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# Astigmatic Mode Matching Sensing for the next gravitational wave detectors.

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VIR-0547A-22

# Summary

## 01. Introduction

Why Astigmatic Mode Matching Sensing?

## 02. Theoretical model

How to formulate the Mode Mismatch between an Astigmatic Gaussian Mode and Linear Optical Cavity

## 03. Experimental Setup

A small overview of the experimental setup

## 04. First Results

First Astigmatic Mismatch Measured

## 05. What's next

How to improve the setup and next step for the validation of the method



# Why Astigmatism

## Today Limit to the total Mode Matching

During the commission of Virgo, we reached target of maximum Mode Matching for the ITF/OMC[1] and the SQZ/Filter Cavity.[2]

The residual losses are mainly given by the small astigmatism of the beam

For Virgo post05 we want to reduce the losses to less than 2%

## Tomorrow Underground Detectors

The underground site is a strong limitation for the design of beam expander and telescopes of Einstein Telescope

New studies are prosing solution with not negligible angles of incident on the Curved Mirror

This will expose the beam to astigmatic aberration

[1] E. Polini on behalf of the DET sub-system, New High Finesse AdV+ Output Mode Cleaner (OMC) - LVK meeting, VIR-0261A-21

[2] Virgo Log Entry <https://logbook.virgo-gw.eu/virgo/?r=53468>

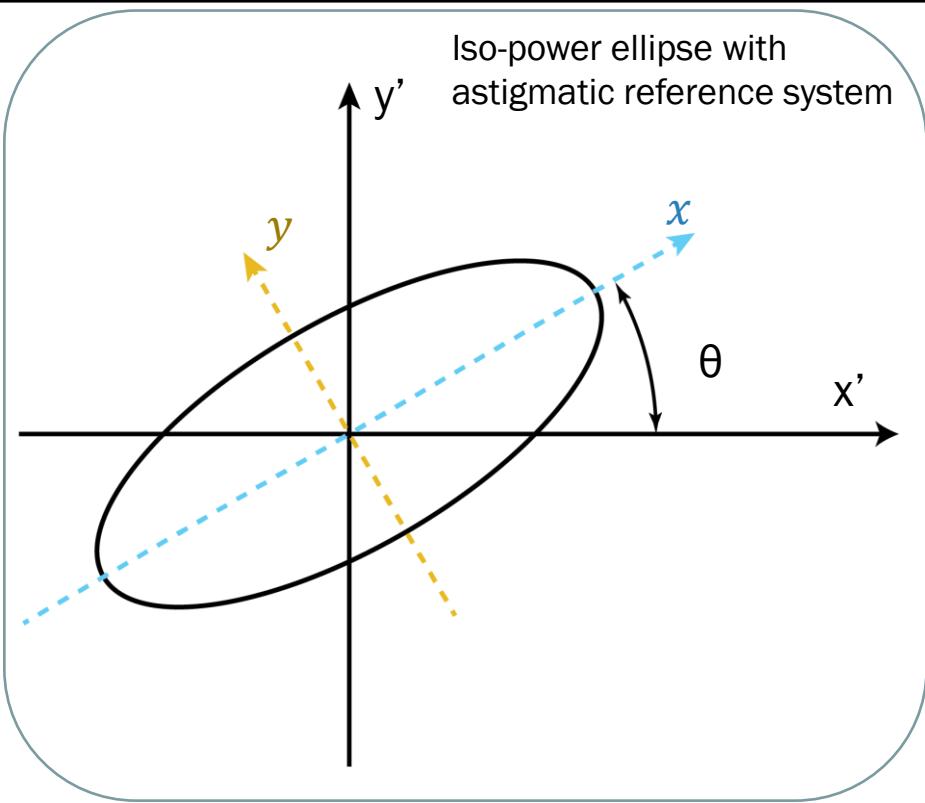
[3] Rowlinson, S., et All (2021). Feasibility study of beam-expanding telescopes in the interferometer arms for the Einstein Telescope. *Physical Review D*, 103(2), 23004.

[4] DeSalvo, R., et All (2022). Angled beam expander telescopes for the Michelson beams in third generation gravitational wave observatories. *Classical and Quantum Gravity*, 39(4), 045008

# Introduction

Simple Astigmatic Gaussian Beam can be represented as product of two Bi-dimensional Gaussian Mode

$$U_{00}(x, z | q_x, q_y) = \underbrace{\sqrt[4]{\frac{2}{\pi w_x^2(z)}} \exp\left[\frac{-x^2}{w_x^2(z)}\right]}_{XZ - plane} \exp\left[\frac{i}{2}\Delta\psi_x - ik\frac{x^2}{2R_x(z)}\right]$$

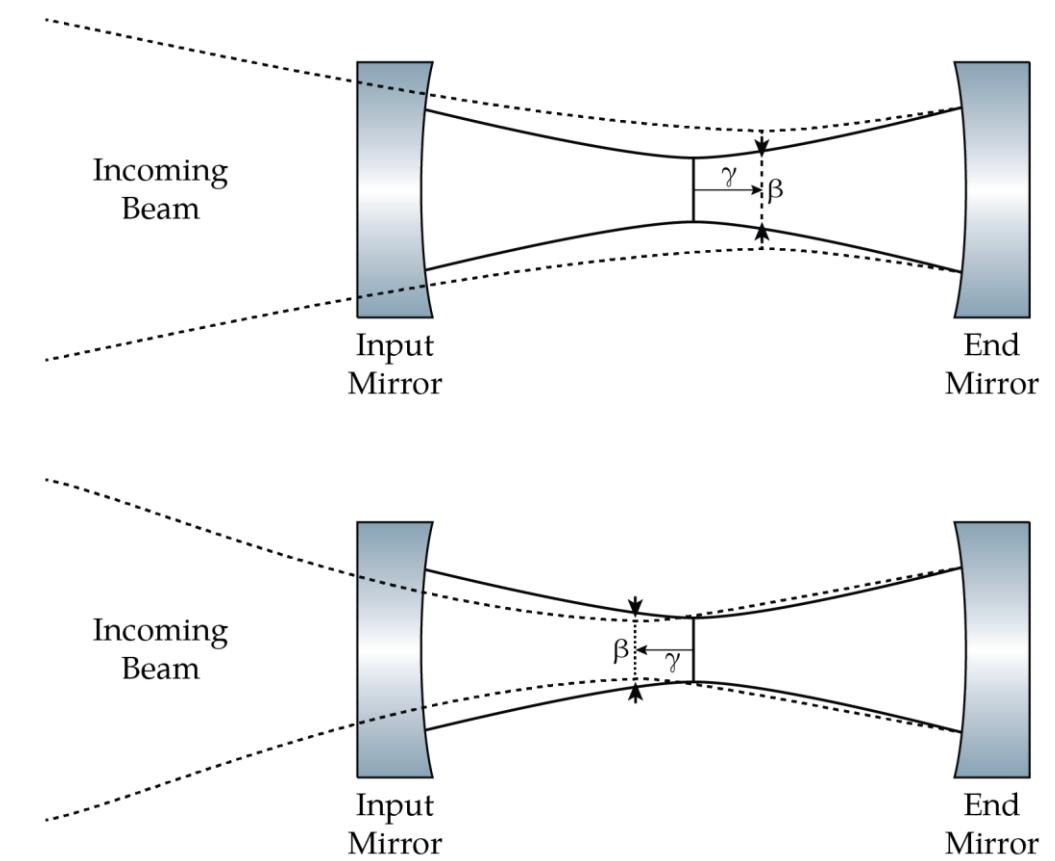
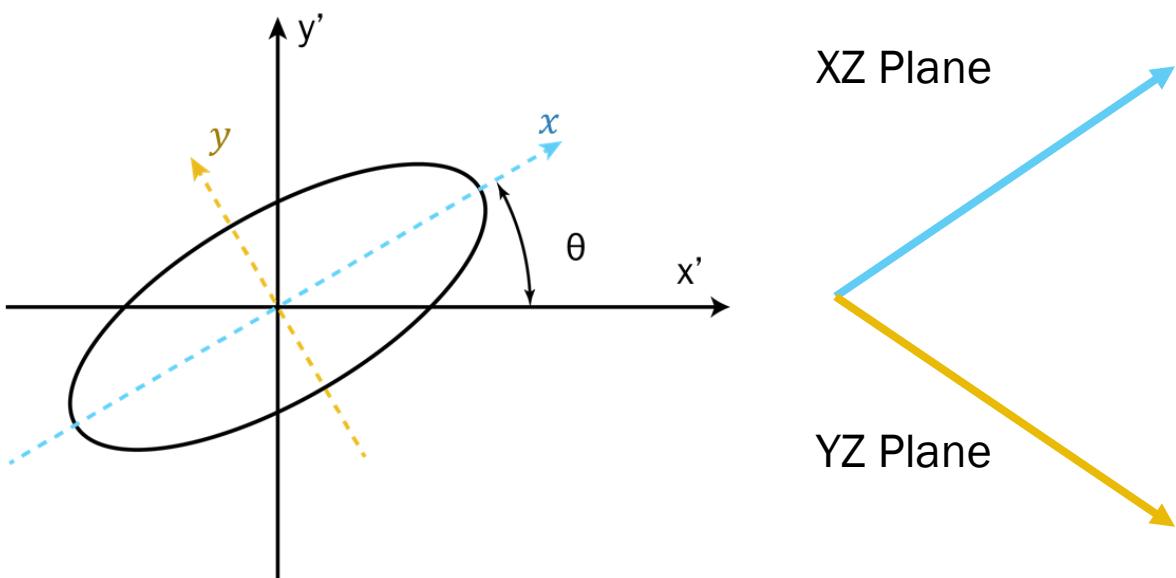


Using an appropriated reference system rotated by and angle  $\theta$

$$\begin{aligned} x &= x' \cos(\theta) - y' \sin(\theta) \\ y &= x' \sin(\theta) + y' \cos(\theta) \end{aligned}$$

$$\underbrace{\sqrt[4]{\frac{2}{\pi w_y^2(z)}} \exp\left[\frac{-y^2}{w_y^2(z)}\right]}_{YZ - plane} \exp\left[\frac{i}{2}\Delta\psi_y - ik\frac{y^2}{2R_y(z)}\right]$$

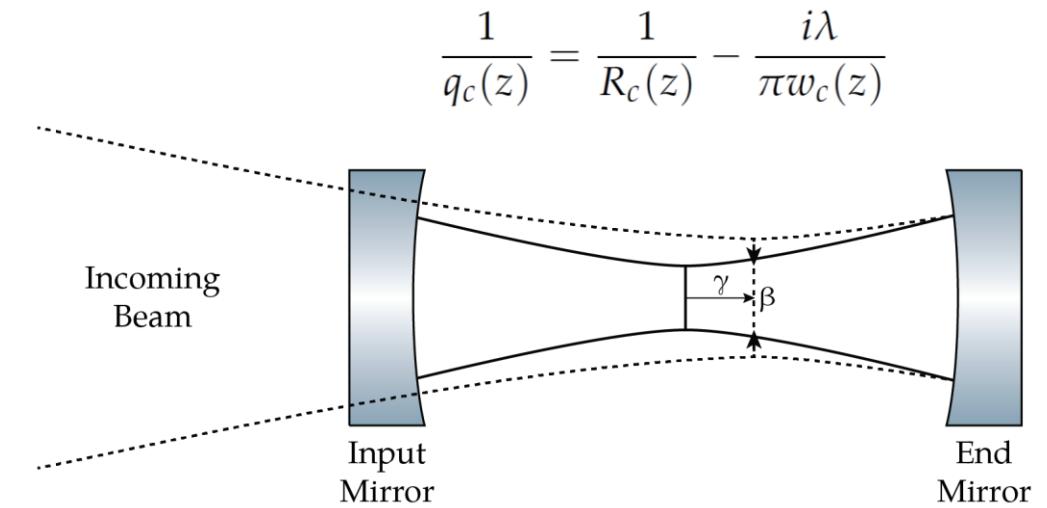
# Theoretical Model



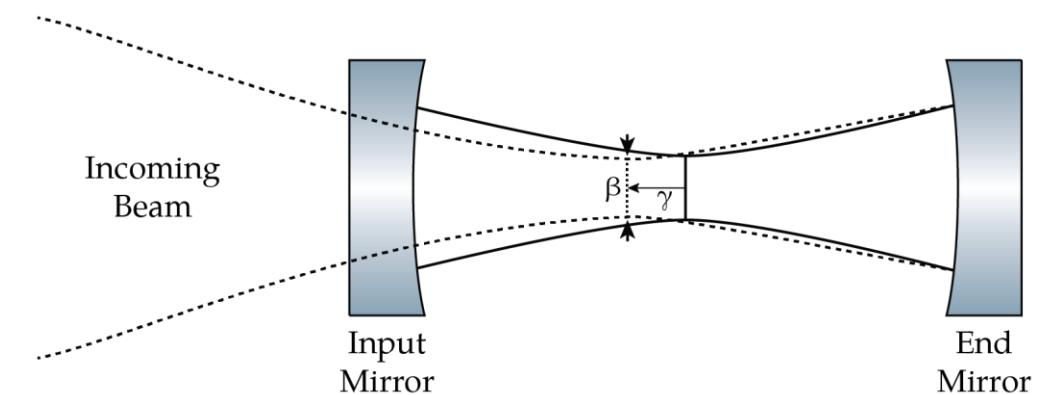
# Theoretical Model

## Bi-dimensional Mode Matching

$$\underbrace{\sqrt[4]{\frac{2}{\pi w_x^2(z)}} \exp\left[\frac{-x^2}{w_x^2(z)}\right] \exp\left[\frac{i}{2}\Delta\psi_x - ik\frac{x^2}{2R_x(z)}\right]}_{XZ-plane} \xrightarrow{\text{Plane X}} \frac{1}{q_x(z)} = \frac{1}{R_x(z)} - \frac{i\lambda}{\pi w_x(z)}$$



$$\underbrace{\sqrt[4]{\frac{2}{\pi w_y^2(z)}} \exp\left[\frac{-y^2}{w_y^2(z)}\right] \exp\left[\frac{i}{2}\Delta\psi_y - ik\frac{y^2}{2R_y(z)}\right]}_{YZ-plane} \xrightarrow{\text{Plane Y}} \frac{1}{q_y(z)} = \frac{1}{R_y(z)} - \frac{i\lambda}{\pi w_y(z)}$$



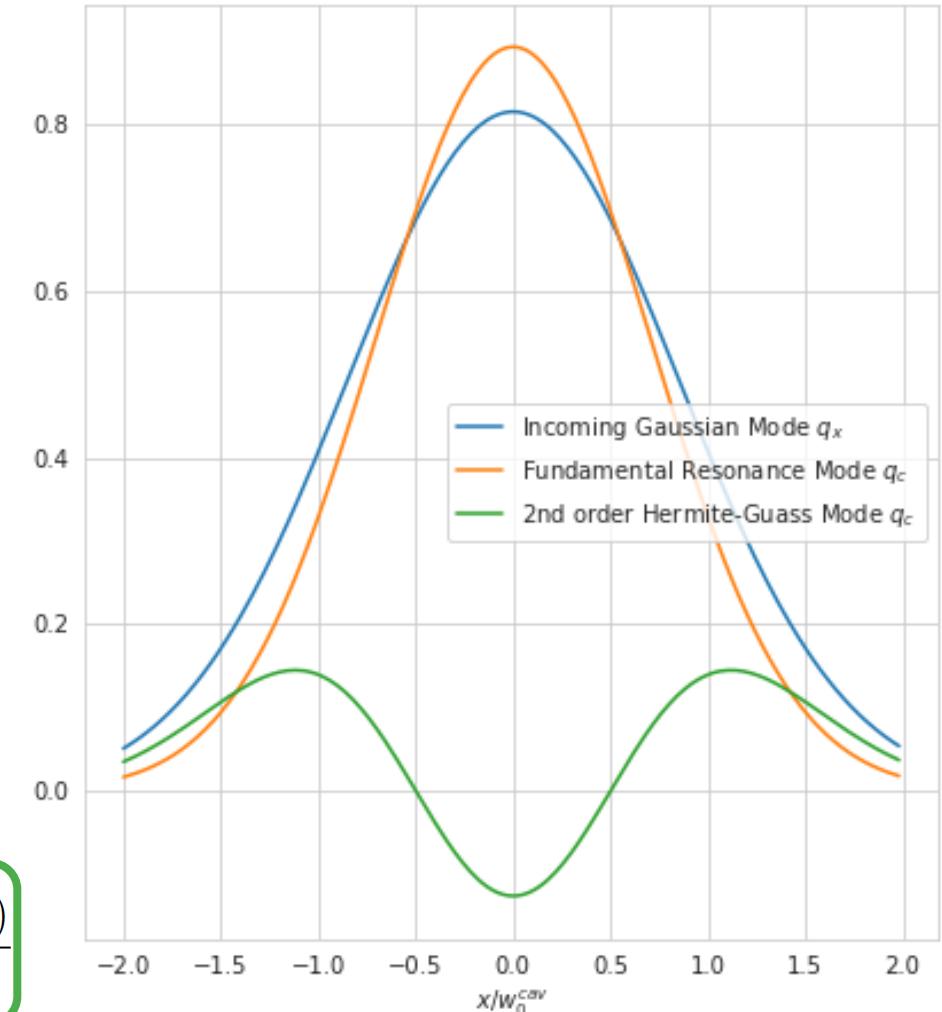
# Theoretical Model

## Anderson approach to bidimensional mismatch

$$\underbrace{\sqrt[4]{\frac{2}{\pi w_x^2(z)}} \exp\left[\frac{-x^2}{w_x^2(z)}\right] \exp\left[\frac{i}{2}\Delta\psi_x - ik\frac{x^2}{2R_x(z)}\right]}_{XZ - plane}$$

Taylor Expansion around  $w_0$  and  $z_0$

$$u_0(x|\bar{w}_0 + \delta w_0, \bar{z}_0 + \delta z_0) = u_0(x|\bar{w}_0, \bar{z}_0) + \underbrace{\left(\frac{\delta w_0}{\bar{w}_0} + i\frac{\delta z_0}{2\bar{z}_R}\right)}_{\epsilon_x} \frac{u_2(x|\bar{w}_0, \bar{z}_0)}{\sqrt{2}}$$



[1] Anderson, D. Z. (1984). Alignment of resonant optical cavities. *Applied Optics*, 23(17), 2944. <https://doi.org/10.1364/AO.23.002944>

# Theoretical Model

## Three dimensional Astigmatic Mismatch

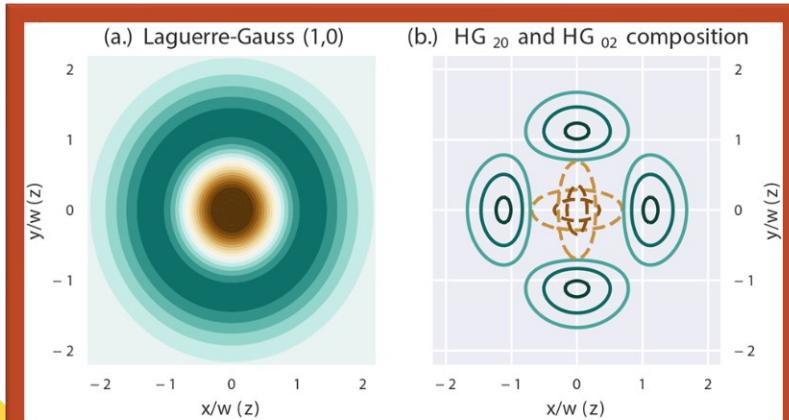
$$\underbrace{\left( \frac{\delta w_0}{\bar{w}_0} + i \frac{\delta z_0}{2\bar{z}_R} \right)}_{\epsilon_x}$$

Common  $\langle \epsilon \rangle = \frac{(\epsilon_x + \epsilon_y)}{2}$

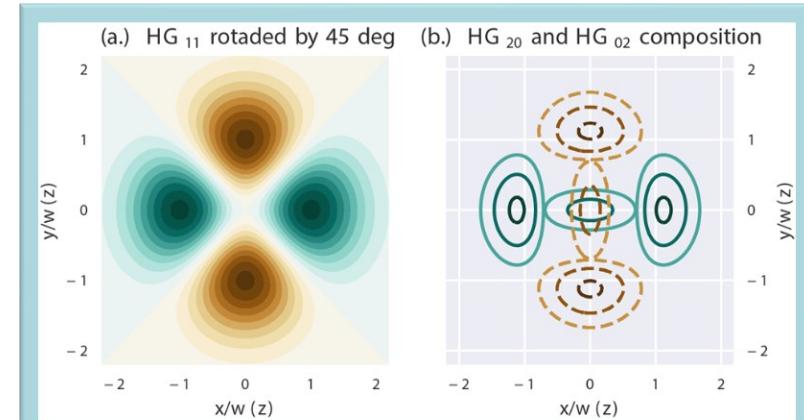
Differential  $\delta \epsilon = \frac{(\epsilon_x - \epsilon_y)}{2}$

$$\Psi_{in} \simeq \mathbf{U}_{00}(x, y|q_c) + \langle \epsilon \rangle \frac{1}{\sqrt{2}} (\mathbf{U}_{20}(x, y|q_c) + \mathbf{U}_{02}(x, y|q_c)) + \delta \epsilon \frac{1}{\sqrt{2}} (\mathbf{U}_{20}(x, y|q_c) - \mathbf{U}_{02}(x, y|q_c))$$

Spherical Mismatch



Astigmatic Mismatch



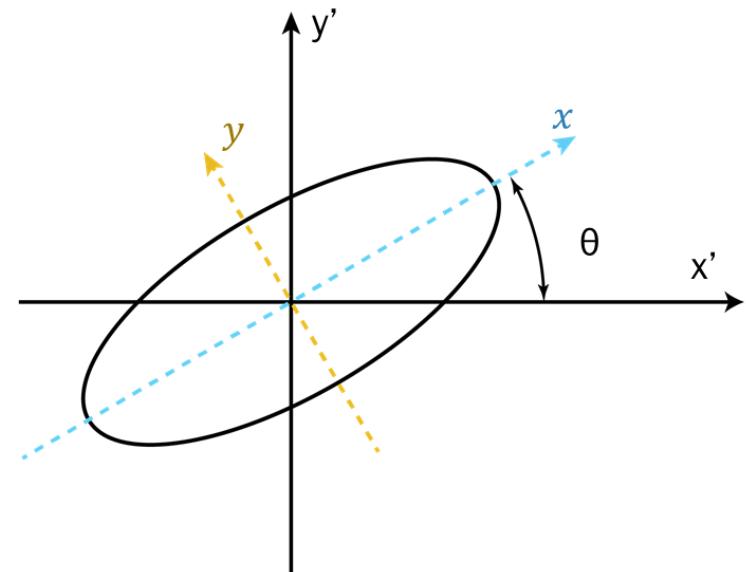
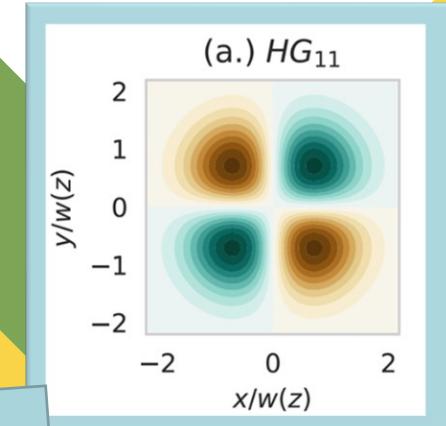
# Theoretical Model

## Astigmatic Ellipse Rotation

$$\Psi_{in} \simeq \mathbf{U}_{00}(x, y|q_c) + \langle \epsilon \rangle \frac{1}{\sqrt{2}} (\mathbf{U}_{20}(x, y|q_c) + \mathbf{U}_{02}(x, y|q_c)) + \delta \epsilon \frac{1}{\sqrt{2}} (\mathbf{U}_{20}(x, y|q_c) - \mathbf{U}_{02}(x, y|q_c))$$

$$x = x' \cos(\theta) - y' \sin(\theta)$$
$$y = x' \sin(\theta) + y' \cos(\theta)$$

$$\mathbf{U}_{20}(x, y) = \mathbf{U}_{20}(x', y') \cos^2 \theta + \mathbf{U}_{02}(x', y') \sin^2 \theta - \sqrt{2} \mathbf{U}_{11}(x', y') \cos \theta \sin \theta$$
$$\mathbf{U}_{02}(x, y) = \mathbf{U}_{20}(x', y') \sin^2 \theta + \mathbf{U}_{02}(x', y') \cos^2 \theta + \sqrt{2} \mathbf{U}_{11}(x', y') \cos \theta \sin \theta$$



# Theoretical Model

## High Order Mode Expansion

$$\text{Common } \langle \epsilon \rangle = \frac{(\epsilon_x + \epsilon_y)}{2}$$

$$\text{Differential } \delta\epsilon = \frac{(\epsilon_x - \epsilon_y)}{2}$$

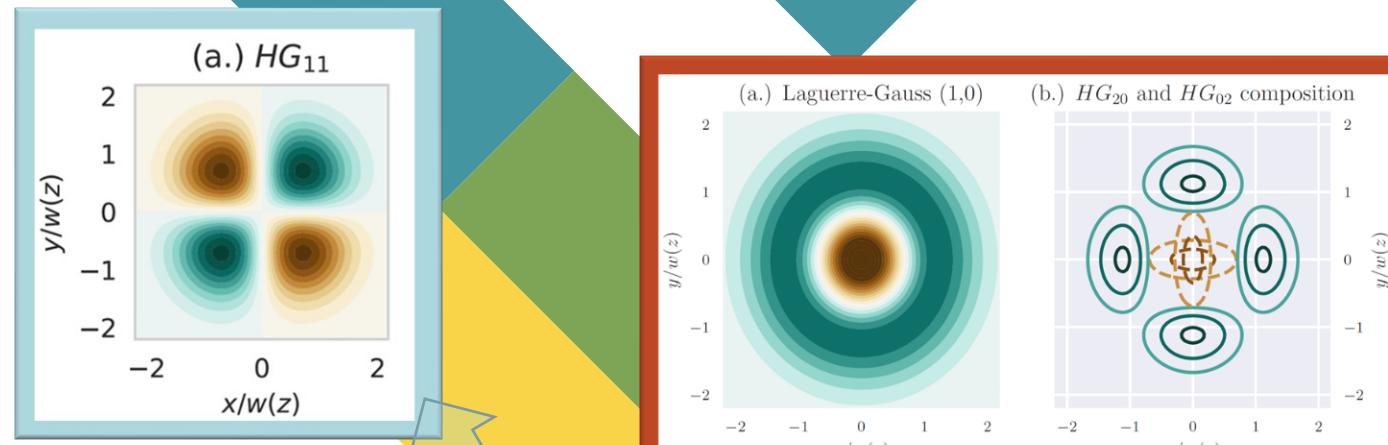
$$\underbrace{\left( \frac{\delta w_0}{\bar{w}_0} + i \frac{\delta z_0}{2\bar{z}_R} \right)}_{\epsilon_x}$$

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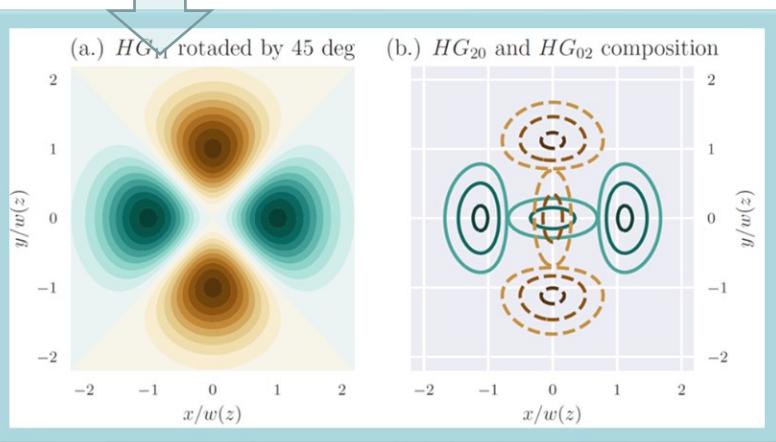
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$$\Psi_{in} = \mathbf{U}_{00}(x', y' | q_c) + \delta\epsilon \cdot \left[ \frac{U_{20}(x', y' | q_c) - U_{02}(x', y' | q_c)}{\sqrt{2}} \cos 2\theta - U_{11}(x', y' | q_c) \sin 2\theta \right] + \langle \epsilon \rangle \cdot \frac{U_{20}(x', y' | q_c) + U_{02}(x', y' | q_c)}{\sqrt{2}}$$



# Astigmatic Mismatch

## Parameters

### Spherical Mismatch

- Average Waist Dimension  $\beta = \frac{\langle \omega_0^{x,y} \rangle - \omega_0^{cav}}{\omega_0^{cav}}$
- Average Waist Position  $\gamma = \frac{\langle z_0^{x,y} \rangle - z_0^{cav}}{2z_R^{cav}}$

$$\langle \epsilon \rangle \cdot \frac{U_{20}(x', y' | q_c) + U_{02}(x', y' | q_c)}{\sqrt{2}}$$

$$\langle \epsilon \rangle = \beta + i\gamma$$
$$\delta\epsilon = \alpha + i\eta$$

### Astigmatic Mismatch

- Waist Dimension Difference  $\alpha = \frac{\omega_0^x - \omega_0^y}{\omega_0^{cav}}$
  - Waist Position Difference  $\eta = \frac{z_0^x - z_0^y}{2z_R^{cav}}$
  - Orientation Axis  $\theta$
- $$\delta\epsilon \cdot \left[ \frac{U_{20}(x', y' | q_c) - U_{02}(x', y' | q_c)}{\sqrt{2}} \cos 2\theta - U_{11}(x', y' | q_c) \sin 2\theta \right]$$

# Spherical Mismatch

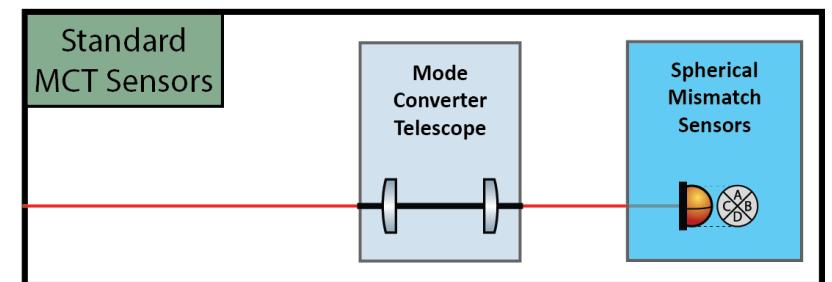
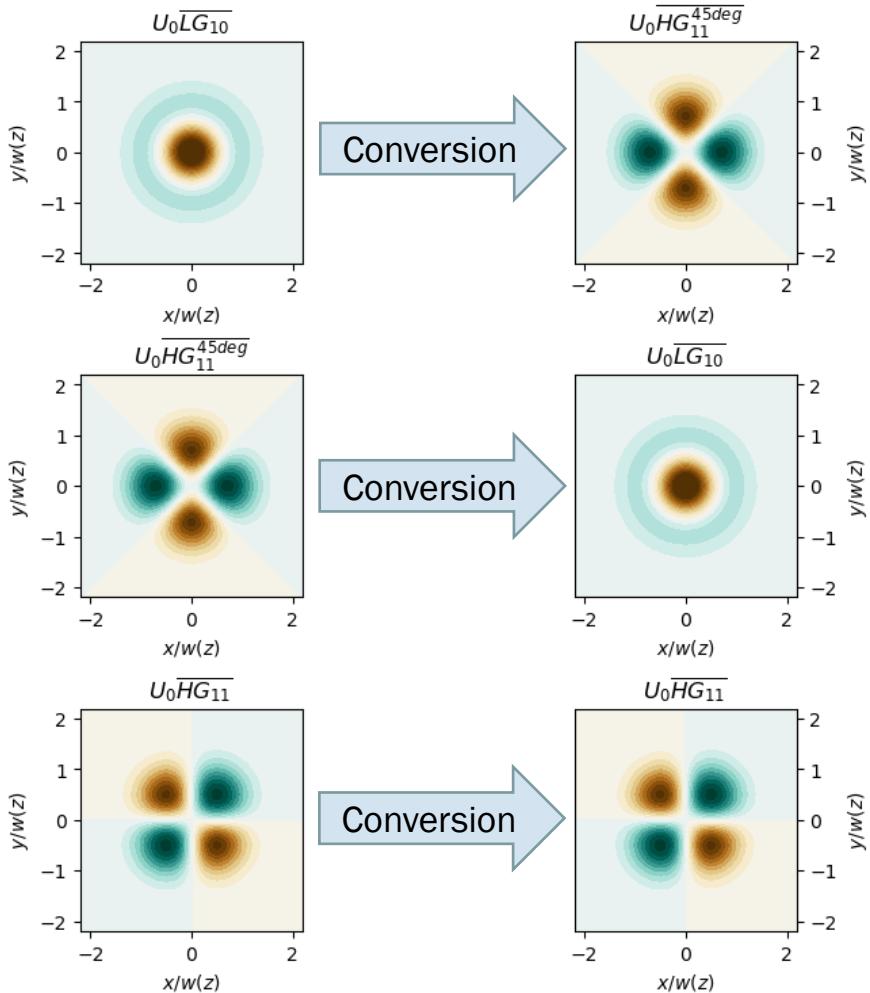
## Standard Mode Converter Technique

### Spherical Mismatch

The standard Mode Converter Telescope technique [1] is sensible only to Common/Spherical Mismatch

- ❖ LG10 converted in HG11 rotated by 45 deg
- ❖ HG<sub>11</sub><sup>45deg</sup> converted in LG10
- ❖ HG11 is not converted

[1] Magaña-Sandoval, F., et Al. (2019). Sensing optical cavity mismatch with a mode-converter and quadrant photodiode. Physical Review D, 100(10), 1–12.



# Astigmatic Mismatch

## Standard Mode Converter Technique

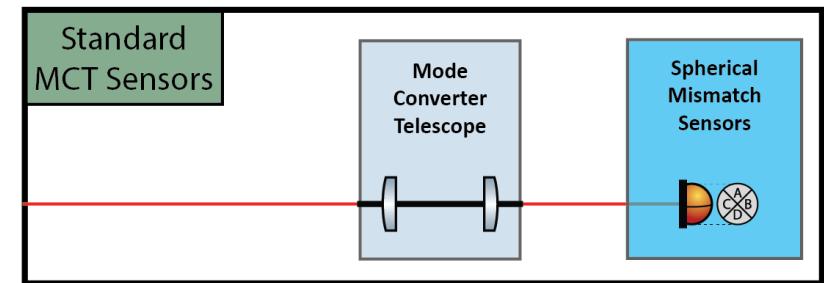
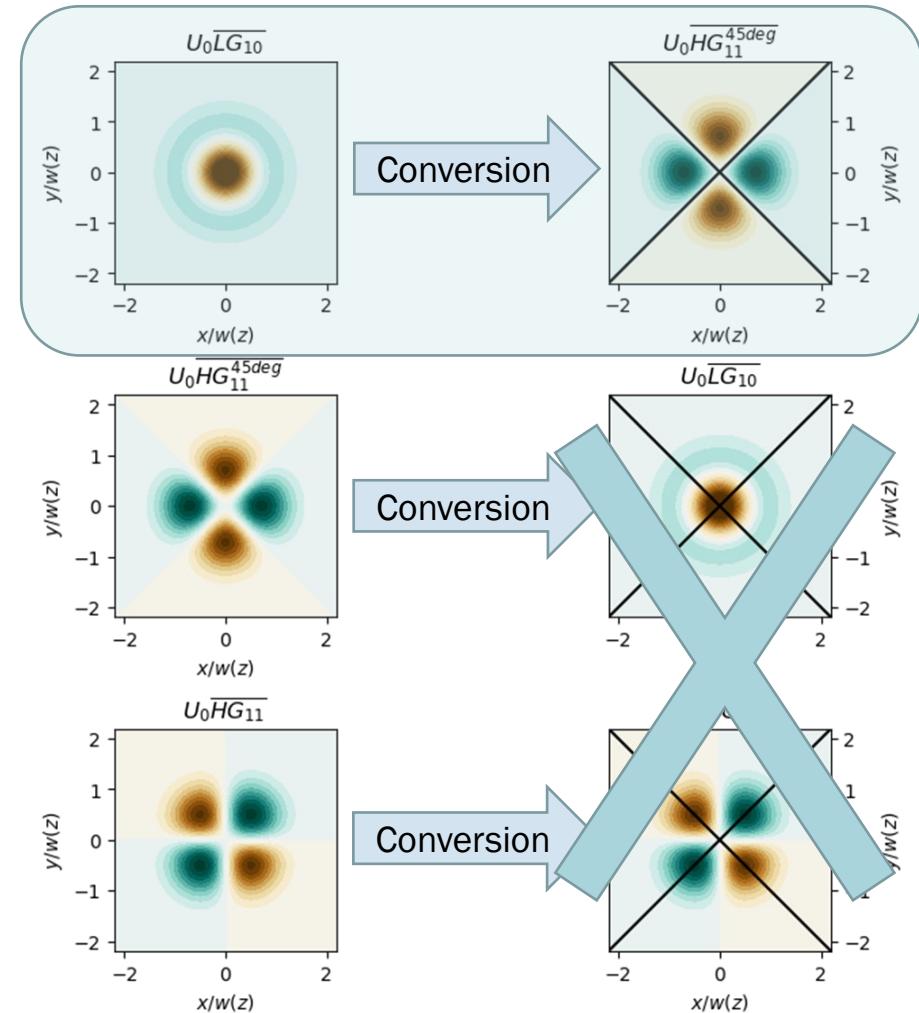
Common Spherical Astigmatic Mismatch

### Spherical Mismatch

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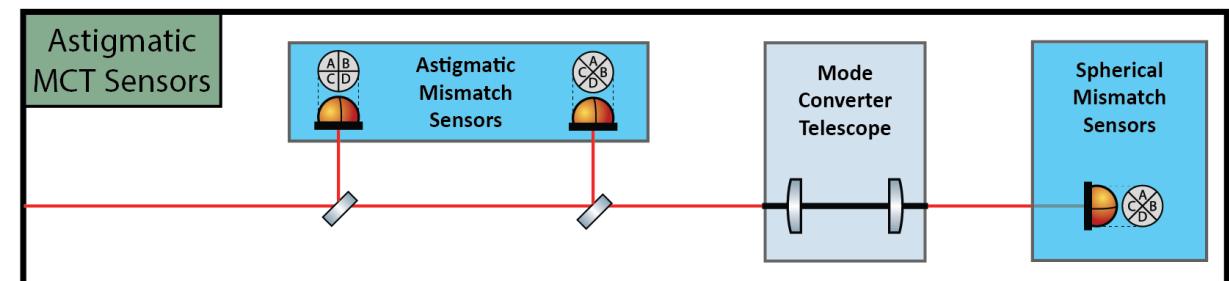
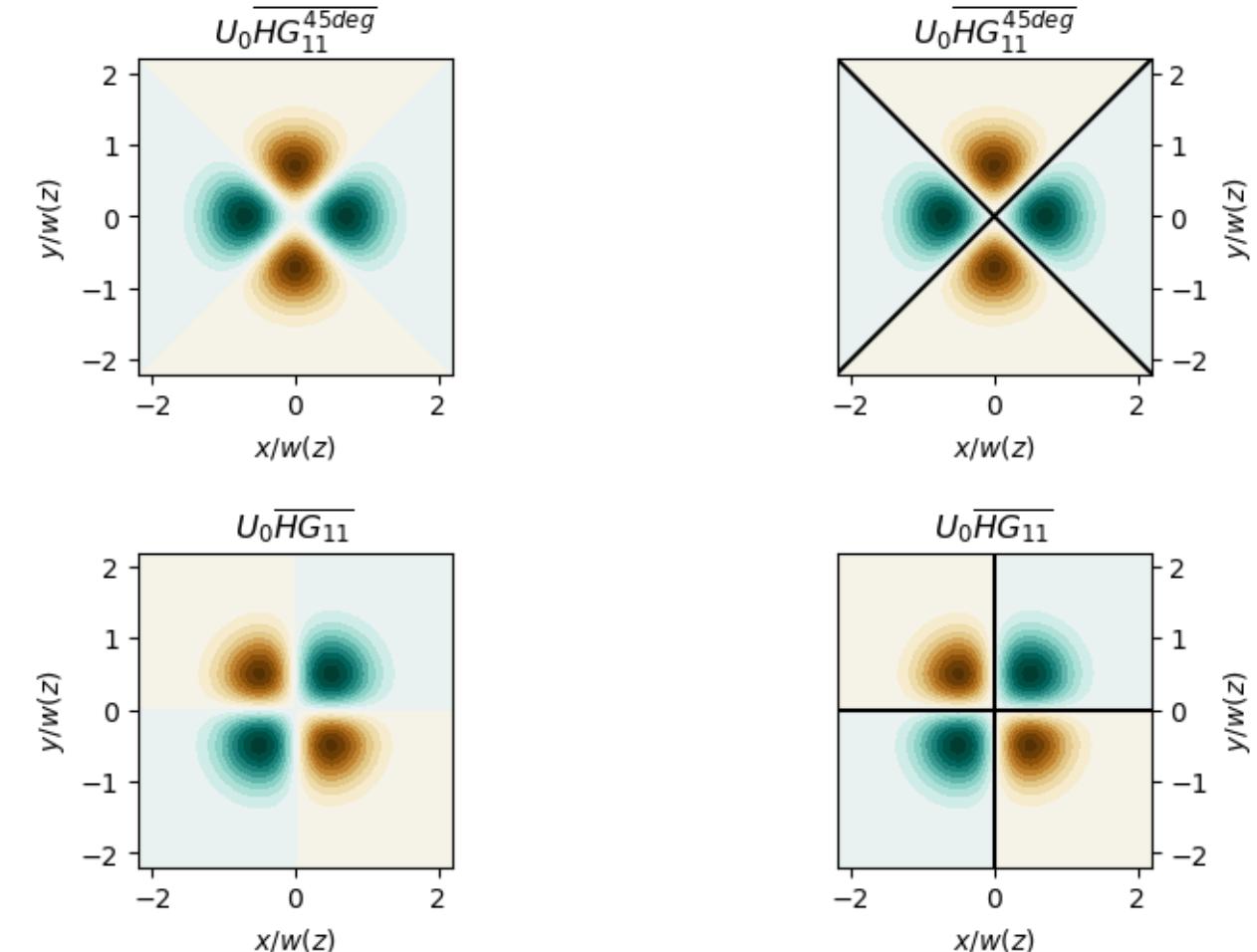
# Astigmatic Mismatch

## Upgrade of Mode Converter Technique

### Astigmatic Mismatch

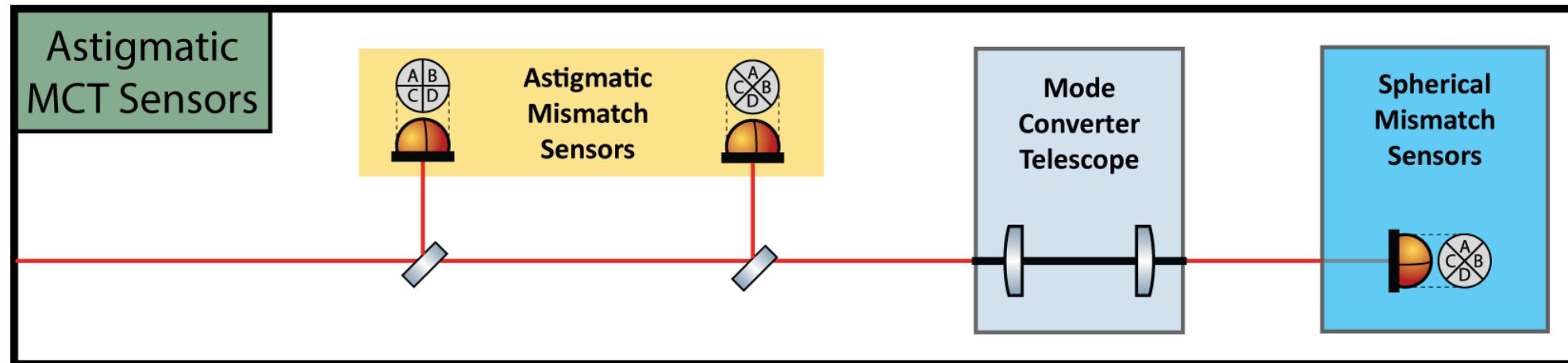
To detect the Astigmatic mismatch, we need to measure the beat-note of the HG11 and HG $11^{45\text{deg}}$

- ❖ Detectors before the Mode Converter Telescope
- ❖ Quadrant Photodiodes rotated by 45 deg with x/y
- ❖ Quadrant Photodiodes aligned with x/y

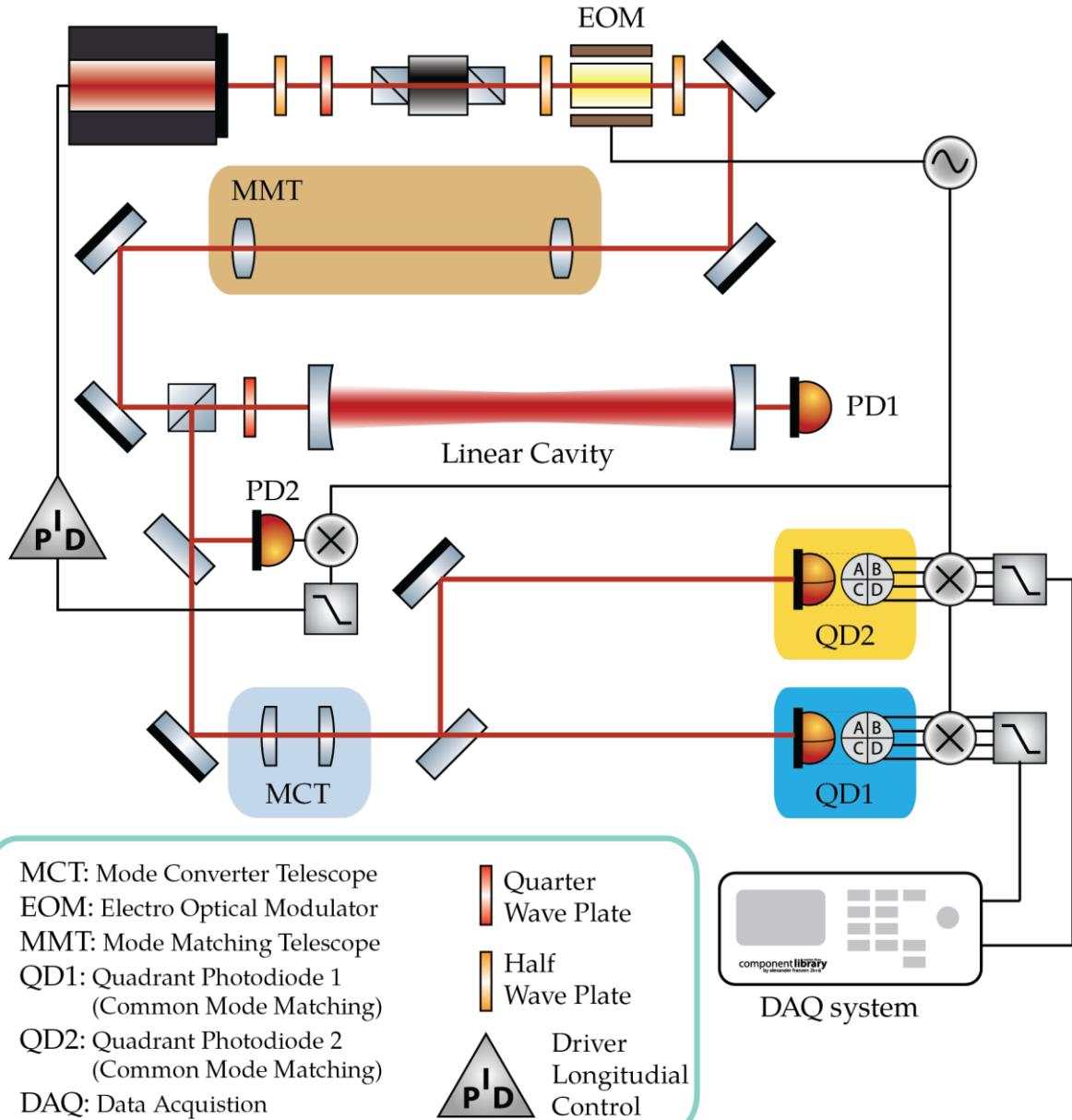


# Astigmatic Mismatch

## Upgrade of Mode Converter Technique



	Quadrant Photodiodes with Mode Converter Telescope	Quadrant Photodiodes Oriented 45 deg with P/T convention	Quadrant Photodiodes Oriented with P/T convention
Gouy Phase 0	$\beta = \frac{\langle \omega_0^{x,y} \rangle - \omega_0^{cav}}{\omega_0^{cav}}$	$(\alpha = \frac{\omega_0^x - \omega_0^y}{\omega_0^{cav}}) \cos \theta$	$(\alpha = \frac{\omega_0^x - \omega_0^y}{\omega_0^{cav}}) \sin \theta$
Gouy Phase 45 deg	$\gamma = \frac{\langle z_0^{x,y} \rangle - z_0^{cav}}{2z_R^{cav}}$	$(\eta = \frac{z_0^x - z_0^y}{2z_R^{cav}}) \cos \theta$	$(\eta = \frac{z_0^x - z_0^y}{2z_R^{cav}}) \sin \theta$



# Simplified Optical Setup

## Validation of Mode Converter Telescope Setup

- ❖ Two Quadrants Photodiodes for the Spherical Mismatch sensing
- ❖ Spherical Mode Matching Telescope used to decouple the Mismatch degree of freedom
- ❖ Average Waist Dimension  $\beta = \frac{\langle \omega_0^{x,y} \rangle - \omega_0^{cav}}{\omega_0^{cav}}$
- ❖ Average Waist Position  $\gamma = \frac{\langle z_0^{x,y} \rangle - z_0^{cav}}{2z_R^{cav}}$



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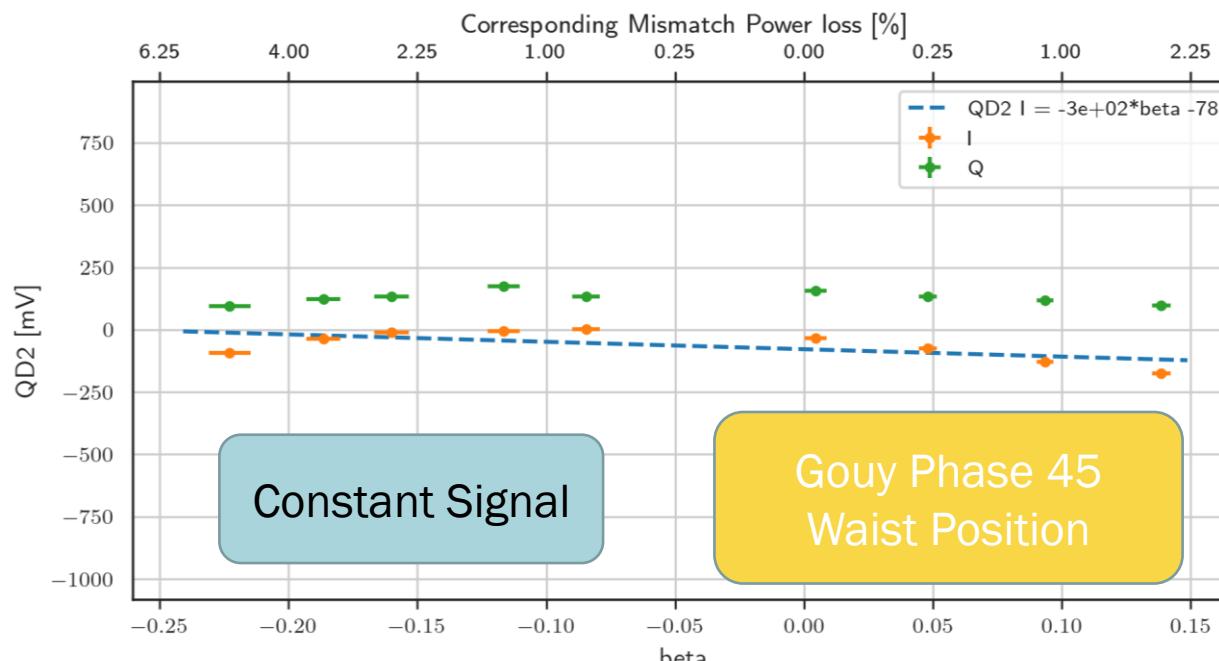
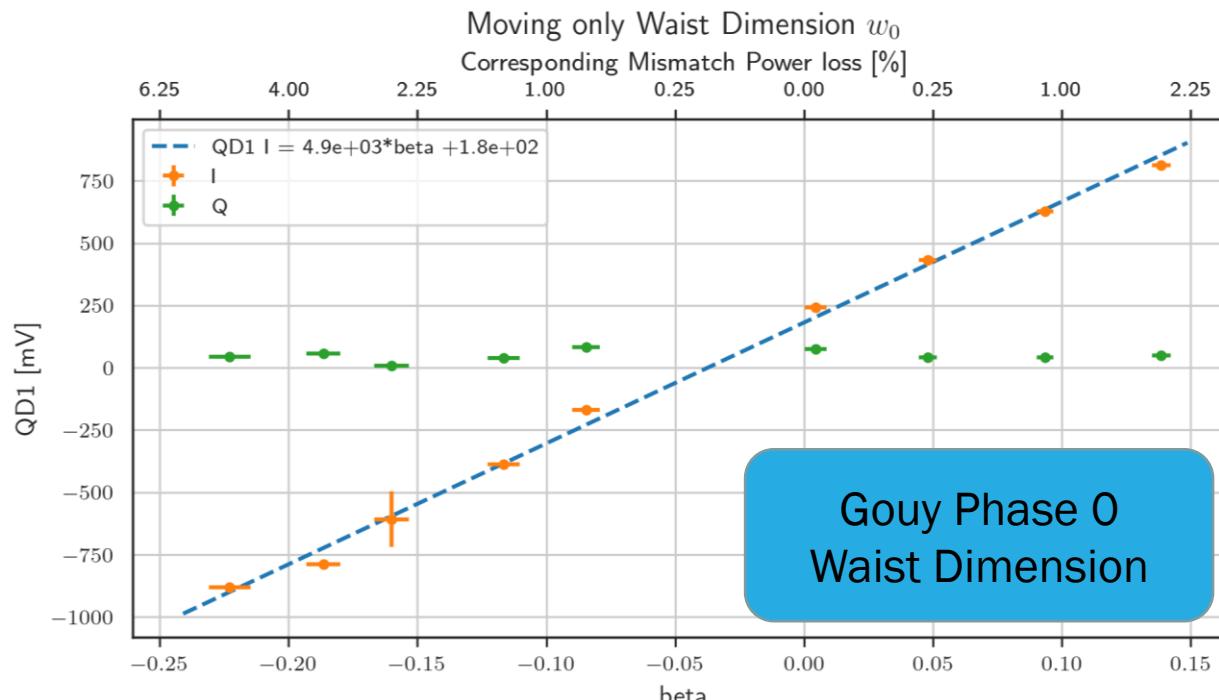
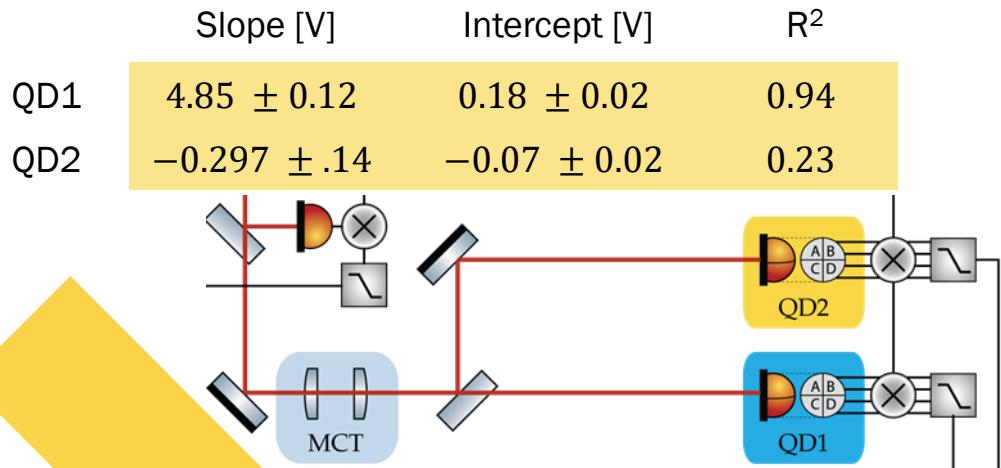
# Spherical Mismatch

## Validation of the Mode Converter Technique

### Spherical Waist Dimension

Quadrant Photodiode 1 is sensitive to change of

$$\beta = \frac{\langle \omega_0^{x,y} \rangle - \omega_0^{cav}}{\omega_0^{cav}}$$



# Spherical Mismatch

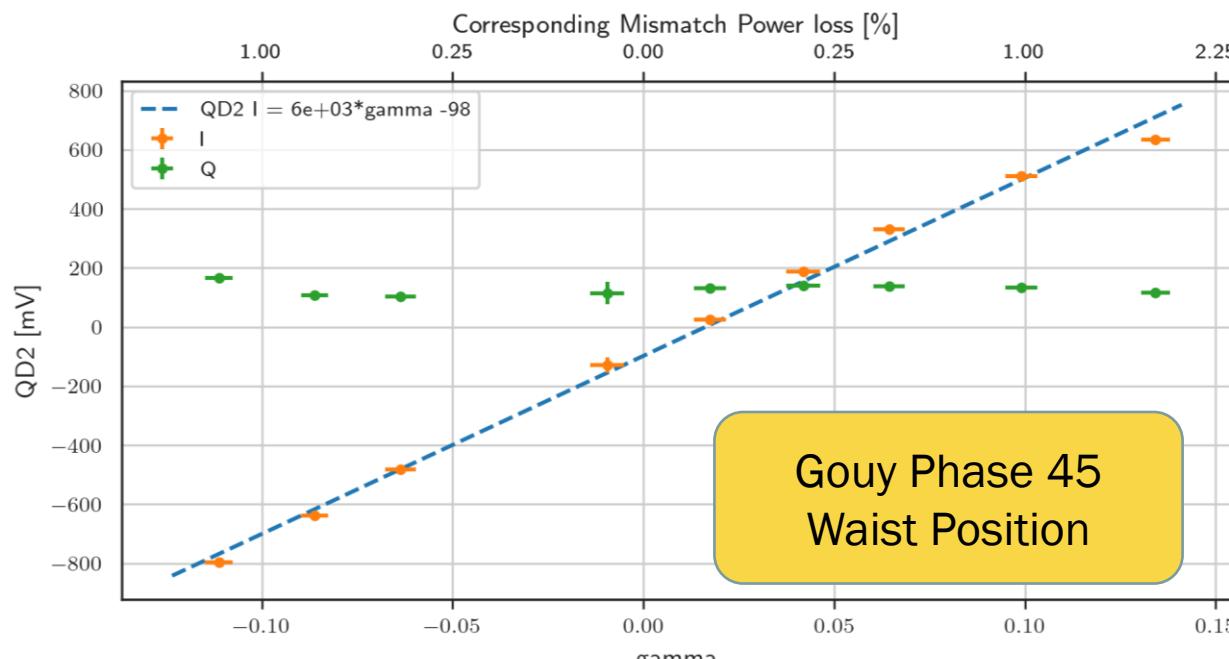
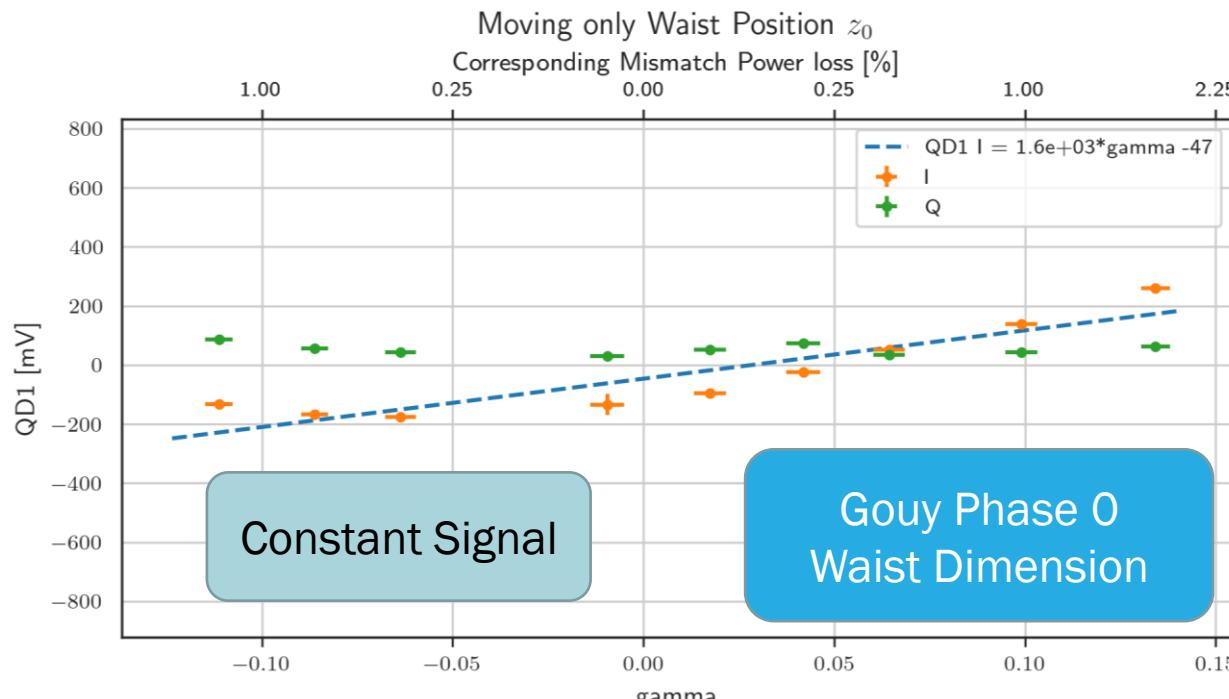
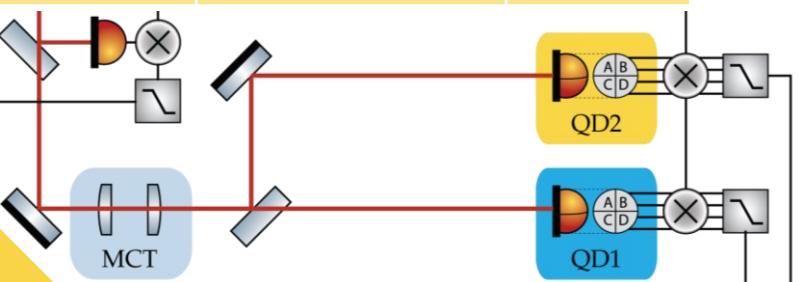
## Validation of the Mode Converter Technique

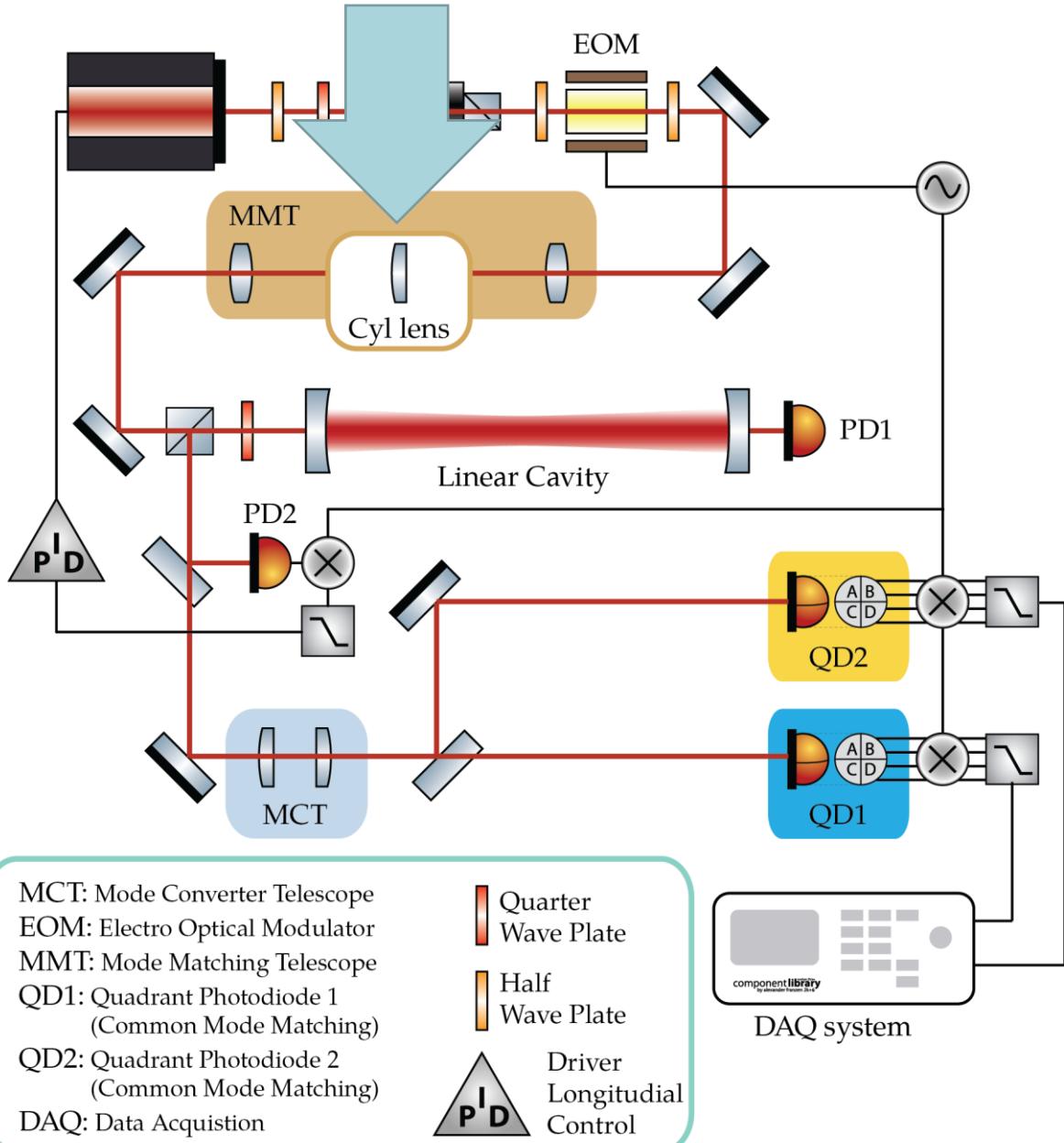
### Spherical Waist Position

Quadrant Photodiode 2 is sensitive to change of

$$\gamma = \frac{\langle z_0^{x,y} \rangle - z_0^{cav}}{2z_R^{cav}}$$

	Slope [V]	Intercept [V]	R <sup>2</sup>
QD1	1.63 ± 0.28	-0.05 ± 0.02	0.59
QD2	6.026 ± .17	-0.10 ± 0.01	0.93





# Simplified Optical Setup

## Validation of Mode Converter Telescope Setup

- Average Waist Dimension  $\beta = \frac{\langle \omega_0^{x,y} \rangle - \omega_0^{cav}}{\omega_0^{cav}}$
- Average Waist Position  $\gamma = \frac{\langle z_0^{x,y} \rangle - z_0^{cav}}{2z_R^{cav}}$
- No sensitivity to Astigmatism
- We add a single Cylindrical Lens in the Mode Matching Telescope



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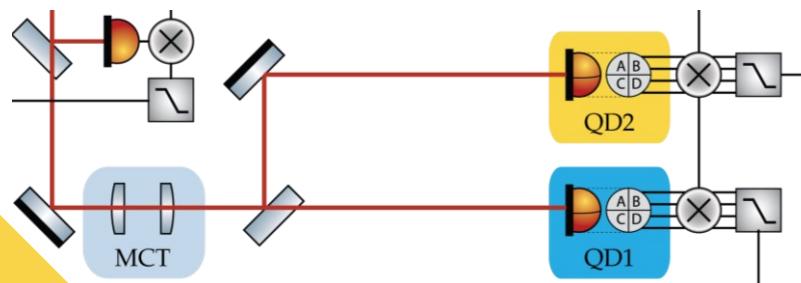
# Astigmatic Mismatch

## Validation of the Mode Converter Technique

### Unsensitivity to Astigmatic Mismatch

$$\text{QD1 Constant} \iff \beta = \frac{\langle \omega_0^{x,y} \rangle - \omega_0^{\text{cav}}}{\omega_0^{\text{cav}}} = \text{Const}$$

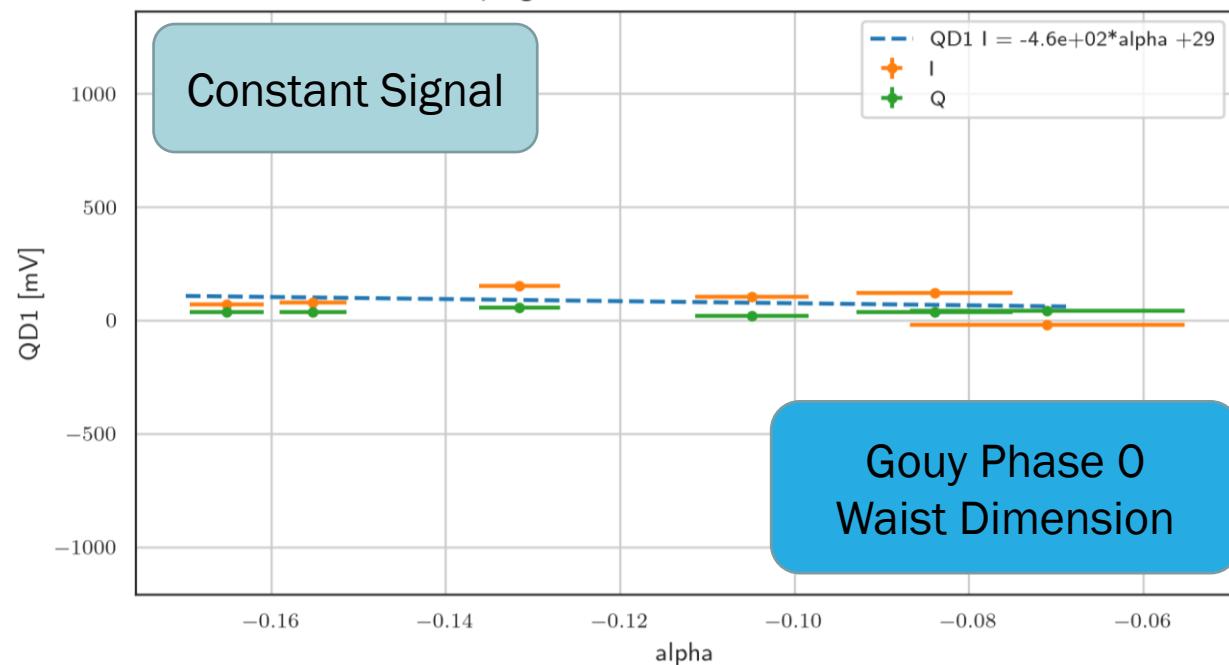
	Slope [V]	Intercept [V]	R <sup>2</sup>
QD1	-0.46 ± 0.73	0.03 ± 0.09	0.05
QD2	19.1 ± 6.4	-2.4 ± 0.78	0.45



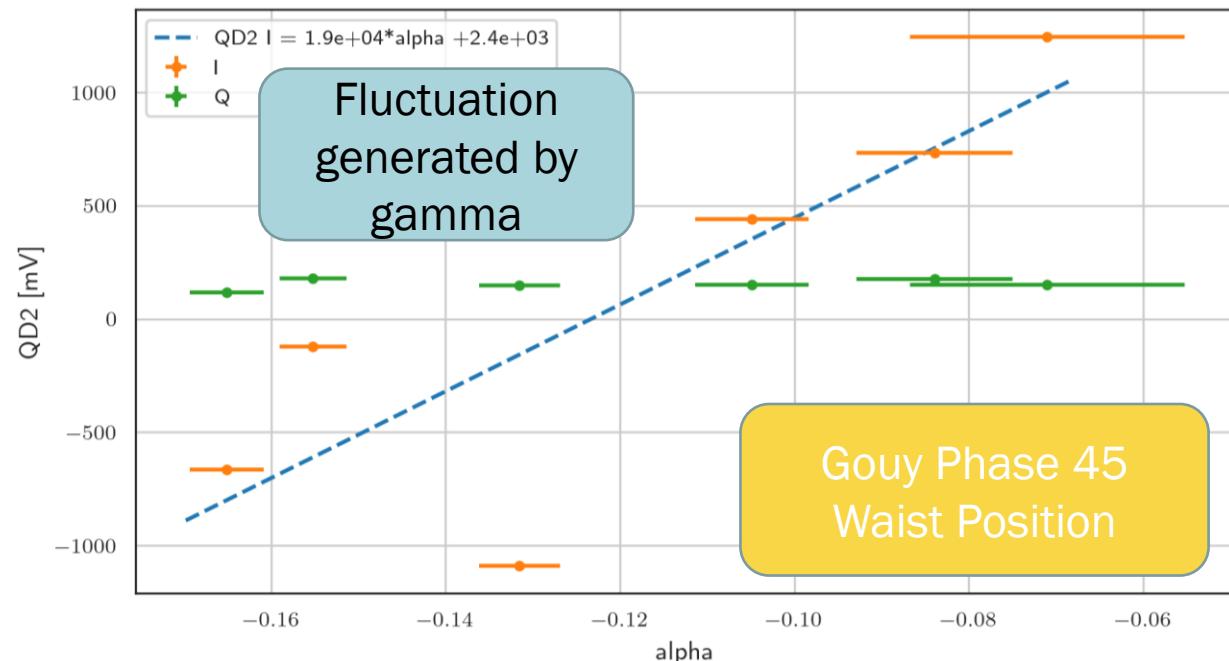
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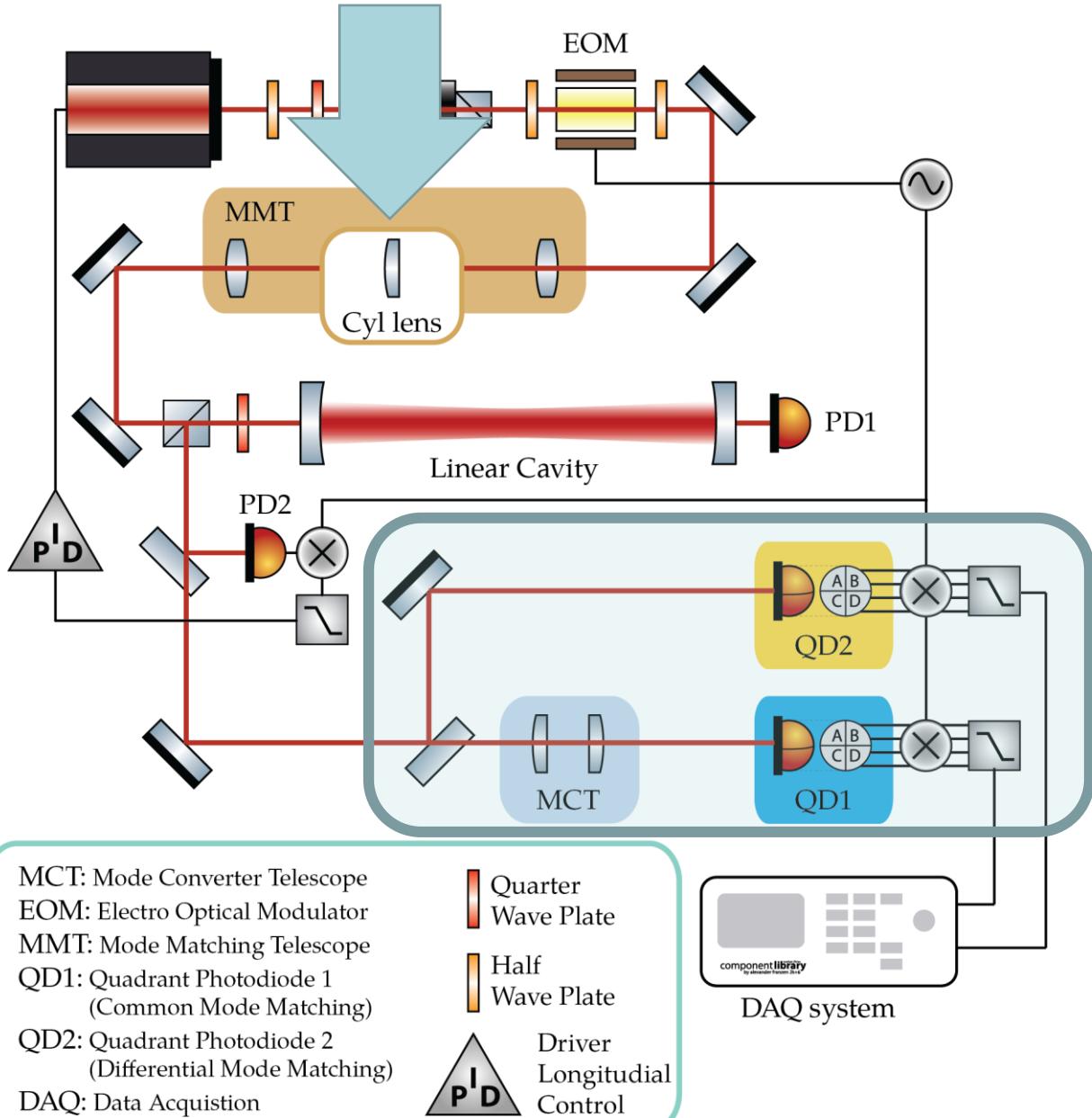
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Keeping Waist Dimension  $w_0$  Constant

Gouy Phase 0  
Waist Dimension



Gouy Phase 45  
Waist Position



# Simplified Optical Setup

## Setup for the Astigmatic Validation

- ❖ Quadrants Photodiode QD2 reads the beat-note before the Mode Converter Telescope

1. Waist Dimension Difference

$$\alpha = \frac{\omega_0^x - \omega_0^y}{\omega_0^{cav}}$$

2. Waist Position Difference

$$\eta = \frac{z_0^x - z_0^y}{2z_R^{cav}}$$

3. Orientation Axis

$$\theta$$



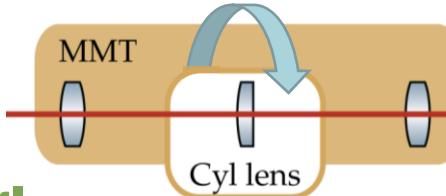
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# Astigmatic Mismatch

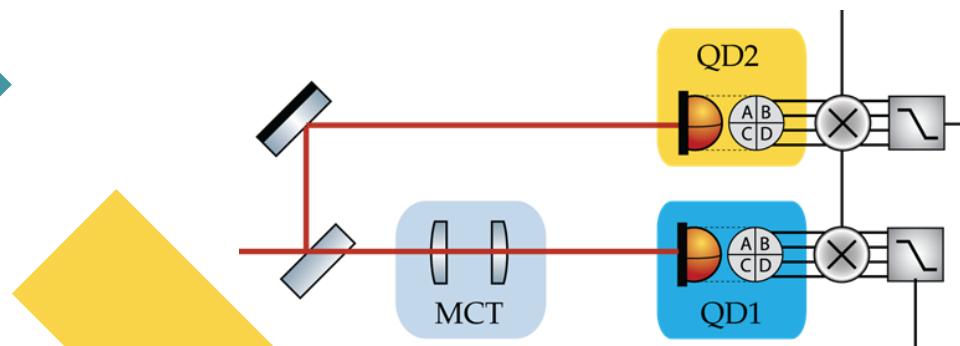
## First astigmatic Parameter



## Astigmatic Ellipse Angle

We verified the Angle dependency by rotating the cylindrical Lens in the Mode Matching telescope

	Amplitude [mV]	Theta 0 [deg]	Intercept [V]	R <sup>2</sup>
QD2	260 ± 7	286 ± 2	1 ± 5	0.89

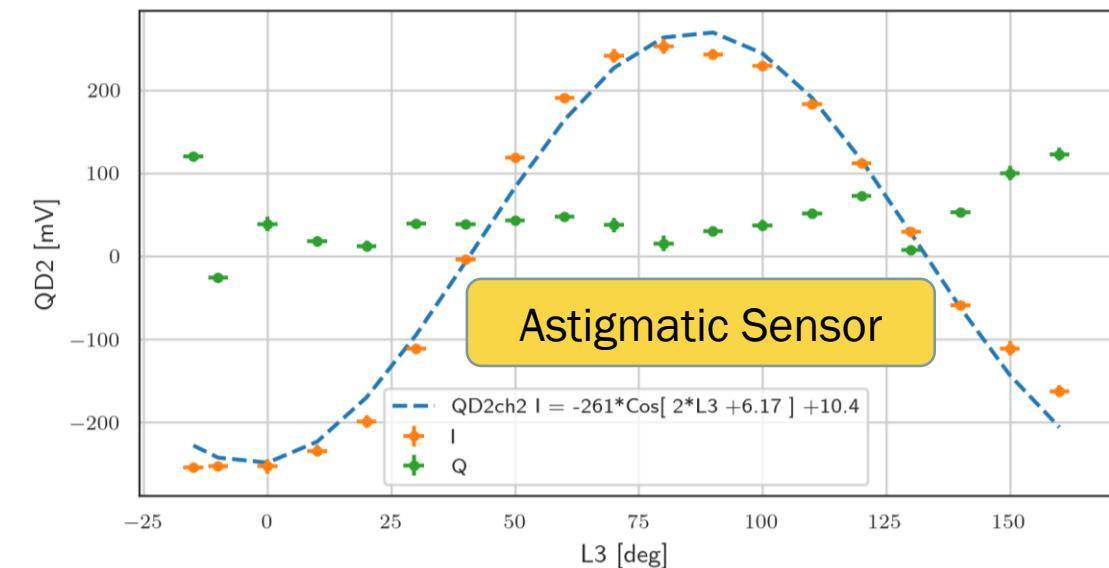
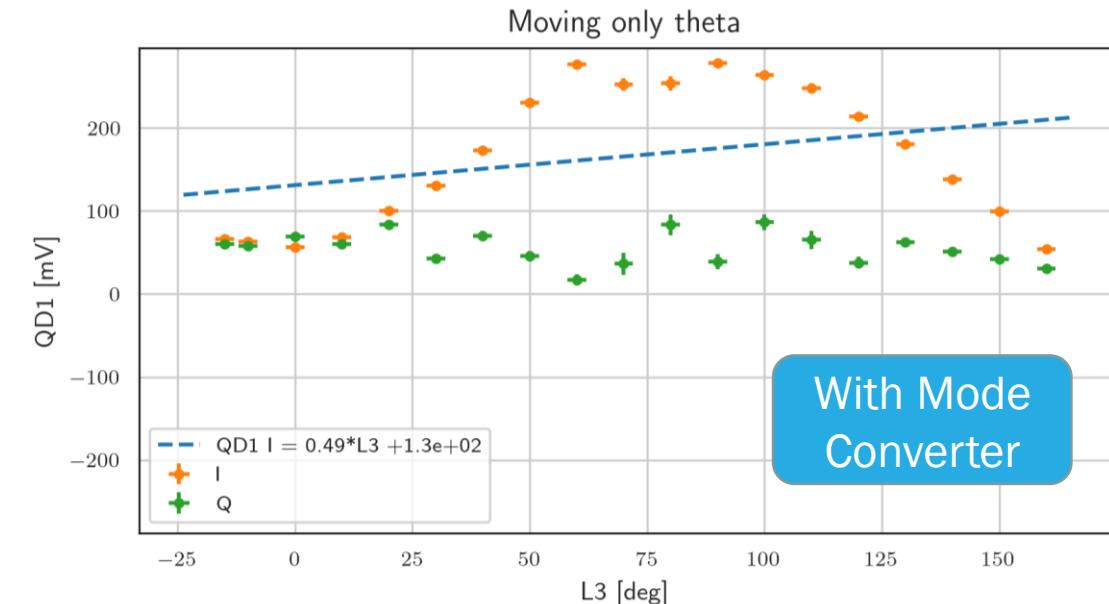


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$$\alpha = 0.13 \pm 0.02$$



$$\delta\epsilon \cdot \frac{u_{20}(x, y | q_c) - u_{02}(x, y | q_c)}{\sqrt{2}} \cos 2\theta$$

# Astigmatic Mismatch

## First Results

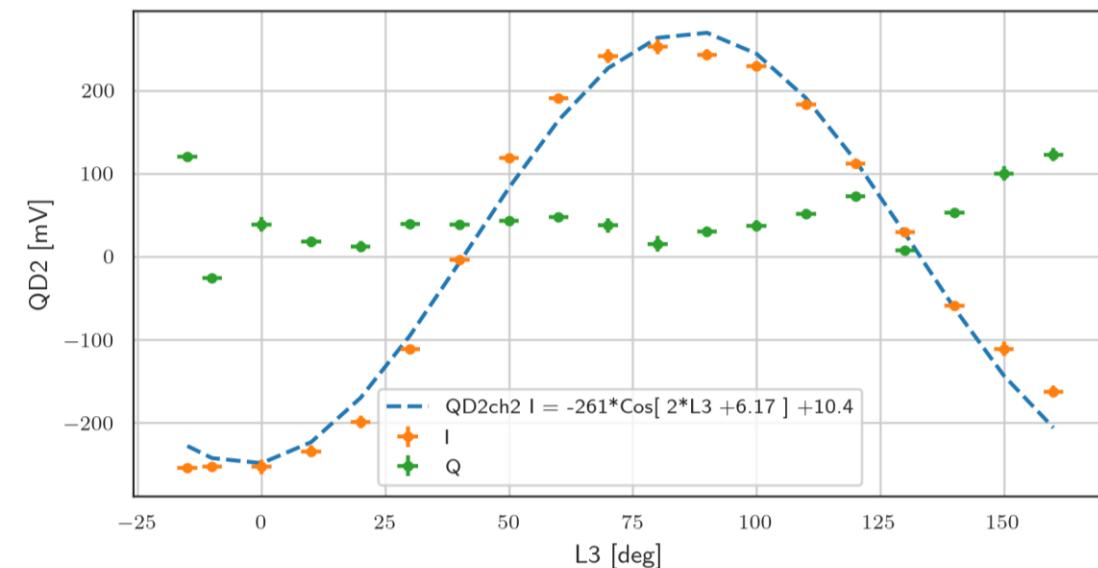
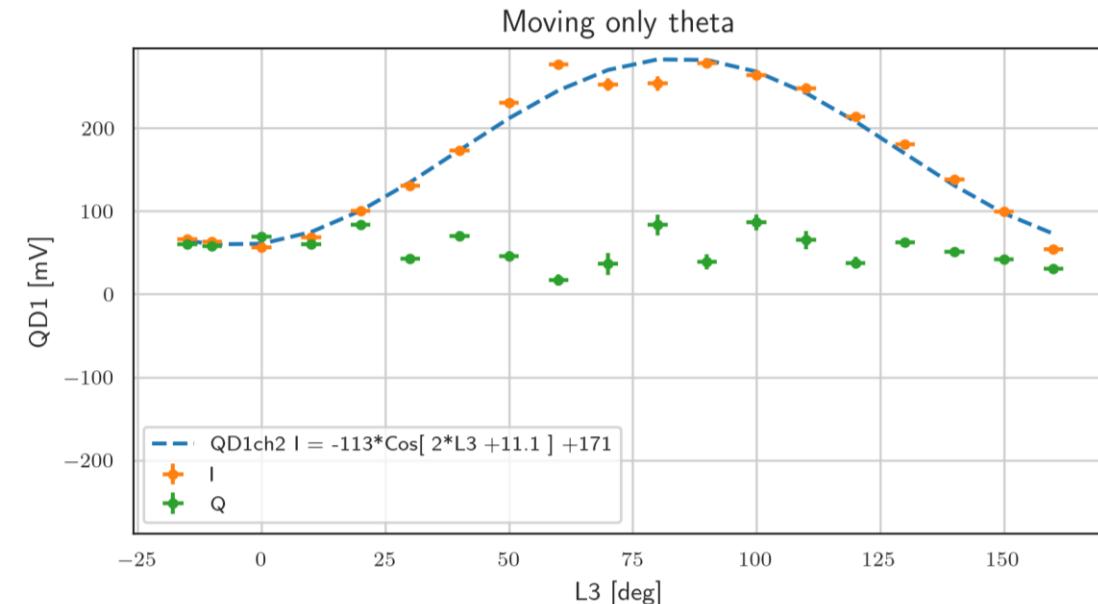
### Astigmatic Mode Matching

Mode Converter Telescope Tuning is not perfect and part of the HG11<sup>45deg</sup> is not completely converted in a LG10

$$\alpha = 0.13 \pm 0.02$$

$$\varphi = \tan^{-1} \left[ \frac{A}{QD1ch2[\alpha]} \right] \approx (89.8 \pm 8.7)deg$$

	Amplitude [mV]	Theta 0 [deg]	Intercept [V]	R <sup>2</sup>
QD1	113 ± 4	11 ± 2	171 ± 3	0.84
QD2	260 ± 7	6 ± 2	1 ± 5	0.89



# Summary

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## Astigmatic Mismatch Model

We propose the theoretical framework for the Astigmatic wavefront sensor

## Astigmatic Mismatch Sensing

We define an optical scheme to detect the 5 degree of freedom of the Simple Astigmatic Mode Matching

## Experimental Results

We verified the limit of the current Technology

We make the first validation measurement of the new method

## Future

### Theoretical Mode

1. General Astigmatic Gaussian Beam
2. Astigmatic Cavities

### Experimental Setup

1. Design, installation and characterization of astigmatic mode matching Telescope
2. Installation of other 4 Quadrant Photodiode sensors

# Padova-Trento team

## Hardware



### Trento



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**Antonio Perreca**

Professor  
University of Trento



**Andrea Grimaldi**

PhD Student  
University of Trento



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