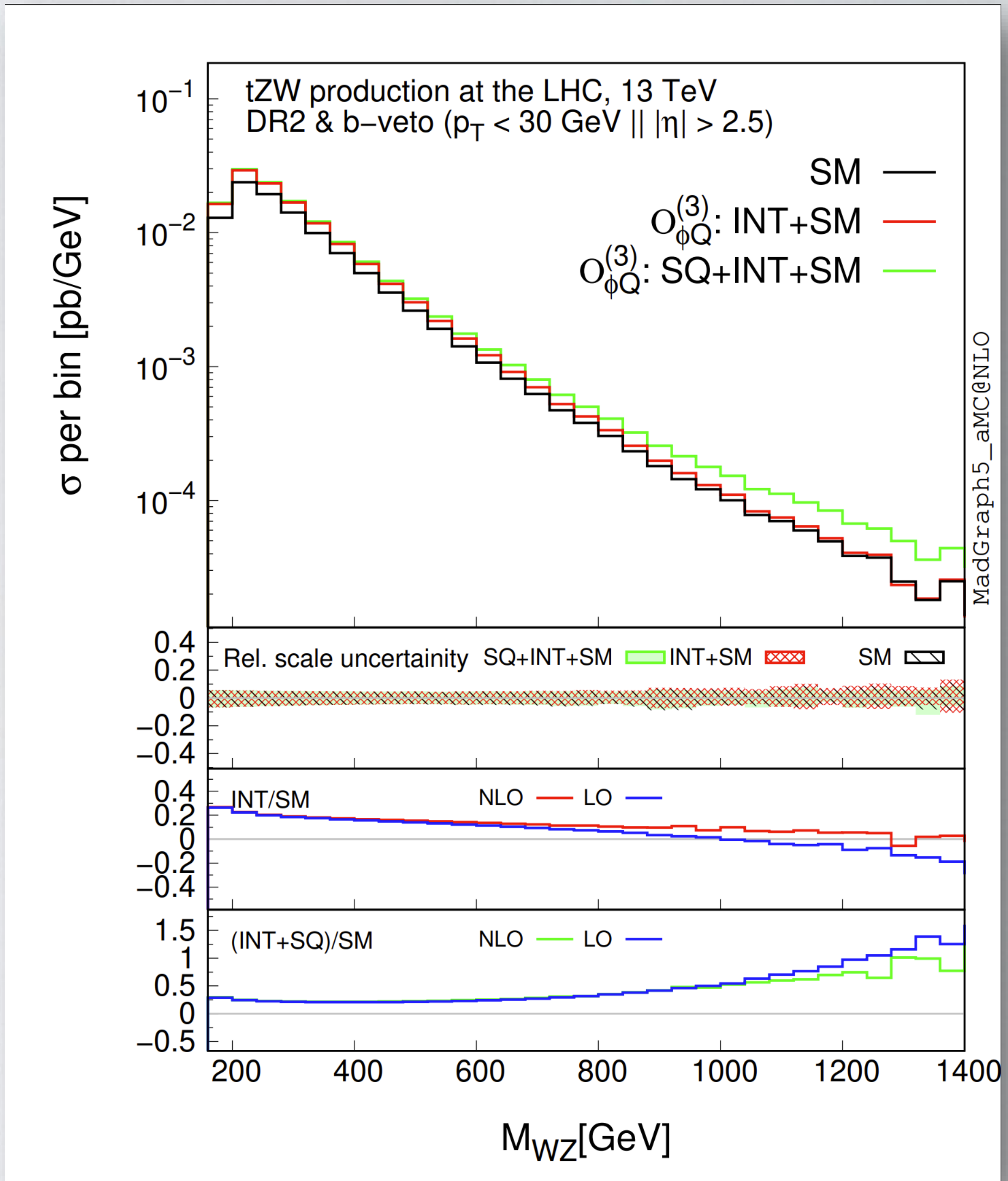
PRECISION IN PARTICLE PHYSICS AT HADRON COLLIDERS



PRECISION FOR COLLIDER PHENOMENOLOGY

- Precision in particle physics offers a valid path to find New Physics, in the form of small deviations from predictions made within the Standard Model.

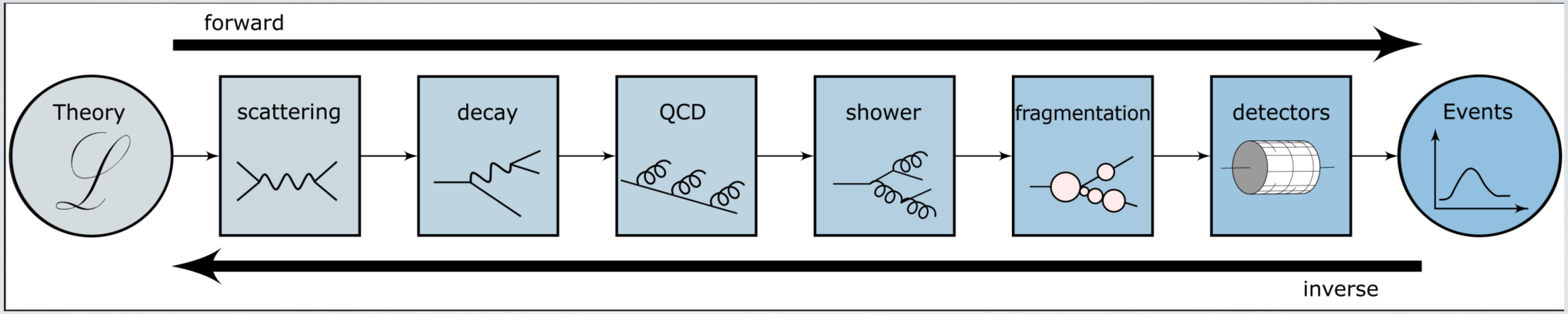




El Faham, Maltoni, Mimasu, Zaro, 2021

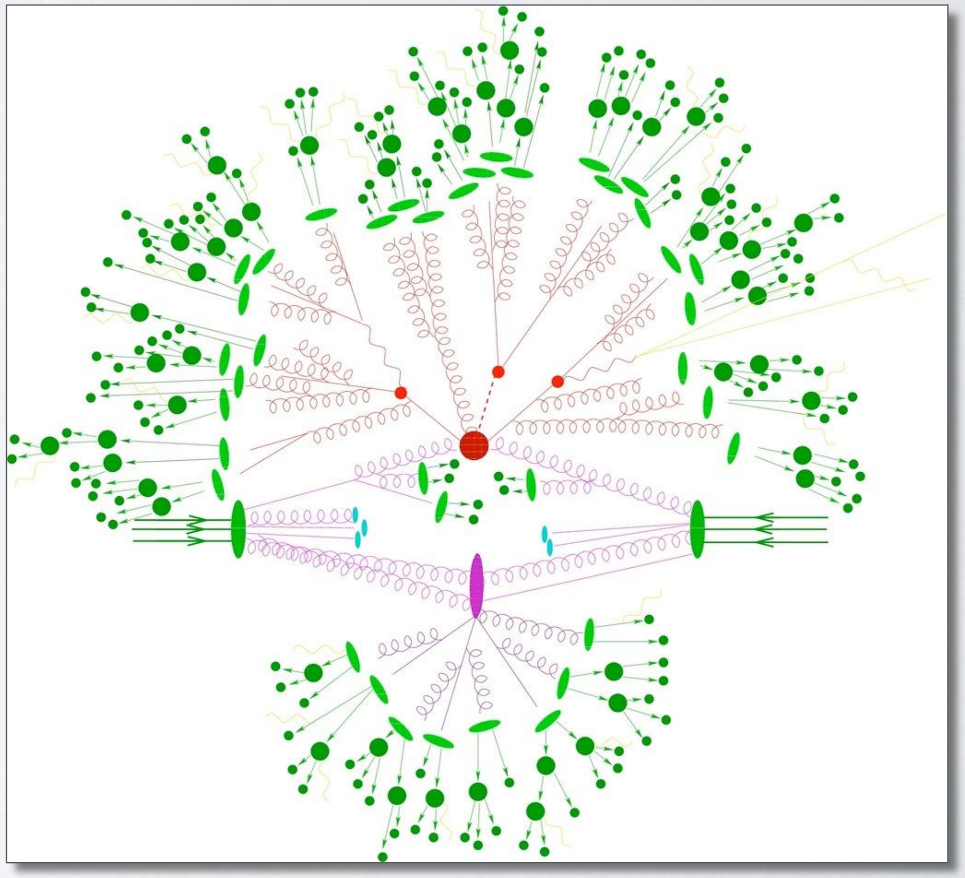
- Highly non-trivial task! Several ingredients are necessary.

PRECISION FOR COLLIDER PHENOMENOLOGY



(Figure from 2203.07460)

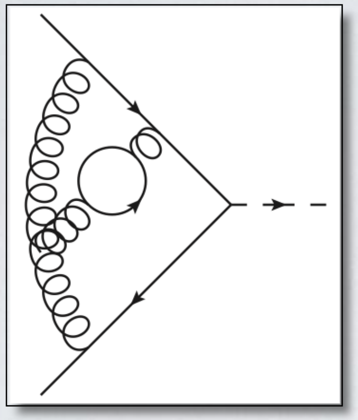
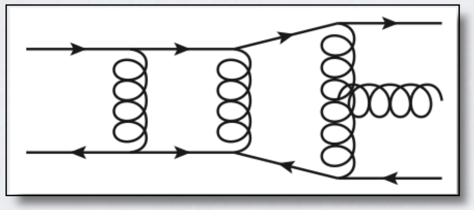
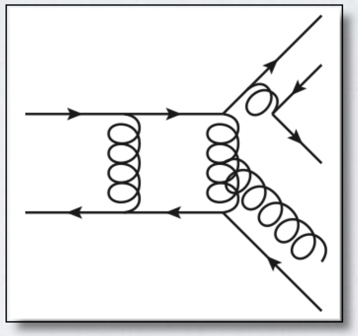
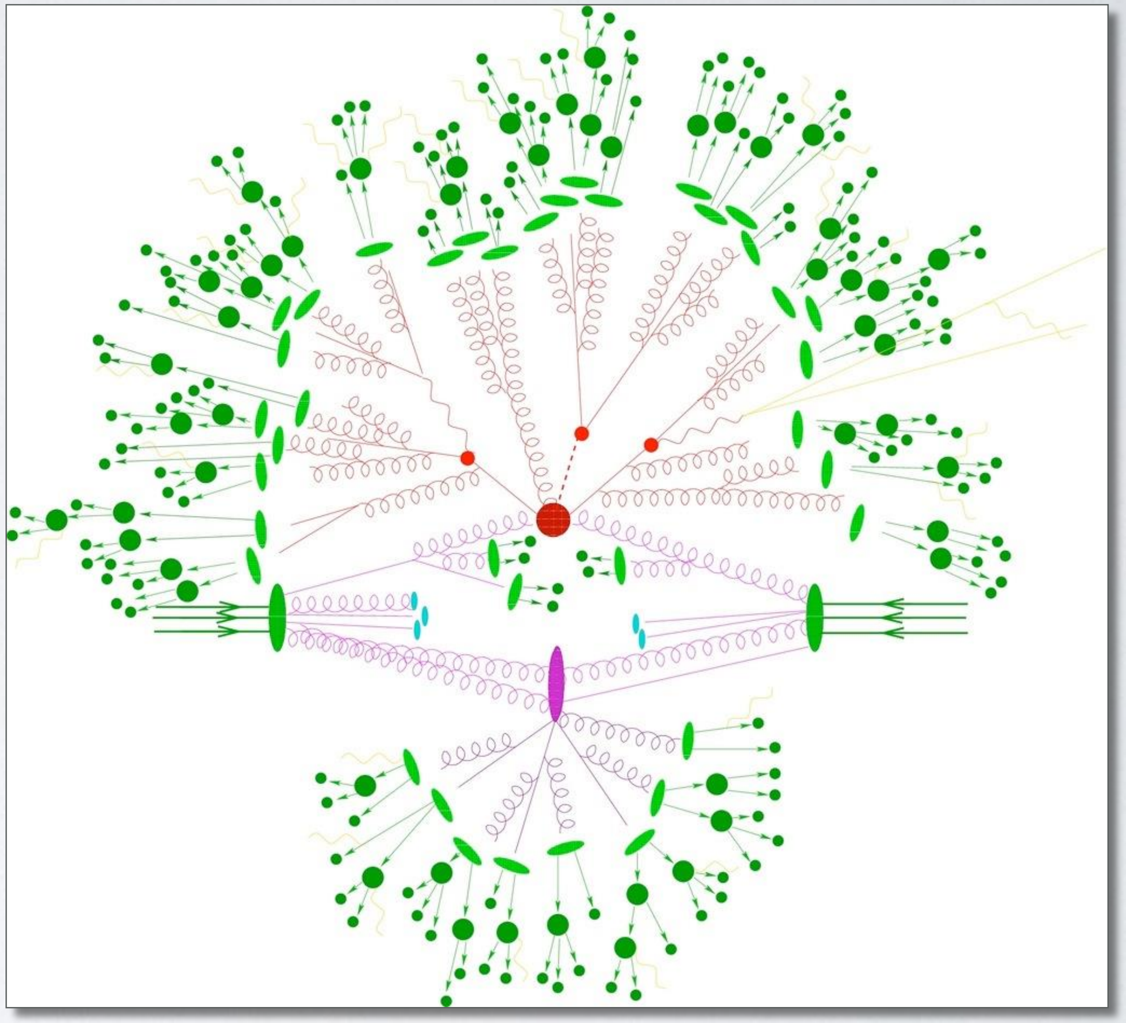
- Here we focus (mostly) on the first step in this chain: the perturbative calculation of hard scattering kernels. This task alone involves an incredible amount of work:



- QCD corrections
- Mixed QCD-EW correction
- Multi-loop and multi-leg processes
- Large logarithms
- SM vs SMEFT
- …

PRECISION FOR COLLIDER PHENOMENOLOGY

- Hard scattering processes are calculated in perturbation theory.

- Going beyond NNLO and N3LO is difficult, yet necessary to match the precision of current and forthcoming experiments!
- Loop and phase space integrals:
  - Analytic vs numerical evaluation
  - Space of functions
  - Infrared divergences
  - Large logarithms

PRECISION FOR COLLIDER PHENOMENOLOGY

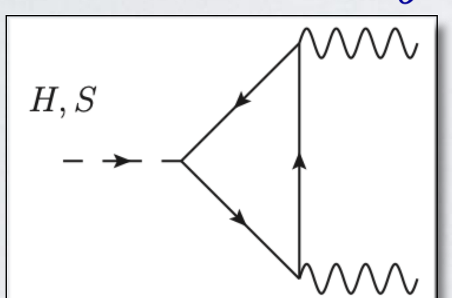
- The presence of largely different scales gives rise to large logarithms:

$$d\sigma \sim 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \ldots$$

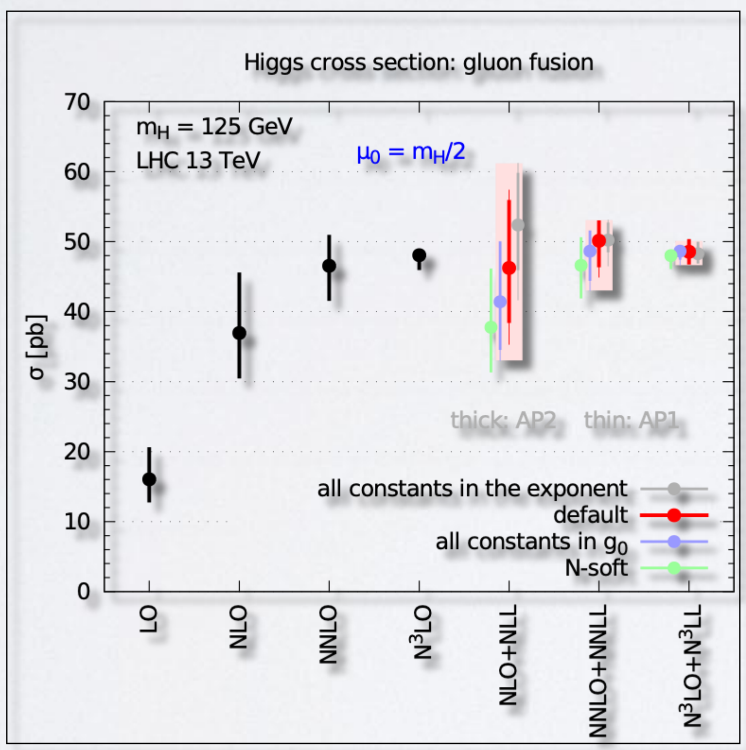
or

$$d\sigma \sim 1 + \alpha_s(L + 1) + \alpha_s^2(L^2 + L + 1) + \ldots$$

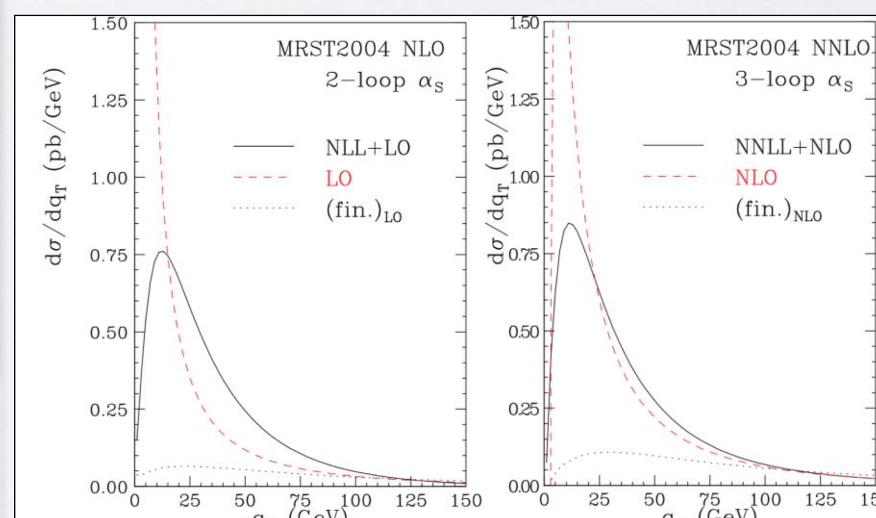
$$\sim \log^2 \frac{m_H^2}{m_b^2}$$



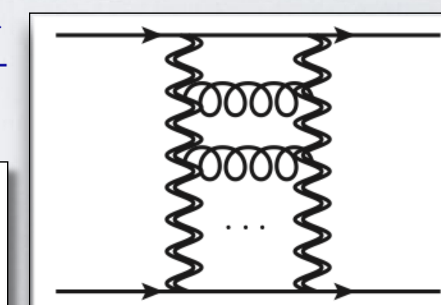
$$\sim \log^2(1-z)$$



$$\sim \log \frac{m_H^2}{p_T^2}$$



$$\sim \log \frac{s}{-t}$$



- Large logarithms spoil the convergence of the perturbative series:

$\rightarrow$ need resummation.

PRECISION FOR COLLIDER PHENOMENOLOGY

- My work within the Fellini project deals with developing new calculation techniques for resummation.
- Interesting task: it requires to understand all order properties of gauge theories.
- As such, it feeds into several aspects of quantum field theory, providing also important results for fixed order perturbation theory and effective field theories.
- I will illustrate these aspects focusing on two cases:

Scattering in the high-energy limit

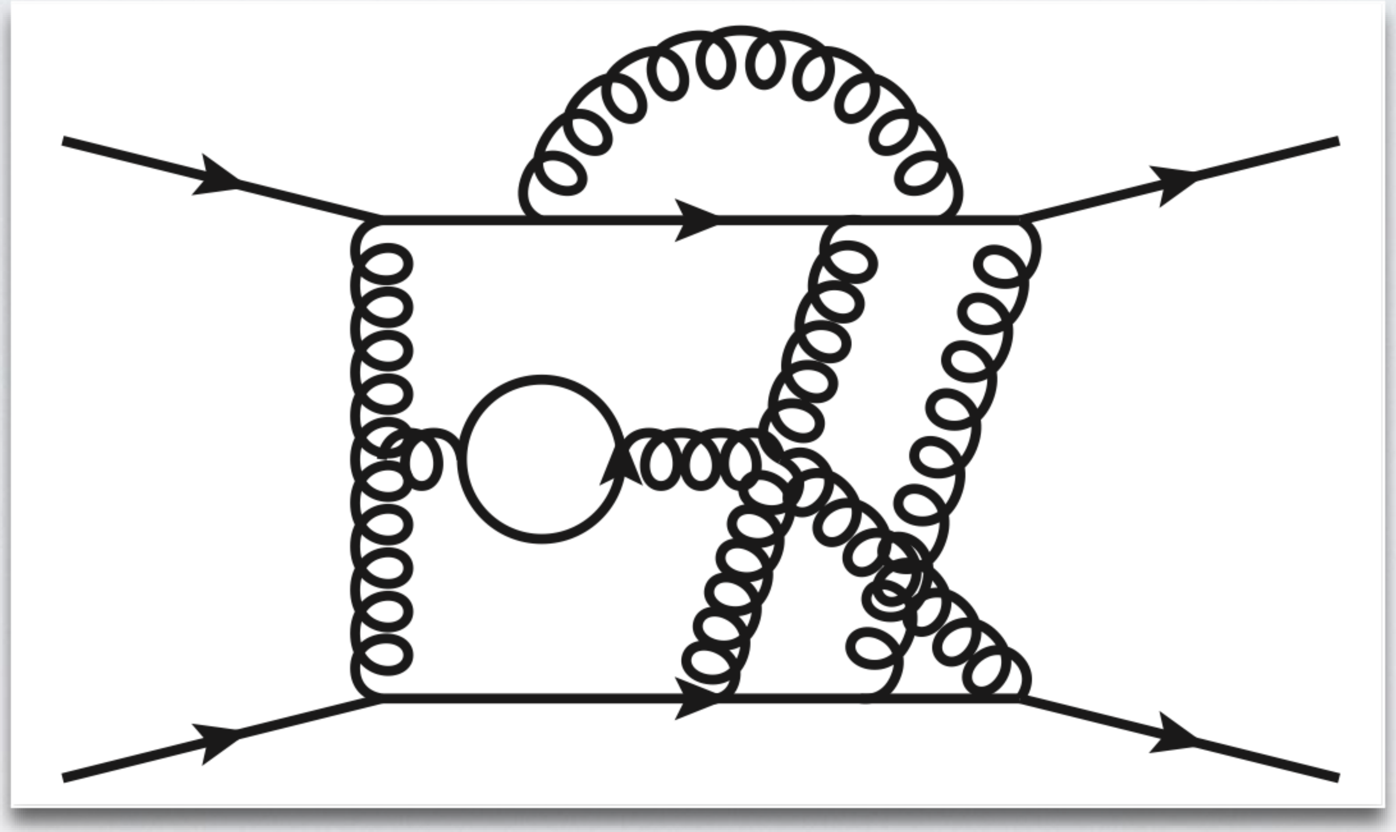
implications for fixed order PT:
→ Infrared fivergences
→ Analytic structure

Scattering near threshold

implications for phenomenology and EFTs

- In both cases, we have developed new theories which allows to systematically calculate large logarithms;
- In turn, we have been able to clarify/solve long standing problems.

SCATTERING
IN THE HIGH-ENERGY LIMIT



HIGH-ENERGY LIMIT

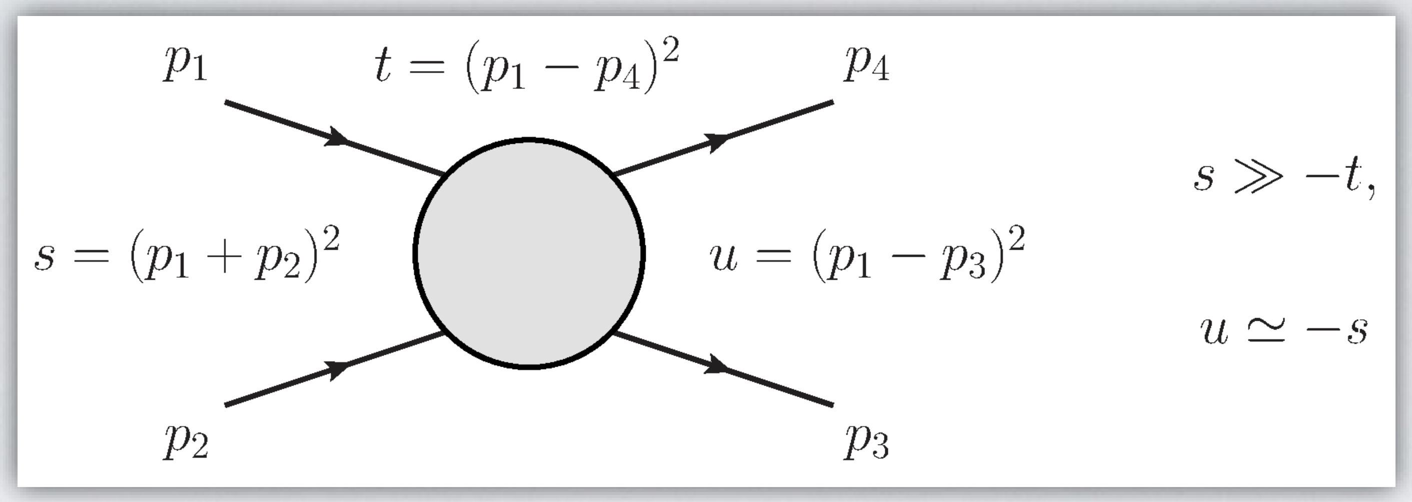
- Very interesting theoretical problem:
  - toy model for full amplitude, yet
    → retain rich dynamic in the 2D transverse plane,
    → non-trivial function spaces;
  - Understand the high-energy QCD asymptotic in terms of Regge poles and cuts;
  - predict amplitudes and other observables in overlapping limits:
    → soft limit, infrared divergences.

MRK in N=4 SYM:
Dixon, Pennington, Duhr, 2012;
Del Duca, Dixon, Pennington, Duhr, 2013;
Del Duca, Druc, Drummond, Duhr, Dulat, Marzucca, Papathanasiou, Verbeek 2019

- Relevant for phenomenology at the LHC and future colliders:
  - perturbative phenomenology of forward scattering, e.g.
    → Deep inelastic scattering/saturation (small x = Regge, large Q^2 = perturbative),
    → Mueller-Navelet: pp → X+2jets, forward and backward.

See e.g. Andersen, Smillie, 2011; Andersen, Medley Smillie, 2016; Andersen, Hapola, Maier, Smillie, 2017; …

TWO-PARTON SCATTERING AMPLITUDES



- Expansion in the strong coupling and in towers of (large) logarithms:

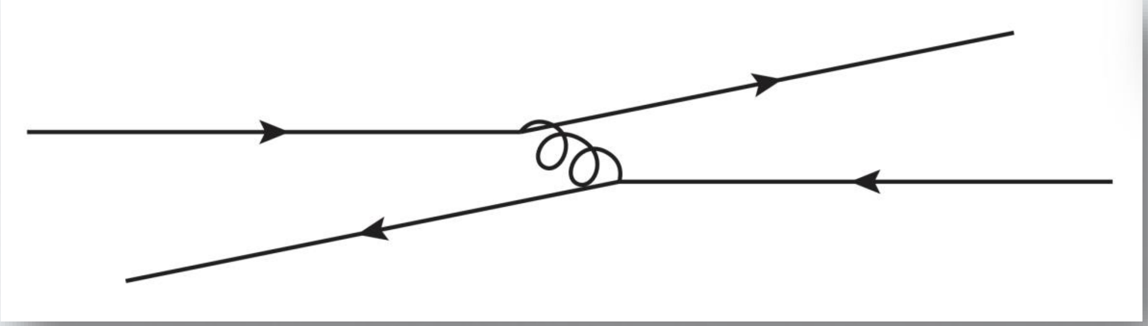
$$\mathcal{M}_{ij\to ij} = \mathcal{M}^{(0)} + \frac{\alpha_s}{\pi} \log \frac{s}{-t} \mathcal{M}^{(1,1)} + \frac{\alpha_s}{\pi} \mathcal{M}^{(1,0)} + \left(\frac{\alpha_s}{\pi}\right)^2 \log^2 \frac{s}{-t} \mathcal{M}^{(2,2)} + \left(\frac{\alpha_s}{\pi}\right)^2 \log \frac{s}{-t} \mathcal{M}^{(2,1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{M}^{(2,0)} + \ldots$$

LL NLL NNLL

- Results: developed a theory for the calculation of amplitudes in the high-energy limit;
- The amplitude is calculated to a given logarithmic accuracy in terms of iterated solution of the Balitsky-JIMWLK evolution equation.

FROM BALITSKY-JIMWLK TO AMPLITUDES

- The physical picture: high-energy limit = forward scattering:

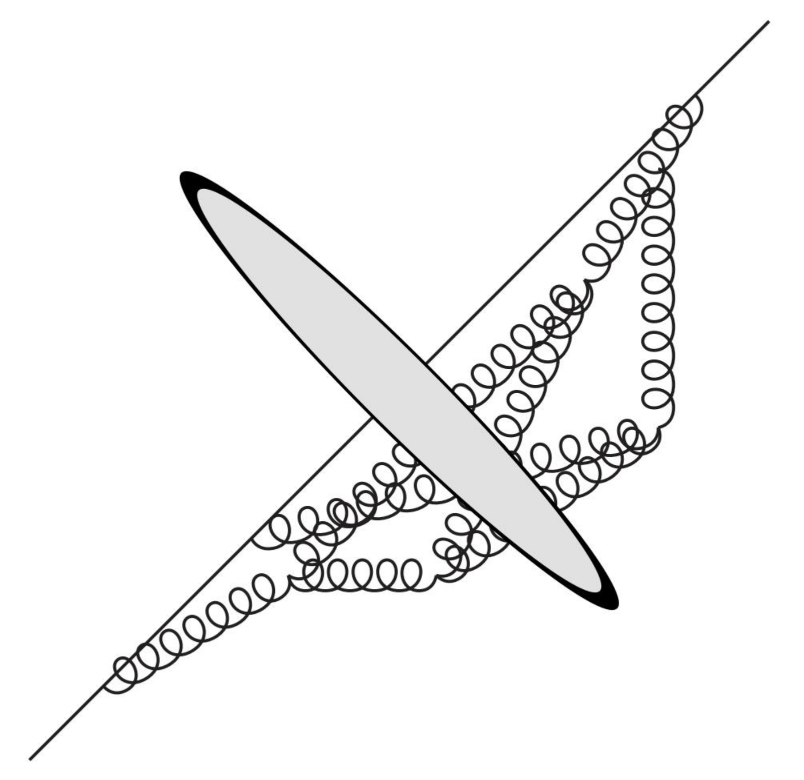


Korchemskaya, Korchemsky, 1994, 1996; Babansky, Balitsky, 2002; Caron-Huot, 2013

- To leading power, the fast projectile and target described in terms of Wilson lines:

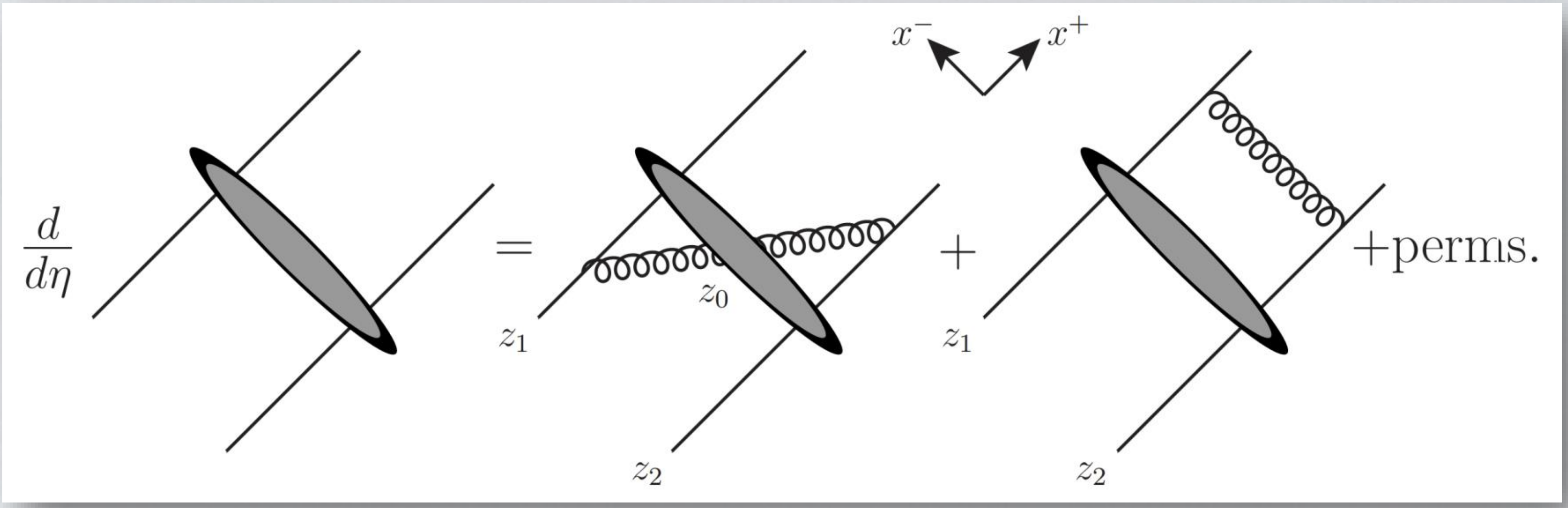
$$U(z_\perp) = \mathcal{P}\exp\left[ig_s\int_{-\infty}^{+\infty} A_+^a(x^+, x^-{=}0, z_\perp)dx^+T^a\right].$$

- Upon evolution in energy (rapidity), emitted radiation gives additional Wilson lines!



$$\eta = L \equiv \log\left|\frac{s}{t}\right| - i\frac{\pi}{2}.$$

FROM BALITSKY-JIMWLK TO AMPLITUDES



- This is expressed by the (non linear!) Balitsky-JIMWLK evolution equation:

$$\frac{d}{d\eta}UU \sim g_s^2 \int d^2 z_0 \, K(z_0, z_1, z_2) \left[U(z_0)UU - UU \right].$$

***NLL:* Balitsky Chirilli, 2013;**
Kovner, Lublinsky, Mulian, 2013, 2014, 2016;
(some) NNLL: Caron-Huot, Gardi, Vernazza, 2017.

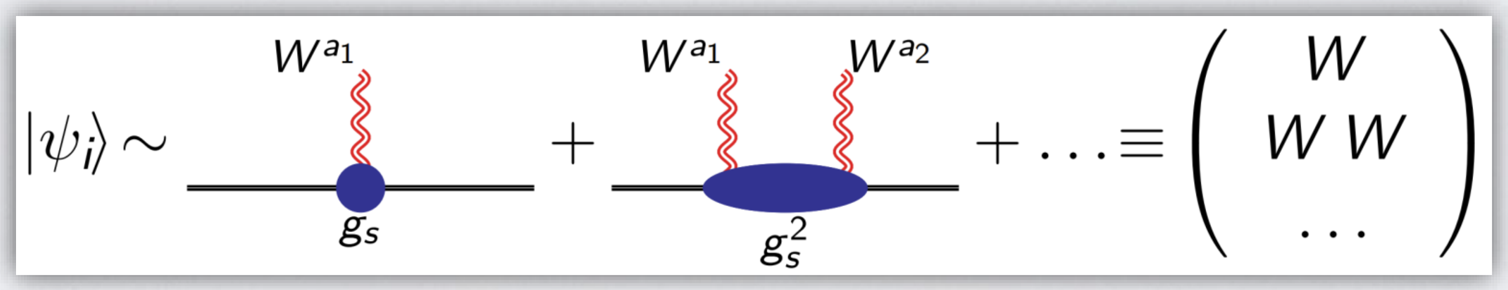
- Shock = Lorentz-contracted target;
- 45° lines = fast projectile partons;
- Each parton crossing the shock gets a Wilson line
- Evolution in rapidity resums the high-energy log: $\eta = L \equiv \log\left|\frac{s}{t}\right| - i\frac{\pi}{2}.$

FROM BALITSKY-JIMWLK TO AMPLITUDES

- The Balitsky-JIMWLK equation is non-linear: leads to the phenomenon of saturation.
- For scattering amplitudes, we can consider the dilute regime: expand Wilson lines around unity in an effective degree of freedom dubbed as "Reggeon":

$$U^{\eta}(z_\perp) = \mathcal{P}\exp\left[ig_s\,\mathbf{T}^a\int_{-\infty}^{+\infty}dx^+\,A_+^a(x^+,x^-=0,z_\perp)\right] \equiv e^{ig_s\,\mathbf{T}^a\,W^a(z_\perp)}\,.$$

- Scattering states (target and projectile) are expanded in Reggeon fields W^a:



- Evolution in rapidity resums the high-energy log:

Caron-Huot, 2013,
Caron-Huot, Gardi, LV, 2017

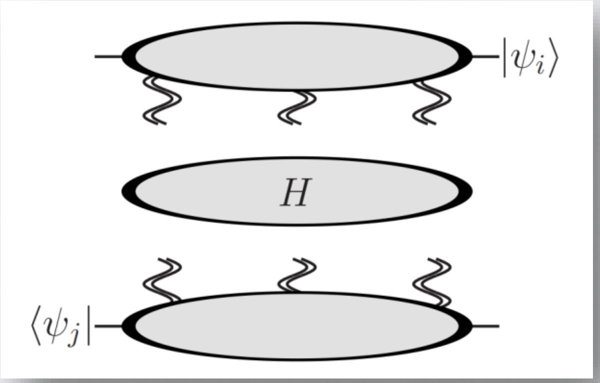
$$\frac{d}{dL}|\psi_i\rangle = -H|\psi_i\rangle.$$

H = Balitsky-JIMWLK Hamiltonian

- Scattering amplitude: expectation value of Wilson lines evolved to equal rapidity:

$$\frac{i}{2s}\frac{1}{Z_iZ_j}\mathcal{M}_{ij\to ij} = \langle\psi_j|e^{-LH}|\psi_i\rangle.$$

(Z_i = collinear poles)



FROM BALITSKY-JIMWLK TO AMPLITUDES

- We obtain the amplitude as an iterated integral over the Balitsky-JIMWLK kernel.
- For instance, in case of two Reggeon exchange one has

$$\hat{\mathcal{M}}^{(+,\ell)}_{\rm NLL} = -i\pi \frac{(B_0)^\ell}{(\ell-1)!} \int [{\rm D}k]\, \frac{p^2}{k^2(k-p)^2}\, \Omega^{(\ell-1)}(p,k)\, \mathbf{T}^2_{s-u}\, \mathcal{M}^{(0)}, \quad B_0 = e^{\epsilon\gamma_{\rm E}} \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}.$$

- One rung = apply once the BFKL kernel on the "target averaged wave function":

$$\Omega^{(\ell-1)}(p,k) = \hat{H}\, \Omega^{(\ell-2)}(p,k), \qquad \hat{H} = (2C_A - \mathbf{T}_t^2)\, \hat{H}_{\rm i} + (C_A - \mathbf{T}_t^2)\, \hat{H}_{\rm m}$$

- "Integration" part:

Caron-Huot, Gardi, Reichel, LV, 2017,2020

$$\hat{H}_{\rm i}\, \Psi(p,k) = \int [{\rm D}k']\, f(p,k,k') \Big[\Psi(p,k') - \Psi(p,k)\Big],$$

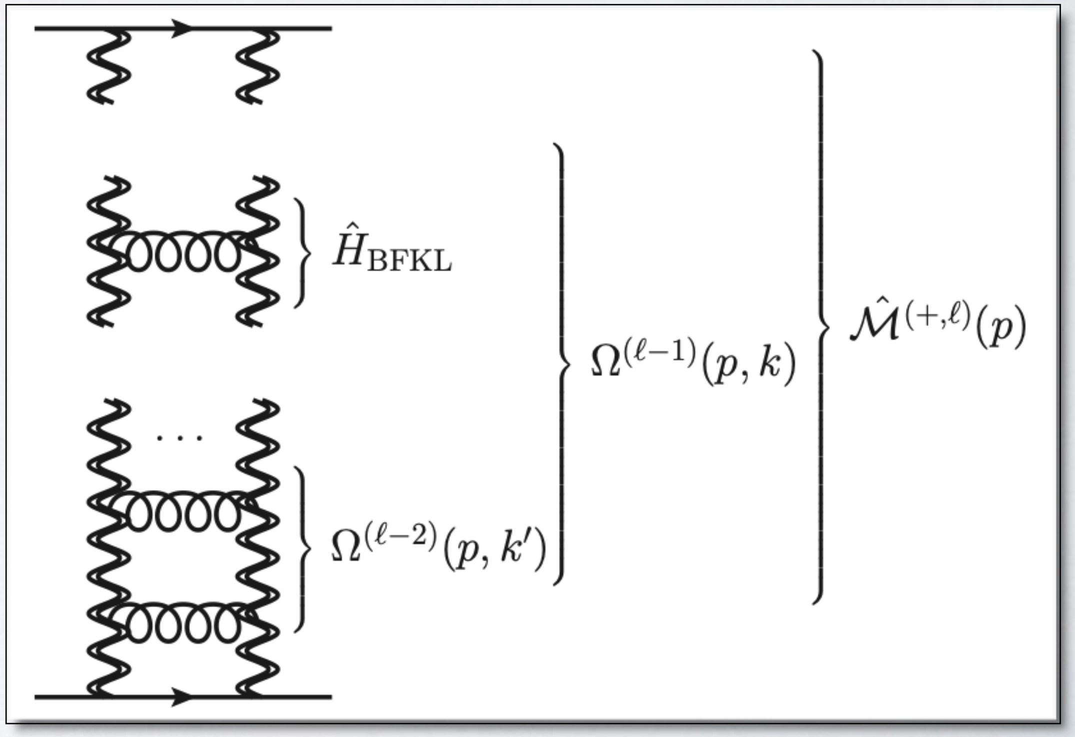
$$f(p,k',k) = \frac{k'^2}{k^2(k-k')^2} + \frac{(p-k')^2}{(p-k)^2(k-k')^2} - \frac{p^2}{k^2(p-k)^2}.$$

- "Multiplication" part:

$$\hat{H}_{\rm m} \Psi(p,k) = \frac{1}{2\epsilon}\left[2 - \left(\frac{p^2}{k^2}\right)^\epsilon - \left(\frac{p^2}{(p-k)^2}\right)^\epsilon\right] \Psi(p,k).$$

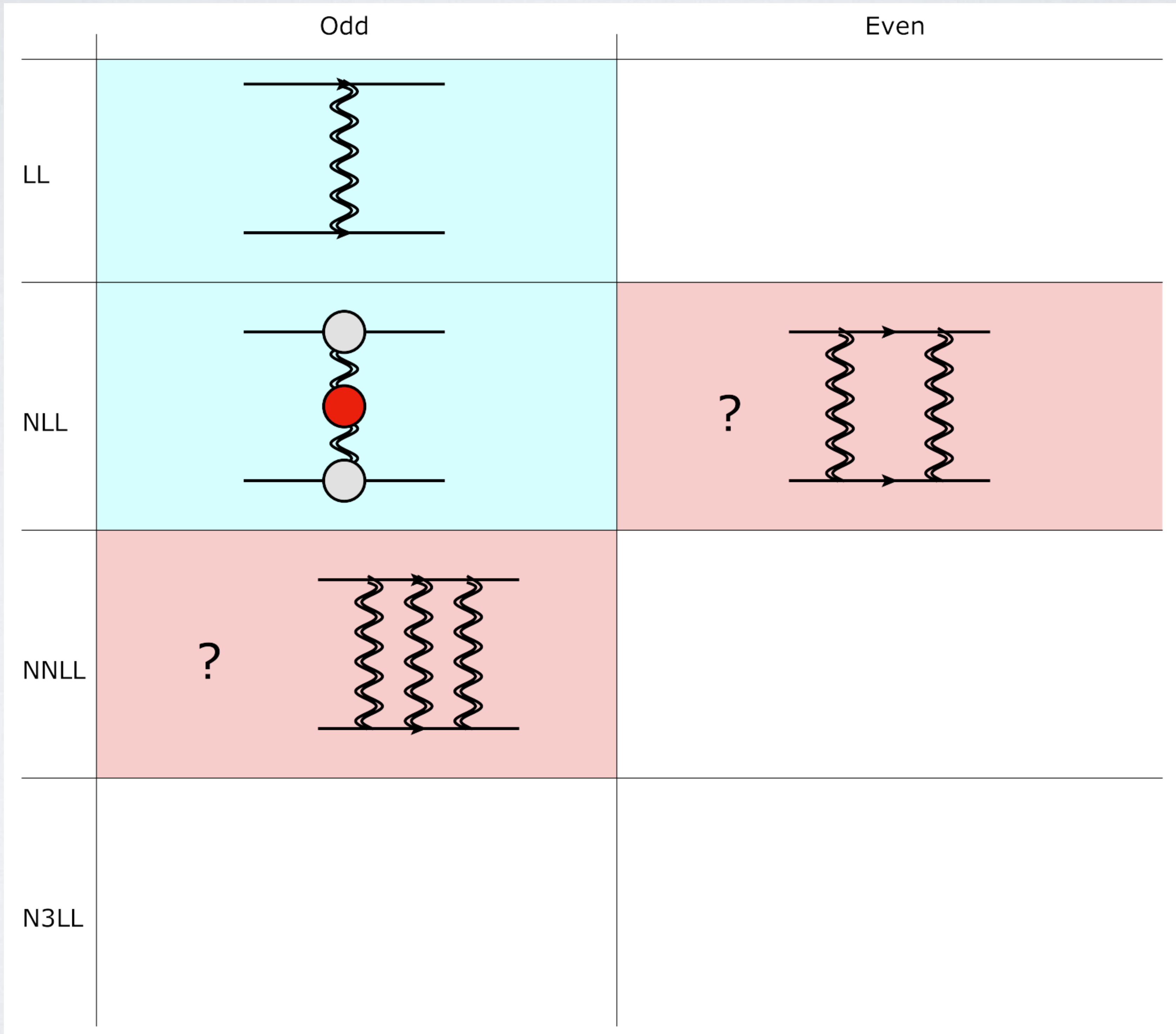
- Initial condition

$$\Omega^{(0)}(p,k) = 1.$$



TWO PARTON SCATTERING AMPLITUDES

- Status pre ~ 2014:



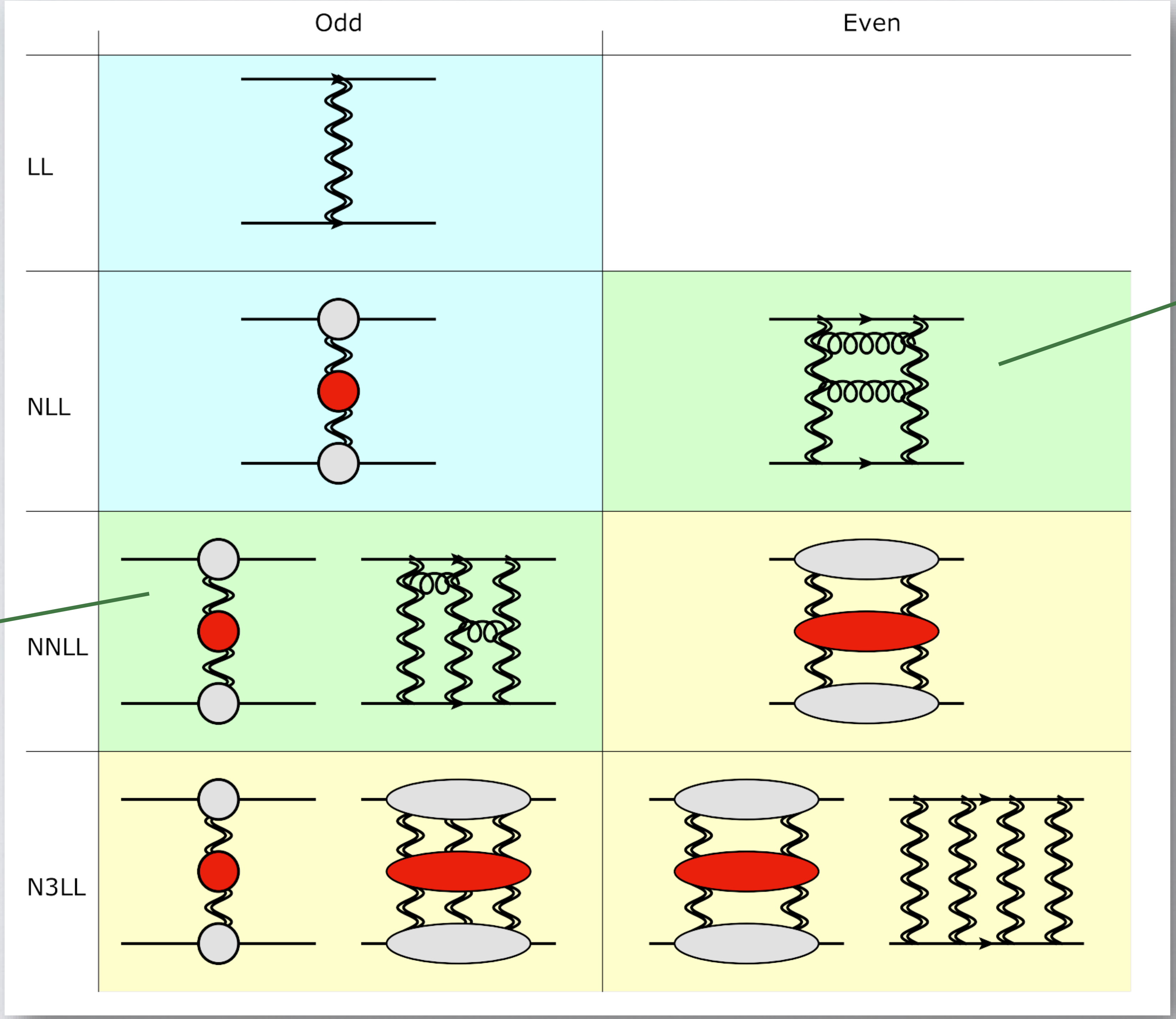
TWO PARTON SCATTERING AMPLITUDES

- Developed a framework for the calculation of amplitudes in the high-energy limit;
- Systematic relation between logarithmic accuracy and number of Reggeons.

Analysed to 2 loops in Del Duca, Falcioni, Magnea, Vernazza 2014;

Calculated to 3 loops in Caron-Huot, Gardi, LV, 2017;

Calculated to 4 loops in Falcioni, Gardi, Milloy, LV, 2020; Falcioni, Gardi, Maher, Milloy, LV, 2021.



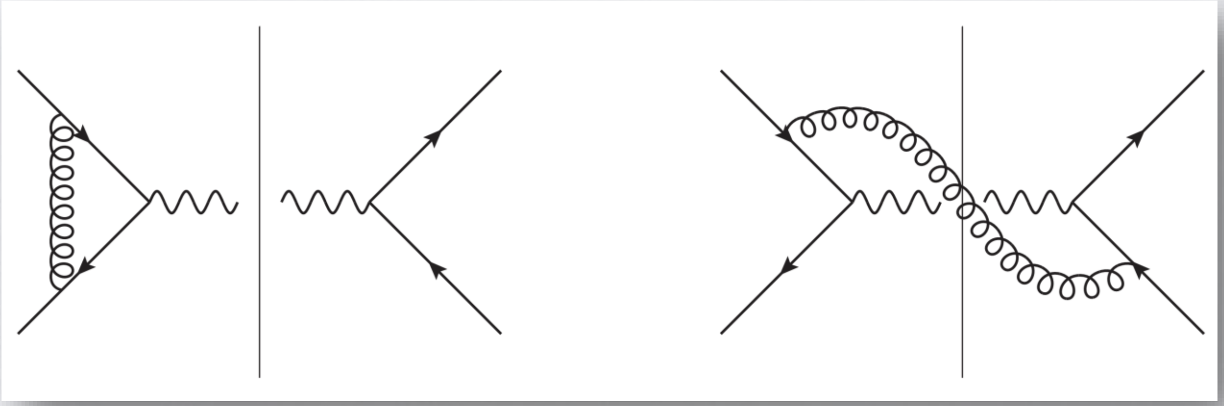
IR rivergences calculated to all orders in Caron-Huot, Gardi, Reichel, LV, 2017;

Finite terms calculated to 13 loops in Caron-Huot, Gardi, Reichel, LV, 2020.

APPLICATION: INFRARED DIVERGENCES

- Individual terms of matrix element squared are infrared divergent;
- Infrared divergences cancel in the sum over equivalent final (and initial) states.

$$\frac{d\sigma_{\mathrm{NLO}}}{dX} = \int d\Phi_n \, V \, \delta_n(X) + \int d\Phi_{n+1} \, R \, \delta_{n+1}(X).$$



See for instance
Agarwal, Magnea, Signorile-Signorile, Tripathi, 2021.

- In practice, need to construct counterterms for both terms.

$$\frac{d\sigma_{\mathrm{NLO}}}{dX} = \int d\Phi_n \Big(V + I \Big) \delta_n(X) + \int \Big(d\Phi_{n+1} \, R \, \delta_{n+1}(X) - d\widehat{\Phi}_{n+1} \, \overline{K} \, \delta_n(X) \Big), \qquad I = \int d\widehat{\Phi}_{\mathrm{rad}} \, \overline{K}.$$

- Structure of infrared divergences is universal: depends on features of soft and collinear radiation in a gauge theory. A lot of work has been devoted to constraint it.

APPLICATION: INFRARED DIVERGENCES

- The infrared divergences of amplitudes are controlled by a renormalization group equation:

$$\mathcal{M}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right) = \mathbf{Z}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right)\mathcal{H}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right),$$

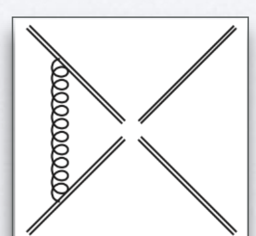
- where $\boldsymbol{Z}_n$ is given as a path-ordered exponential of the soft-anomalous dimension:

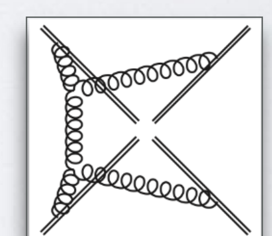
$$\mathbf{Z}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right) = \mathcal{P}\exp\left\{-\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\,\mathbf{\Gamma}_n\left(\{p_i\},\lambda,\alpha_s(\lambda^2)\right)\right\},$$

Becher, Neubert, 2009; Gardi, Magnea, 2009

- The soft anomalous dimension for scattering of massless partons is an operator in color space given by

$$\mathbf{\Gamma}_n\left(\{p_i\},\lambda,\alpha_s(\lambda^2)\right) = \mathbf{\Gamma}_n^{\text{dip.}}\left(\{p_i\},\lambda,\alpha_s(\lambda^2)\right) + \mathbf{\Delta}_n\left(\{\rho_{ijkl}\}\right).$$

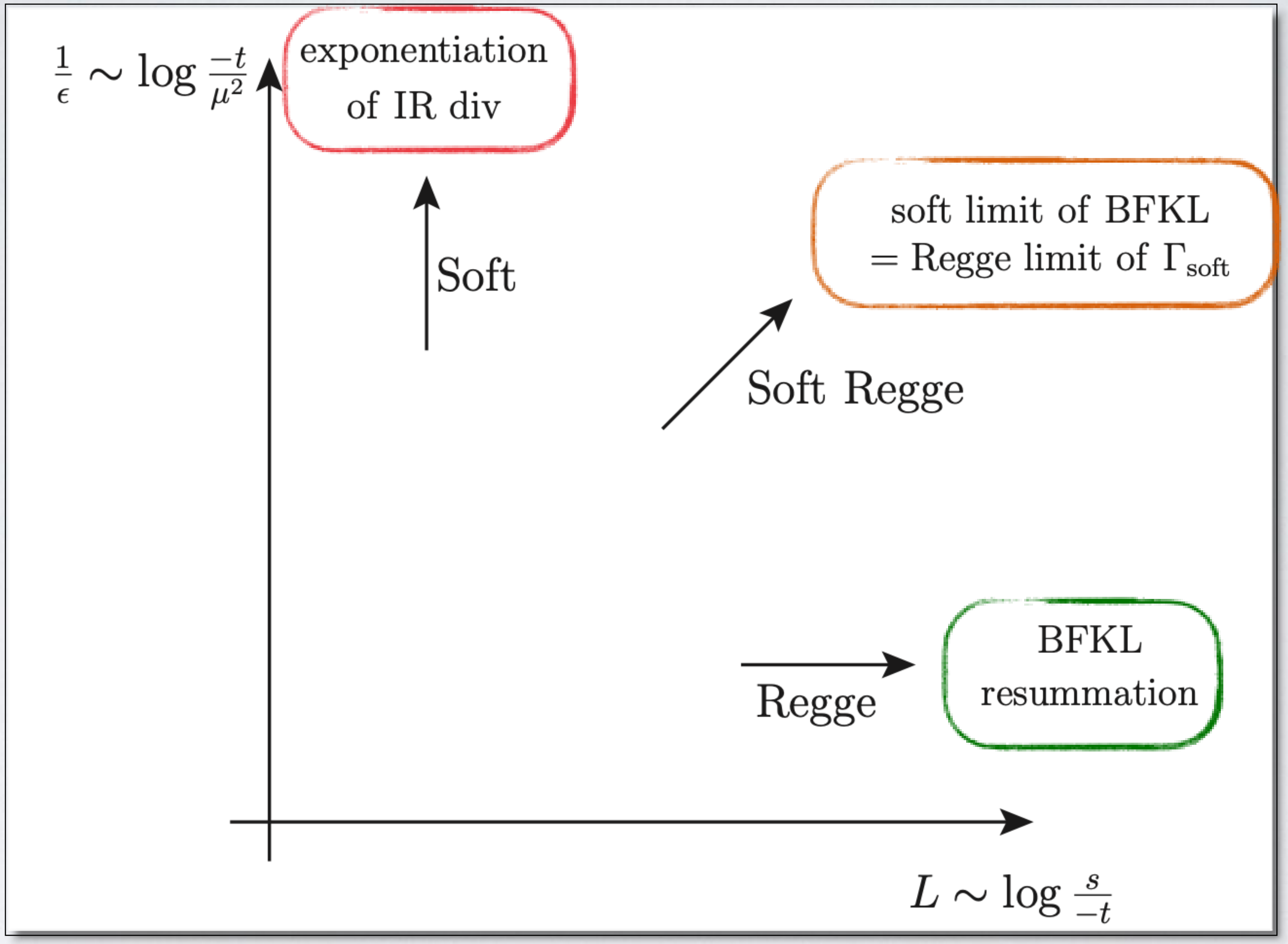




- In the past years a lot of work has been devoted to calculate/constrain $\boldsymbol{\Delta}_n$.

Dixon, Gardi, Magnea, 2009; Del Duca, Duhr, Gardi, Magnea, White, 2011; Neubert, LV, 2012; Caron-Huot, 2013; Almelid, Duhr, Gardi, 2015, 2016; Caron-Huot, Gardi, LV, 2017; Almelid, Duhr, Gardi, McLeod, White, 2017; Becher, Neubert, 2019; Magnea 2021; Falcioni, Gardi, Maher, Milloy, Vernazza 2021.

APPLICATION: INFRARED DIVERGENCES



- Use amplitudes calculated in the high-energy limit to extract the soft anomalous dimension in that limit;
- Bootstrap the result to constrain the structure of infrared divergences in general kinematic.

APPLICATION: INFRARED DIVERGENCES

Re	L^0	L^1	L^2	L^3	L^4	L^5	L^6
α_s^1	$\frac{1}{4}\widehat{\gamma}_K^{(1)} \ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i + \sum_{i=1}^4 \gamma_i^{(1)}$	$\frac{1}{2}\widehat{\gamma}_K^{(1)}\mathbf{T}_t^2$					
α_s^2	$\frac{1}{4}\widehat{\gamma}_K^{(2)} \ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i + \sum_{i=1}^4 \gamma_i^{(2)}$	$\frac{1}{2}\widehat{\gamma}_K^{(2)}\mathbf{T}_t^2$	0				
α_s^3	$\frac{1}{4}\widehat{\gamma}_K^{(3)} \ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i + \sum_{i=1}^4 \gamma_i^{(3)} + \Delta^{(+,3,0)}$	$\frac{1}{2}\widehat{\gamma}_K^{(3)}\mathbf{T}_t^2$	0	0			
α_s^4			$\Delta^{(+,4,2)}$	0	0		
α_s^5					0	0	
α_s^6						0	0

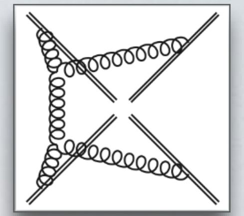
Im	L^0	L^1	L^2	L^3	L^4	L^5	L^6
α_s^1	$\frac{1}{2}\widehat{\gamma}_K^{(1)} i\pi\mathbf{T}_{s-u}^2$	0					
α_s^2	$\frac{1}{2}\widehat{\gamma}_K^{(2)} i\pi\mathbf{T}_{s-u}^2$	0	0				
α_s^3	$\frac{1}{2}\widehat{\gamma}_K^{(3)} i\pi\mathbf{T}_{s-u}^2 + \Delta^{(-,3,0)}$	$\Delta^{(-,3,1)}$	0	0			
α_s^4				$\Delta^{(-,4,3)}$	0		
α_s^5					$\Delta^{(-,5,4)}$	0	
α_s^6						$\Delta^{(-,6,5)}$	0

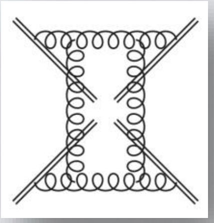
Caron-Huot, Gardi, LV, 2017; Caron-Huot, Gardi, Reichel, LV, 2017; Gardi, Falcioni, Milloy, LV, 2020; Gardi, Falcioni, Maher, Milloy, LV, 2021.

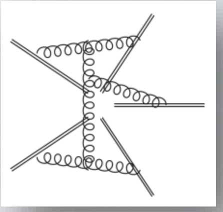
APPLICATION: INFRARED DIVERGENCES

- Structure of the soft anomalous dimension in general kinematic up to four loops:

$$\begin{aligned}\mathbf{\Gamma}_n\big(\{s_{ij}\},\mu,\alpha_s(\mu^2)\big) =& -\frac{\gamma_K(\alpha_s)}{4}\sum_{(i,j)}\mathbf{T}_i\cdot\mathbf{T}_i\,\log\frac{-s_{ij}}{\mu^2}+\sum_i\gamma_i(\alpha_s)\\ &+f(\alpha_s)\sum_{(i,j,k)}\mathcal{T}_{iikj}+\sum_{(i,j,k,l)}\mathcal{T}_{ijkl}\,\mathcal{F}(\beta_{ijlk},\beta_{iklj};\alpha_s)\\ &-\sum_R\frac{g^R(\alpha_s)}{2}\Bigg[\sum_{(i,j)}\big(\mathcal{D}^R_{iijj}+2\mathcal{D}^R_{iiij}\big)\ln\frac{-s_{ij}}{\mu^2}+\sum_{(i,j,k)}\mathcal{D}^R_{ijkk}\ln\frac{-s_{ij}}{\mu^2}\Bigg]\\ &+\sum_R\sum_{(i,j,k,l)}\mathcal{D}^R_{ijkl}\mathcal{G}^R(\beta_{ijlk},\beta_{iklj};\alpha_s)+\sum_{(i,j,k,l)}\mathcal{T}_{ijkli}\mathcal{H}_1(\beta_{ijlk},\beta_{iklj};\alpha_s)\\ &+\sum_{(i,j,k,l,m)}\mathcal{T}_{ijklm}\mathcal{H}_2(\beta_{ijkl},\beta_{ijmk},\beta_{ikmj},\beta_{jiml},\beta_{jlmi};\alpha_s)+\mathcal{O}(\alpha_s^5).\end{aligned}$$







- From the Regge limit we obtain constrains, useful for a bootstrap approach:

Gardi, Falcioni, Maher, Milloy, LV, 2021.

Signature even				Signature odd			
	L^3	L^2	L^1 (conj.)		L^3	L^2	L^1
$\mathcal{F}_A^{(+,4)}$	0	$-\frac{C_A}{8}\zeta_2\zeta_3$	0	$\mathcal{F}_A^{(-,4)}$	$i\pi\frac{C_A}{24}\zeta_3$	?	?
$\mathcal{F}_F^{(+,4)}$	0	0	0	$\mathcal{F}_F^{(-,4)}$	0	?	?
$\mathcal{G}_A^{(+,4)}$	0	$\frac{1}{2}\zeta_2\zeta_3$	$\frac{1}{6}g_A^{(4)}$				
$\mathcal{G}_F^{(+,4)}$	0	0	$\frac{1}{6}g_F^{(4)}$				
$\mathcal{H}_1^{(+,4)}$	0	0	0	$\mathcal{H}_1^{(-,4)}$	0	?	?
				$\tilde{\mathcal{H}}_1^{(-,4)}$	0	?	?

See e.g. Almelid, Duhr, Gardi, McLeod, White, 2017

APPLICATION: REGGE POLE AND CUT

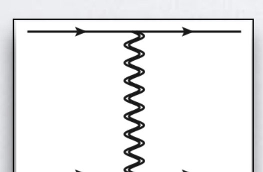
- Before the development of QCD and perturbation theory, scattering amplitudes have been studied as an analytic function in the complex angular momentum plane.
- In this context, the amplitude is expected to be given in terms of Regge pole and cut:

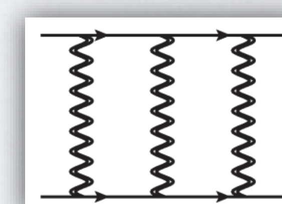
$$A_{LL} \propto \underbrace{\frac{s^{\alpha_g(t)}}{t}}_{\text{“Regge pole”}}, \qquad A_{\rm NLL} \propto \underbrace{\int d\nu\, c(\nu)\, s^{E(\nu)}}_{\text{“Regge cut”}}.$$

Regge, Gribov ~ 1960; Lipatov; Fadin, Kuraev, Lipatov 1976.

- Before our studies it was possible to identify the Regge pole contribution (given in terms of the Regge trajectory) only up to NLL; starting at NNLL, the contribution of Regge pole and cut mix, leading to ambiguities.
- Our results allows to identify these contributions unambiguously, thus relating concepts of analyticity and integrability with the modern perturbation theory:

$$\mathcal{M}^{(-)}_{ij\to ij} = \underbrace{\mathcal{M}^{(-)\,\rm SR}_{ij\to ij} + \mathcal{M}^{(-)\,\rm MR}_{ij\to ij}\Big|_{\rm planar}} + \underbrace{\mathcal{M}^{(-)\,\rm MR}_{ij\to ij}\Big|_{\rm nonplanar}}$$
$$= \qquad \mathcal{M}^{(-)\,\rm pole}_{ij\to ij} \qquad + \qquad \mathcal{M}^{(-)\,\rm cut}_{ij\to ij}.$$





APPLICATION: REGGE POLE AND CUT

- With our definition we are able to extract unambiguously the Regge trajectory at three loops, matching our calculation of the Regge-cut contribution with the recent calculations of two-parton scattering at three loops in QCD:

Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi, 2021

$$\mathcal{M}^{(-)}_{ij\to ij} = Z_i(t)\,\bar{D}_i(t)\,Z_j(t)\,\bar{D}_j(t)\left[\left(\frac{-s}{-t}\right)^{C_A\tilde{\alpha}_g(t)} + \left(\frac{-u}{-t}\right)^{C_A\tilde{\alpha}_g(t)}\right]\mathcal{M}^{\rm tree}_{ij\to ij} + \sum_{n=2}^{\infty}\frac{\alpha_s}{4\pi}L^{n-2}\mathcal{M}^{(-,n,n-2)\,\rm cut}_{ij\to ij},$$

with

$$\hat{\tilde{\alpha}}^{(3)}_g = K^{(3)} + C_A^2\left(\frac{297029}{93312} - \frac{799\zeta_2}{1296} - \frac{833\zeta_3}{216} - \frac{77\zeta_4}{192} + \frac{5}{24}\zeta_2\zeta_3 + \frac{\zeta_5}{4}\right) + C_A n_f\left(\frac{103\zeta_2}{1296} + \frac{139\zeta_3}{144} - \frac{5\zeta_4}{96} - \frac{31313}{46656}\right)$$
$$+ C_F n_f\left(\frac{19\zeta_3}{72} + \frac{\zeta_4}{8} - \frac{1711}{3456}\right) + n_f^2\left(\frac{29}{1458} - \frac{2\zeta_3}{27}\right) + \mathcal{O}(\epsilon),$$

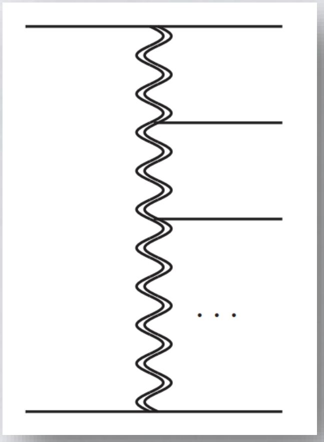
$$K_{\rm cusp}(\alpha_s(\mu^2)) \equiv -\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\Gamma^{\rm cusp}_A(\alpha_s(\lambda^2))\,.$$

Gardi, Falcioni, Maher, Milloy, LV, 2021.

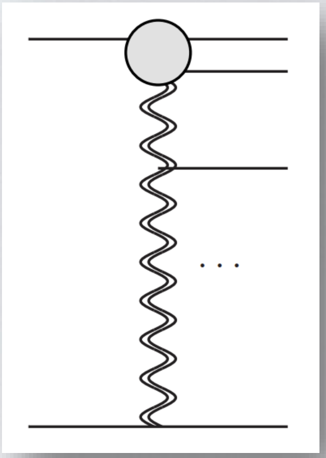
- The Regge-pole contribution is universal among all two-parton scattering processes, but theory dependent (i.e. different in N=4 SYM, QCD, etc);
- The Regge–cut contribution is different for each channel but depends only on the action of color operators in the gauge theory considered.

HIGH ENERGY LIMIT: PERSPECTIVE

- Complete NNLL calculation of two-parton scattering amplitudes;
- Extend the shockwave formalism to Multi-Regge kinematics:



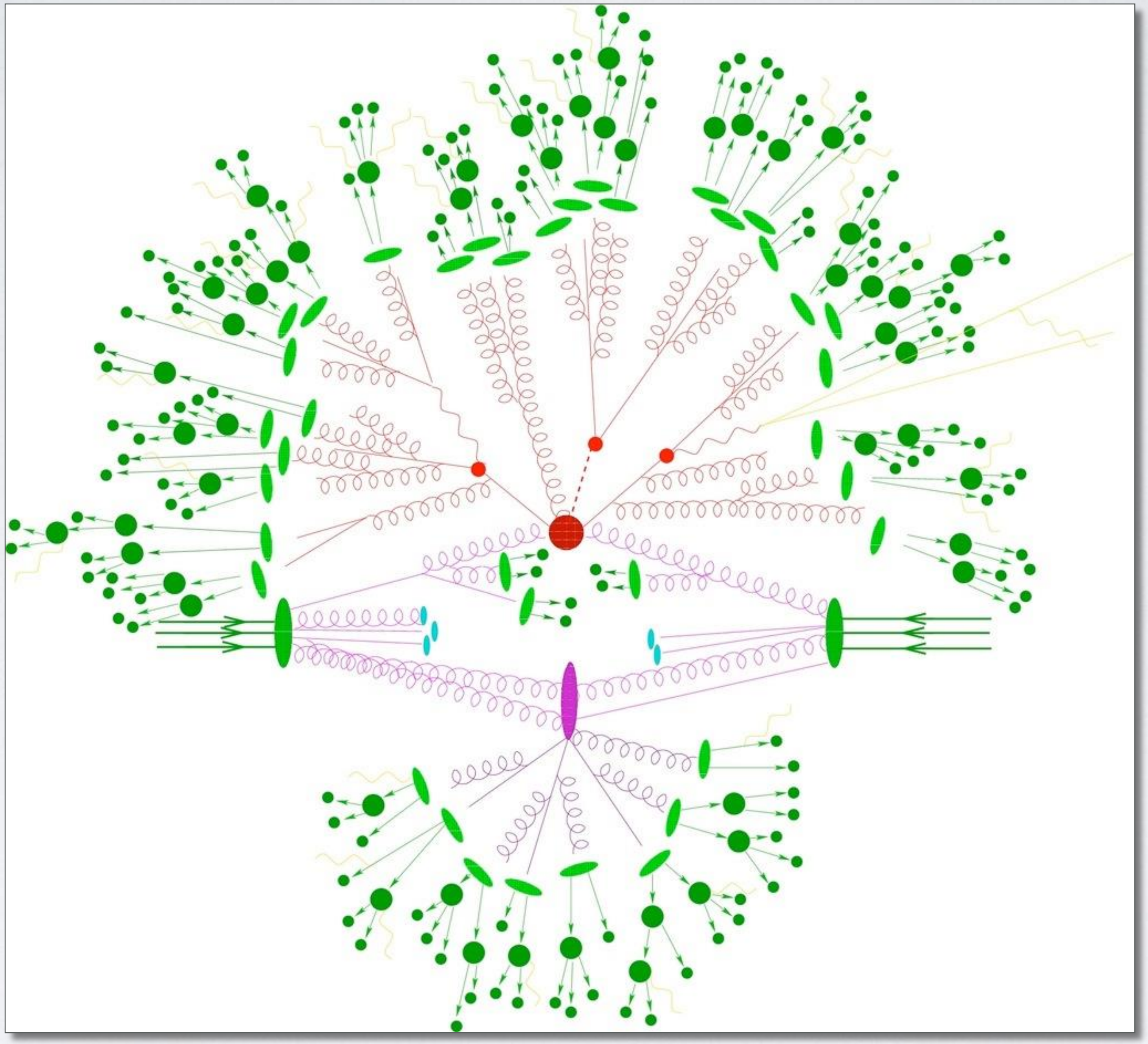
(See for instance Caron-Huot, Chicherin, Henn, Zhang, Zoia, 2020).



(See for instance Canay,Del Duca, 2021).

- Provides useful input for the perturbative calculation of multi-leg processes;
- Further constrain the soft anomalous dimension;
- Phenomenology …

PARTICLE SCATTERING NEAR THRESHOLD

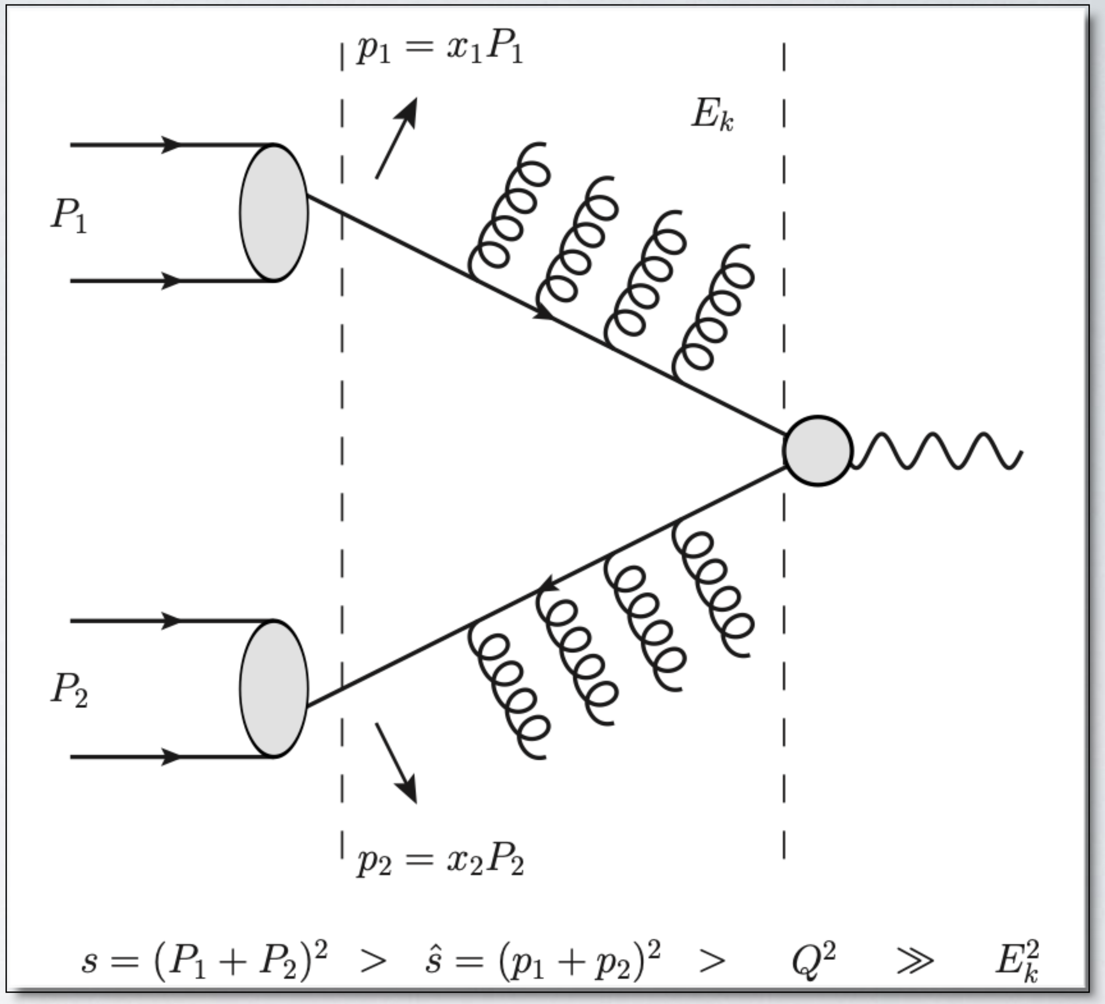


PARTICLE SCATTERING NEAR THRESHOLD

- Consider the DY invariant mass distribution:

$$\frac{d\sigma}{dQ^2} = \tau\,\tilde{\sigma}_0(Q^2)\int_\tau^1 \frac{dz}{z}\,\mathcal{L}_{ab}\left(\frac{\tau}{z}\right)\Delta_{ab}(z),$$

$$\mathcal{L}_{ab}(y) = \int_y^1 \frac{dx}{x}\, f_{a/A}(x)\, f_{b/B}\left(\frac{y}{x}\right).$$



- Near partonic threshold:

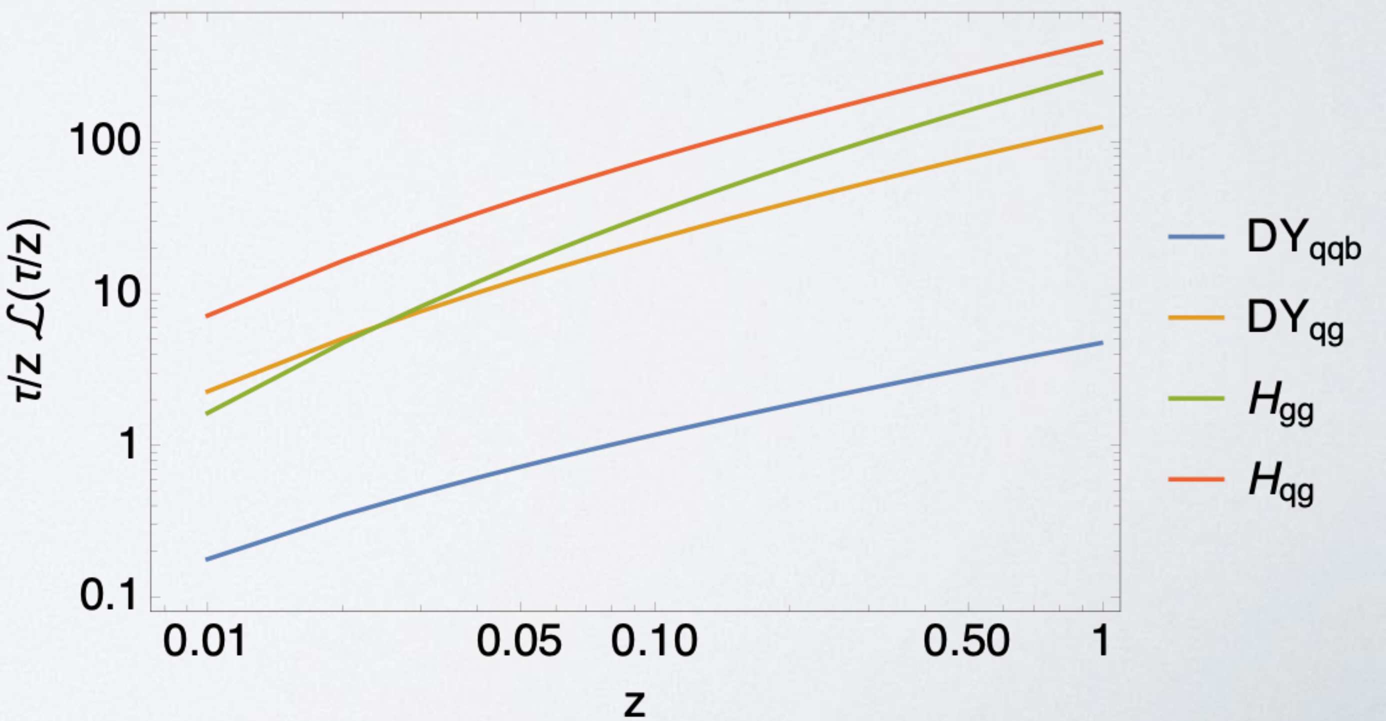
$$\tau = \frac{Q^2}{s}, \quad z = \frac{Q^2}{\hat{s}}, \quad (z \geq \tau), \quad z \to 1,$$

the partonic cross section has the singular expansion

$$\Delta_{ab}(z) = \sum_{n=0}^{\infty} \alpha_s^n \Bigg[c_n \delta(1-z) + \sum_{m=0}^{2n-1}\left(c_{nm}\left[\frac{\ln^m(1-z)}{1-z}\right]_+ + d_{nm}\ln^m(1-z)\right) + \ldots \Bigg].$$

LP terms

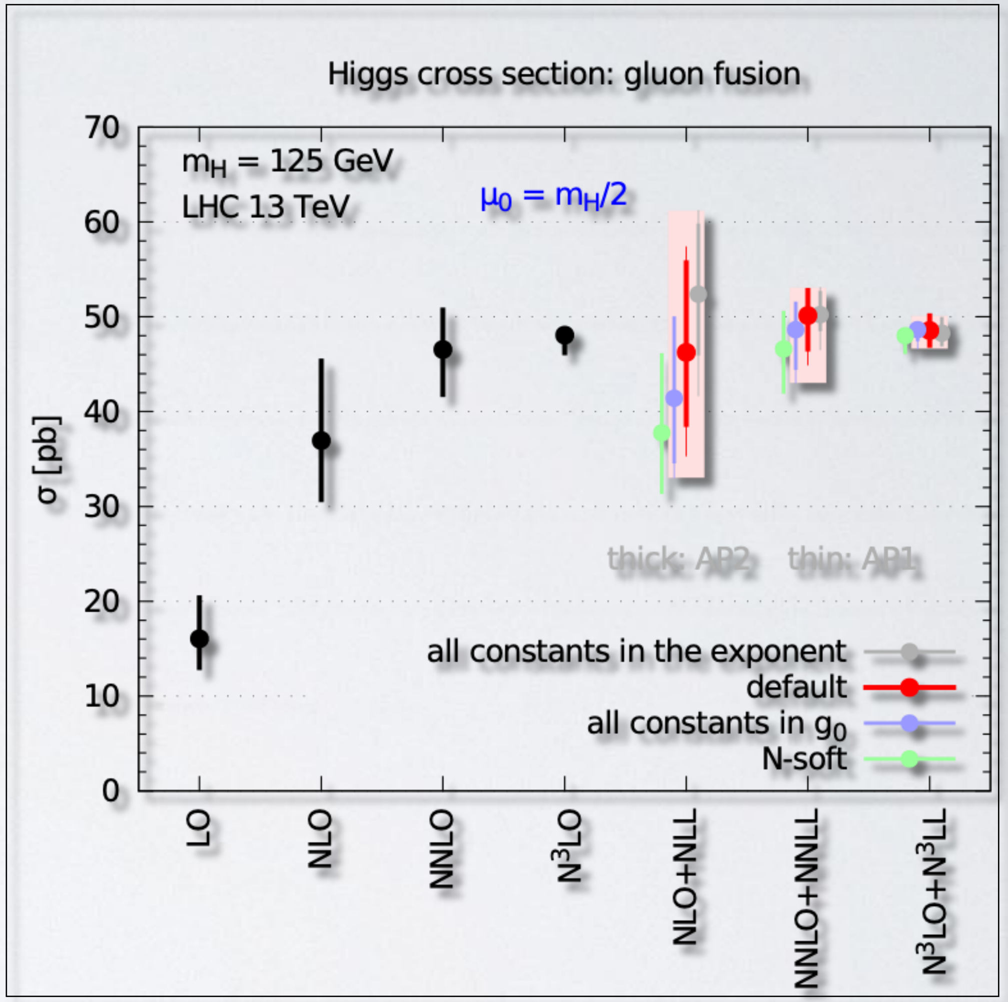
NLP terms



PARTICLE SCATTERING NEAR THRESHOLD: LP

$$\frac{d\sigma}{dz} = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{m=0}^{2n-1} c_{nm}^{(-1)} \left.\frac{\log^m(1-z)}{1-z}\right|_+ + \dots$$

- Large threshold logarithms spoil the reliability of the perturbative expansion and needs to be resummed
- Resummation of LP logarithms is well established: it relies on factorization and exponentiation properties of soft radiation.
- The resummation of threshold logarithms leads to a more reliable perturbative expansion.
- More relevant for the production of heavy final states ($HH,\ t\bar{t},\ t\bar{t}W,\ t\bar{t}H,\ \dots$)



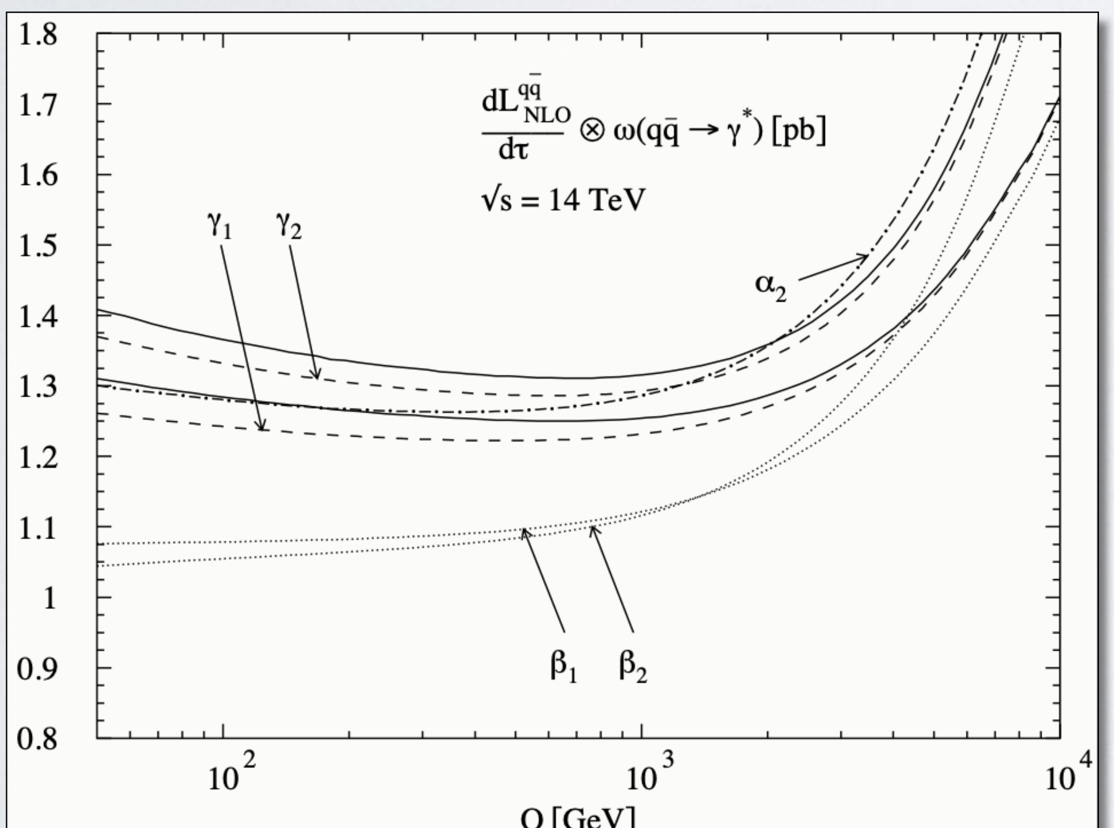
Bonvini, Marzani, Muselli, Rottoli 2016

PARTICLE SCATTERING NEAR THRESHOLD: NLP

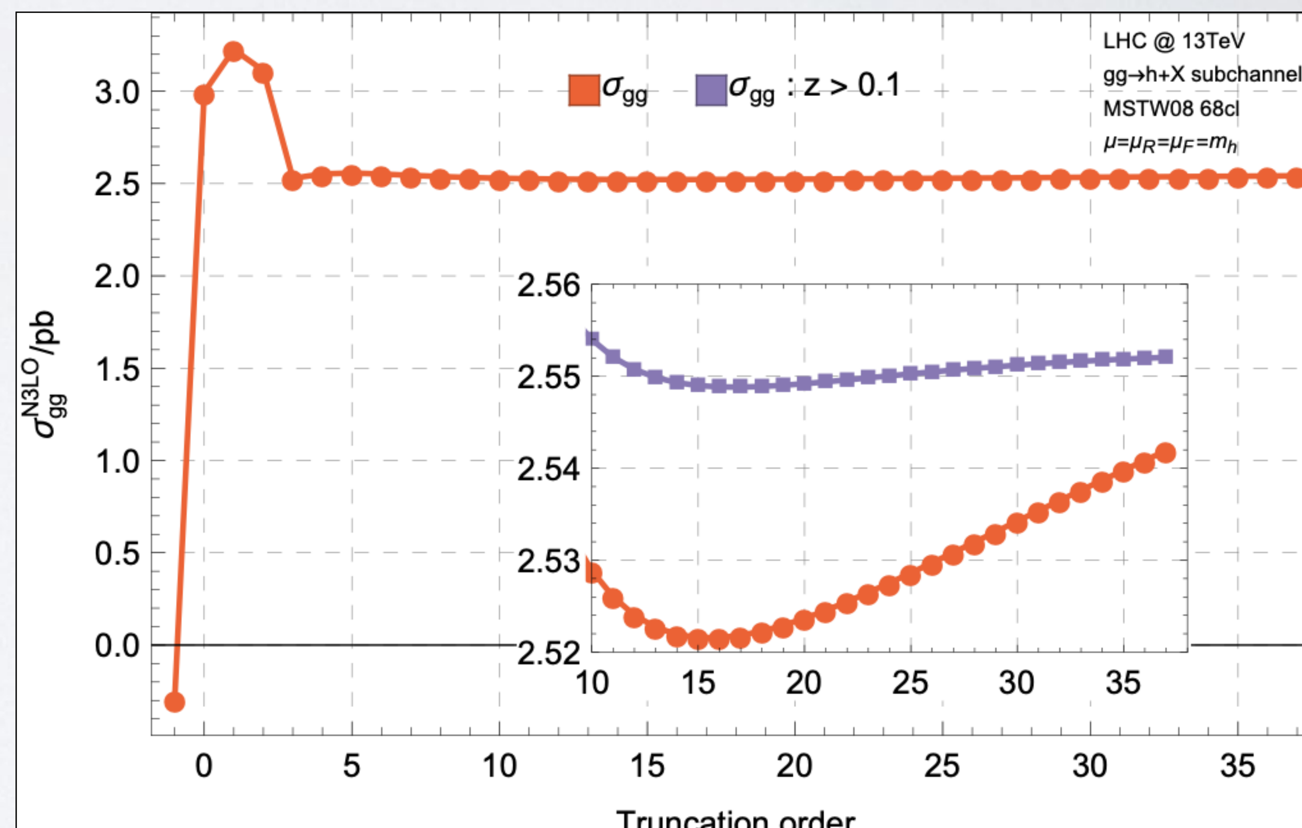
- What about NLP and higher power terms?

$$\Delta_{ab}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right].$$

- Can be relevant for precision physics!
- Interesting problem: probes all-order structures beyond the semi-classical approximation.



Kramer, Laenen, Spira, 1998



Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 2015

FACTORIZATION AND RESUMMATION AT NLP

$$\frac{d\sigma}{d\xi} \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n \delta(\xi) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(\xi)}{\xi}\right]_+ + d_{nm} \ln^m(\xi) \right) + \ldots \right].$$

- Understanding the factorization and resummation of large logarithms at next-to-leading power (NLP) has been subject of intense work in the past few years!

- Drell-Yan, Higgs and DIS near threshold

  Del Duca, 1990; Bonocore, Laenen, Magnea, LV, White, 2014, 2015, 2016;

  Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019;

  van Beekveld, Beenakker, Laenen, White, 2019; van Beekveld, Laenen, Sinninghe Damsté, LV, 2021;

  Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;

  Beneke, Broggio, Jaskiewicz, LV, 2019;

  Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019, 2020.

- Operators and Anomalous dimensions

  Larkoski, Neill, Stewart 2014;

  Moult, Stewart, Vita 2017; Feige, Kolodrubetz, Moult, Stewart 2017;

  Beneke, Garny, Szafron, Wang, 2017, 2018, 2019.

- Thrust

  Moult, Stewart, Vita, Zhu 2018, 2019.

- pT and Rapidity logarithms

  Ebert, Moult, Stewart, Tackmann, Vita, 2018,

  Moult, Vita Yan 2019;

  Cieri, Oleari, Rocco, 2019; Oleari, Rocco 2020.

- Mass effects

  Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang, Fleming, 2020;

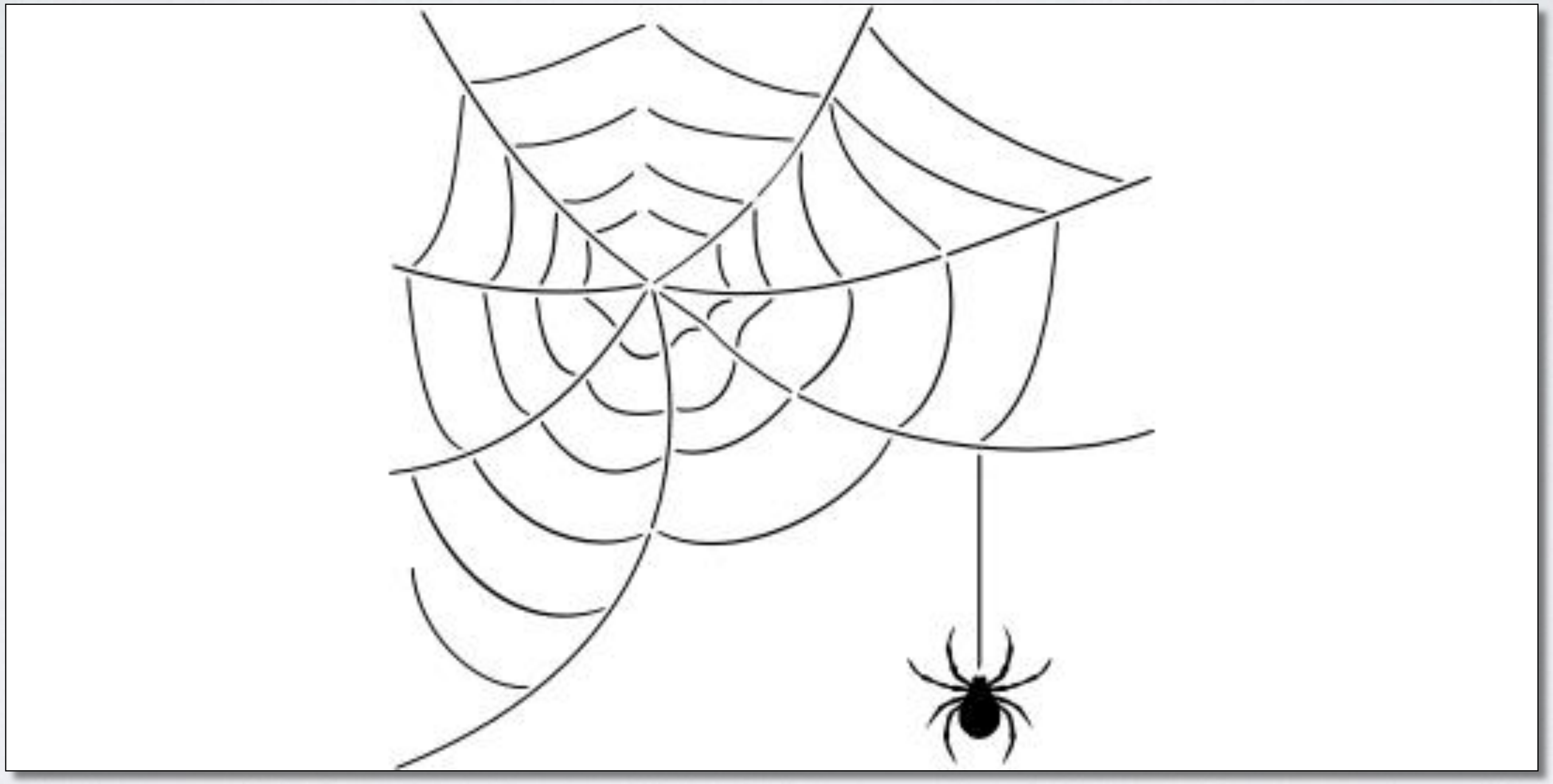
  Liu, Mecaj, Neubert, Wang, 2020;

  Anastasiou, Penin, 2020.

And many more!

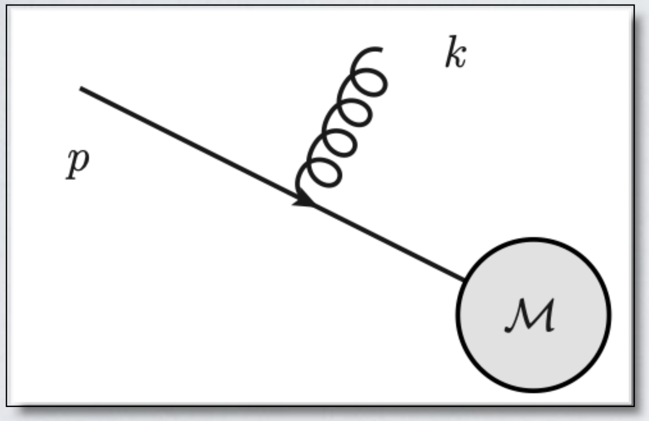
[O(50 publications) and counting]

SCATTERING NEAR THRESHOLD: LP VS NLP



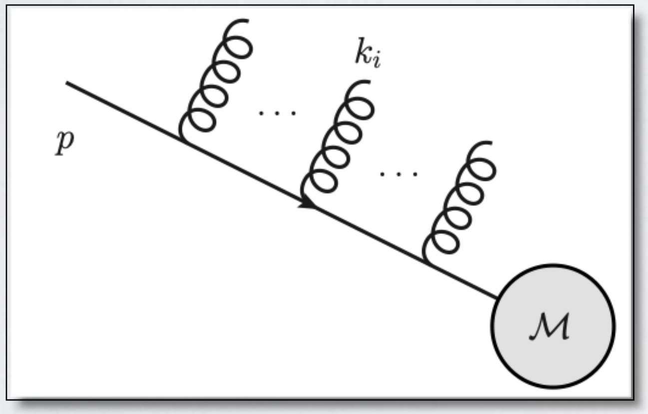
FACTORIZATION OF SOFT GLUONS AT LP

- Emission of soft gluons from an energetic parton (quark):



$$= \mathcal{M}\, \frac{\not{p} - \not{k}}{2p \cdot k}\, \gamma^\mu\, T^A\, u(p) \;\sim\; \mathcal{M}\, \frac{p^\mu}{p \cdot k}\, T^A\, u(p).$$

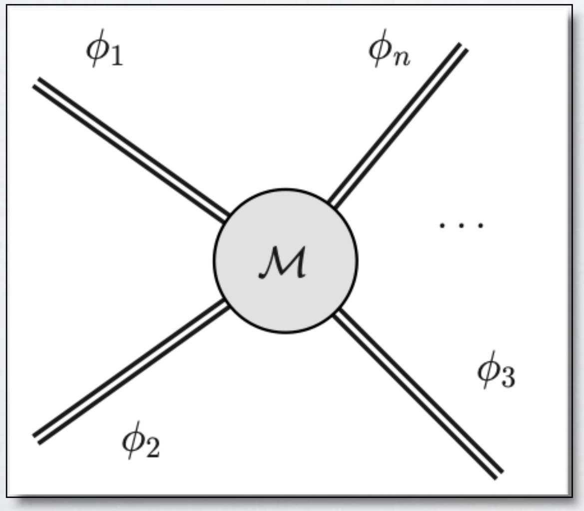
- Emission of multiple soft gluons factorises:



$$\sim \mathcal{M}\, \mathcal{S}\, u(p), \qquad \mathcal{S} = \langle 0 | \Phi_\beta(-\infty, 0) | 0 \rangle,$$

$$\Phi_\beta(\lambda_1, \lambda_2) = \mathcal{P} \exp\left\{ i\, g_s \int_{\lambda_1}^{\lambda_2} d\lambda\; \beta \cdot A(\lambda\beta) \right\}.$$

- In general



$$\sim \mathcal{M}\, \mathcal{S}\, u(p_1) \bar{v}(p_2) \ldots \bar{u}(p_n),$$

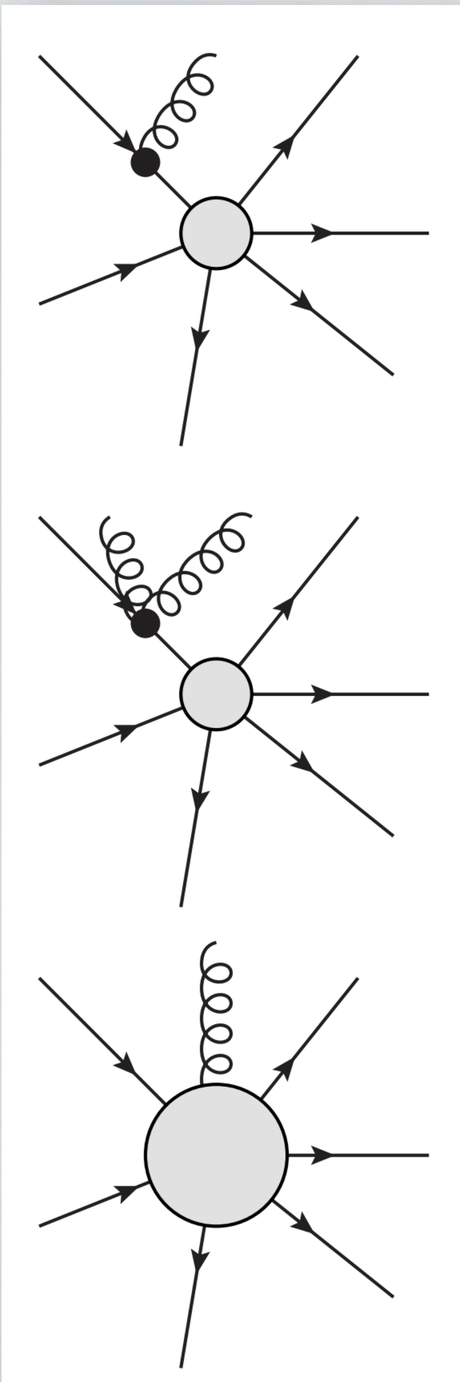
$$\mathcal{S} = \langle 0 | \Phi_1 \ldots \Phi_n | 0 \rangle \sim e^{\mathcal{W}_E}.$$

Collins, Soper,Sterman, 1989;

Gardi, Laenen, Stavenga, White, 2010;
Gardi, Smillie, White, 2013

FACTORIZATION OF SOFT GLUONS BEYOND LP

- One needs to take into account several effects:

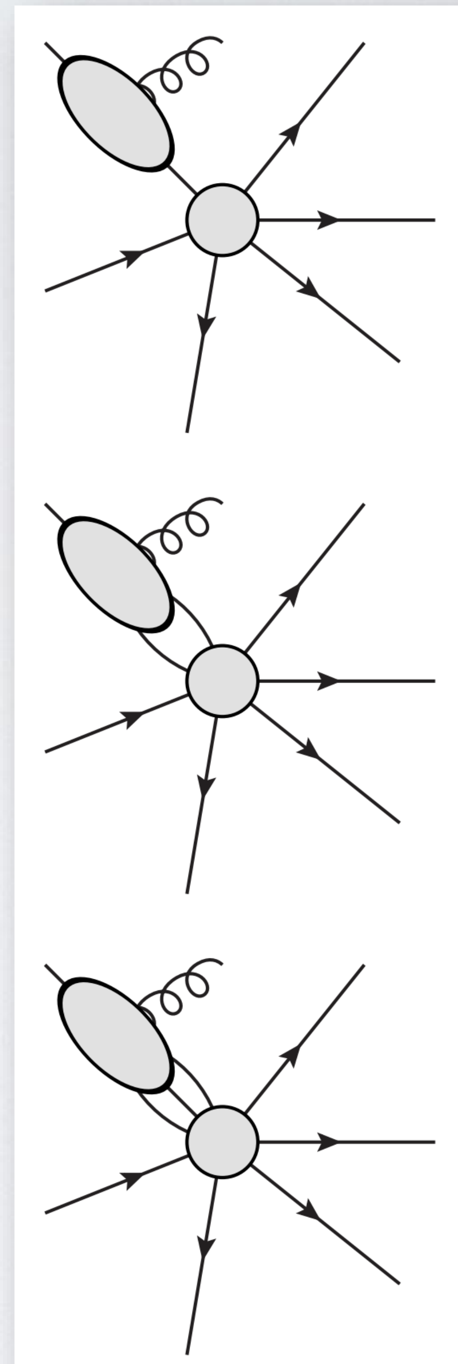


- Emission of soft gluons beyond the eikonal approximation, for instance sensitive to the spin of the emitting particle

Laenen, Magnea, Stavenga, White, 2009, 2010; Bonocore, Laenen, Magnea, LV, White, 2016.

- The soft emission resolve the hard interaction (LBK theorem)

Low 1958, Burnett,Kroll 1968



- Emission of soft gluons from a cluster of collinear particles: one finds several types of "radiative jets".

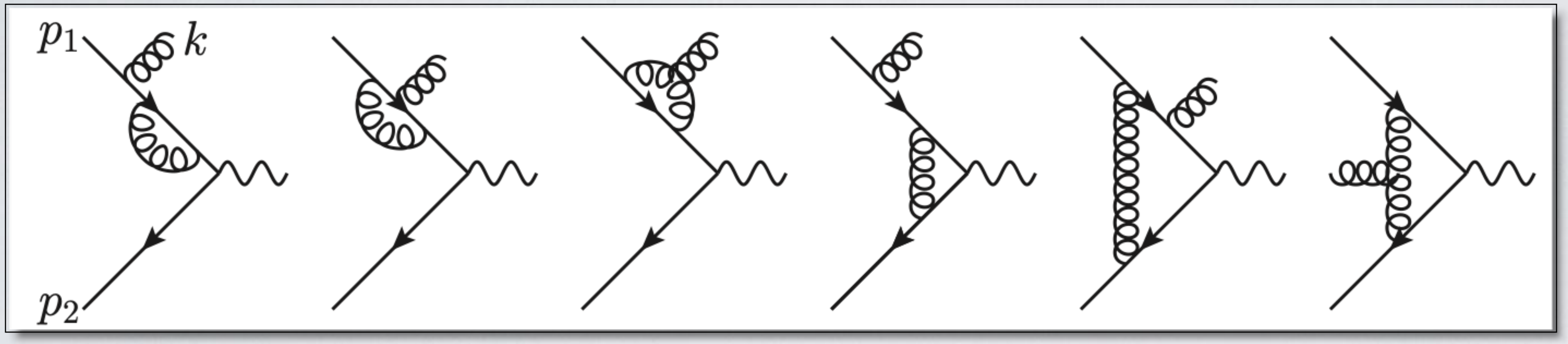
Del Duca 1990;

Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016;

Gervais 2017;

Laenen, Sinninghe-Damsté, LV, Waalewijn, Zoppi, 2020

FACTORIZATION OF SOFT GLUONS BEYOND LP



$$\hat{s} = (p_1 + p_2)^2, \quad t = (p_1 - k)^2, \quad u = (p_2 - k)^2.$$

- Virtual gluons gives non-analytical contributions ∝ to the scales of the problem: NLP

$$|\mathcal{M}|^2 \propto C_F^2 \Big\{ \overset{\text{NLP}}{\frac{\hat{s}(t+u)}{tu}} \overset{\text{hard}}{\left(\frac{\mu^2}{-\hat{s}}\right)^{\epsilon}} \left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon} + \dots\right) + \left[\overset{\text{NLP}}{\frac{s}{t}} \overset{\text{coll.}}{\left(\frac{\mu^2}{-t}\right)^{\epsilon}} + \overset{\text{NLP}}{\frac{\hat{s}}{u}} \overset{\text{anti-coll.}}{\left(\frac{\mu^2}{-u}\right)^{\epsilon}} \right] \left(-\frac{2}{\epsilon} + \dots\right) \Big\}$$

$$+ C_A C_F \overset{\text{NLP}}{\frac{\hat{s}(t+u)}{tu}} \overset{\text{soft}}{\left(\frac{\hat{s}\,\mu^2}{t\,u}\right)^{\epsilon}} \left(-\frac{1}{\epsilon^2} + \dots\right) + \left[\overset{\text{NLP}}{\frac{\hat{s}}{t}} \overset{\text{coll.}}{\left(\frac{\mu^2}{-t}\right)^{\epsilon}} + \overset{\text{NLP}}{\frac{\hat{s}}{u}} \overset{\text{anti-coll.}}{\left(\frac{\mu^2}{-u}\right)^{\epsilon}} \right] \left(-\frac{5}{2} + \dots\right) \Big\} + \dots$$

↓

Factorization?

↓

Bonocore, Laenen, Magnea, LV, White, 2014

$$S\left[\frac{\hat{s}\,\mu^2}{t\,u}, \epsilon\right] \times J\left[\frac{\mu^2}{-t}, \epsilon\right] \times \bar{J}\left[\frac{\mu^2}{-u}, \epsilon\right] \times H\left[\frac{\mu^2}{-\hat{s}}, \epsilon\right]$$

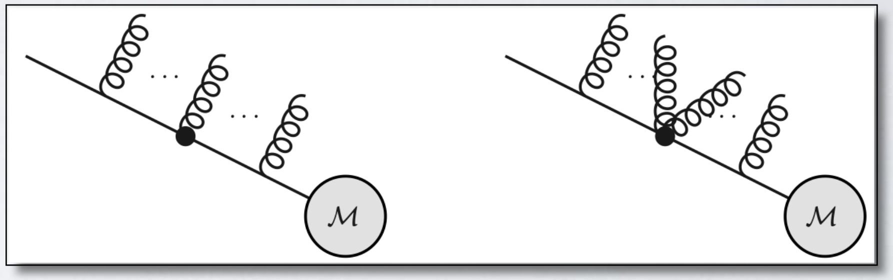
- Need an effective approach to take into account hard, collinear and soft modes.
- Two approaches: ~ Diagrammatic; ~ Soft Collinear Effective Field Theory.

DIAGRAMMATIC APPROACH

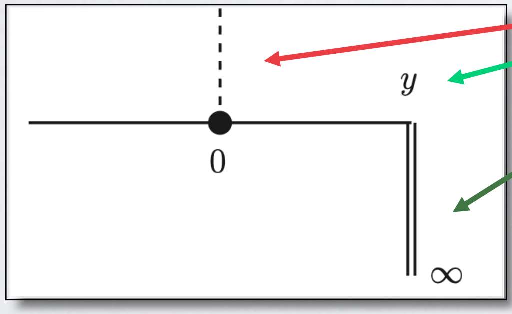
- Describe momentum regions in terms of universal functions in QCD:
- For instance, for Drell-Yan we have
  - "Derivative" of the non-radiative amplitude,
  - "Generalized" soft function,
  - "Radiative" jet function,

Low 1958, Burnett,Kroll 1968

$$\frac{k^\nu}{p_l\cdot k}\left[p_{l,\nu}\frac{\partial}{\partial p_l^\mu} - p_{l,\mu}\frac{\partial}{\partial p_l^\nu}\right]\mathcal{A}(p_j)$$



$$J_{\mu,a}\,(p,n,k)\,u(p) \,=\, \int d^dy\;\mathrm{e}^{-\mathrm{i}(p-k)\cdot y}\;\langle 0\,|\,\Phi_n(\infty,y)\,\psi(y)\,j_{\mu,a}(0)\,|\,p\rangle\;.$$



Del Duca 1990,

Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016

$$F_p(-\infty,0)=\mathcal{P}\exp\left[g\int\frac{d^dk}{(2\pi)^d}A_\mu(k)\left(-\frac{p^\mu}{p\cdot k}+\frac{k^\mu}{2p\cdot k}-k^2\frac{p^\mu}{2(p\cdot k)^2}\right.\right.$$
$$\left.\left.-\frac{\mathrm{i}k_\nu\Sigma^{\nu\mu}}{p\cdot k}\right)+\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dl}{(2\pi)^d}A_\mu(k)A_\nu(l)\left(\frac{\eta^{\mu\nu}}{2p\cdot(k+l)}+\ldots\right)\right].$$

Laenen, Magnea, Stavenga, White, 2009, 2010

Bonocore, Laenen, Magnea, LV, White,2016

- One has

$$\mathcal{A}_{\mu,a}(p_j,k)\,=\,\sum_{i=1}^{2}\left(\frac{1}{2}\,\widetilde{\mathcal{S}}_{\mu,a}(p_j,k)+g\,\mathbf{T}_{i,a}\,G^{\nu}_{i,\mu}\,\frac{\partial}{\partial p_i^\nu}+J_{\mu,a}\,(p_i,n_i,k)\right)\mathcal{A}(p_j)-\mathcal{A}^{\widetilde{\mathcal{J}}}_{\mu,a}(p_j,k)\,,$$

for $n_1 = p_2$, $n_2 = p_1$.

(Removes soft-collinear overlap in the radiative jet)

SOFT-COLLINEAR EFFECTIVE FIELD THEORY

- Effective Lagrangian and operators made of collinear and soft fields.

$$\mathcal{L}_{\text{SCET}} = \sum_i \mathcal{L}_{c_i} + \mathcal{L}_s,$$

Bauer, Fleming, Pirjol, Stewart, 2000,2001; Beneke, Chapovsky, Diehl, Feldmann, 2002; Hill, Neubert 2002.

$$\mathcal{O}_n = \int dt_1 \ldots dt_n \, \mathcal{C}(t_1, \ldots, t_n) \, \phi_1(t_1 n_{1+}) \ldots \phi_n(t_n n_{n+}).$$

- Constructed to reproduce a scattering process as obtained with the method of regions.
- The cross section factorizes into a hard scattering kernel, and matrix elements of soft and collinear fields.

$$\sigma \sim \mathcal{H} \otimes \mathcal{J}_1 \otimes \ldots \otimes \mathcal{J}_n \otimes \mathcal{S}.$$

Hard matching coefficient

Jet functions – matrix elements of collinear fields

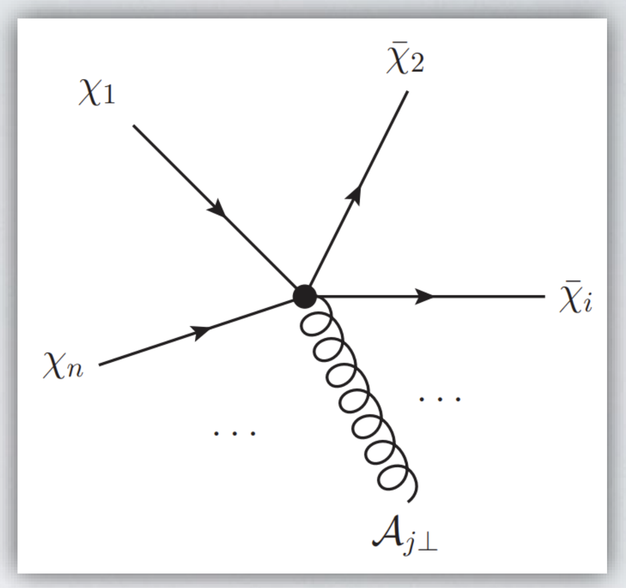
Soft function – matrix element of soft fields

- Renormalize UV divergences of EFT operators and obtain renormalization group equations.
- Each function depends on a single scale: solving the RGE resums large logarithms.

See e.g. Becher, Neubert 2006

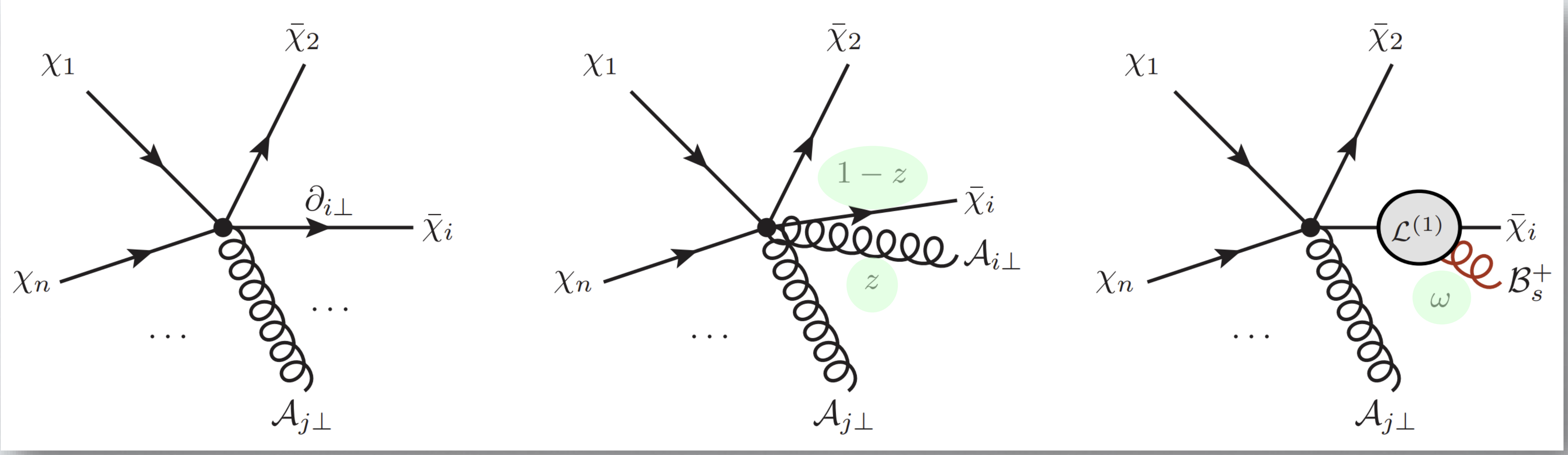
FACTORIZATION IN SCET: LP VS NLP

- Leading power (LP):
  - N-jet operators;
  - Soft-collinear decoupling.



- Next-to-leading power (NLP):
  - Kinematic suppression;
  - Multi-particle emission along the same collinear direction;
  - No soft-collinear decoupling.

Beneke, Garny, Szafron, Wang, 2017,2018



DRELL-YAN AT NLP IN SCET

- Schematic factorization formula at NLP: we expect

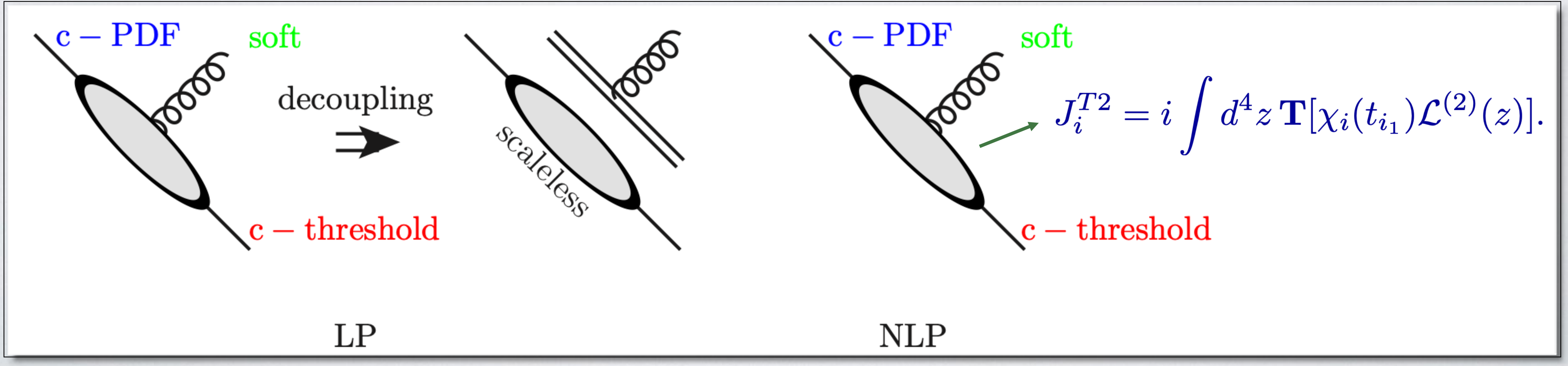
$$\frac{d\sigma_{\rm DY}}{dQ^2} = \frac{4\pi\alpha_{\rm EM}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}^{\rm NLP}_{q\bar{q}}(z),$$

where

$$\hat{\sigma}_{NLP} = \sum_{\rm terms} \left[C \otimes J \otimes \bar{J} \right] \otimes S,$$

  - C is the hard Wilson matching coefficient,
  - S is a *generalized* soft function,
  - J is a new collinear function.

- The collinear function is trivial at LP, because all threshold collinear modes are scaleless.
- The collinear scale is induced by the injection of a soft momentum.



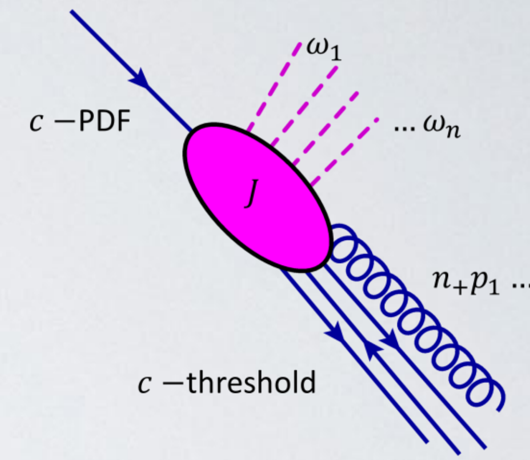
DRELL-YAN AT NLP IN SCET

- This is easily generalized at any subleading power: there can be many Lagrangian insertions, each with its own $\boldsymbol{\omega}_i$ conjugate to the large component of the collinear momentum.

$$i^m \int \{d^4 z_j\} \;\; \mathbf{T}\Big[\{\psi_c(t_k n_+)\} \times \Big\{\mathcal{L}^{(l)}(z_j)\Big\}\Big]$$

$$= 2\pi \sum_i \int du \int \{dz_{j-}\}\; \tilde{J}_i\left(\{t_k\}, u; \{z_{j-}\}\right)\;\; \chi_c^{\mathrm{PDF}}(un_+)\; \mathfrak{s}_i(\{z_{j-}\}),$$

Collinear matrix element



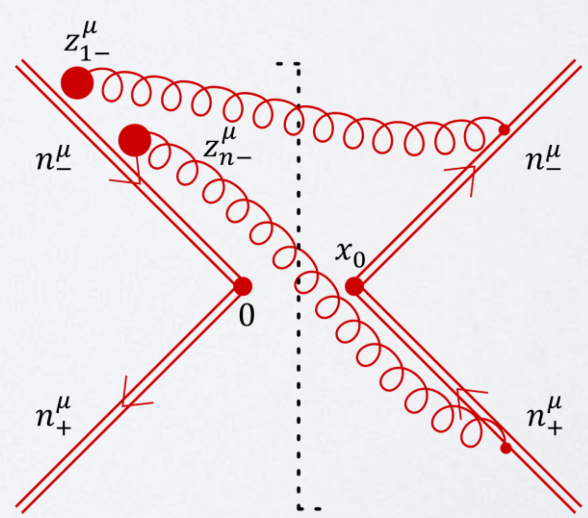
- After taking the matrix element squared, this gives a generalized soft functions:

$$S(\Omega,\omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2} \left(\prod_{j=1}^{n} \int \frac{d(z_{-j})}{4\pi} e^{-i\omega_j z_{-j}}\right)$$

$$\times\; \mathrm{Tr}\langle 0|\bar{\mathbf{T}}\big[(Y_+^\dagger Y_-)(x^0)\big]\mathbf{T}\big[(Y_-^\dagger Y_+)(x^0) \times \mathcal{L}_s^n(z_{1-}) \times \ldots \times \mathcal{L}_s^n(z_{n-})\big]|0\rangle.$$

which are equivalent to the generalized Wilson lines built in terms of NLP webs in the diagrammatic approach.

Beneke, Broggio, Jaskiewicz, LV, 2019



DRELL-YAN AT NLP IN SCET

- Up to NLP one has:

$$\Delta^{dyn}_{\rm NLP}(z) = -\frac{2}{(1-\epsilon)}\, Q\left[\left(\frac{\not{n}_-}{4}\right)\gamma_{\perp\rho}\left(\frac{\not{n}_+}{4}\right)\gamma^\rho_\perp\right]_{\beta\gamma}$$
$$\times \int d(n_+p)\, C^{A0,A0}(n_+p, x_b n_- p_B)\, C^{*A0A0}(x_a\, n_+p_A,\, x_b n_- p_B)$$
$$\times \sum_{i=1}^{5}\int \{d\omega_j\}\; J_i(n_+p, x_a\, n_+p_A; \{\omega_j\})\; S_i(\Omega; \{\omega_j\}) + \text{h.c.}\,.$$

Beneke, Broggio, Jaskiewicz, LV, 2019, 2020

- The convolution is regularized by dimensional regularization. For resummation, we treat the two object independently, and expand in ϵ prior to performing the convolution:

$$\int d\omega\; \underbrace{\left(n_+p\,\omega\right)^{-\epsilon}}_{\text{collinear piece}}\; \underbrace{\frac{1}{\omega^{1+\epsilon}}\,\frac{1}{(\Omega-\omega)^\epsilon}}_{\text{soft piece}}\,.$$

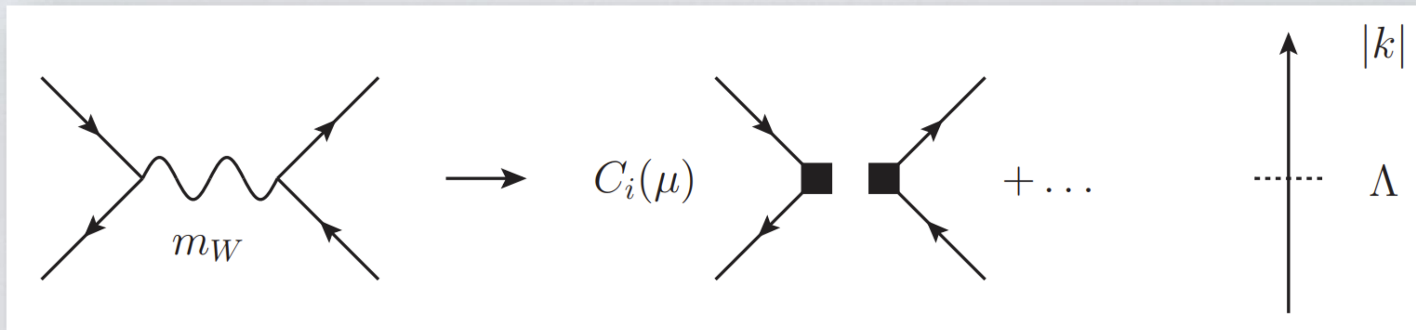
***Studies in:** Moult, Stewart, Vita, Zhu, 2019; Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020; Liu, Mecaj, Neubert, Wang, Fleming, 2019, 2020;*

but the convolution is endpoint divergent in d=4!

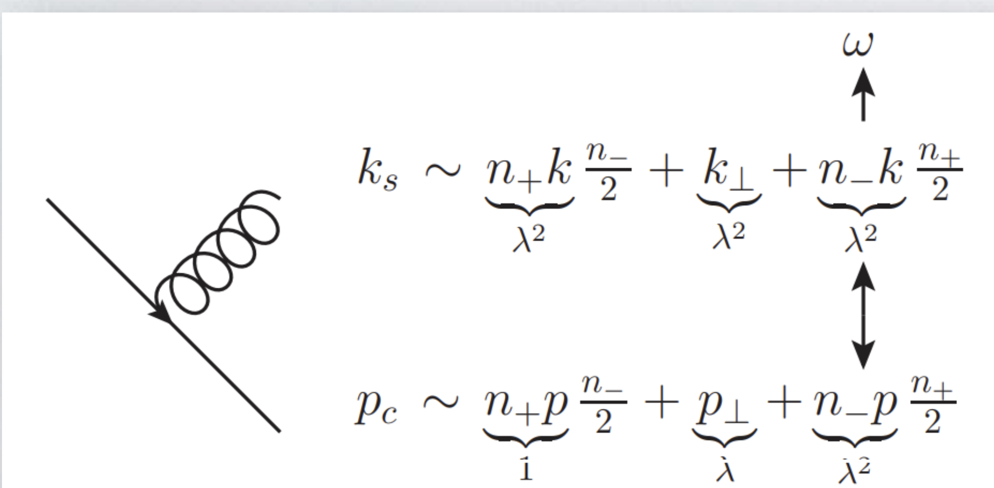
- This is actually an issue affecting in general any non-local effective field theory, such as SCET: resummation near threshold at NLP provides a perturbative, well-defined framework where to study and possibly solve the issue!

DRELL-YAN AT NLP IN SCET

- "Standard" EFTs:



- Non-local EFTs:



$$\xi_c(z) \to Y_+(z_-)\xi_c^{(0)}(z), \qquad Y_\pm = \mathbf{P}\exp\left[ig_s \int_{-\infty}^{0} ds\, n_\mp A_s(x + sn_\mp)\right],$$

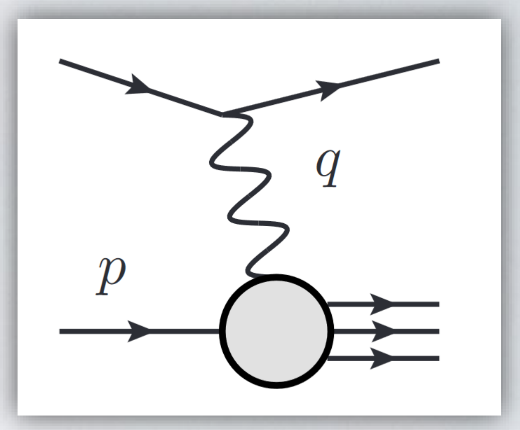
$$\text{At LP}: \qquad \bar{\xi}_c\, in_-D \frac{\not{n}_+}{2}\, \xi_c \to \bar{\xi}_c^{(0)}\, in_-D^{(0)} \frac{\not{n}_+}{2}\, \xi_c^{(0)}.$$

- At LP convolutions become trivial thanks to the "decoupling transformation": soft-collinear interactions decouple at LP.
- Beyond LP this does not occur, and convolutions are unavoidable. Endpoint divergences potentially spoil factorization.

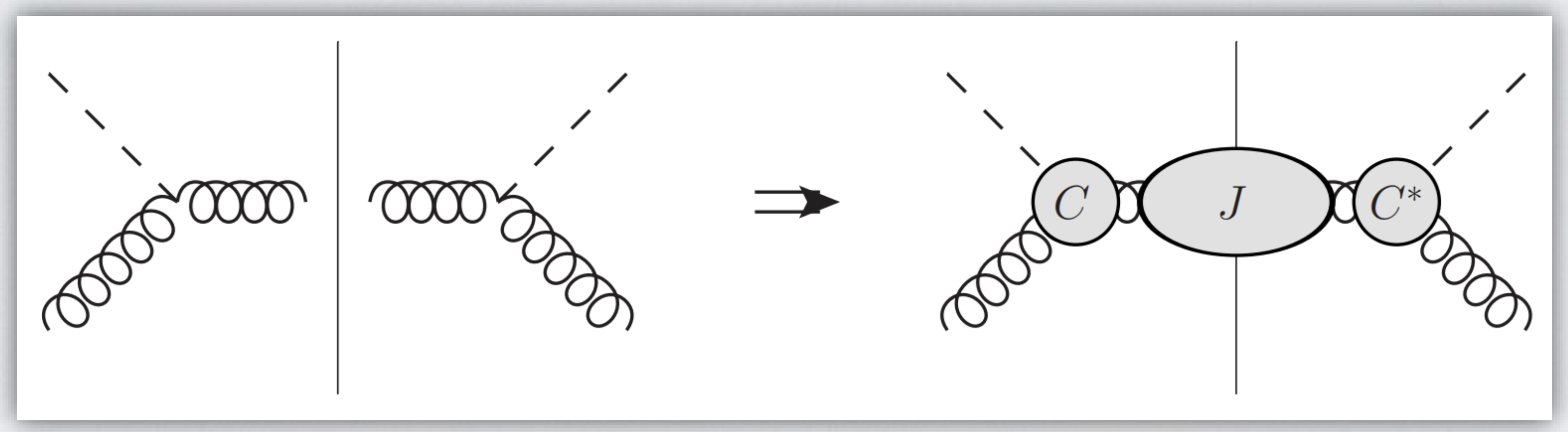
DEEP INELASTIC SCATTERING

- The problem of endpoint divergences is typical at NLP. Consider for instance Deep inelastic scattering (DIS) near threshold:

$$Q^2 \gg P_X^2 \sim Q^2(1-x), \qquad \text{with} \qquad x \equiv \frac{Q^2}{2p\cdot q} \to 1.$$



- Factorization and resummation well understood at LP:



Sterman 1987; Catani, Trentadue 1989; Korchemsky, Marchesini, 1993; Moch, Vermaseren, Vogt 2005; Becher, Neubert, Pecjak, 2007

$$W_\phi = \frac{1}{8\pi Q^2}\int d^4x\, e^{iq\cdot x}\,\langle N(P)|\big[G^A_{\mu\nu}G^{\mu\nu A}\big](x)\big[G^B_{\rho\sigma}G^{\rho\sigma B}\big](0)|N(P)\rangle$$

$$= |C(Q^2,\mu)|^2\int_x^1 \frac{d\xi}{\xi}\, J\Big(Q^2\frac{1-\xi}{\xi},\mu\Big)\,\frac{x}{\xi} f_g\Big(\frac{x}{\xi},\mu\Big).$$

Short-distance coefficient and jet function are single scale object – resummation obtained by solving the corresponding RGE.

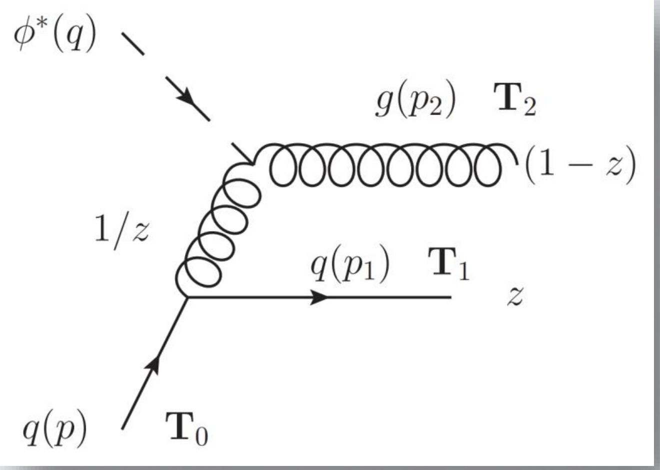
DIS: OFF-DIAGONAL CHANNEL

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

- The off-diagonal channel $q(p) + \phi^*(q) \to X(p_X)$ contributes to DIS at NLP. Consider the partonic structure function

$$W_{\phi,q}\big|_{q\phi^*\to qg} = \int_0^1 dz \left(\frac{\mu^2}{s_{qg} z\bar{z}}\right)^\epsilon \mathcal{P}_{qg}(s_{qg},z)\Big|_{s_{qg}=Q^2\frac{1-x}{x}}, \quad \mathcal{P}_{qg}(s_{qg},z) \equiv \frac{e^{\gamma_E\epsilon}Q^2}{16\pi^2\Gamma(1-\epsilon)} \frac{|\mathcal{M}_{q\phi^*\to qg}|^2}{|\mathcal{M}_0|^2}.$$

with momentum fraction $z \equiv \dfrac{n_-p_1}{n_-p_1 + n_-p_2}$, and $\bar{z} = 1 - z$.



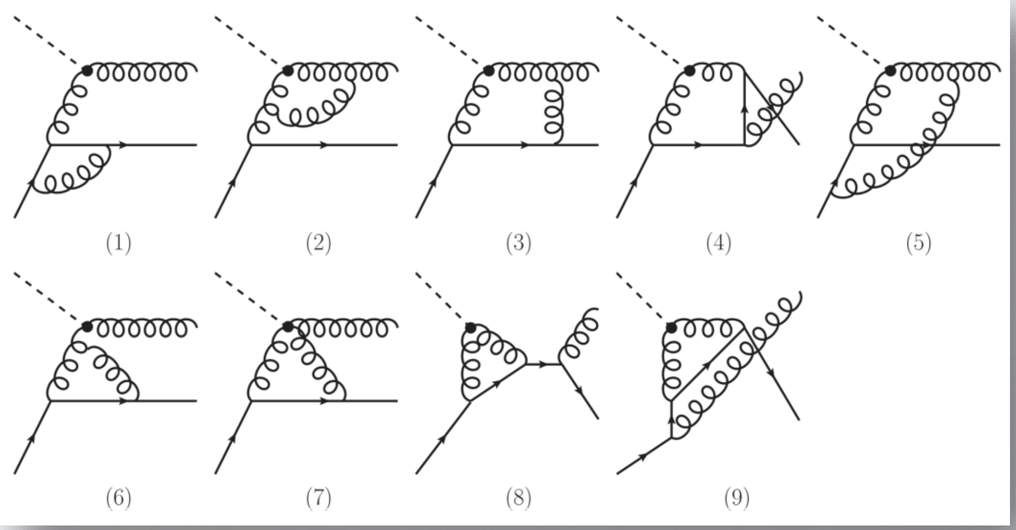
- At LO one has

$$\mathcal{P}_{qg}(s_{qg})|_{\text{tree}} = \frac{\alpha_s C_F}{2\pi}\frac{\bar{z}^2}{z}, \quad \Rightarrow \quad W_{\phi,q}\big|^{\text{NLP}}_{\mathcal{O}(\alpha_s),\,\text{leading pole}} = -\frac{1}{\epsilon}\frac{\alpha_s C_F}{2\pi}\left(\frac{\mu^2}{Q^2(1-x)}\right)^\epsilon.$$

The single pole originate from $z \to 0$, due to the $1/z$ of the momentum distribution function.

- At NLO:

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}}\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}$$
$$\cdot\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^\epsilon\right.$$
$$\left.+\,\mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^\epsilon - \left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \left(\frac{\mu^2}{zs_{qg}}\right)^\epsilon\right]\right) + \mathcal{O}(\epsilon^{-1}).$$



ON THE ENDPOINT DIVERGENCES

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\mathrm{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\mathrm{tree}}\,\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^\epsilon + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^\epsilon - \left(\frac{\mu^2}{zQ^2}\right)^\epsilon + \left(\frac{\mu^2}{zs_{qg}}\right)^\epsilon\right]\right) + \mathcal{O}(\epsilon^{-1}).$$

- The **T1.T2** term contains a single pole, but: promoted to leading pole after integration!
- Compare exact integration:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(1-z^{-\epsilon}\right) = -\frac{1}{2\epsilon^3},$$

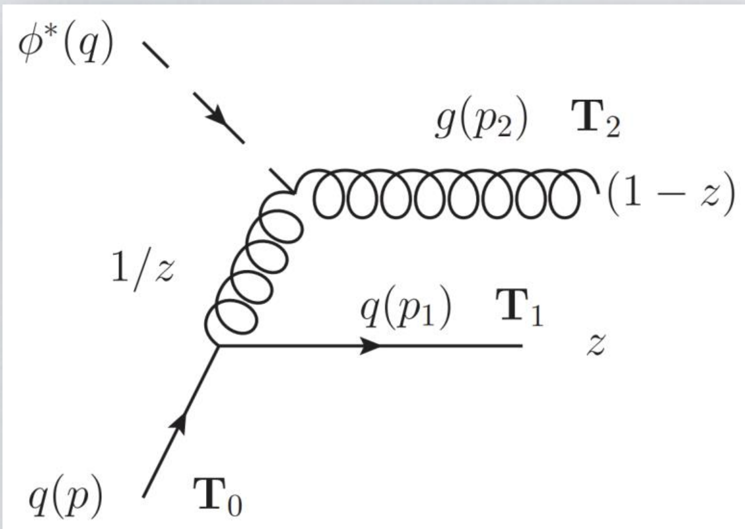
vs integration after expansion:

$$\frac{1}{\epsilon^2}\int_0^1 dz\,\frac{1}{z^{1+\epsilon}}\left(\epsilon\ln z - \frac{\epsilon^2}{2!}\ln^2 z + \frac{\epsilon^2}{3!}\ln^3 z + \cdots\right) = -\frac{1}{\epsilon^3} + \frac{1}{\epsilon^3} - \frac{1}{\epsilon^3} + \cdots\,.$$

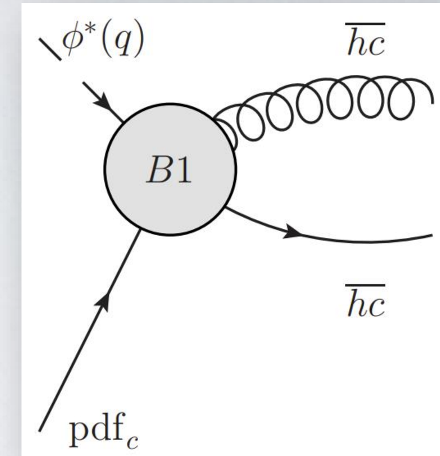
- Expansion in ε not possible before integration!
- The pole associated to **T1.T2** does not originate from the standard cups anomalous dimension.

BREACKDOWN OF FACTORIZATION NEAR THE ENDPOINT

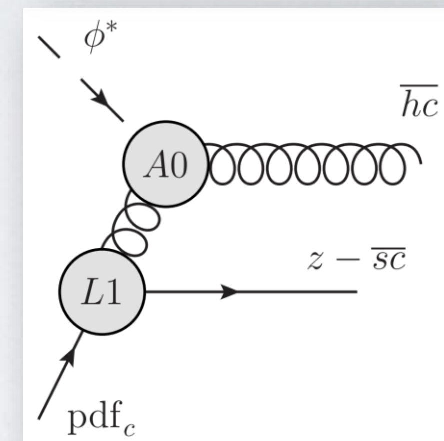
- What happens for $z \to 0$?



For z ~ 1 intermediate propagator is hard



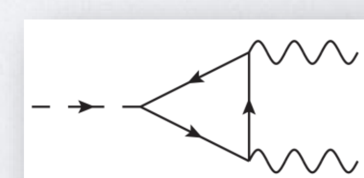
For z « 1 intermediate propagator cannot be integrated out



- Dynamic scale: zQ^2.
- In the endpoint region new counting parameter, $\lambda^2 \ll z \ll 1$.
- New modes contribute: need “z-SCET”.
- z-modes are non-physical! Not related to external scales of the problem.
- Need re-factorization:

$$\underbrace{C^{B1}(Q,z)}_{\text{multi-scale function}}\, J^{B1}(z) \xrightarrow{z\to 0} C^{A0}(Q^2) \int d^4x\, \mathbf{T}\Big[J^{A0}, \mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = \underbrace{C^{A0}(Q^2) D^{B1}(zQ^2,\mu^2)}_{\text{single-scale functions}}\, J^{B1}_{z-\overline{sc}}.$$

- Similar re-factorization proven in Liu, Mecaj, Neubert, Wang 2020.



DIS FACTORIZATION

- Re-factorization is nontrivial: needs to be embedded in a complete EFT description of DIS:
- Physical modes:

"z-SCET" is here (→ Hard)

Perturbative modes

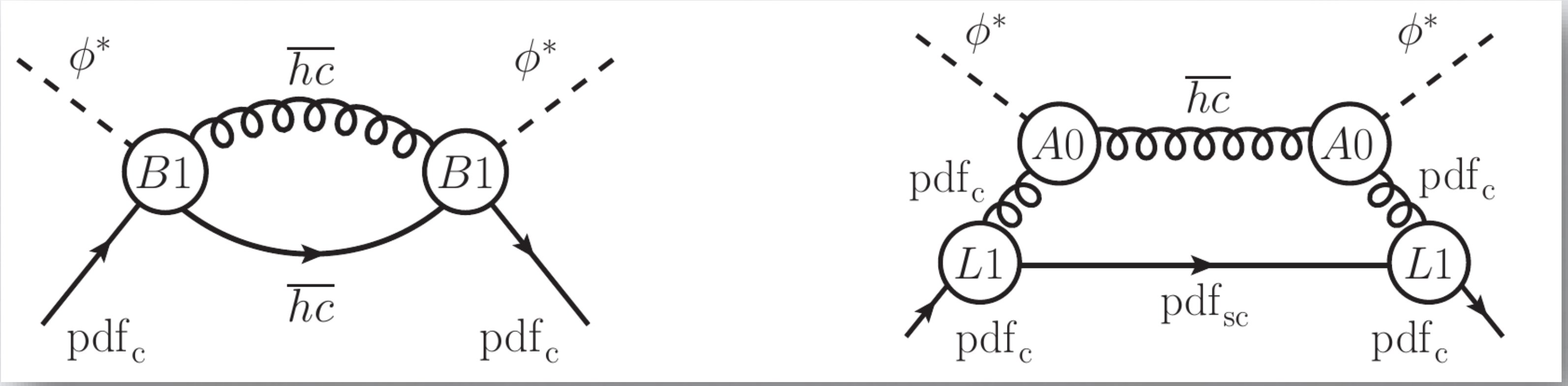
Hard:	$p^2 = Q^2,$	
Hard-collinear:	$p^2 = Q^2\lambda^2 = Q^2/N,$	

Non-perturbative modes

Collinear:	$p^2 = \Lambda^2,$	
Soft-collinear:	$p^2 = \Lambda^2\lambda^2 = \Lambda^2/N.$	

$$g(N) \equiv \int_0^1 dx\, x^{N-1}\, g(x),$$

$$x \to 1 \;\Leftrightarrow\; N \to \infty.$$



Time-ordered product contribution

B-type current contribution

- Both terms contain endpoint divergences in the convolution integral.
- We could reshuffle factorization theorem;
  → however, use d-dimensional consistency conditions to start with. → ***Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020***

D-DIMENSIONAL CONSISTENCY CONDITIONS

- Hadronic structure function is finite:

$$W = \sum_i W_{\phi,i}\, f_i = \sum_i \tilde{C}_{\phi,k} \tilde{f}_k \,, \qquad \text{with} \qquad \tilde{f}_k = Z_{ki} f_i, \quad W_{\phi,i} = \tilde{C}_{\phi,k} Z_{ki}.$$

- Focus on the bare functions: at NLP one has:

$$\sum_i (W_{\phi,i} f_i)^{NLP} = W_{\phi,q}^{NLP} f_q^{LP} + W_{\phi,\bar{q}}^{NLP} f_{\bar{q}}^{LP} + W_{\phi,g}^{NLP} f_g^{LP} + W_{\phi,g}^{LP} f_g^{NLP} \,.$$

- In d-dimensions: the general expansion of the cross section reads

$$\sum_i (W_{\phi,i} f_i)^{NLP} = f_q(\Lambda) \times \frac{1}{N} \sum_{n=1} \left(\frac{\alpha_s}{4\pi}\right)^n \frac{1}{\epsilon^{2n-1}} \sum_{k=0}^{n} \sum_{j=0}^{n} c_{kj}^{(n)}(\epsilon) \left(\frac{\mu^{2n} N^j}{Q^{2k} \Lambda^{2(n-k)}}\right)^{\epsilon} + \, f_{\bar{q}}(\Lambda), \; f_g(\Lambda) \text{ terms} \,.$$

- In this equation:

Each hard loop gives $\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}$, each hard-collinear loop gives $\left(\frac{\mu^2}{Q^2} N\right)^{\epsilon}$,

Each collinear loop gives $\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}$, each soft-collinear loop gives $\left(\frac{\mu^2}{\Lambda^2} N\right)^{\epsilon}$.

- Invoking cancellation of poles gives a series of constraints on the coefficients $c_{kj}^{(n)}$.

D-DIMENSIONAL CONSISTENCY CONDITIONS

- One finds that there are only n independent coefficients, one per loop in a given region!
- Consider $c_{n1}^{(n)}$: this is the n-loop hard region. Assume exponentiation of 1-loop result:

$$\mathcal{P}_{qg}(s_{qg},z)|_{1-\text{loop}} = \mathcal{P}_{qg}(s_{qg},z)|_{\text{tree}} \frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(\mathbf{T}_1\cdot\mathbf{T}_0\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \mathbf{T}_2\cdot\mathbf{T}_0\left(\frac{\mu^2}{\bar{z}Q^2}\right)^{\epsilon} + \mathbf{T}_1\cdot\mathbf{T}_2\left[\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} - \left(\frac{\mu^2}{zQ^2}\right)^{\epsilon} + \left(\frac{\mu^2}{zs_{qg}}\right)^{\epsilon}\right]\right) + \mathcal{O}(\epsilon^{-1}).$$

Similar conjecture "soft quark Sudakov" in Moult, Stewart, Vita, Zhu, 2019.

- Restricting to the hard region and substituting color operators one has

$$\mathcal{P}_{qg,\text{hard}}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A - C_F)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right].$$

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

- With $f_i(\mu) = U_{ij}(\mu) f_j(\Lambda)$ one has

$$\sum_i (W_{\phi,i} f_i)^{NLP}\Big|_{\propto f_q(\Lambda)} = \left(W_{\phi,q}^{NLP} U_{qq}^{LP} + W_{\phi,g}^{LP} U_{gq}^{NLP}\right) f_q(\Lambda)\,.$$

(Reproduces earlier conjecture by Vogt, 2010)

- Inserting the result above in the end one has

$$W_{\phi,q}^{NLP,LP} = -\frac{1}{2N}\frac{C_F}{C_F - C_A}\frac{\epsilon N^{\epsilon}}{N^{\epsilon}-1}\left(\exp\left[\frac{\alpha_s C_F}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(N^{\epsilon}-1)\right] - \exp\left[\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{Q^2}\right)^{\epsilon}(N^{\epsilon}-1)\right]\right),$$

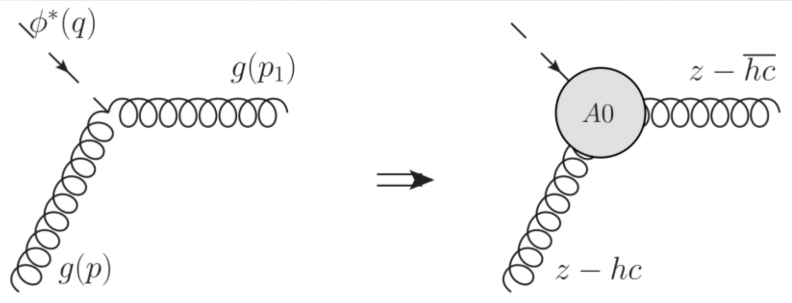
$$U_{gq}^{NLP,LP} = -\frac{1}{2N}\frac{C_F}{C_F - C_A}\frac{\epsilon N^{\epsilon}}{N^{\epsilon}-1}\left(\exp\left[-\frac{\alpha_s C_F}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}(N^{\epsilon}-1)\right] - \exp\left[-\frac{\alpha_s C_A}{\pi}\frac{1}{\epsilon^2}\left(\frac{\mu^2}{\Lambda^2}\right)^{\epsilon}(N^{\epsilon}-1)\right]\right).$$

RESUMMATION FROM RE-FACTORIZATION: A GLIMPSE

- Is it possible to achieve this in SCET? Another look at re-factorization:

$$C^{B1}(Q,z)J^{B1}(z) \stackrel{z\to 0}{\longrightarrow} C^{A0}(Q^2)\int d^4x\,\mathbf{T}\Big[J^{A0},\mathcal{L}_{\xi q_{z-\overline{sc}}}(x)\Big] = C^{A0}(Q^2)D^{B1}(zQ^2,\mu^2)J^{B1}_{z-\overline{sc}}.$$

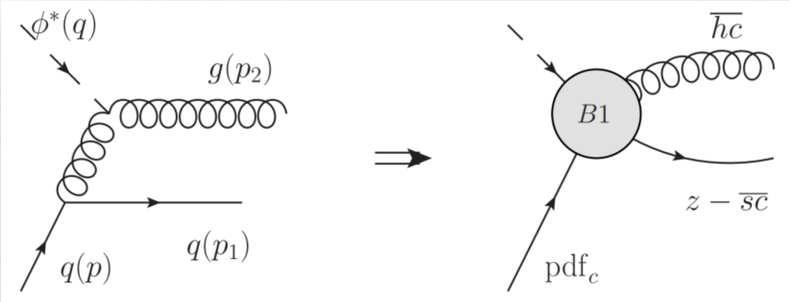
- Integrate out hard modes (solve RGEs in d-dimensions)



$$\frac{d}{d\ln\mu}C^{A0}(Q^2,\mu^2) = \frac{\alpha_s C_A}{\pi}\ln\frac{Q^2}{\mu^2}\,C^{A0}(Q^2,\mu^2)\,.$$

$$\Rightarrow \qquad \left[C^{A0}\left(Q^2,\mu^2\right)\right]_{\rm bare} = C^{A0}\left(Q^2,Q^2\right)\exp\left[-\frac{\alpha_s C_A}{2\pi}\frac{1}{\epsilon^2}\left(\frac{Q^2}{\mu^2}\right)^{-\epsilon}\right].$$

- Integrate out z-hardcollinear modes



$$\frac{d}{d\ln\mu}D^{B1}\left(zQ^2,\mu^2\right) = \frac{\alpha_s}{\pi}\left(C_F-C_A\right)\ln\frac{zQ^2}{\mu^2}\,D^{B1}\left(zQ^2,\mu^2\right).$$

$$\Rightarrow \qquad \left[D^{B1}\left(zQ^2,\mu^2\right)\right]_{\rm bare} = D^{B1}\left(zQ^2,zQ^2\right)\exp\left[-\frac{\alpha_s}{2\pi}\left(C_F-C_A\right)\frac{1}{\epsilon^2}\left(\frac{zQ^2}{\mu^2}\right)^{-\epsilon}\right].$$

- This reproduces

$$\mathcal{P}_{qg,\rm hard}(s_{qg},z) = \frac{\alpha_s C_F}{2\pi}\frac{1}{z}\exp\left[\frac{\alpha_s}{\pi}\frac{1}{\epsilon^2}\left(-C_A\left(\frac{\mu^2}{Q^2}\right)^{\epsilon} + (C_A-C_F)\left(\frac{\mu^2}{zQ^2}\right)^{\epsilon}\right)\right].$$

Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

OFF-DIAGONAL DIS: THE DIAGRAMMATIC WAY

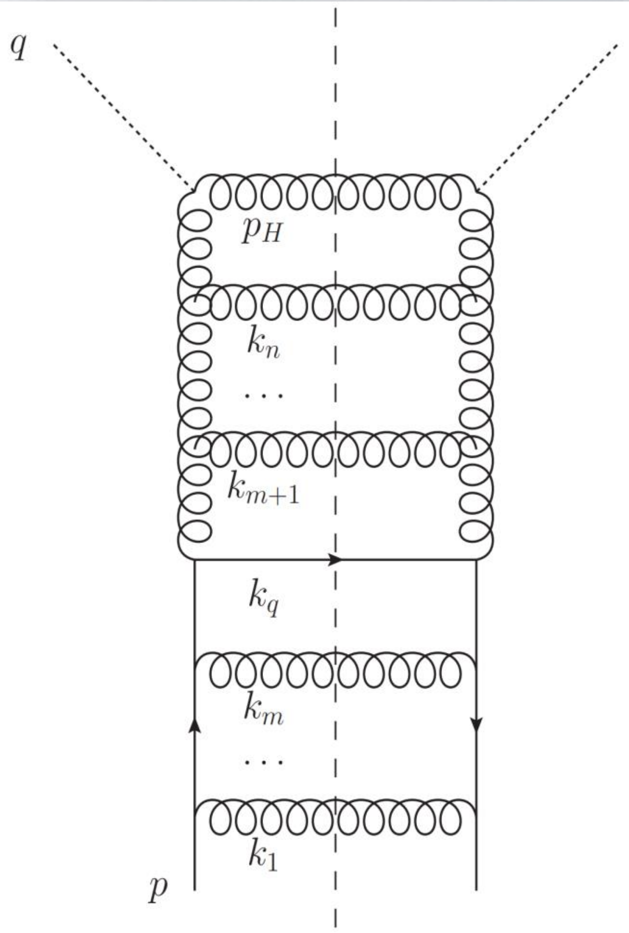
- The tower of coefficient in the soft real emissions is particularly suitable to be determined with diagrammatic methods. It can be determined based on the following considerations:

- In a physical polarization gauge in which

Gribov, Lipatov, 1972; Dokshitzer, Diakonov, Troian, 1980; Dokshitzer, Khoze, Mueller, Troian, 1991.

$$\sum_{\text{pols.}} \epsilon^\dagger_\mu(k)\epsilon_\nu(k) = -\eta_{\mu\nu} + \frac{k_\mu c_\nu + k_\nu c_\mu}{c \cdot k}\,, \qquad c = q + xp.$$

only ladder diagrams contribute to the LLs.



- The power suppression is given by the soft quark polarization sum; gluon emissions are eikonal (LP).
- Phase space can be also approximated to LP, and factorizes in Laplace space.
- The full result is found requiring that virtual corrections modify the real emission contributions at each order, removing singularities which are simultaneously soft and collinear.
- In the end one recover the previous result

$$W_{\phi,q}\Big|_{\text{LL}} = -\frac{2a_s C_F}{\epsilon}\frac{N^\epsilon}{N}\frac{1}{C_F - C_A}\left(\frac{4a_s(N^\epsilon-1)}{\epsilon^2}\right)^{-1}\left\{\exp\left[\frac{4a_s C_F(N^\epsilon-1)}{\epsilon^2}\right] - \exp\left[\frac{4a_s C_A(N^\epsilon-1)}{\epsilon^2}\right]\right\},$$

van Beekveld, LV, White 2021

OFF-DIAGONAL DIS: THE DIAGRAMMATIC WAY

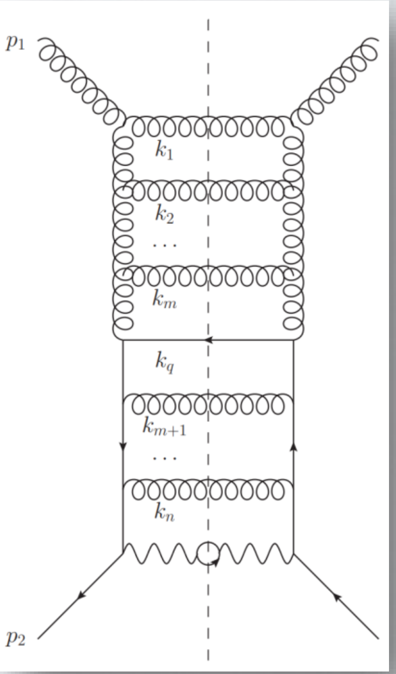
- The same procedures can be easily adapted to the subleading qg channel in Drell-Yan (and Higgs production).
- Consistency conditions can be studied to determine the smallest set of parameters necessary to determine the whole partonic cross section;
- The set of parameters can be determined
  - by assuming exponentiation of a given region, justified within a refactorization approach.
  - by direct calculation of the ladder diagrams contributing to the real emission.
- Either way, one in the end reproduces an earlier conjecture in Lo Presti, Almasy, Vogt 2014:

Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang, 2020 (unpublished)

van Beekveld, LV, White 2021

$$W_{\mathrm{DY},g\bar{q}}\Big|_{\mathrm{LL}} = -\frac{T_R}{2(C_F - C_A)}\frac{1}{N}\frac{\epsilon(N^{\epsilon-1})}{N^\epsilon - 1}\exp\left[\frac{4a_sC_F(N^\epsilon-1)}{\epsilon^2}\right] \times\left\{\exp\left[\frac{4a_sC_FN^\epsilon(N^\epsilon-1)}{\epsilon^2}\right] - \exp\left[\frac{4a_sC_AN^\epsilon(N^\epsilon-1)}{\epsilon^2}\right]\right\},$$

$$\widetilde{C}_{\mathrm{DY},g\bar{q}}\Big|_{\mathrm{LL}} = \frac{T_R}{C_A - C_F}\frac{1}{2N\ln N}\left[e^{8C_Fa_s\ln^2 N}\mathcal{B}_0[4a_s(C_A-C_F)\ln^2 N] - e^{(2C_F+6C_A)a_s\ln^2 N}\right].$$

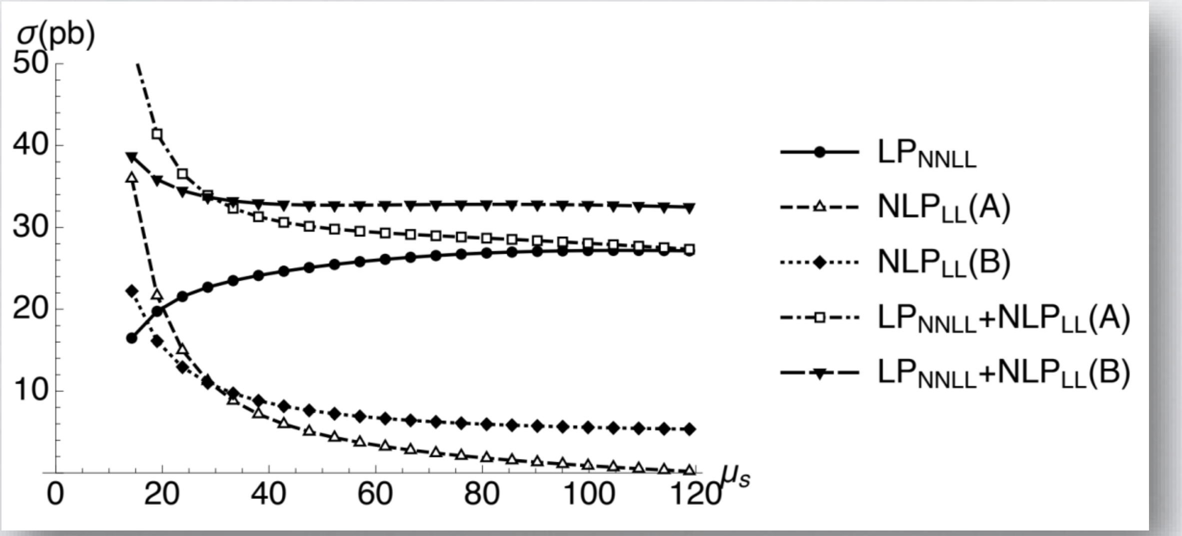


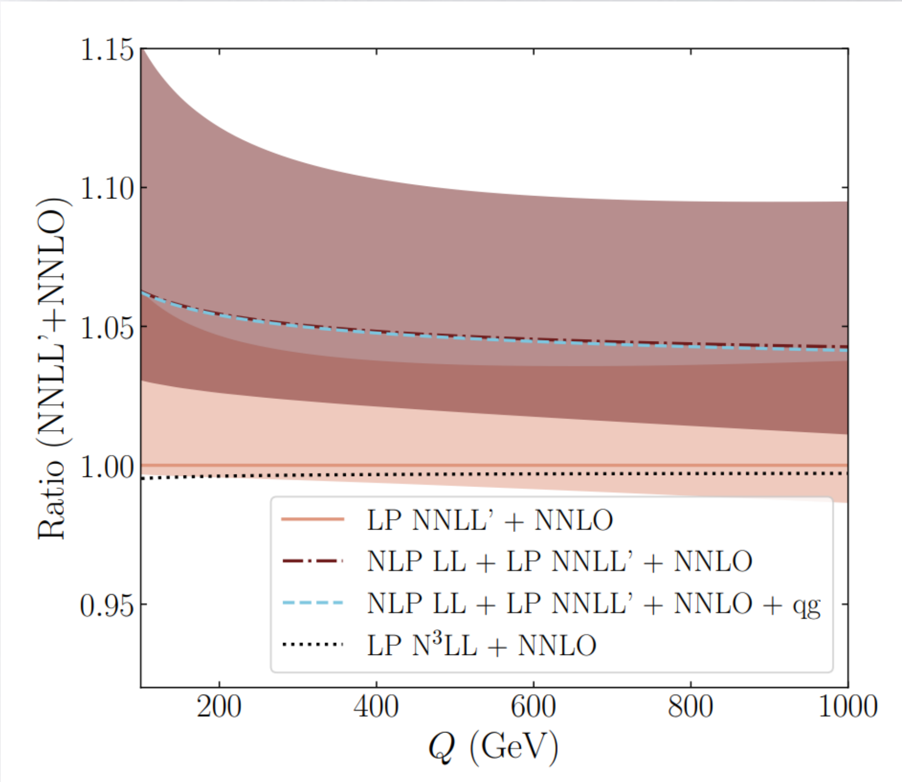
LL RESUMMATION AT NLP

- For leading channels like $q\bar{q}$ in Drell-Yan or gg in Higgs production, it turns out that the collinear function contributes only starting at NLL accuracy.
- This means that at LL accuracy only the hard and soft functions contribute. The divergent contribution problem can be easily overcome, and LLs can be resummed.

SCET: Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;
Diagrammatic: Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019

***Phenomenological analysis in:* Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019;**
van Beekveld, Laenen, Sinninghe Damsté, LV, 2021.





PERSPECTIVES

- The resummation of large leading logarithms at NLP is now under control, both in the diagonal (quark-antiquark, gluon-gluon) and off-diagonal (quark-gluon) channels, in electroweak annihilation processes (Drell-Yan, Higgs production, etc) and DIS.

- The next step is to formalize the refactorization process, such as to allow for a systematic resummation at NLP, beyond leading logarithmic accuracy.

- These result will be applied to produce phenomenological analysis of relevant processes for the LHC;

- On the other hand, knowledge gained in understanding the structure of large logarithms at NLP near threshold will be useful to extend resummation at NLP to other kinematic limits (small pT, small β, etc).

CONCLUSION

- One well-known issue on the path to precision physics is the summation of large logarithm to all order in perturbation theory.
- This is an interesting problem, which requires to understand all-order properties of gauge- and effective field theories. In turn, information obtained in this way feeds into several other problem of perturbative quantum field theory.
- We have developed two complete frameworks.

→ The first deals with large logarithms in the high-energy limit. We have developed methods, based on the shockwave formalism, which allows us to calculate scattering amplitudes up to NNLL in the high-energy logarithms. In this way we obtain information relevant for understanding also the structure of infrared divergences in gauge theories, and analytic properties of scattering amplitudes.

→ The second addresses the resummation of large logarithms at NLP near threshold. Working with a diagrammatic approach, and with methods based on SCET, we are able to resum leading logarithms at NLP in electroweak annihilation processes and DIS.