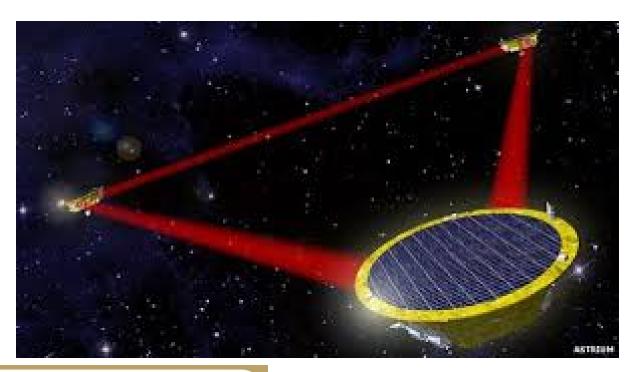
End-to-End simulation of LISA interferometric link

C.P.Sasso & G.Mana



INRIM - Strada delle cacce, 91 10135 Torino ITALY www.inrim.it

c.sasso@inrim.it



















143 Laboratories

130.000 m² campus





29 M€ di founded reserach projects

225
reserachers
and
technicians







Avogadro project – measuring N_A

Crystal density method

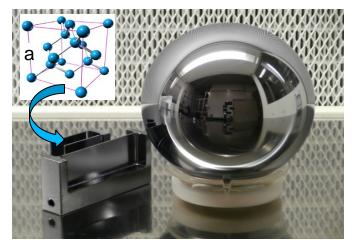
$$N_{\rm A} = \frac{M}{m(Si)} = \frac{8M}{\rho_m a^3}$$

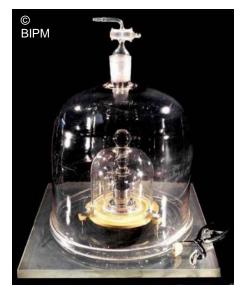
M Molar mass of Si₂₈;

 p_m mean density m_{sp}/V_{sp} ;

volume of the unit cell;

8 Atoms in the unit cell;





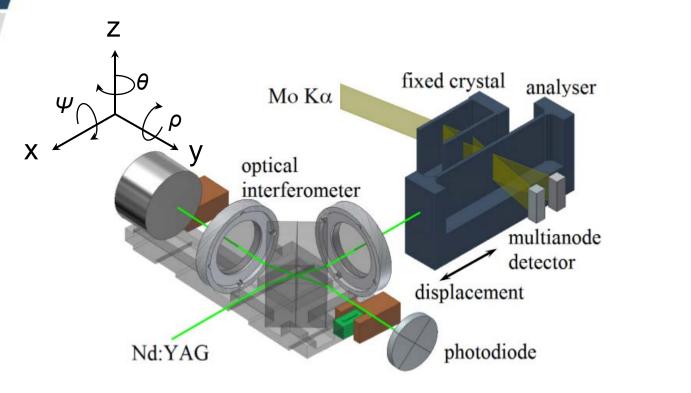






Unit cell volume

Combining x-ray and optical interferometry



 ρ, ϑ nrad

 Ψ µrad

x pm

y, z nm (frazioni di)

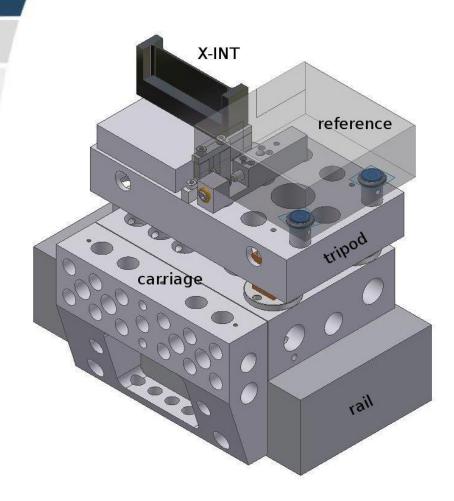
Active digital controls on the 6 mechanical DoF

Pitch (ρ) , Yaw (ϑ) and displacement x – Optical interferometry;

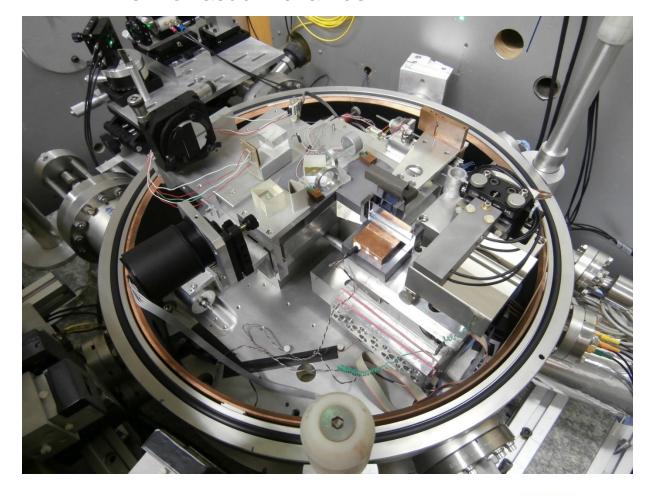
Roll (Ψ) and translation z-y – capacitive sensors (less demanding);



Unit cell volume



Thermo vacuum chamber



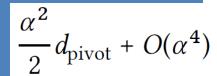


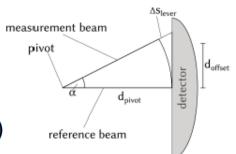
LISA - Phase noise

- Wavefront changes of the beam at the detector light passing at different angle in the telescope – optics instabilities
- Jitter
 – far field: not perfectly spherical wave coupling between angle of jitter and the aberrations on the detector TILT TO LENGTH COUPLING

Geometrical effect

Local interferometer



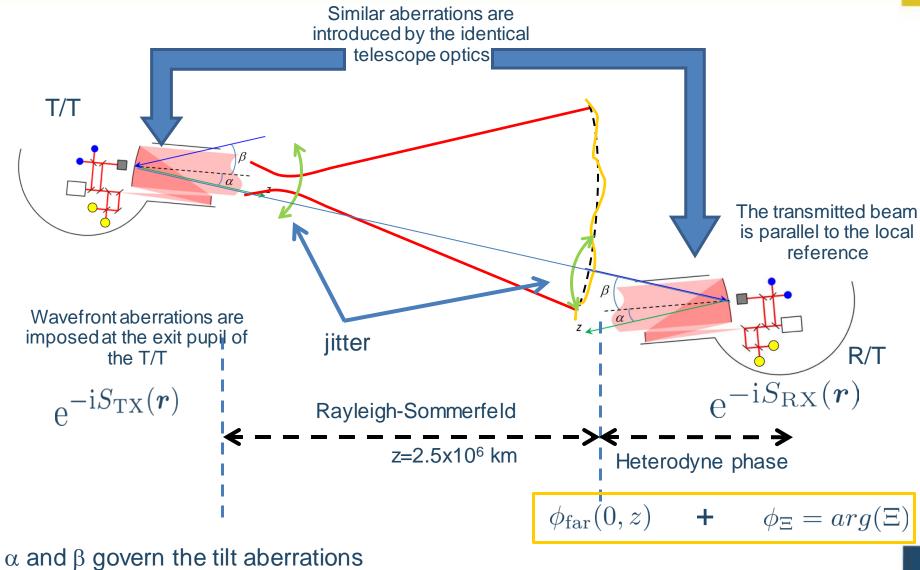


Instabilities of the path-length in the telescope



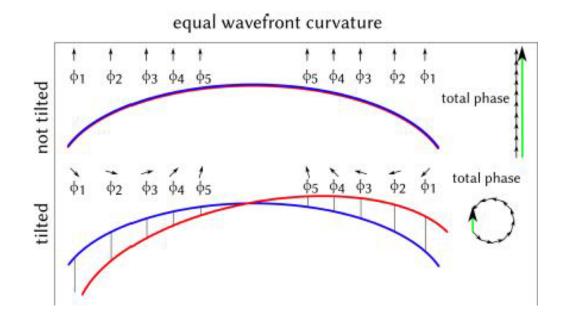


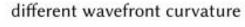


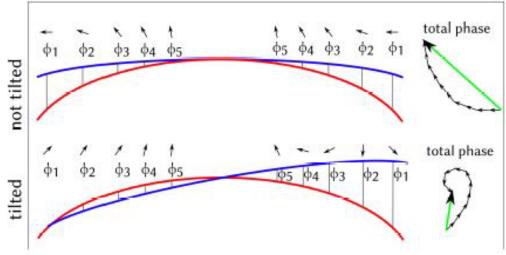




Non-geometric tilt to length noise







Sönke Schuster, M. Sc. Ph.D. thesis 2017







Project founded by ESA and coordinated by Thales Alenia Space Italy (Coord'r S. Mottini)

Metrology Telescope Design for a Gravitational Wave Observatory(MTD)

INRIM's WP: Criteria for beam quality and performance assessment applicable to the

Metrology Telescope for a GWO

PAPER • OPEN ACCESS

Coupling of wavefront errors and jitter in the LISA interferometer: far-field propagation

To cite this article: C P Sasso et al 2018 Class. Quantum Grav. 35 185013

PAPER

Coupling of wavefront errors and pointing jitter in the LISA interferometer: misalignment of the interfering wavefronts

C P Sasso¹ D, G Mana¹ D and S Mottini²
Published 13 November 2018 • © 2018 IOP Publishing Ltd
Classical and Quantum Gravity, Volume 35, Number 24
Citation C P Sasso *et al* 2018 *Class. Quantum Grav.* 35 245002

Research Article

Vol. 27, No. 12 | 10 Jun 2019 | OPTICS EXPRESS 16855

Optics EXPRESS

Telescope jitters and phase noise in the LISA interferometer

CARLO PAOLO SASSO,1,* GIOVANNI MANA,1 AND SERGIO MOTTINI2





Assumptions and constraints



Telescopes are identical



Scalar and paraxial optics



No dynamical effect on the beam propagation

Transfer function of jitter into phase is frequency independent



Telescopes point ahead to compensate the relative motions of the satellites



The telescope operates between two conjugate pupils w/o translation on the detector



RMS deviation from flatness of w/f is used as a quality parameter





Design numbers



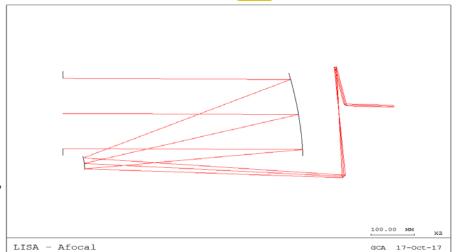
$$> \lambda = 1064 \text{ nm}$$



Telescopes off-axis, afocal

telescope main mirror: 300mm diameter

exit pupil: 2.24mm magnification 134x





$$S_{
m IFO}^{1/2}=10~{
m pm\over \sqrt{
m Hz}}\sqrt{1+\left({2~{
m mHz}\over f}
ight)^4}$$
 in the frequency range from 0.1 mHz to 0.1 Hz



ightharpoonup Jitter 10 nrad \sqrt{Hz}



Point ahead angle about 1 µrad





Mathematical model

Far field

Propagation of a truncated Gaussian beam $E(\mathbf{r}, z; t) = e^{-r^2/w_{\text{TX}}^2} e^{-iS_{\text{TX}}(\mathbf{r})} e^{-i(kz - \omega t)}$ The beam has a Gaussian intensity profile but a "almost" flat wavefront

Rayleigh-Sommerfeld integral



$$u(\boldsymbol{r};z) = \frac{\mathrm{i}k\mathrm{e}^{-\mathrm{i}kr^2/(2z)}}{2\pi z} \int_{\mathcal{M}} \mathrm{e}^{\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{\xi}/z} \mathrm{e}^{-\xi^2/w_{\mathrm{TX}}^2} \mathrm{e}^{-\mathrm{i}S_{\mathrm{TX}}(\boldsymbol{\xi})} \,\mathrm{d}\boldsymbol{\xi}$$

Primary mirror of the telescope

kernel is an additional defocus

Spherical wave...aberrated

$$u(\boldsymbol{r};z) \approx \frac{\mathrm{i}k|u(0;z)|\mathrm{e}^{-\mathrm{i}[kr^2/(2z)+\phi_{\mathrm{far}}]}}{2\pi z}$$

On-axis advance or delay

$$\phi_{\text{far}} = \arg[u(0; z)]$$





Mathematical model cont.

Heterodyne interferometry

$$E_1(\boldsymbol{r};t) = \mathrm{e}^{-r^2/w_1^2} \mathrm{e}^{-\mathrm{i}S_1(\boldsymbol{r})}$$
 Local refrence

$$E_2(\boldsymbol{r};t) = \mathrm{e}^{-r^2/w_2^2} \mathrm{e}^{-\mathrm{i}S_2(\boldsymbol{r})} \mathrm{e}^{-\mathrm{i}(kz+\phi_{\mathrm{far}}+\Omega t)}$$
 Received



Gaussian intensity & aberrated 'flat' profile

$$\Xi = \int_{\mathcal{D}} u_1^*(\boldsymbol{r}) u_2(\boldsymbol{r}) \, \mathrm{d}\boldsymbol{r} = \int_0^{r_D} r \mathrm{e}^{-2r^2/w_{\mathrm{RX}}^2} \left(\int_0^{2\pi} \mathrm{e}^{-\mathrm{i}S_{\mathrm{RX}}(\boldsymbol{r})} \, \mathrm{d}\boldsymbol{\theta} \right) \, \mathrm{d}\boldsymbol{r}$$
 detector area
$$S_{\mathrm{RX}}(\boldsymbol{r}) = S_2(\boldsymbol{r}) - S_1(\boldsymbol{r})$$
 Small deviation from

flatness

$$\phi_{\Xi} = \arg(\Xi)$$







Mathematical model cont.



Far field

$$\phi_{\text{far}} = \arg[u(0;z)]$$

We consider only the on axis value

$$u(0;z) = \int_{\mathcal{M}} e^{-\xi^2/w_{\text{TX}}^2} e^{iS_{\text{TX}}(\xi)} d\xi = \int_0^{r_{\text{TX}}} \xi e^{-\xi^2/w_{\text{TX}}^2} \left(\int_0^{2\pi} e^{-iS_{\text{TX}}(\xi)} d\theta \right) d\xi$$



> Heterodyne interferometry

$$\phi_{\Xi} = \arg(\Xi)$$

Resemble each other

$$\Xi = \int_{\mathcal{D}} u_1^*(\boldsymbol{r}) u_2(\boldsymbol{r}) d\boldsymbol{r} = \int_0^{r_D} r e^{-2r^2/w_{RX}^2} \left(\int_0^{2\pi} e^{-iS_{RX}(\boldsymbol{r})} d\theta \right) dr$$







$$S_{\text{TX,RX}}(\mathbf{r}) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} z_n^m R_n^{|m|}(\rho) e^{im\theta} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} z_n^m Z_n^m(\rho, \theta)$$

First order terms govern the tilt

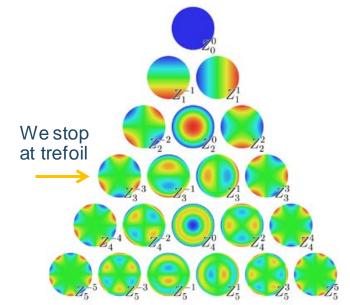
$$z_1^{-1} \rho e^{-i\theta} + z_1^1 \rho e^{i\theta} = 2|z_1^1| \rho \cos(\theta + \theta_1^1)$$

$$\begin{cases} kr_{\rm TX}\alpha\rho\cos(\theta+\theta_1^1) & \text{transmitted wavefront} \\ kr_{\rm RX}\beta\rho\cos(\theta+\theta_1^1) & \text{detected wavefronts} \end{cases}$$

$$r_{RX} = Mr_D$$
 effective radius

 $lpha=2|z_1^1|/(kr_{
m TX})$ is null when transmission occurs along z axis

$$eta=2|z_1^1|/(kr_{
m RX})$$
 is null when the local beam is parallel to the trans. beam











Tilt-to-Length coupling

Analysis is limited to the lowest order coupling between TILT and the other aberrations

$$\phi_{\text{far}} = \arg[u(0;z)] \approx a_{00} + a_{10}\zeta_x + a_{20}\zeta_x^2 + a_{01}\zeta_y + a_{02}\zeta_y^2 + a_{11}\zeta_x\zeta_y$$

$$\zeta_x = k r_{\rm TX} \alpha_x / 2$$

$$\zeta_y = k r_{\rm TX} \alpha_y / 2$$

Coefficients depend in a complicated way on the normalized beam waist and aberrations amplitudes

$$\phi_{\Xi} = \arg(\Xi) \approx b_{00} + b_{10}\eta_x + b_{20}\eta_x^2 + b_{01}\eta_y + b_{02}\eta_y^2 + b_{11}\eta_x\eta_y$$

$$\eta_x = k r_{\rm RX} \beta_x / 2$$

$$\eta_y = k r_{\rm RX} \beta_y / 2$$

Minimization of the TTL coupling is always possible

by finding the transmission/receiving directions that nullify the sensitivity to jitter (Sasso C. et al. Class. Quantum Grav. 35 (2018) 245002)





Noise transfer function ${\cal H}$



Noise variance link (Normal uncorrelated jitter)

$$\sigma_{\phi}^{2} \approx \left[|\nabla_{\alpha} \phi_{\text{far}}|_{0}^{2} + |\nabla_{\beta} \phi_{\Xi}|_{0}^{2} \right] \sigma_{\text{jitter}}^{2} / 4$$
$$\phi = \phi_{\text{far}} + \phi_{\Xi}$$

$$\mathcal{H}^{2} \approx \frac{1}{4} \left[|\nabla_{\alpha} \phi_{\text{far}}|_{0}^{2} + |\nabla_{\beta} \phi_{\Xi}|_{0}^{2} \right]$$

$$= \left(\frac{k w_{\text{TX}}}{4} \right)^{2} \left[(a_{10} + 2a_{20}\zeta_{0x} + a_{11}\zeta_{0y})^{2} w_{\text{TX}}^{\prime - 2} + (a_{01} + 2a_{02}\zeta_{0y} + a_{11}\zeta_{0x})^{2} w_{\text{TX}}^{\prime - 2} \right]$$

$$+ (b_{10} + 2b_{20}\eta_{0x} + b_{11}\eta_{0y})^{2} w_{\text{RX}}^{\prime - 2} + (b_{01} + 2b_{02}\eta_{0y} + b_{11}\eta_{0x})^{2} w_{\text{RX}}^{\prime - 2} \right]$$





Results: Monte Carlo simul.'s



Amplitude averaging (with RMS of the amplitude upper bounded...)

$$\sigma_S^2 = \sum_{n=2}^{\infty} \sum_{m=0}^{n} \frac{c_n^m |z_n^m|^2}{n+1}$$

$$\sigma_S^2 = 75 \, \mathrm{mrad} \Longrightarrow \lambda/16$$
 PV amplit.

A single parameter to assess the quality of the telescope optics

Sensitivity coefficients $\langle \mathcal{H}^2 \rangle_{z_n^m} \approx \left[\overline{g_0(w'_{\mathrm{TX}})} + \overline{g_0(w'_{\mathrm{RX}})} \right] \sigma_S^4 + \overline{g_2(w'_{\mathrm{TX}})} \sigma_S^2 \alpha_0^2 + \overline{g_2(w'_{\mathrm{RX}})} \sigma_S^2 \beta_0^2$

do not depend on Zernike polynomials amplitude

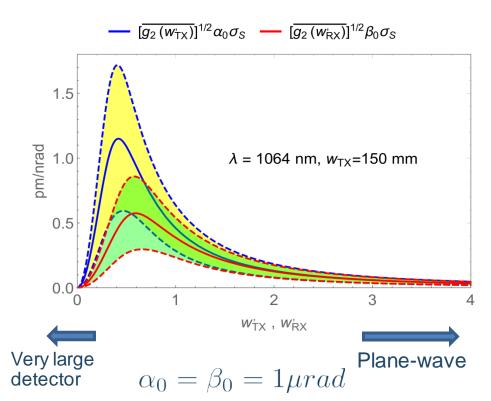




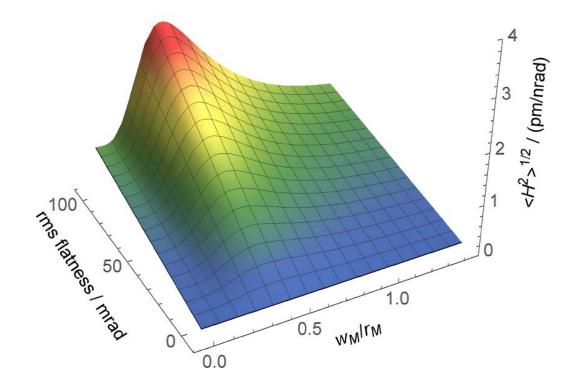


Results: Monte Carlo simul.'s





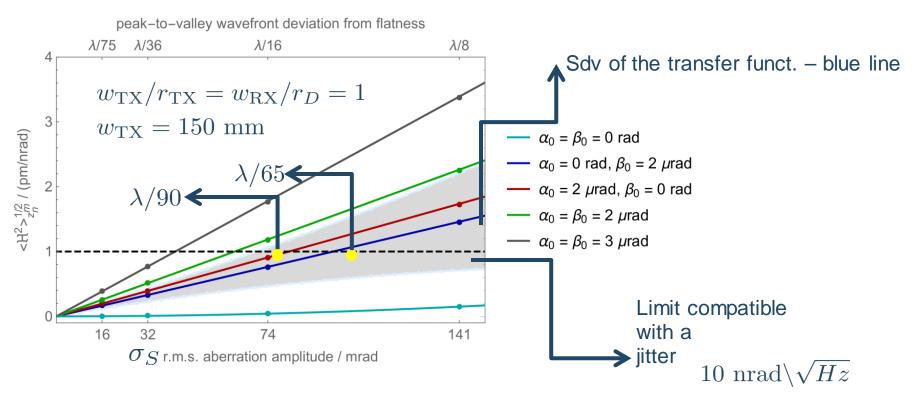








Results: Monte Carlo simul.'s



Dots are numerical integrations – solid lines are analytic formulas





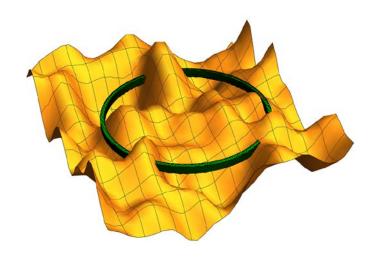
Thank you for the attention

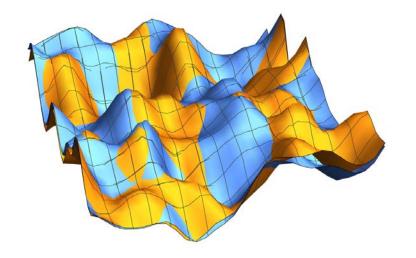
THE END



Transmitted waveform - High frequency aberrations

$$u(0;z) = \int_{\mathcal{M}} e^{-\xi^2/w_{\text{TX}}^2} e^{iS_{\text{TX}}(\xi)} d\xi = \int_0^{r_{\text{TX}}} \xi e^{-\xi^2/w_{\text{TX}}^2} \left(\int_0^{2\pi} e^{-iS_{\text{TX}}(\xi)} d\theta \right) d\xi$$





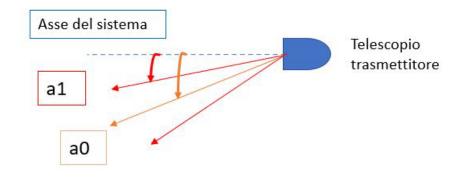
Numerical computation of the Rayleigh-Sommerfeld integral

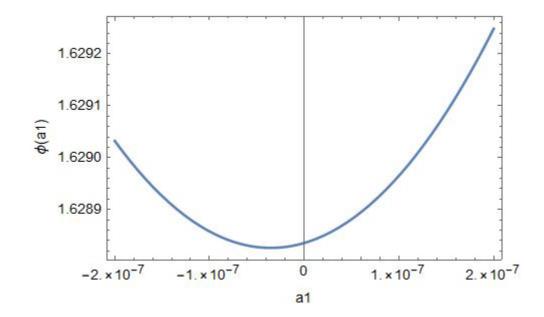
Analytical integral through DCT decomposition





• Jitter angle: $A=a0\pm a1$ ampiezza del Tilt $a0=\lesssim 1~\mu rad \qquad a0=500~nrad$ $a1=10\sim 20~nrad$









- Specchio
$$\frac{\lambda}{20}$$

$$\Delta \phi \sim 10^{-4} rad \rightarrow \Delta s \sim 10^{-11} m$$

- Specchio
$$\frac{\lambda}{50}$$

$$\Delta \phi \sim 10^{-5} rad \rightarrow \Delta s \sim 10^{-12} m$$

