Quantum Membrane and Gravitational Waves

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Outline

- What are the signatures of black holes?
- Alternatives to black holes.
- "Quantising" the membrane paradigm.
- Observational implications.
- Future prospects.

Reference

- **SC**, Maggio Mazumdar and Pani, arXiv: 2202.09111.

Why Black Holes?

- Black holes can be constructed from normal matter, using simple collapse scenarios.

- Black holes are unique and have universal properties.
  [Heusler, Black Hole Uniqueness Theorem (Cambridge University Press)]

- Black holes behave as thermodynamic objects with temperature and entropy — Hint towards quantum gravity. **[Bekenstein, Phys. Rev. D 7, 2333 (1973)]**

- Black holes are stable under all possible perturbations.

- Observation of shadows from Event Horizon Telescope and the ringdown signals from LIGO and VIRGO are definitely consistent with the existence of Black Holes.

- Consistency with general relativity is another story.

But ...

- Despite being the simplest objects, there are issues.
- **Singularity**: All black hole spacetimes have a singular region/point → breakdown of the theory.
- **Loss of Predictability**: Most of the black holes inherit Cauchy horizon → future cannot be determined.

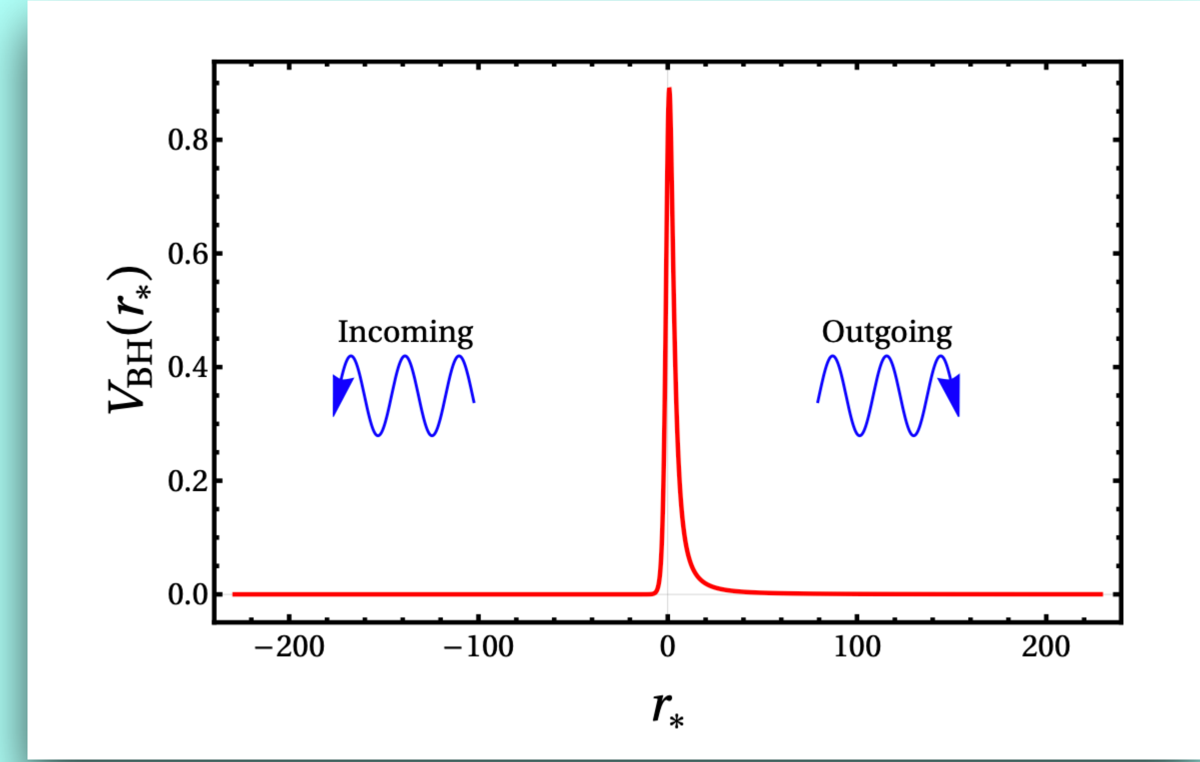
  [Poisson and Israel, Phys. Rev. D 41, 1796 (1990)]
- **Information Loss Paradox**: The existence of thermal radiation results into loss of information.

  [Hawking, Commun. Math. Phys. 43, 199 (1975)]

  [SC and Lochan, Universe 3, 55 (2017)]
- All of these suggest that we may need to look for alternatives — curing these problems and yet remaining consistent with experiments.

Black Hole Hypothesis

- Does the existence of a photon sphere implies the existence of a black hole? **[Cardoso et. al., Phys. Rev. Lett. 116, 171101 (2016)]**
- The ringdown is governed by the perturbation of the photon sphere alone. **[Cardoso et. al., Phys. Rev. D 79, 064016 (2009)]**
- Structure beneath the photon sphere can not be probed directly.
- Can such objects exist? What will be their observational properties?



[Figure Courtesy: Biswas, Rahman and SC, arXiv:2205.14743]

Exotic Matter

- Raychaudhuri equation guarantees that normal matter cannot cure singularities ⟶ require exotic matter.
- The consistency with observations, require any alternatives to have

$$\frac{1}{2} > \left(\frac{M}{R}\right) > \frac{1}{3}$$

- The limiting stellar configuration, with normal matter, being (Buchdahl limit):

$$\left(\frac{M}{R}\right) \leq \frac{4}{9}$$

- Recent shadow measurement argues that Buchdahl limit must be violated ⟶ exotic matter is necessary.

Only Exotic Matter?

- Are these exotic matters stable → ergo-region instability, enhanced superradiant instability, for rotating objects.

[Cardoso et. al., Phys. Rev. D 77, 124044 (2008)]

- Can quantum effects play any role?

[Maggio et. al., Phys. Rev. D 96, 104047 (2017)]

- Area quantised black holes are generic predictions of theories of quantum gravity and these have non-trivial physics at horizons.

[Abedi et. al., Phys. Rev. D 96, 082004 (2017)]

- The basic point is to modify the horizon itself by a reflective membrane as quantum effects are taken into account.

[Dey, SC and Afshordi, Phys. Rev. D 101, 104014 (2020)]

- What about membrane paradigm and its quantum version?

[Price and Thorne, Phys. Rev. D 33, 915 (1986)]

[Maggio et. al., Phys. Rev. D 102, 064053 (2020)]

“Quantum” Membrane

- The membrane is assumed to be constructed out of a large number of harmonic oscillators at their ground states.

[SC et. al., arXiv: 2202.09111]

- The wave function of the membrane is governed by

$$\Psi(\epsilon) = A \exp\left(-\frac{\epsilon^2}{2\sigma^2}\right) \qquad |A|^2 = \frac{2}{\sigma\sqrt{\pi}}$$

- Classically the membrane will be located at $\langle\hat{\epsilon}\rangle$, with a fluctuation

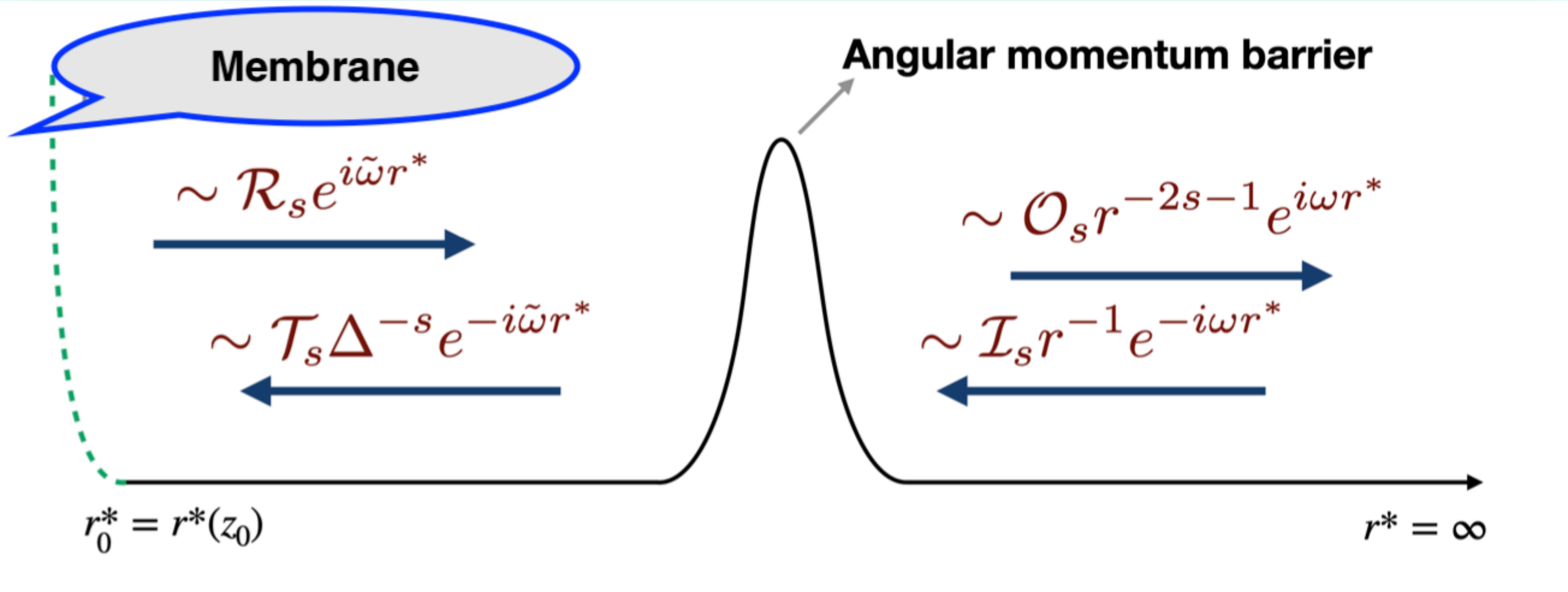
$$\langle\hat{\epsilon}^2\rangle - \langle\hat{\epsilon}\rangle^2 = \sigma^2\left(\frac{1}{2} - \frac{1}{\pi}\right)$$

- Classical limit ($\hbar \rightarrow 0$), corresponds to $\sigma \rightarrow 0$.

- The quantum membrane will have an energy-momentum tensor

$$\hat{T}_{ab} = \rho\hat{u}_a\hat{u}_b + \left(p - \zeta\hat{\Theta}\right)\hat{\gamma}_{ab} - 2\eta\hat{\sigma}_{ab} \qquad \rho = \rho_0 + \delta\rho \qquad p = p_0 + \delta p$$

Basic Structure



[Figure Courtesy: Dey, SC and Afshordi, Phys. Rev. D 101, 104014 (2020)]

"Quantum" Matter = Geometry

- The properties of matter get related to the geometry by the junction conditions: $[[K_{ab} - K h_{ab}]] = -8\pi \langle \hat{T}_{ab} \rangle$ and $[[h_{ab}]] = 0$ on $R = r_+ + \langle \hat{\epsilon} \rangle$.
- In absence of perturbations, the energy density and pressure becomes,

$$\rho_0 = -\frac{f(R)^{3/2}}{4\pi R} \left[\frac{1}{f(R) + \frac{1}{2} f''(r_+) \left(\langle \hat{\epsilon}^2 \rangle - \langle \hat{\epsilon} \rangle^2 \right) + \mathcal{O}(\tilde{\sigma}^3)} \right]$$

$$p_0 = \frac{R\left[2f(R) + Rf'(R)\right]}{16\pi \sqrt{f(R)} \left[R^2 + \left(\langle \hat{\epsilon}^2 \rangle - \langle \hat{\epsilon} \rangle^2 \right)\right]}$$

- These must be perturbed due to perturbation of the metric and the governing equation will be $\delta K_{ab} - K \delta h_{ab} = -8\pi \langle \delta \hat{T}_{ab} \rangle$.
- For simplicity we will consider axial gravitational perturbation, which in the Regge-Wheeler gauge has only two independent components $\delta g_{t\phi}$ and $\delta g_{r\phi}$.

Not Purely Ingoing

- For axial perturbation of static and spherically symmetric spacetime, with $-g_{tt} = g^{rr}$, the Regge-Wheeler choice provides the following boundary condition at $\boxed{R = r_+ + \langle \hat{\epsilon} \rangle}$. [SC et. al., arXiv: 2202.09111]

$$\boxed{\begin{aligned} i\omega\psi(R) = \frac{\eta}{(\rho_0 + p_0)\sqrt{f(R)}} \Bigg[& V_{\rm axial}(R)\psi(R) - \frac{1}{R}\frac{d\psi(R)}{dx}\left[Rf'(R) - 2f(R)\right] \\ & - \frac{4f(R)}{R}\left(\frac{d\psi(R)}{dx} + \frac{f(R)}{R}\psi(R)\right)\left(1 + \frac{4\pi\rho_0 R}{\sqrt{f(R)}}\right)\Bigg]. \end{aligned}}$$

- As $R \to r_+$, the above condition reduces to,

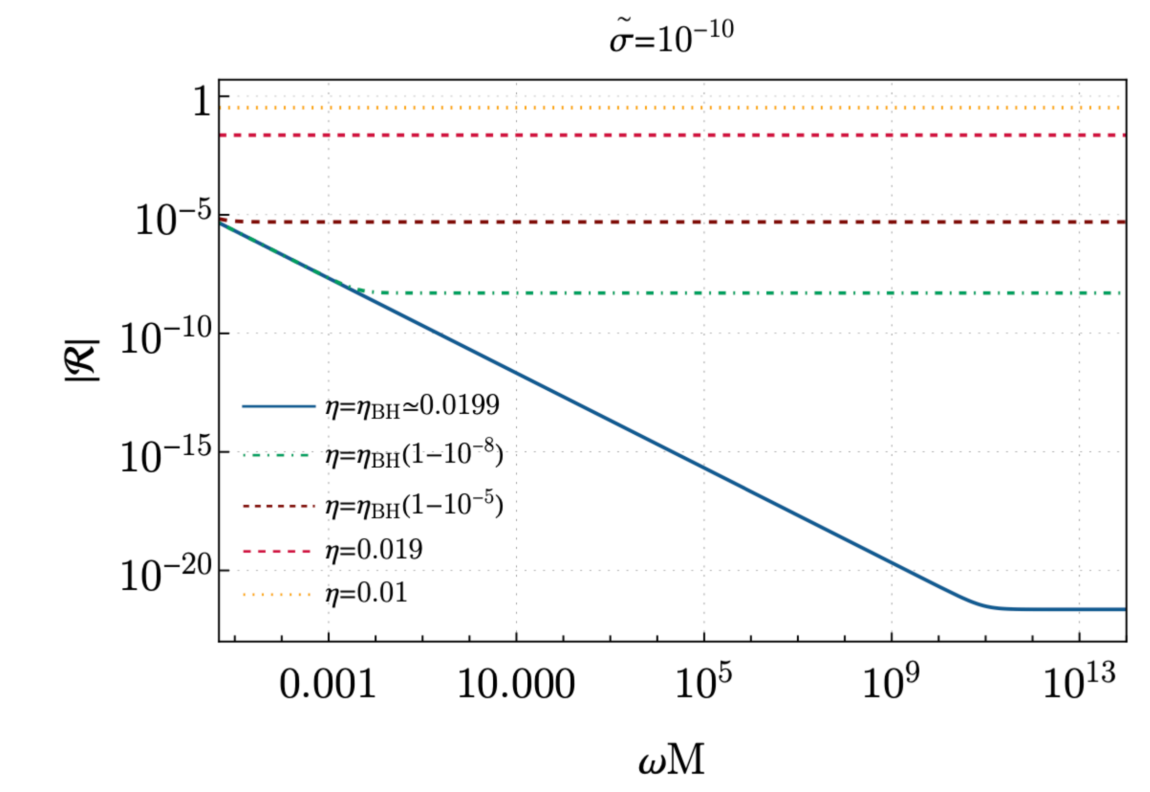
$$\boxed{i\omega\psi(R) = -16\pi\eta\frac{d\psi(R)}{dx}}$$

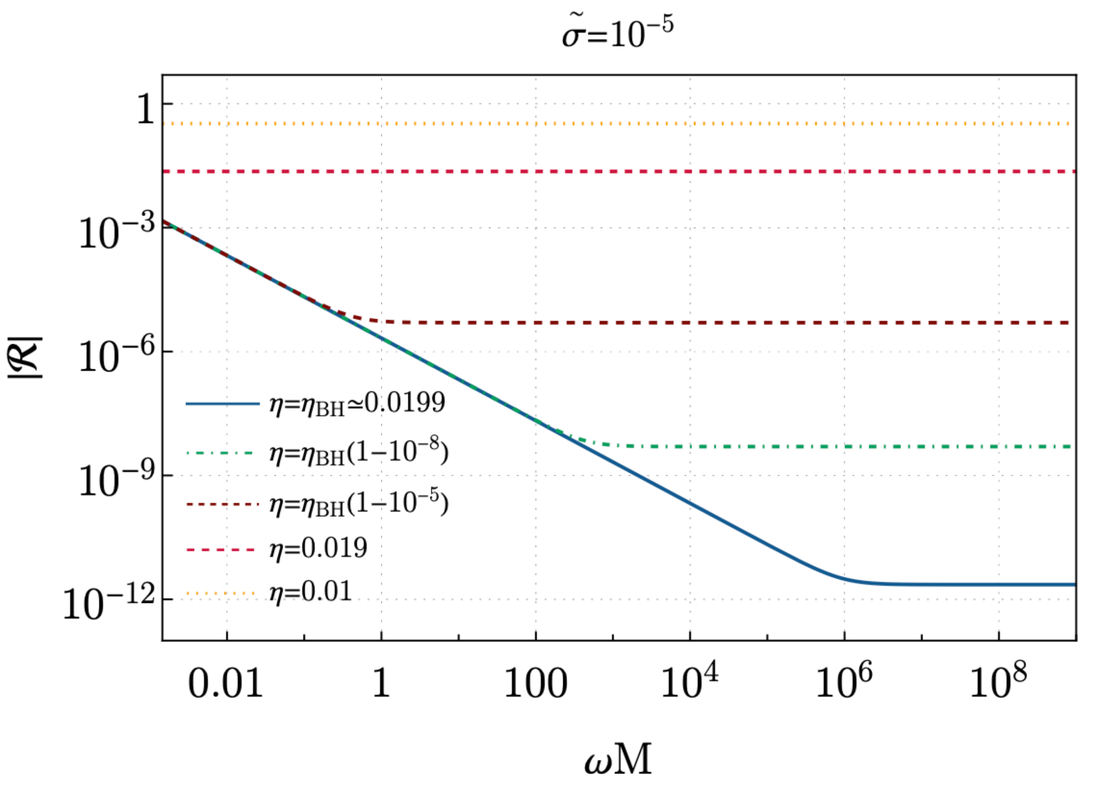
- This is equivalent to purely ingoing waves at the horizon, which will not be the case in general.

Reflecting "quantum" membrane

[SC et. al., arXiv: 2202.09111]

- The boundary condition allows existence of ingoing as well as outgoing waves near the membrane, such that, $$\psi_{\rm M} = e^{-i\omega x} + \mathcal{R} e^{i\omega x}$$
- In appropriate limits, $$|\mathcal{R}|^2 \sim \left(\frac{1-\eta/\eta_{\rm BH}}{1+\eta/\eta_{\rm BH}}\right)^2 + \frac{16384\,[\ell(\ell+1)-3]^2\,\pi^3\eta^4\tilde{\sigma}^2}{(1+\eta/\eta_{\rm BH})^4\,\omega^2 M^2}$$

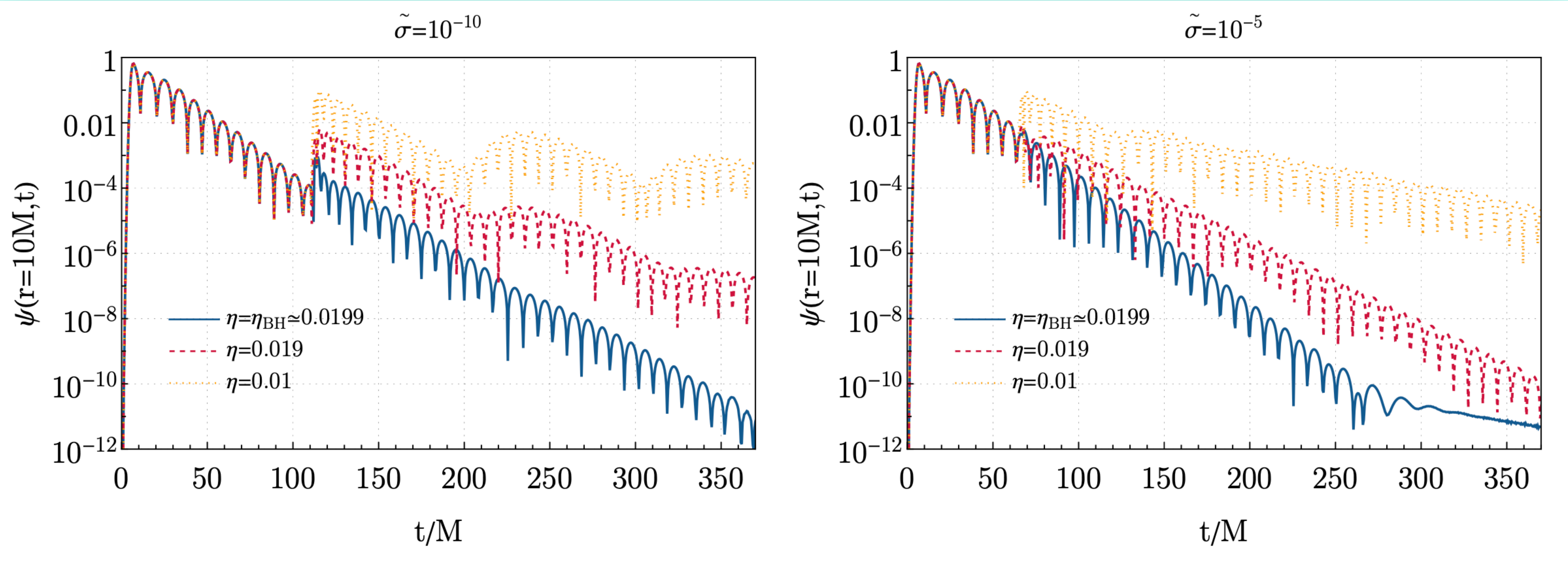




Ringdown Waveform

- As the effective classical membrane nears the horizon, there are pronounced echoes. [SC et. al., arXiv: 2202.09111]

- The time delay is consistent with size of the membrane.

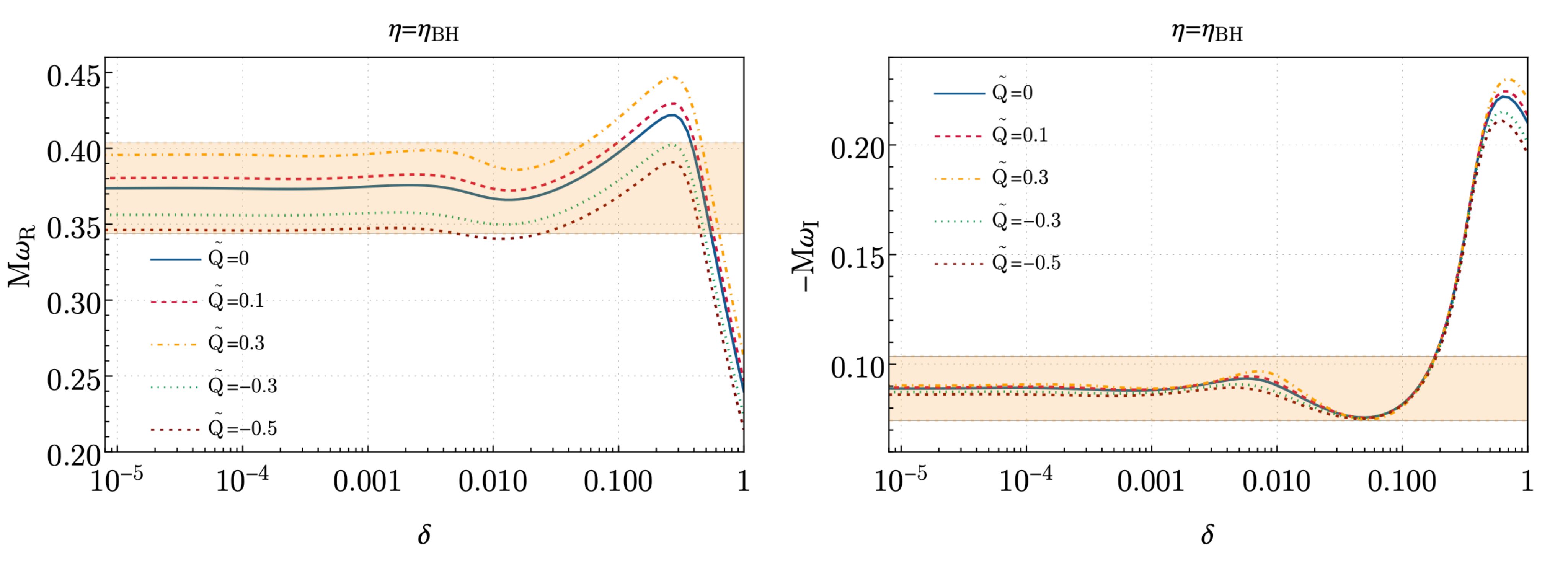


Other Possibilities

- There are various other possibilities to consider, e.g., we may consider a classical membrane, but a quantum corrected black hole.

- This scenario is best achieved by considering higher dimensional black holes, where the AdS/CFT correspondence ensures that the four dimensional black holes are quantum corrected.

[SC et. al., arXiv: 2202.09111]



Conclusion-I

- We have provided a model involving quantum harmonic oscillator, with non-trivial reflectivity at the horizon.
- The reflectivity depends on the properties of the membrane and its quantum nature.
- There are echoes in the ringdown signal of gravitational waves, originating from “quantum” membrane.

Conclusion-II

- Similar conclusions hold true for a classical membrane, but with the properties of “quantum” corrected black holes.
- For the future, we are considering effects of such “quantum” membrane on the inspiral part of the gravitational wave signal, in particular, for tidal deformations.
- Improving the “quantum” model from ground state oscillators to coherent state.

Thank You