Atmospheric contribution to Newtonian noise for gravitational wave detectors

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Outline

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- Modeling the atmospheric contribution to Newtonian noise
    - Homogeneous and isotropic turbulence (frozen approximation)
    - Homogeneous and isotropic turbulence (finite correlation time)
    - Inhomogeneous model
- Results

Motivations

Among the several noise sources, Newtonian noise (NN) is one of the most subtle

NN is the fluctuation of the gravitational acceleration felt by the detector

It is caused by local density fluctuations in the nearby environment

Motivations

Two main NN countributions: seismic and atmospheric

Seismic contributions to NN has been extensively studied

Atmospheric NN has been studied only for second generation detectors considering homogeneous and isotropic turbulence in the frozen approximation [T. Creighton, 2008]

The models—Understanding NN

Consider a density fluctuation $\delta\rho(\mathbf{r}, t)$. It produces a fluctuation in the gravitational acceleration in the detector given by

$$\delta\mathbf{a}(\mathbf{r}, t) = G_N \frac{\delta\rho(\mathbf{r}, t)}{r^2} \Delta V \hat{\mathbf{r}} \equiv G(\mathbf{r})\delta\rho(\mathbf{r}, t)\Delta V \hat{\mathbf{r}}$$

This has to be summed over the contribution of the whole atmosphere and projected onto the detector arm

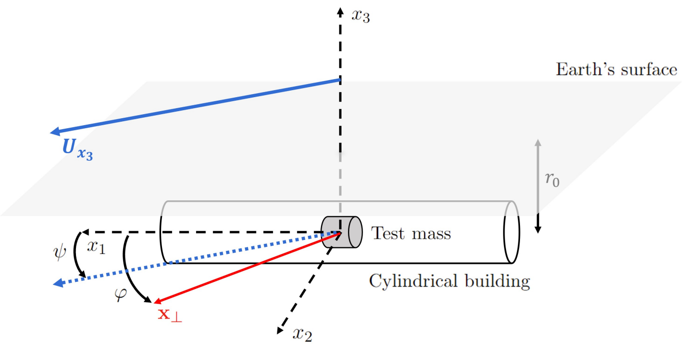
Direct evaluation of δa is hopeless, therefore we could try to compute the spectral density, which can be expressed as

$$\begin{aligned} S_h(\omega) &\propto \frac{1}{\omega^4} \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} \langle \delta a(t) \delta a(0) \rangle \\ &\propto \frac{1}{\omega^4} \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t} \int_V \mathrm{d}^3\mathbf{r} \int_V \mathrm{d}^3\mathbf{r}' G(\mathbf{r}) G(\mathbf{r}') \langle \delta\rho(\mathbf{r}, t) \delta\rho(\mathbf{r}', 0) \rangle \end{aligned}$$

The models—Geometry of the detector

In order to compute the spectral density of the noise, we need to specify a particular geometry

In order to study the effect of the noise for underground detectors, we chose the geometry sketched below



The models—Atmospheric NN by temperature fluctuations

Atmospheric NN has two main contributions: pressure fluctuations [P. R. Saulson, 1984] and temperature fluctuations [T. Creighton, 2008]

Temperature fluctuations are expected to be dominant

$$\delta\rho(\mathbf{r}, t) = -\frac{\rho_0}{T_0}\delta T(\mathbf{r}, t)$$

$$\langle \delta\rho(\mathbf{r}, t)\delta\rho(\mathbf{r}', 0)\rangle \propto \langle \delta T(\mathbf{r}, t)\delta T(\mathbf{r}', 0)\rangle$$

Thus, atmospheric NN can be computed once the correlation function $\langle \delta T(\mathbf{r}, t)\delta T(\mathbf{r}', 0)\rangle$ is specified

The models—HI turbulence

Homogeneous and isotropic turbulence obeys the Kolmogorov scaling [A. N. Kolmogorov, 1991]

$$\langle \delta T(\mathbf{r}, t) \delta T(\mathbf{r}', 0) \rangle \propto l^{11/3}$$

$l =$ typical vortex size

Frozen approximation: the decay time τ of turbulent structures is much larger than the wind advection time, i.e. $\tau \gg \omega / U$

We neglect energy loss

Finite correlation time: the decay time of turbulent structures is comparable with or smaller than the wind advection time

We cannot neglect energy loss

The models—Inhomogeneous model

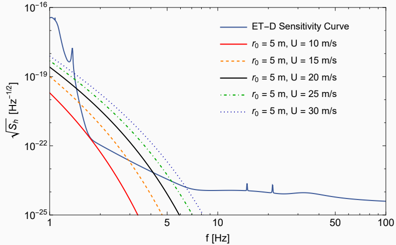
Turbulence is intrinsically inhomogeneous along the vertical axis

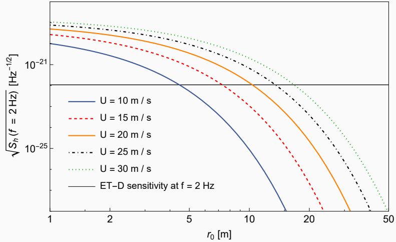
The wind speed U becomes a function of the height

$$U \equiv U(x_3) \sim \log(x_3/z_0)$$

The correlation function becomes then inhomogeneous along the vertical direction and Kolmogorov scaling is not valid

z_0 (m)	Terrain surface characteristics
1.0	city
0.8	forest
0.2	bushes
0.05	farmland (open appearance)
0.008	mown grass
0.005	bare soil (smooth)
0.0003	sand surfaces (smooth)





Exponential suppression for large frequencies f

Exponential suppression for great detector's depth r_0

Monotonical growth with the wind speed U

Noise signal above ET sensitivity for sufficiently large wind

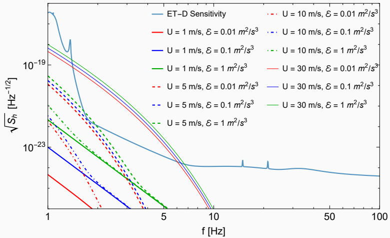
Results—HI turbulence, finite correlation time

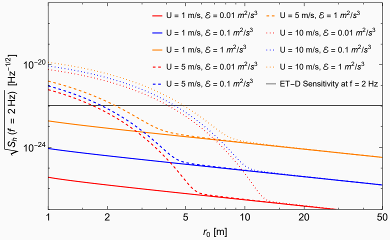
$\mathcal{E}$: energy loss rate per unit mass

Two regions: *exponential suppression* and *power-law behaviour*

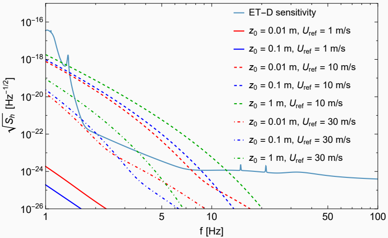
Exponential suppression: wind advection dominates, independent of $\mathcal{E}$

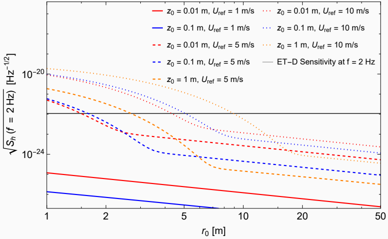
Power-law behaviour: vortex decay dominates, independent of U





Results—Inhomogeneous model





U_{ref}: wind speed at the reference height of 10 m

z_0: roughness of the terrain

Exponential suppression: wind advection dominates, almost independent of z_0

Power-law behaviour: vortex decay dominates, dependent on the parameters of the system

The frequency at which the power-law behaviour starts dominating strongly depends on z_0

Conclusions

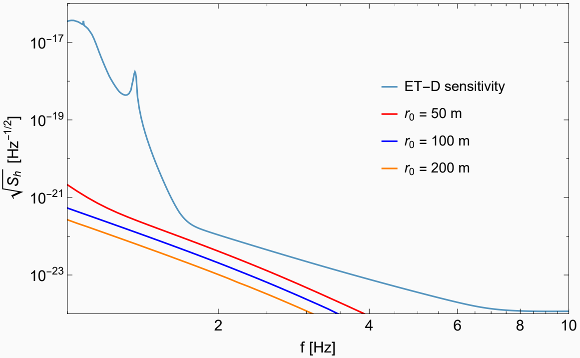
We modelled the atmospheric contribution to NN in three regimes: (a) HI, frozen turbulence; (b) HI, finite correlation time; (c) inhomogeneous turbulence

Atmospheric NN is always suppressed for large frequencies and for large depths, while it grows with the wind speed

We found two typical behaviours:

- Exponential suppression, when wind advection dominates
- Power-law behaviour, when vortex decay dominates
    - $1/r_0$ behaviour for large r_0

Conclusions



Passive mitigation could be insufficient to suppress atmospheric Newtonian noise

For instance, the ratio between the ET sensitivity and the noise curve for $r_0 = 200$ m is less than a factor of ~ 20

Backup slides

Models—Factorization

We want to compute the temperature correlation function

$$\langle \delta T(\mathbf{r},t), \delta T(\mathbf{r}',0)\rangle = \langle \delta T(\mathbf{r}-\mathbf{U}t,t), \delta T(\mathbf{r}'-\mathbf{U}t,0)\rangle_U = \int \mathrm{d}^3\mathbf{k}\, \mathrm{e}^{-\mathrm{i}\omega t} C_\omega(\mathbf{k})$$

We assume a factorization of the spatial and temporal part of the correlation function in the Fourier space, i.e.

$$C_\omega(\mathbf{k}) = f(\mathbf{k}) h[\tau(\omega - \mathbf{k}\cdot\mathbf{U})]$$

Frozen approximation: $h[\tau(\omega - \mathbf{k}\cdot\mathbf{U})] = 2\pi\delta(\omega - \mathbf{k}\cdot\mathbf{U})$

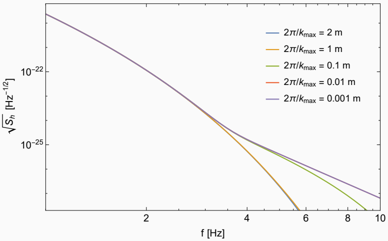
Finite correlation time: $h[\tau(\omega - \mathbf{k}\cdot\mathbf{U})] = \mathrm{e}^{-\tau^2(\omega-\mathbf{k}\cdot\mathbf{U})^2}$

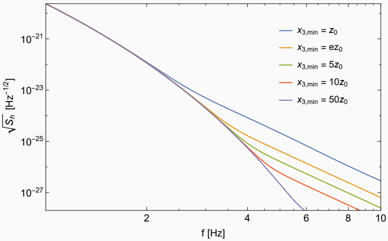
The last choice is arbitrary but reasonable

Dependence on the cutoffs on the wavevector and on the height

$k_{\max}$: upper cutoff of the integrals over k

$x_{3,\min}$: lower cutoff of the integrals over x_3 (vertical direction)





Dependence on ψ

ψ: angle between the detector arm and the wind direction (assumed to be uniform)