

Accretion disks around supermassive black-hole binaries can break

Davide Gerosa

University of Milano-Bicocca

Jun 17, 2022

EuCAPT workshop: Gravitational wave
probes of black hole environments

Rome, Italy

with: G. Rosotti, R. Barbieri, R. Nealon, E. Ragusa,
N. Steinle, G. Lodato, B. Veronesi



European Research Council

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Supermassive BHs: the road to merger

Begelman, Blandford, Rees 1980 (seminal) Colpi 2014 (review)

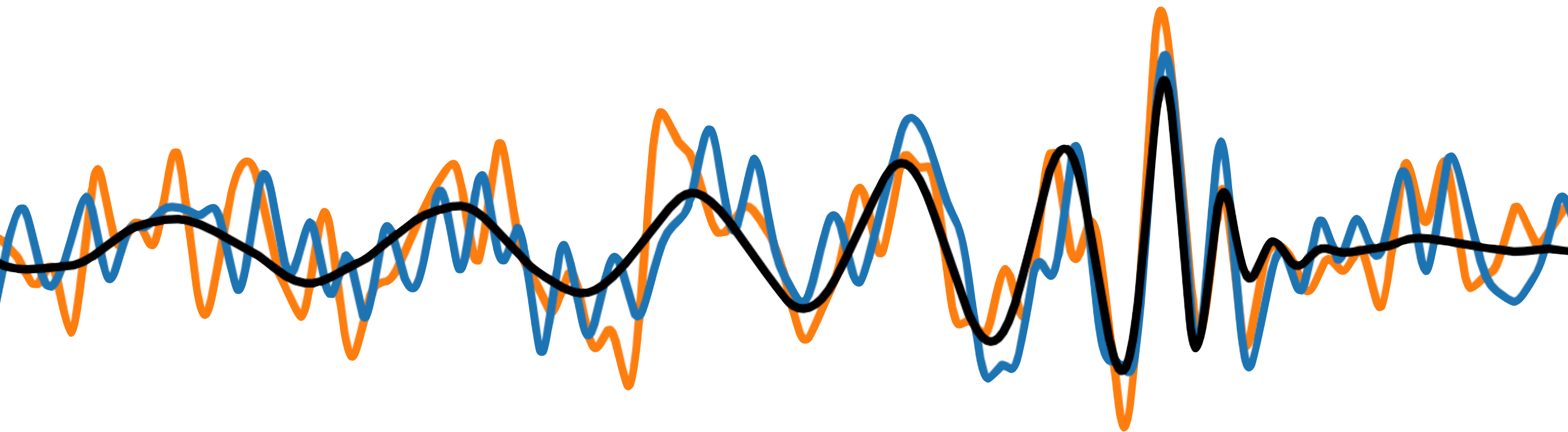
1. Galaxy mergers → SMBHs left at **~kpc** scale.
2. Dynamical friction → Down to **~10 pc**. Binary formation
3. Star scattering → Loss-cone refilling, triaxiality. Down to **~1pc**
Merritt Poon 2004
4. Gas-driven phase → Possibly **~0.01pc**.
5. Gravitational-waves → $t_{\text{GW}} \sim r^4$... prompt merger **if** $r \sim 0.01 \text{ pc}$
Peters & Mathews 1963
6. Merger → **LISA detection!**
7. Recoil: ejections? → up to 5000 Km/s > **escape speed** of galaxies
Gonzales et al. 2007; Campanelli et al. 2007



Do gravitational waves remember the disk?

Can't really say with the masses, but **spins are the secret**

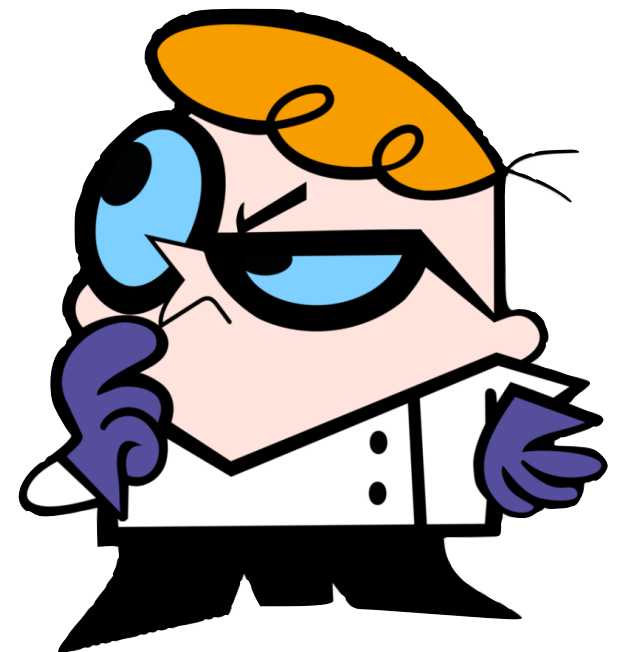
- If misaligned: relativistic spin precession in the LISA band
- Unique signal morphology
- Measurable with LISA's signal-to-noise ratios
- Superkicks strongly correlated with spin dynamics



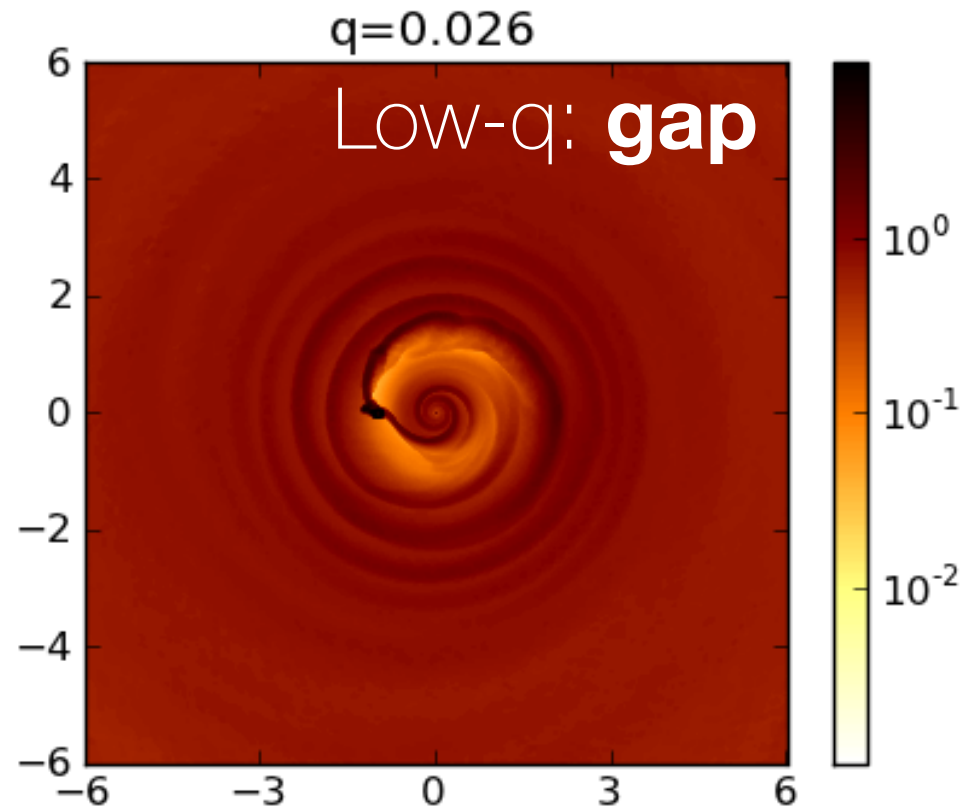
Step 1 [arXiv:1503.06807](https://arxiv.org/abs/1503.06807)

What are the timescales involved?

Inspiral and spin alignment take place simultaneously in the race towards merger



Gas-driven migration



Inspiral:
circumbinary disk

Alignment:
individual disks

Definitions

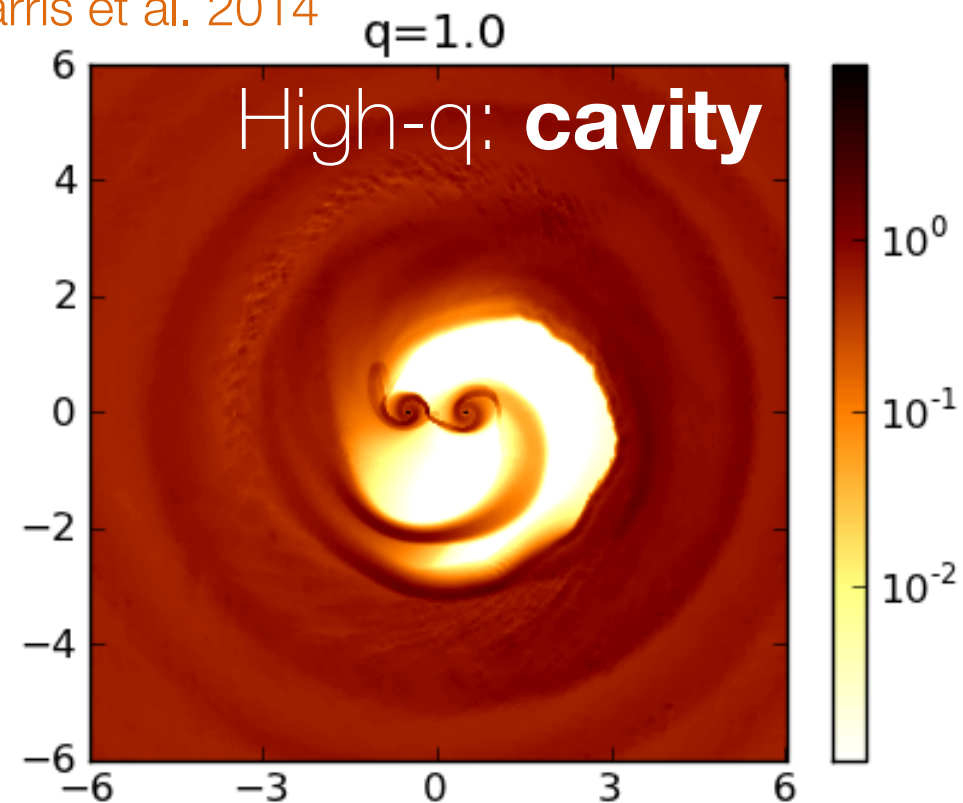
Primary BH: M_1

Secondary BH: M_2

Mass ratio: $q = M_2/M_1 \leq 1$

Disk mass: $M(R)$
within the binary separation

Farris et al. 2014



$$t_{\text{in}}(R) = \frac{M_1}{M_1 + M_2} \frac{M_2 + M(R)}{M(R)} t_{\nu}(R)$$

High-q: speed up
Rafikov 2013

Low-q: viscous time
Armitage & Natarajan 2002

Massive secondary: slow down
Syer & Clarke 1995; Ivanov et al. 1999; Lodato et al. 2009

Gas disks can align the spins

Bardeen-Petterson effect

Bardeen & Petterson 1975; Rees 1978

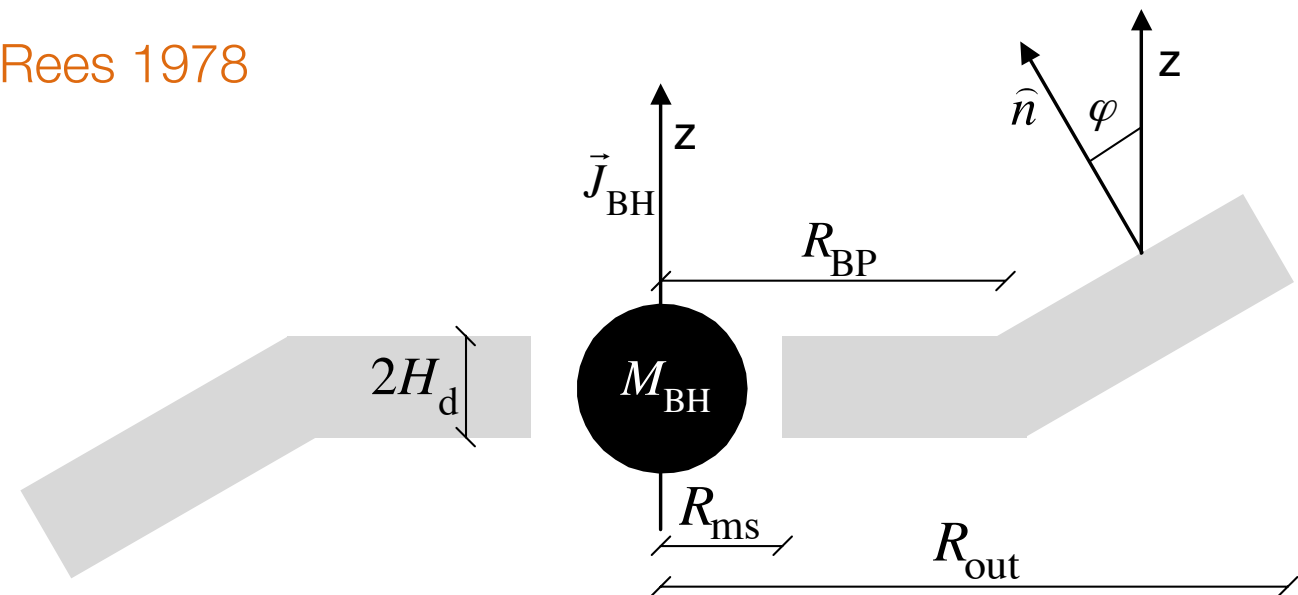
1. Lense-Thirring (GR) precession:
inner disk quickly aligns

$$R_{\text{BP}} \simeq \left(\frac{\chi}{\alpha_2}\right)^{2/3} \left(\frac{H}{R}\right)^{4/3} \left(\frac{GM}{c^2}\right) \sim 10^{-3} \text{pc}$$

2. Reaction: the **outer disk** pulls the BH spin on a timescale

$$t_{\text{align}} \simeq \frac{M}{\dot{M}} \alpha \left(\frac{\chi}{\alpha_2} \frac{H}{R}\right)^{2/3} \simeq 5 \text{ Myr}$$

Scheuer Feiler 1996
Natarajan Pringle 1998
Martin+ 2007, 2009
Lodato **DG** 2012

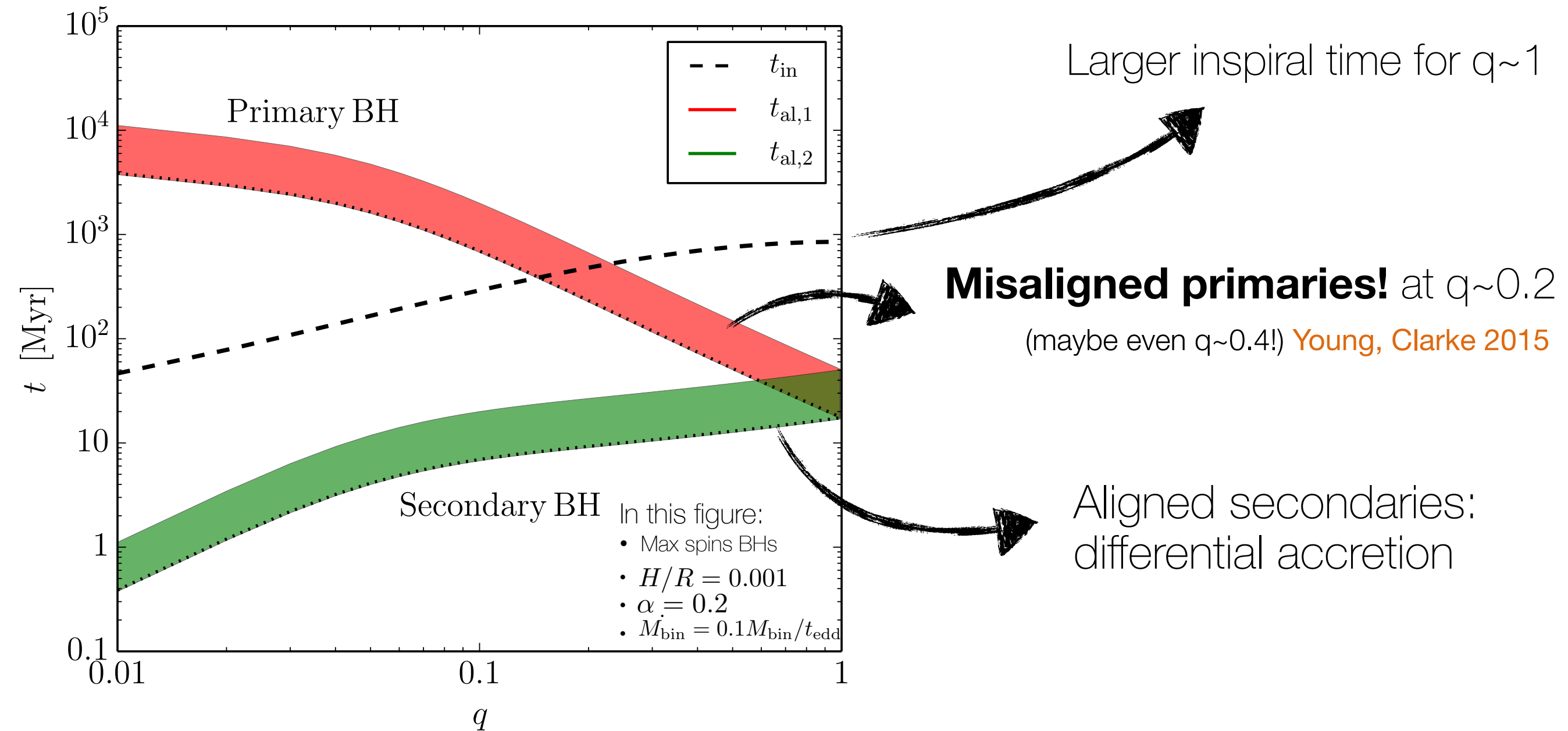


Adapted from Caproni et al. 2006

In a binary: is there enough time to align the spins?

A tale of three timescales

DG+ 2015



Using cosmologically-motivated distributions: **~10% of misaligned primaries!**

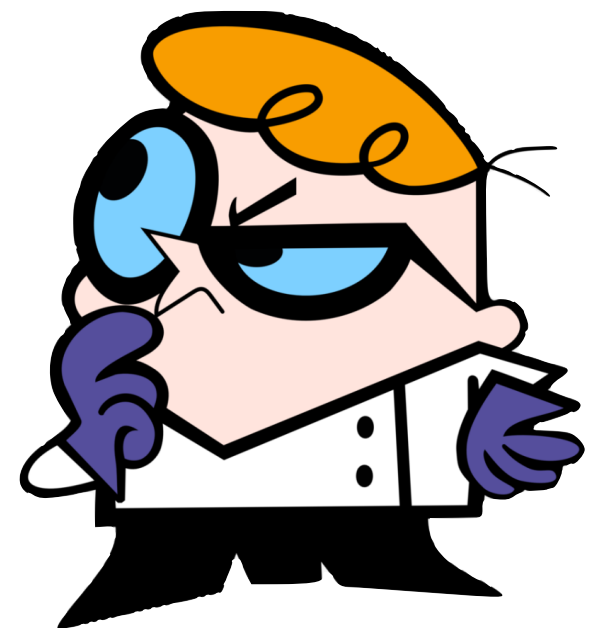
[Arun et al. \(LISA team\) 2008](#); [Sesana+ 2011](#)

Small secondaries prevent primaries from aligning!

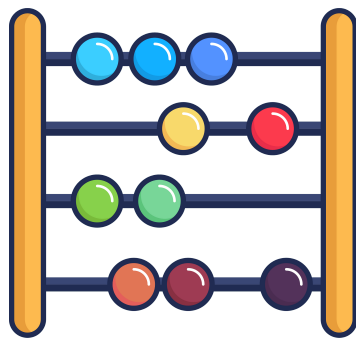
Step 2 [arXiv:2004.02894](https://arxiv.org/abs/2004.02894)

How to go beyond timescales and predict the residual misalignments?

A systematic approach to solve for structure of the disks
and the backreaction onto the BH



Warped accretion disks



Pringle 1992

Mass conservation

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[R^{1/2} \frac{\partial}{\partial R} \left(\nu_1 \Sigma R^{1/2} \right) \right] + \frac{1}{R} \frac{\partial}{\partial R} \left[\nu_2 \Sigma R^2 \left| \frac{\partial \hat{\mathbf{L}}}{\partial R} \right|^2 \right]$$

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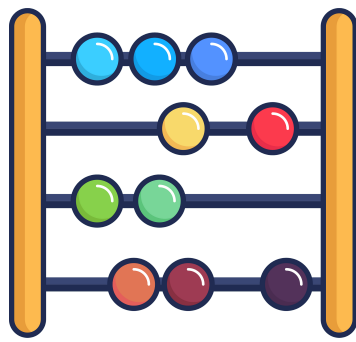
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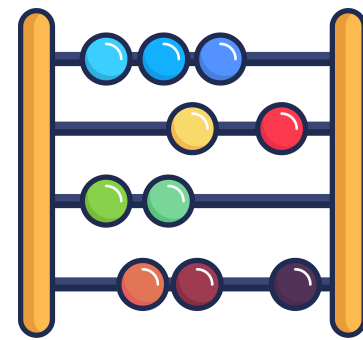
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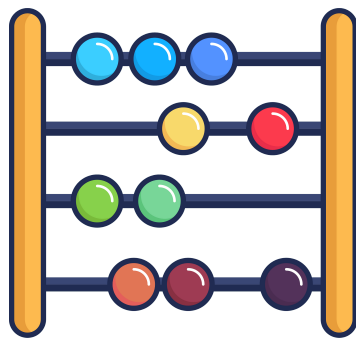
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Pringle 1992,
Lodato Price 2010

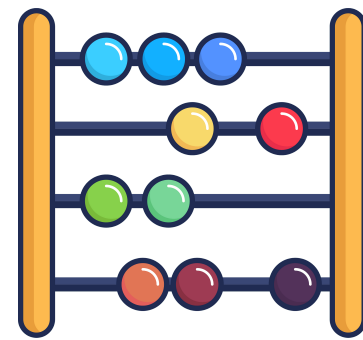
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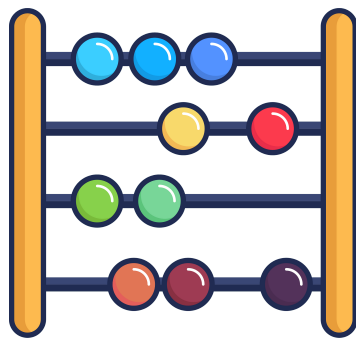
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An iterative boundary value problem

- Look for stationary solutions
- Inner disk is aligned with the BH, outer disk aligned with the binary
- But viscosities are non linear! $\nu = \nu(\alpha, \partial\hat{\mathbf{L}}/\partial R)$
Ogilvie 1999, Ogilvie Latter 2013
- A **new iterative scheme**, which simultaneously solves for the viscosity and the disk dynamics

For a **single** BH, the solution is self similar

Scheuer Feiler 1996, Martin+ 2007

$$R_{\text{LT}} = \frac{4G^2 M^2 \chi}{c^3 \alpha \nu_0 \zeta}$$

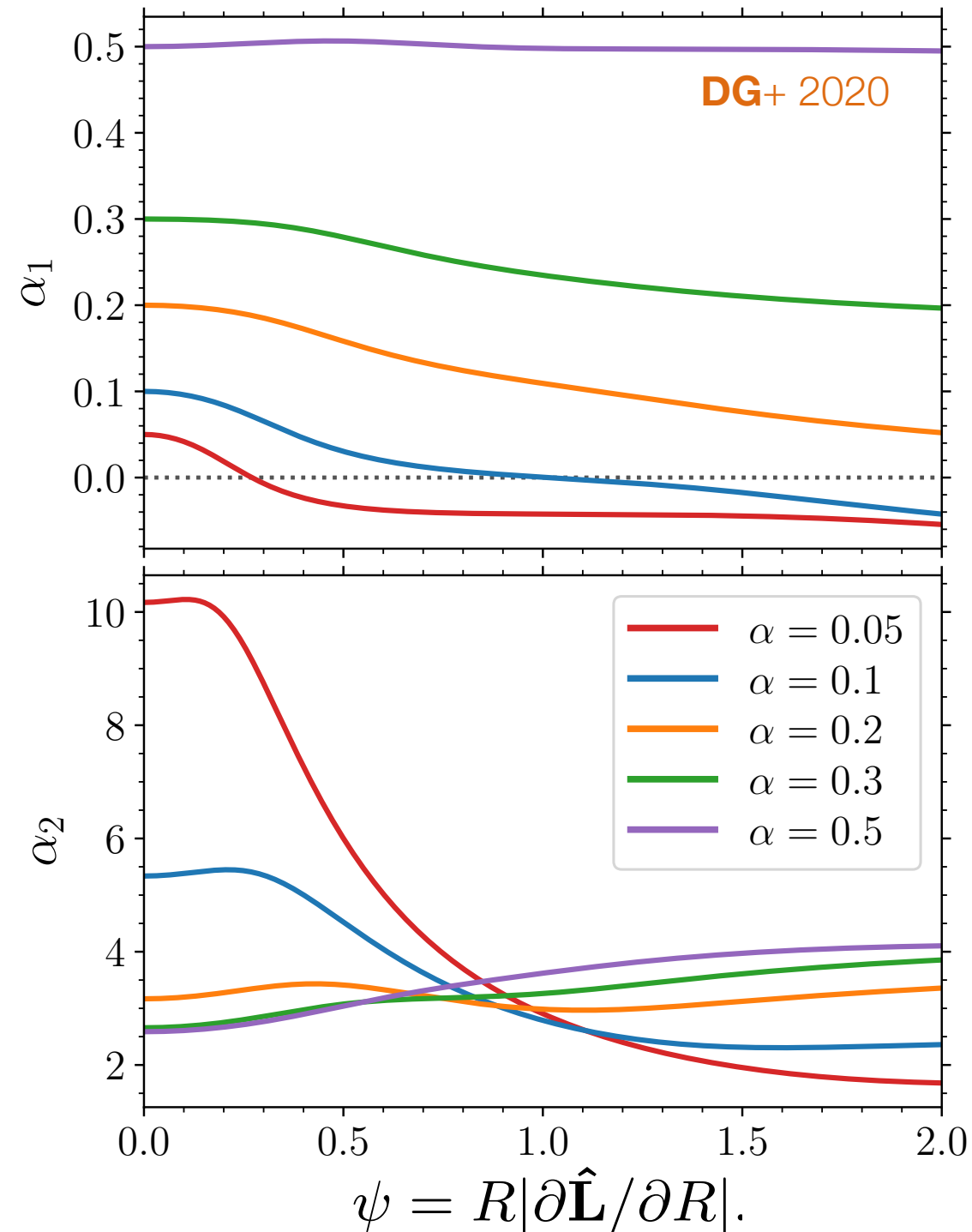
For a **binary**, there's an additional parameter

Tremaine Davis 2014, DG+ 2020

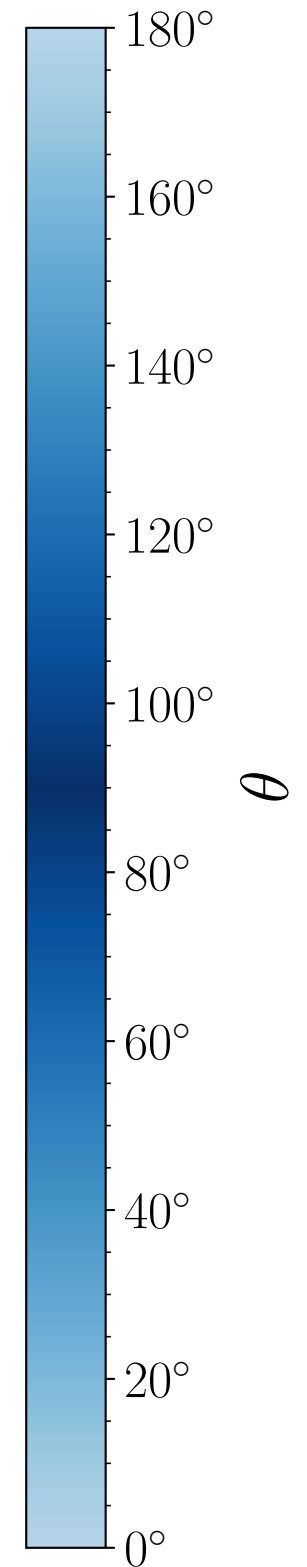
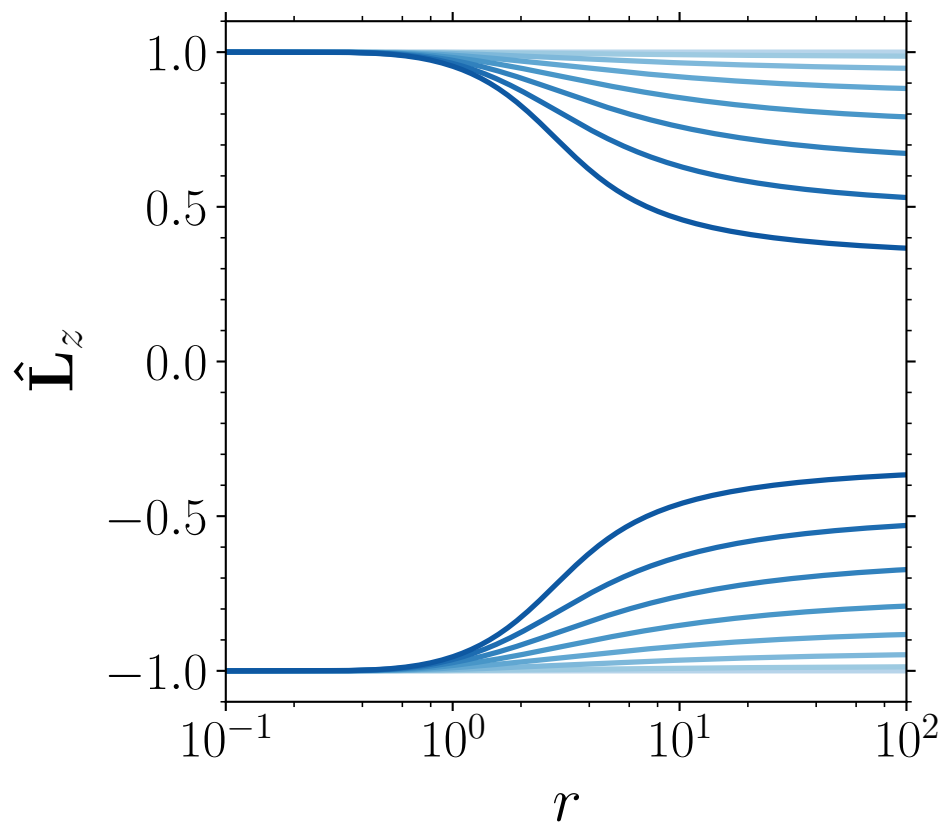
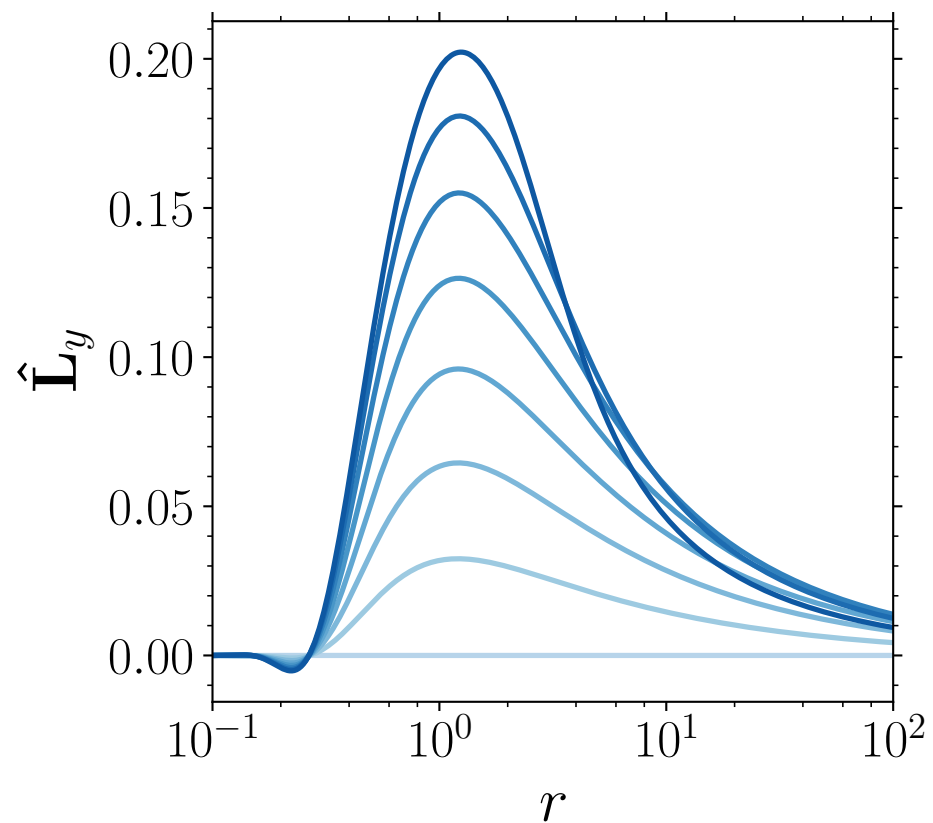
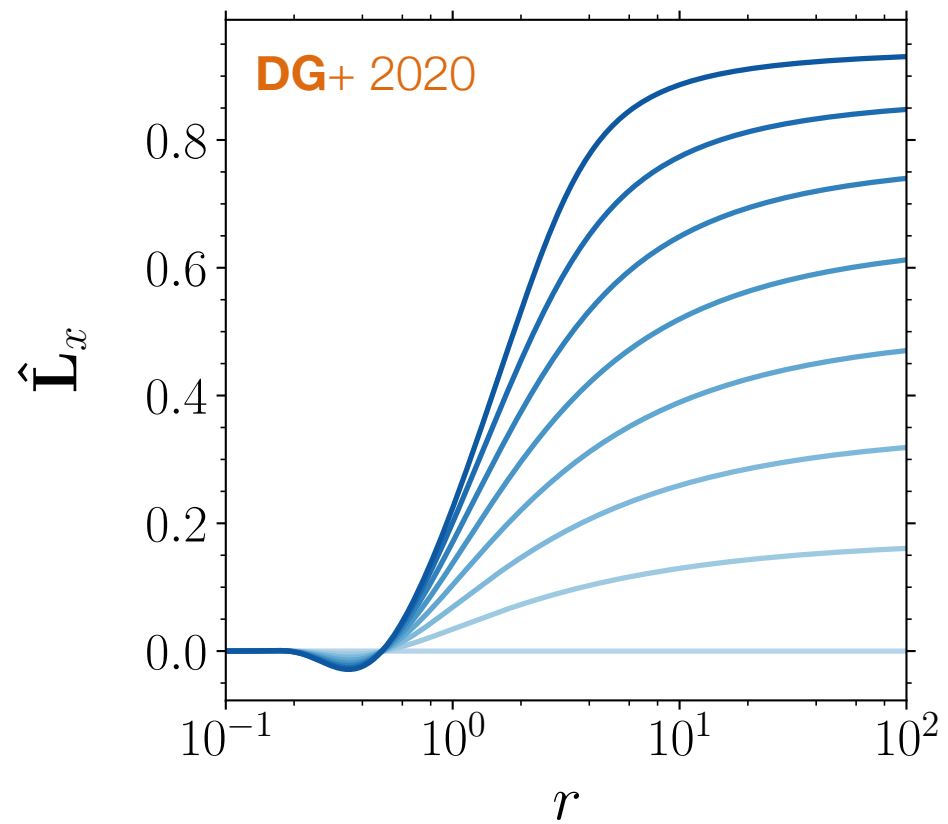
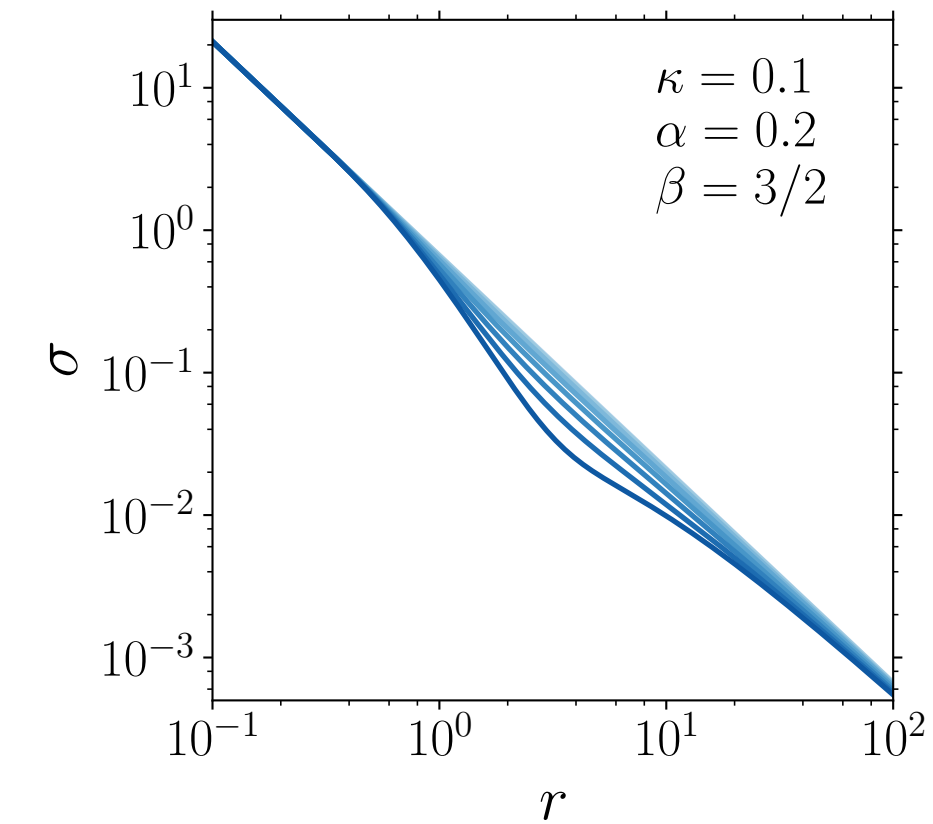
$$R_{\text{tid}} = \left(\frac{3}{2} \frac{\sqrt{GM}}{GM_\star} R_\star^3 \alpha \nu_0 \zeta \right)^{2/7}$$

$$\kappa = \left(\frac{R_{\text{tid}}}{R_{\text{LT}}} \right)^{-7/2}$$

binary separation



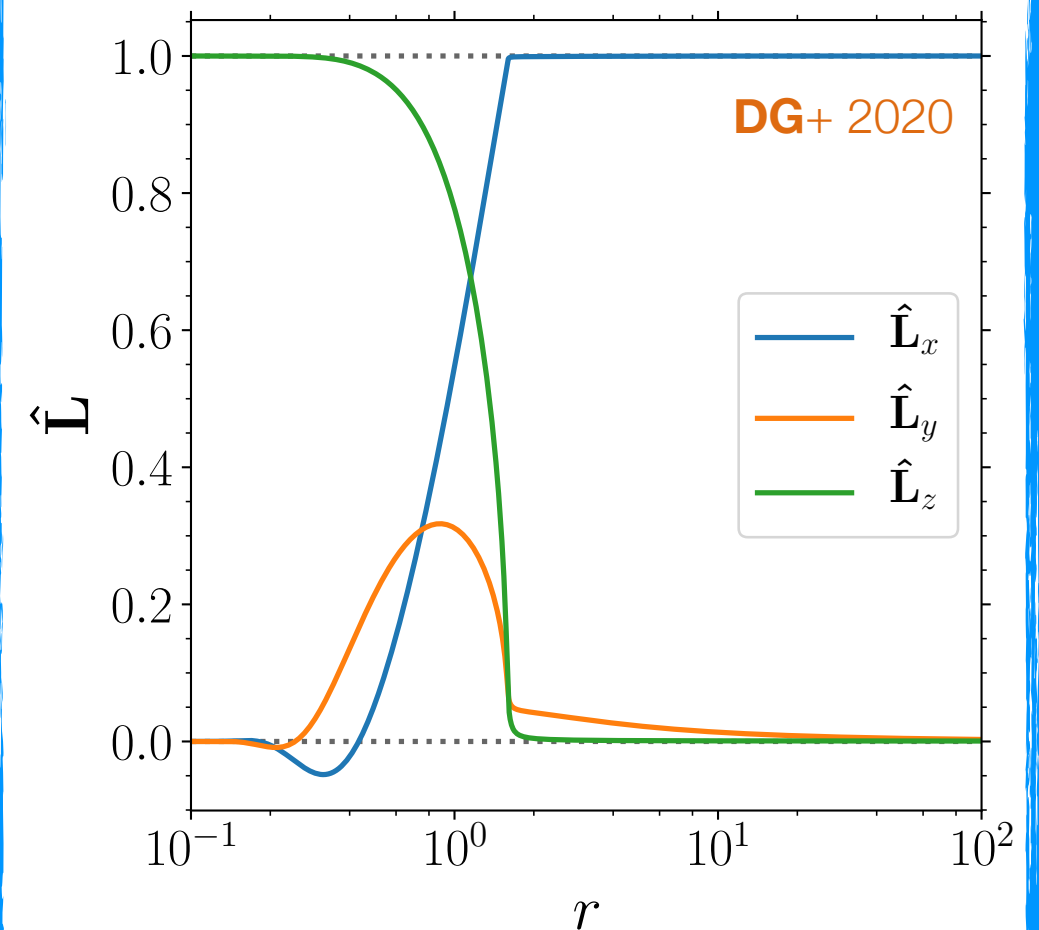
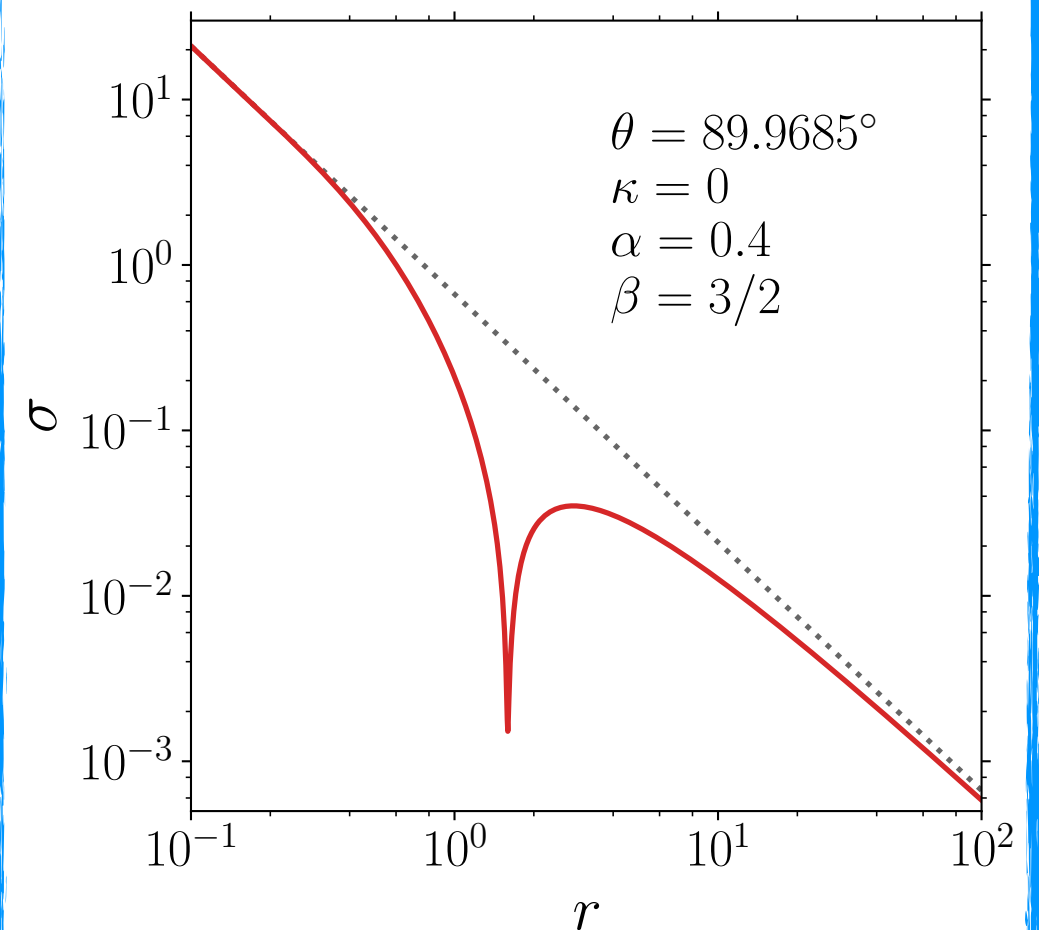
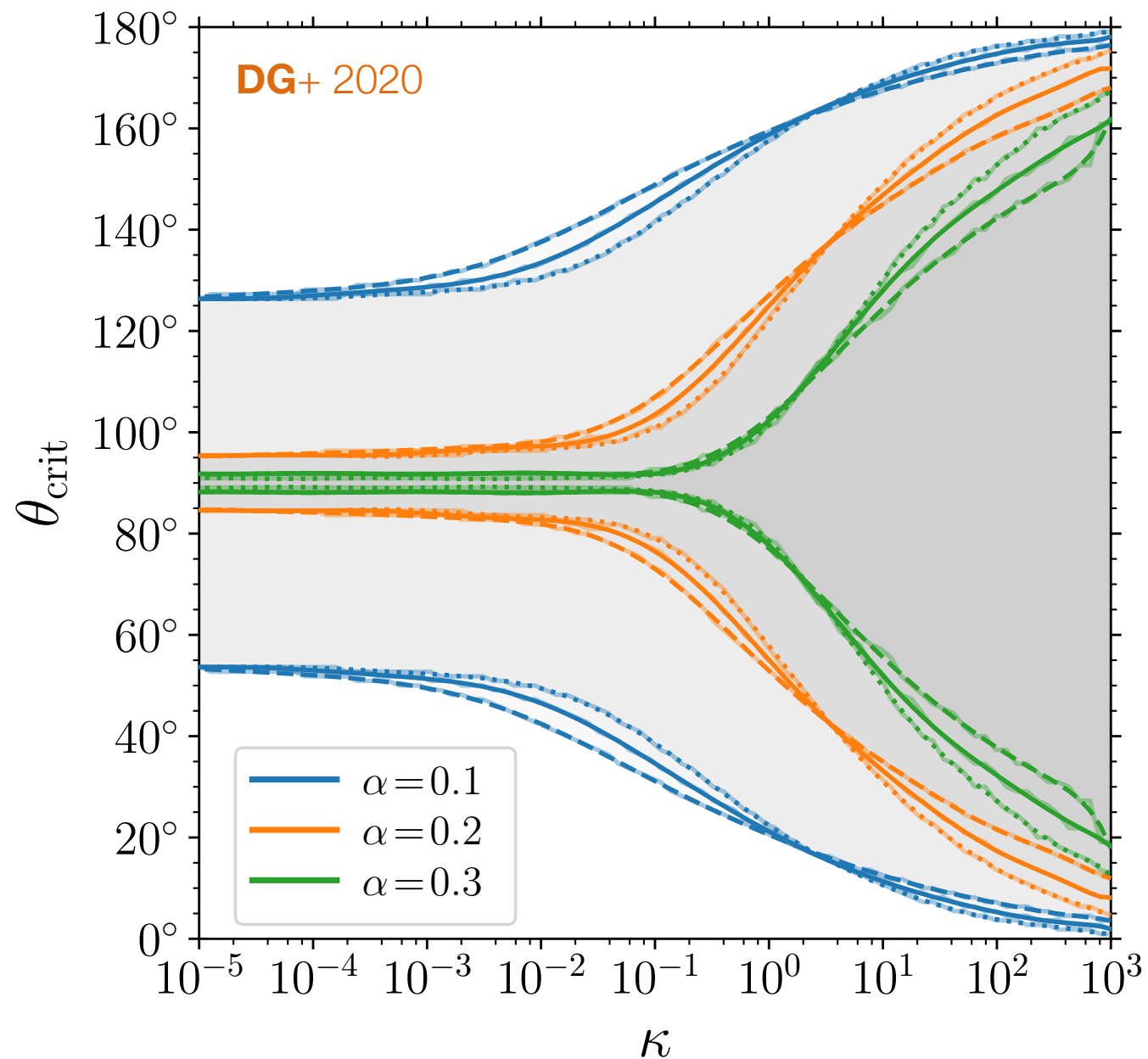
The shape of the disk



Critical obliquity

(these are the individual disks, not circumbinary)

- Solutions ceases to exists!
- Strong indication this is a physical effects
- Hints previously reported [Tremaine and Davis 2014](#)



Coupled alignment and inspiral

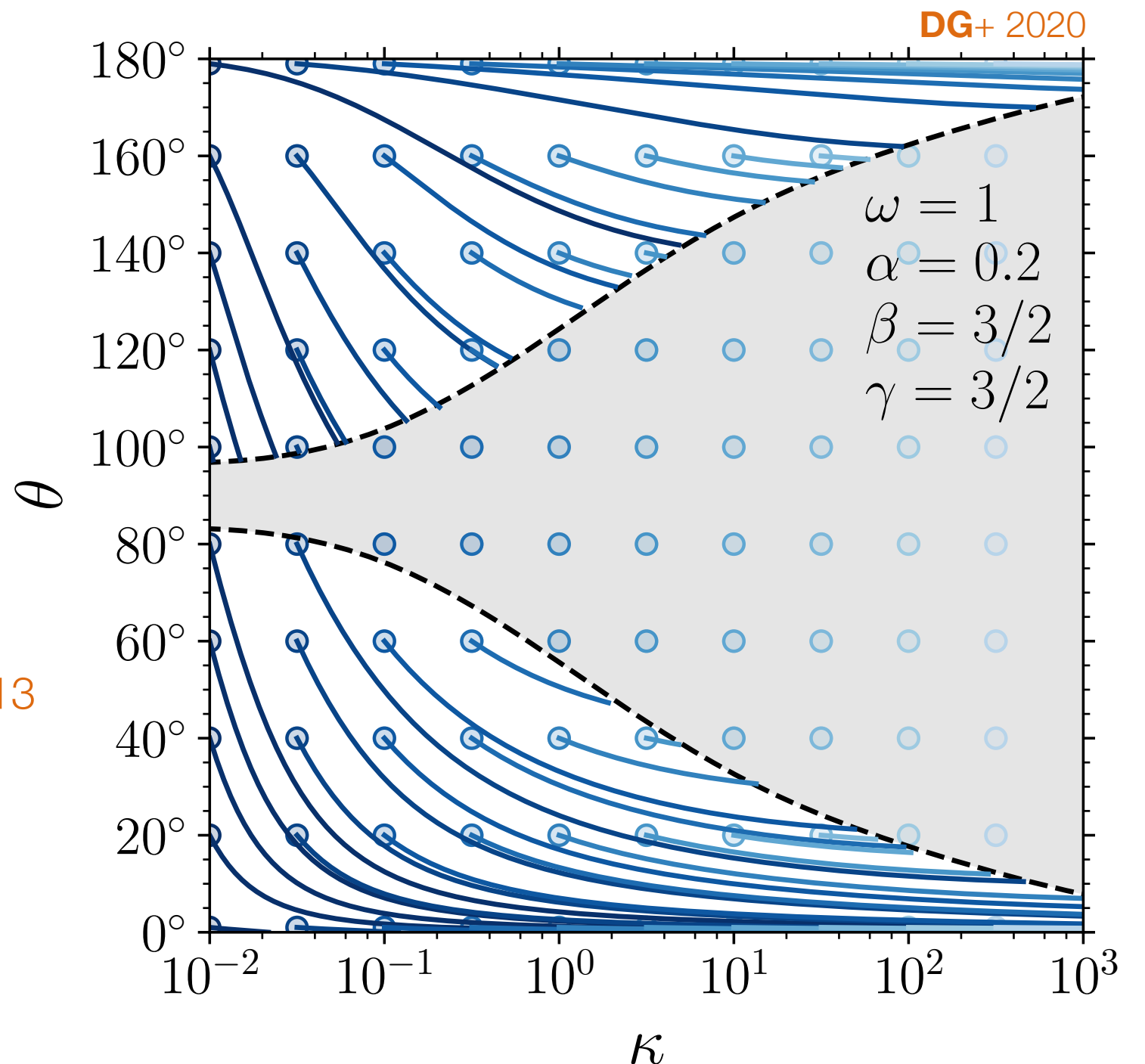
Back reaction from the disk onto the BH

$$\frac{d \cos \theta}{d \ln \kappa} = -\omega \kappa^{-\gamma/3} \int_{r_{\min}}^{r_{\max}} (\hat{\mathbf{J}} \times \hat{\mathbf{L}}) \cdot \hat{\mathbf{L}}_{\star} \frac{\sigma}{r^{3/2}} dr$$



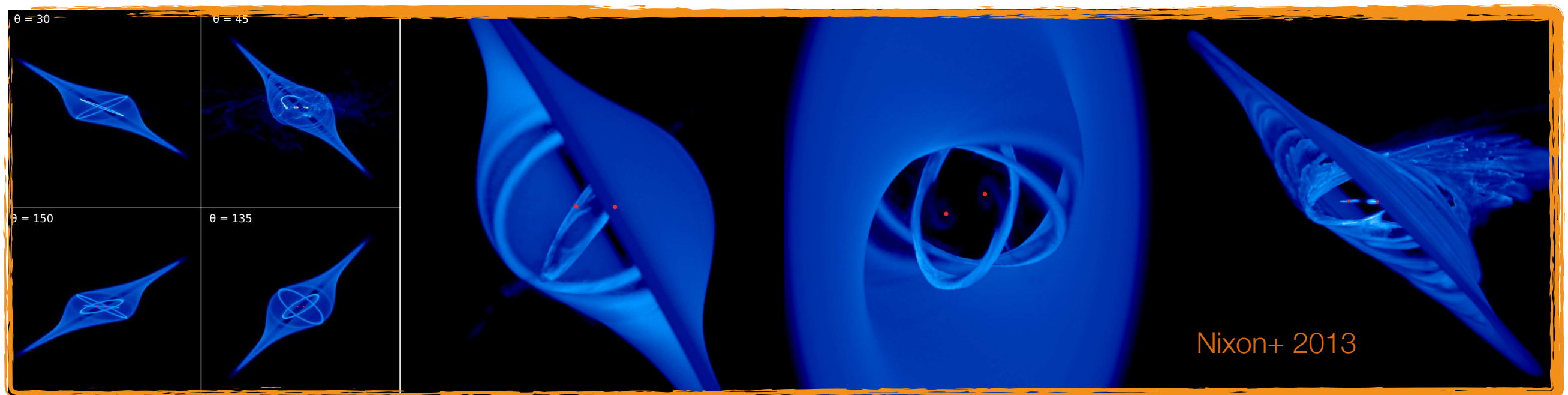
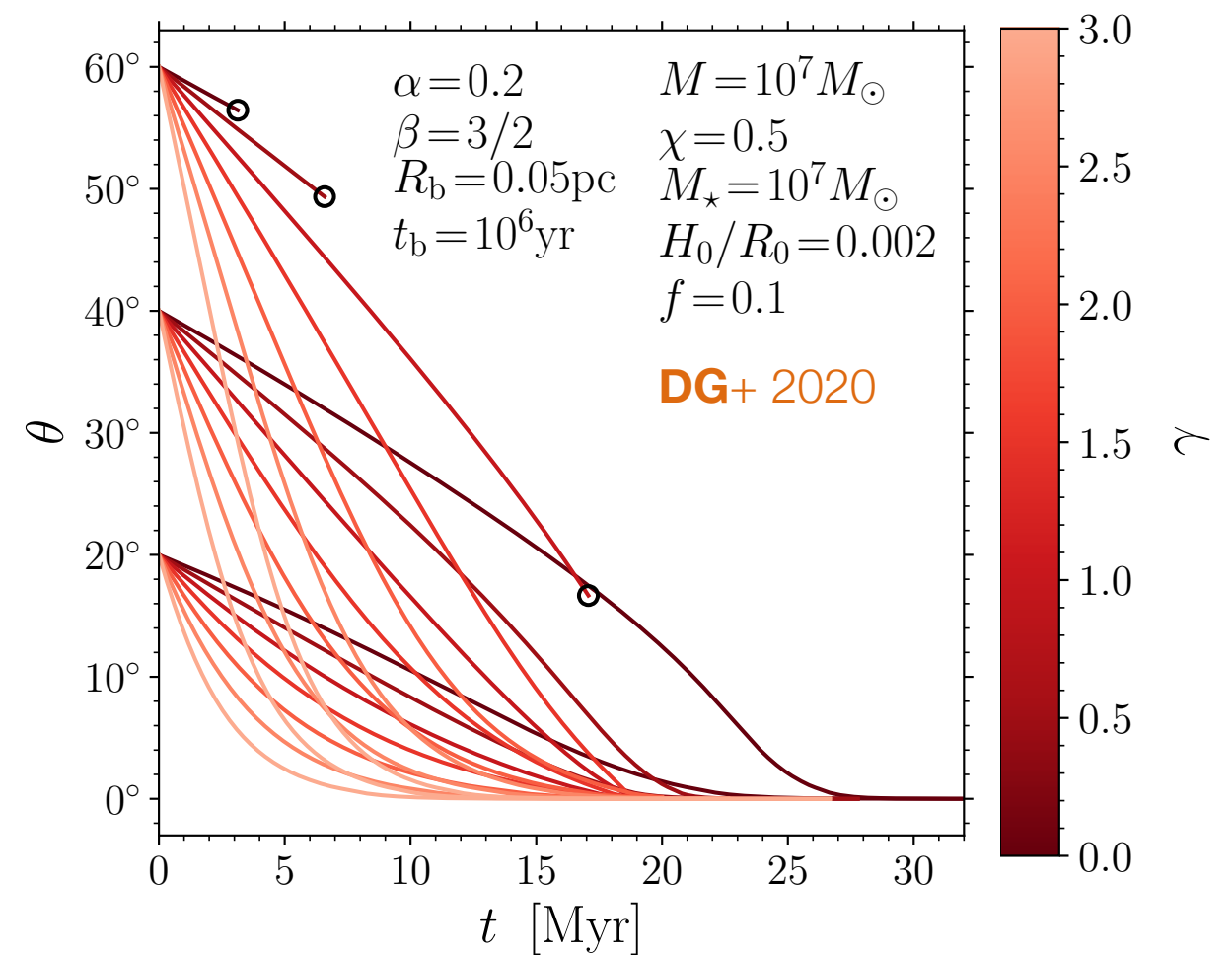
A history of solutions, just one more dimensionless parameter: **migration** ω

- Companion speeds up the alignment: warp radius moves inwards, material is misaligned closer to the BH [Miller and Krolik 2013](#)
- Warp non linearities are important! Surface density depletion right at the warp radius [DG+ 2020](#)



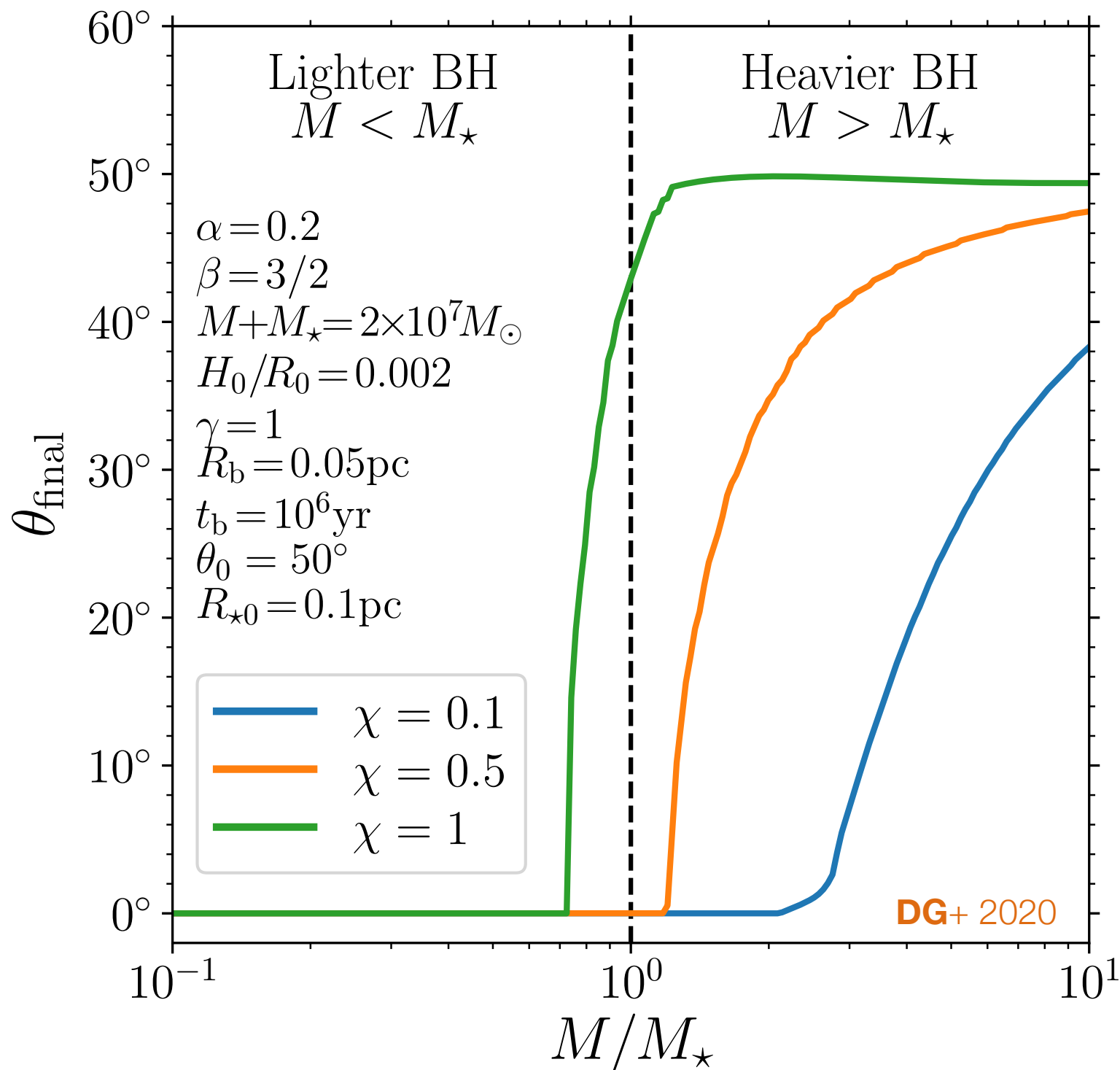
A secular path to disk breaking?

- Do physical disks surrounding BH in binaries actually break?
- Something as simple as the inspiral might do it!
- What happens to the disk at the breaking point?
- And **what happens to the black holes?**



Disk physics with gravitational waves?

Residual angle at criticality



- Secondary BH align quickly
- Primary BH does not accrete much, hence become critical.
- Do they stay misaligned till merger?



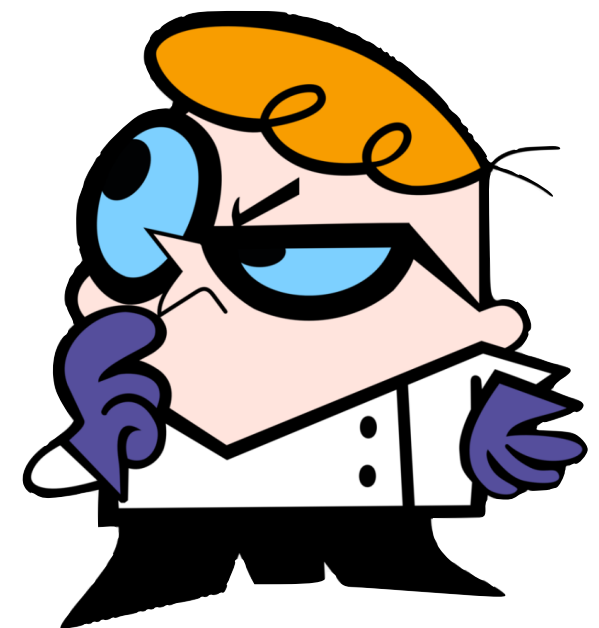
Steinle's talk

Putting this together with relativistic evolutions and LISA predictions!

Step 3 [arXiv:2111.08065](https://arxiv.org/abs/2111.08065)

Is the critical obliquity really there?

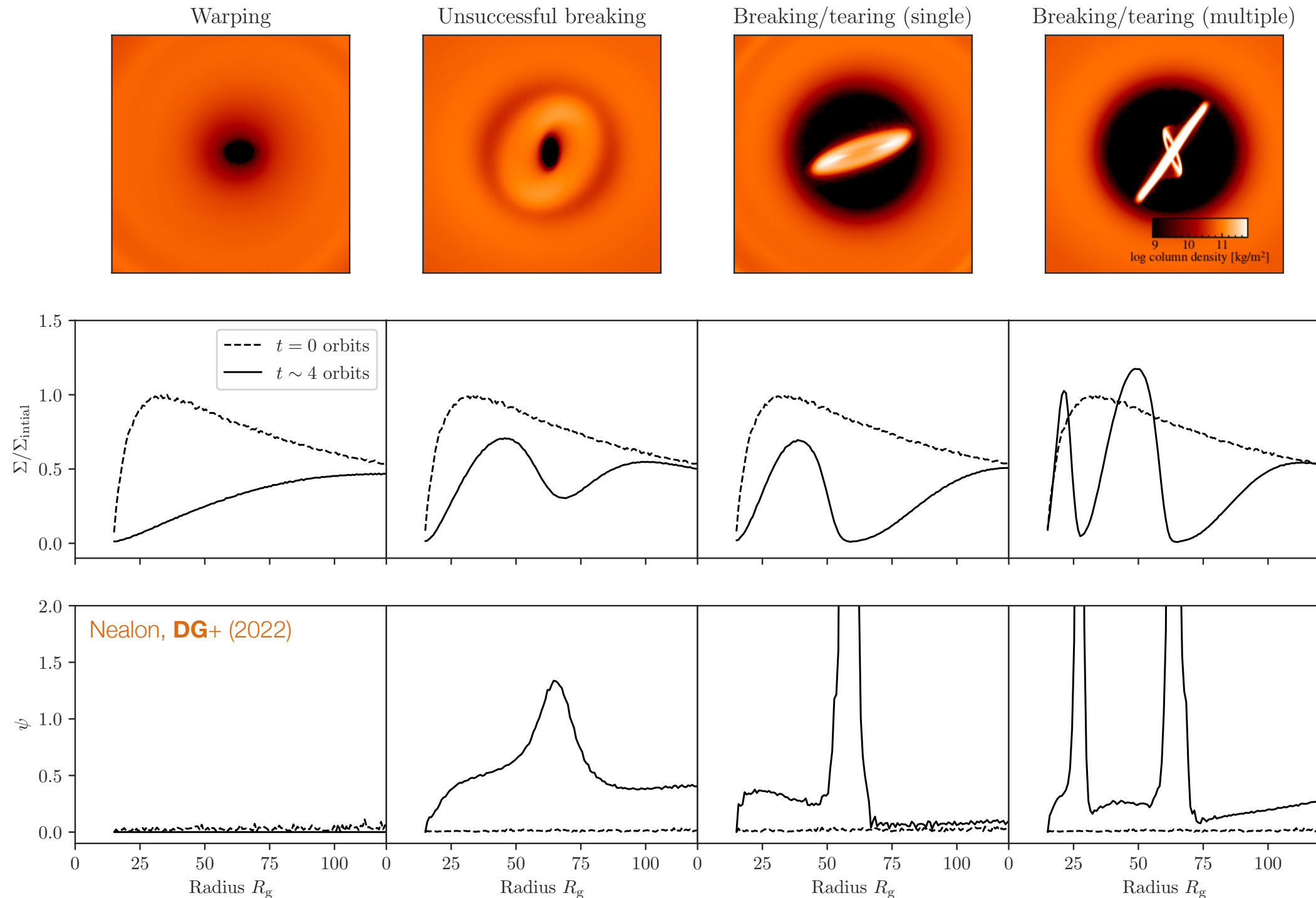
Check my 1D stationary solutions against 3D SPH



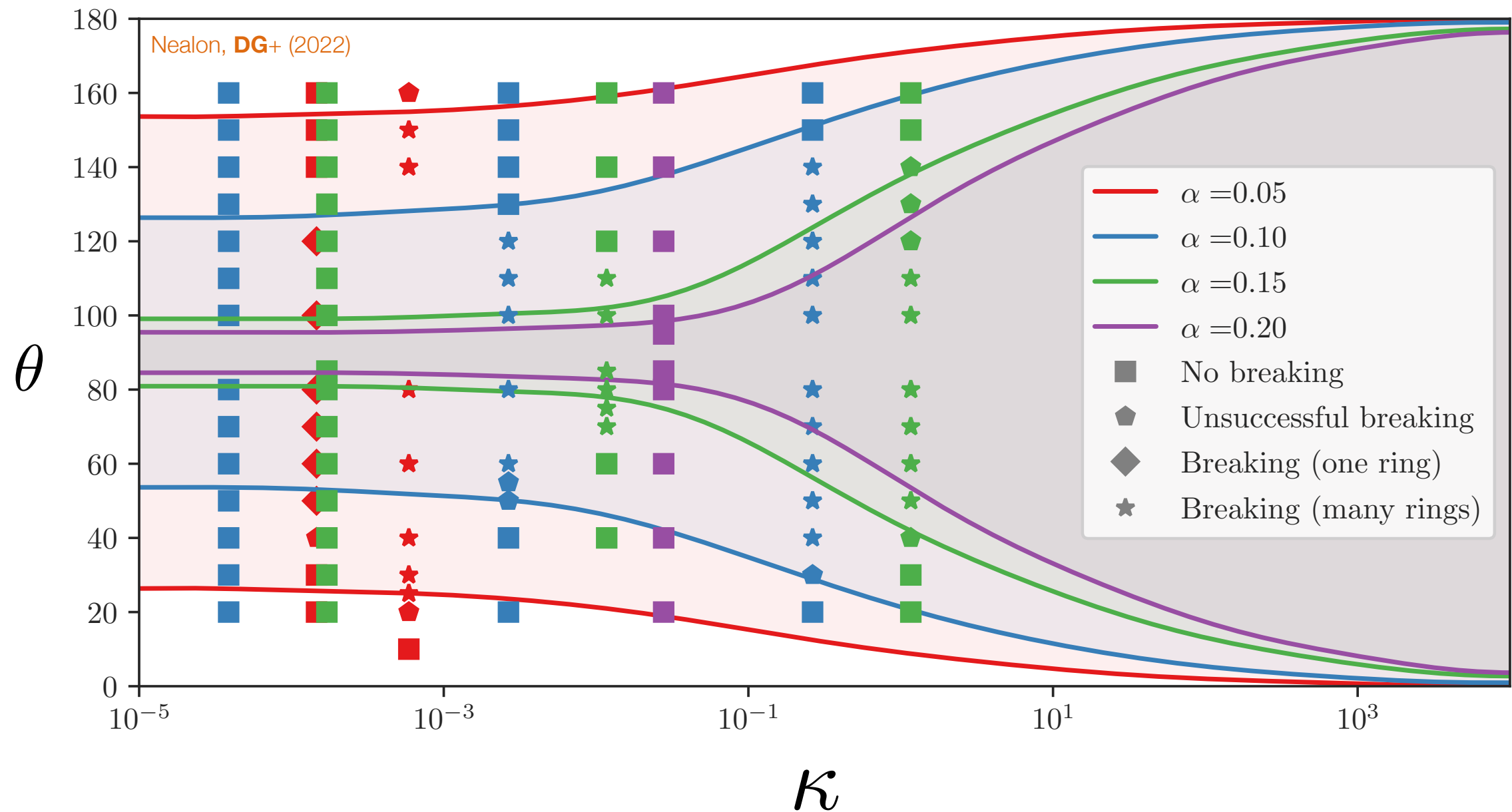
Tackling the problem head on

Nealon, **DG+** (2022)

Large suite of >70 SPH runs



A surprising agreement...



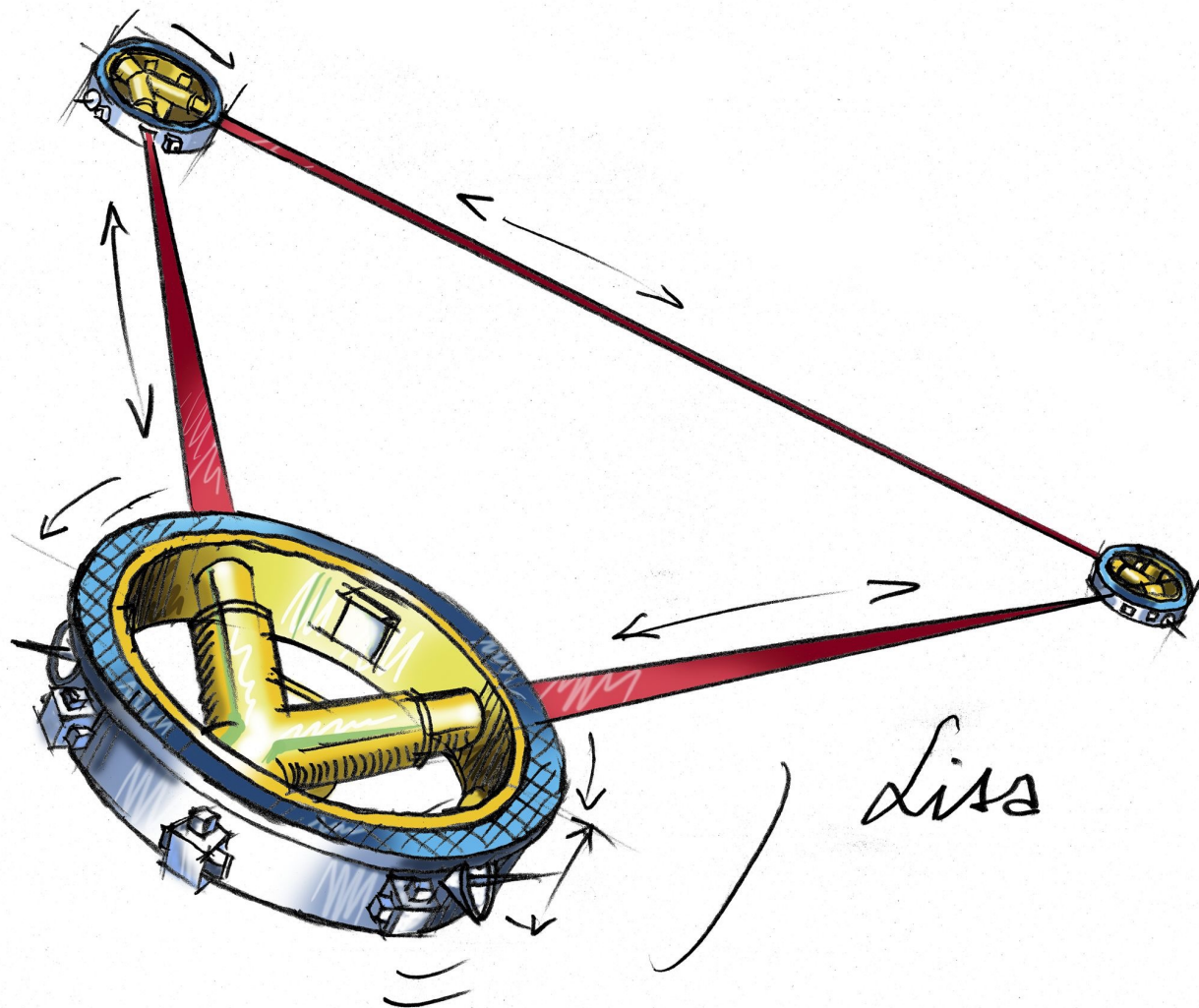
...with some interesting 3D effects on top

(e.g. spirals can prevent breaking)

Can LISA teach us something about disk warps?

Haven't really tackled the LISA problem yet...

but **now we have all the ingredients we need...**



DG+, arXiv:1503.06807

DG+ arXiv:2004.02894

Nealon, **DG+** arXiv:2111.08065

Steinle, **DG** arXiv:22xx.soon

Next episode in this story...

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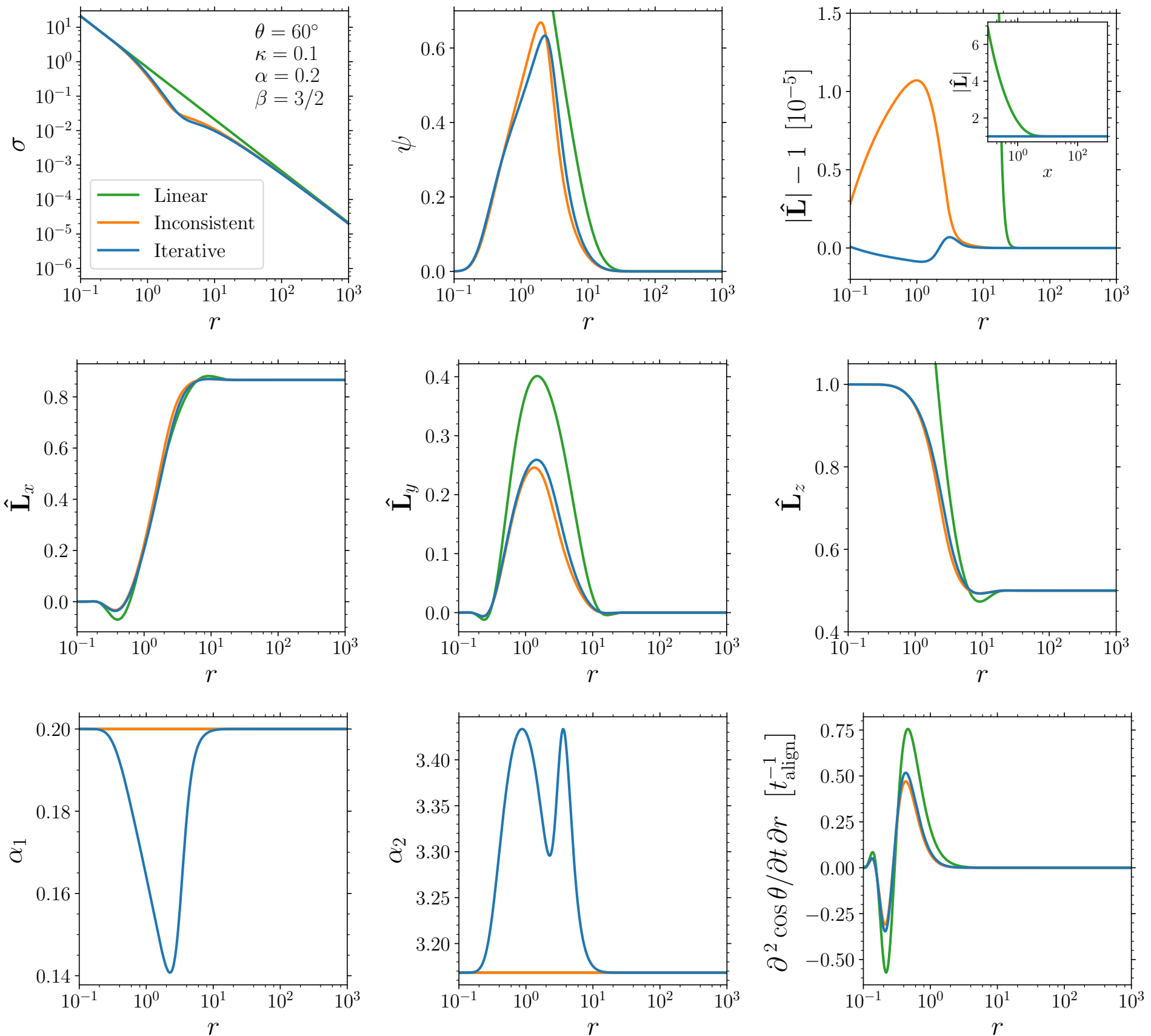


davide.gerosa@unimib.it
www.davidegerosa.com

Backup: scalings

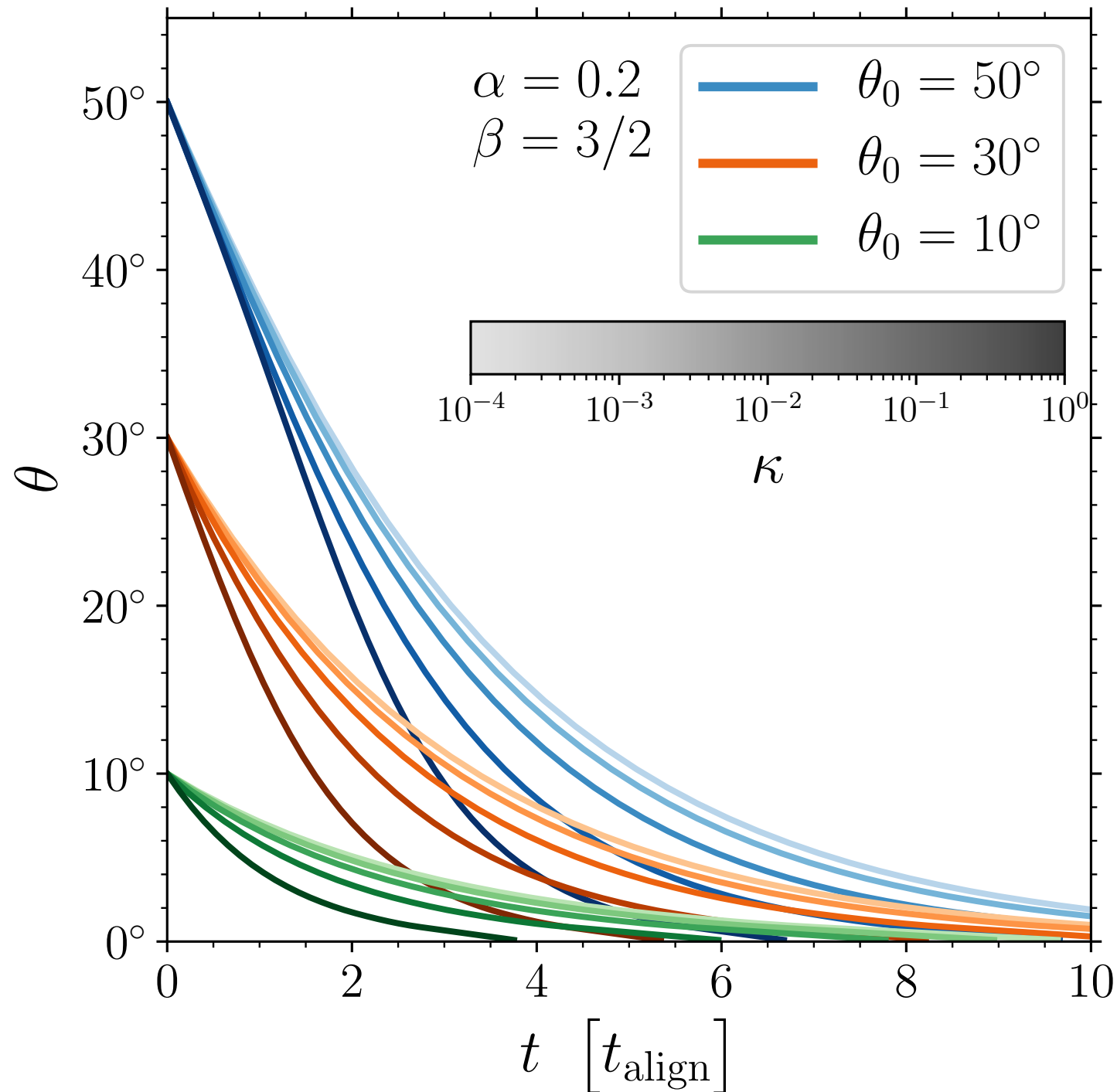
$$\begin{aligned} R_{\text{LT}} &\simeq 1.6 \times 10^{-3} \left(\frac{M}{10^7 M_{\odot}} \right) \left(\frac{\chi}{0.5} \right)^{2/3} \left(\frac{H_0/R_0}{0.002} \right)^{-4/3} \\ &\quad \times \left(\frac{\alpha}{0.2} \right)^{-2} \left[\frac{\zeta}{1/(2 \times 0.2^2)} \right]^{-2} \text{pc}, \\ \kappa &\simeq 0.66 \left(\frac{M}{10^7 M_{\odot}} \right)^2 \left(\frac{\chi}{0.5} \right)^2 \left(\frac{M_{\star}}{10^7 M_{\odot}} \right) \left(\frac{R_{\star}}{0.1 \text{pc}} \right)^{-3} \\ &\quad \times \left(\frac{H_0/R_0}{0.002} \right)^{-6} \left(\frac{\alpha}{0.2} \right)^{-3} \left[\frac{\zeta}{1/(2 \times 0.2^2)} \right]^{-3}, \\ \omega &\simeq (0.54 \times 10^{0.55\gamma}) \left(\frac{M}{10^7 M_{\odot}} \right)^{-1+2\gamma/3} \left(\frac{\chi}{0.5} \right)^{2(\gamma-1)/3} \\ &\quad \times \left(\frac{M_{\star}}{10^7 M_{\odot}} \right)^{1+\gamma/3} \left(\frac{R_b}{0.05 \text{pc}} \right)^{-\gamma} \left(\frac{t_b}{10^6 \text{yr}} \right) \\ &\quad \times \left(\frac{H_0/R_0}{0.002} \right)^{-2(\gamma+1/3)} \left(\frac{\alpha}{0.2} \right)^{-\gamma-1/3} \left[\frac{\zeta}{1/(2 \times 0.2^2)} \right]^{2/3-\gamma} \end{aligned}$$

Backup: linear vs nonlinear

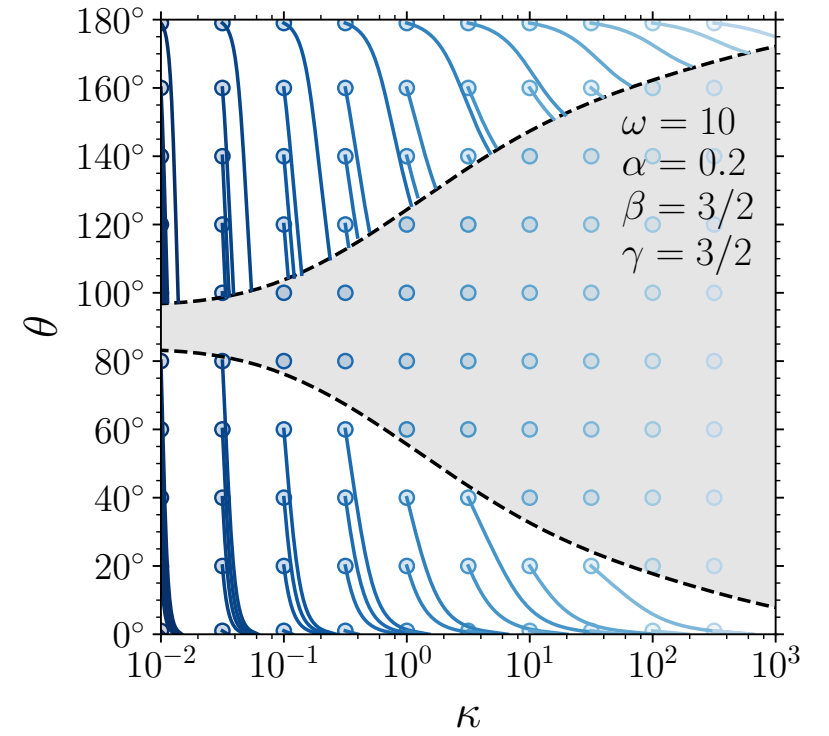
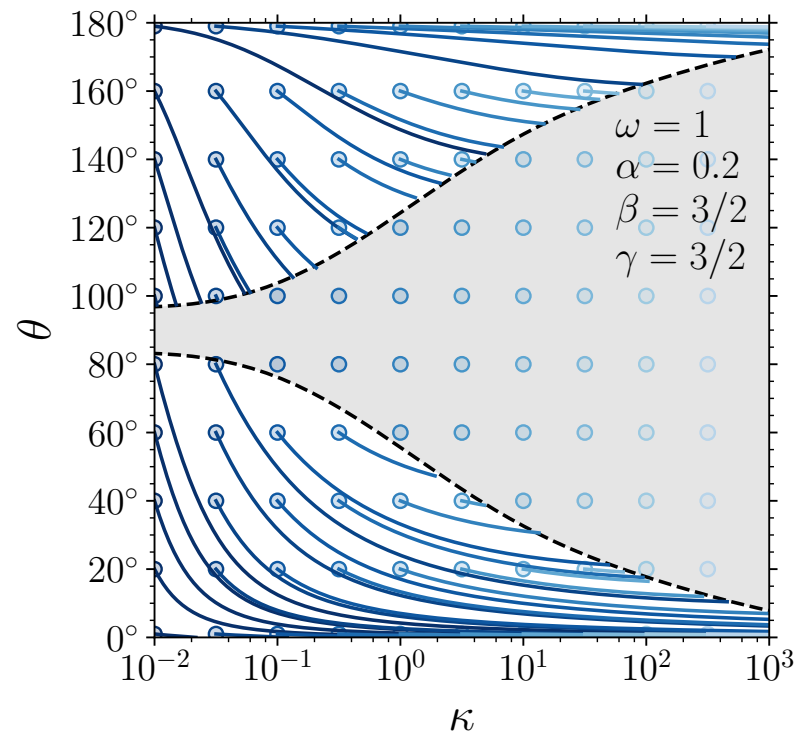
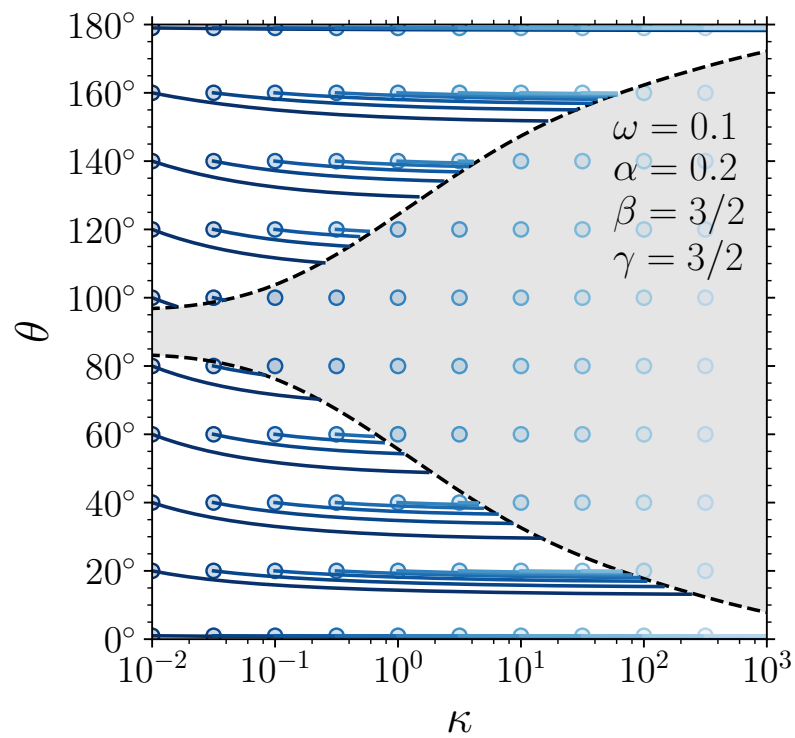


Backup: backreaction

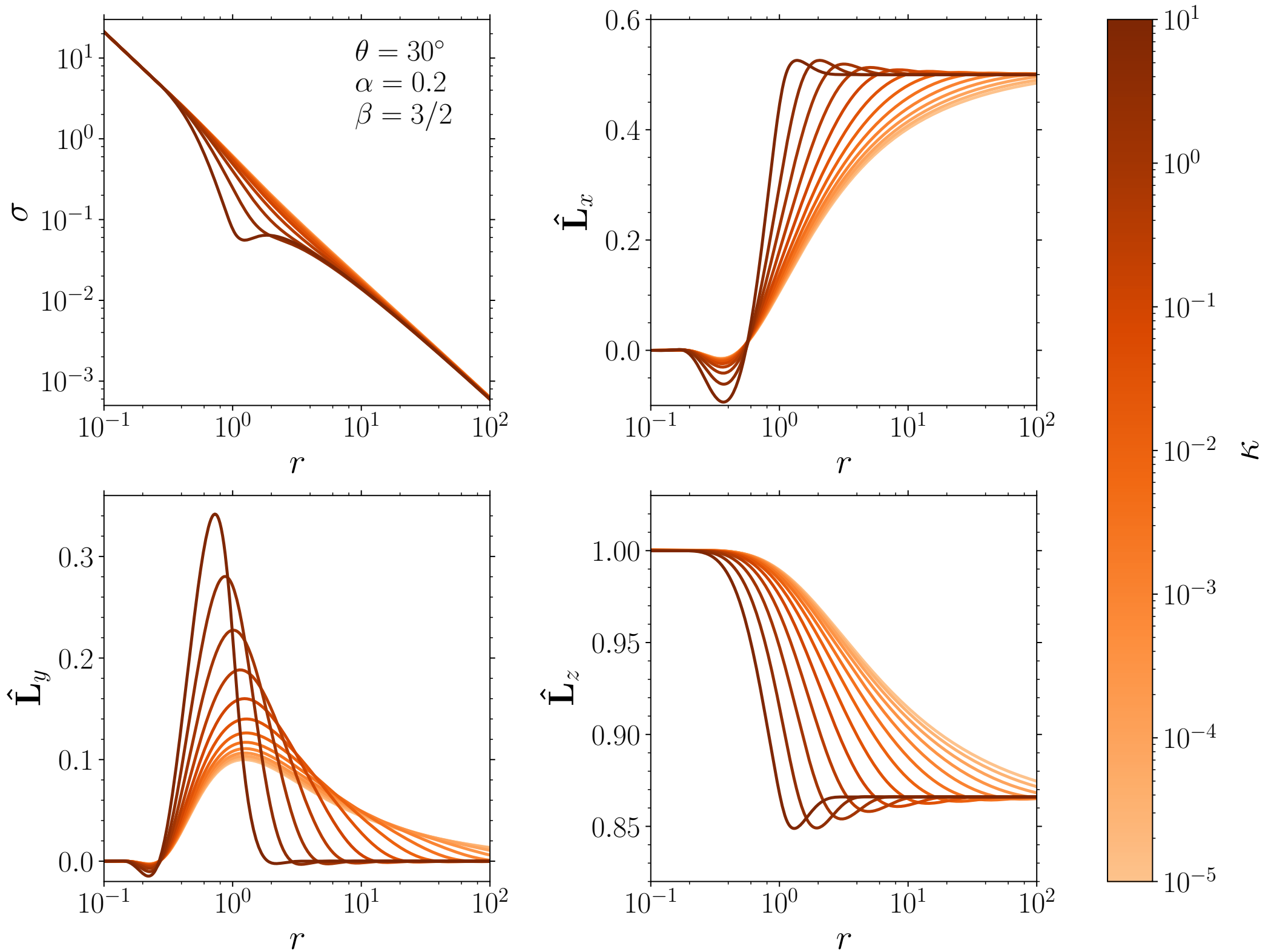
$$\frac{d \cos \theta}{dt} = \frac{d\hat{\mathbf{J}}}{dt} \cdot \hat{\mathbf{L}}_{\star} = -\frac{1}{t_{\text{align}}} \int_{r_{\text{min}}}^{r_{\text{max}}} (\hat{\mathbf{J}} \times \hat{\mathbf{L}}) \cdot \hat{\mathbf{L}}_{\star} \frac{\sigma}{r^{3/2}} dr$$



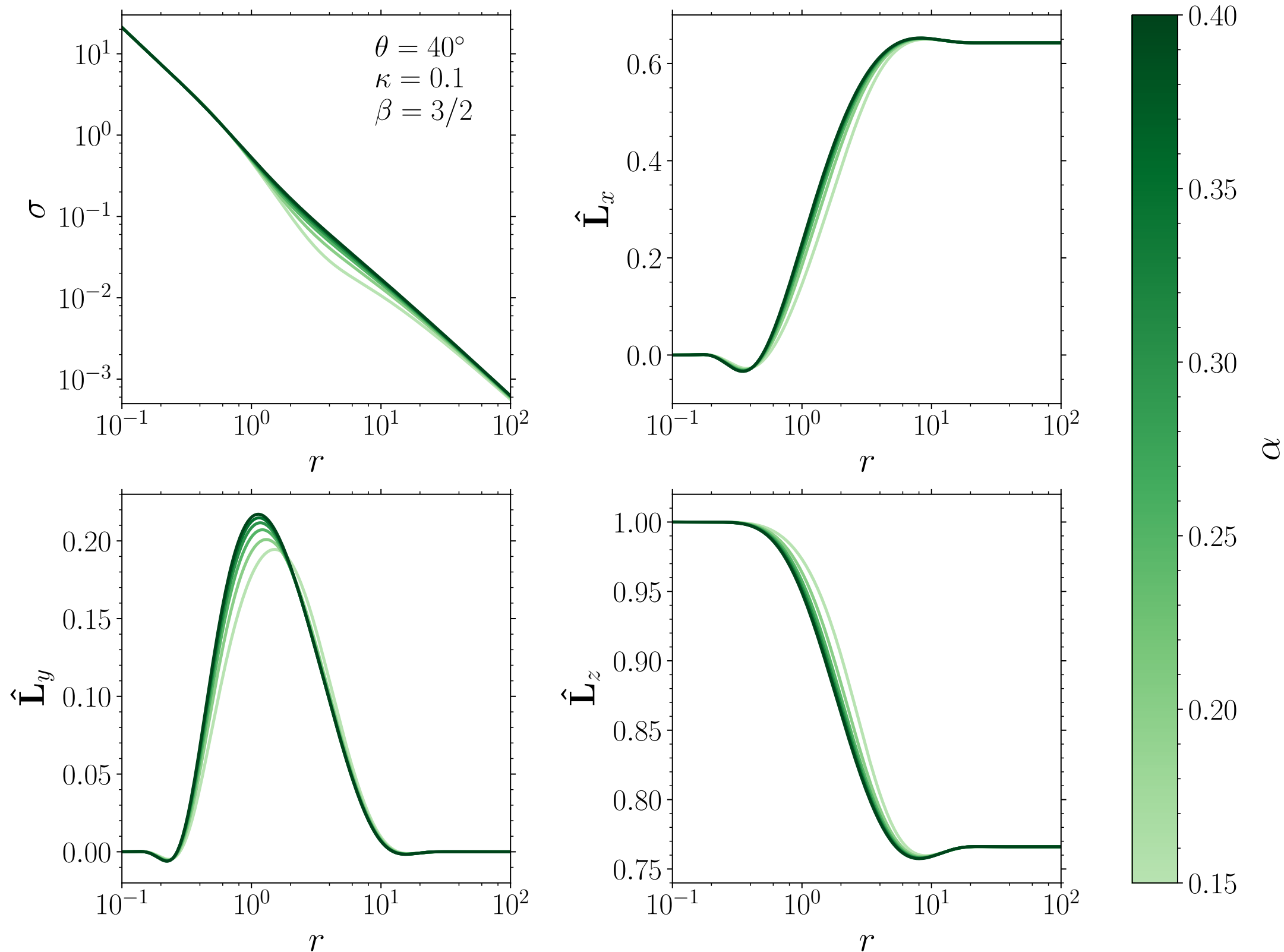
Backup: impact of the migration parameter



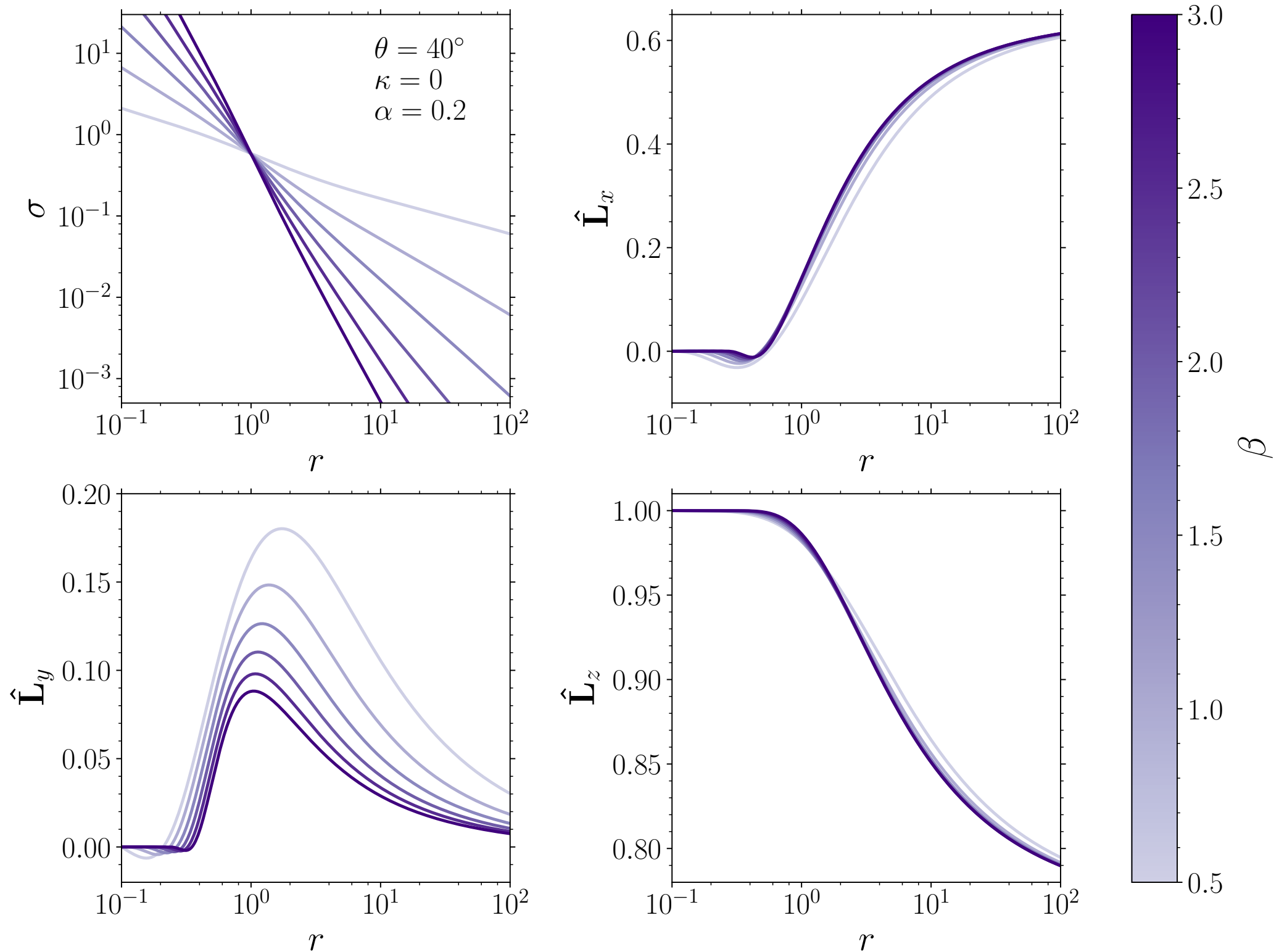
Backup: disk structure



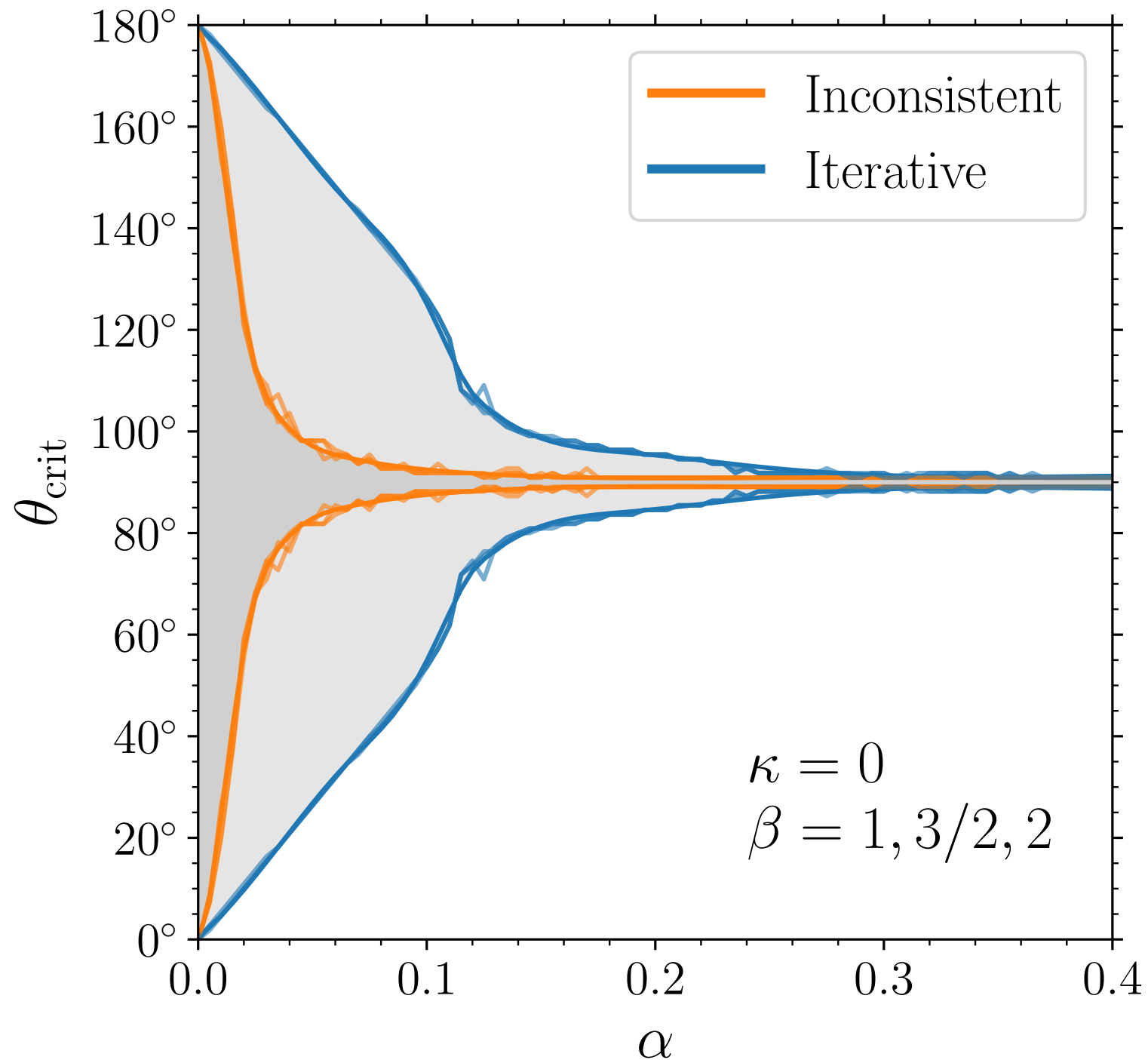
Backup: disk structure



Backup: disk structure



Backup: importance of iterative procedure



Backup: no solution region

