Dispersive determination of the HVP contribution to the muon $g-2$

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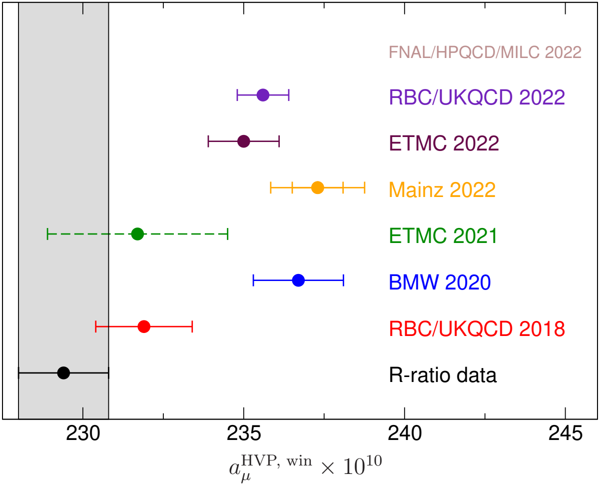
What the hell is going on with HVP?



In this talk: no new answers, but old ones to frequently asked questions, and some more perspectives

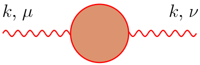
More details: TI workshop at Higgs Centre

For more details of recent developments, see website of the Fifth Plenary Workshop of the **Muon $g-2$ Theory Initiative** at the Higgs Centre in Edinburgh

$\hookrightarrow$ https://indico.ph.ed.ac.uk/event/112/

Hadronic vacuum polarization: a reminder

- General principles yield **direct connection with experiment**
  - **Gauge invariance**



$$= -i\big(k^2 g^{\mu\nu} - k^\mu k^\nu\big)\Pi(k^2)$$

  - **Analyticity**

$$\Pi_{\text{ren}} = \Pi(k^2) - \Pi(0) = \frac{k^2}{\pi}\int\limits_{4M_\pi^2}^{\infty} ds \frac{\operatorname{Im}\Pi(s)}{s(s-k^2)}$$

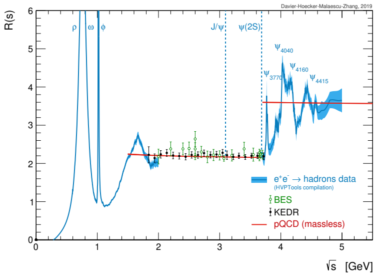
  - **Unitarity**

$$\operatorname{Im}\Pi(s) = -\frac{s}{4\pi\alpha}\sigma_{\text{tot}}\big(e^+e^- \to \text{hadrons}\big) = -\frac{\alpha}{3}R_{\text{had}}(s)$$

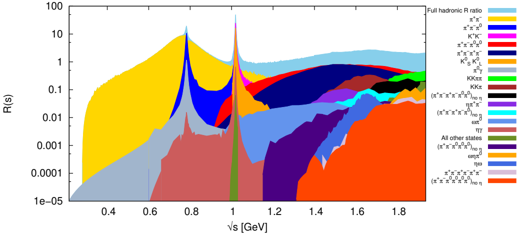
Master formula for HVP contribution to a_μ

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s)$$

Hadronic vacuum polarization from e^+e^- data



Davier, Hoecker, Malaescu, Zhang 2019



Keshavarzi, Nomura, Teubner 2018

- Decades-long effort to measure e^+e^- cross sections
  - cross sections defined photon-inclusively

    $\hookrightarrow$ threshold $s_{\text{thr}} = M_{\pi^0}^2$ due to $\pi^0\gamma$ channel
  - up to about 2 GeV: sum of exclusive channels
  - above: inclusive data + narrow resonances + pQCD
- **Tensions in the data**: most notably between KLOE and BaBar 2π data

  $\hookrightarrow$ extensive discussion in WP of current status and consequences

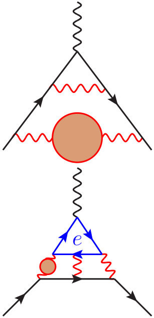
Data-driven determination of HVP: our recommendation

HVP from e^+e^- data

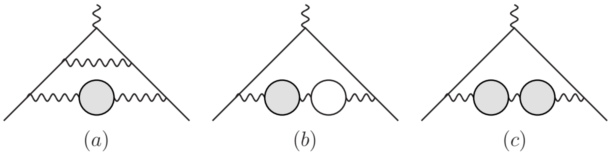
$$a_\mu^{\text{HVP, LO}} = 6931(28)_{\text{exp}}(28)_{\text{sys}}(7)_{\text{DV+QCD}} \times 10^{-11} = 6931(40) \times 10^{-11}$$

$$a_\mu^{\text{HVP}} = 6845(40) \times 10^{-11}$$

- DV+QCD: comparison of inclusive data and pQCD in transition region
- Sensitivity of the data is better than the quoted error
  $\hookrightarrow$ would get $4.2\sigma \to 4.8\sigma$ when ignoring additional systematics
- Systematic effect dominated by [fit w/o KLOE - fit w/o BaBar]/2
- a_μ^{HVP} includes NLO Calmet et al. 1976 and NNLO Kurz et al. 2014 iterations



FAQ 1: do e^+e^- data and lattice really measure the same thing?



- Conventions for **bare cross section**
  - Includes radiative intermediate states and final-state radiation: $\pi^0\gamma$, $\eta\gamma$, $\pi\pi\gamma$, ...
  - Initial-state radiation and VP subtracted to avoid double counting
- NLO HVP insertions

$$a_\mu^{\text{HVP, NLO}} \simeq [\underbrace{-20.7}_{(a)} + \underbrace{10.6}_{(b)} + \underbrace{0.3}_{(c)}] \times 10^{-10} = -9.8 \times 10^{-10}$$

$\hookrightarrow$ dominant VP effect from leptons, HVP iteration very small

- Important point: **no need to specify hadronic resonances**

  $\hookrightarrow$ calculation set up in terms of decay channels

FAQ 1: do e^+e^- data and lattice really measure the same thing?

- HVP in subtraction determined iteratively (converges with α) and self-consistently

$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha_{\text{lep}}(q^2) - \Delta\alpha_{\text{had}}(q^2)} \qquad \Delta\alpha_{\text{had}}(q^2) = -\frac{\alpha q^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(s-q^2)}$$

- Subtlety for very narrow $c\bar{c}$ and $b\bar{b}$ resonances (ω and ϕ perfectly fine)

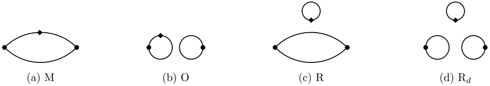
  $\hookrightarrow$ Dyson series does not converge Jegerlehner

- Solution: take out resonance that is being corrected in R_{had} in VP undressing
- How to match all of this on the lattice?
- Need to calculate all sorts of **isospin-breaking (IB) corrections**

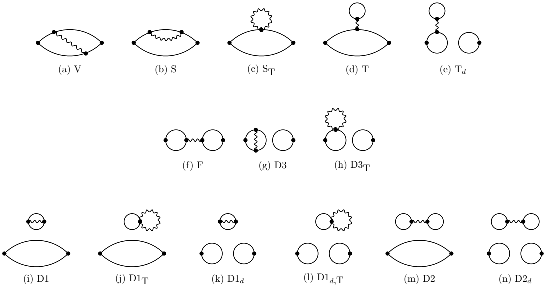
  $\hookrightarrow$ e^2 (QED) and $\delta = m_u - m_d$ (strong IB) corrections

FAQ 1: do e^+e^- data and lattice really measure the same thing?

- **Strong isospin breaking** $\propto m_u - m_d$



- **QED effects** $\propto \alpha$



plots from Gülpers et al. 2018

- Diagram (f) F critical for consistent VP subtraction

  $\hookrightarrow$ same diagram without additional gluons is subtracted RBC/UKQCD 2018

FAQ 1: do e^+e^- data and lattice really measure the same thing?

	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
ρ–ω mixing	–	0.05(0)	–	0.83(6)	–	2.79(11)	–	3.68(17)
FSR (2π)	0.11(0)	–	1.17(1)	–	3.14(3)	–	4.42(4)	–
M_{π^0} vs. $M_{\pi^\pm}$ (2π)	0.04(1)	–	−0.09(7)	–	−7.62(14)	–	−7.67(22)	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
kaon mass (K^+K^-)	−0.29(1)	0.44(2)	−1.71(9)	2.63(14)	−1.24(6)	1.91(10)	−3.24(17)	4.98(26)
kaon mass ($\bar{K}^0K^0$)	0.00(0)	−0.41(2)	−0.01(0)	−2.44(12)	−0.01(0)	−1.78(9)	−0.02(0)	−4.62(23)
total	0.14(1)	0.08(3)	1.61(12)	1.02(20)	−2.44(16)	2.92(17)	−0.68(29)	4.04(39)
BMWc 2020	–	–	−0.09(6)	0.52(4)	–	–	−1.5(6)	1.9(1.2)
RBC/UKQCD 2018	–	–	0.0(2)	0.1(3)	–	–	−1.0(6.6)	10.6(8.0)
JLM 2021	–	–	–	–	–	–	–	3.32(89)

- Note: error estimates only refer to the effects included
  ↪ **additional channels missing** (most relevant for SD and int window)
- Reasonable agreement with BMWc 2020, RBC/UKQCD 2018, and James, Lewis, Maltman 2021
  ↪ if anything, the result would become even larger with pheno estimates

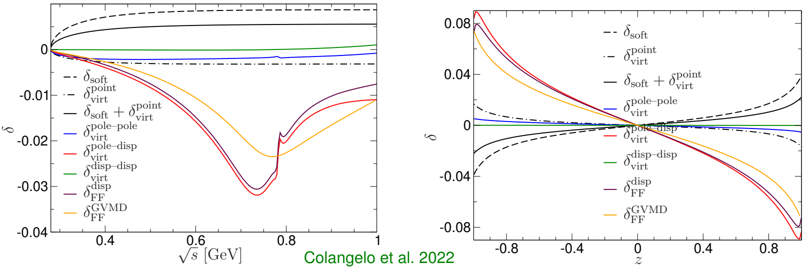
FAQ 2: can we trust radiative corrections/MC generators?

- Typical objection: can we really trust scalar QED in the MC generator?
- Report by Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

  $\hookrightarrow$ Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data (0912.0749)
- Never just use scalar QED, include pion form factor wherever possible

  $\hookrightarrow$ **FsQED** talk by G. Colangelo
- From the point of view of dispersion relations, this captures the **leading infrared enhanced effects**
- Existing NLO calculations do not point to (significant) center-of-mass-energy dependent effects Campanario et al. 2019
- Could there be subtleties in how the form factor is implemented or from pion rescattering?

FAQ 2: can we trust radiative corrections/MC generators?



Colangelo et al. 2022

- Test case: **forward–backward asymmetry** (*C*-odd)
- Large corrections found in GVMD model Ignatov, Lee 2022
- Can be reproduced using dispersion relations

  $\hookrightarrow$ effect still comes from **infrared enhanced contributions**
- Relevant effects for the *C*-even contribution? talk by G. Colangelo

FAQ 3: what about the τ data?

- Why did people stop using $\tau \to \pi\pi\nu_\tau$ data?
  - Better precision from e^+e^-
  - **IB corrections not under sufficient control**
- If this issue could be solved, would yield very useful cross check

  $\hookrightarrow$ new data at least on spectrum from Belle II
- New developments from the lattice talk by M. Bruno at Edinburgh

  $\hookrightarrow$ re-using HLbL lattice data
- Long-distance QED (G_{EM}) still taken from phenomenology for the time being

  $\hookrightarrow$ dispersive methods?

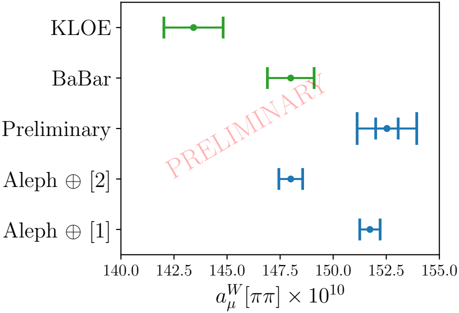
FAQ 3: what about the τ data?

talk by M. Bruno at Edinburgh

WINDOW FEVER - τ

my PRELIMINARY analysis of exp. + latt. data

- only exp. errs, no attempt at estimating sys. errs for [1] and [2]
- LQCD syst. errs require further investigation/improvements



Isospin-breaking:

[1]: w/o $\rho\gamma$ mixing

[2]: w/ $\rho\gamma$ mixing

What is $\rho\gamma$? too much to say, too little time to explain everything...

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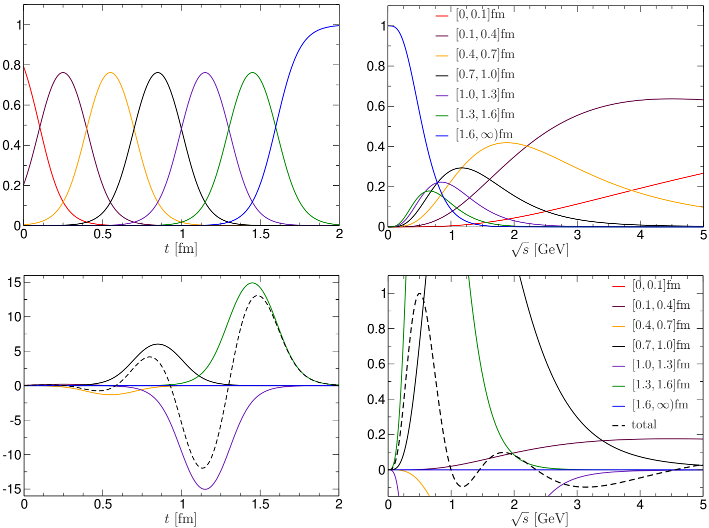
Where to go from here?

- For Run 2+3 result of E989 (spring 2023): lattice vs. e^+e^- will not be resolved
- Aim for WP update: produce a lattice-QCD "method average" in analogy to e^+e^-

  $\hookrightarrow$ **robust quantification of tension in intermediate window**
- Beyond:
  - Lattice side: improved calculations (so far still only BMWc for full HVP), more windows

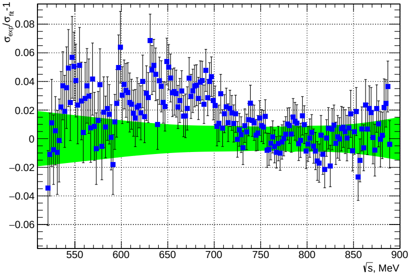
    talks by T. Blum, B. Toth
  - New e^+e^- data talk by A. Denig
  - Scrutiny of radiative corrections talk by G. Colangelo
  - Potentially τ data to be resurrected as a viable cross check
  - If all that does not change anything: new physics in e^+e^- data? talk by L. Di Luzio

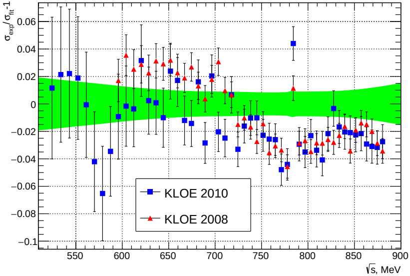
Window quantities: the inverse Laplace problem



Colangelo et al. 2022

↪ localization in energy entails **strong cancellation in Euclidean time**





BaBar vs. SND 20 2004.00263 KLOE vs. SND 20

- New data from SND experiment not yet included in WP20 number
  ↪ lie between BaBar and KLOE
- **More $\pi\pi$ data to come** from: CMD3, BESIII, BaBar, Belle II
- New data for 3π: BESIII, BaBar
- New data on inclusive region: BESIII (slight tension with pQCD)
- **MUonE project**: space-like HVP from μe scattering

2π channel: isospin breaking and ω mass

	χ^2/dof	p-value	M_ω [MeV]	$10^3 \times \text{Re}\,\epsilon_\omega$	δ_ϵ [°]	$10^{10} \times a_\mu^{\pi\pi}\|_{\leq 1\,\text{GeV}}$
SND06	1.40	5.3%	781.49(32)(2)	2.03(5)(2)		499.7(6.9)(4.1)
	1.08	35%	782.11(32)(2)	1.98(4)(2)	8.5(2.3)(0.3)	497.8(6.1)(4.9)
CMD-2	1.18	14%	781.98(29)(1)	1.88(6)(2)		496.9(4.0)(2.3)
	1.01	45%	782.64(33)(4)	1.85(6)(4)	11.4(3.1)(1.0)	495.8(3.7)(4.2)
BaBar	1.14	5.7%	781.86(14)(1)	2.04(3)(2)		501.9(3.3)(2.0)
	1.14	5.5%	781.93(18)(4)	2.03(4)(1)	1.3(1.9)(0.7)	501.9(3.3)(1.8)
KLOE''	1.20	3.1%	781.81(16)(3)	1.98(4)(1)		491.8(2.1)(1.8)
	1.13	10%	782.42(23)(5)	1.95(4)(2)	6.1(1.7)(0.6)	490.8(2.0)(1.7)
BESIII	1.12	25%	782.18(51)(7)	2.01(19)(9)		490.8(4.8)(3.9)
	1.02	44%	783.05(60)(2)	1.99(19)(7)	17.6(6.9)(1.2)	490.3(4.5)(3.1)
SND20	2.93	3.3×10^{-7}	781.79(30)(6)	2.04(6)(3)		494.2(6.7)(9.0)
	1.87	4.1×10^{-3}	782.37(28)(6)	2.02(5)(2)	10.1(2.4)(1.4)	494.9(5.3)(3.1)

Colangelo et al. 2022

- Mysteries in the fit:
  - Phase of the ρ–ω mixing parameter varies widely among experiments
  - Resulting value of M_ω at odds with 3π, $\pi^0\gamma$ channel

$\hookrightarrow$ hopefully forthcoming data will shed some light

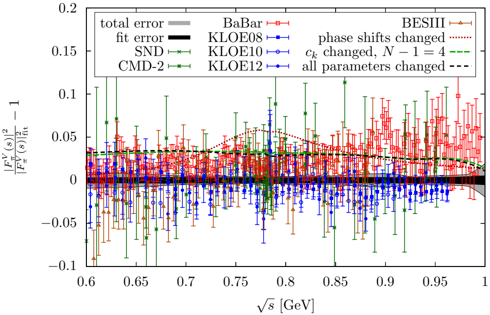
Relation to global electroweak fit

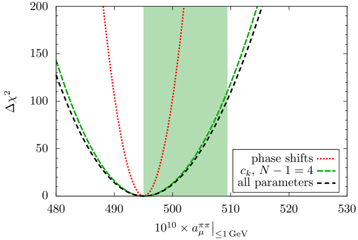
Hadronic running of α

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ enters as input in **global electroweak fit**
  $\hookrightarrow$ integral weighted more strongly towards high energy Passera, Marciano, Sirlin 2008
- Changes in $R_{\text{had}}(s)$ have to occur at low energies, $\lesssim 2\,\text{GeV}$ Crivellin et al. 2020, Keshavarzi et al. 2020, Malaescu et al. 2020
- This seems to happen for BMWc calculation (translated from the space-like), with only moderate increase of tensions in the electroweak fit ($\sim 1.8\sigma \to 2.4\sigma$)
  $\hookrightarrow$ need **large changes in low-energy cross section**
- Similar conclusion from Mainz 2022 calculation of hadronic running

Changing the $\pi\pi$ cross section below 1 GeV



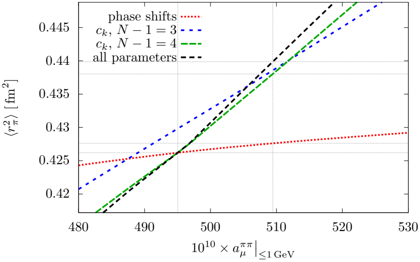


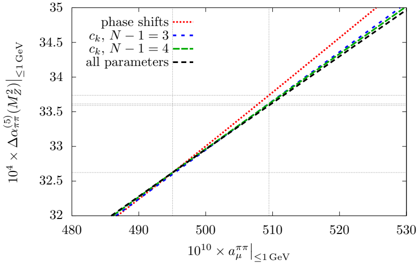
Colangelo, MH, Stoffer 2020

- Changes in 2π cross section **cannot be arbitrary** due to analyticity/unitarity constraints, but increase is actually possible
- Three scenarios:
  1. "Low-energy" scenario: **$\pi\pi$ phase shifts**
  2. "High-energy" scenario: **conformal polynomial**
  3. Combined scenario

$\hookrightarrow$ 2. and 3. lead to uniform shift, 1. concentrated in ρ region

Correlations





Correlations with other observables:

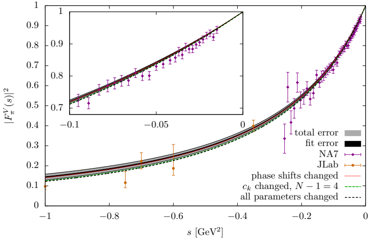
- **Pion charge radius $\langle r_\pi^2 \rangle$**

  $\hookrightarrow$ significant change in scenarios 2. and 3.

  $\hookrightarrow$ can be tested in lattice QCD

- **Hadronic running of α**

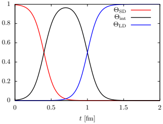
- **Space-like pion form factor**

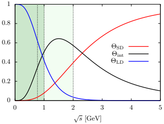


What can we conclude about the difference at the moment?

4 Window quantities

Some insights from the window quantities





- using form of weight functions: at least $\sim 40\%$ from **above 1 GeV**
- assumptions:
  - rather uniform shifts in low-energy $\pi\pi$ region
  - no significant negative shifts

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5 Conclusions

Conclusions

- **window quantities** and **analyticity constraints** point at an effect $\lesssim 8 \times 10^{-10}$ below 1 GeV, $\gtrsim 6 \times 10^{-10}$ above 1 GeV
- more detailed analysis might be possible with additional windows and knowledge of **correlations**

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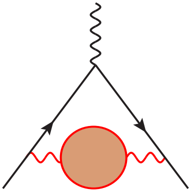
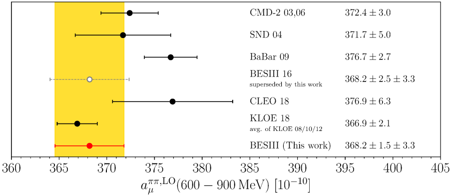
talk by P. Stoffer at Edinburgh

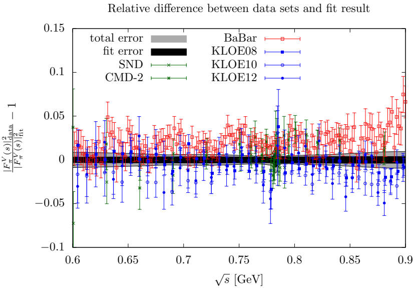
Summary and outlook

- **Muon $g-2$**: where do we stand?
  - E989 to improve experimental result by another factor 3
    $\hookrightarrow$ Run 6 with μ^+ approved
  - For HLbL agreement between lattice and phenomenology
    $\hookrightarrow$ another factor 2 looks feasible
  - New e^+e^- data and lattice calculations forthcoming
    $\hookrightarrow$ window observables for sharper comparisons
  - For prospects see also Snowmass contribution 2203.15810
  - WP update in preparation, aimed for Run 2+3 result



Cross checks from analyticity and unitarity





BESIII 2009.05011

Colangelo, MH, Stoffer 2018

- For "simple" channels $e^+e^- \to 2\pi, 3\pi$ can derive form of the cross section from **general principles of QCD** (analyticity, unitarity, crossing symmetry)

  $\hookrightarrow$ strong cross check on the data sets (covering about 80% of HVP)

- Uncovered an error in the covariance matrix of BESIII 16 (now corrected), all other data sets passed the tests

Merging procedure

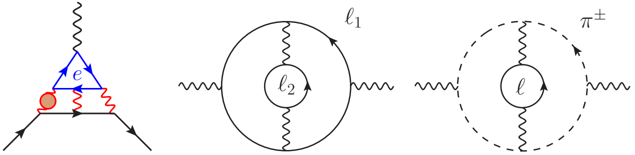
- How to deal with tensions?

  $\hookrightarrow$ extensive discussion at TI workshops
- Errors **systematics dominated**

  $\hookrightarrow$ scale factor not adequate/sufficient
- There was broad consensus to adopt **conservative error estimates**
- **Merging procedure**
  - Take average of central values from different analyses channel by channel (including analyticity/unitarity constraints)
  - In each channel: take biggest uncertainty from DHMZ/KNT, add half their difference as additional systematic effect
  - Exception: in 2π channel this additional systematic uncertainty taken as [fit w/o KLOE - fit w/o BaBar]/2
  - Take interchannel correlations from DHMZ analysis

$\hookrightarrow$ **covers tensions in the data and accounts for different methodologies for the combination of data sets**

A note on higher-order hadronic effects



- Generic scaling of $\mathcal{O}(\alpha^4)$ effects: $\left(\frac{\alpha}{\pi}\right)^4 \simeq 3 \times 10^{-11}$
- Enhancements (numerical or $\log \frac{m_e}{m_\mu}$) can make such effects relevant Kurz et al. 2014
- NLO HLbL small Colangelo et al. 2014
- Mixed hadronic and leptonic contributions with inner electron potentially dangerous

  $\hookrightarrow$ could affect LO HVP via radiation of e^+e^- pairs, but $\lesssim 1 \times 10^{-11}$ MH, Teubner 2022

Lattice QCD calculations of HVP

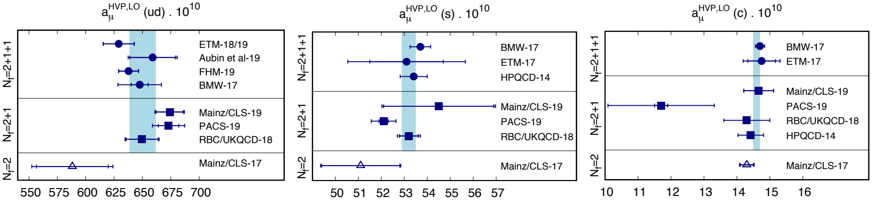
HVP from lattice QCD

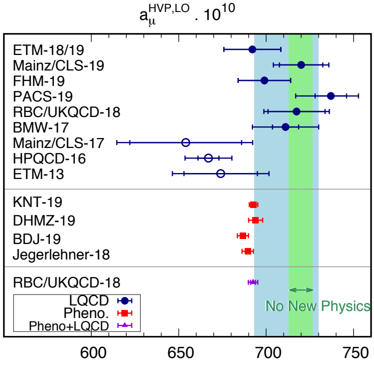
$$a_\mu^{\text{HVP, LO}} = a_{\mu,\,\text{conn}}^{\text{HVP, LO}}(ud) + \sum_{q=s,c,b} a_{\mu,\,\text{conn}}^{\text{HVP, LO}}(q) + a_{\mu,\,\text{disc}}^{\text{HVP, LO}} + a_{\mu,\,\text{IB}}^{\text{HVP, LO}}$$

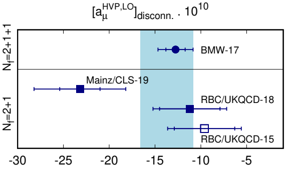
$$= 7116(184) \times 10^{-11}$$

- Basic differences to data-driven approach:
  - Calculation in **space-like**, not time-like kinematics
  - **Decomposition by flavor**, not hadronic channel
  - Disconnected diagrams and isospin breaking calculated as corrections
- WP discussion includes:
  - Detailed discussion of computational strategy (e.g., schemes for isospin breaking)
  - Comparisons of calculations available as of the deadline 31 March, 2020
  - Averages of subquantities and total HVP

HVP from lattice QCD: averages







+ isospin-breaking corrections

↪ many different calculations required for full HVP

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)\times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

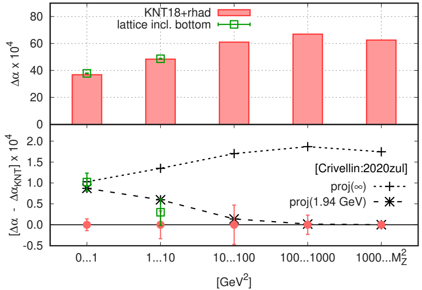
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)=\frac{\alpha M_Z^2}{3\pi}P\int_{s_{\text{thr}}}^{\infty}ds\frac{R_{\text{had}}(s)}{s(M_Z^2-s)}$$

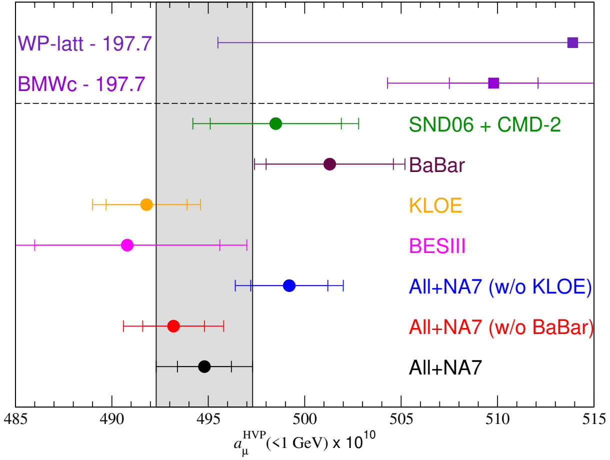
- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)=\frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2)+\frac{\alpha}{\pi}\big(\hat{\Pi}(M_Z^2)-\hat{\Pi}(-M_Z^2)\big)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020



Assumption: suppose all changes occur in $\pi\pi$ channel below 1 GeV

$\hookrightarrow$ $a_\mu^{\text{total}}[\text{WP20}] - a_\mu^{2\pi,<1\text{ GeV}}[\text{WP20}] = 197.7 \times 10^{-10}$