MUonE: Theory Status







- Manoj Kumar Mandal
 - INFN Padova

Flavour Changing and Conservation Process 23rd September, 2022

G.A. 754496



 $a_{\mu}^{EXP} = (116592061 \pm 41) \times 10^{-11} [0.35ppm] \text{ WA}$



Motivation

 $a_{\mu}^{EXP} = (116592089 \pm 63) \times 10^{-11} [0.54ppm]$ BNL E821 $a_{\mu}^{EXP} = (116592040 \pm 54) \times 10^{-11} [0.46ppm]$ FNAL E989 Run 1

Anomalous Magnetic Moment of Muon



$$a_{\mu}^{\rm SM} = a_{\mu}^{\rm QED} + a_{\mu}^{\rm EW} + a_{\mu}^{\rm had}$$

$$a_{\mu}^{\rm SM} = 116591810(43) \times 10^{-11}$$

$$a_{\mu}^{\rm had} = a_{\mu}^{\rm HVP} + a_{\mu}^{\rm HLbL}$$

Motivation

Leading hadronic contribution computed via the usual dispersive (timelike) formula:



$$a_{\mu}^{\text{HLO}} = \frac{1}{4\pi^3} \int_{m_{\pi}^2}^{\infty} ds \, K(s) \, \sigma_{\text{had}}^{(0)}(s)$$
$$K(s) = \int_0^1 dx \, \frac{x^2 \, (1-x)}{x^2 + (1-x) \, \left(s/m_{\mu}^2\right)}$$

• Alternatively, simply exchanging the x and s integrations:



Lautrup, Peterman, de Rafael, 1972

$\Delta \alpha_{had}(t)$ is the hadronic contribution to the space-like running of a: proposal to measure a_{μ}^{HLO} via scattering data!

M Passera Edinburgh Sep 2022

Carloni Calame, MP, Trentadue, Venanzoni, 2015

$$(-x) \Delta \alpha_{\text{had}}[t(x)]$$



Smooth integrand Inclusive measurement Direct interplay with lattice QCD



MUonE Experiment

$$K(s) = \int_{0}^{1} dx \frac{x^{2}}{x^{2} + (1 - x^{2})^{2}}$$



 $\Delta \alpha_{had}(t)$ Can be measured via the elastic scattering

Muon-Electron Scattering at 10 PPM

Eur. Phys. J. C (2020) 80:591 https://doi.org/10.1140/epjc/s10052-020-8138-9

THE EUROPEAN **PHYSICAL JOURNAL C**

Review

Theory for muon-electron scattering @ 10 ppm

A report of the MUonE theory initiative

P. Banerjee¹, C. M. Carloni Calame², M. Chiesa³, S. Di Vita⁴, T. Engel^{1,5}, M. Fael⁶, S. Laporta^{7,8}, P. Mastrolia^{7,8}, G. Montagna^{2,9}, O. Nicrosini², G. Ossola¹⁰, M. Passera⁸, F. Piccinini², A. Primo⁵, J. Ronca¹¹, A. Signer^{1,5,a}, W. J. Torres Bobadilla¹¹, L. Trentadue^{12,13}, Y. Ulrich^{1,5}, G. Venanzoni¹⁴





[Banerjee, Engel, Signer, Ulrich (2020)] [Banerjee, Engel, Schalch, Signer, Ulrich (2021)] [Budassi, Carloni Calame, Chiesa, Del Pio, Hasan, Montagna, Nicrosini, Piccinini (2021)] [Carloni Calame, Chiesa, Hasan, Montagna,

Nicrosini, Piccinini (2020)]

[Masiero, Paradisi, Passera (2020)] [Dev, Rodejohann, Xu, Zhang (2020)]

New Physics Effect









Muon-Electron Scattering at NLO



Full set of QED NLO corrections computed and checked

Fully differential fixed-order MC @ NLO ready

Muon Electron Scattering with Multiple Electromagnetic Radiation (MESMER) github.com/cm-cc/mesmer

EW NLO corrections known but not required at 10ppm level

[Alacevich, Carloni Calame, Chiesa, Montagna, Nicrosini, Piccinini (2019)]



$$\frac{\sigma_{\rm EW}}{\sigma_{\rm QED}} \sim \left(\frac{s}{M_Z^2}\right)^2 \sim 10^{-5}$$

Muon-Electron Scattering [NNLO]



Double Virtual

$$\int \left[rac{VV_4}{\epsilon^4} + rac{VV_3}{\epsilon^3} + rac{VV_2}{\epsilon^2} + rac{VV_1}{\epsilon^1} + VV_0
ight]d\phi_2 \qquad \qquad \int \left[rac{RV_2}{\epsilon^2} + rac{VV_2}{\epsilon^2} +$$





Real Virtual

Double Real

$$\left[rac{RV_1}{\epsilon^1} + RV_0
ight] d\phi_3$$

$$\int \left[RR_{0}
ight] d\phi_{4}$$

Double Virtual

Di-Muon Production

The two-loop four-fermion scattering amplitude in QED

R. Bonciani,^{1,*} A. Broggio,^{2,†} S. Di Vita,^{3,4} A. Ferroglia,^{5,6,‡} M. K. Mandal,^{7,8,§} P. Mastrolia,^{8,7,¶} L. Mattiazzi,^{7,8,∥} A. Primo,^{9,**} J. Ronca,^{10,††} U. Schubert,^{11,‡‡} W. J. Torres Bobadilla,^{12,§§} and F. Tramontano^{10,¶¶}



Crossing





1-Loop Diagrams

6 Diagrams

$$\mathcal{M}^{(1)} = A^{(1)} + n_l \, B_l^{(1)} + n_h \, C_h^{(1)}$$



[Bonciani, Broggio, Di Vita, Ferroglia, MKM, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Torres Bobadilla, Tramontano (2021)]



2-Loop Diagrams

69 Diagrams

0 Due to Furry's Theorem

$$\mathcal{M}^{(2)} = A^{(2)} + n_l B_l^{(2)} + n_h C_h^{(2)} + n_l^2 D_l^{(2)}$$
$$+ n_h n_l E_{hl}^{(2)} + n_h^2 F_h^{(2)}$$



Computation of the Loop Amplitude

Mathematica Based Package AIDA



[Mastrolia, Peraro, Primo, Ronca, Torres Bobadilla (To be Published)]

$$\mathcal{M}_{\mathrm{b}}^{(n)} = (S_{\epsilon})^n \int \prod_{i=1}^n \frac{d^d k_i}{(2\pi)^d} \sum_G \frac{N_G}{\prod_{\sigma \in G} D_{\sigma}}$$

$$\mathcal{M}_{\mathrm{b}}^{(n)} = \mathbb{C}^{(n)} \cdot \mathbf{I}^{(n)}$$

Master Integrals

Master Integrals

The differential equation method has been the most successful in the computation of the MIs



[Kotikov (1990)] [Gehrmann, Remiddi (1999)]

[Henn (2013)] [Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Schubert, Tancredi (2014)]

[Bonciani, Ferroglia, Gehrmann, von Manteuffel (2008-13)]

[Mastrolia, Passera, Primo, Schubert (2017)]

[Di Vita, Laporta, Mastrolia, Primo, Schubert (2018)]

[MKM, Mastrolia, Ronca, Torres Bobadilla (2022)]

Generalized Polylogarithms

$$G(w_n, \dots, w_1; \tau) \equiv \int_0^\tau \frac{dt}{t - w_n} G(w_{n-1}, \dots, w_1; t)$$
$$G(w_1; t) \equiv \log(1 - t/w_1)$$



UV Renormalization

Renormalized Amplitude

$$\mathcal{A}(\alpha) = 4\pi\alpha \left[\mathcal{A}^{(0)} + \left(\frac{\alpha}{\pi}\right) \mathcal{A}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \mathcal{A}^{(2)} \right]$$

$$\delta Z_f^{(2)} = n_h \left(\frac{L_\mu}{8} + \frac{1}{16\epsilon} - \frac{5}{96} \right)$$

[Czakon, Mitov, Moch (2007)]

$$\begin{aligned} \mathcal{A}^{(0)} &= \mathcal{A}_{\rm b}^{(0)} \\ \mathcal{A}^{(1)} &= \mathcal{A}_{\rm b}^{(1)} + \left(\delta Z_{\alpha}^{(1)} + \delta Z_{F}^{(1)}\right) \mathcal{A}_{\rm b}^{(0)} \\ \mathcal{A}^{(2)} &= \mathcal{A}_{\rm b}^{(2)} + \left(2\delta Z_{\alpha}^{(1)} + \delta Z_{F}^{(1)}\right) \mathcal{A}_{\rm b}^{(1)} \\ &+ \left(\delta Z_{\alpha}^{(2)} + \delta Z_{F}^{(2)} + \delta Z_{f}^{(2)} + \delta Z_{F}^{(1)} \delta Z_{\alpha}^{(1)}\right) \mathcal{A}_{\rm b}^{(0)} \\ &+ \delta Z_{M}^{(1)} \mathcal{A}_{\rm b}^{(1,\text{mass CT})} \end{aligned}$$



IR Factorization

$$\mathcal{M}^{(1)}\Big|_{\text{poles}} = \frac{1}{2} Z_1^{\text{IR}} \mathcal{M}^{(0)}\Big|_{\text{poles}}$$
$$\mathcal{M}^{(2)}\Big|_{\text{poles}} = \frac{1}{8} \left[\left(Z_2^{\text{IR}} - \left(Z_1^{\text{IR}} \right)^2 \right) \mathcal{M}^{(0)} + 2 Z_1^{\text{IR}} \mathcal{M}^{(1)} \right] \Big|_{\text{poles}}$$

IR Renormalization Factor

$$\ln Z_{\rm IR} = \frac{\alpha}{4\pi} \left(\frac{\Gamma_0'}{4\epsilon^2} + \frac{\Gamma_0}{2\epsilon} \right) + \left(\frac{\alpha}{4\pi} \right)^2 \left(-\frac{3\beta_0 \Gamma_0'}{16\epsilon^3} + \frac{\Gamma_1'}{16\epsilon^3} \right)^2$$

$$\Gamma = \gamma_{\text{cusp}}(\alpha) \ln\left(-\frac{s}{\mu^2}\right) + 2\gamma_{\text{cusp}}(\alpha) \ln\left(\frac{t}{u}\right) + \frac{1}{2} \sum_{\alpha} \frac{1}{2} \left(\frac{s}{u}\right) + \frac{1}{2} \sum_{\alpha} \frac{1}{2} \left(\frac{s}{u}\right) + \frac{1}{2} \sum_{\alpha} \frac{1}{2} \sum_{\alpha}$$

Cusp Anomalous dimension



Anomalous dimension



Numerical Result of the 2-Loop Amplitude

[Bonciani, Broggio, Di Vita, Ferroglia, MKM, Mastrolia, Mattiazzi, Primo, Ronca, Schubert, Torres Bobadilla, Tramontano (2021)]

◆ At 2 Loop there are 4063 GPLs up to weight 4 $G(w_n,\ldots,$

• 18 Letters $w_i = w_i(x, y, z)$

$$-t/M^2 = x$$

 $-s/M^2 = (1 - y)^2$
 $-(u - M^2)/(t - M^2)$

The GPLs are evaluated by Ginac [PolyLogTools interface] and HandyG

We have obtained complete agreement between the predicted IR poles and the 2-Loop UV renormalized amplitude

 \mathbf{V} We recovered the Abelian part of the QCD result of $q \bar{q}
ightarrow t \bar{t}$ at 1- and 2-Loop

 $\mathbf{M}^{n}h$ contributions were checked independently [Fael, Passera (2019)] [Fael (2018)]

$$(w_1; \tau) \equiv \int_0^\tau \frac{dt}{t - w_n} G(w_{n-1}, \dots, w_1; t)$$



[Vollinga, Weinzierl (2004)] [Duhr, Dulat (2019)] [Naterop, Signer, Ulrich (2019)]

[Czakon (2008)]

[Bonciani, Ferroglia, Gehrmann, Maitre, Studerus (2008)] [Bärnreuther, Czakon, Fiedler (2014)] [MKM, Mastrolia, Ronca, Torres Bobadilla (2022)]



Towards Complete NNLO

Towards Complete NNLO cross-section





Towards Complete NNLO cross-section [VV]

[Broggio, Engel, Ferroglia, MKM, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Ulrich, Zoller (In Progress)]



- The complete two loop virtual amplitude is available with massless electron.
- Leading electron mass effects are restored through massification
- The two loop virtual correction can be divided as
 - Diagrams with closed fermionic loop [VP]
 - Diagrams with no fermionic loops
 - Correction on the electron line [electronic]
 - Correction on the muon line [muonic]
 - Mixed corrections [mixed]

[Mitov, Moch (2006)] [Penin (2006)] [Becher, Melnikov (2007)] [Engel, Gnendiger, Signer, Ulrich (2018)]

Massification



VP contribution computed exactly with electron mass

Known using massive vertex formfactor

Known using massive vertex formfactor

Employed the idea of Massification





Towards Complete NNLO cross-section [VV]

[Broggio, Engel, Ferroglia, MKM, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Ulrich, Zoller (In Progress)]

[Mitov, Moch (2006)] [Penin (2006)] [Becher, Melnikov (2007)] [Engel, Gnendiger, Signer, Ulrich (2018)]

Massification



Known using massive vertex formfactor

Known using massive vertex formfactor

Employed the idea of Massification







[Fael, Passera (2019)] **VP** contribution has been checked independently [Fael (2018)]

Vertex corrections has been checked independently

Carloni Calame, Chiesa, Hasan, Montagna, Nicrosini, Piccinini (2020)



Towards Complete NNLO cross-section [RV+RR]



Computed via OpenLoops assisted with next-to-soft stabilization

[Banerjee, Engel, Schalch, Signer, Ulrich (2021)] One Loop generalization of the LBK theorem helped to obtain the soft expansion to NLP [Engel, Signer, Ulrich (2021)]

MAll components have been implemented within the McMule Framework for their efficient and stable evaluation Banerjee, Coutinho, Engel, Gurgone, Hagelstein, Kollatzsch, Naterop, Rocco, Schalch, Sharkovska, Signer, Ulrich

M Dedicated comparison with the MESMER from Pavia group for the Real-virtual and double Real contribution Budassi, Carloni Calame, Chiesa, Del Pio, Hasan, Montagna, Nicrosini, Piccinini (2021)



Buccioni, Pozzorini, Zoller





Towards Complete NNLO cross-section

[Broggio, Engel, Ferroglia, MKM, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Ulrich, Zoller (In Progress)]



On-Shell Renormalization scheme







Total Cross-section



The results are split into purely electronic, muonic, mixed, and VP corrections

+All three leptons as well as the hadronic contribution are included in the VP

[Broggio, Engel, Ferroglia, MKM, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Ulrich, Zoller (In Progress)]

$\sigma/\mu b$	$\delta K^{(i)}\%$	$\delta K^{(i)}\%$
S2	S 1	S2
106.44356		
-4.66041	-0.57505	-4.37830
-0.16016	-0.20108	-0.15047
-0.16133	-0.02670	-0.15156
0.01597	0.01500	0.01500
1.57105	1.47594	1.47594
103.03269		
0.06594	0.00083	0.06400
0.01925	0.00088	0.01869
0.00001	-0.00004	0.00001
-0.07376	-0.01300	-0.0715
103.10397		

$$K^{(i)} = 1 + \delta K^{(i)} = -\frac{1}{\sigma}$$

$$\sigma_2 = \sigma^{(0)} + \sigma^{(1)} + \sigma_1 = \sigma^{(0)} + \sigma^{(1)}$$







Differential distribution for θ_e



[Broggio, Engel, Ferroglia, MKM, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Ulrich, Zoller (In Progress)]

S2





Differential distribution for t_{μ}

[Broggio, Engel, Ferroglia, MKM, Mastrolia, Passera, Rocco, Ronca, Signer, Torres Bobadilla, Ulrich, Zoller (In Progress)]





MUonE : Dominant Three-Loop Corrections



Massive vector form factors to three loops computed very recently [Fael, Lange, Schönwald, Steinhauser (2022)]

Subtraction scheme is ready: FKS3 [Engel, Signer, Ulrich (2019)]

M Dedicated N3LO kick-off workshop, Aug 2022 at IPPP, Durham to investigate the feasibility

[Buccioni, Pozzorini, Zoller (2017)]

Openloop can be used

Next-To-Soft Stabilization will be useful

[Engel, Signer, Ulrich (2021)] [Banerjee, Engel, Schalch, Signer, Ulrich (2021)]

Y. Ulrich, https://conference.ippp.dur.ac.uk/event/1104



$a_{\mu}^{HVP}(110) = -98.3 (7) \times 10^{11}$



$$a_{\mu}^{\text{HVP}}(\text{LO}) = -\frac{\alpha}{\pi^2} \int_{-\infty}^{0} \int_{-\infty}^{0} \frac{dt}{dt} \prod_{h \in \mathcal{X}} \prod_{h \in \mathcal{X}} \frac{dt}{dt} \prod_{h \in \mathcal{$$

$$a_{\mu}^{(4a)} = \left(\frac{\alpha}{\pi}\right)^{2} \int_{0}^{1} dx \,\kappa^{(4)}(x) \,\Delta\alpha_{h}(t(x)), \qquad \kappa^{(4)}(x)_{2} = \frac{1}{2} \int_{0}^{1} dx \,\kappa^{(4)}(x) \,\Delta\alpha_{h}(t(x)), \qquad \kappa^{(4)}(x) = \frac{1}{2} \int_{0}^{1} dx \,\kappa^{(4)}(x) \,\Delta\alpha_{h}(t(x)), \qquad \kappa^{(4)}(x) = \frac{1}{2} \int_{0}^{1} \frac{1}{2} \int_{0}^{1} dx \,\kappa^{(4)}(x) \,\Delta\alpha_{h}(t(x)), \qquad \kappa^{(4)}(x) = \frac{1}{2} \int_{0}^{1} \frac$$

[Nesterenko (2021)] [Balzani, Laporta, Passera (2021)]





HVP NNLO contribution to Muon g-2 with MUonE



M The NNLO space-like kernels are available for MUonE

The NLO & NNLO space-like kernels can also be used in lattice QCD computation

[Balzani, Laporta, Passera (2021)]

[Kurz, Liu, Marquard, Steinhauser (2014)]

$$a_{\mu}^{\mathrm{HVP}}(\mathrm{NNLO}) = 12.4(1) \times 10^{10}$$

Timelike Region



MUonE Theory Papers

C.M. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni, PLB 746 (2015) 325 [1504.02228 [hep-ph]] P. Mastrolia, M. Passera, A. Primo, U. Schubert, JHEP 1711 (2017) 198 [1709.07435 [hep-ph]] S. Di Vita, S. Laporta, P. Mastrolia, A. Primo, U. Schubert, JHEP 1809 (2018) 016 [1806.08241 [hep-ph]] M. Alacevich et al, JHEP 02 (2019) 155 [1811.06743 [hep-ph]] M. Fael, JHEP 1902 (2019) 027 [1808.08233 [hep-ph]] M. Fael and M. Passera, PRL 122 (2019) 192001 [1901.03106 [hep-ph]] P. Banerjee et al. [The MUonE TI], EPJC 80 (2020) 591 [2004.13663 [hep-ph]] C. M. Carloni Calame et al, JHEP 11 (2020) 028 [2007.01586 [hep-ph]] P. Banerjee, T. Engel, A. Signer, Y. Ulrich, SciPost Phys. 9 (2020) 027 [2007.01654 [hep-ph]] R. Bonciani et al, PRL 128 (2022) 022002 [2106.13179 [hep-ph]] E. Budassi et al, JHEP 11 (2021) 098 [2109.14606 [hep-ph]] A.V. Nesterenko, JPG 49 (2022) 055001 [2112.05009 [hep-ph]] E. Balzani, S. Laporta, M. Passera, arXiv:2112.05704 [hep-ph] M. Fael, F. Lange, K. Schönwald, M. Steinhauser, PRL128 (2022) 172003 [2202.05276 [hep-ph]] M. Fael, F. Lange, K. Schönwald, M. Steinhauser, PRD 106 (2022) 034029 [2207.00027 [hep-ph]] D. Greynat and E. de Rafael, JHEP 05 (2022) 084 [2202.10810 [hep-ph]] E. Budassi, C.M. Carloni Calame, C.L. Del Pio, F. Piccinini, PLB 829 (2022) 137138 [2203.01639 [hep-ph]]

NP:

P.S.B. Dev, W. Rodejohann, X. J. Xu, Y. Zhang, JHEP05 (2020), 053 [2002.04822 [hep-ph]] A. Masiero, P. Paradisi, M. Passera, PRD102 (2020) 7, 075013 [2002.05418 [hep-ph]] U. Schubert, C. Williams, PRD100 (2019) 3, 035030 K. Asai, K. Hamaguchi, N. Nagata, S.Y. Tseng, J. Wada, arXiv:2109.10093 [hep-ph] I. Galon, D. Shih, I.R. Wang, arXiv:2202.08843 [hep-ph] G. Grilli di Cortona, E. Nardi, PRD 105 (2022) 11, L111701 [2204.04227 [hep-ph]]

Conclusion

Independent determination of the HVP contribution to the anomalous magnetic moment of Muon compared to dispersive and lattice results

Fully differential fixed-order MC @ NLO ready

W NLO corrections known but not required at 10ppm level

Fully analytic evaluation of the two-loop QED amplitude completed

Section A section of the two-loop QED amplitude has been done and combined with Real-Virtual and Real-Real piece

Fixe independent MC (MESMER, McMule) at NNLO in progress

First step towards investigating dominant N3LO corrections

New NLO and NNLO space-like kernels for HVP are ready

Possible new physics searches, etc

Thank You

Back up

Scattering Process

 $e^{-}(p_1)\mu^{\pm}(p_2) \to e^{-}(p_3)\mu^{\pm}(p_4) +$

✦ Elastic events are required

$$\theta_{\mu}^{\rm el^*} = \pi - \theta_e^{\rm el^*}$$

$$\tan \theta_{\mu}^{\text{el}} = \frac{2 \tan \theta_{e}^{\text{el}}}{(1 + \gamma^{2} \tan^{2} \theta_{e}^{\text{el}})(1 + g_{\mu}^{*}) - 2}$$
$$g_{\mu}^{*} = \frac{E_{2}m + M^{2}}{E_{2}m + m^{2}} \qquad \gamma = \frac{E_{2} + m}{\sqrt{s}}$$

$$X(p_5)$$

$$s = m^2 + M^2 + 2mE_2$$



Budassi, Carloni Calame, Chiesa, Del Pio, Hasan, Montagna, Nicrosini, Piccinini (2021)

Budassi, Carloni Calame, Del Pio, Piccinini (2022)

LAB Frame

