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GRAvitational-wave Science&technology Symposium

The 5n-vector ensemble method for detecting GWs from known pulsars

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CWs targeted search

NO evidence of CW signal in the LIGO/Virgo data

- 236 pulsars, O2+O3 data : arXiv:2111.13106

How to improve the detection probability?

- Considering an ensemble of pulsars Giazotto et al. 1997 *Phys.Rev.D* 55
- F-stat Chen et al 2016 Phys.Rev.D 94,
- Bayesian method Pitkin et al 2018 Phys.Rev.D 98
- 5n-vector method
  - D'Onofrio et al. 2021 Class. Quantum Grav.38 13502
  - D'Onofrio et al. 2022 Phys. Rev. D 105, 063012
- Stochastic Targeted search arXiv:2203.03536

ENSEMBLE ANALYSIS combines the effects of weak sources that are individually undetectable

- Multiple test $H_0 = H_0^{(1)} \cup H_0^{(2)} \cup ... \cup H_0^{(N)}$
- Linearly combining the single pulsar detection statistics
- S_i defined by the 5n-vector method in [1], **"normalized"** to be compared

  Gaussian noise : $S_i \sim Gamma(x; 2, 1)$
- **Rank truncation method [2]**

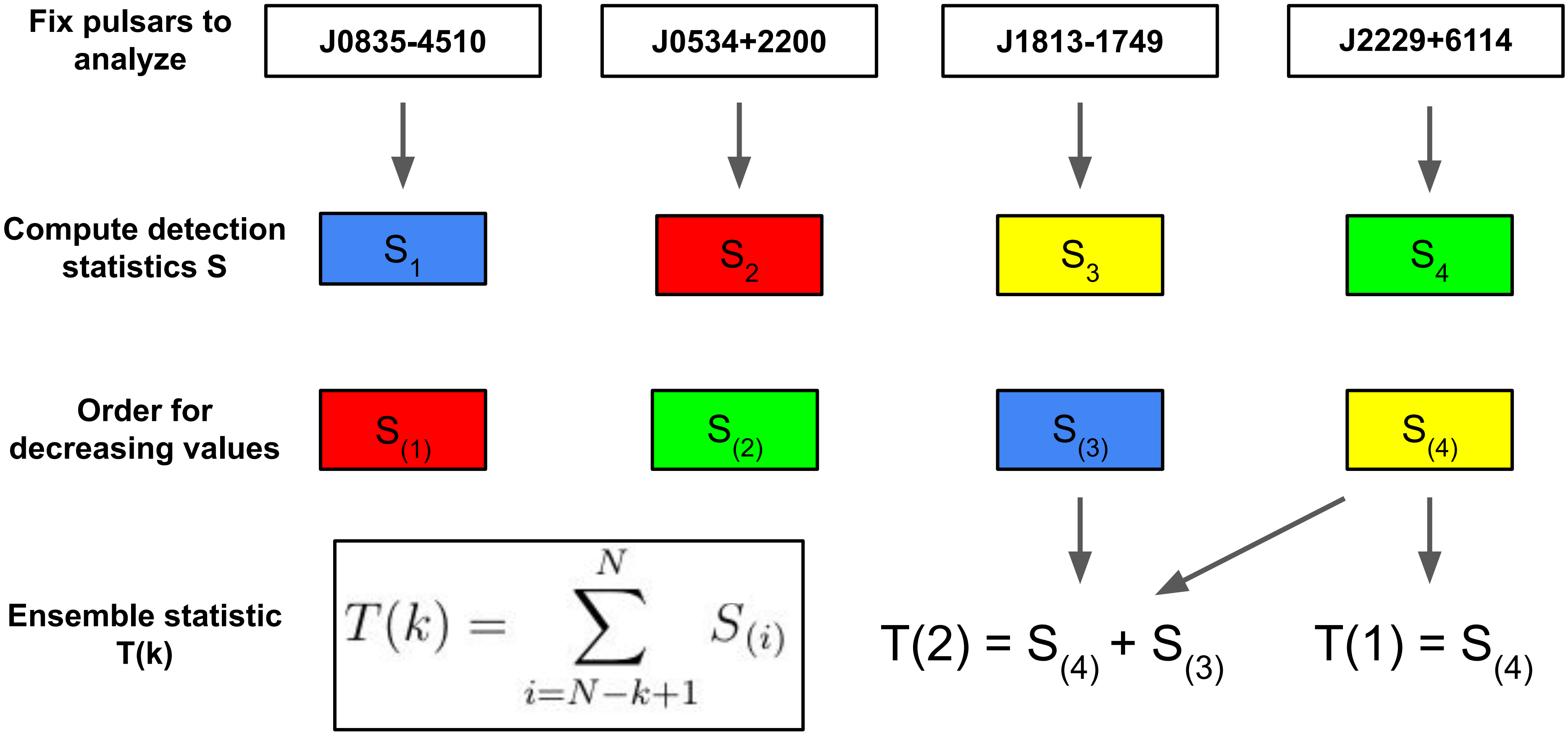
[1] P Astone et al 2012 J. Phys.: Conf. Ser.363 012038
[2] D. Zaykin et al., Gen. Ep. 22 (2002)

Ensemble procedure

- Simplest way to define an ensemble statistic : take the sum of the S statistics
  - with ~200 pulsars, how many signals can be near the detection thr?
- To maximize the det. prob. we need to estimate signals “strength”
- Use single pulsar p-values in a real analysis!
- Organize for increasing p-values is equal to organize for decreasing values of the statistics

$$\overline{S}_{(1)} < \overline{S}_{(2)} < \ldots < \overline{S}_{(N)}$$

$$T(k) = \sum_{i=N-k+1}^{N} \overline{S}_{(i)}$$ → **Sum of order statistics**


Fix pulsars to analyze
J0835-4510
J0534+2200
J1813-1749
J2229+6114
Compute detection statistics S
S_1
S_2
S_3
S_4
Order for decreasing values
$S_{(1)}$
$S_{(2)}$
$S_{(3)}$
$S_{(4)}$
Ensemble statistic T(k)
$T(k) = \sum_{i=N-k+1}^{N} S_{(i)}$
$T(2) = S_{(4)} + S_{(3)}$
$T(1) = S_{(4)}$

P-value of ensemble

The idea is to construct a p-value of ensemble for the T(k) statistics as a function of k

To reconstruct T(k) noise distribution:

1) Gaussian noise case

a) Fix N

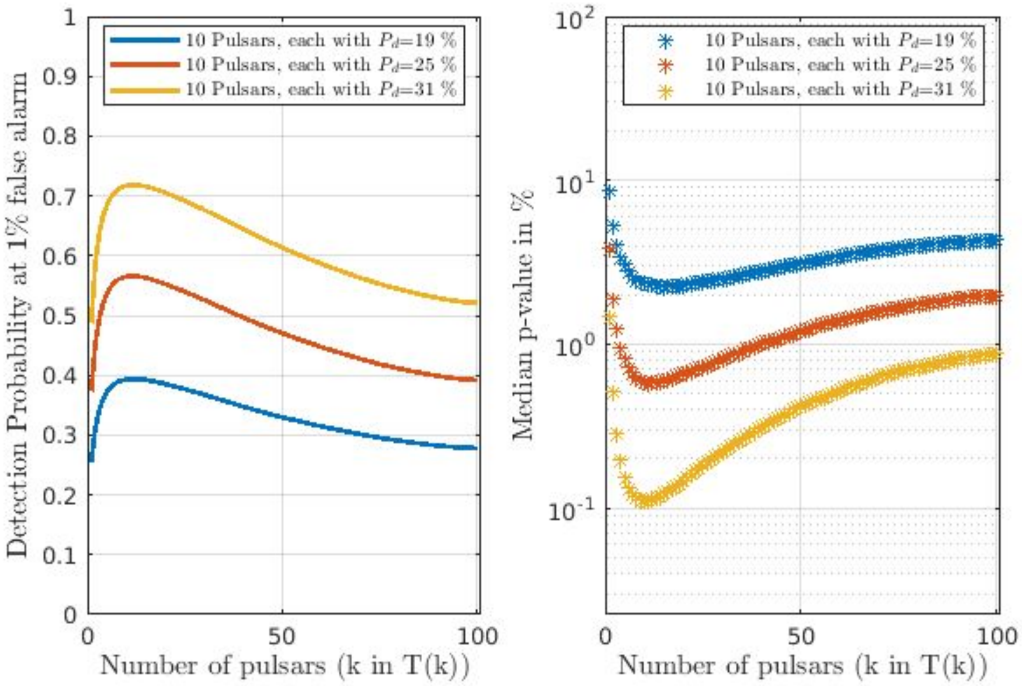
b) S ~ Gamma(2,1) NOISE

S ~ χ2(Λ ,4) SIGNAL

c) Monte Carlo procedure

d) T(k) distribution for each k

e) Sensitivity test



P-value of ensemble

The idea is to construct a p-value of ensemble for the T(k) statistics as a function of k

To reconstruct T(k) noise distribution:

1) Gaussian noise case →Sensitivity test

2) Real case →S distribution from off-source frequencies in a band (tenth of Hz) near the GW frequency

a) BSD framework [1]

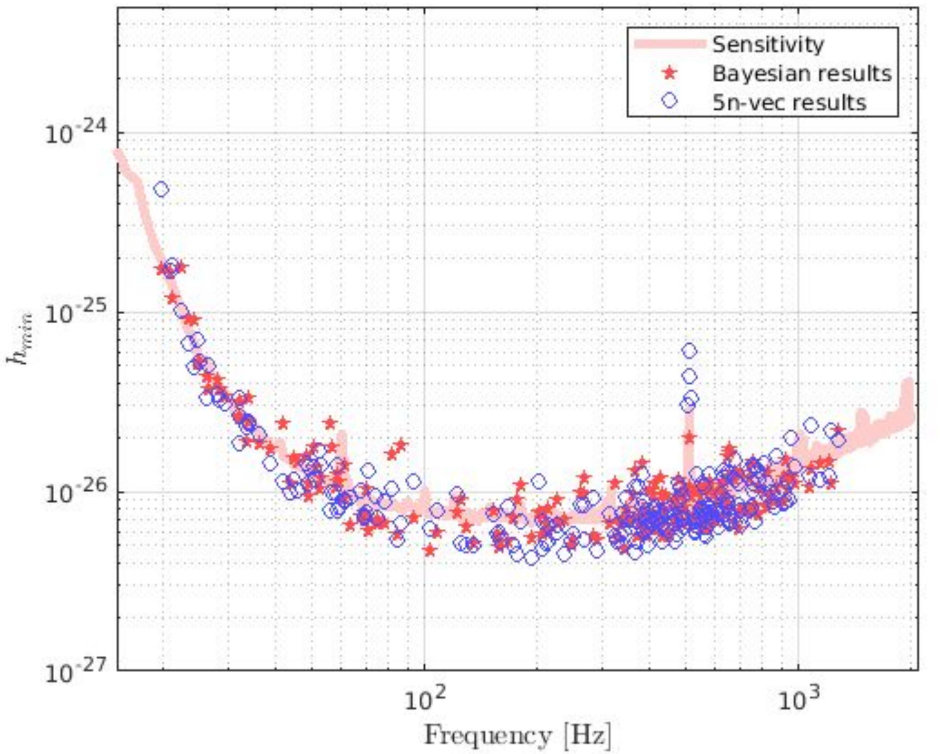
b) Generalize the Monte Carlo procedure starting from the experimental S distribution for each pulsar

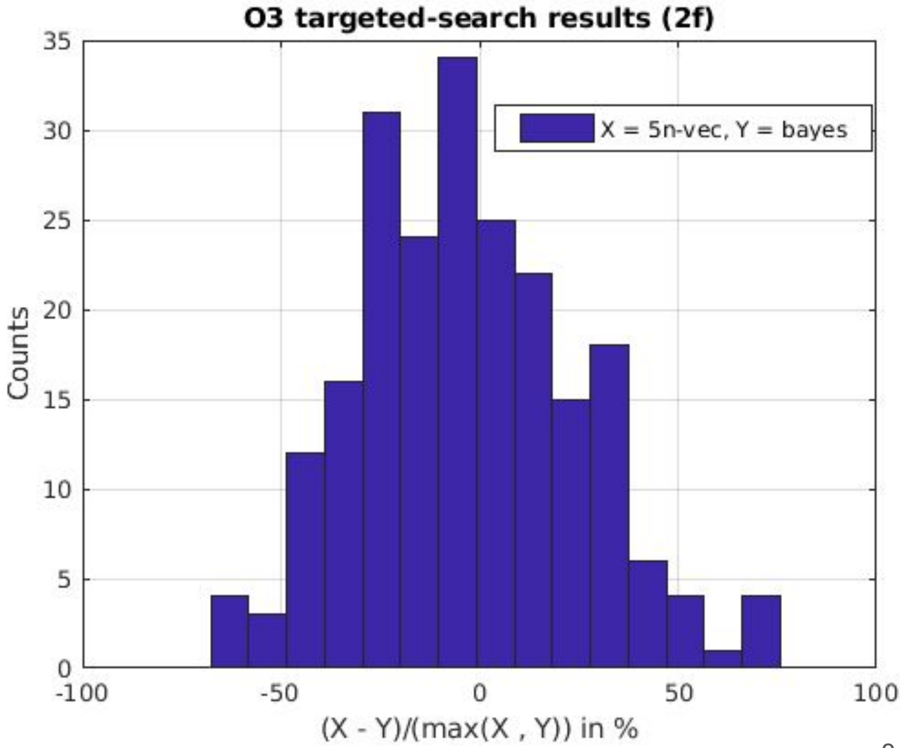
[1] O J Piccinni *et al* 2019 *Class. Quantum Grav.* 36 015008

Preliminary results !

Single pulsar analysis

219 pulsars used in O3 targeted search (R. Abbott et al. 2021)
First results on binary systems for the 5n-vector method
LIGO-Virgo O3 datasets



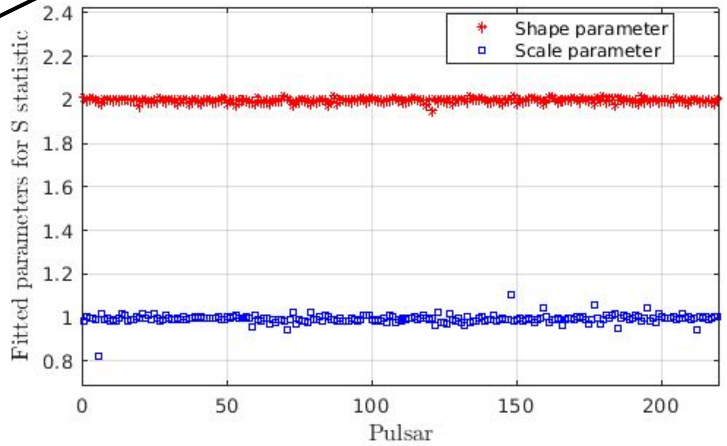


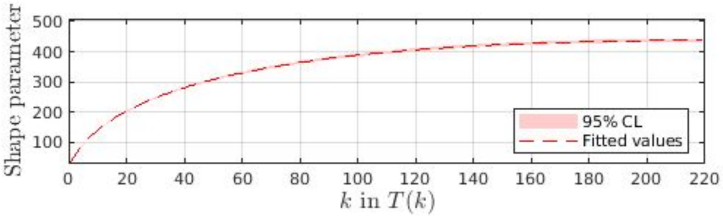
Relative error in %

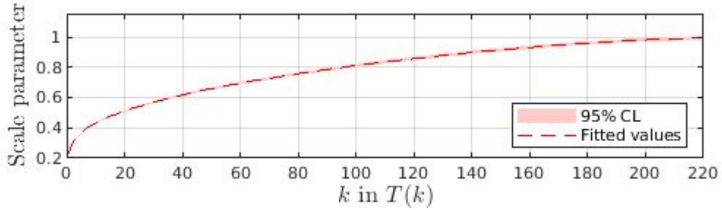
Preliminary results !

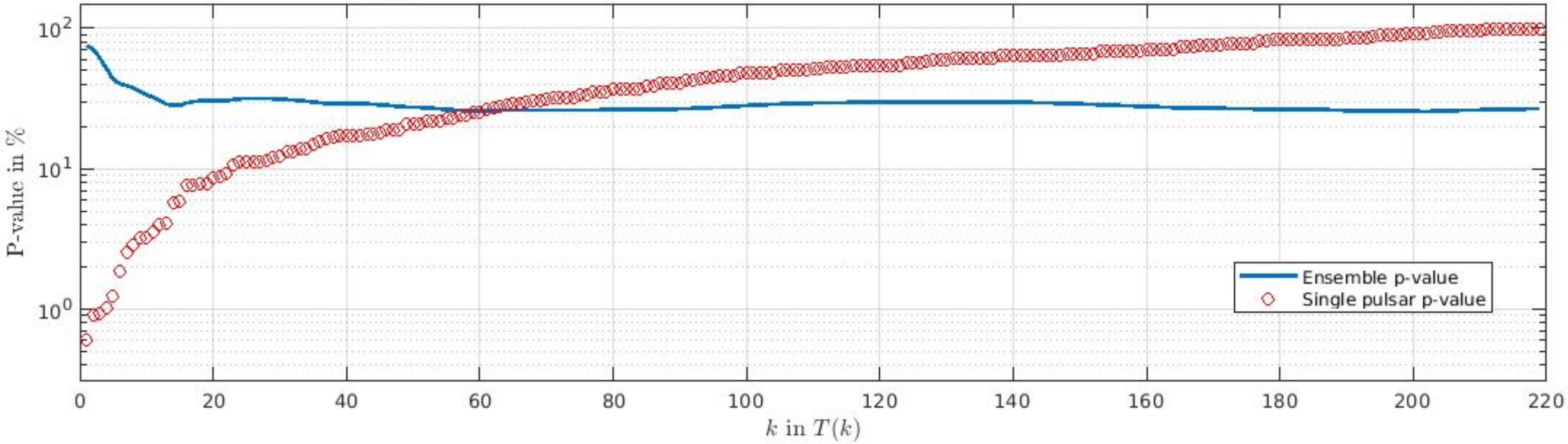
Ensemble analysis

Known pulsars in O3 targeted search





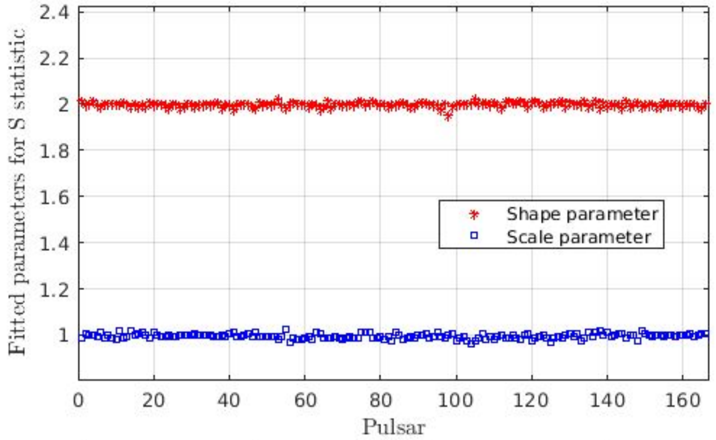


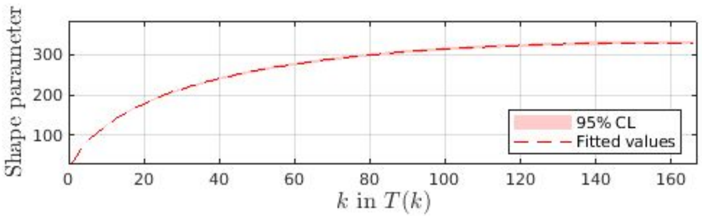


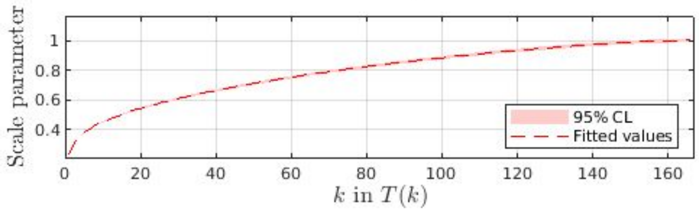
Preliminary results !

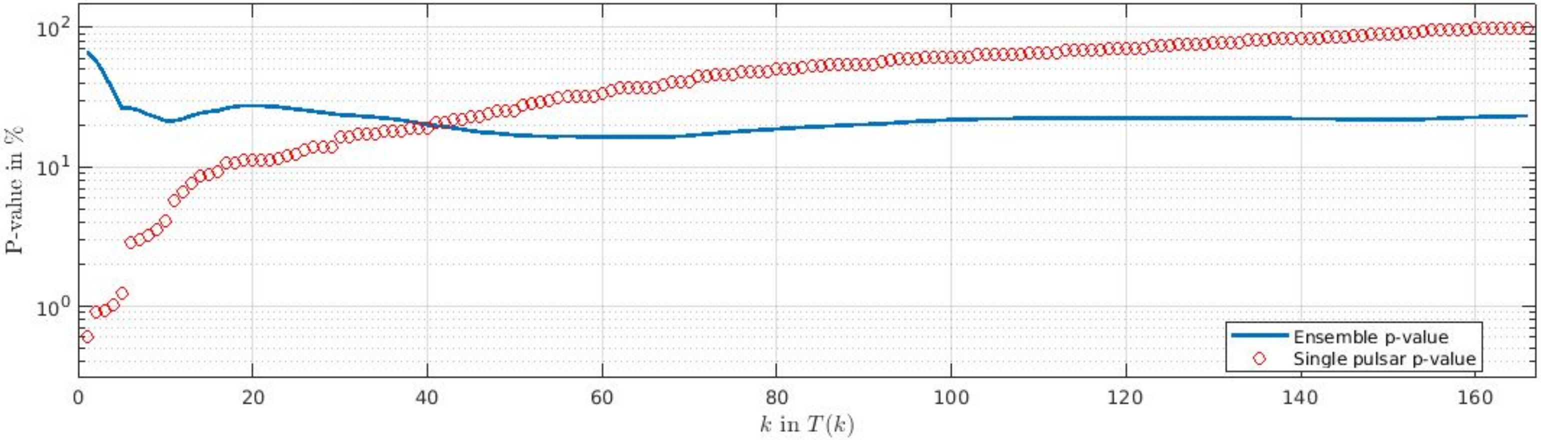
Ensemble analysis

Millisecond pulsars





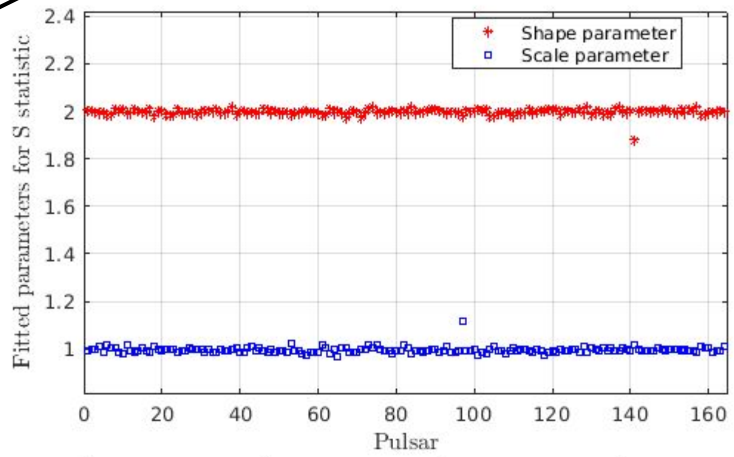


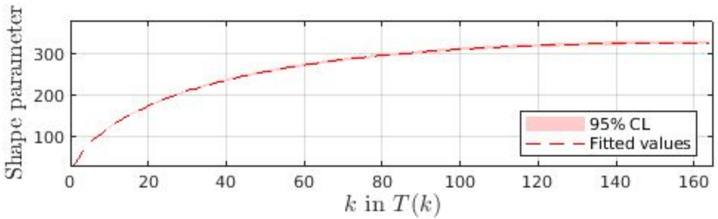


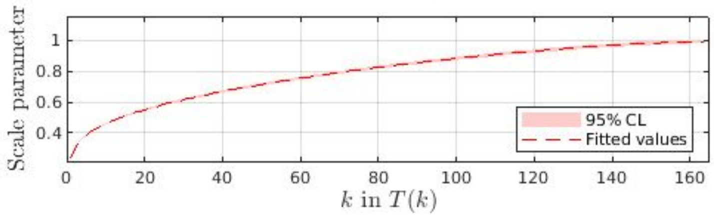
Preliminary results !

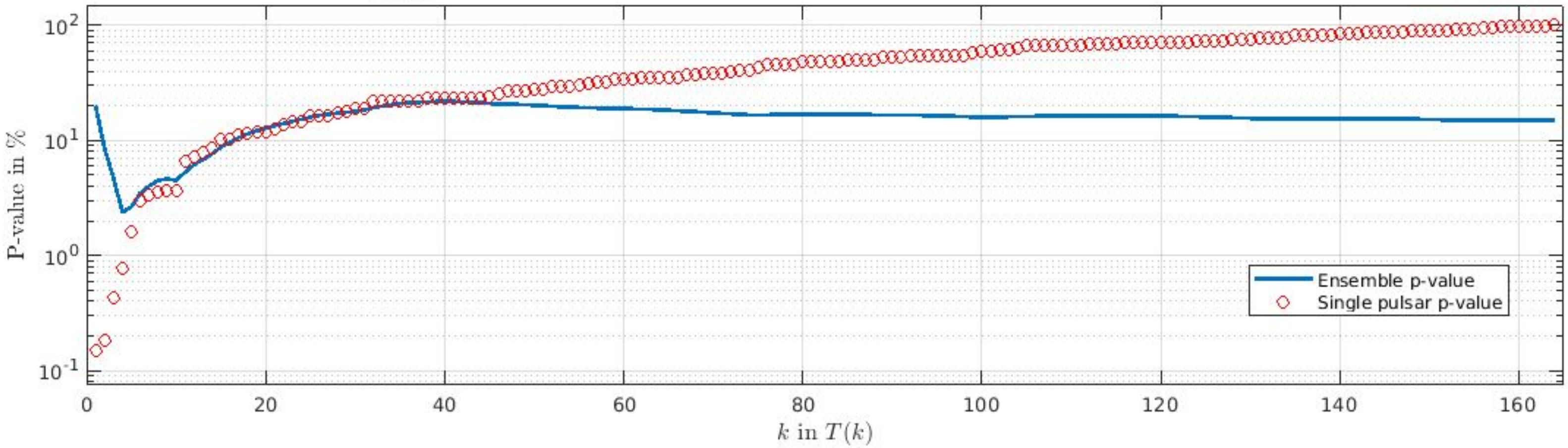
Ensemble analysis

Millisecond pulsars in LIGO detectors









Upper limit

- T(k) signals distributions can not be analytically computed
- BUT ----> T(N) ~ non-central χ2 r.v. with parameter Λ

$$\Lambda = \sum_{i=1}^{N} 2 \cdot H_{0,i}^2 \left(\frac{|\mathbf{A}_i^+|^4 \cdot |H_{+,i}|^2}{\sum_{j=1}^n \sigma_j^2 \cdot T_j \cdot |\mathbf{A}_{j,i}^+|^2} + \frac{|\mathbf{A}_i^\times|^4 \cdot |H_{\times,i}|^2}{\sum_{k=1}^n \sigma_k^2 \cdot T_k \cdot |\mathbf{A}_{k,i}^\times|^2} \right)$$

$$P(\Lambda|T^*) \propto P(T^*|\Lambda)P(\Lambda)$$

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$$P(\Lambda|T^*) \propto P(T^*|\Lambda)P(\Lambda)$$

- Going hierarchical →hyper-parameter α (e.g. mean value for exponential dist.)
- Hypothesis: we will assume a common distribution for the ellipticities

1) $$P(\alpha|T^*) \propto \int P(T^*|\Lambda)P(\Lambda|\alpha)P(\alpha)d\Lambda$$

2) $$P(\alpha|T^*) \propto \left(\prod_{i=1}^{N} \int P(S_i|H_i)P(H_i|\alpha)dH_i \right) P(\alpha)$$

Summary

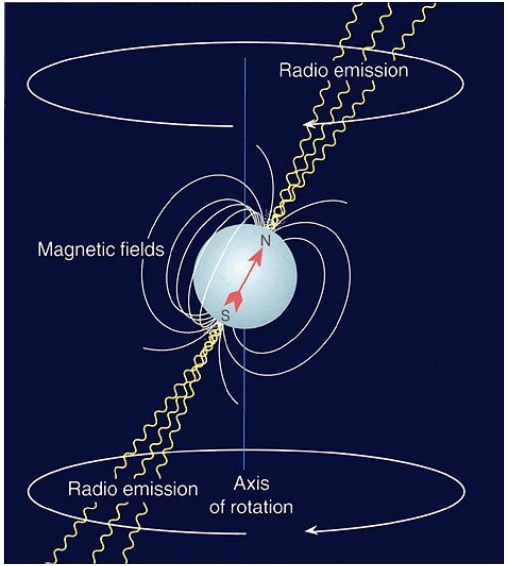
- I propose a multi-detector ensemble procedure to combine the single pulsar statistics that increases the detection probability for CW signals.
- This procedure is a rank-truncation method since I combine together the top-ranked statistics according to single pulsar p-values.
- Preliminary results for O3 data show no evidence of CW signal from ensemble. Upper limits can be set on population parameters.
- Pipeline review on-going for application to O4 data.
- The procedure can be easily generalized to different pipelines/measurements/fields for sub-threshold signal detections.

Thank you!

Continuous gravitational waves (CWs)

Isolated spinning neutron stars with non axi-symmetric mass distribution are possible sources of CWs.

- CWs are always present in GW detectors.
- CW frequency is linked to source rotational frequency
- CW amplitude is expected much weaker than that generated by binary BH/NS collisions



8 parameters for CW signal :

f_{rot} $\dot{f}_{rot}$ α δ → **EXTRINSIC**

h_0 ϕ η ψ → **INTRINSIC**

Different strategies considering source assumptions:

- **Targeted search**; → accurate ephemeris → "KNOWN PULSAR"
- Narrow-band search;
- Directed search;
- All-sky search;

CW Signal

Source as triaxial neutron star rotating around a principal axis of inertia : $f_{gw} = 2f_{rot}$

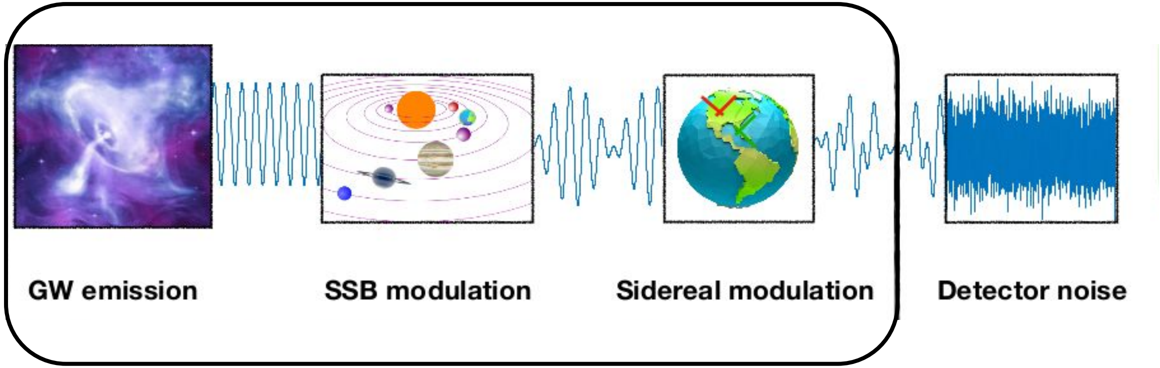
$$h_0 = \frac{16\pi^2 G}{c^4}\frac{I_z \epsilon f_{rot}^2}{d} \simeq 4.23 \cdot 10^{-26} \left[\frac{1Kpc}{d}\right] \left[\frac{I_z}{10^{38} Kg \cdot m^2}\right] \left[\frac{\epsilon}{10^{-6}}\right] \left[\frac{f_{rot}}{100Hz}\right]^2$$

con

$$\epsilon = \frac{|I_x - I_y|}{I_z}$$

$$h_0^{SD} = \frac{1}{d}\left(\frac{5}{2}\frac{GI_z}{c^3}\frac{\dot{f}_{rot}}{f_{rot}}\right)^{1/2}$$

→ **Spin-down limit**: theoretical upper limit

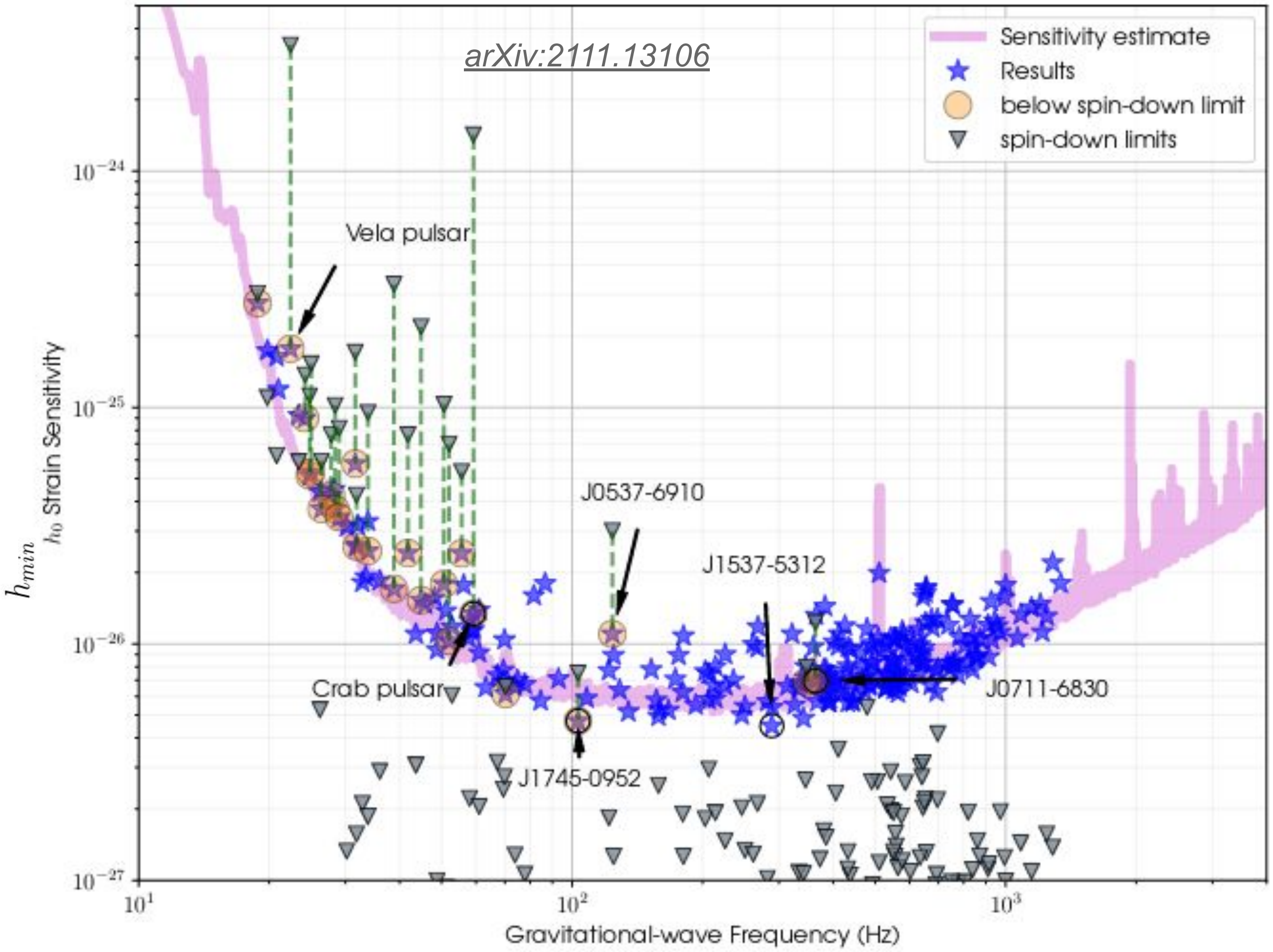


- Doppler correction
- Spin-down correction

At the detector, CW signal is splitted in 5 frequencies (antenna pattern) :

$$f_{gw}, \quad f_{gw} \pm \Omega, \quad f_{gw} \pm 2\Omega$$

O3 Targeted Search



- **Targeted search** on 236 pulsars (O1-O2-O3 data, LHO LLO V)
- Minimum detectable signal (Targeted search):

$$h_{min} \approx 11\sqrt{\frac{S_h(f)}{T_{obs}}}$$

- No evidence of CW signal
- *Astrophys.J.Lett. 902 L21* **Gravitational-wave constraints on the equatorial ellipticity of millisecond pulsars**

Tools

- **5-vector method**, matched filter in frequency domain

$$x(t) = h(t) + n(t)$$

$$h(t) = H_0(H_+A^+ + H_\times A^\times)e^{\jmath\omega_0 t+\gamma_0}$$

$$H_+ = \frac{\cos 2\psi - \jmath\eta\sin 2\psi}{\sqrt{1+\eta^2}} \qquad H_\times = \frac{\sin 2\psi + \jmath\eta\cos 2\psi}{\sqrt{1+\eta^2}}$$

$$A_+ = a_0 + a_{1c}\cos\Omega t + a_{1s}\sin\Omega t + a_{2c}\cos 2\Omega t + a_{2s}\sin 2\Omega t$$

$$A_\times = b_{1c}\cos\Omega t + b_{1s}\sin\Omega t + b_{2c}\cos 2\Omega t + b_{2s}\sin 2\Omega t$$

It can be rewritten in terms of Signal 5-VECs $\mathbf{A}^+ \quad \mathbf{A}^\times$

$$\hat{H}_{+/x} = \frac{\mathbf{X}\cdot\mathbf{A}^{+/x}}{|\mathbf{A}^{+/x}|^2} \rightarrow H_0 e^{i\gamma} H_{+/x}$$

- **5n-vector method,** extension to a network of n detectors

$$\mathbf{X} = [\mathbf{X}_L, \mathbf{X}_H] \qquad \mathbf{A}^+ = [\mathbf{A}_L^+, \mathbf{A}_H^+] \qquad \mathbf{A}^\times = [\mathbf{A}_L^\times, \mathbf{A}_H^\times]$$

$$\tilde{S} = |\mathbf{A}^+|^4|\hat{H}_+|^2 + |\mathbf{A}^\times|^4|\hat{H}_\times|^2$$ → 5n-vec definition

$$S = \frac{|\mathbf{A}^+|^4}{\sum_{j=1}^n \sigma_j^2\cdot T_j\cdot|\mathbf{A}_j^+|^2}|\hat{H}_+|^2 + \frac{|\mathbf{A}^\times|^4}{\sum_{j=1}^n \sigma_j^2\cdot T_j\cdot|\mathbf{A}_j^\times|^2}|\hat{H}_\times|^2$$ → “normalized” definition

Fix pulsars to analyze i.e. N !

Experimental S distribution

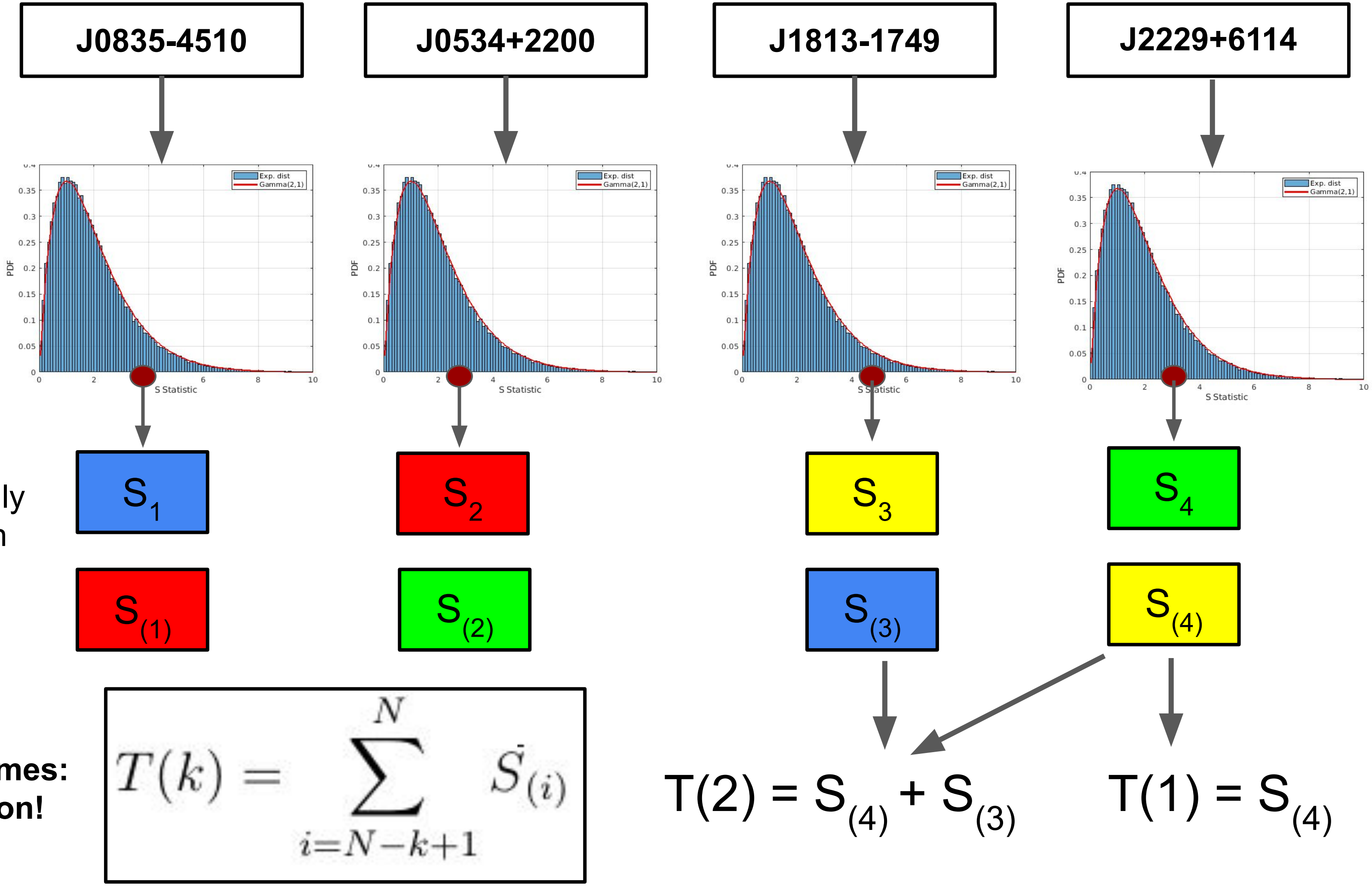
Monte Carlo procedure:

1. Select randomly a point from each distribution

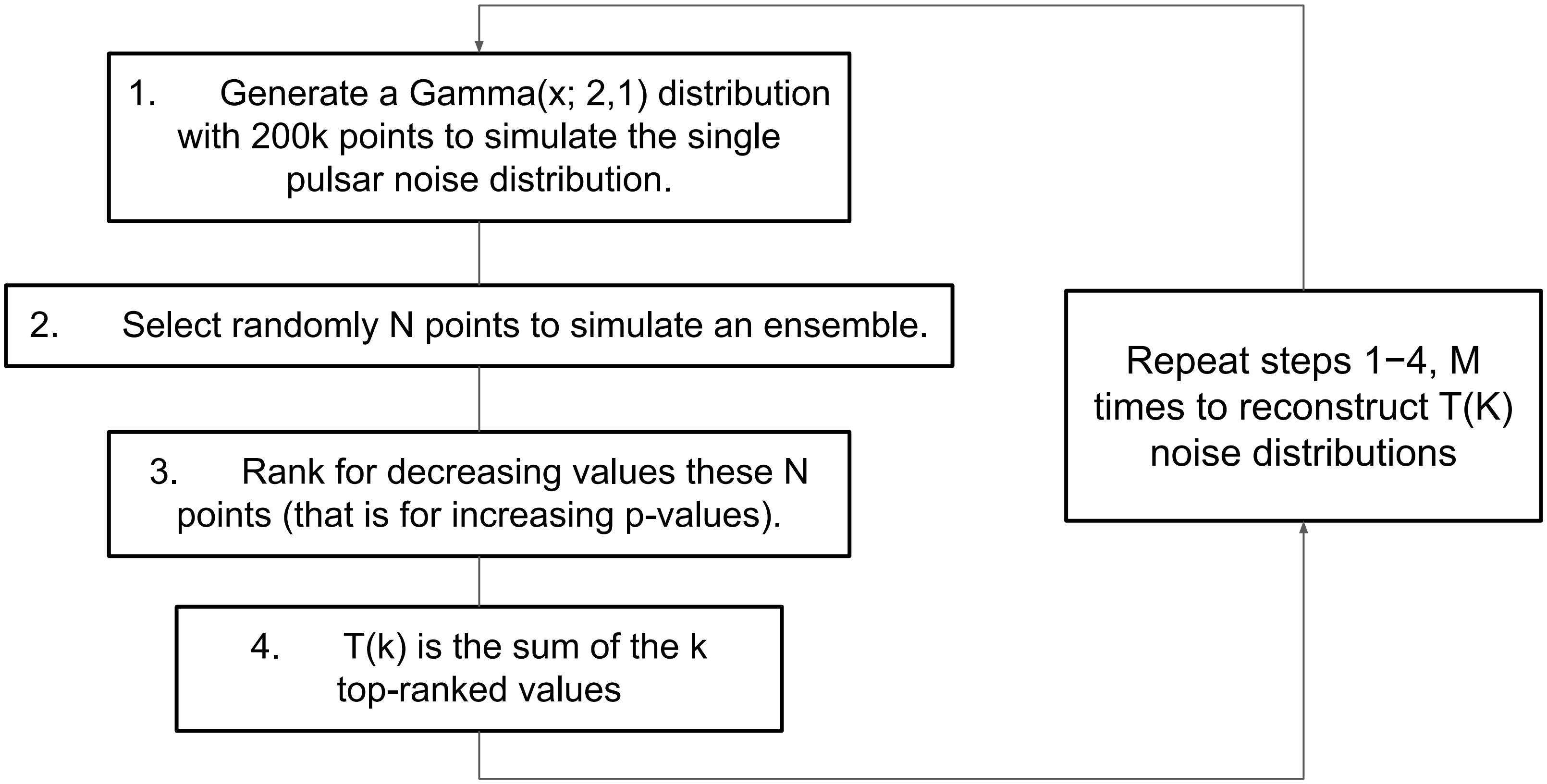
2. Order

3. Partial sum

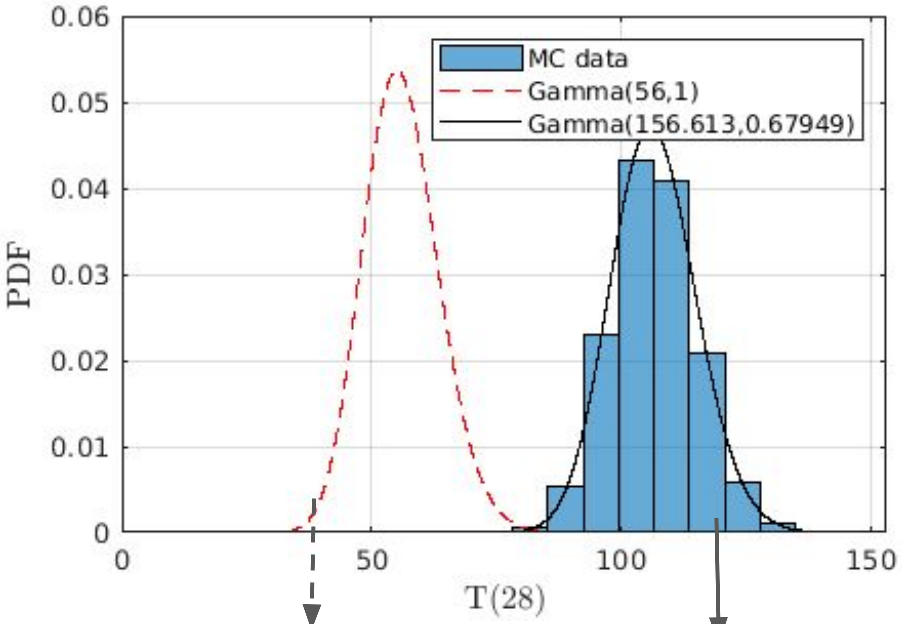
Ripet 1-2-3 M times: T(k) distribution!

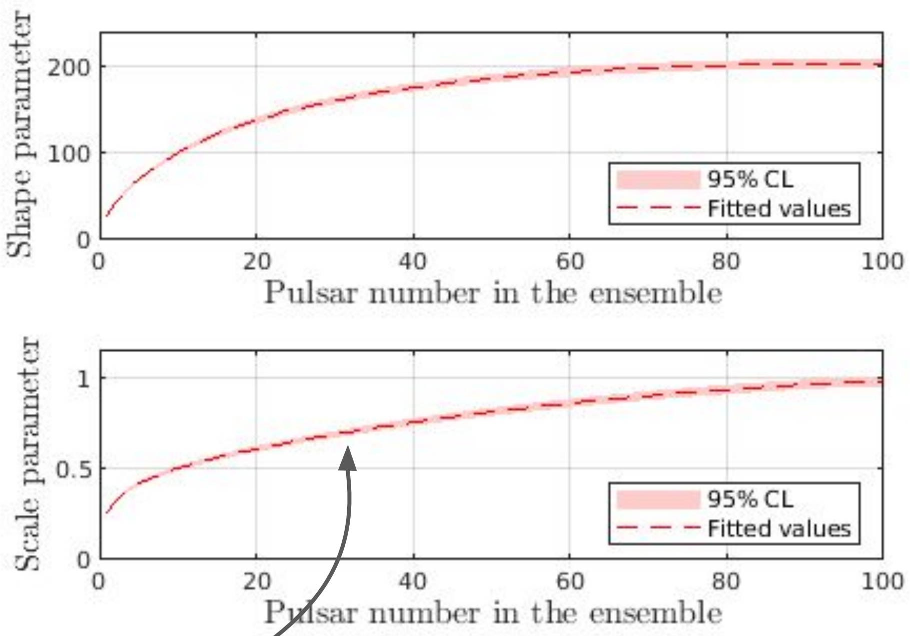


- MC_noise.m


1. Generate a Gamma(x; 2,1) distribution with 200k points to simulate the single pulsar noise distribution.
2. Select randomly N points to simulate an ensemble.
3. Rank for decreasing values these N points (that is for increasing p-values).
4. T(k) is the sum of the k top-ranked values
Repeat steps 1−4, M times to reconstruct T(K) noise distributions

noise_par = MC_noise(100, 'K' , 28)

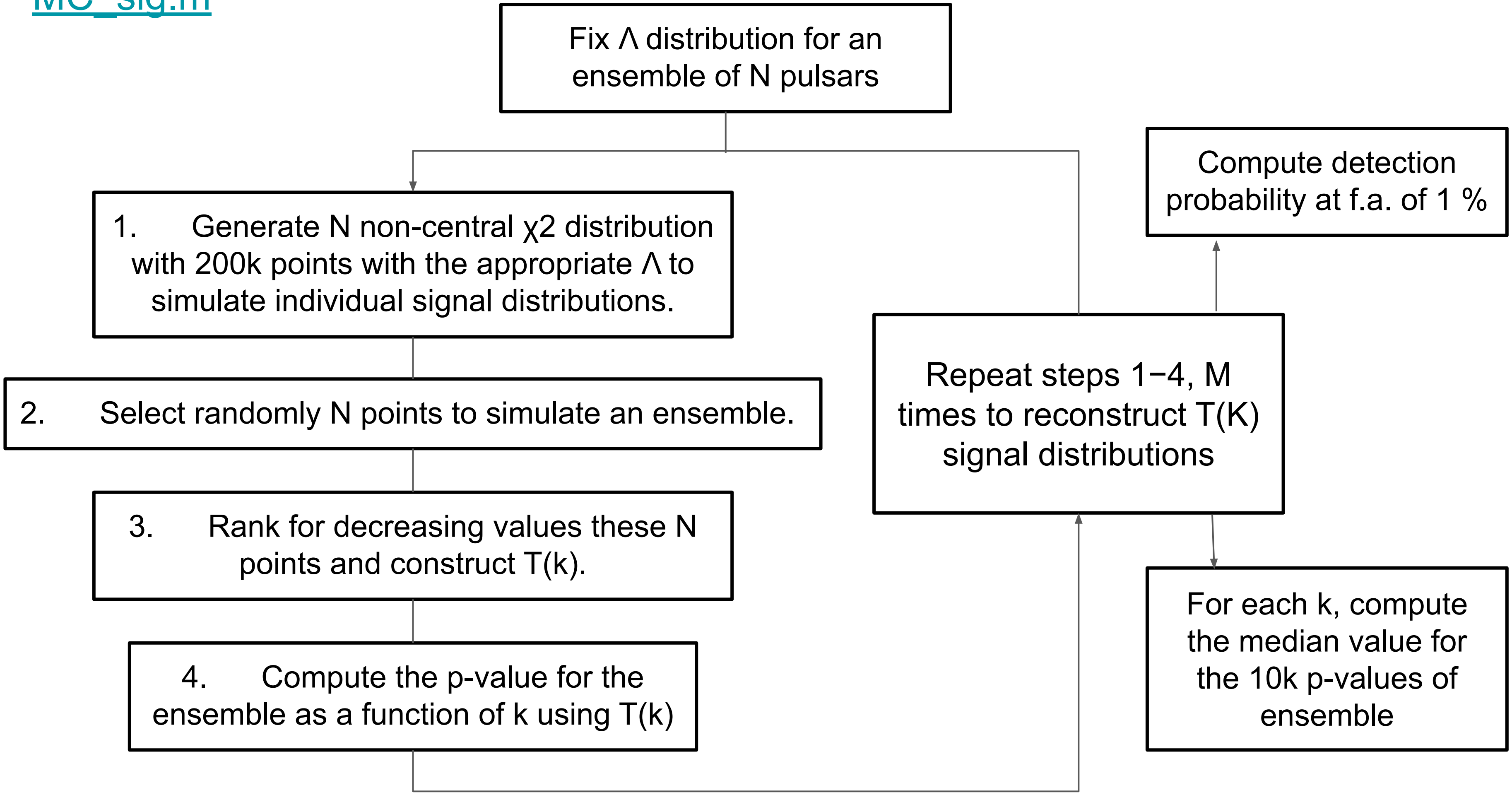




Simple sum of gamma random variable

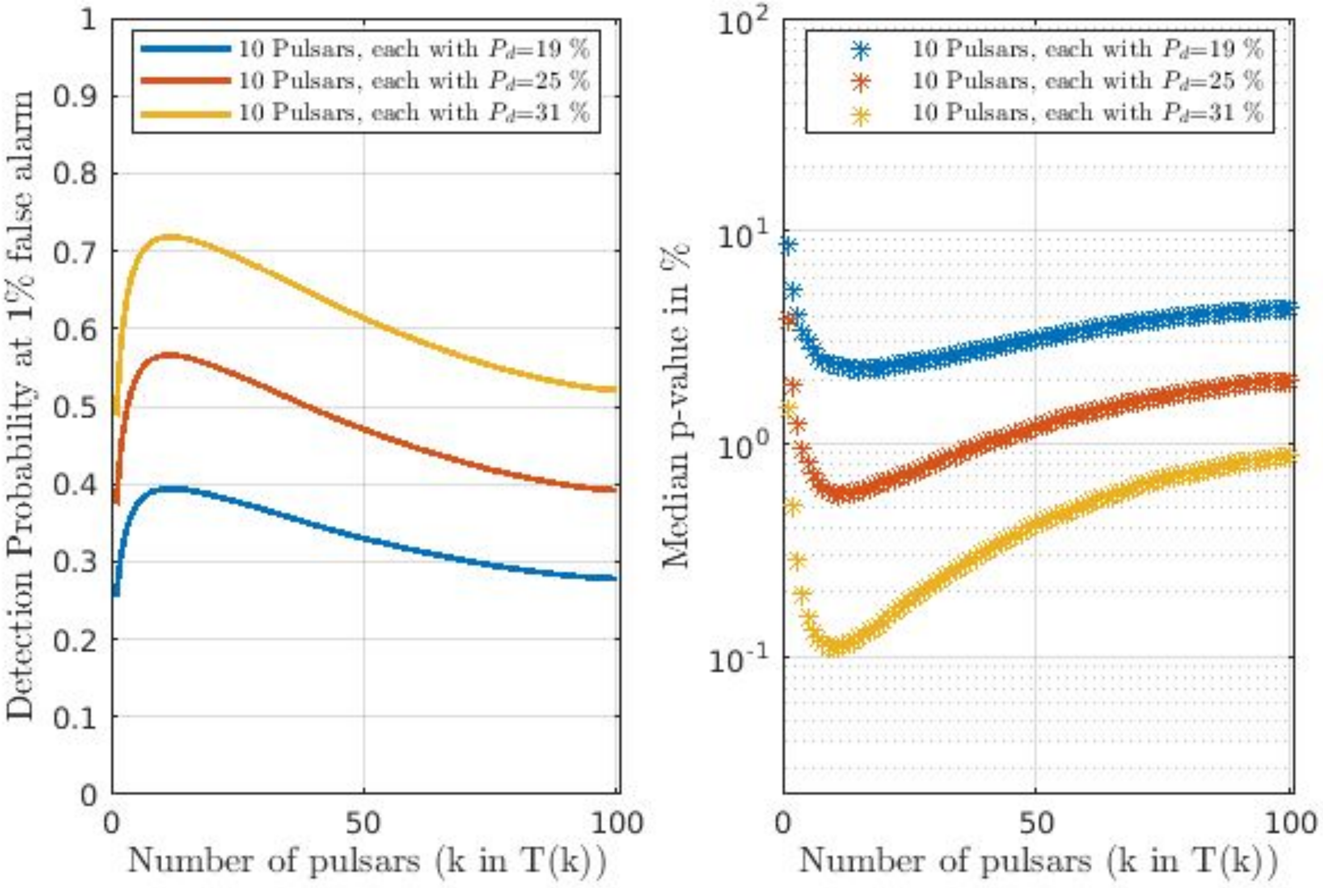
Fit using a Gamma (mle() matlab function)

- MC_sig.m

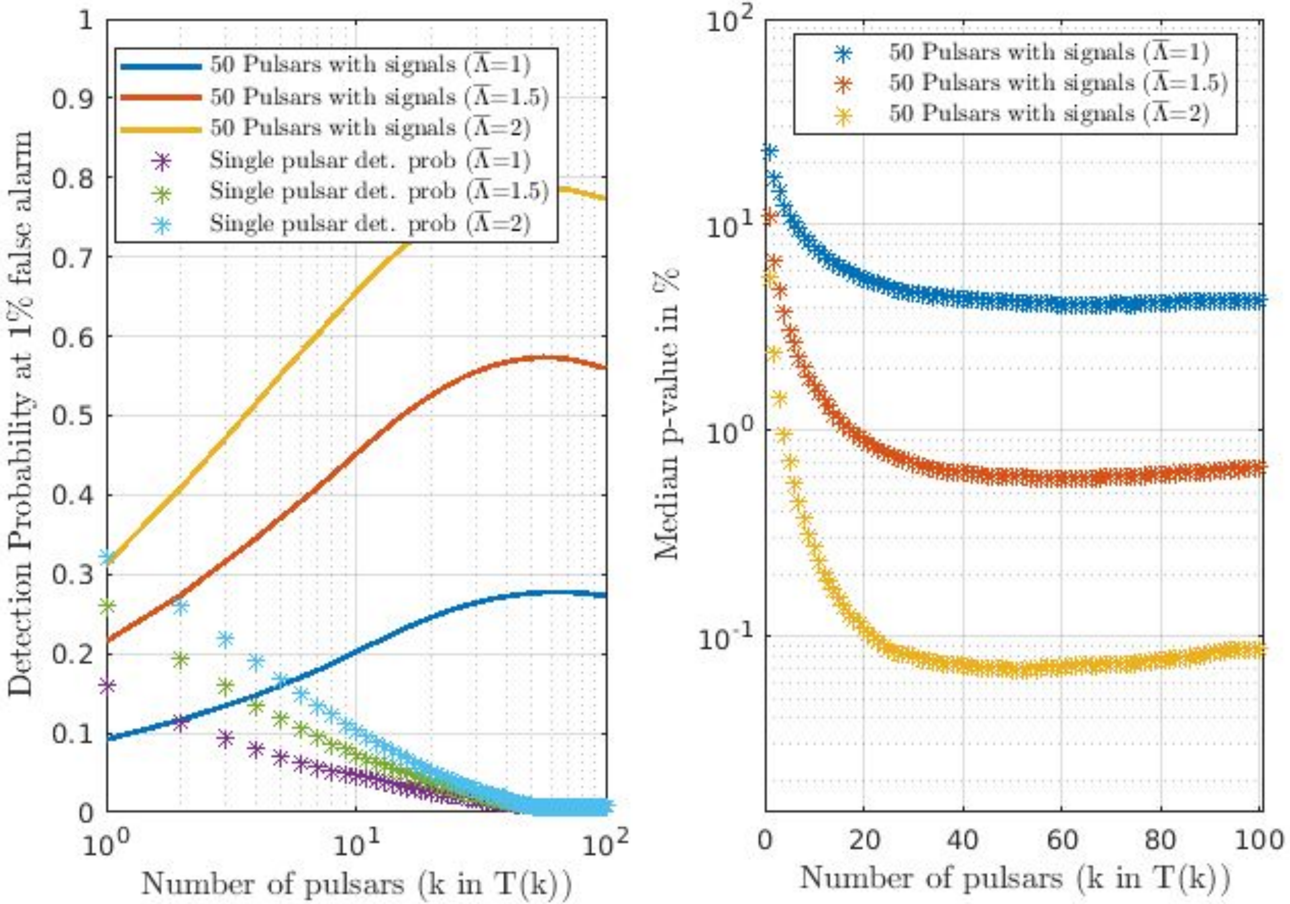


Fix Λ distribution for an ensemble of N pulsars
1. Generate N non-central χ2 distribution with 200k points with the appropriate Λ to simulate individual signal distributions.
2. Select randomly N points to simulate an ensemble.
3. Rank for decreasing values these N points and construct T(k).
4. Compute the p-value for the ensemble as a function of k using T(k)
Repeat steps 1−4, M times to reconstruct T(K) signal distributions
Compute detection probability at f.a. of 1 %
For each k, compute the median value for the 10k p-values of ensemble

```
[median_pv_MC, pd,pd_S]=MC_sig(noise_par,'ParDist','flat','ParValue',[5,6,7],'SigNum',10);
```



[median_pv_MC, pd,pd_S]=MC_sig(noise_par,'ParDist','exp','ParValue',[1,1.5,2],'SigNum',50);



- ensemble_analysis.m