PROBING BLACK HOLES WITH QUASI-NORMAL MODES

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WHAT ARE (QUASI-)NORMAL MODES?

Basic example: a sum of (damped) oscillations (e.g. guitar string!)

$$y(t) = \sum_n A_n \exp\left(i\omega_n t\right),$$

- excitation coefficients depend on initial data (how one perturbs the system)
- QNMs **only** depend on unperturbed system (inherent property)

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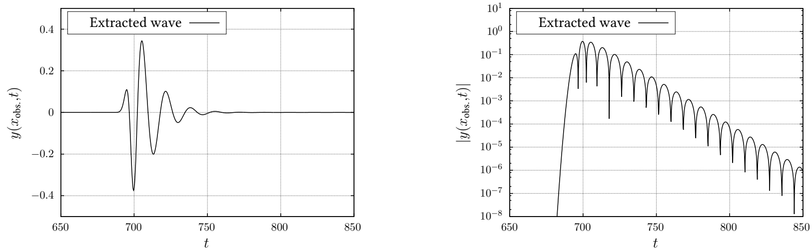


Time evolution of perturbations that scattered with a black hole as observed far away.

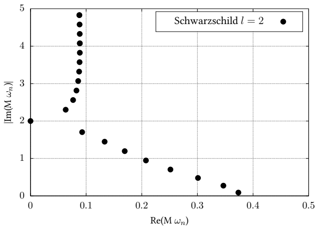
SCHWARZSCHILD PERTURBATION POTENTIALS

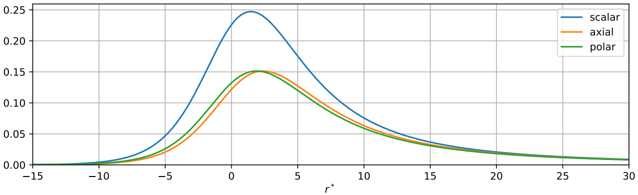
$$\frac{\mathrm{d}^2\psi}{\mathrm{d}r^{*2}} + \left[\omega^2 - V(r)\right]\psi = 0$$

$$V_l^{\mathrm{sc.}}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2M}{r^3}\right)$$

$$V_l^{\mathrm{ax.}}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right)$$

$$V_l^{\mathrm{po.}}(r) = \left(1 - \frac{2M}{r}\right)\frac{2\lambda^2(\lambda+1)r^3 + 6\lambda^2 M r^2 + 18\lambda M^2 r + 18M^3}{r^3(\lambda r + 3M)^2}$$





Perturbation potentials for $l = 2$ as function of the tortoise coordinate r^*.

SOME COMMENTS FOR GR AND BEYOND

Qualitative features of GR (vacuum):

- perturbation equations are separable
- no additional fields or couplings
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Possible impact of environmental effects:

- presence of matter modifies the effective potential
- additional fields could be viewed as environmental effect

MOTIVATION

Extraction of QNMs from binary black hole mergers:

- commonly done for NR simulations (e.g. Giesler et al. [1], Bhagwat et al.[2])
- real (current) data highly non-trivial (e.g. Isi et al.[3], Cotesta et al. [4])
- highly rewarding to test Kerr hypothesis
- **What would one learn if QNMs deviate from Kerr?**

[1]Giesler et al., Phys.Rev.X 9 041060 (2019)
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Assume a few/"all" QNMs would be known from observations:

- Can one recover the effective potential?
- Can one recover the presence of coupling functions to additional fields?
- How do measurement uncertainties impact the reconstruction?
- **What are robust "features"?** (PN $\sim M/r$ like expansion?)

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METHODS OVERVIEW

We adopt the parameterized framework to study non-rotating black holes[5, 6]:

- coupled equations are prototype for rotation and alternative theories

$$f \frac{\mathrm{d}}{\mathrm{d}r}\left(f \frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}r}\right) + \left[\omega^2 - f\mathbf{V}\right]\mathbf{\Phi} = 0, \tag{1}$$

- with $f(r) = 1 - r_0/r$, with r_0 being the location of the event horizon and

$$\mathbf{\Phi} = \left[\Phi^{\text{scalar}}, \Phi^{\text{polar}}, \Phi^{\text{axial}}, \ldots\right]. \tag{2}$$

- it is assumed the potential can be written as (“natural extension” to GR)

$$V_{ij} = V_{ij}^{\mathrm{GR}} + \delta V_{ij}, \qquad \delta V_{ij} = \frac{1}{r_H^2} \sum_{k=0}^{\infty} \alpha_{ij}^{(k)} \left(\frac{r_H}{r}\right)^k \tag{3}$$

[5] Cardoso et al., PRD 99 104077 (2019)
[6] McManus et al., PRD 100 044061 (2019)

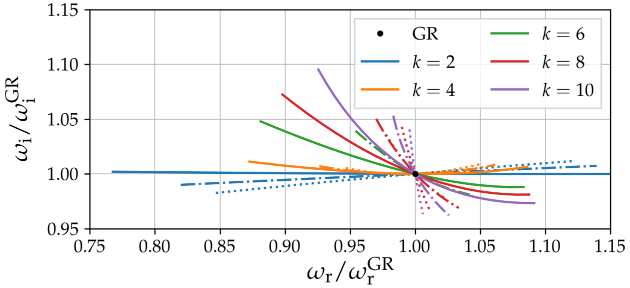
METHODS OVERVIEW

Parametrized framework relates ω with $\alpha^{(k)}$ in a simple way

- approximate expression up to quadratic order in $\alpha^{(k)}$

$$\omega = \omega^0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \alpha_{ij}^{(k)} \partial_\omega \alpha_{pq}^{(s)} d_{(k)}^{ij} d_{(s)}^{pq} + \frac{1}{2} \alpha_{ij}^{(k)} \alpha_{pq}^{(s)} e_{(ks)}^{ijpq} \tag{4}$$

- coefficients $d_{(k)}$, $e_{(ks)}$ are universal, $\alpha^{(k)}$ theory dependent or free parameters
- we **extended coefficients** to compute the $n = 1,2$ **overtones**

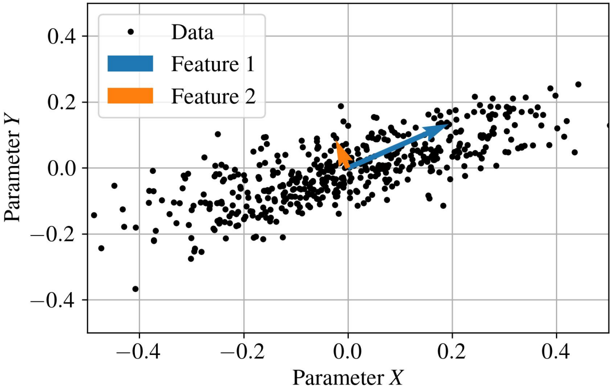


Modifications to the QNM spectrum. SV, Franchini and Barausse, PRD 105 084046 (2022).

METHOD OVERVIEW: PCA IN A NUTSHELL

What is principal component analysis (PCA)?

- PCA is a well known tool for linear problems
- finds most "**efficient basis**" describing the "**main features**" of data



ASSUMPTIONS AND RESULTS

Remarks on our likelihood and QNM "data" (full equations in paper)

- "post processing" idea, assume you know the spectrum directly
- precision of QNMs as proof of principle here $1\,\%$
- marginalize over black hole mass (r_{H} with $5\,\%$)
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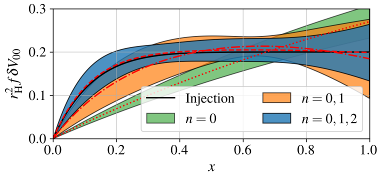

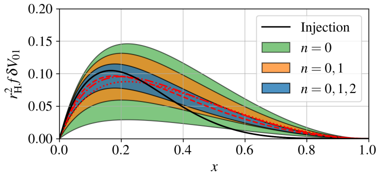

Left: Injection and PCA reconstruction of parametrized deviations of the axial potential only.
Right: Injection and PCA reconstruction of parametrized deviations of only symmetric coupling functions, injecting dynamical Chern-Simons as example.
SV, Franchini and Barausse, PRD 105 084046 (2022).

FOLLOW-UP QUESTIONS

- Can one **constrain individual** $\alpha^{(k)}$ and their correlations with an MCMC?
- Are constraints for **single parameters** robust **versus many parameters**?
- What is the **role of priors** for the studied parameters?
- Do possible **correlations of the provided QNMs** play a big role?
- Are there more **"intuitive" features**?

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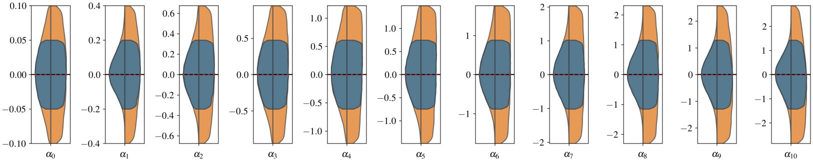
WKB method connects derivatives of potential peak with QNMs[7]

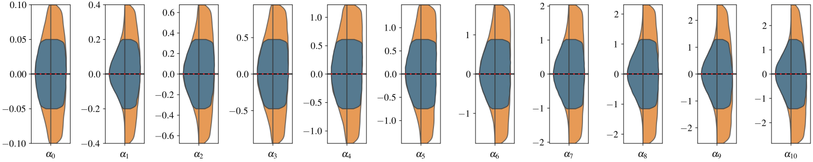
$$\omega_n^2 = V^{(0)} + i\sqrt{-2V^{(2)}}\left(n + \frac{1}{2}\right) + \sum_i \tilde{\Lambda}_i(n) \tag{5}$$

Results



Inject GR $n = 0{,}1$ QNMs $\vec{\omega}$ with relative errors of $1.0\,\%, 4.7\,\%, 3.4\,\%, 8.2\,\%$. We then sample r_{H} and $\alpha^{(k)}$ for $k = 0 \ldots 10$. Left violins show rH with one specific $\alpha^{(k)}$ varied at a time, right violins show rH varied with all $\alpha^{(k)}$ at the same time. Different colors correspond to different prior ranges for $\alpha^{(k)}$.

RESULTS



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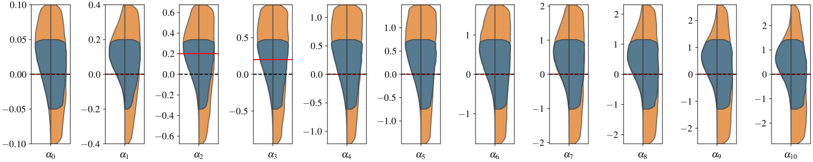
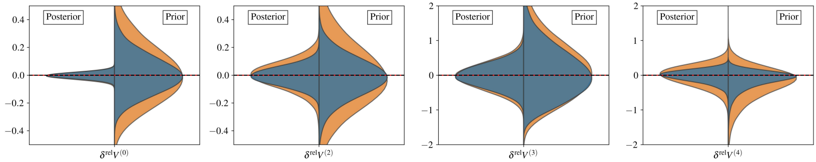


Inject mock non-GR $n = 0,1$ QNMs $\vec{\omega}$ with non-zero $\alpha^{(k)}$ for $k = 2$ and $k = 3$ (shown in red).

RESULTS



Samlpling derivatives of the effective potential with respect to tortoise coordinate from the “all $\alpha^{(k)}$” posterior distributions (left sides) versus sampling from the priors of $\alpha^{(k)}$ (right sides). **GR injection**.

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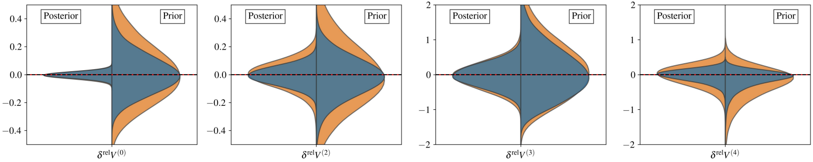

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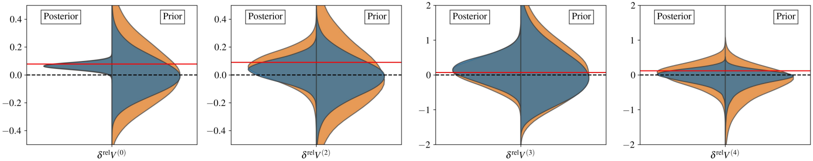

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The Good:

- **in principle clean test**, less uncertain astrophysics (compared to EHT)
- more precise data becomes available (**quest of overtones**)
- semi-analytic techniques and statistical methods can be combined

Open Problems:

- theory specific tests for **rotating** black holes very hard
- parametrized background metrics exist, but “dynamics” with rotation unclear
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Grazie mille, thank you very much!