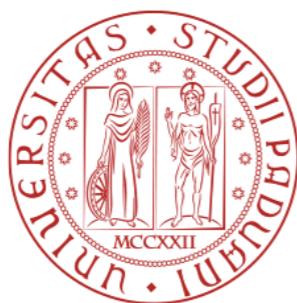


# The new muon $g-2$ puzzle: can the *BMW* lattice result and low-energy $e^+e^-$ data be reconciled by new physics ?

FCCP2022 - Capri Island - 24.09.2022

Luca Di Luzio

800  
ANNI  
1222-2022



UNIVERSITÀ  
DEGLI STUDI  
DI PADOVA



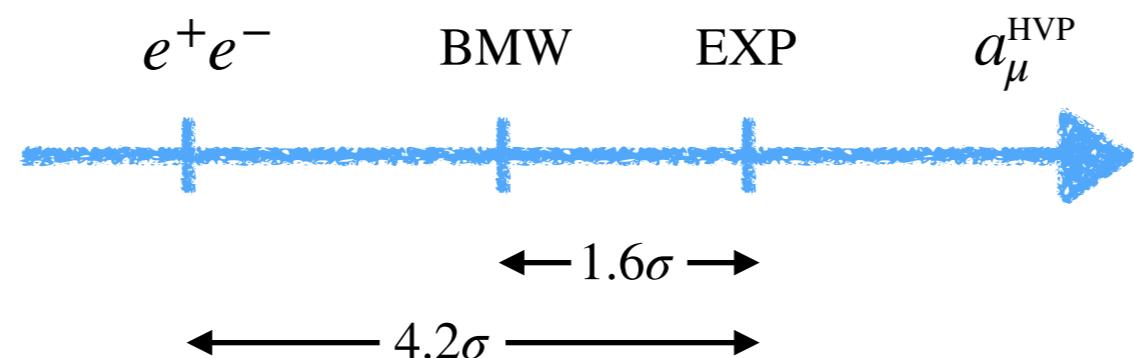
Dipartimento di Fisica e  
Astronomia  
"Galileo Galilei"



# Plan of the Talk

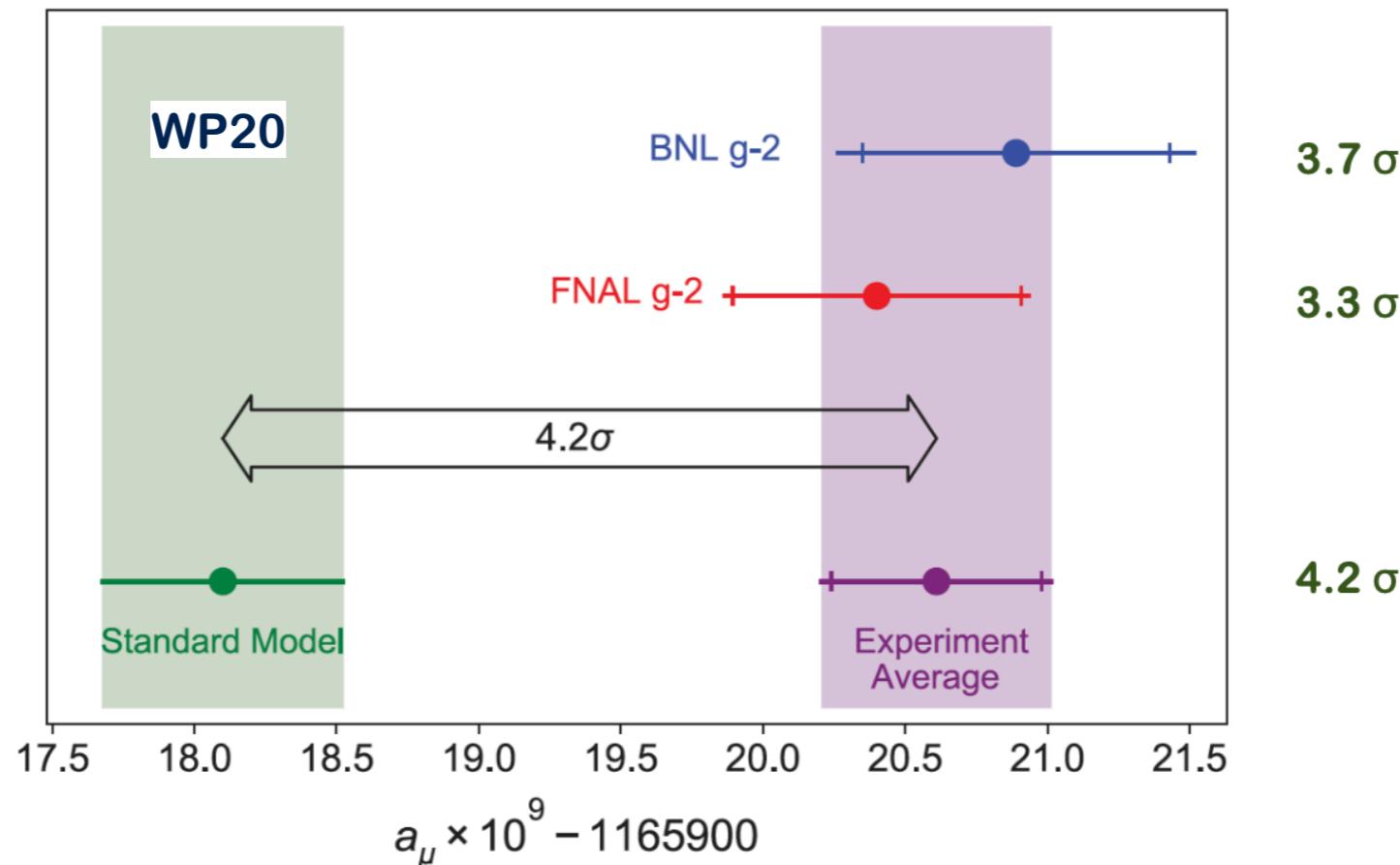
- Status of the muon g-2 as of mid 2022
  - Breakdown of the SM contributions [Courtesy of M. Passera]
  - Hadron Vacuum Polarization (HVP) & the “new g-2 puzzle”
- New physics (NP) behind the “new g-2 puzzle”? [Based on  
LDL, Masiero, Paradisi, Passera 2112.08312]

Working assumption: both lattice (BMW) and  $e^+e^-$  data are correct  $\rightarrow$  NP hiding in  $e^+e^-$  data ?



# FNAL confirms BNL

WP20 = White Paper  
of the Muon g-2  
Theory Initiative:  
arXiv:2006.04822



$$a_\mu^{\text{EXP}} = (116592089 \pm 63) \times 10^{-11} [0.54\text{ppm}] \quad \text{BNL E821}$$

$$a_\mu^{\text{EXP}} = (116592040 \pm 54) \times 10^{-11} [0.46\text{ppm}] \quad \text{FNAL E989 Run 1}$$

$$a_\mu^{\text{EXP}} = (116592061 \pm 41) \times 10^{-11} [0.35\text{ppm}] \quad \text{WA}$$

- FNAL aims at  $16 \times 10^{-11}$  (first 5 runs completed)
- Muon g-2 proposal at J-PARC: Phase-I with similar BNL precision

# QED contribution

$$a_\mu^{\text{QED}} = (1/2) (\alpha/\pi) \text{ [Schwinger, 1948]}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

[Sommerfield; Petermann; Suura&Wichmann '57; Elend '66]

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

[Remiddi, Laporta, Barbieri...; Czarnecki, Skrzypek '99]

$$+ 130.8780 (60) (\alpha/\pi)^4$$

[Kinoshita et al. '81-'15; Steinhauser et al. '13-'16; Laporta '17]

$$+ 750.86 (88) (\alpha/\pi)^5 \text{ [Kinoshita et al. '90-'19]}$$

$$a_\mu^{\text{QED}} = 116584718.931 (19)(100)(23) \times 10^{-11}$$

mainly from 4-loop coeff. unc.

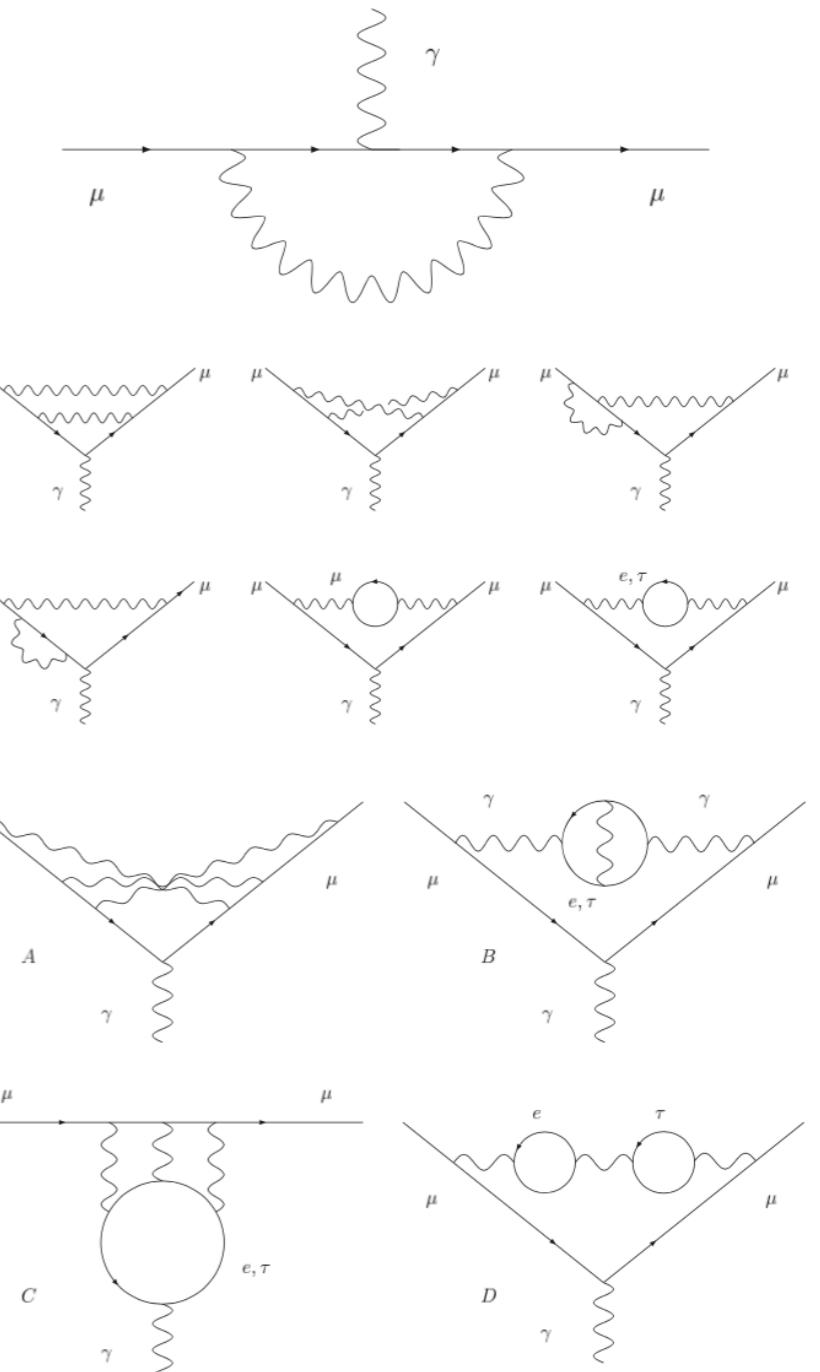
6-loop

from  $\alpha(\text{Cs})$

$\alpha = 1/137.035999046(27)$  [0.2ppb] Parker et al 2018

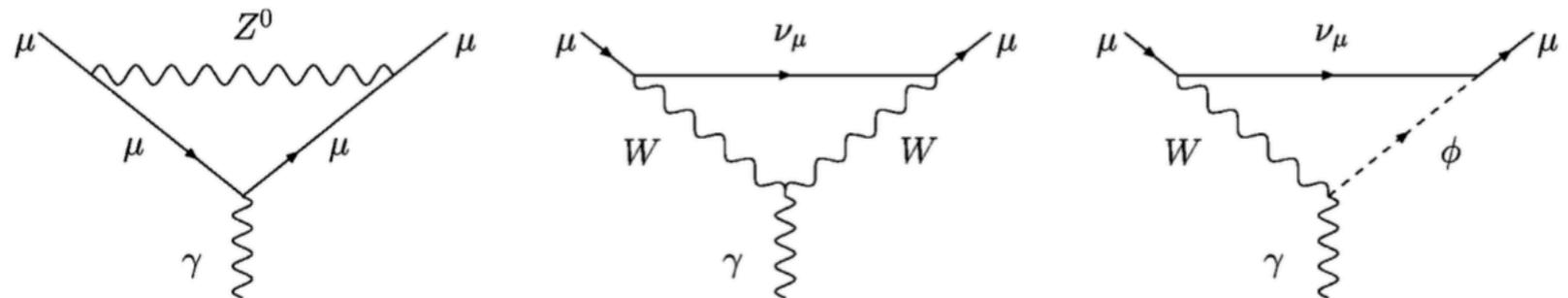
WP20 value

[WP20  $\equiv$  T. Aoyama *et al.*, Phys. Rept. '20]



# EW contribution

- One-loop term



$$a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} (1 - 4 \sin^2 \theta_W)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiw, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

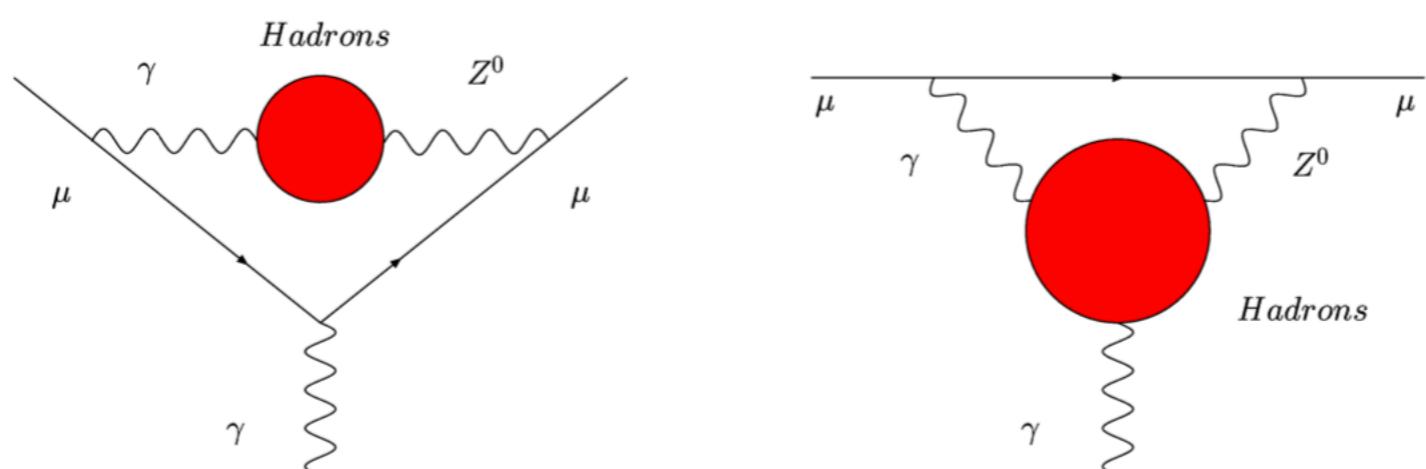
- One-loop plus higher-order terms

**$a_\mu^{\text{EW}} = 153.6 (1.0) \times 10^{-11}$**

Hadronic loop uncertainties (and 3-loop nonleading logs).

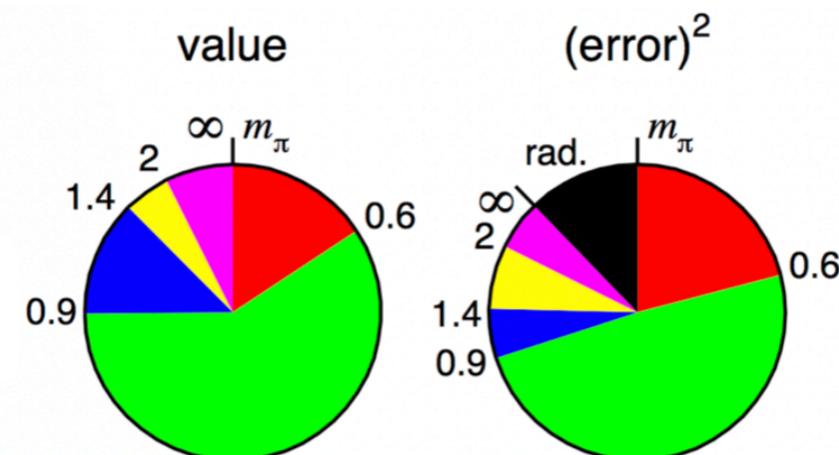
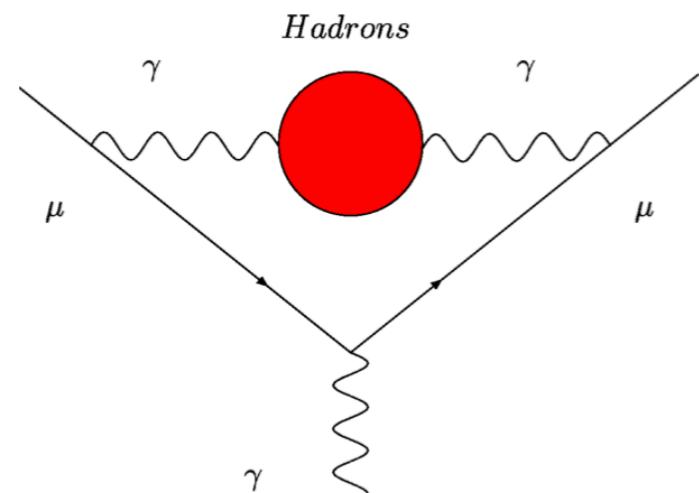
WP20 value

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.



# Hadronic LO contribution

- Hadron Vacuum Polarization (HVP)



Keshavarzi, Nomura, Teubner 2018

$$\text{Im } \text{---} \text{---} \text{---} \sim \left| \text{---} \text{---} \text{---} \right|^2 \sim \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$$

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{1}{4\pi^3} \int_{m_\pi^2}^\infty ds K(s) \sigma_{\text{had}}(s) \quad K(s) \approx m_\mu^2/3s \quad \text{for} \quad \sqrt{s} \gg m_\mu$$

$a_\mu^{\text{HLO}} = 6895 (33) \times 10^{-11}$

F. Jegerlehner, arXiv:1711.06089

$= 6939 (40) \times 10^{-11}$

Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

$= 6928 (24) \times 10^{-11}$

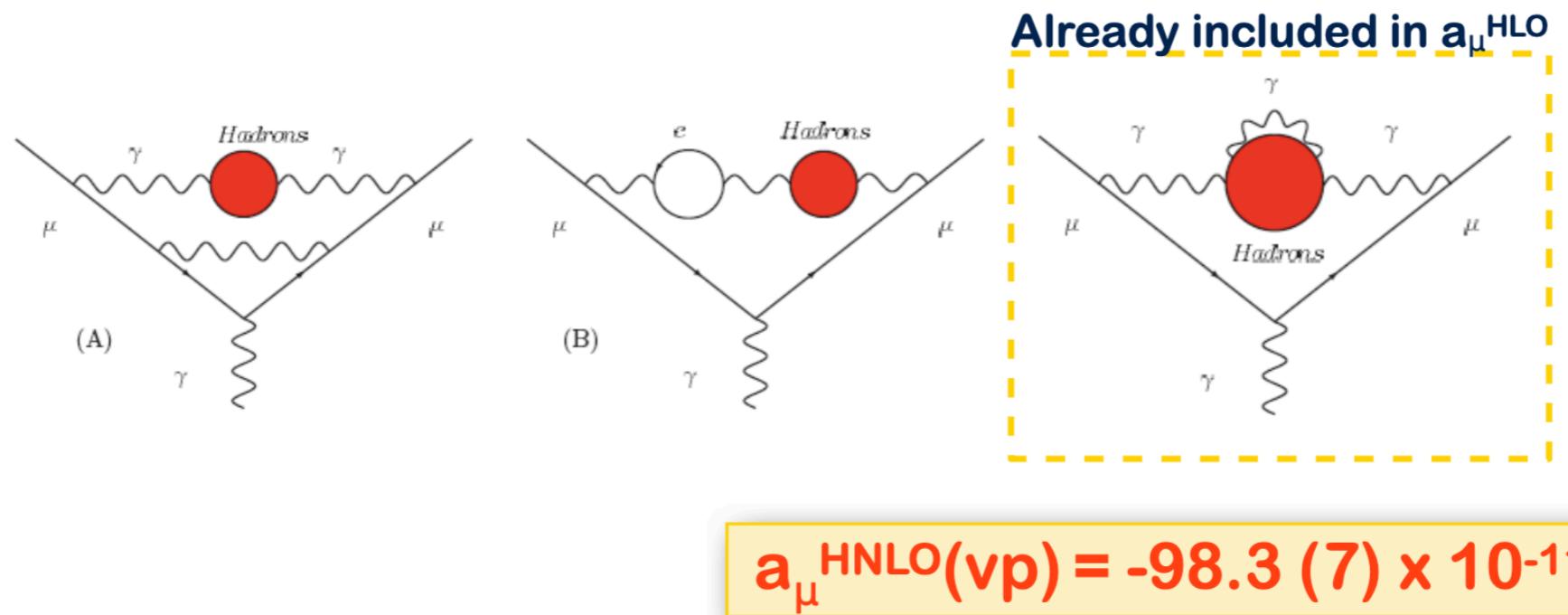
Keshavarzi, Nomura, Teubner, arXiv:1911.00367

$= 6931 (40) \times 10^{-11} (0.6\%)$

WP20 value

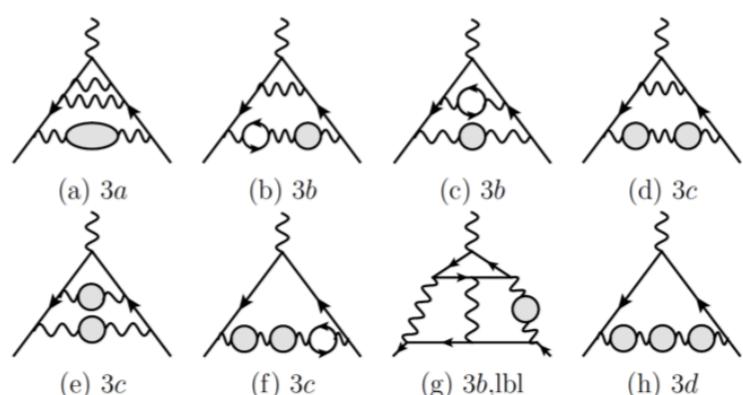
# Higher-order HVP contribution

- $\mathcal{O}(\alpha^3)$  contributions of diagrams containing HVP insertions



Krause '96; Keshavarzi, Nomura, Teubner 2019; WP20.

- $\mathcal{O}(\alpha^4)$  contributions of diagrams containing HVP insertions



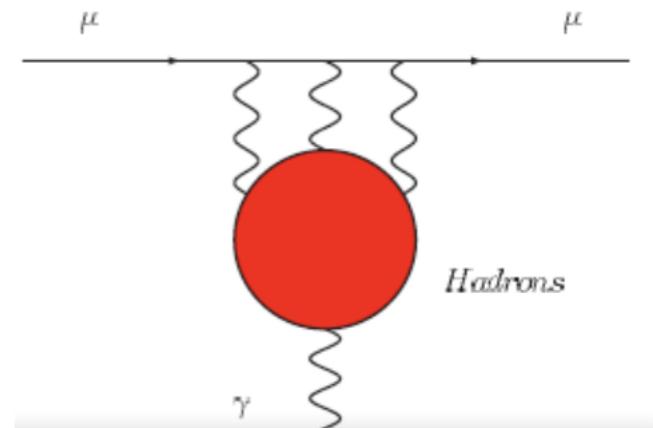
$a_\mu^{\text{HNNLO(vp)}} = 12.4(1) \times 10^{-11}$

Kurz, Liu, Marquard, Steinhauser 2014

# Hadronic LbL contribution

- Hadronic light-by-light at  $\mathcal{O}(a^3)$

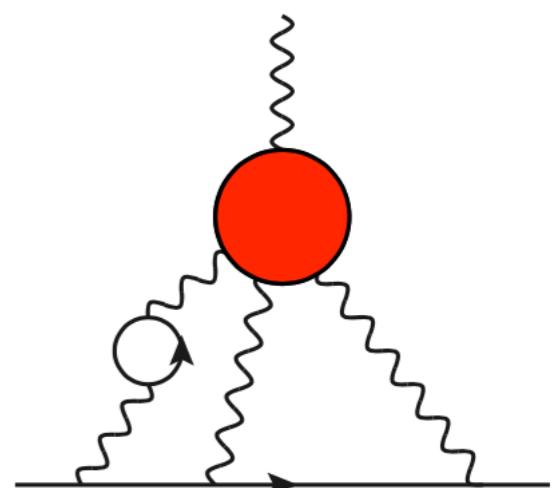
$a_\mu^{\text{HNLO}}( \mathbf{b} ) =$	<b>80 (40) <math>\times 10^{-11}</math></b>	Knecht & Nyffeler '02
=	<b>136 (25) <math>\times 10^{-11}</math></b>	Melnikov & Vainshtein '03
=	<b>105 (26) <math>\times 10^{-11}</math></b>	Prades, de Rafael, Vainshtein '09
=	<b>100 (29) <math>\times 10^{-11}</math></b>	Jegerlehner, arXiv:1705.00263
=	<b>92 (19) <math>\times 10^{-11}</math></b>	WP20 (phenomenology)



- Hadronic light-by-light at  $\mathcal{O}(a^4)$

$$a_\mu^{\text{HNNLO}}(|\mathbf{b}|) = 2 (1) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; WP20



# Breakdown of SM contributions

- $a_\mu$  from WP20 (w/o BMWc lattice result)

[Colangelo EPS-HEP2021 proceeding]

Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [3]
Experiment (FNAL)	116 592 040(54)	Ref. [1]
Experiment (World-Average)	116 592 061(41)	
HVP LO ( $e^+e^-$ )	6931(40)	Refs. [6–11]
HVP NLO ( $e^+e^-$ )	-98.3(7)	Ref. [11]
HVP NNLO ( $e^+e^-$ )	12.4(1)	Ref. [12]
HVP LO (lattice, $udsc$ )	7116(184)	Refs. [13–21]
HLbL (phenomenology)	92(19)	Refs. [22–34]
HLbL NLO (phenomenology)	2(1)	Ref. [35]
HLbL (lattice, $uds$ )	79(35)	Ref. [36]
HLbL (phenomenology + lattice)	90(17)	
QED	116 584 718.931(104)	Refs. [37, 38]
Electroweak	153.6(1.0)	Refs. [39, 40]
HVP ( $e^+e^-$ , LO + NLO + NNLO)	6845(40)	
HLbL (phenomenology + lattice + NLO)	92(18)	
Total SM Value	116 591 810(43)	
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

# Breakdown of SM contributions

- $a_\mu$  from WP20 (w/o BMWc lattice result)

[Colangelo EPS-HEP2021 proceeding]

Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [3]
Experiment (FNAL)	116 592 040(54)	Ref. [1]
Experiment (World-Average)	116 592 061(41)	
HVP LO ( $e^+e^-$ )	6931(40)	Refs. [6–11]
HVP NLO ( $e^+e^-$ )	-98.3(7)	Ref. [11]
HVP NNLO ( $e^+e^-$ )	12.4(1)	Ref. [12]
HVP LO (lattice, $udsc$ )	7116(184)	Refs. [13–21]
HLbL (phenomenology)	92(19)	Refs. [22–34]
HLbL NLO (phenomenology)	2(1)	Ref. [35]
HLbL (lattice, $uds$ )	79(35)	Ref. [36]
HLbL (phenomenology + lattice)	90(17)	
QED	116 584 718.931(104)	Refs. [37, 38]
Electroweak	153.6(1.0)	Refs. [39, 40]
HVP ( $e^+e^-$ , LO + NLO + NNLO)	6845(40)	
HLbL (phenomenology + lattice + NLO)	92(18)	
Total SM Value	116 591 810(43)	
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	



HVP LO is the bottle-neck of the SM prediction

# Breakdown of SM contributions

- $a_\mu$  from WP20 (w/o BMWc lattice result)

$$\Delta a_\mu = \mathbf{a}_\mu^{\text{EXP}} - \mathbf{a}_\mu^{\text{SM}} \equiv a_\mu^{\text{NP}} = 251(59) \times 10^{-11} \quad (\mathbf{4.2\sigma \text{ discrepancy!}})$$

$$\underbrace{(0.1)_{\text{QED}}, \quad (1)_{\text{EW}}, \quad (18)_{\text{HLbL}}, \quad (40)_{\text{HVP}}, \quad (41)_{\delta a_\mu^{\text{EXP}}}}_{(43)_{\text{TH}}}.$$

$(\delta a_\mu^{\text{EXP}} \approx 16 \times 10^{-11}$  by the E989 Muon g-2 exp. in a few years)

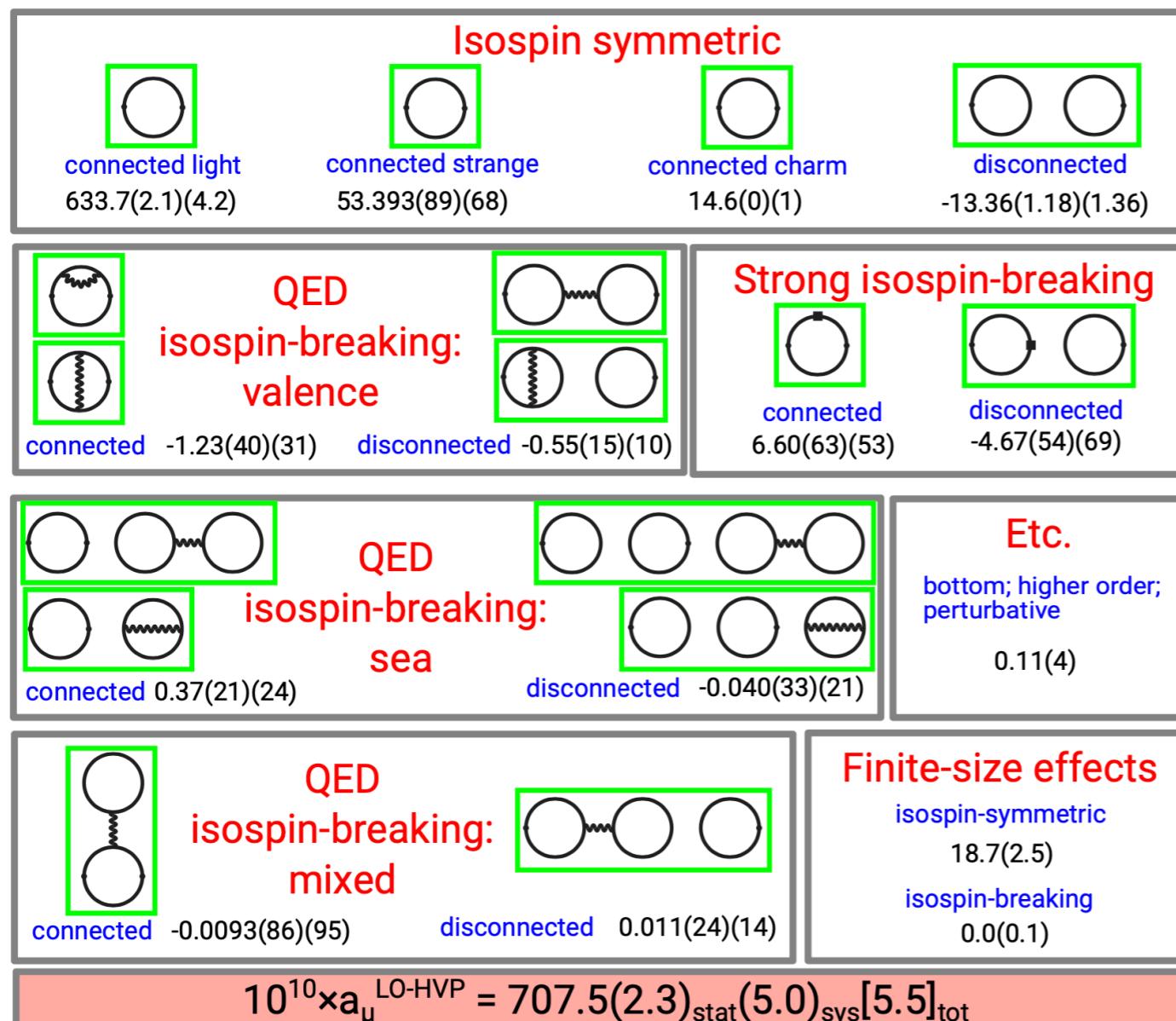
→ HVP LO is the bottle-neck of the SM prediction

# LO HVP from lattice QCD

**Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:**

$$a_\mu^{\text{HLO}} = 7075(23)_{\text{stat}}(50)_{\text{syst}} \times 10^{-11}.$$

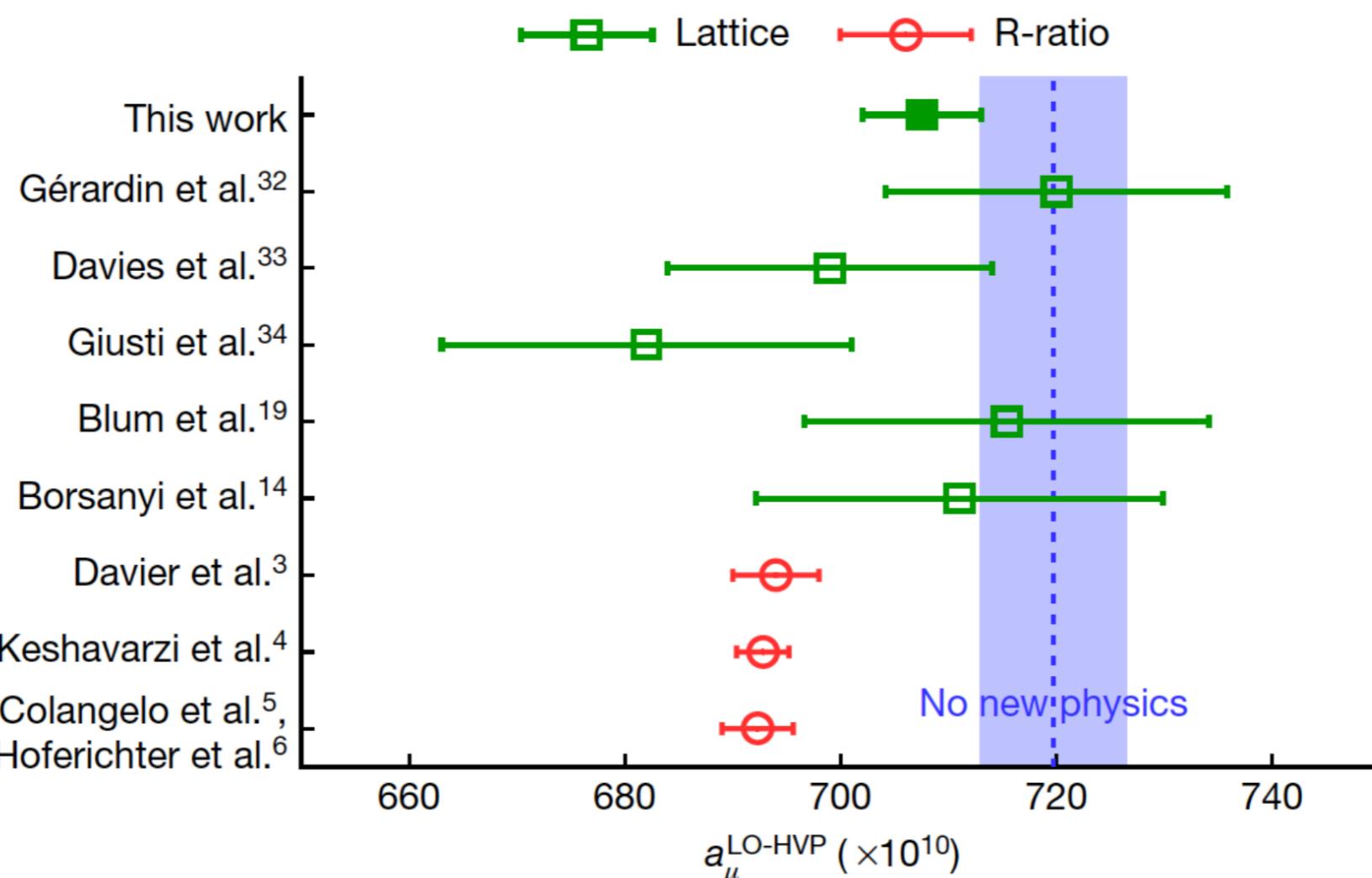
**55**



Borsanyi et al (BMWc), Nature 2021

# LO HVP from lattice QCD

Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:  
 $a_\mu^{\text{HLO}} = \underbrace{7075(23)_{\text{stat}}(50)_{\text{syst}}}_{55} \times 10^{-11}$ . Some tension with dispersive evaluations. BMWc 2021

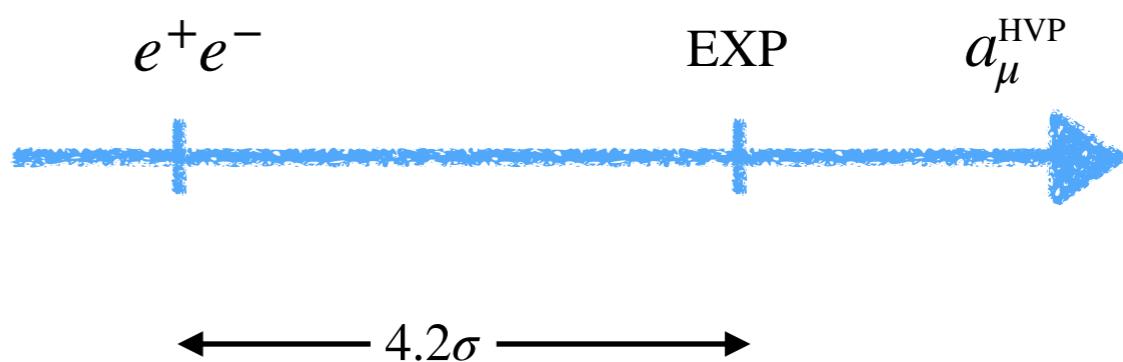


Borsanyi et al (BMWc), Nature 2021

# “old vs. new” muon g-2 puzzle

$$(a_\mu^{\text{HVP}})_{\text{EXP}} = a_\mu^{\text{EXP}} - a_\mu^{\text{SM, rest}}$$

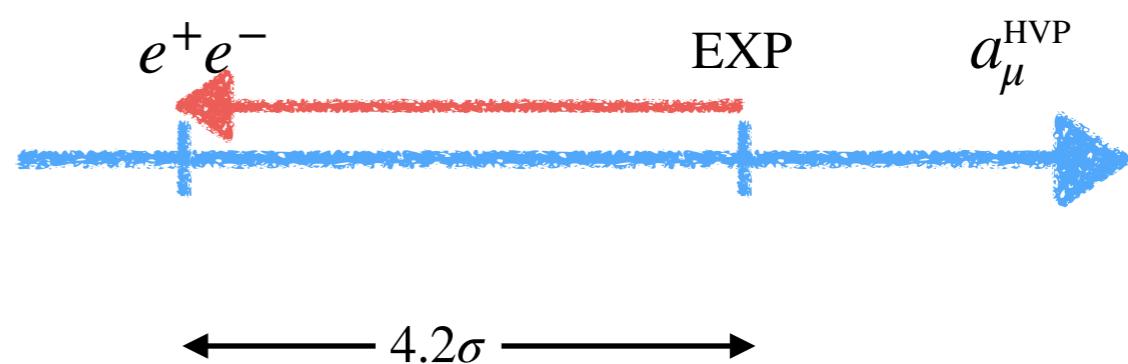
$$(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}} = 6931(40) \times 10^{-11}$$



# “old vs. new” muon g-2 puzzle

$$(a_\mu^{\text{HVP}})_{\text{EXP}} = a_\mu^{\text{EXP}} - a_\mu^{\text{SM, rest}} - a_\mu^{\text{NP}}$$

$$(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}} = 6931(40) \times 10^{-11}$$



“old puzzle”: a direct NP contribution to  $a_\mu$  is required in order to match the EXP value

$$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} \equiv a_\mu^{\text{NP}} \approx (a_\mu^{\text{SM}})_{\text{EW}} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 200 \times 10^{-11}$$

- ▶ NP is at the weak scale ( $\Lambda \approx v$ ) and weakly coupled to SM particles.
- ▶ NP is very heavy ( $\Lambda \gg v$ ) and strongly coupled to SM particles.
- ▶ NP is very light ( $\Lambda \lesssim 1 \text{ GeV}$ ) and feebly coupled to SM particles.

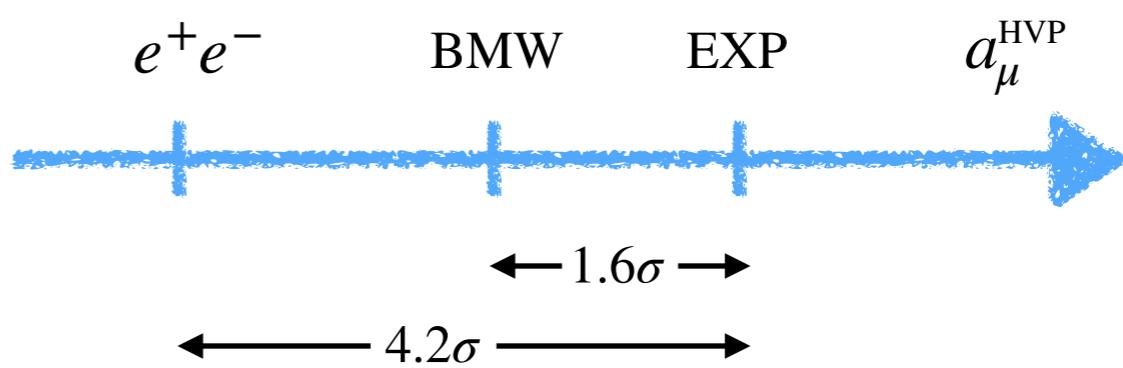
O(few x 100) papers

# “old vs. new” muon g-2 puzzle

$$(a_\mu^{\text{HVP}})_{\text{EXP}} = a_\mu^{\text{EXP}} - a_\mu^{\text{SM, rest}}$$

$$(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}} = 6931(40) \times 10^{-11}$$

$$(a_\mu^{\text{HVP}})_{\text{BMW}} = 7075(55) \times 10^{-11}$$



“new puzzle”: if BMW is correct, the “old” g-2 discrepancy ( $4.2\sigma$ ) would be basically gone

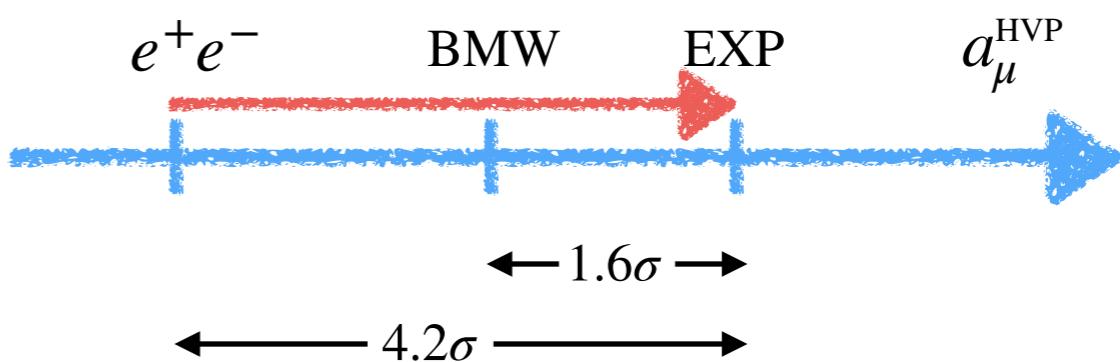
→ however, this brings in a new tension with  $e^+e^-$  data ( $2.2\sigma$ )

# “old vs. new” muon g-2 puzzle

$$(a_\mu^{\text{HVP}})_{\text{EXP}} = a_\mu^{\text{EXP}} - a_\mu^{\text{SM, rest}}$$

$$(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}} = 6931(40) \times 10^{-11}$$

$$(a_\mu^{\text{HVP}})_{\text{BMW}} = 7075(55) \times 10^{-11}$$



“new puzzle”: if BMW is correct, the “old” g-2 discrepancy ( $4.2\sigma$ ) would be basically gone

→ however, this brings in a new tension with  $e^+e^-$  data ( $2.2\sigma$ )

Here, NP in  $\sigma_{\text{had}}(e^+e^- \rightarrow \text{hadrons})$  such that

[LDL, Masiero, Paradisi, Passera 2112.08312]

1.  $(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}} \approx (a_\mu^{\text{HVP}})_{\text{EXP}}$
2. the approximate agreement between BMW and EXP is not spoiled
3. w/o a direct contribution  $a_\mu^{\text{NP}}$  (i.e. NP not in muons)

# Muon g-2 $\rightleftharpoons$ $\Delta\alpha$ connection

- Can  $\Delta a_\mu$  be due to a missing contribution in  $\sigma_{\text{had}}$  ?

[Marciano, Passera, Sirlin 0804.1142 & 1001.4528; Crivellin, Hoferichter, Manzari, Montull 2003.04886;  
Keshavarzi, Marciano, Passera, Sirlin 2006.12666; de Rafael 2006.13880; Malaescu, Schott 2008.08107;  
Colangelo, Hoferichter, Stoffer 2010.07943]

# Muon g-2 $\rightleftarrows$ $\Delta\alpha$ connection

- Can  $\Delta a_\mu$  be due to a missing contribution in  $\sigma_{\text{had}}$ ?

→ an upward shift of  $\sigma_{\text{had}}$  induces an increase of  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha_{\text{lep}}(M_Z) - \Delta\alpha_{\text{had}}^{(5)}(M_Z) - \Delta\alpha_{\text{top}}(M_Z)}$$

$$a_\mu^{\text{HLO}} \simeq \frac{m_\mu^2}{12\pi^3} \int_{4m_\pi^2}^\infty ds \frac{\sigma(s)}{s}, \quad \Delta\alpha_{\text{had}}^{(5)} = \frac{M_Z^2}{4\pi\alpha^2} \int_{4m_\pi^2}^\infty ds \frac{\sigma(s)}{M_Z^2 - s}$$

- disfavoured by the EW fit (at about  $2\sigma$ ), if the shift happens at  $\sqrt{s} \gtrsim 1 \text{ GeV}$

→ selects light NP inducing a sub-GeV modification of  $\sigma_{\text{had}}$

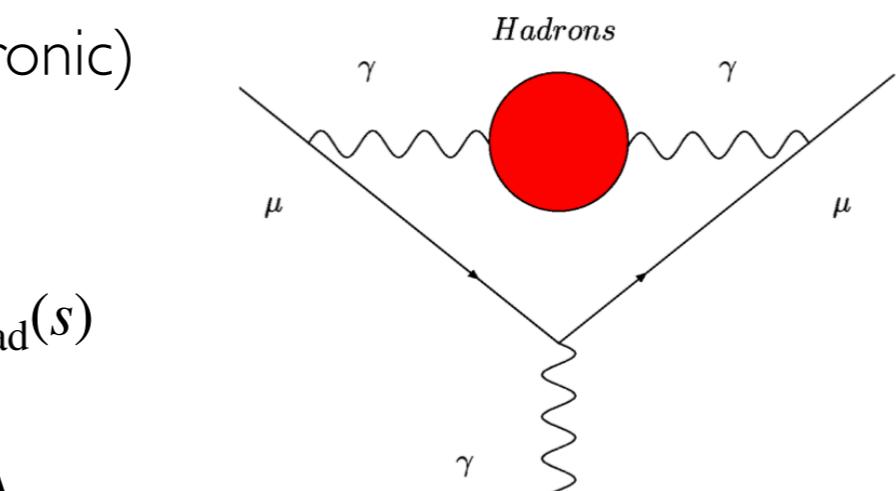
# A closer look at HVP LO

- dominated by  $e^+e^- \rightarrow \pi^+\pi^-$  channel (70% of the full hadronic)

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^{\infty} \frac{ds}{s} K(s) \text{Im } \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$

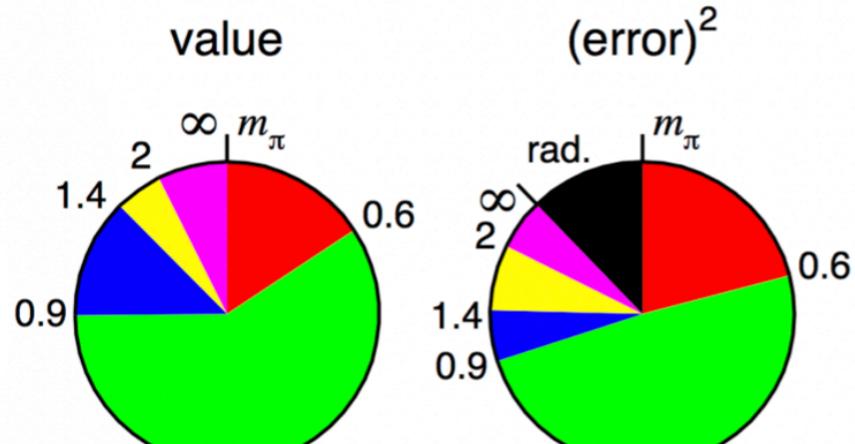
dispersion relations

optical theorem

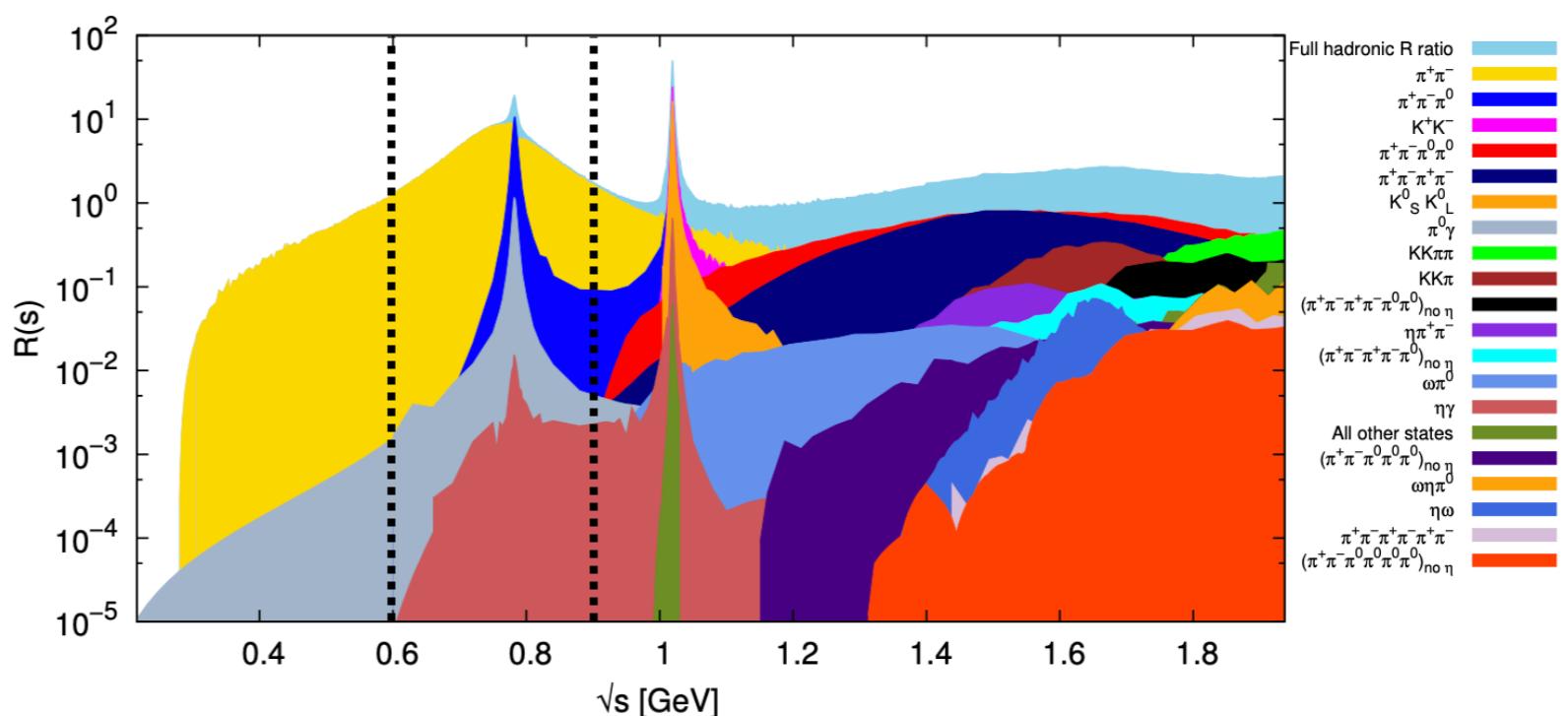


kernel function

$$K(s) \approx m_\mu^2/3s \quad \text{for} \quad \sqrt{s} \gg m_\mu$$



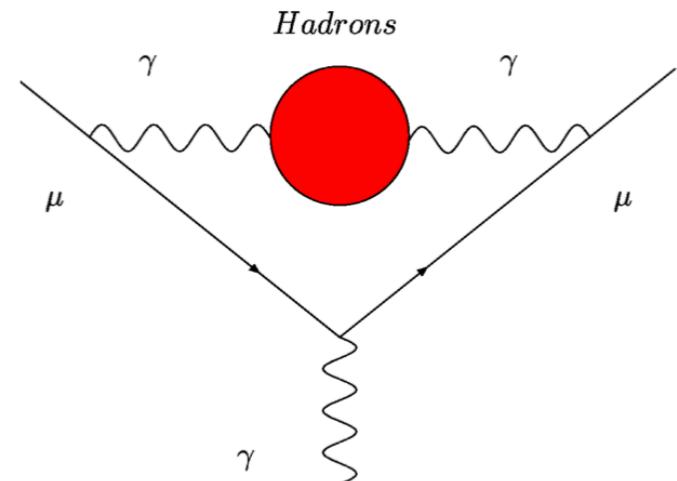
Keshavarzi, Nomura, Teubner 2018



# A closer look at HVP LO

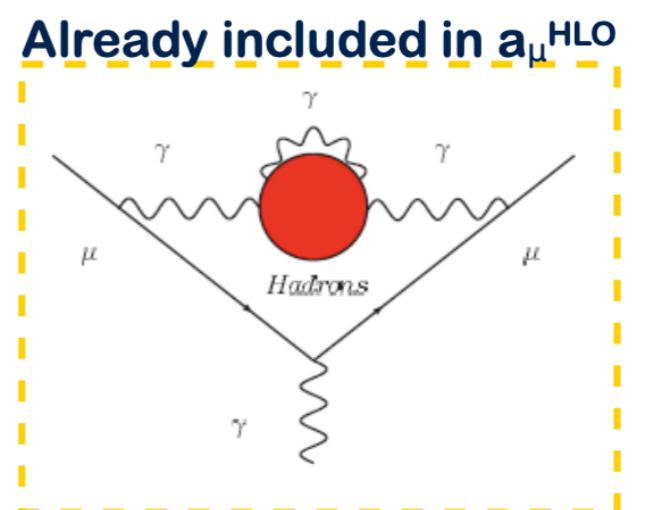
- dominated by  $e^+e^- \rightarrow \pi^+\pi^-$  channel (70% of the full hadronic)

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^\infty \frac{ds}{s} K(s) \text{Im } \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^\infty ds K(s) \sigma_{\text{had}}(s)$$



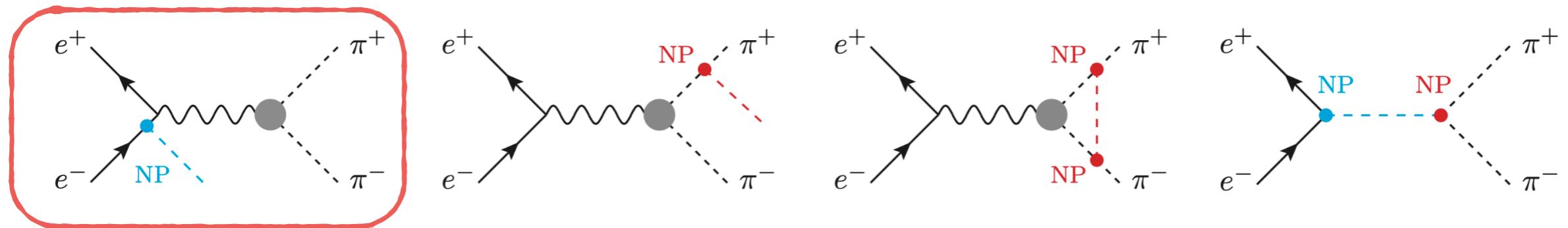
- what is  $\sigma_{\text{had}}(s)$  ?

- Includes Final State Radiation (FSR)
- Initial State Radiation (ISR) and Vacuum Polarization are subtracted



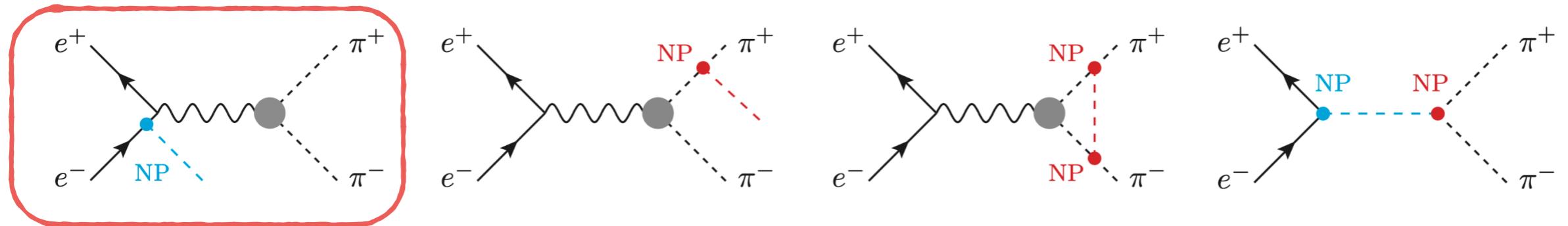
$$(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{FSR}} \approx 50 \times 10^{-11}$$

# Light NP in $\sigma_{\text{had}}$



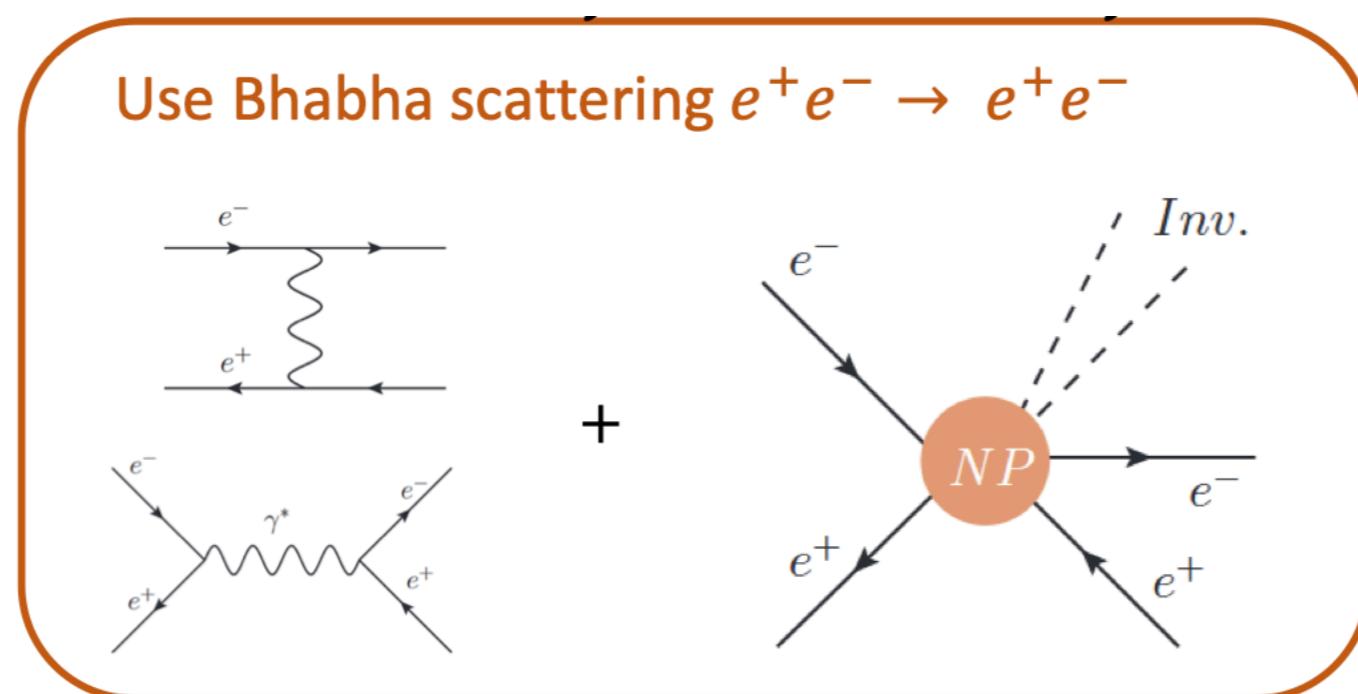
- I. NP coupled only to electrons → severe bounds / ISR should not be included into  $\sigma_{\text{had}}(s)$

# Light NP in $\sigma_{\text{had}}$



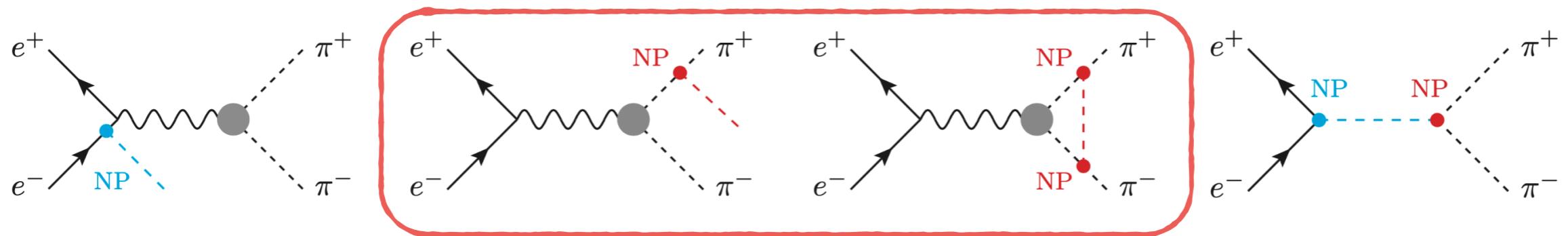
- I. NP coupled only to electrons → severe bounds / ISR should not be included into  $\sigma_{\text{had}}(s)$

[See however Darmé, Grilli di Cortona, Nardi 21|12.09|39  
can NP in Bhabha scattering affect KLOE luminosity? → backup slides]



[Darmé ICHEP 2022]

# Light NP in $\sigma_{\text{had}}$



## 2. NP coupled only to hadrons

FSR effects due to NP should be included into  $\sigma_{\text{had}}(s)$ , not easy to be accounted for...  
(depend on exp. cuts and mass of NP)

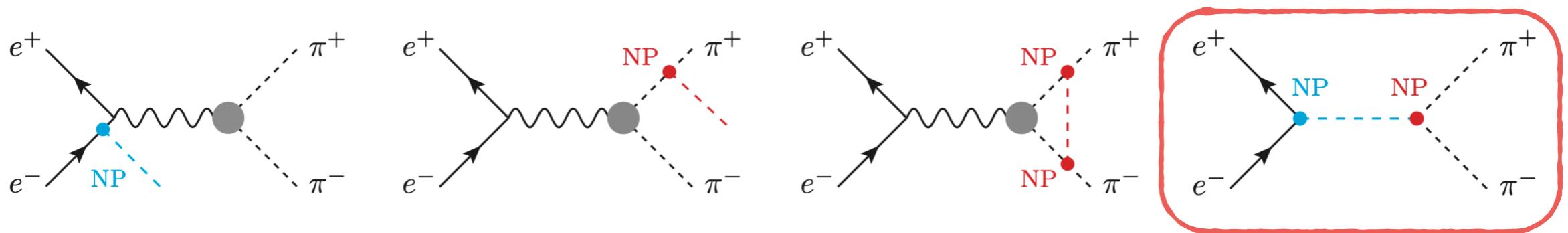


however, we know that in the QED case

$$(a_\mu^{\text{HVP}})_{e^+e^-}^{\text{FSR}} \approx 50 \times 10^{-11} \quad \longleftrightarrow \quad |(a_\mu^{\text{HVP}})_{\text{BMW}} - (a_\mu^{\text{HVP}})_{e^+e^-}^{\text{WP20}}| \approx 150 \times 10^{-11}$$

and, moreover, we expect an extra  $(g_{\text{NP}}/e)^2 \ll 1$  suppression

# Light NP in $\sigma_{\text{had}}$



3. NP coupled both to **hadrons** and **electrons**

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^\infty \frac{ds}{s} K(s) \text{Im } \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^\infty ds K(s) \sigma_{\text{had}}(s)$$



optical theorem

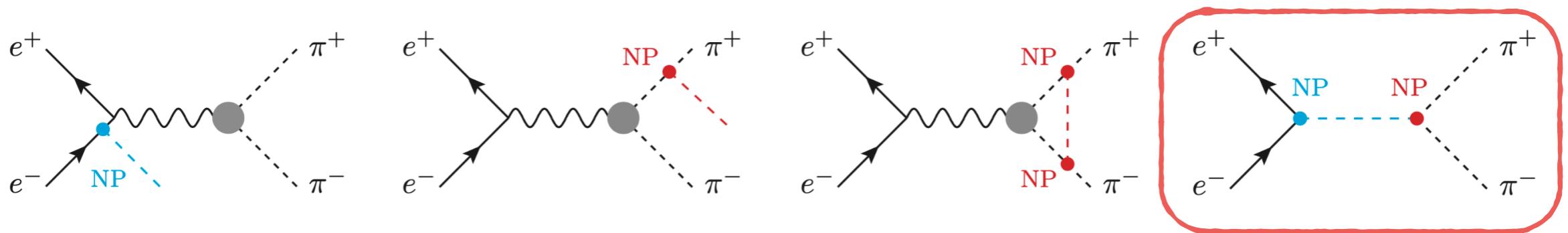
$$\sigma_{\text{had}} = \sigma_{\text{had}}^{\text{SM}} + \Delta\sigma_{\text{had}}^{\text{NP}}$$



experimental x-section

$$\text{Im } \text{---} \sim \left| \text{---} \right|^2 \sim \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$$

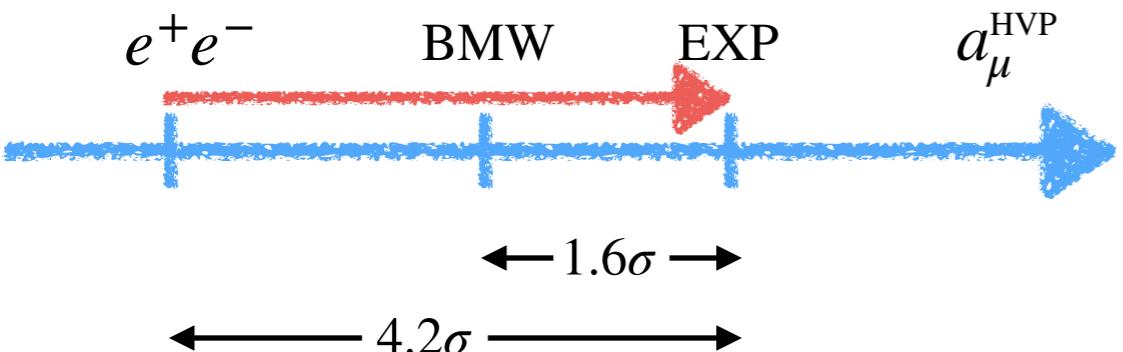
# Light NP in $\sigma_{\text{had}}$



3. NP coupled both to **hadrons** and **electrons**

$$(a_\mu^{\text{HVP}})_{e^+e^-} = \frac{\alpha}{\pi^2} \int_{m_{\pi^0}^2}^\infty \frac{ds}{s} K(s) \text{Im } \Pi_{\text{had}}(s) = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^\infty ds K(s) \sigma_{\text{had}}(s)$$

$$\sigma_{\text{had}} = \sigma_{\text{had}}^{\text{SM}} + \Delta\sigma_{\text{had}}^{\text{NP}}$$



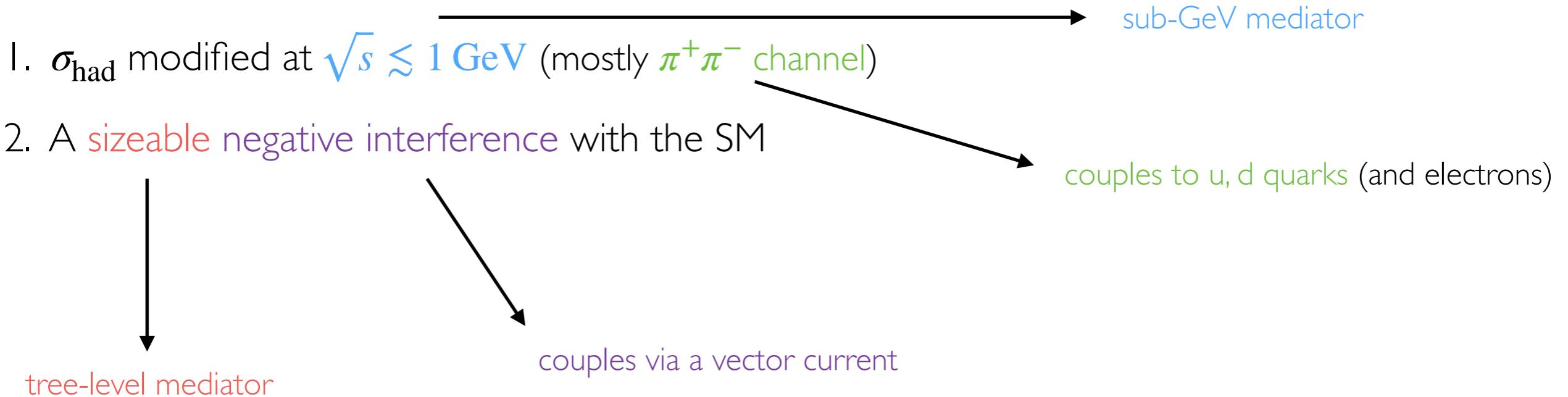
$$\sigma_{\text{had}} - \Delta\sigma_{\text{had}}^{\text{NP}}$$

should be “subtracted” by NP,  
since NP does not contribute to HVP at the LO  
(but it contributes at the LO to the x-section)

→ a positive sift on  $(a_\mu^{\text{HVP}})_{e^+e^-}$  requires  $\Delta\sigma_{\text{had}}^{\text{NP}} < 0$  (negative interference)

# Basically, a unique scenario

- Requirements:



→ a light spin-1 mediator with vector couplings to first generation SM fermions

$$\mathcal{L}_{Z'} \supset (g_V^e \bar{e} \gamma^\mu e + g_V^q \bar{q} \gamma^\mu q) Z'_\mu \quad q = u, d \quad m_{Z'} \lesssim 1 \text{ GeV}$$

# $Z'$ shift on $(a_\mu^{\text{HVP}})_{e^+e^-}$

- Neglecting iso-spin breaking corrections due to NP

$$\frac{\sigma_{\pi\pi}^{\text{SM+NP}}}{\sigma_{\pi\pi}^{\text{SM}}} = \left| 1 + \frac{g_V^e(g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + im_{Z'}\Gamma_{Z'}} \right|^2$$

- Requiring that the shift in the x-section saturates the g-2 discrepancy

$$\Delta a_\mu = \frac{1}{4\pi^3} \int_{s_{\text{exp}}}^\infty ds K(s) (-\Delta\sigma_{\text{had}}^{\text{NP}}(s))$$



$\sqrt{s_{\text{exp}}} \approx 0.3 \text{ GeV}$   
for  $\pi^+\pi^-$  channel

$$\Delta\sigma_{\text{had}}^{\text{NP}}(s) \approx \sigma_{\pi\pi}^{\text{SM}}(s) \times \frac{2\epsilon s(s - m_{Z'}^2) + \epsilon^2 s^2}{(s - m_{Z'}^2)^2 + m_{Z'}^4 \gamma^2}$$

$$\epsilon \equiv g_V^e(g_V^u - g_V^d)/e^2$$

$$\gamma \equiv \Gamma_{Z'}/m_{Z'}$$

# $Z'$ shift on $(a_\mu^{\text{HVP}})_{e^+e^-}$

- Typical benchmarks solving the g-2 discrepancy

$$\Delta a_\mu = \frac{1}{4\pi^3} \int_{s_{\text{exp}}}^\infty ds K(s) (-\Delta\sigma_{\text{had}}^{\text{NP}}(s))$$

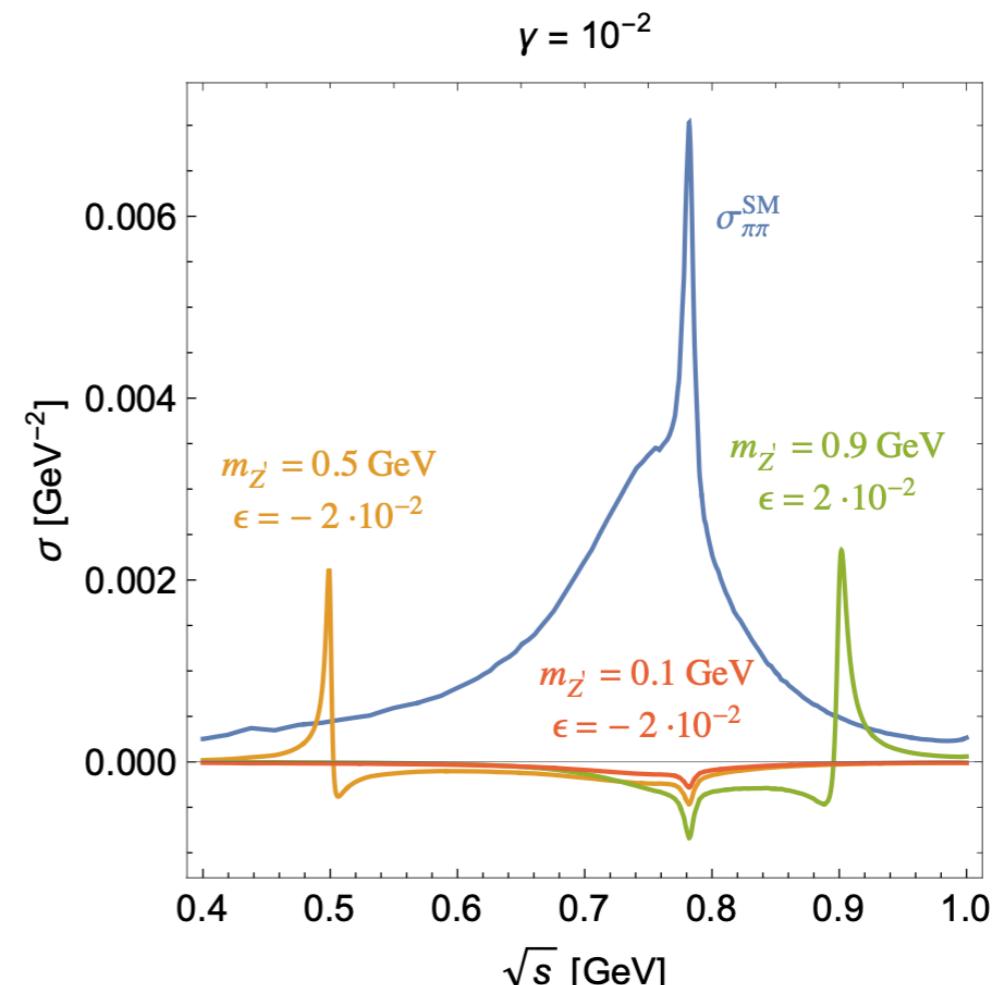


$\sqrt{s_{\text{exp}}} \approx 0.3 \text{ GeV}$   
for  $\pi^+\pi^-$  channel

$$\Delta\sigma_{\text{had}}^{\text{NP}}(s) \approx \sigma_{\pi\pi}^{\text{SM}}(s) \times \frac{2\epsilon s(s - m_{Z'}^2) + \epsilon^2 s^2}{(s - m_{Z'}^2)^2 + m_{Z'}^4 \gamma^2}$$

$$\epsilon \equiv g_V^e (g_V^u - g_V^d)/e^2$$

$$\gamma \equiv \Gamma_{Z'}/m_{Z'}$$



# $Z'$ constraints

## I. Semi-leptonic processes

$e^+e^- \rightarrow q\bar{q}$  has been measured with per-cent accuracy at LEP-II

$$\frac{\sigma_{qq}^{\text{SM+NP}}}{\sigma_{qq}^{\text{SM}}} \approx 1 + 2 \frac{g_V^e g_V^q}{e^2 Q_q} \quad \xrightarrow{\hspace{1cm}} \quad |g_V^e g_V^q| \lesssim 4.6 \cdot 10^{-4} |Q_q| \quad (\epsilon \lesssim 3.3 \cdot 10^{-3})$$

# $Z'$ constraints

I. Semi-leptonic processes

2. Leptonic processes

- for  $m_{Z'} \lesssim 0.3$  GeV ( $Z' \rightarrow e^+e^-$  is the main decay mode)

$$e^+e^- \rightarrow \gamma Z' @ BaBar \quad \longrightarrow \quad g_V^e \lesssim 2 \cdot 10^{-4}$$

- for  $m_{Z'} \gtrsim$  MeV

$$\text{electron g-2} \quad \longrightarrow \quad |g_V^e| \lesssim 10^{-2} (m_{Z'}/0.5 \text{ GeV})$$

# $Z'$ constraints

1. Semi-leptonic processes
2. Leptonic processes
3. Iso-spin breaking observables

charged vs. neutral pion mass<sup>2</sup> difference  $\Delta m^2 = m_{\pi^+}^2 - m_{\pi^0}^2$

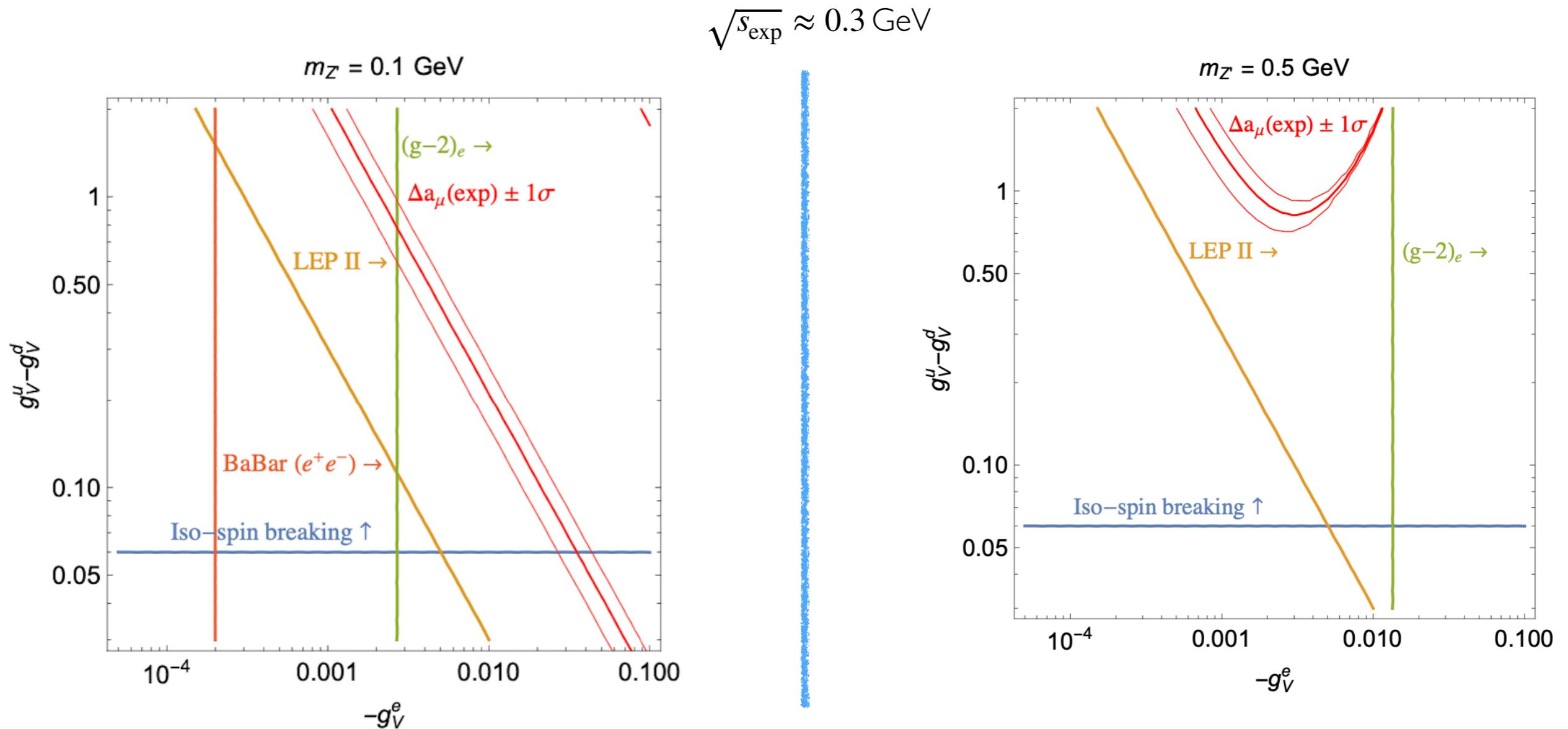
$$(\Delta m^2)_{Z'} \sim \frac{(g_V^u - g_V^d)^2}{(4\pi)^2} \Lambda_\chi^2 \quad (\Lambda_\chi \approx 1 \text{ GeV})$$



$$|g_V^u - g_V^d| \lesssim 0.06$$

[Rescaling lattice QCD calculation of Frezzotti et al 2112.01066]

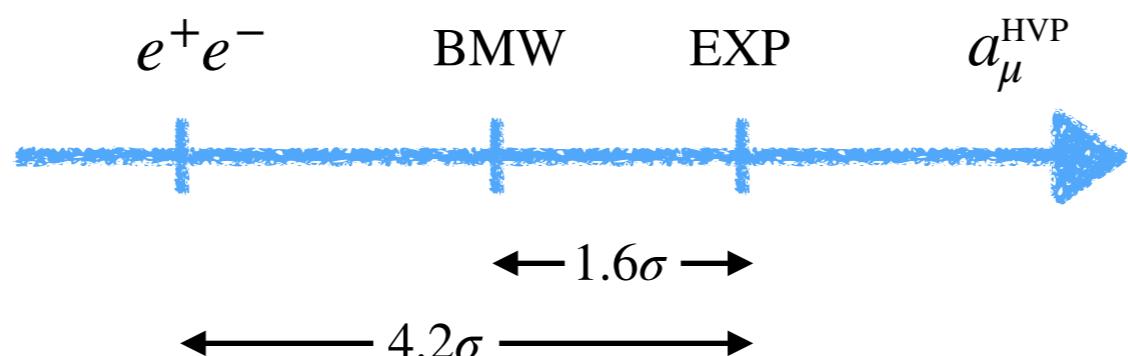
# $Z'$ constraints



at least two independent bounds preventing to solve the “new muon g-2 puzzle”

# Conclusions

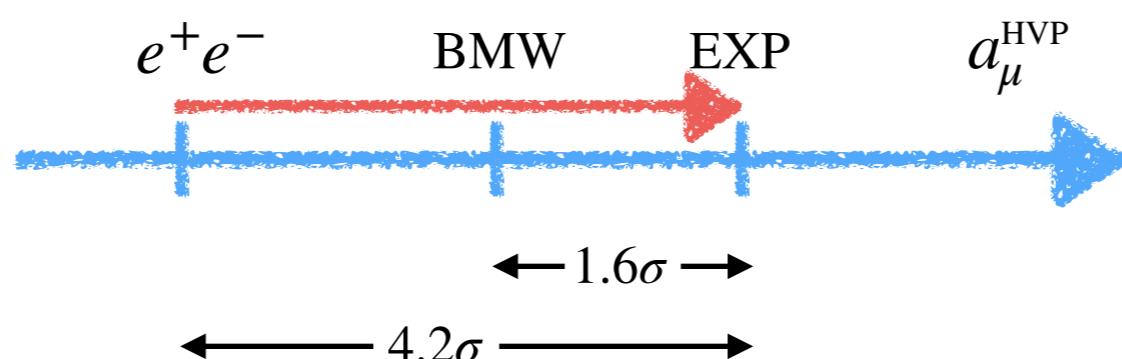
- Fermilab's Muon g-2 experiment confirms BNL's result
  - The BMWc lattice result weakens the exp-SM discrepancy, but brings in a tension with  $e^+e^-$  data
- “new muon g-2 puzzle”



# Conclusions

- Fermilab's Muon g-2 experiment confirms BNL's result
- The BMWc lattice result weakens the exp-SM discrepancy, but brings in a tension with  $e^+e^-$  data

→ “new muon g-2 puzzle”



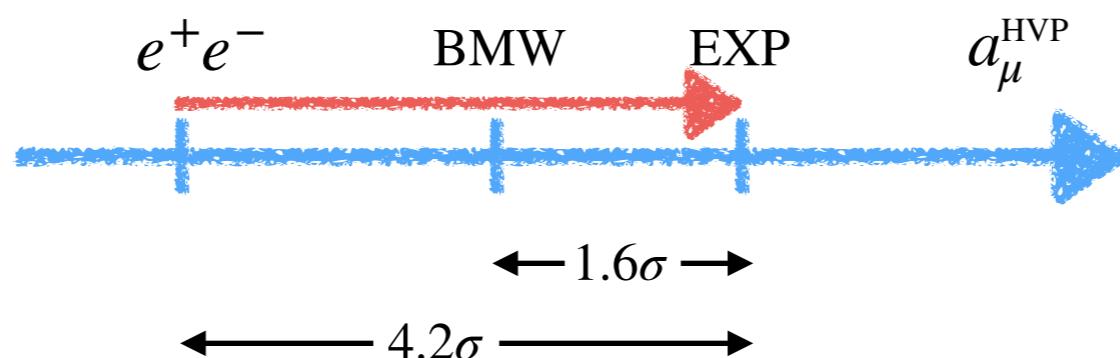
- Here, we considered the possibility this is due to NP (not in muons) that modifies  $\sigma_{\text{had}}$ 

→ excluded by a number of exp. constraints

# Conclusions

- Fermilab's Muon g-2 experiment confirms BNL's result
- The BMWc lattice result weakens the exp-SM discrepancy, but brings in a tension with  $e^+e^-$  data

→ “new muon g-2 puzzle”



- Here, we considered the possibility this is due to NP (not in muons) that modifies  $\sigma_{\text{had}}$

→ excluded by a number of exp. constraints

other ways in which NP can address this puzzle ? →

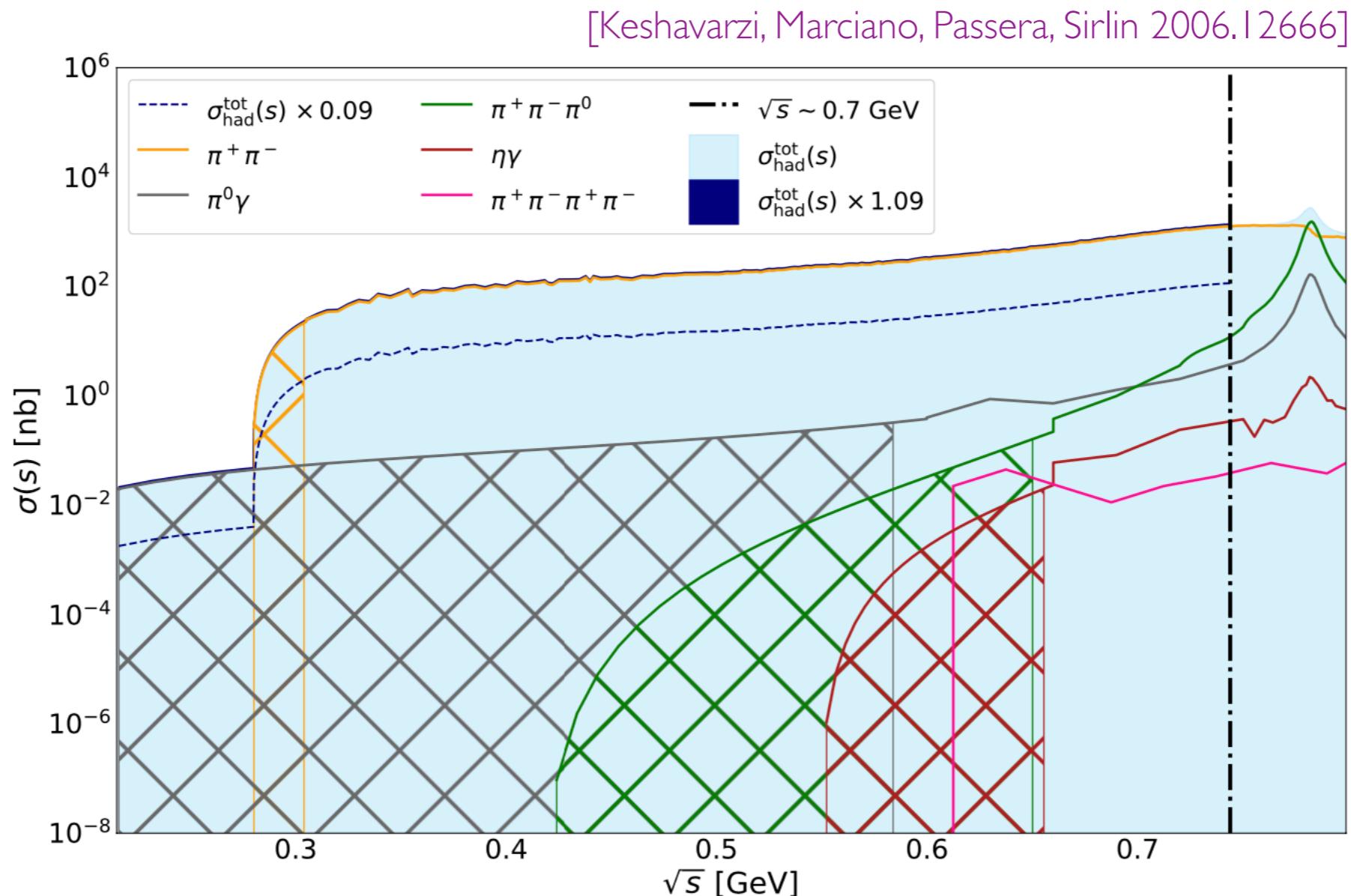
[Darmé, Grilli di Cortona, Nardi 2112.09139  
NP in Bhabha scattering? → backup slides]

- Alternative confirmations of HVP contribution will be crucial (lattice, MUonE, ...)

# Backup slides

*Thank you for your attention !*

# $\sigma_{\text{had}}$ : theory vs exp. region



# $\sigma_{\text{had}}$ data

[WP20, 2006.04822]

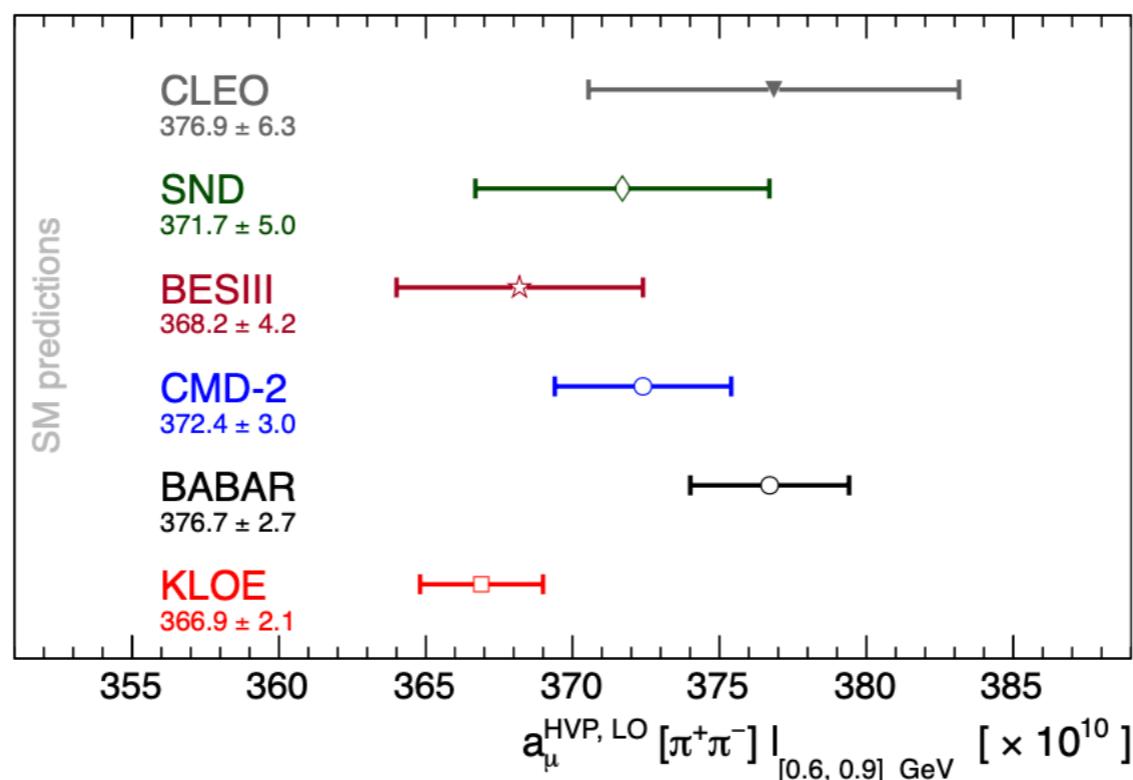


Figure 15: Comparison of results for  $a_\mu^{\text{HVP, LO}}[\pi\pi]$ , evaluated between 0.6 GeV and 0.9 GeV for the various experiments.

# $\sigma_{\text{had}}$ data

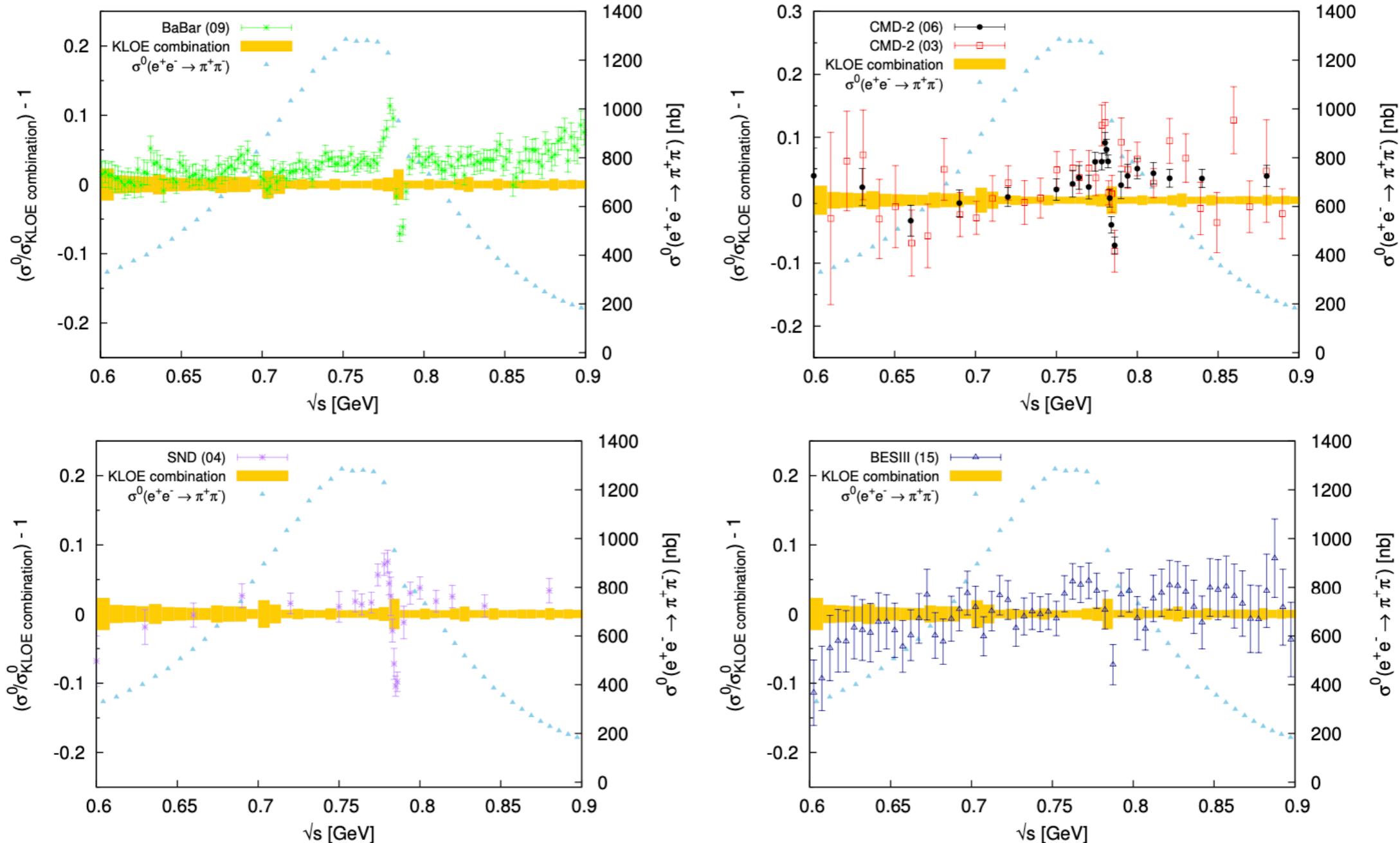


Figure 13: The  $\pi^+\pi^-$  cross section from the KLOE combination compared to the BABAR, CMD-2, SND, and BESIII data points in the 0.6–0.9 GeV range [82]. The KLOE combination is represented by the yellow band. The uncertainties shown are the diagonal statistical and systematic uncertainties summed in quadrature. Reprinted from Ref. [82].

# Pion matrix element

- Vector form factor (VFF) defined via

$$\langle \pi^\pm(p') | J_{\text{em}}^\mu(0) | \pi^\pm(p) \rangle = \pm(p' + p)^\mu F_\pi^V(q^2) \quad J_{\text{em}}^\mu = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d$$
$$q = p' - p$$

Using iso-spin and C invariance

$$\langle \pi^\pm | J_{\text{em}}^\mu | \pi^\pm \rangle = \langle \pi^\pm | \bar{u}\gamma^\mu u | \pi^\pm \rangle = -\langle \pi^\pm | \bar{d}\gamma^\mu d | \pi^\pm \rangle$$

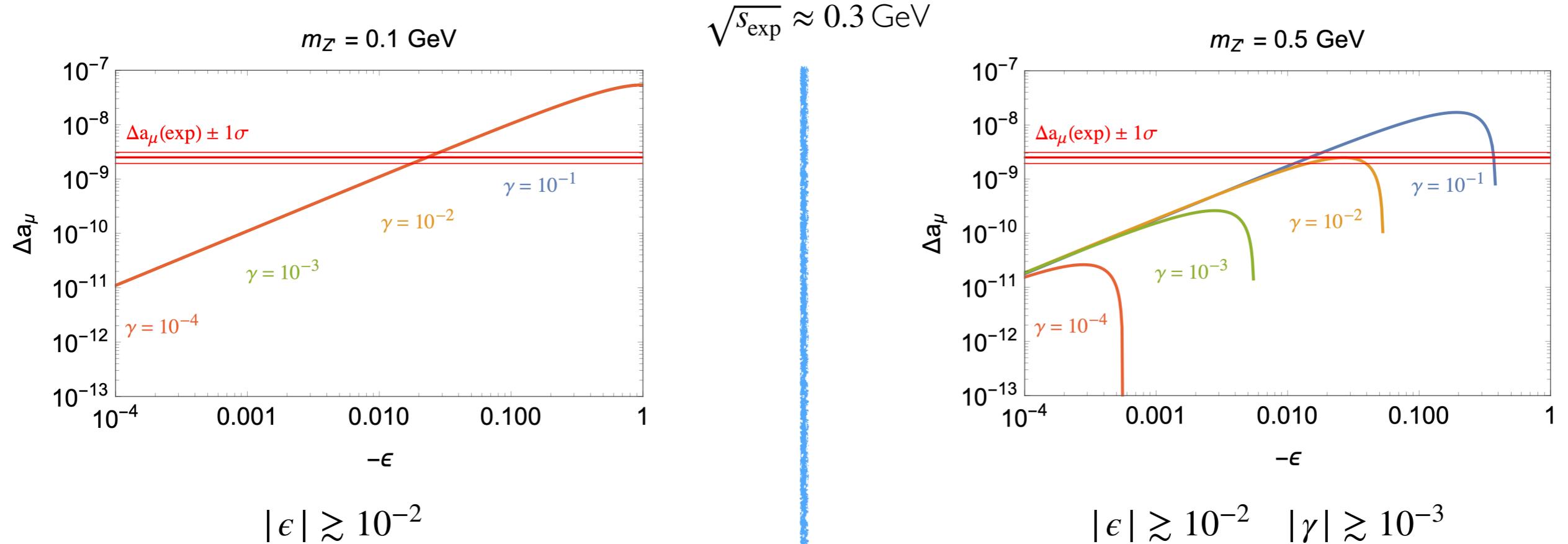
we can cast the matrix element of the  $Z'$  quark current in terms of the VFF

$$\langle \pi^\pm(p') | J_{Z'}^\mu(0) | \pi^\pm(p) \rangle = \pm(p' + p)^\mu F_\pi^V(q^2)(g_V^u - g_V^d) \quad J_{Z'}^\mu = g_V^u \bar{u}\gamma^\mu u + g_V^d \bar{d}\gamma^\mu d$$

$$\frac{\sigma_{\pi\pi}^{\text{SM+NP}}}{\sigma_{\pi\pi}^{\text{SM}}} = \left| 1 + \frac{g_V^e(g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + im_{Z'}\Gamma_{Z'}} \right|^2$$

# $Z'$ shift on $(a_\mu^{\text{HVP}})_{e^+e^-}$

- Typical benchmarks solving the g-2 discrepancy



$$\epsilon \equiv g_V^e (g_V^u - g_V^d) / e^2$$

$$\gamma \equiv \Gamma_{Z'} / m_{Z'}$$

$$\gamma_{ee} \approx \frac{(g_V^e)^2}{12\pi} = 2.7 \times 10^{-10} \left( \frac{g_V^e}{10^{-4}} \right)^2 \quad \gamma_{\pi\pi} = \frac{(g_V^u - g_V^d)^2}{48\pi} |F_\pi^V(m_{Z'}^2)|^2 \left( 1 - \frac{4m_\pi^2}{m_{Z'}^2} \right)^{3/2}$$

# NP in Bhabha scattering ?

- What if the measurement of the KLOE luminosity is affected by NP ?

[Darmé, Grilli di Cortona,  
Nardi 21 [2.09] [39]]

$$\mathcal{L}_{e^+e^-}^{\text{SM}} = \frac{N_{\text{Bha}}}{\sigma_{\text{eff}}^{\text{SM}}}$$



$$\mathcal{L}_{e^+e^-} = \mathcal{L}_{e^+e^-}^{\text{SM}} \frac{\sigma_{\text{eff}}^{\text{SM}}}{\sigma_{\text{eff}}}$$

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}^{\text{SM}} (1 + \delta_R)$$

$$\sigma_{\text{had}} \propto N_{\text{had}} / \mathcal{L}_{e^+e^-}$$



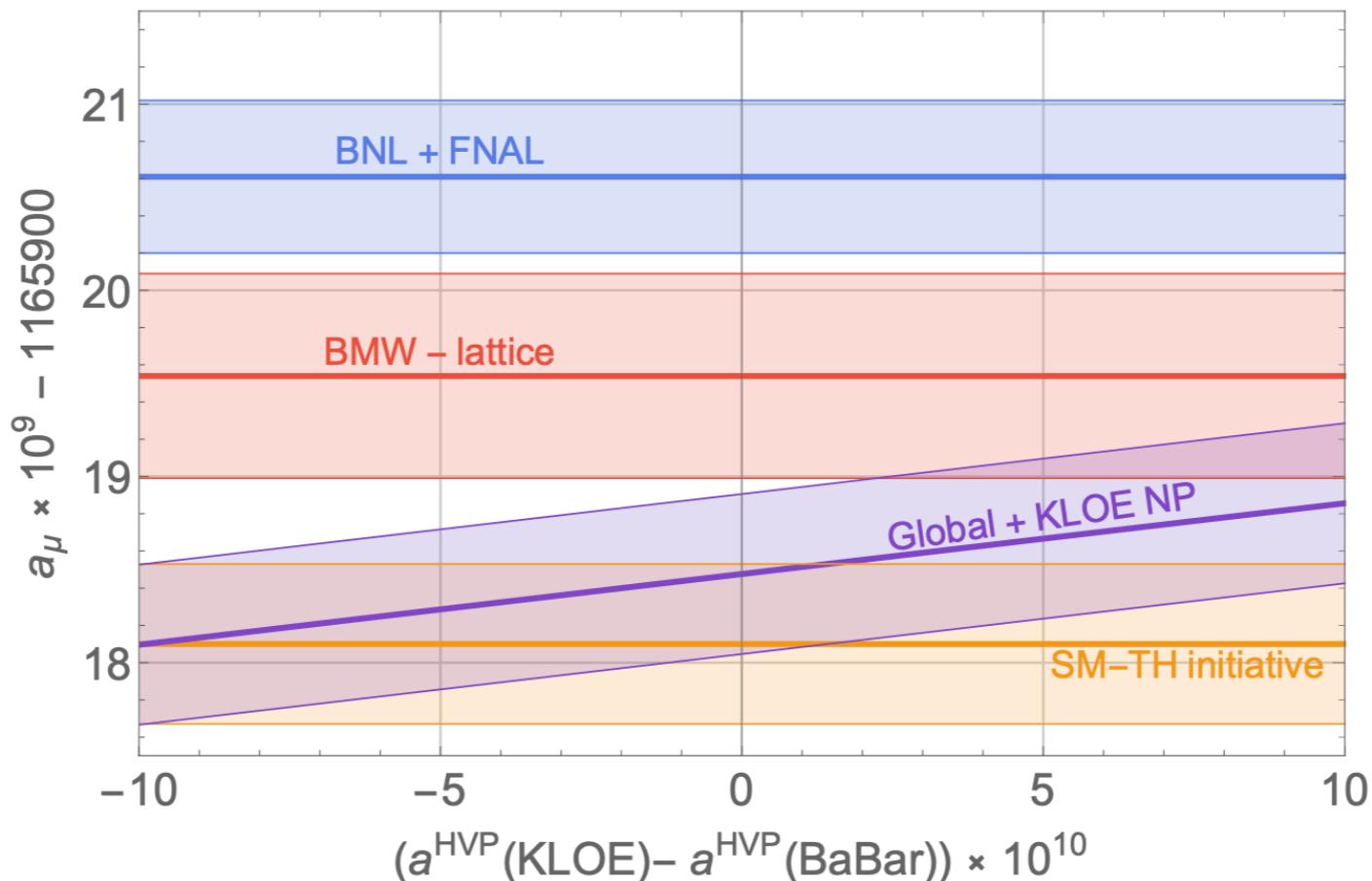
$$\sigma_{\text{had}} \rightarrow \sigma_{\text{had}} (1 + \delta_R)$$

$$a_\mu^{\text{LO,HVP}} \rightarrow a_\mu^{\text{LO,HVP}} (1 + \delta_R)$$

# NP in Bhabha scattering ?

- What if the measurement of the KLOE luminosity is affected by NP ?

[Darmé, Grilli di Cortona,  
Nardi 21 [2.09] 39]

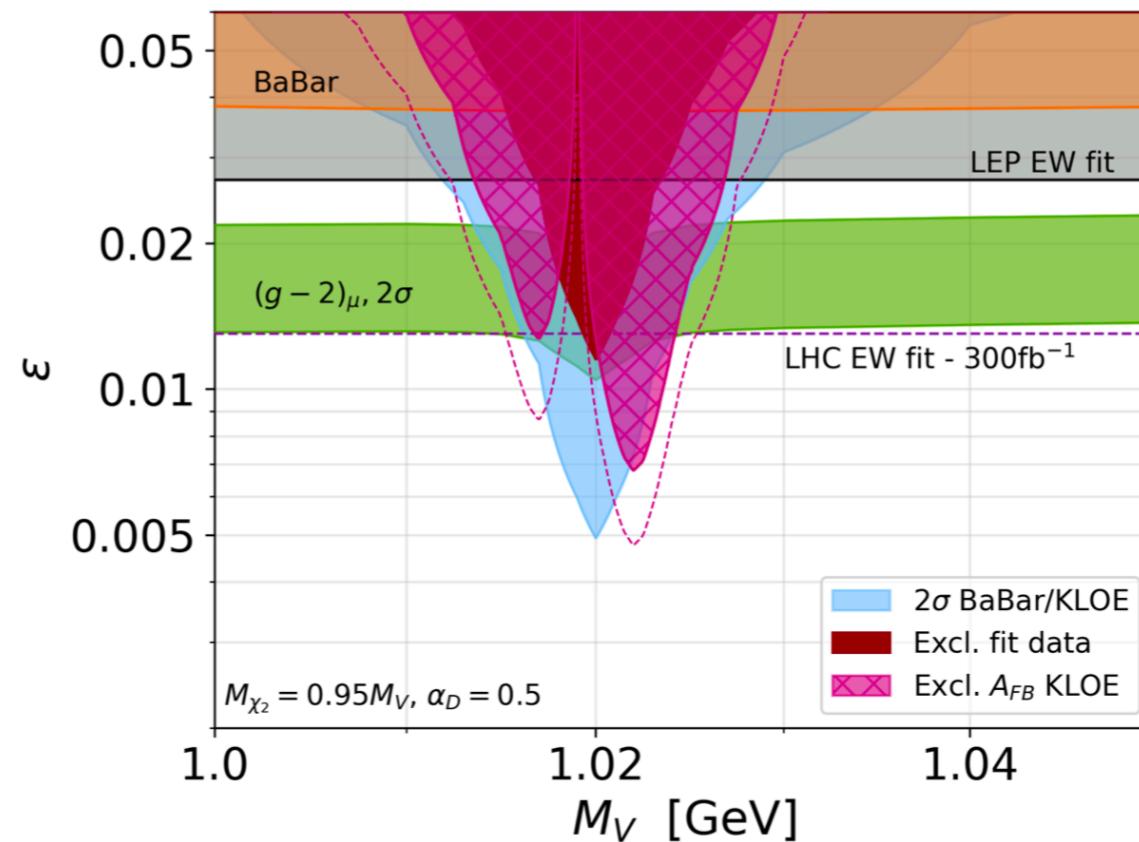


**Figure 1.** Theoretical prediction for  $a_\mu$  obtained by modifying the KLOE result in the data driven global fit to  $a_\mu^{\text{LO,HVP}}$  (oblique violet band). The blue band corresponds to the combined BNL and FNAL experimental results, the red band to the prediction obtained with the BMW lattice estimate of  $a_\mu^{\text{LO,HVP}}$ , and the orange band to the one obtained from  $\sigma_{\text{had}}$  without modifications of the KLOE results. The width of the bands represents  $1\sigma$  uncertainties.

# NP in Bhabha scattering ?

- What if the measurement of the KLOE luminosity is affected by NP ?

[Darmé, Grilli di Cortona,  
Nardi 21 | 2.09 | 39]

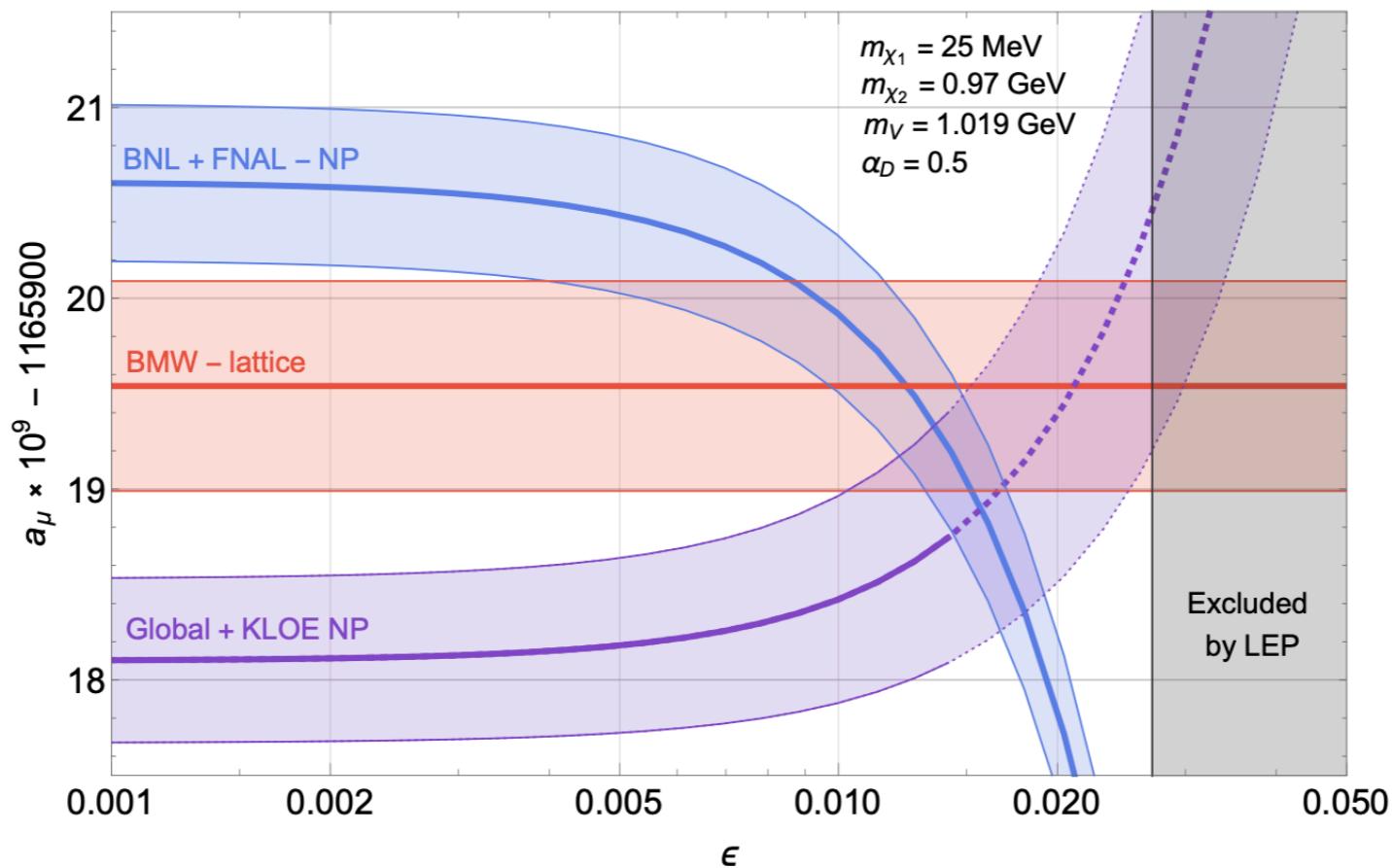


**Figure 3.** Parameter range compatible at  $2\sigma$  with the experimental measurement of  $\Delta a_\mu$  (green region) resulting from a redetermination of the KLOE luminosity, for  $\alpha_D = 0.5$ ,  $m_{\chi_2} = 0.95m_V$  and  $m_{\chi_1} = 25$  MeV. In the blue region the KLOE and BaBar results for  $\sigma_{\text{had}}$  are brought into agreement at  $2\sigma$ . The red region corresponds to a shift of the KLOE measurement in tension with BaBar (and with the other experiments) at more than  $2\sigma$ . The limit from the electroweak fit at LEP (gray band), the projection for LHC run-3 [73] (violet dashed line), and the recasting of the BaBar limit [51] (orange band) are also shown (see text). The hatched magenta region corresponds to the conservative  $2\sigma$  exclusion from  $\Delta A_{FB}$ , while the magenta dashed line corresponds to the more aggressive exclusion limit

# NP in Bhabha scattering ?

- What if the measurement of the KLOE luminosity is affected by NP ?

[Darmé, Grilli di Cortona, Nardi 21 [2.09] [39]]



**Figure 4.** Theoretical prediction (purple) for  $a_\mu$  as a function of  $\epsilon$  for a dark photon model with  $m_{\chi_1} = 25$  MeV,  $m_{\chi_2} = 0.9$  GeV,  $m_V = 1.019$  GeV and  $\alpha_D = 0.5$ . The dashed purple curve denotes the region where the KLOE and BaBar results are more than  $2\sigma$  away. The blue band corresponds to the combined BNL and FNAL experimental results after subtracting the direct NP contribution from the dark photon. The red band shows the prediction obtained with the BMW lattice estimate of  $a_\mu^{\text{LO,HVP}}$ . The width of the bands represents  $1\sigma$  uncertainties. The grey region is excluded by LEP.