Mode matching sensing through RF Higher Order Modulation
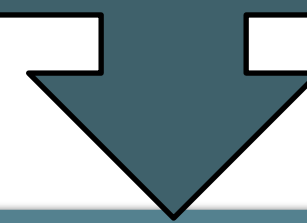
G. Chiarini, PhD student

G. Ciani, Professor

GRASS 2022
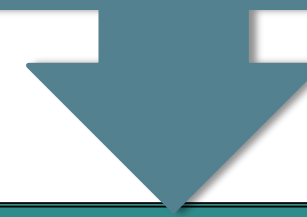
WHY MODE MATCHING?

One of the main losses in optical cavities is the mode mismatch:

not perfect accordance between the waist size and position of the input beam and the one supported by the cavity

need an online sensing technique to obtain informations on the level of mismatch and on the causes

in AdV+ will improve the FDS squeezing level

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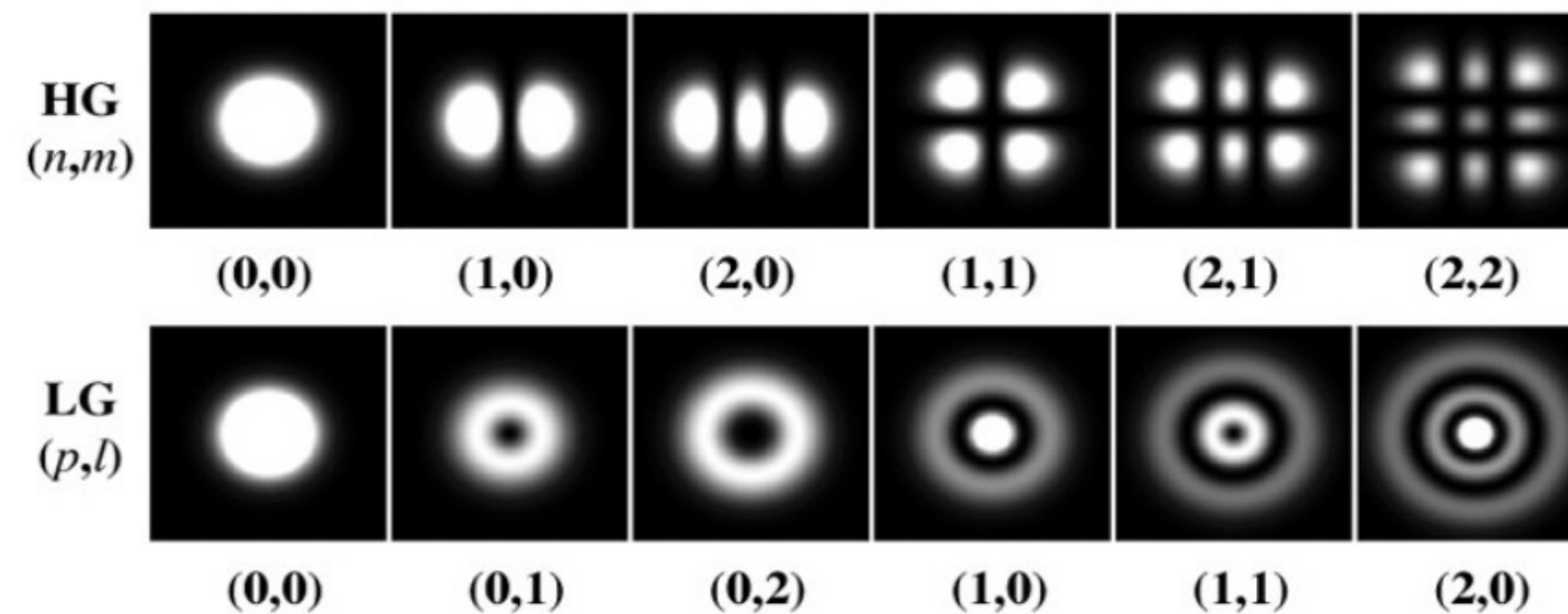
THE MODE-MISMATCH PROBLEM

Mode Matching

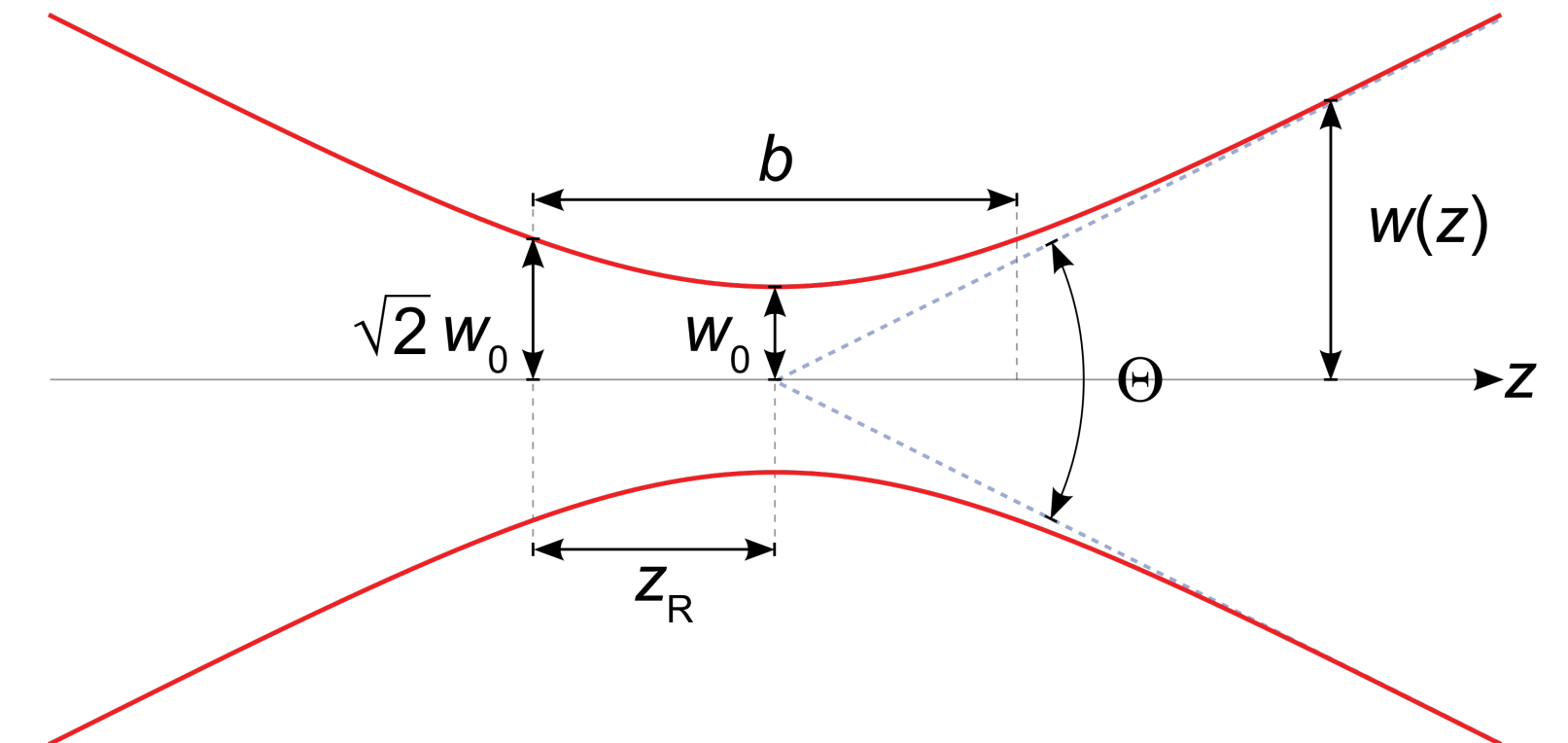Helmotz equation: $\nabla^2 f = -k^2 f$

Taking f as $\tilde{E}(x, y, z) \equiv \tilde{u}(x, y, z)e^{jkz}$ with the paraxial approximation $|\frac{\partial^2 \tilde{u}}{\partial^2 z}| \ll |2k \frac{\partial \tilde{u}}{\partial z}|$

$$\frac{\partial^2 \tilde{u}}{\partial^2 x} + \frac{\partial^2 \tilde{u}}{\partial^2 y} = 2ik \frac{\partial \tilde{u}}{\partial z}$$
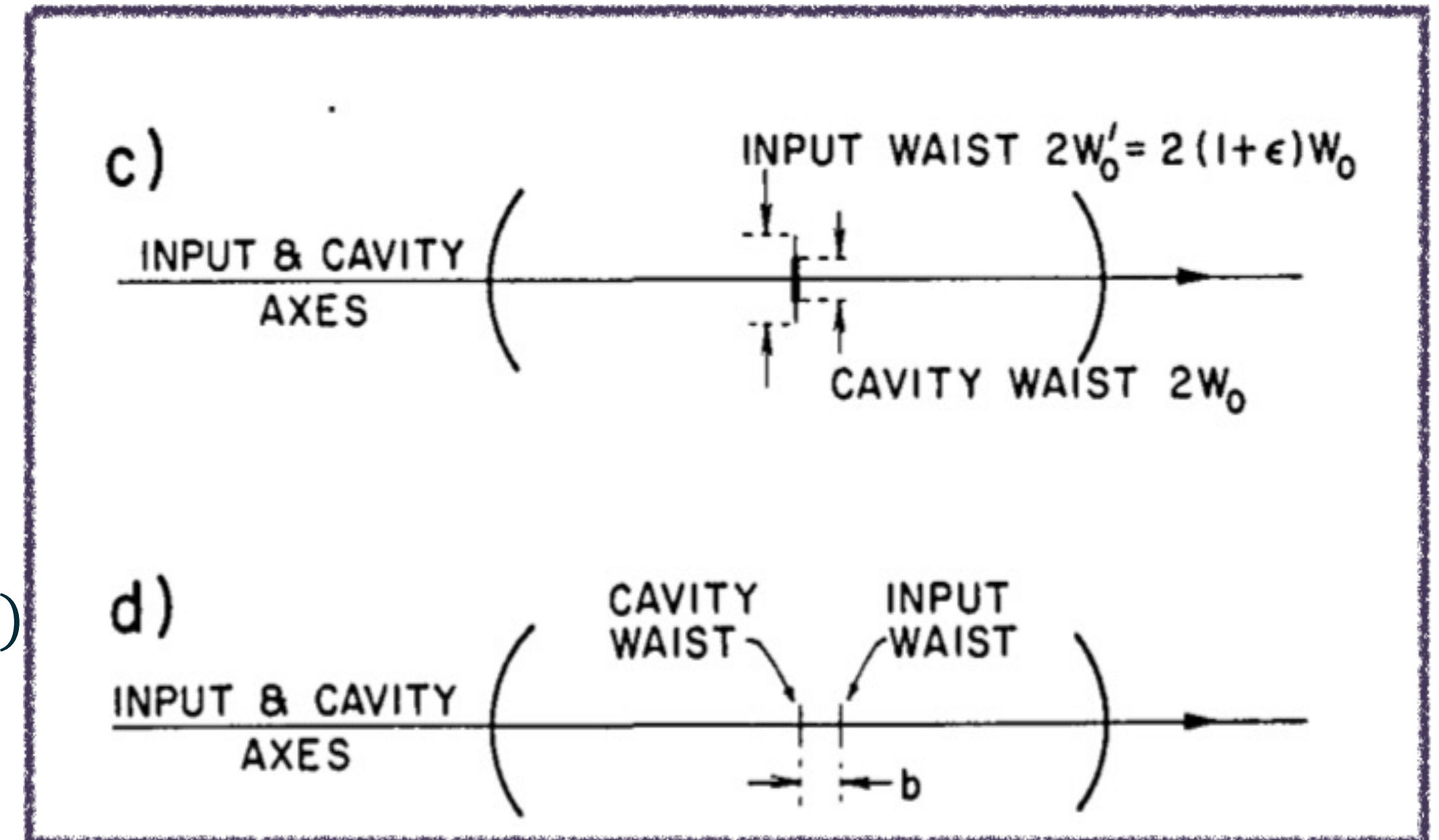
Hermite-Gauss modes

Laguerre-Gauss modes

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THE MODE-MISMATCH PROBLEM

Mode Matching

Mode Mismatch: discrepancy in waist size and position between the fundamental mode of the input beam and the one supported by the cavity

$$\beta \qquad \gamma$$

$$LG_{00}(w_0 + \delta w, z_0 + \delta z) = LG_{00}(w_0, z_0) - \left(\frac{\delta w}{w_0} - i\frac{\delta z}{2z_R}\right)LG_{10}(w_0, z_0)$$

Waist size mismatch   **Waist position mismatch**

c) INPUT WAIST $2W_0' = 2(1+\epsilon)W_0$

INPUT & CAVITY AXES

CAVITY WAIST $2W_0$

d) CAVITY WAIST   INPUT WAIST

INPUT & CAVITY AXES

b

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MODE MISMATCH SENSING

Method

How can we sense the mismatch of optical cavities?

We can use the LG10 reflected by the cavity when locked in the fundamental mode
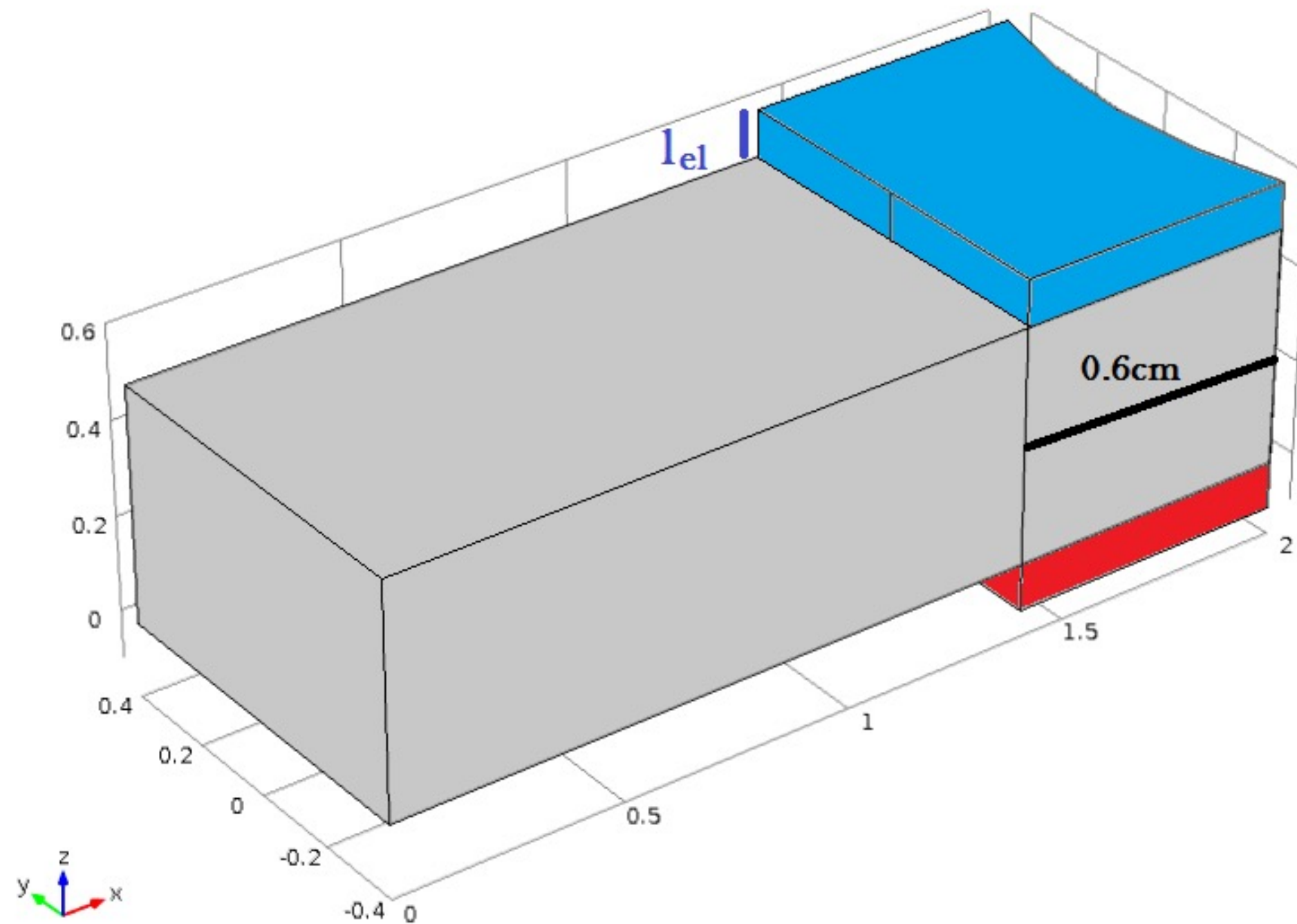
Beating with LG10 sideband to overcome orthogonality

I/Q demodulation using signal from a single element PD (the same for PDH)

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EOL PROJECT

Simulations

LG10 sidebands generated by modulating a lensing element: Electro-Optical Lens

1. Lithium Niobate (LiNbO$_3$) uniform crystal
   x=2cm, y=0.8cm, z=0.5cm
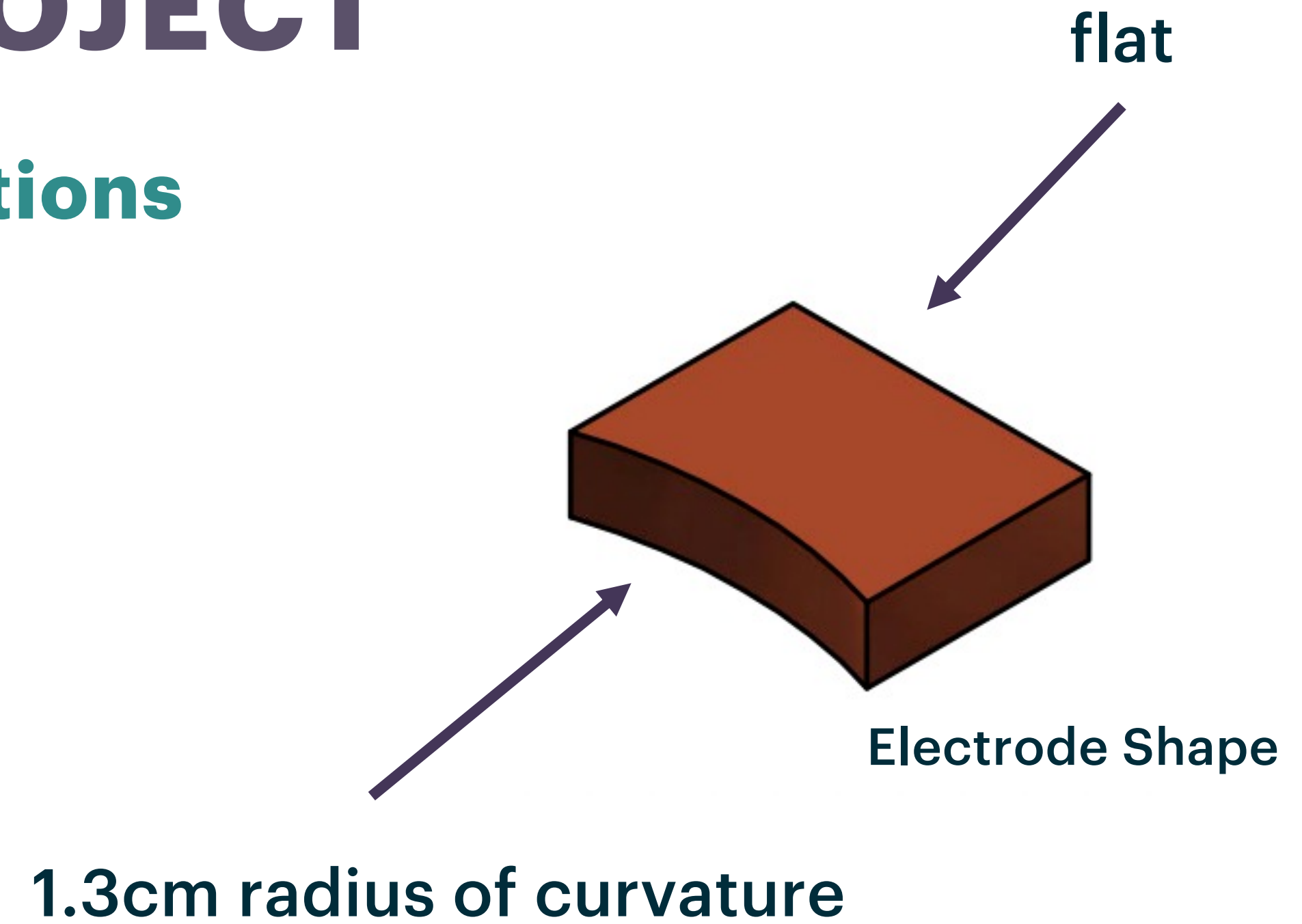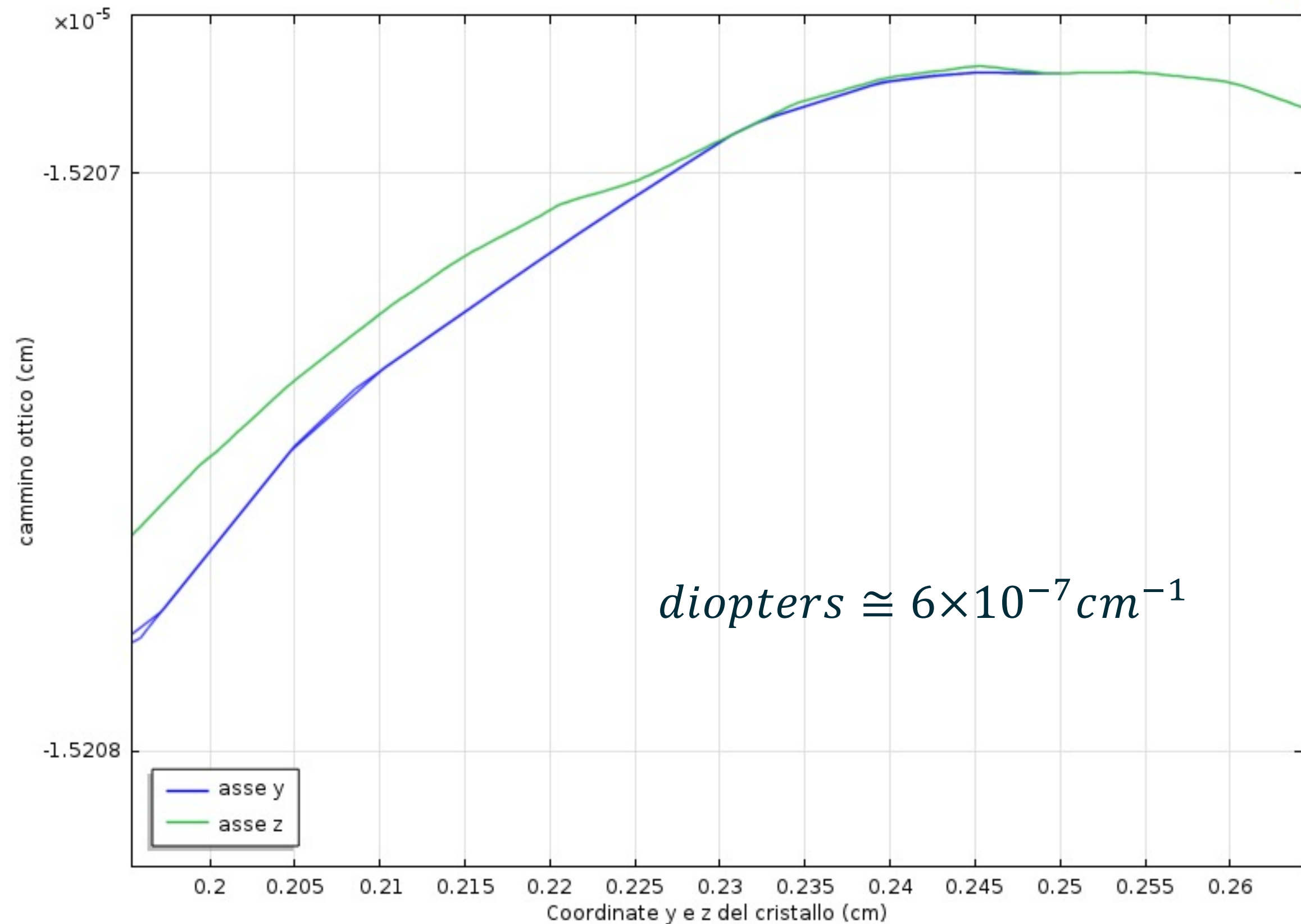2. Electrode with a particular shape (effect on the borders)

Pockels effect: refractive index change in the crystal when an electric field is applied to it

$$\Delta n(E_z) = \frac{1}{2} n_e^3 r_{33} E_z$$

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EOL PROJECT

Simulations

flat

Optical path difference vs y,z-coord of the crystal

$$diopters \cong 6 \times 10^{-7} cm^{-1}$$

Electrode Shape

1.3cm radius of curvature

The shape of the electrodes was simulated (COMSOL) in order to have a spherical profile of the phase delay at the exit of the EOL
x=0.6cm, y=0.8cm, z=0.05cm

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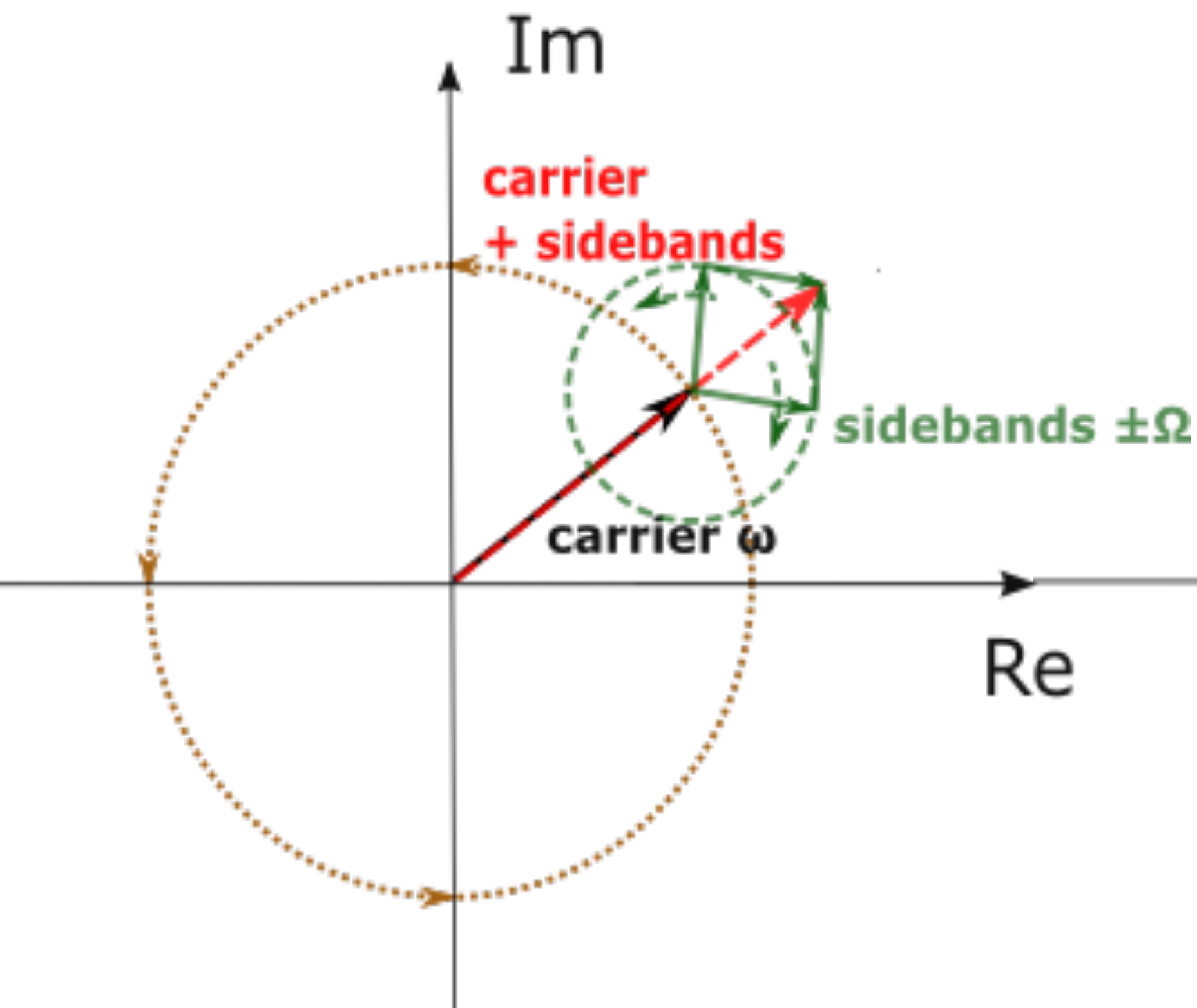
EOL RF MODULATION

After the EOL:

$$\frac{\delta w_0}{w_0^{IN}} = B = \frac{m_B}{2}\left(e^{i\Omega t} + e^{-i\Omega t}\right)$$

$$\frac{\delta z_0}{2z_R^{IN}} = G = \frac{m_G}{2}\left(e^{i\Omega t} + e^{-i\Omega t}\right)$$
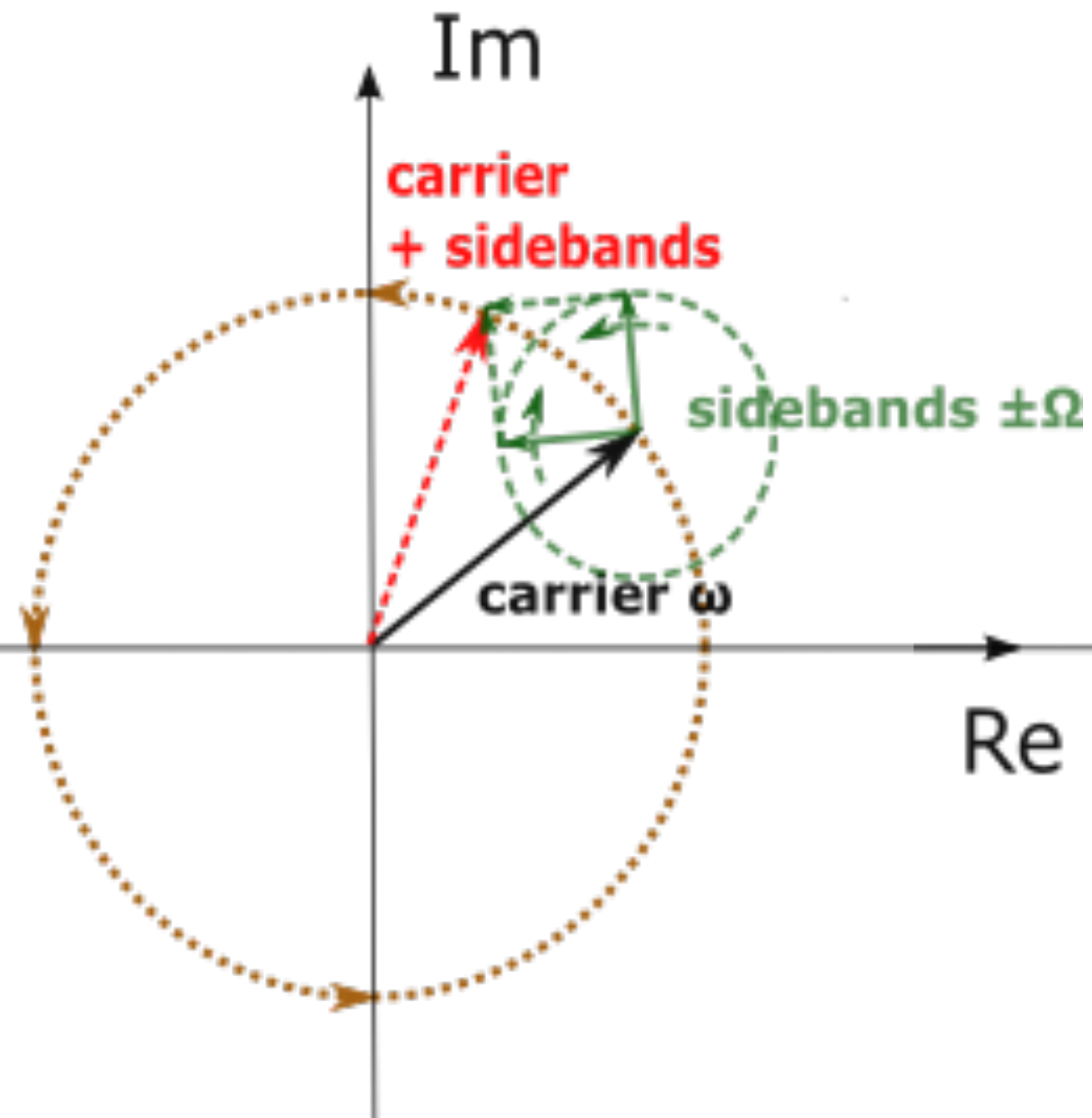
m_B and m_G are the modulation depths

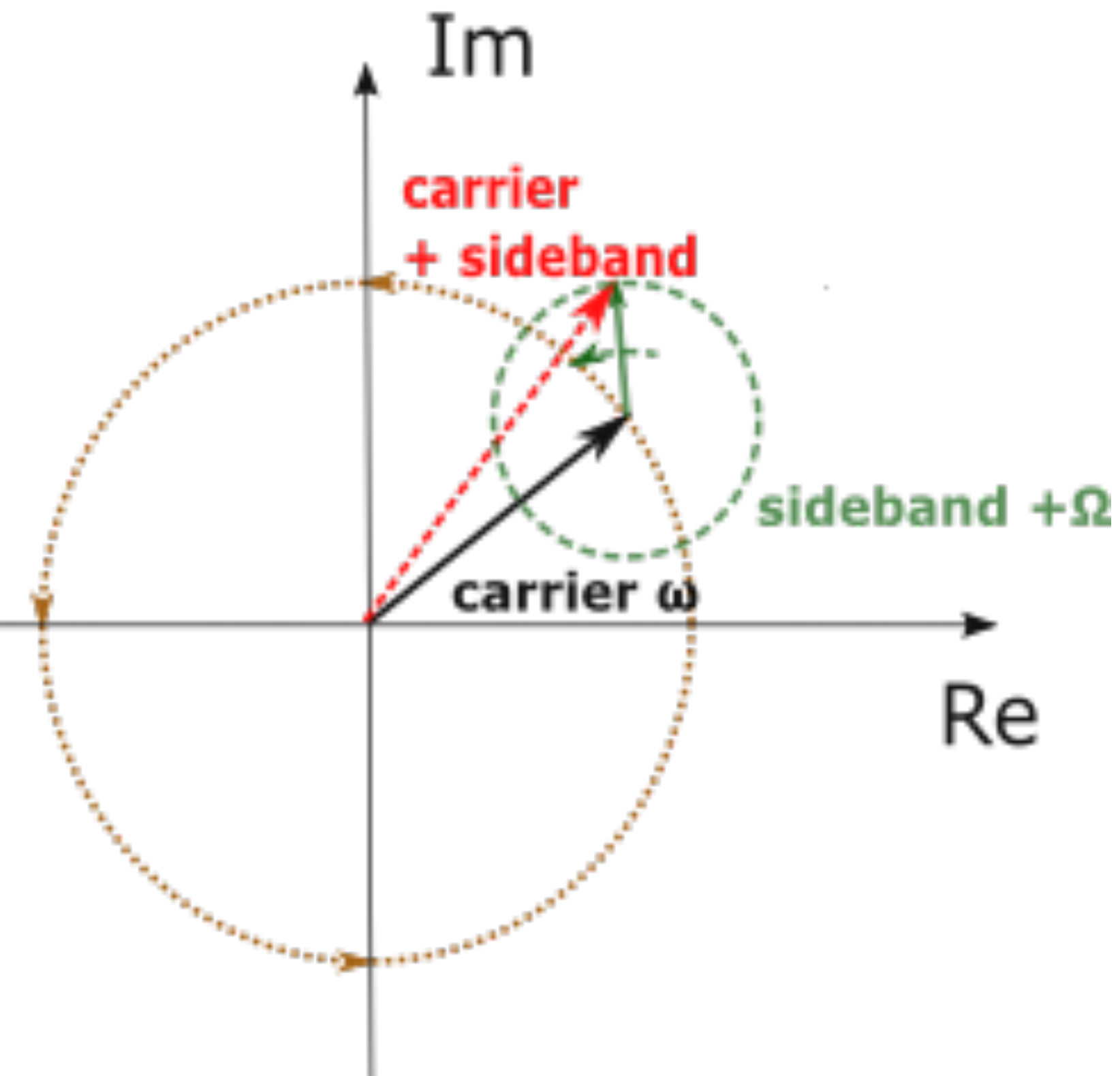**phase modulation at frequency Ω, to have amplitude modulation we need to break the sideband symmetry**

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EOL RF MODULATION

Amplitude modulation

Phase modulation

Modulation symmetry breaking

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EOL RF MODULATION

$FSR = 83.3 MHz$

$\Omega = 2HOMs = 30.75 MHz$
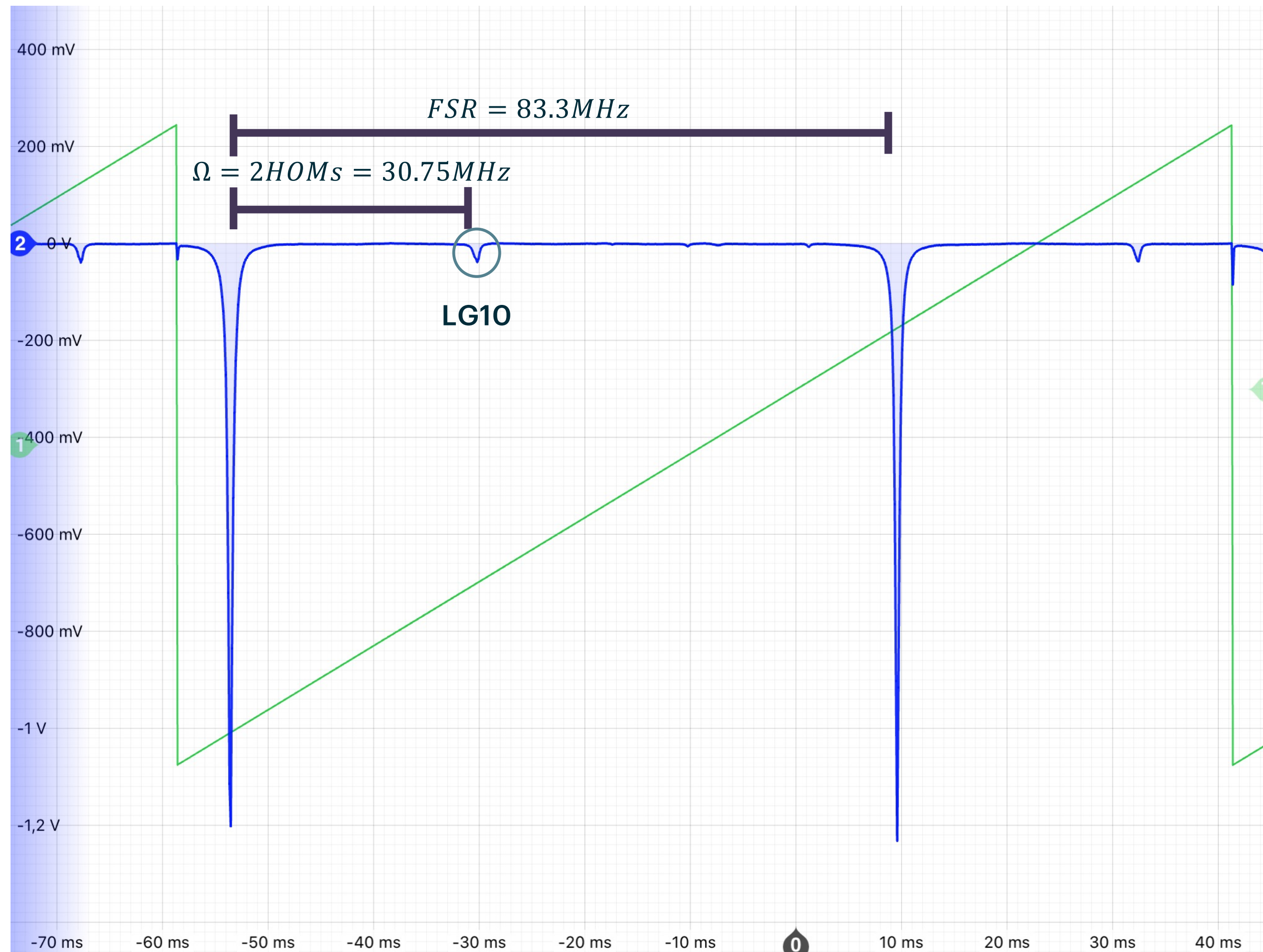
LG10

Pratically the modulation frequency needs to match twice the Higher Order Modes (HOM) Spacing Frequency of the cavity
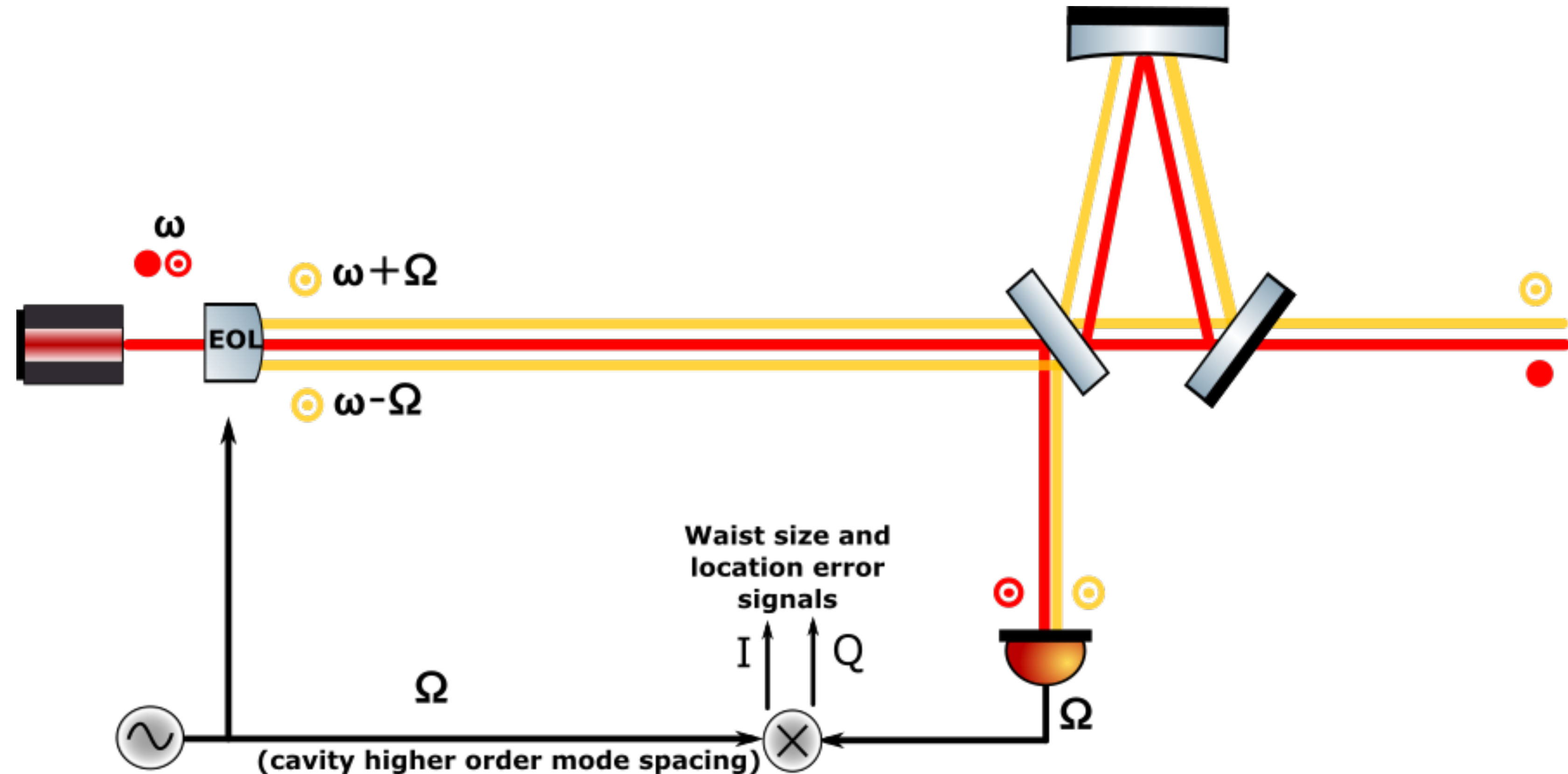
one sideband resonant in the cavity at the LG10 frequency

$$HOM = \frac{FSR \arccos\left(\sqrt{g3}\right)}{\pi}$$
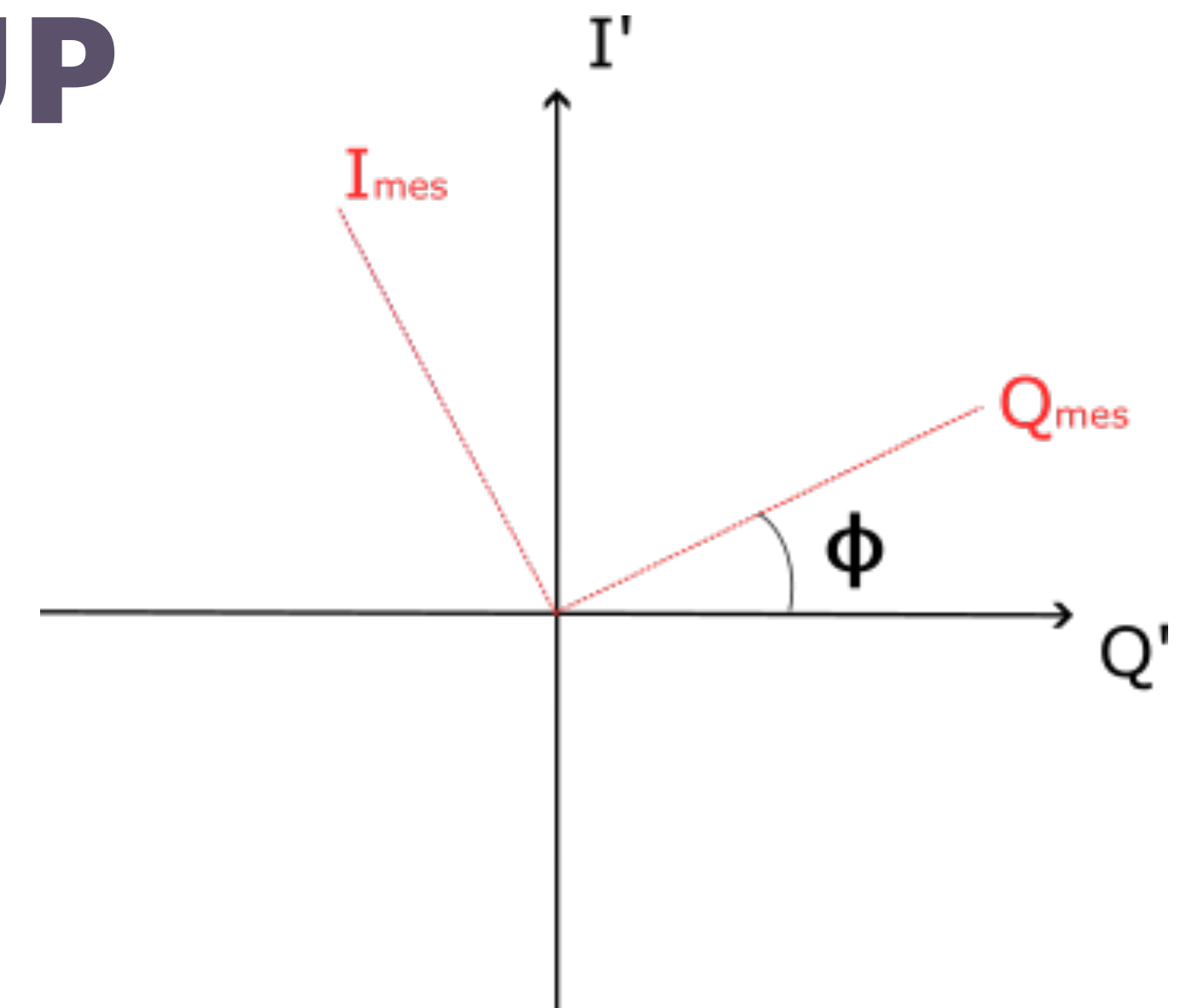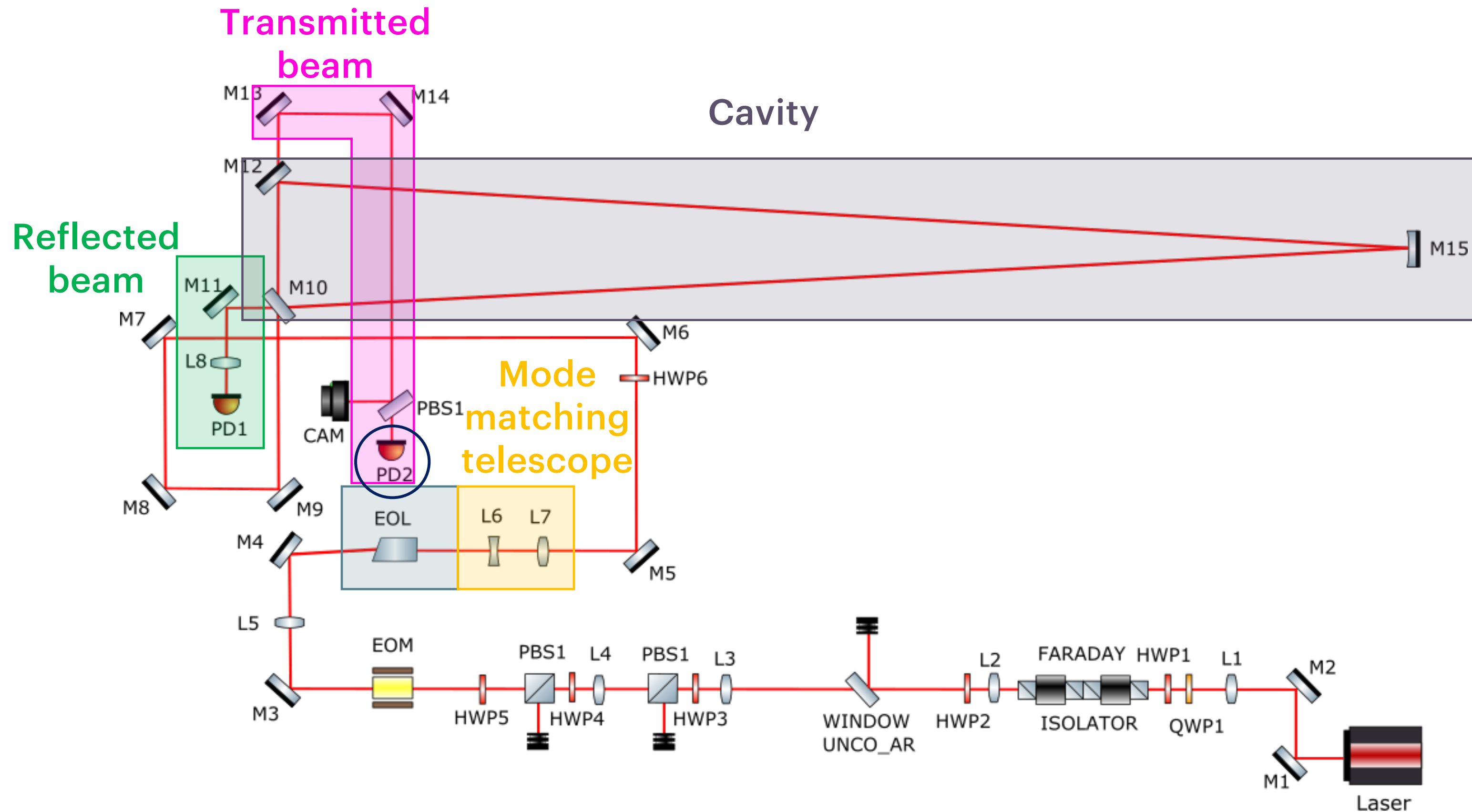
SIMPLIFIED OPTICAL SETUP

Modulation frequency matches the Higher Order Modes (HOM) Spacing Frequency of the cavity which is $\gtrsim MHz$

→ can't use mechanical actuators → Electro-Optic Lens (EOL)

REAL OPTICAL SETUP
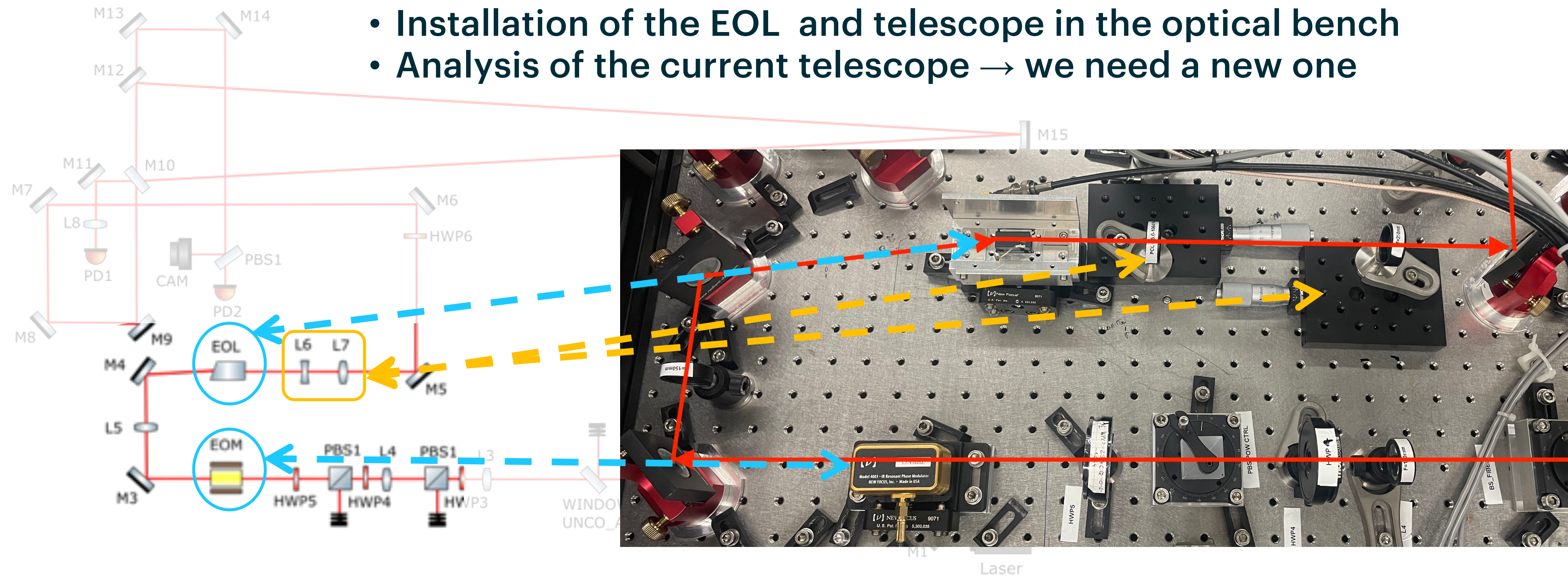

I/Q demodulation:

$$Q = I_0(\gamma m_G + \beta m_B)$$

$$I = I_0(\beta m_G - \gamma m_B)$$

use mode matching telescope to modify **γ** and **β** independently

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REAL OPTICAL SETUP

Preliminary work: setup configuration

- Locking of the cavity with the PDH signal (EOM)
- Installation of the EOL and telescope in the optical bench
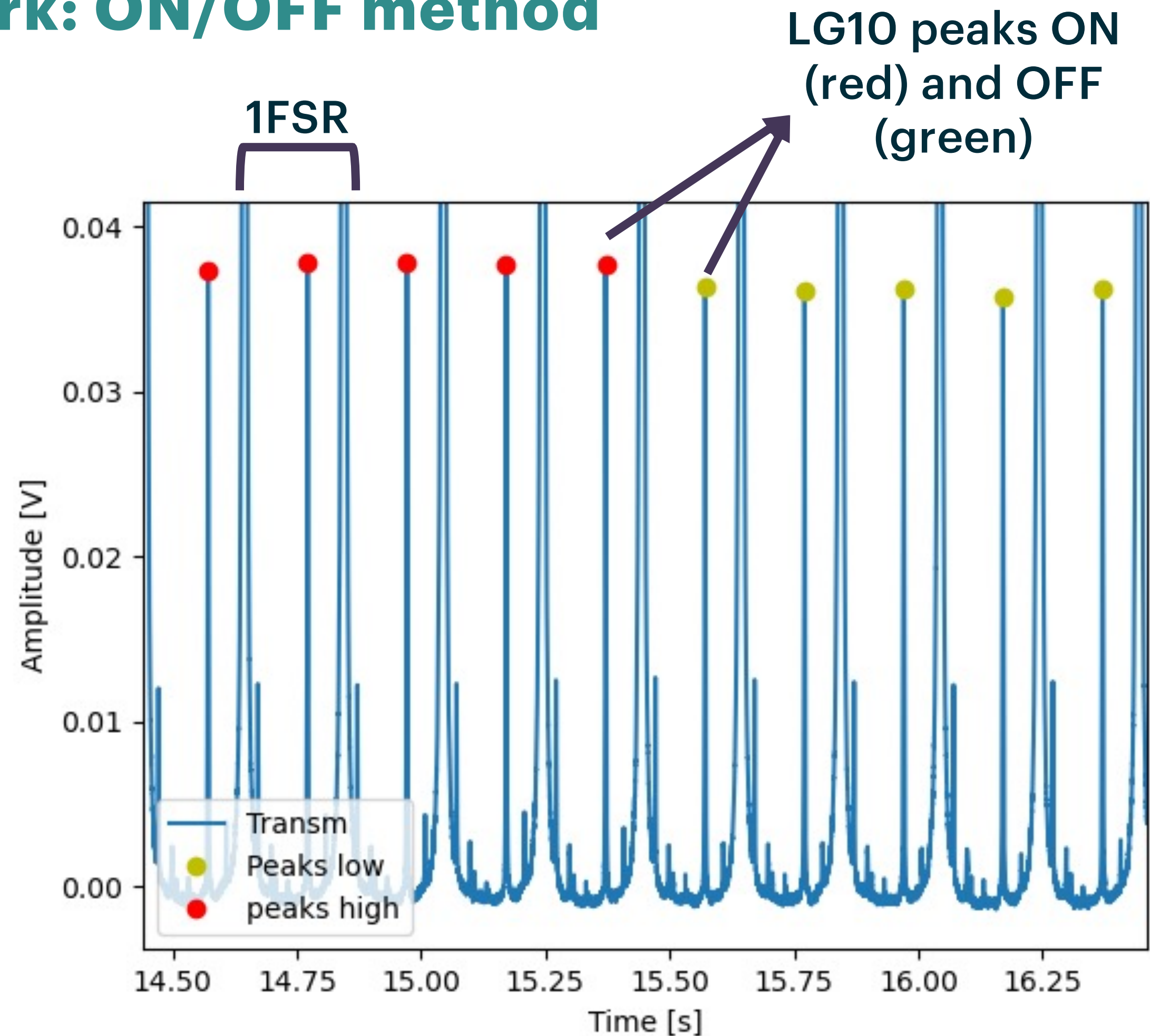- Analysis of the current telescope → we need a new one

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QUASI-STATIC MEASUREMENTS

Preliminary work: ON/OFF method

Through this method we want to see the amplitude modulation due to the EOL presence

Slow modulation to simulate ON/OFF behaviour with square wave:
- frequency 0.5Hz
- amplitude 400V
- scanning of the cavity

LG10 peaks ON (red) and OFF (green)

1FSR

QUASI-STATIC MEASUREMENTS

Preliminary work: ON/OFF method

Through this method we want to see the amplitude modulation due to the EOL presence
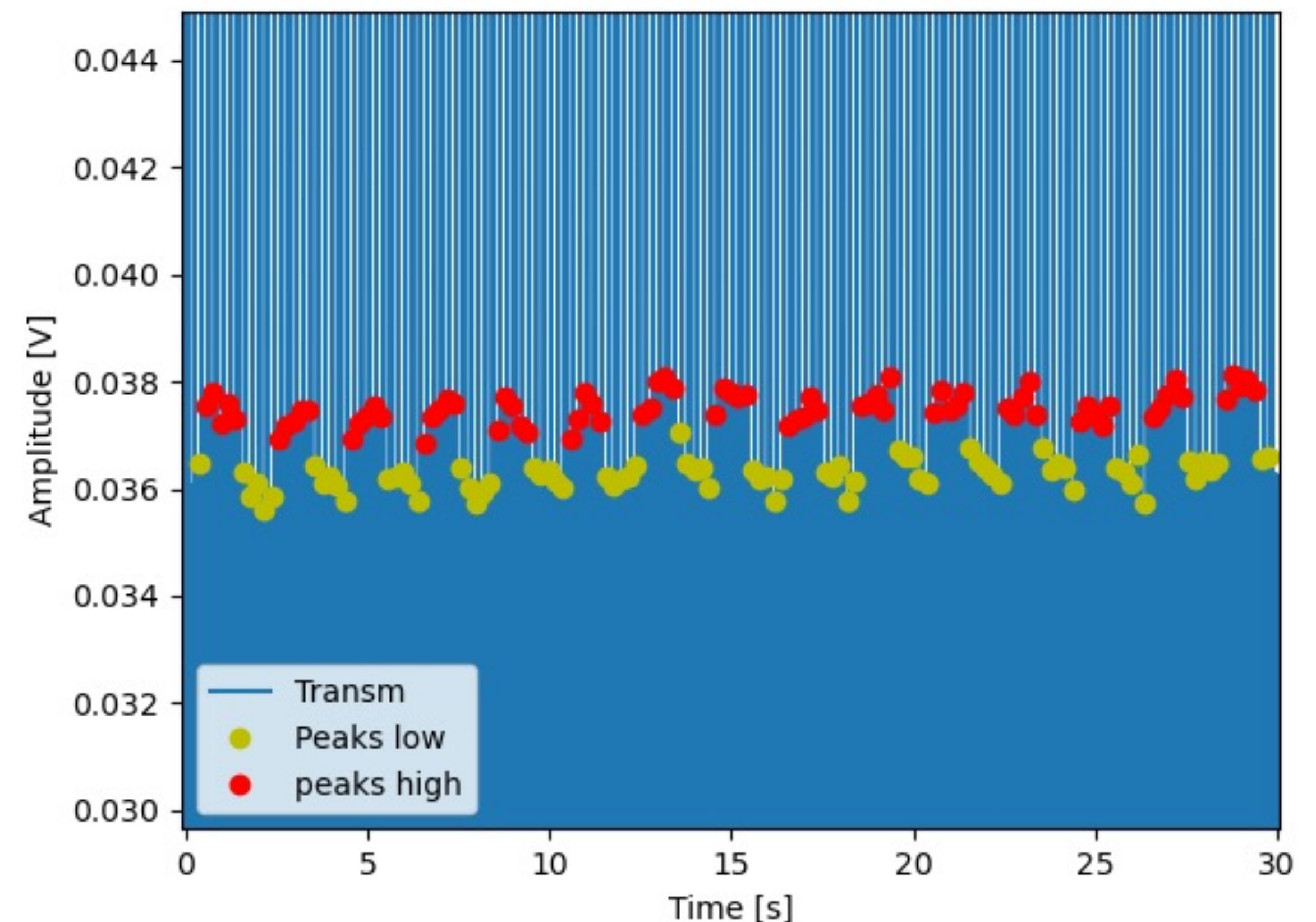
Slow modulation to simulate ON/OFF behaviour with square wave:

- frequency 0.5Hz
- amplitude 400V
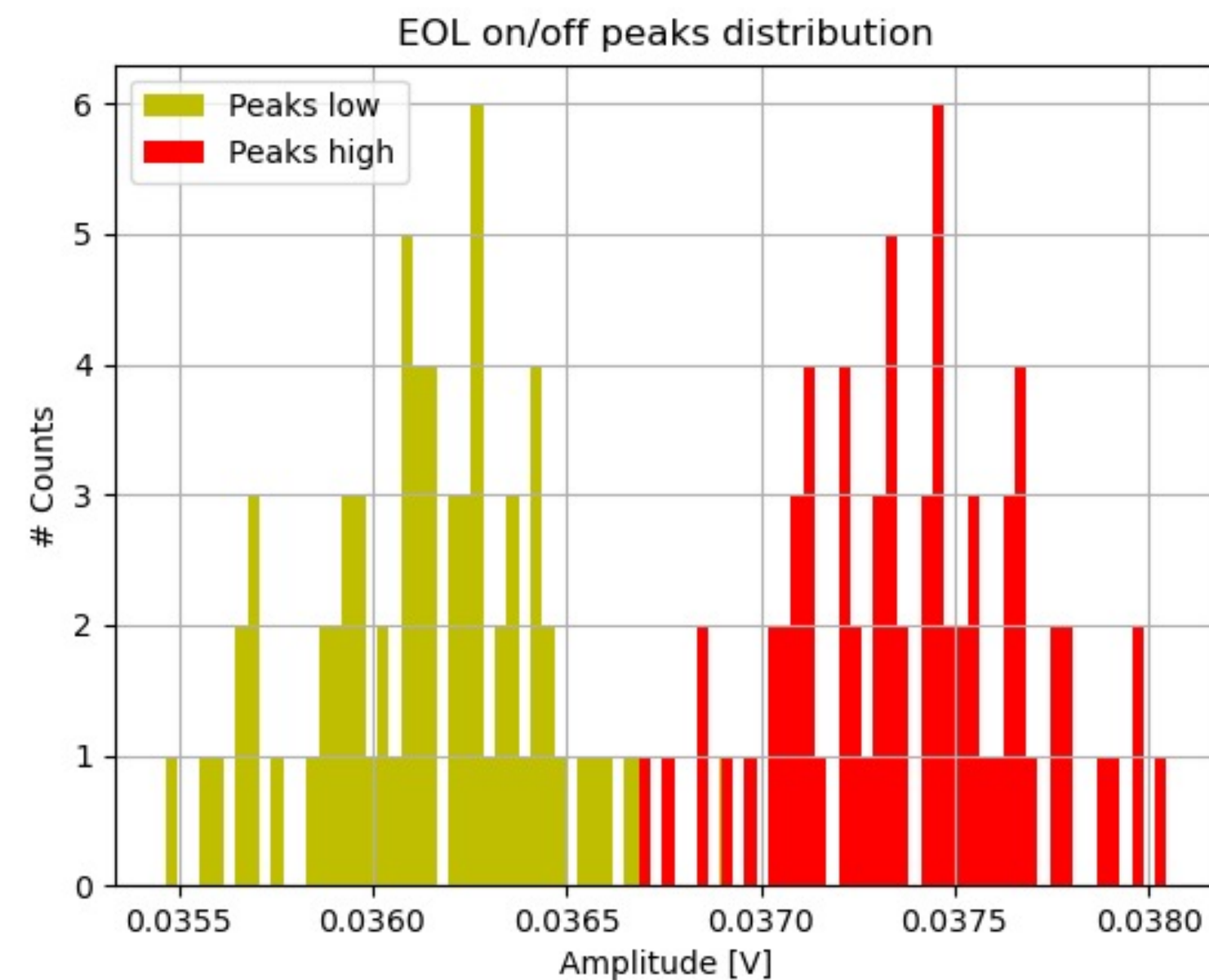- scanning of the cavity

3% increasing between ON/OFF

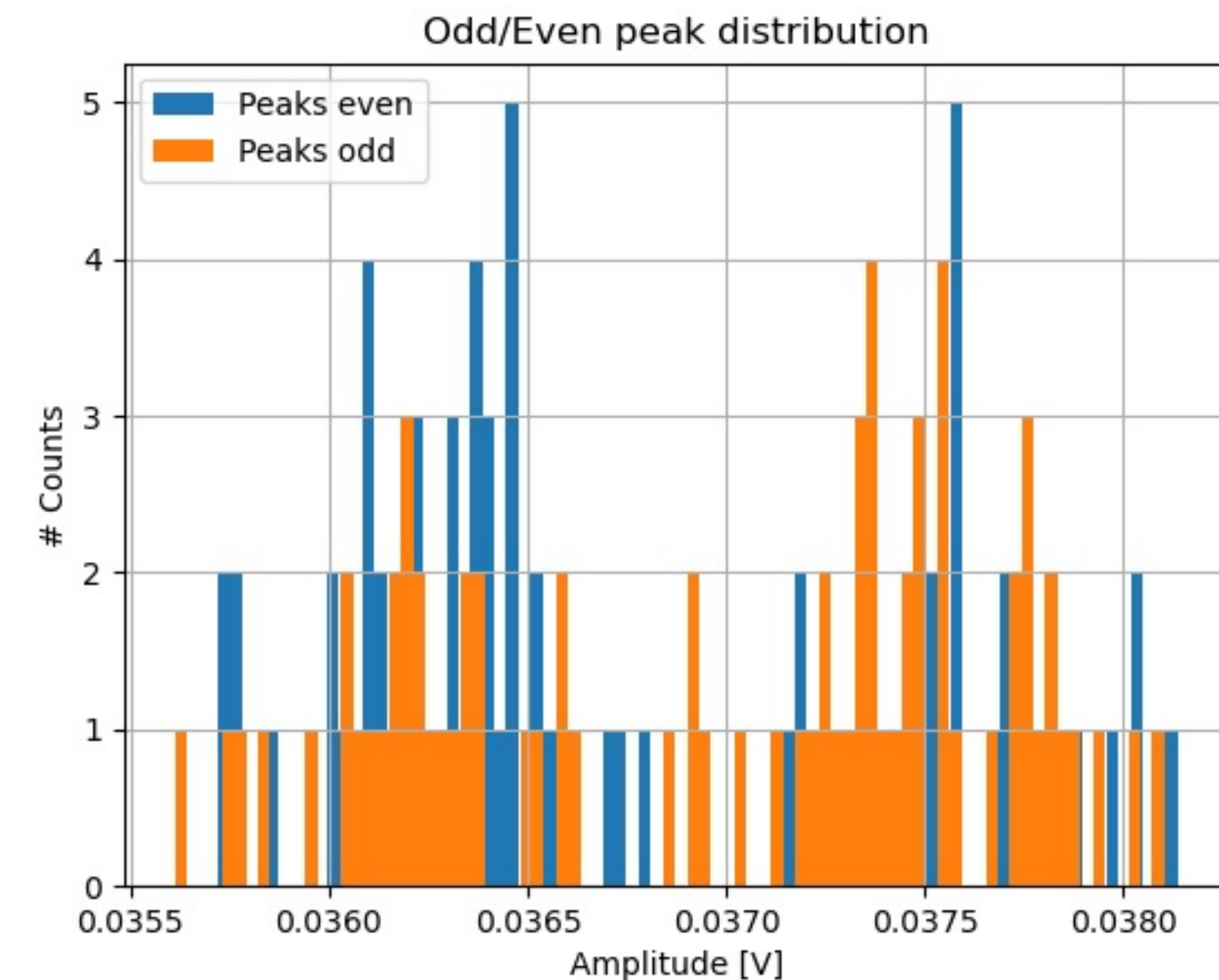30 seconds of cavity scan:we can see the modulation of the LG10 peaks (red-ON, green-OFF)

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QUASI-STATIC MEASUREMENTS

Preliminary work: ON/OFF method
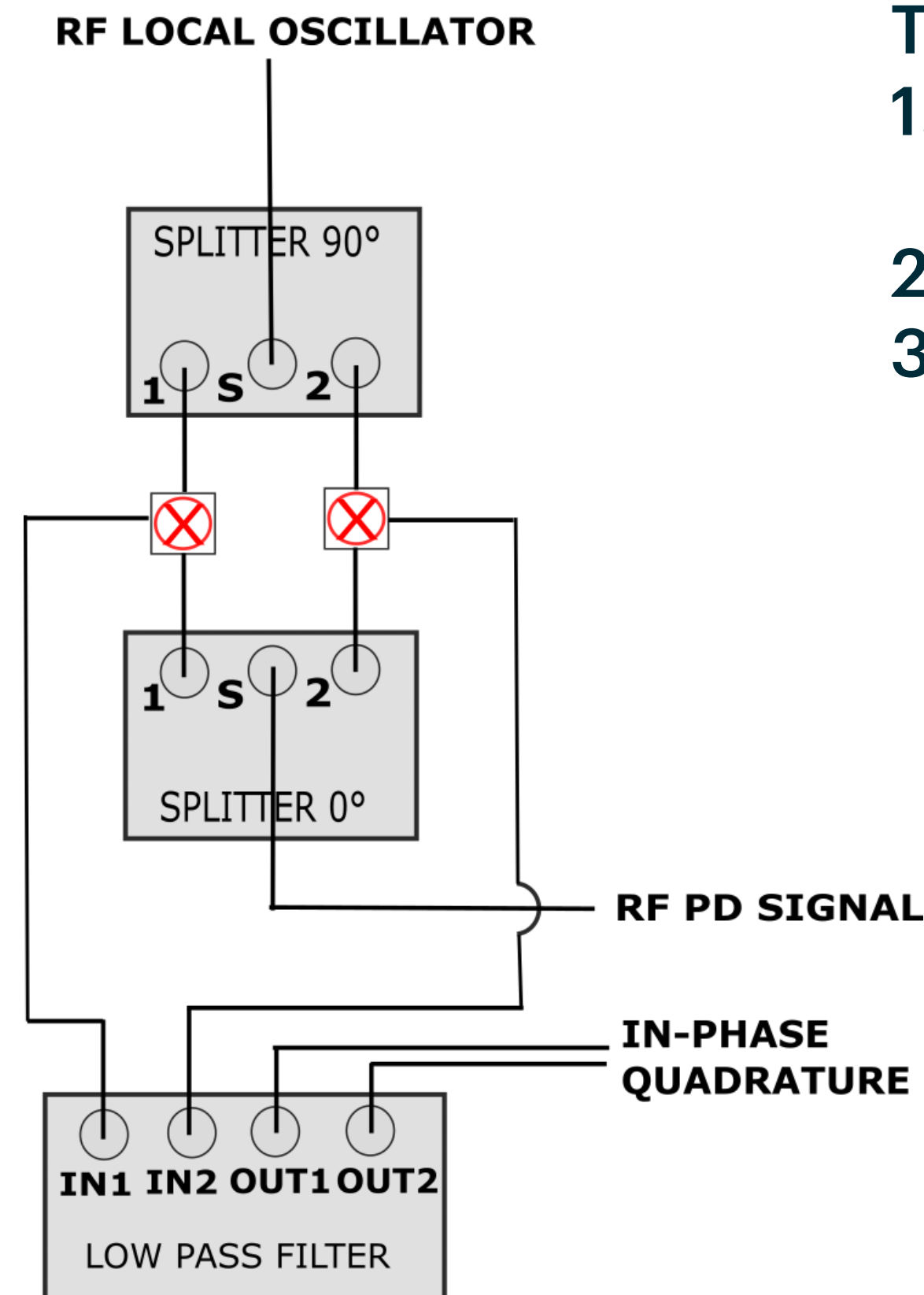
Distribution of the ON/OFF peaks

Distribution of the odd/even peaks

The odd/even analysis tells us that the difference between the ON/OFF distributions is not due to random flucutations of the peaks
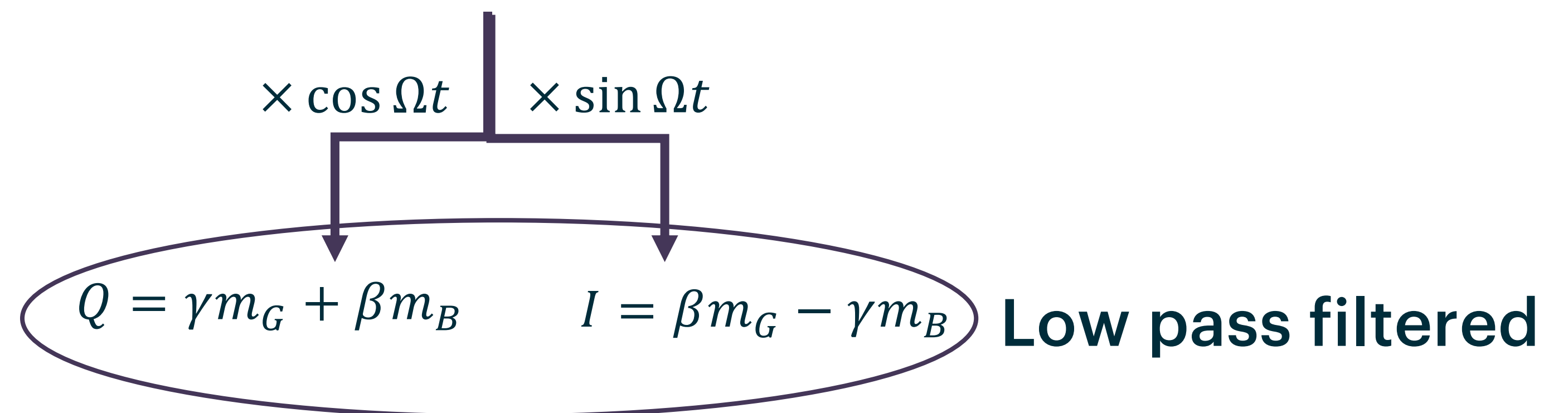
I/Q DEMODULATION CHAIN

Preliminary work: configuration

RF LOCAL OSCILLATOR

SPLITTER 90°

1 S 2

SPLITTER 0°

1 S 2

RF PD SIGNAL

IN-PHASE QUADRATURE
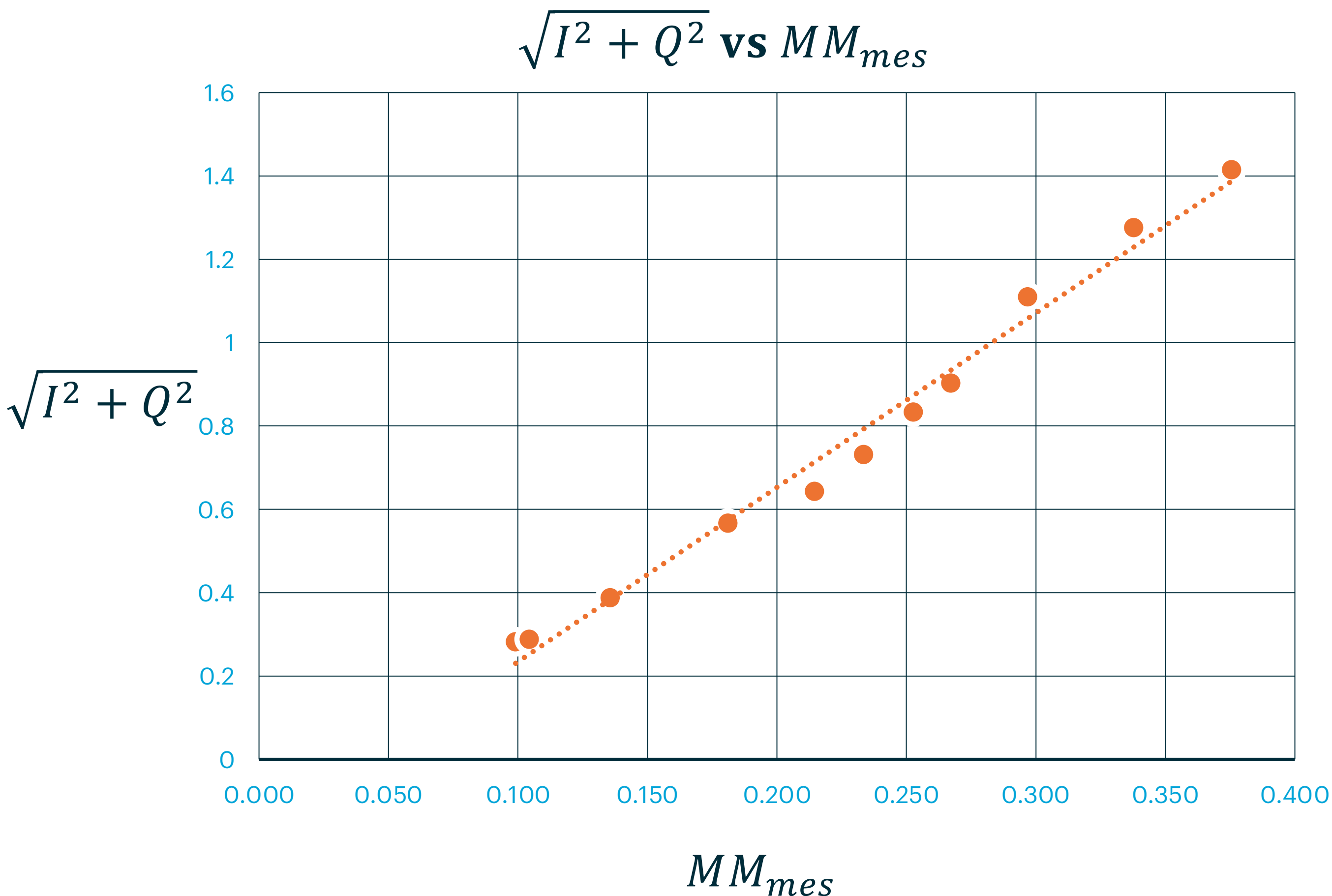
IN1 IN2 OUT1 OUT2

LOW PASS FILTER

THE I/Q CHAIN:
1. LOCAL OSCILLATOR AT MODULATION FREQUENCY IS SPLITTED AT 90°
2. RF SIGNAL FROM PD IN REFLECTION IS SPLITTED AT 0°
3. THE SPLITTED SIGNALS ARE MIXED AND THEN LOW PASS FILTERED

$$\frac{I_{PD}^{\Omega}}{I_0} = (\gamma m_G + \beta m_B)\cos\Omega t + (\beta m_G - \gamma m_B)\sin\Omega t$$

$\times \cos\Omega t$ $\times \sin\Omega t$

$$Q = \gamma m_G + \beta m_B \qquad I = \beta m_G - \gamma m_B$$

Low pass filtered

I/Q DEMODULATION CHAIN

Preliminary work: first measurements

$\sqrt{I^2 + Q^2}$ **vs** MM_{mes}

Sperimental demonstration of:

$$\sqrt{I^2 + Q^2} \propto MM$$

which comes from:

$$\sqrt{I^2 + Q^2} = \sqrt{(\gamma^2 + \beta^2)(m_B^2 + m_G^2)}$$

$$MM = \sqrt{\frac{LG10}{LG00}} = \sqrt{\gamma^2 + \beta^2}$$

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SUMMARY

Method:
- generating LG10 sidebands at the LG10 carrier resonating frequency (EOL)
- locking the cavity with the PDH technique (EOM)
- with the same PD of the PDH, detect the beating between the reflected LG10 sideband and LG10 carrier
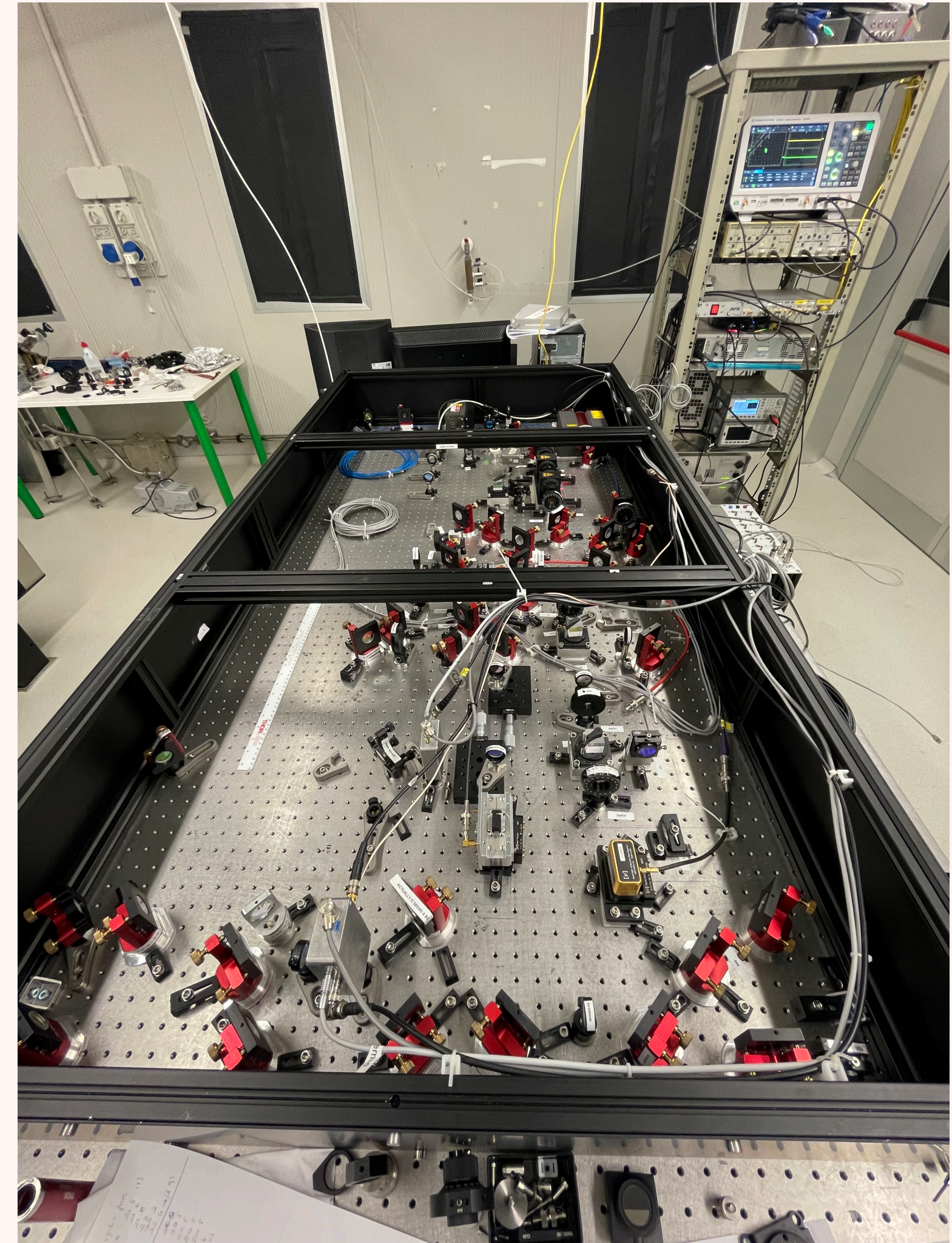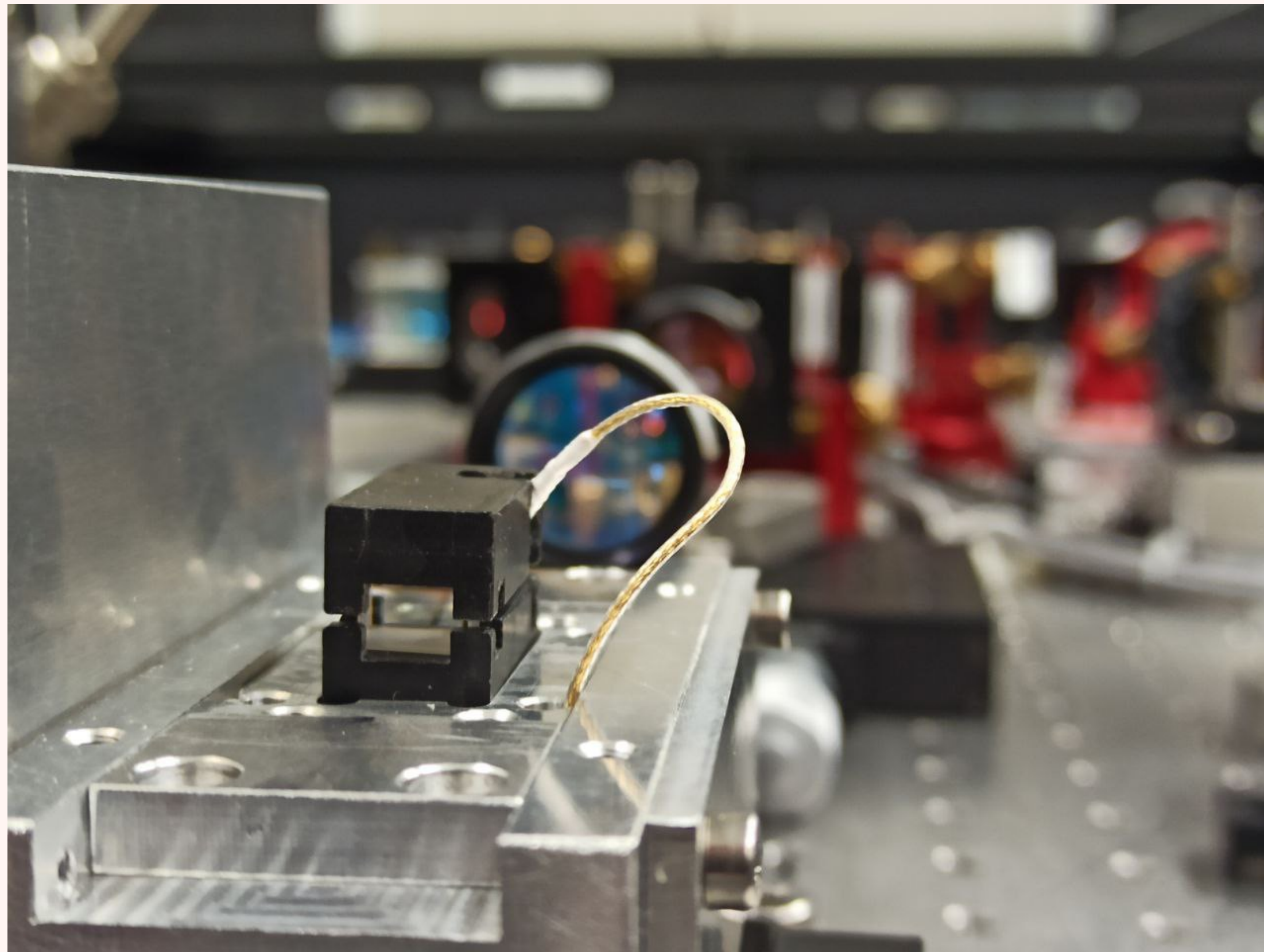- demodulation through the I/Q method and analysis

What we did so far:
- simulated the electrode shapes to have a spherical lens (COMSOL)
- installation of the EOL + locking of the cavity
- Quasi-static study to demonstrate the EOL amplitude modulation of LG10 (+3%)
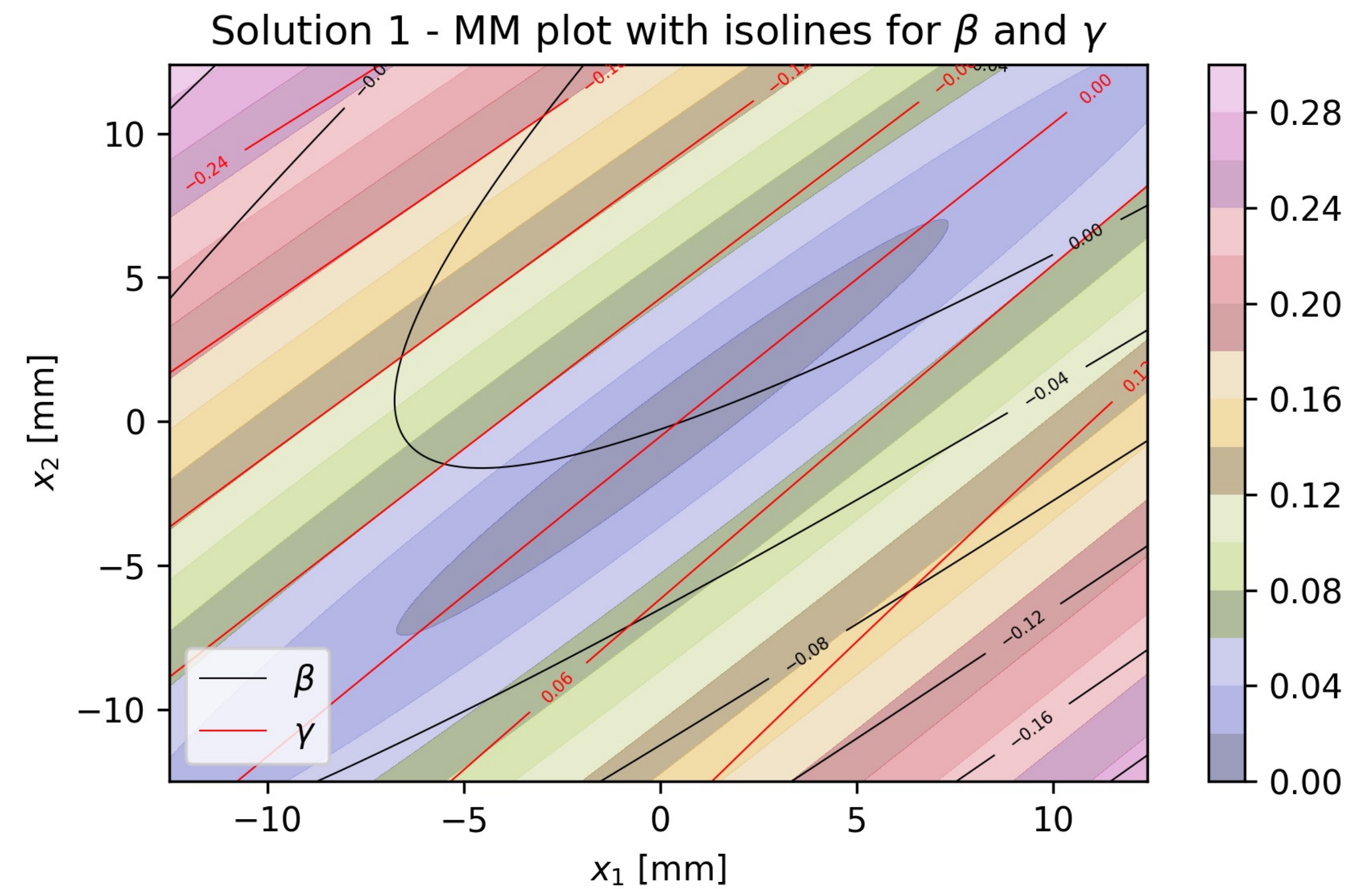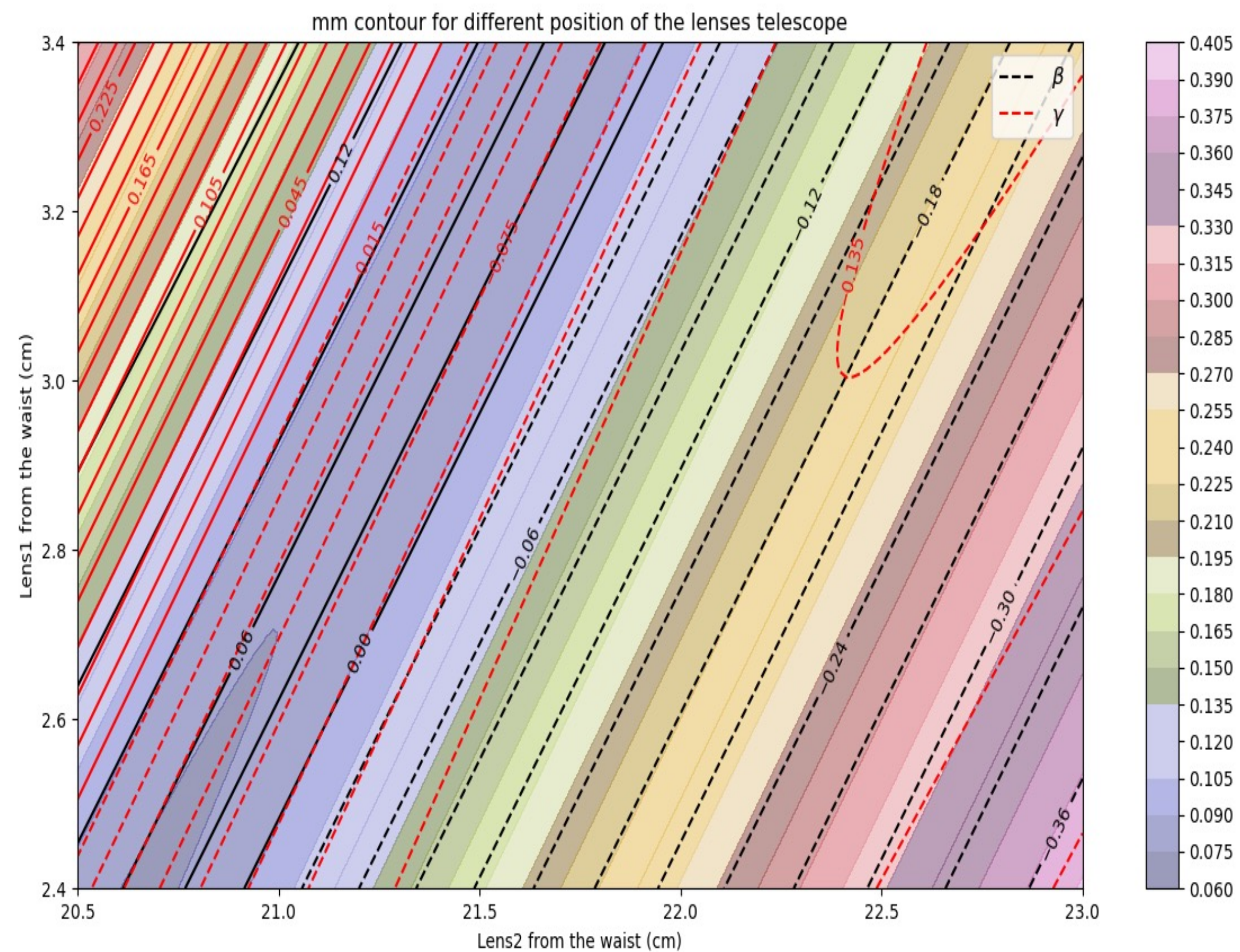- sperimental demonstration of $\sqrt{I^2 + Q^2} \propto MM$

What we will do:
- independent study of γ and β contributions through the mode matching telescope
- I/Q analysis for different values of mode matching

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THANK YOU FOR THE ATTENTION

TELESCOPE CONTOUR PLOT

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OTHERS