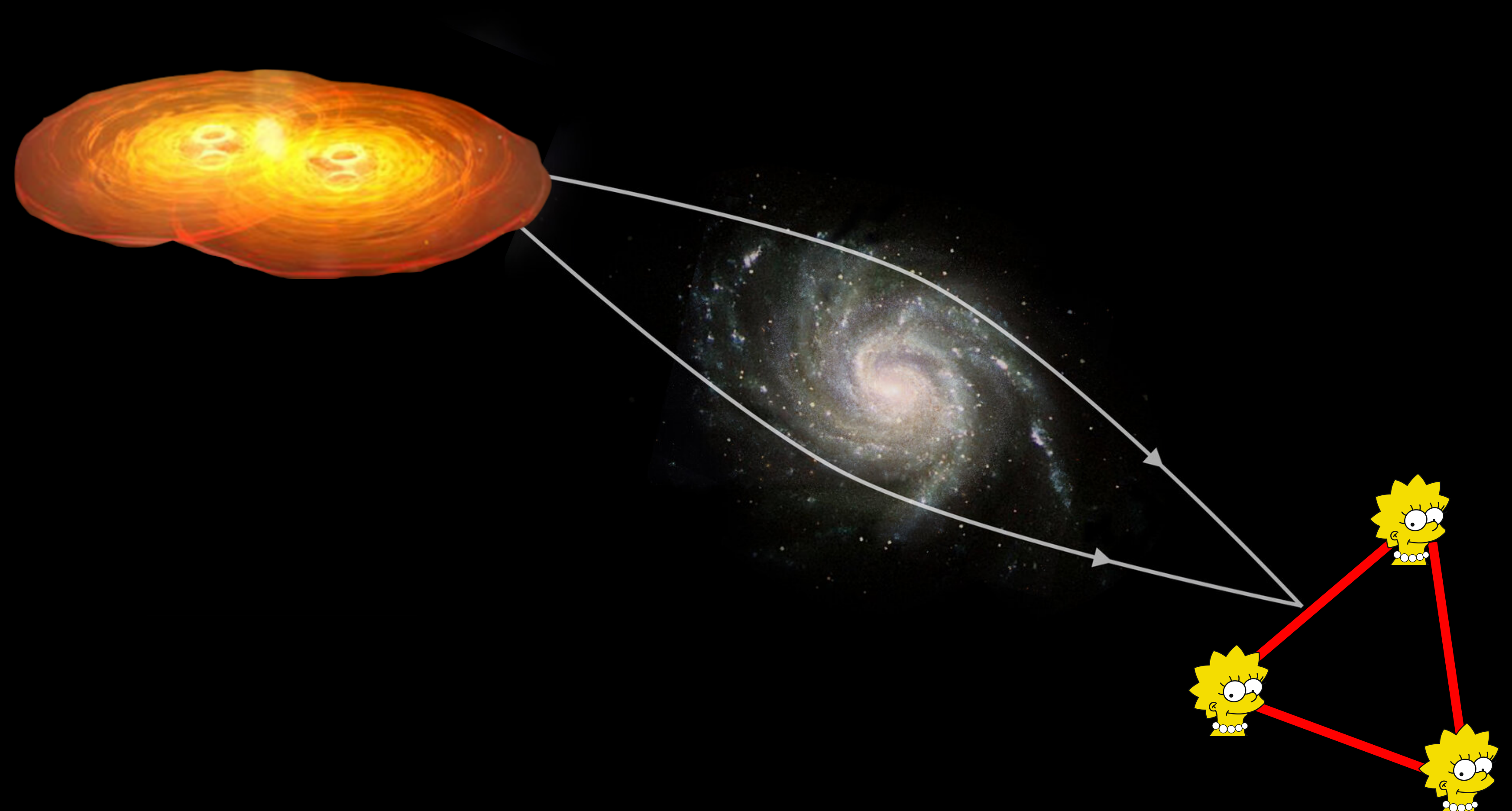
Observability of lensing of gravitational waves from massive black hole binaries with LISA



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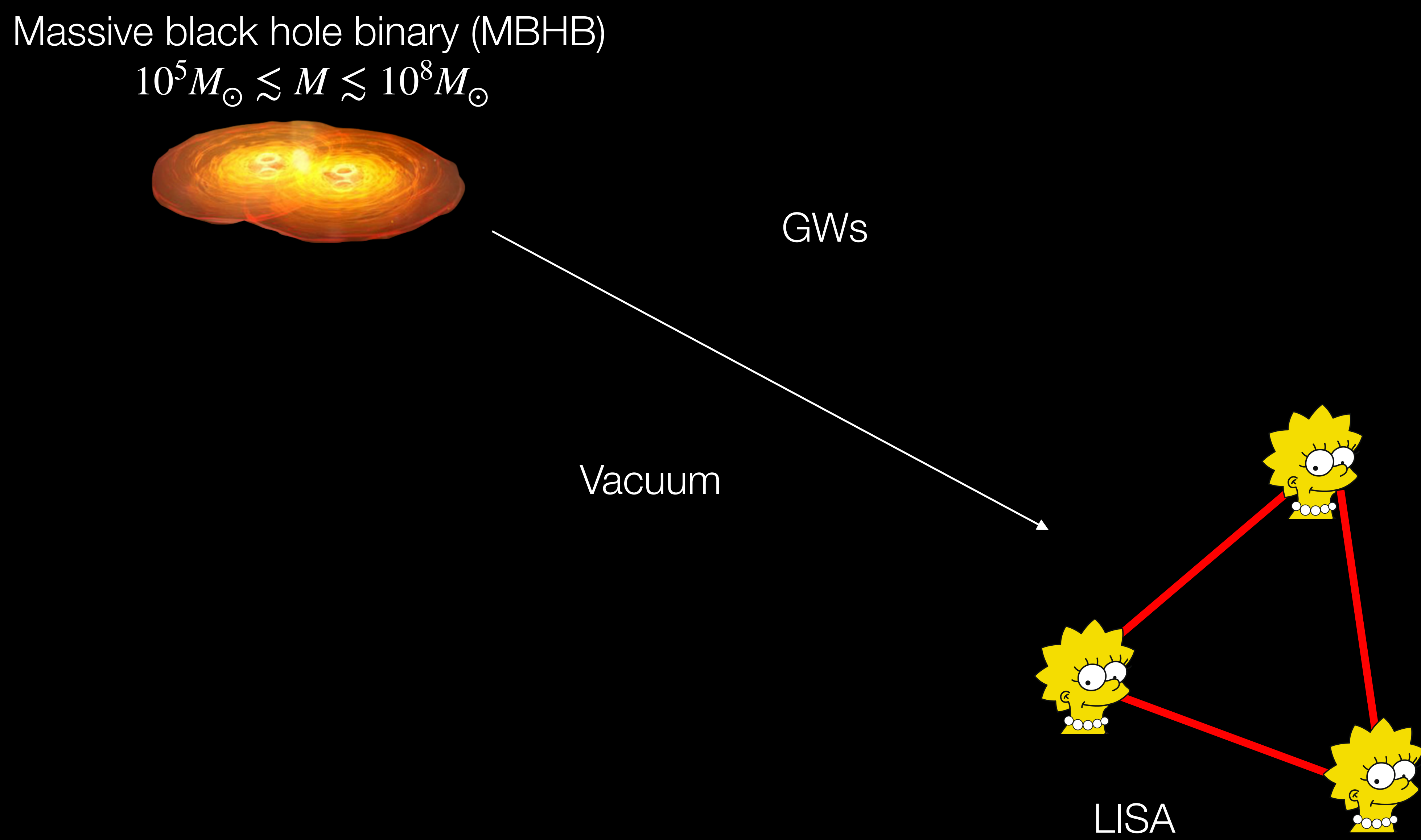
Roberto Cotesta

See the paper on arXiv:2206.02803 by
Mesut Çalışkan, Lingyuan Ji, Roberto Cotesta, Emanuele Berti,
Mark Kamionkowski and Sylvain Marsat

Lensing of gravitational waves (GWs)



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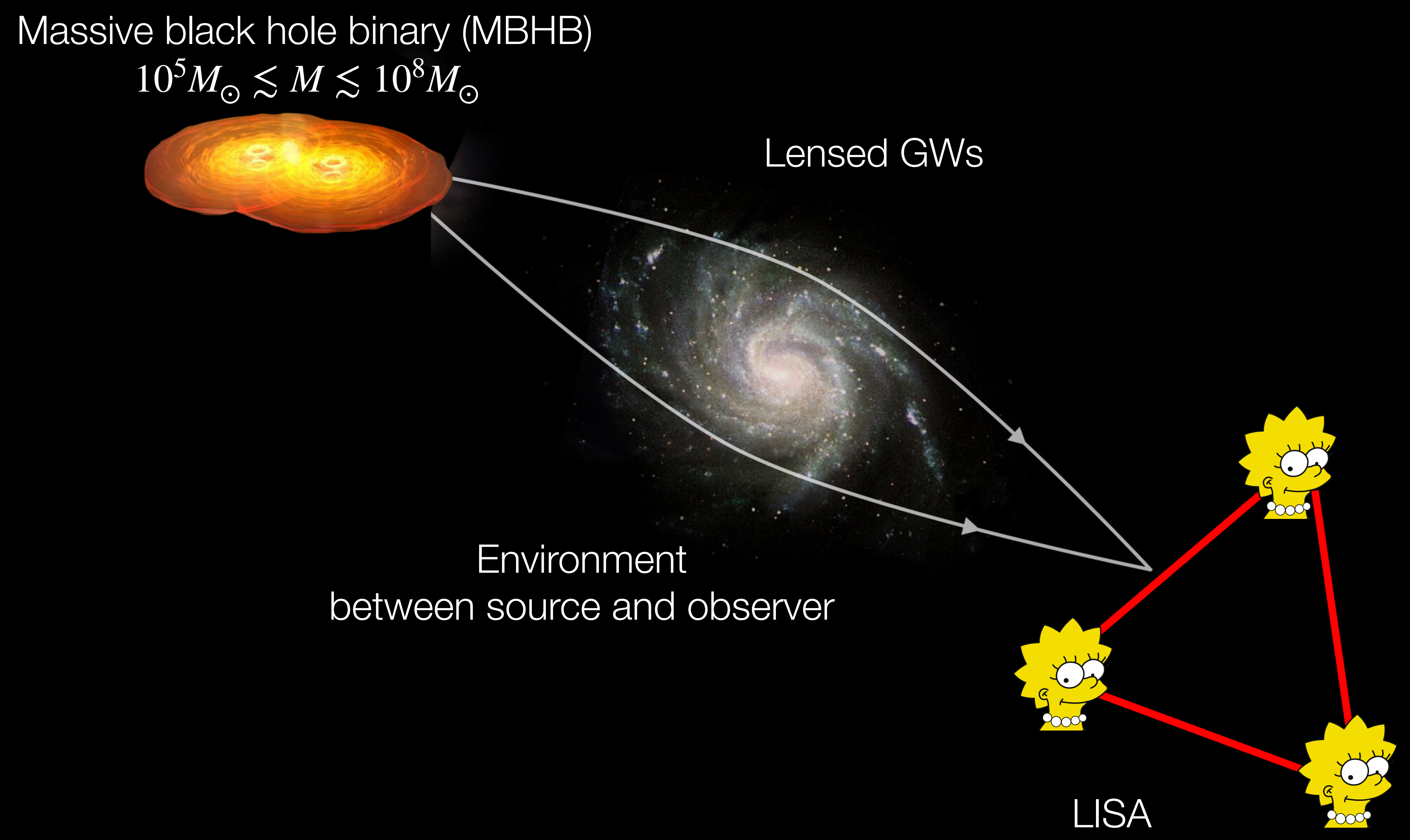

Lensing of gravitational waves (GWs)

$$\tilde{h}^{\mathrm{L}}(f;\theta^{\mathrm{S}},\theta^{\mathrm{L}}) = F(f,\mathbf{y})\,\tilde{h}(f;\theta^{\mathrm{S}})$$

$\theta^{\mathrm{S}} \equiv$ parameters of the source

$\theta^{\mathrm{L}} = \{M_{\mathrm{L}}, \mathbf{y}\}$

$M_{\mathrm{L}} \equiv$ mass of the lens

$\mathbf{y} \equiv$ coordinates of the source in source plane

$F(f,\mathbf{y}) \equiv$ diffraction integral

$$F(f,\mathbf{y}) \equiv \frac{D_{\mathrm{S}}(1+z_{\mathrm{L}})\xi_0^2}{D_{\mathrm{L}}D_{\mathrm{LS}}}\frac{f}{i}\int \mathrm{d}^2\mathbf{x}\ \exp[2\pi i f t_{\mathrm{d}}(\mathbf{x},\mathbf{y})]$$

$D_{\mathrm{S}} \equiv$ distance from the source

$D_{\mathrm{L}} \equiv$ distance from the lens

$z_{\mathrm{L}} \equiv$ redshift of the lens

$D_{\mathrm{LS}} \equiv$ distance lens-source

$t_{\mathrm{d}}(\mathbf{x},\mathbf{y}) \equiv$ time delay function

$\mathbf{x} \equiv$ coordinates of the image in lens plane

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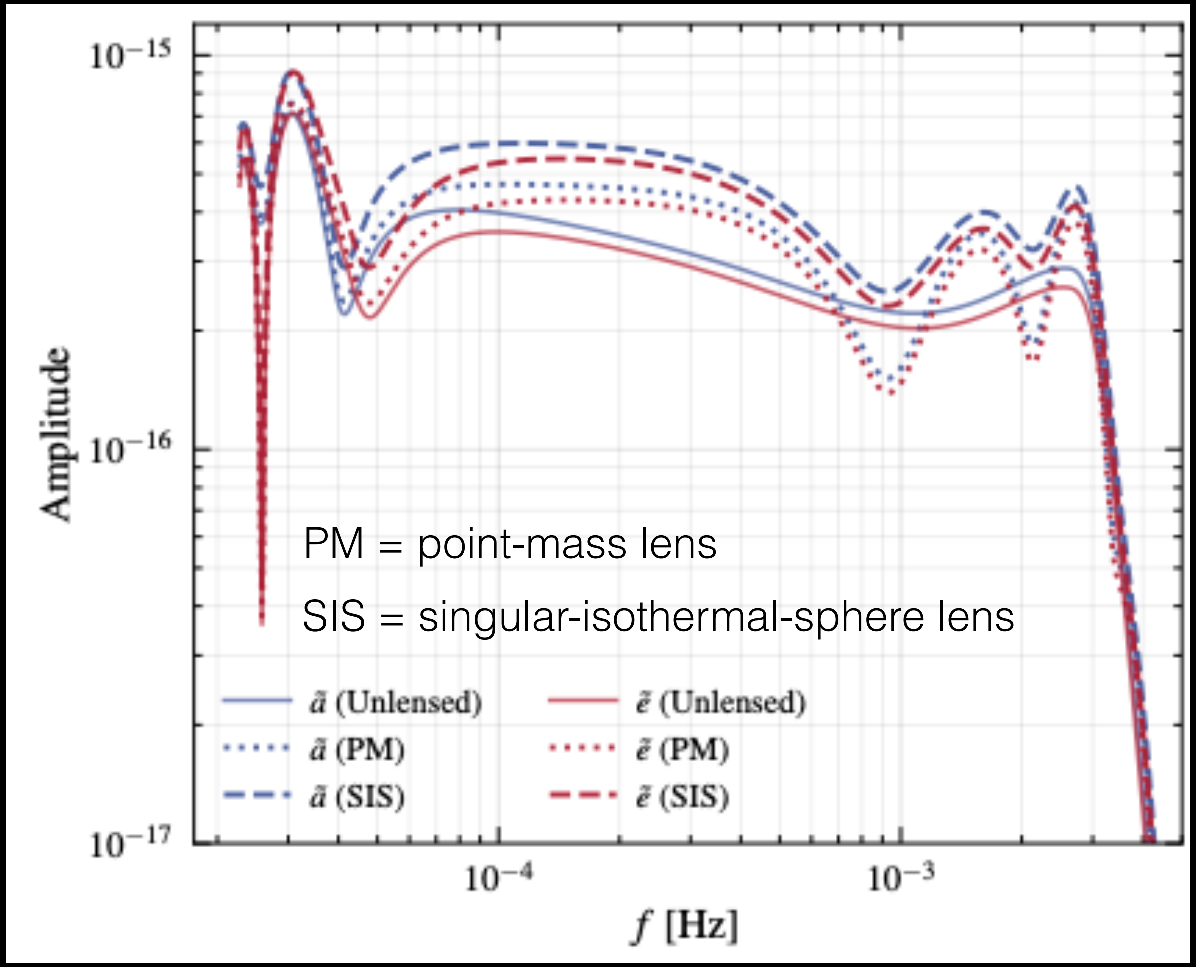

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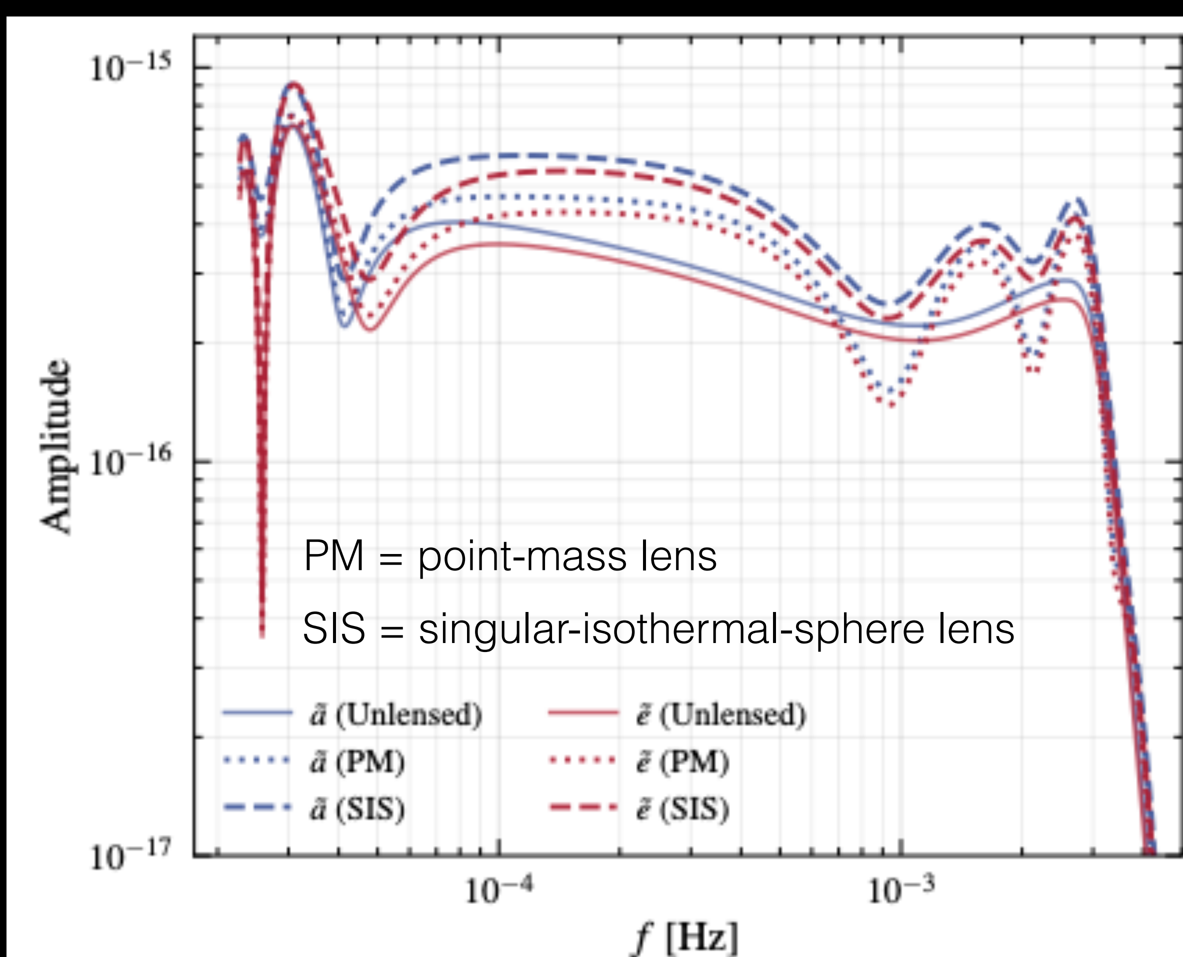


→ **Lensing effects depend on frequency!**

↓

Maybe you are more familiar with…

$$F(f,\mathbf{y}) \approx \sum_j \sqrt{|\mu_j|}\,\exp[2\pi i f t_{\mathrm{d}}(\mathbf{x}_j,\mathbf{y}_j) - i\pi n_j]$$

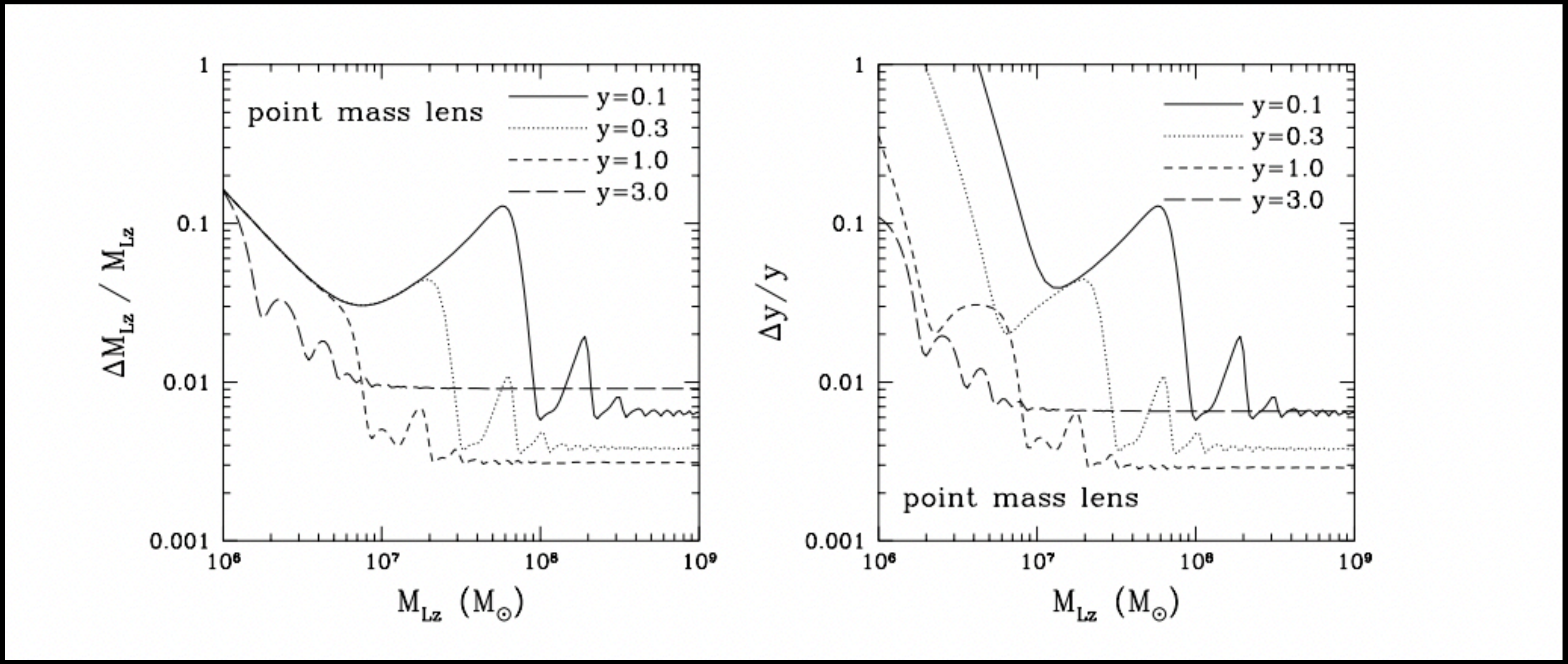
The **geometric optics** limit is **NOT** generically **valid** for **LISA**!

Diffraction effects **important** when

$$M_{\mathrm{L}} \lesssim 10^5\,M_{\odot}/(f/\mathrm{Hz})$$

Can we measure lensing from LISA observations?

YES!



Takahashi and Nakamura (2003)

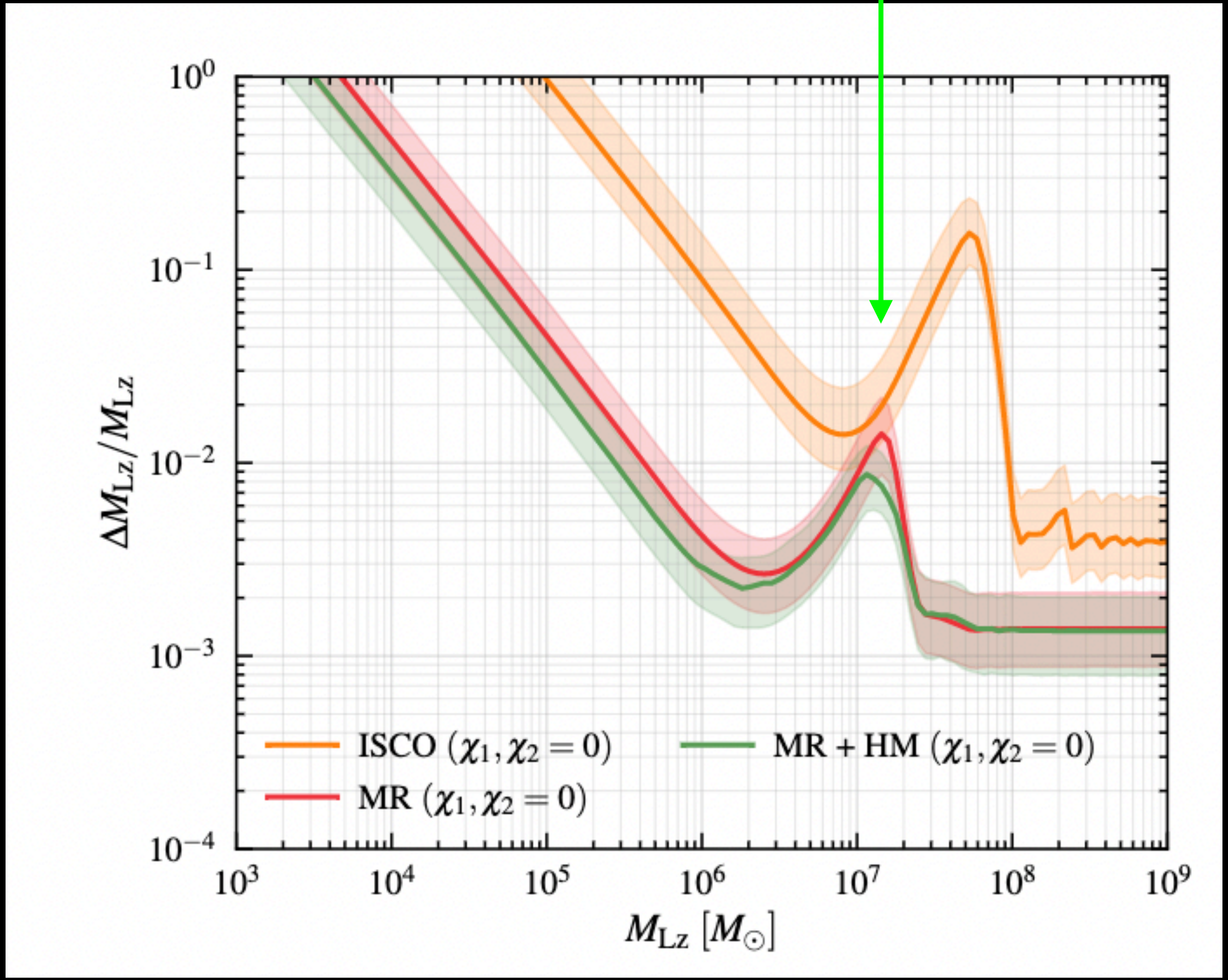
Fisher matrix analysis for non-spinning $10^6 + 10^6 M_\odot$ @ $z_s = 1$

Lensing is measurable when $M_L \gtrsim 10^6 M_\odot, y \leq 10$

See also Neto (2003), Sereno+ (2011), Cusin and Tamanini (2021), LISA Cosmology working group (2022), Gao+ (2022)

Impact of merger-ringdown and higher harmonics

Takahashi and Nakamura used **inspiral-only waveforms without higher harmonics**



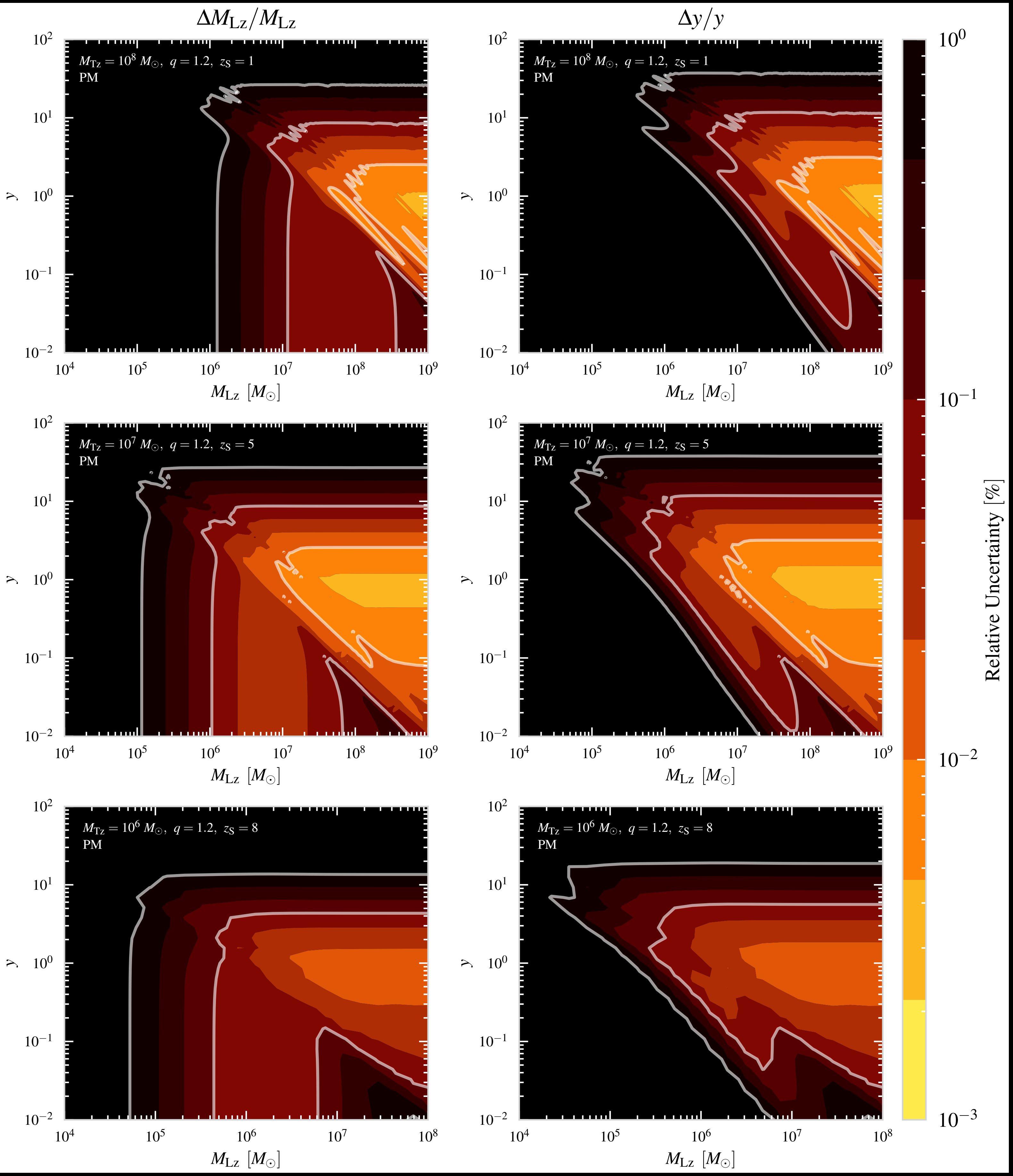
Çalışkan,…**RC** et al.

Fisher matrix analysis for non-spinning $10^6 + 10^6 M_\odot$ @ $z_s = 1, y = 0.1$
with analytic expression of diffraction integral and its derivatives
for Fisher matrix we use lisabeta Marsat+(2020)

Including **merger-ringdown** in the waveform **improves precision** on lens mass measurement by **an order of magnitude**, **higher harmonics** by a **factor of 2**

The **precision** on the measurement of **y** is **improved** by a **factor of 2**

Measurability of lensing effects in MBHBs



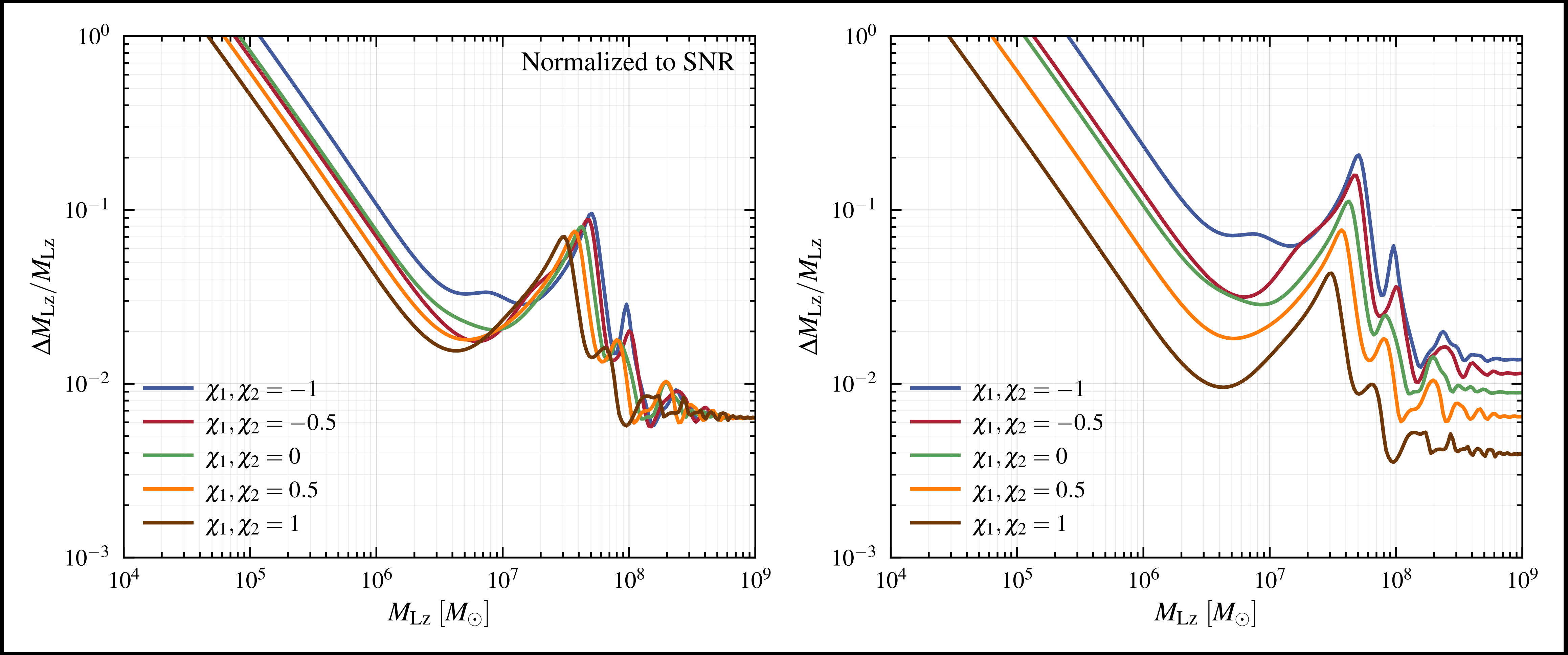
Çalışkan,…**RC** et al.

			$\Delta M_{\rm Lz}/M_{\rm Lz}$		$\Delta y/y$
$M_{\rm Tz}$ $[M_\odot]$	q	z_S	$M_{\rm Lz}^{\rm crit}$ $[M_\odot]$	$y^{\rm crit}$	$y^{\rm crit}$
10^8	1.2	1	$\gtrsim 10^6$	$\lesssim 30$	$\lesssim 40$
10^7	1.2	5	$\gtrsim 10^5$	$\lesssim 30$	$\lesssim 40$
10^6	1.2	8	$\gtrsim 5 \times 10^4$	$\lesssim 15$	$\lesssim 20$

We have results also for SIS lens in partial agreement with Gao+ (2022)

Impact of BH spins

Çalışkan,…**RC** et al.



Fisher matrix analysis for $5.5 \times 10^6 + 4.5 \times 10^6 M_\odot$ @ $z_s = 5, y = 0.1$

Large spins aligned with the angular momentum **improve measurability by an order of magnitude** (a factor of 3 when normalizing by the SNR), when **compared** to **large anti-aligned spins**

In the paper we also show the impact of other parameters of the MBHB on the measurability of lensing effect

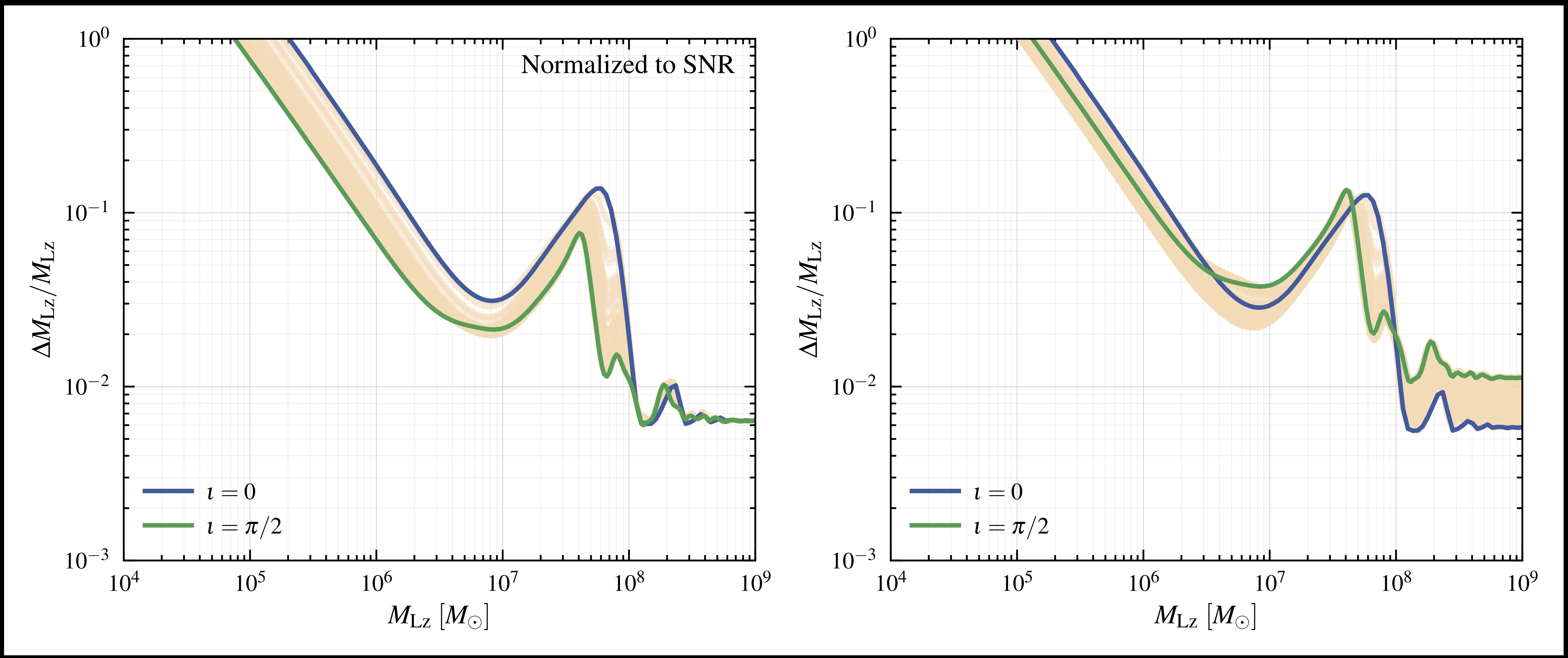
Conclusions and future work

- For MBHBs in LISA we cannot always use the geometric optics approximation, as **diffraction** effects are **important**
- **Lensing** effects are **detectable** for certain **MBHBs** in **LISA**
- **Previous** studies **underestimated by an order of magnitude the precision on the measurement of the lens mass** and by a **factor of 2 that on the impact parameter y**, because of **neglecting** the **merger-ringdown** part of the waveform
- Using Fisher matrix and analytical solution for the diffraction integral and its derivatives, we have analyzed the **measurability of lensing** for "typical" **MBHBs in LISA** using both **PM** and **SIS lens**
- We investigated the effect on all binary parameters (BH spins, source-observer inclination, sky position…) on the measurability of lensing effects
- Use **our Fisher matrix analysis** on a **population of MBHBs** to estimate the number of GW signals with detectable lensing effects **given a population of lenses**

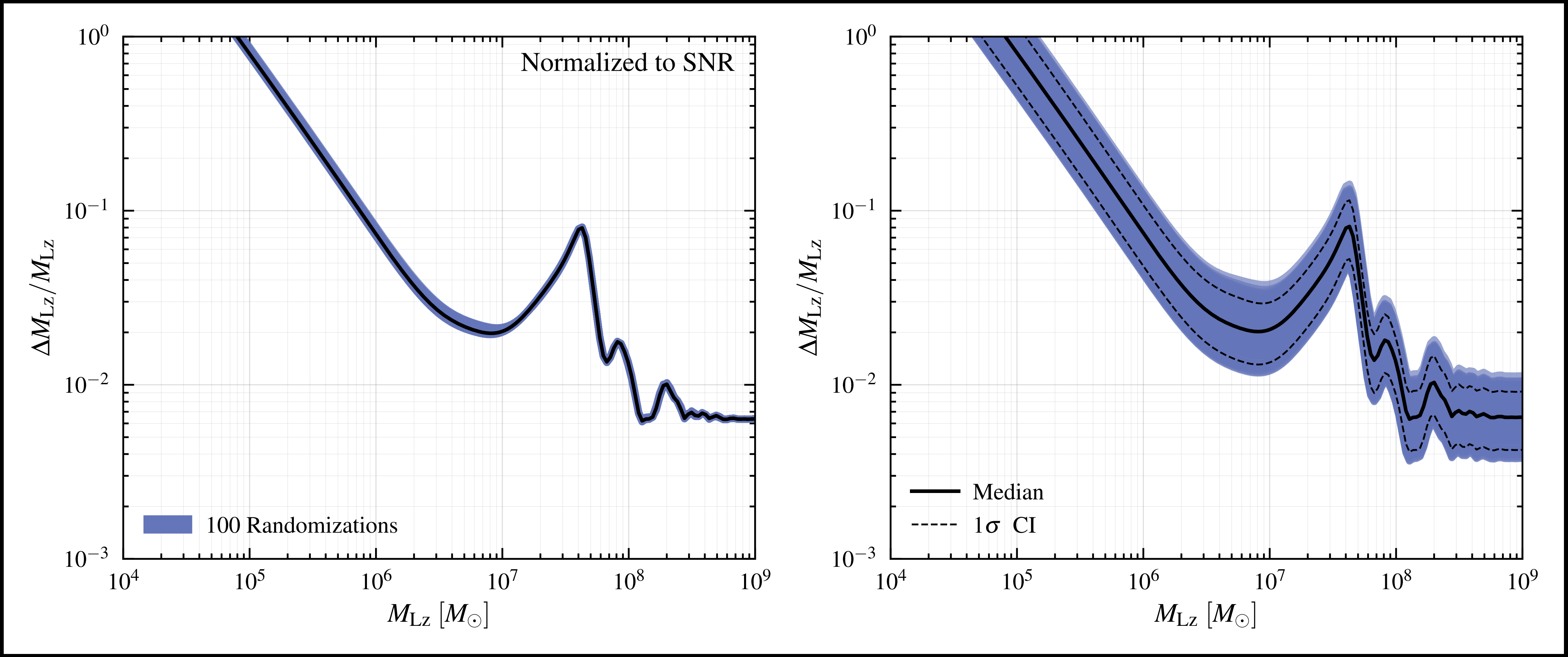
Thanks!

Read this QR code for the paper! arXiv:2206.02803

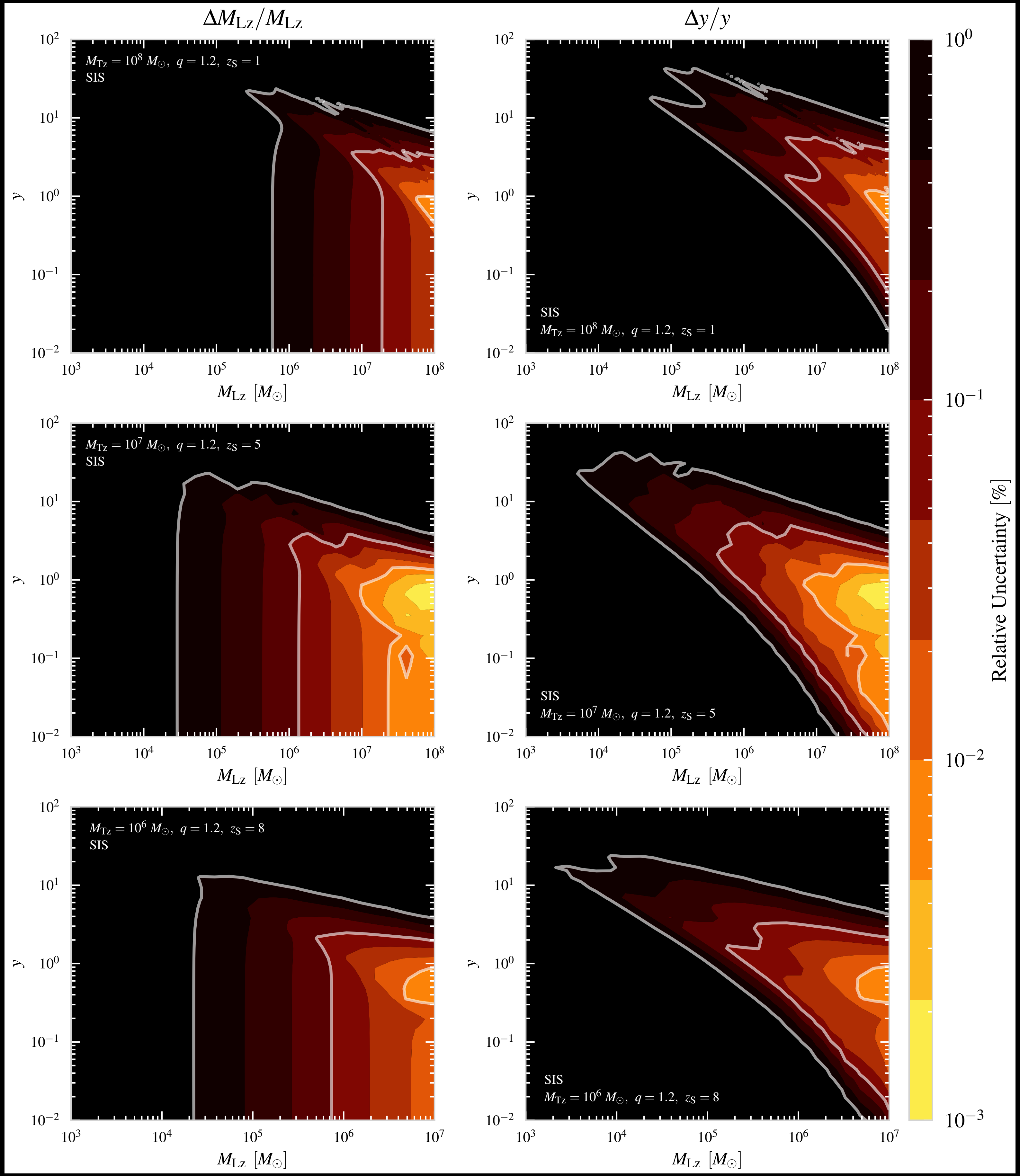
Impact of source-observer angle



Impact of sky-localization and other angles



SIS results



$\Delta M_{\rm Lz}/M_{\rm Lz}$
$\Delta y/y$
$M_{\rm Tz} = 10^8\, M_\odot,\ q = 1.2,\ z_S = 1$
SIS
$M_{\rm Tz} = 10^7\, M_\odot,\ q = 1.2,\ z_S = 5$
$M_{\rm Tz} = 10^6\, M_\odot,\ q = 1.2,\ z_S = 8$
y
$M_{\rm Lz}\ [M_\odot]$
Relative Uncertainty [%]