Hadronic contributions to the muon g-2 in holographic QCD

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Hadronic contributions to muon $g-2$ SM prediction

[Theory Initiative "White Paper", Aoyama et al., 2006.04822]

Hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL) totally dominate theoretical error 43×10^{-11} in a_μ

$$\begin{aligned} a_\mu^{\text{HVP}} &= (6845 \pm 40) \times 10^{-11} \quad \text{(0.6\% error, 3\% discrepancy with BMW)} \\ a_\mu^{\text{HLbL}} &= (92 \pm 19) \times 10^{-11} \quad \text{(20\% uncertainty)} \\ &\supset a_\mu^{\text{HLbL,SDC}} = (15 \pm 10) \times 10^{-11} \quad \text{(67\% uncertainty!)} \\ & \quad\; a_\mu^{\text{HLbL,axials}} = (6 \pm 6) \times 10^{-11} \quad \text{(100\% uncertainty!)} \end{aligned}$$

Experimental error in $a_\mu^{\text{exp}} = (116\,592\,061 \pm 41) \times 10^{-11}$ eventually to be reduced by new runs at FNAL to $\sim 10 \times 10^{-11}$

- Discrepancy between data-driven (dispersive) approaches and lattice calculations needs to be resolved, and accuracy improved!
- Also hadronic light-by-light (HLbL) contribution needs work, especially axials

Holographic QCD: conjectural approximation to strongly coupled (large-N_c) QCD with typical errors of $10..30\%$

$\Rightarrow$ too crude to help with HVP, but interesting for HLbL contributions!

Holographic QCD

True holographic dual to non-supersymmetric and non-conformal large-N_c QCD unknown, but:

1998 Witten succeeded in constructing a string-theoretical dual ("*top-down*") to the low-energy limit of large-N_c QCD from type-IIA superstring theory compactified on one further extra dimension, and

2004 Sakai & Sugimoto found D-brane construction to add chiral quarks in fundamental representation.
However: wrong UV behavior, not even asymptotically AdS

2005ff: Erlich, Katz, Son, Stephanov, ... (HW1)
Hirn, Sanz (HW2) (simpler)
succeeded in constructing phenomenologically interesting models of hadron physics with similar ingredients on simple AdS_5 background ($\leftrightarrow$ conformal symmetry) broken in IR by "hard walls" (HW) or "soft walls" (SW)

$\rightarrow$ "*bottom-up*" holographic QCD

HLbL contribution to muon $g-2$ & holographic QCD

hQCD results for

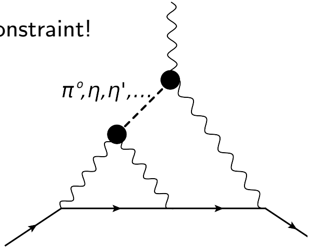
- **single and double virtual (pion) transition form factor** $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

  [Grigoryan, Radyushkin, PRD76,77,78 (2007-8)]
  [Cappiello, Catà, D'Ambrosio, PRD83 (2011)]
  [J. Leutgeb, J. Mager, AR, PRD100 (2019)]

  - comparison with recent low-energy data (BESIII) and lattice results
  - hQCD prediction for $a_\mu^{\pi^0,\eta,\eta'}$

- **Axial vector meson contributions** [J. Leutgeb, AR, PRD101, 1912.01596]
  [Cappiello, Catà, D'Ambrosio, Greynat, Iyer, PRD102, 1912.02779]

  - comparison with $f_1 \to \gamma\gamma^*$ data from L3
  - crucial role in saturation of Melnikov-Vainshtein constraint!
  – first hadronic model to achieve this in chiral limit!
  - hQCD prediction for $a_\mu^{a_1,f_1,f_1',\ldots}$
  - extension to massive quarks:
    Leutgeb, AR, PRD104, 2108.12345
    including U(1)$_A$ anomaly and $\mathbf{m_s \gg m_{u,d}}$:
    Leutgeb, Mager, AR, 221x.xxxxx



+ permutations of $\mu\gamma$-vertices

Bottom-up and top-down holographic QCD

(Axial) vector mesons and pions are described by **5-d YM fields** $\mathcal{F}^{L,R}_{MN} = \mathcal{F}^{V}_{MN} \mp \mathcal{F}^{A}_{MN}$ for global $U(N_f)_L \times U(N_f)_R$ chiral symmetry of boundary theory

$$S_{\rm YM} \propto \frac{1}{g_5^2}\,\mathrm{tr}\int d^4x \int_0^{z_0} dz\, e^{-\Phi(z)} \sqrt{-g}\, g^{PR} g^{QS} \left(\mathcal{F}^{(L)}_{PQ}\mathcal{F}^{(L)}_{RS} + \mathcal{F}^{(R)}_{PQ}\mathcal{F}^{(R)}_{RS}\right),$$

where $P,Q,R,S = 0,\ldots,3,z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

with conformal boundary at $z=0$, and

either sharp cut-off of AdS$_5$ at z_0 (HW) or with nontrivial dilaton $z_0 = \infty$ (SW)
(SS: not asymptotically AdS$_5$, finite z_0 corresponding to point where D8 branes join)

Chiral symmetry breaking either from

- extra bifundamental scalar field [Erlich-Katz-Son-Stephanov 2005] (HW1), or
- through different boundary conditions for vector/axial-vector fields at z_0 [Hirn-Sanz 2005] (HW2), [Sakai-Sugimoto 2004] (SS)

Vector meson dominance (VMD) naturally built in:
photons couple through *bulk-to-boundary propagators of vector gauge fields* whose normalizable modes give (infinite tower of!) vector mesons (ρ, ω, ϕ, . . .)

Anomalous TFFs from holographic QCD

Flavor anomalies follow uniquely from 5-dimensional Chern-Simons term:

$$S^L_{\rm CS} - S^R_{\rm CS}, \quad S_{\rm CS} = \frac{N_c}{24\pi^2} \int \mathrm{tr} \left(\mathcal{B}\mathcal{F}^2 - \frac{i}{2}\mathcal{B}^3\mathcal{F} - \frac{1}{10}\mathcal{B}^5 \right).$$

with infinite tower of vector and axial-vector mesons contained in 5-dimensional $SU(N_f)_L \times SU(N_f)_R$ gauge field $\mathcal{B}^{L,R}_M$; Goldstone bosons of χSB in $\mathcal{B}^{L-R}_5$

- Pion transition form factor for $\pi^0 \to \gamma^*\gamma^*$

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi} \int_0^{z_0} dz\, \mathcal{J}(Q_1, z)\mathcal{J}(Q_2, z)\Psi(z),$$

with *bulk-to-boundary propagator* $\mathcal{J}$ and holographic pion profile Ψ

- The amplitude for axial vector mesons $a^{(n)}_\mu$ decaying into two virtual photons following from the Chern-Simons action has the form

$$\mathcal{M}^a = i\frac{N_c}{4\pi^2}\mathrm{tr}(\mathcal{Q}^2 t^a)\,\epsilon^\mu_{(1)}\epsilon^\nu_{(2)}\epsilon^{*\rho}_A\epsilon_{\mu\nu\rho\sigma}\left[q^\sigma_{(2)}Q_1^2 A_n(Q_1^2, Q_2^2) - q^\sigma_{(1)}Q_2^2 A_n(Q_2^2, Q_1^2)\right],$$

where

$$A_n(Q_1^2, Q_2^2) \quad = \frac{2g_5}{Q_1^2}\int_0^{z_0} dz \left[\frac{d}{dz}\mathcal{J}(Q_1, z)\right]\mathcal{J}(Q_2, z)\psi^A_n(z), \quad n = 1, \dots, \infty$$

- **Landau-Yang theorem** (AV→ $\gamma\gamma$ is forbidden) realized by $\mathcal{J}'(Q, z) = 0$ for $Q^2 = 0$

Short distance constraints on TFFs

Amazingly, hQCD models with asymptotic AdS_5 geometry reproduce
asymptotic momentum dependence of pQCD [Brodsky-Lepage 1979-81]
(not achieved by older phenomenological VMD models)

- **Pseudoscalars** [Grigoryan & Radyushkin, PRD76,77,78 (2007-8)]:

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) \rightarrow \frac{2f_\pi}{Q^2}\sqrt{1-w^2}\int_0^\infty d\xi\,\xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$
$$= \frac{2f_\pi}{Q^2}\left[\frac{1}{w^2}-\frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}\right],$$

with $Q^2=\frac{1}{2}(Q_1^2+Q_2^2)\to\infty$, $w=(Q_1^2-Q_2^2)/(Q_1^2+Q_2^2)$,
corresponding to asymptotic behavior

$$F^\infty(Q^2,0)=\frac{2f_\pi}{Q^2},\qquad F^\infty(Q^2,Q^2)=\frac{2f_\pi}{3Q^2}.$$

- **Axial vector mesons** [J. Leutgeb & AR, 1912.01596]
(agreeing with independent pQCD result by Hoferichter & Stoffer 2004.06127):

$$A_n(Q_1^2,Q_2^2)\rightarrow\frac{12\pi^2F_n^A}{N_cQ^4}\frac{1}{w^4}\left[w(3-2w)+\frac{1}{2}(w+3)(1-w)\ln\frac{1-w}{1+w}\right]$$

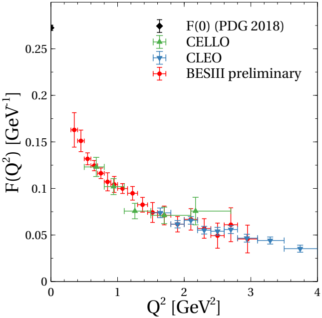
Pion TFF

Pion transition form factor $F_{\pi^0\gamma^*\gamma^*}$ with on-shell pion and virtual photons:

$$\int d^4x e^{-iq_1\cdot x}\langle\pi^0(q_1+q_2)|T J_\mu^{\text{e.m.}}(x)J_\nu^{\text{e.m.}}(0)|0\rangle = \epsilon_{\mu\nu\rho\sigma}q_1^\rho q_2^\sigma F_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2)$$

axial anomaly: $F_{\pi^0\gamma^*\gamma^*}(0,0)=\dfrac{N_c}{12\pi^2 f_\pi}$

Experimental data so far available only for $Q_2^2=0$: [Danilkin et al. 1901.10346]:
(spacelike $Q^2\equiv Q_1^2\equiv -q_1^2$)

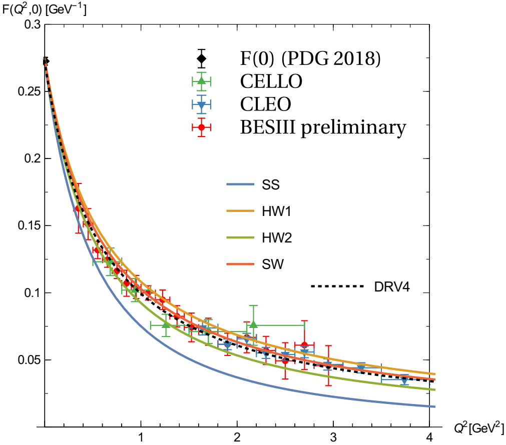


$a_\mu^{\pi^0}$ receives $\sim 85\%$ from $Q_i^2\le 1$ GeV — singly and doubly virtual cases needed!

- Bottom-up hQCD interesting interpolation of correct low and high momentum regime

Holographic pion TFF and experimental data

Comparison with single-virtual TFF from CELLO, CLEO, and BESIII (preliminary):
(hQCD models fitted to match f_π and m_ρ — only 2 free low-energy parameters)

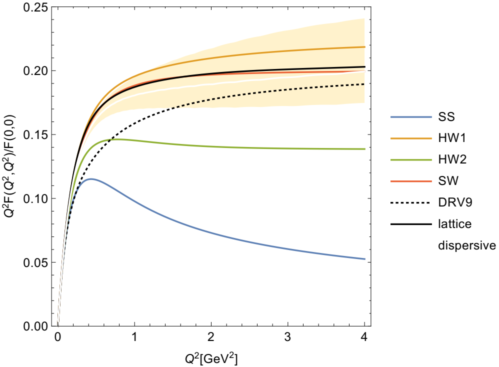


data compilation from Danilkin, Redmer & Vanderhaeghen, 1901.10346

Sakai-Sugimoto model (SS) only good at low Q^2; incorrect short-distance behavior → dropped

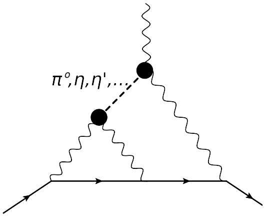
Holographic pion TFF and recent lattice data

No experimental data yet for double-virtual pion TFF, but
• results from dispersive approach M. Hoferichter et al., 1808.04823 and
lattice extrapolations from A. Gérardin, H. B. Meyer, and A. Nyffeler, 1903.09471:



- HW1: (too) quickly approaches LO pQCD result (but NLO $\sim -10\%$ at largish Q^2)
- SW: (fortuitously?) close to lattice (89% of LO pQCD asymptotically)
- SS: wrong asymptotics, but below 0.3 GeV^2 closer to lattice than DRV interpolator

Holographic pion TFF and $a_\mu^{\pi^0}$ predictions



method/model	$a_\mu^{\pi^0} \times 10^{11}$
LMD+V [Nyffeler 2016]	72 ± 12
lattice (Mainz, 2016)	65 ± 8
lattice (Mainz, 2019)	60 ± 4
lattice (Mainz, 2019)+exp.data	62 ± 2
Danilkin et al. (DRV,2019)	56 ± 2
dispersive [WP 2020]	$63.0^{+2.7}_{-2.1}$
HW1 (chiral)	65.2
HW2 (chiral)	56.9
SW (chiral)	59.2
hQCD (HW,SW chiral) [LMR 2019]	61 ± 4
hQCD (HW massive) [LR 2021]	63.6 ± 3.0

(hQCD error estimate: spread of different models)

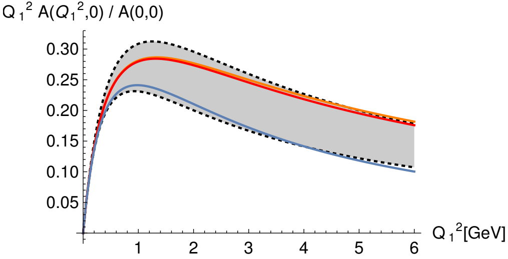
hQCD agrees well with data-driven (dispersive) evaluations and lattice hQCD results ⇒

interesting to evaluate also axial-vector contributions where only simple hadronic models have been used so far, which leads to 100% error estimate in White Paper for their assumed contribution of $6(6) \times 10^{-11}$

Holographic TFFs for axial vector mesons vs. experiments

Shape of single-virtual axial TFF: [J. Leutgeb & AR, 1912.01596]

dipole fit of L3 data for $f_1(1285)$ (gray band) vs. SS, HW1, and HW2 models:



Magnitude: hQCD results:	HW1	HW2
$\lvert A(0,0)\rvert$ [GeV^{-2}]	21.04	16.63

roughly right ballpark compared to experimental data:

$$A(0,0)^{\rm L3\,exp.}_{f_1(1285)} = 16.6(1.5)\,\mathrm{GeV}^{-2}; \qquad A(0,0)_{a_1(1230)} = 19.3(5.0)\,\mathrm{GeV}^{-2}$$

Roig & Sanchez-Puertas, 1910.02881:

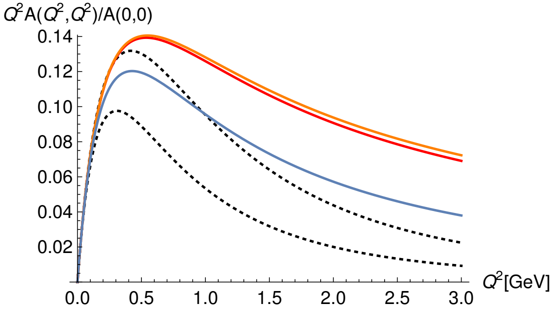
but HW1 somewhat too high

Double-virtual axial vector meson TFF

Holographic results of SS, HW1, and HW2 models quite different than

symmetric dipole model $\frac{A^{\rm PV}(Q_1^2,Q_2^2)}{A(0,0)} = \frac{1}{(1+Q_1^2/\Lambda_D^2)^2(1+Q_2^2/\Lambda_D^2)^2}$ (dashed lines)

used by Pauk & Vanderhaeghen [1401.0832] in their calculation of $a_\mu^{f_1,f_1'}$
which is main basis for SM estimate by Muon g-2 Theory Initiative



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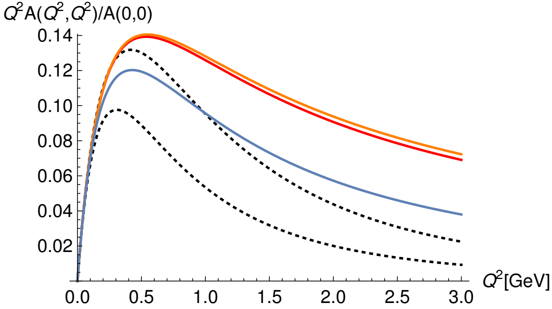


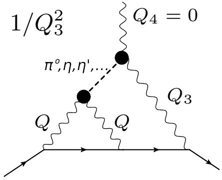
Even more important:
role of axial vector mesons in Melnikov-Vainshtein short-distance constraint

Melnikov-Vainshtein short-distance constraint

Melnikov and Vainshtein [hep-ph/0312226, PRD70(2004)]:
nonrenormalization theorem for axial anomaly implies
short-distance constraint for 4-photon-amplitude (in BTT basis w/ 54 structure functions):

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

each single meson exchange contribution gives 0
because propagator $\sim 1/Q_3^2$ and the two form factors $\sim 1/Q^2$ and $1/Q_3^2$



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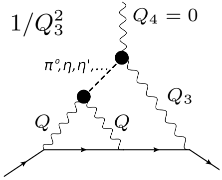

MV model: MV-SDC satisfied by replacing
external TFF by constant on-shell value, leading to
significant (almost +40%) increase of $a_\mu^{\pi^0,\eta,\eta'}$ by $\underline{38 \times 10^{-11}}$

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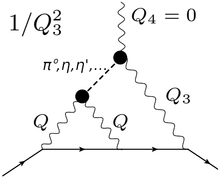

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WP estimate for MV-SDC based on Regge model of infinite tower of excited PS states constructed to saturate MV-SDC with $\underline{\Delta a_\mu^{\rm PS} = 13(6)\times10^{-11}}$ [Colangelo et al., 1910.11881]

But: Excited PS states decouple in chiral large-N limit [MV, 1911.05874]

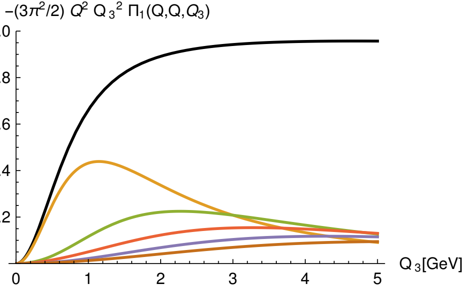
Axial vector contributions to MV-SDC

hQCD comes with infinite tower of axial vector mesons,
MV-SDC satisfied upon complete summation:

large $Q = 50\text{GeV}$ and increasing $Q_3 \ll Q$:

black line: infinite sum

colored lines: first 5 axial vector modes



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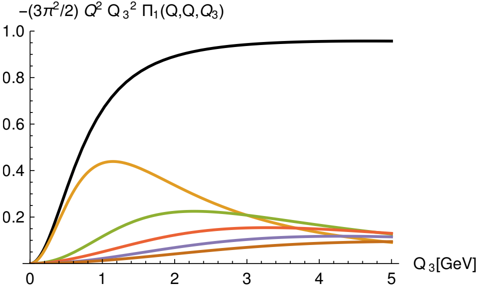


Chiral HW1 model: MV-SDC satisfied *exactly* [Leutgeb & AR, PRD101, 1912.01596]
HW2 model: with fitted f_π, m_ρ MV-SDC at 62% (UV-fit possible with overweight m_ρ)

HW2 case independently also by [Cappiello, Catà, D'Ambrosio et al., PRD102, 1912.02779]

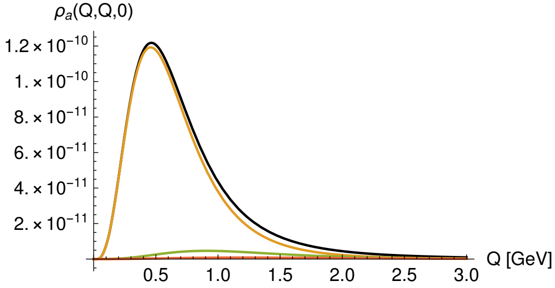
HW1 model with massive quarks [Leutgeb & AR, PRD104, 2108.12345]:

MV-SDC still completely satisfied through tower of axial-vector mesons;
tower of excited massive pions gives subleading contribution $\propto \ln(Q_3^2)/Q_3^4 Q^2$

Total axial-vector contributions to muon $g-2$

$$a_\mu^{AV} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\, \rho_a(Q_1, Q_2, \tau)$$

E.g. at $\tau = 0$:



Strongly dominated by lowest axials, but nonnegligible contribution from higher modes:

	SDC	$j=1$	$j \leq 2$	$j \leq 3$	$j \leq 4$	$j \leq 5$	a_μ^{AV}
HW1	100%	31.4	36.2	37.9	39.1	39.6	40.6 $\times 10^{-11}$
HW2	62%	23.0	26.2	27.4	27.9	28.2	28.7 $\times 10^{-11}$
SS	0% (!)	13.8	14.5	14.7	14.8	14.8	14.8 $\times 10^{-11}$

($\approx$ 60% longitudinal + 40% transverse)

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NB: Experimental data for pion TFF bracketed by HW1 and HW2 models (with HW1 closer)

$\rightarrow$ (chiral) hQCD estimate $\boxed{a_\mu^{\rm AV}[L+T] = \mathbf{35(6)}\,[20(3)+15(3)]\times 10^{-11}}$

cp. White Paper: $a_\mu^{\rm SDC+axials} = \mathbf{21(16)}\,[15(10)+6(6)]\times 10^{-11}$

consistent with hQCD results, but HW1 model which saturates SDC by 100% somewhat higher!

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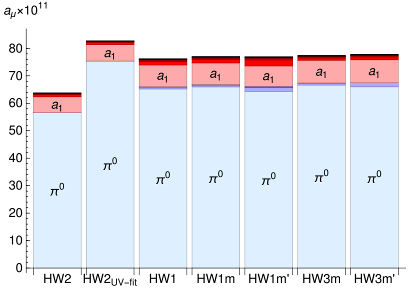
NB: NLO QCD results for TFFs at $\sim 90\%$ of asymptotic value when pQCD becomes applicable
$\rightarrow$ optimal(?): HW1 with g_5^2 reduced by factor 0.9 $\rightarrow a_\mu^{\mathrm{AV,HW1}} \times 0.95$
supported by HVP in HW1 which then increases to within 5% of dispersive result!
[J. Leutgeb, AR, M. Stadlbauer, PRD105, 2203.16508]

HWm models with massive pions

Rigorous inclusion of quark masses in HW1 and HW3 models:
$\rightarrow$ little difference to chiral model with manually inserted pion mass



(darker colors: excited π^{0*}, a_1^*)

HW1m: HW1 with nonzero light quark mass and correct pion mass

HW1m': HW1m with modified scaling dimension of bifundamental scalar, additionally correct $a_1(1230)$ mass, but not mass of $\pi(1300)$

HW3m: HW1m with HW2 boundary conditions

HW3m': HW3m with modified scaling dimension of bifundamental scalar, additionally correct $\pi(1300)$ mass, but not mass of $a_1(1230)$

U(3) symmetric: $\boxed{a_\mu^{\rm AV} = 4a_\mu^{a_1} = \mathbf{39.5(1.6)} \times 10^{-11}}$ for (HW1m$\sim$HW3m)$|_{.9\,\rm SDC}$

(AV masses range from 1230 to 1431 MeV in those models)

Katz-Schwartz: HW1 with realistic m_s and U(1)$_A$ anomaly

E. Katz & M. Schwartz, *An Eta Primer: Solving the U(1) problem with AdS/QCD*, JHEP 08 (2007) 077

proposed hard-wall AdS/QCD Lagrangian including

- besides bifundamental $X \leftrightarrow \bar{q}_i q_j$ with $\langle X_{ij} \rangle = M_{ij} z + \Sigma_{ij} z^3$ and $\Sigma \leftrightarrow \langle \bar{q}_i q_j \rangle$
- also complex scalar $Y \leftrightarrow \alpha(GG + iG\tilde{G})$ with

$$\mathcal{L} \supset \kappa Y^{N_f} \det(X)$$

  accounting for U(1)$_A$ anomalous Ward identities
- essentially independent of κ as long as $\kappa \gg 1$
- only new free parameter: gluon condensate $\Xi \leftrightarrow \langle G^2 \rangle$ in $\langle Y \rangle = C + \Xi z^4$ with OPE $\Rightarrow C = \frac{\sqrt{2N_f}}{2\pi^2}\alpha_s$, $\alpha_s \to 1/\beta_0 \ln(\Lambda_{QCD} z)$, $\Lambda_{QCD} \to z_0^{-1}$
  $\Rightarrow \Xi z^4 \to \Xi z^4 [\ln^2(\Lambda_{QCD} z) + \ldots]$
- realizes Witten-Veneziano mechanism for $m_{\eta'}$
- phase of $Y \leftrightarrow$ pseudoscalar glueball mixing with $\eta^{(\prime)}$

Katz-Schwartz HW1+U(1)$_A$ Model (preliminary!)

[Leutgeb, Mager, AR, in preparation]

$N_f = 2+1$ with $m_s \approx 24.3 m_{u,d}$

Two versions:

I) $g_5 = 2\pi$ such that UV constraints on TFF satisfied to 100%

II) $g_5 = 5.94$ such that f_ρ is fitted ($\approx 90\%$ of asymptotic SDCs)

Tuning of gluon condensate Ξ (neglected by KS) $\rightarrow$ virtually exact fit of m_η and $m_{\eta'}$:

Version I)

	m [MeV]	m–$m^{\rm exp}$ [%]	f^8[MeV]	f^0[MeV]	f_G[MeV]	$\|F(0,0)\|$
π^0	135	(input)	0	0	0	0.276
η	550.4	+0.5%	-100.5	-28.3	29.1	0.275
η'	957.2	-0.04%	39.1	-114	78.7	0.328
PSGB	1927.6	?	-14.3	1.94	30.6	0.162
	m [MeV]	m–$m^{\rm exp}$ [%]	F_A^8/m_A[MeV]	F_A^0/m_A[MeV]	$A^8(0,0)$	$A^0(0,0)$
a_1	1367	+11%	0	0	0	0
f_1	1483	+16%	-177	-33.0	-20.9	-3.40
f_1'	1841	+29%	-25.4	207	-3.38	20.0

$|\Xi| = 0.012\ \mathrm{GeV}^4$

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Version II) (our newest "best guess" regarding a_μ)

	m [MeV]	m–$m^{\exp}$ [%]	f^8[MeV]	f^0[MeV]	f_G[MeV]	$\|F(0,0)\|$
π^0	135	(input)	0	0	0	0.276
η	553.7	+1%	-102.6	-30.5	30.6	0.265
η'	956.6	-0.2%	39.4	-123	86.5	0.303
PSGB	1820	?	-13.6	2.02	25.7	0.196
	m [MeV]	m–$m^{\exp}$ [%]	F_A^8/m_A[MeV]	F_A^0/m_A[MeV]	$A^8(0,0)$	$A^0(0,0)$
a_1	1295	+5%	0	0	0	0
f_1	1411	+10%	-176	-26.5	-19.6	-2.37
f_1'	1856	+30%	-12.4	226	-2.22	18.9

$|\Xi| = 0.016\ \mathrm{GeV}^4$

PS: $f^{8,0}$'s within a few % of χPT values

AV: f_1-f_1' mixing angle $\phi_f - \phi_f^{\mathrm{ideal}}$ about twice as large as indicated by L3 data

(ϕ_f strongly dependent on Ξ; sum $a_\mu^{f_1} + a_\mu^{f_1'}$ rather insensitive)

a_μ in Katz-Schwartz HW1+U(1)$_A$ Model (preliminary!!)

$a_\mu^{\cdots} \times 10^{11}$	I (100% SDC)	II (f_ρ fitted)	WP
π^0	66.0	63.3	$62.6^{+3.0}_{-2.5}$
η	19.6	17.7	16.3(1.4)
η'	16.2	13.5	14.5(1.9)
$PSGB$	0.3	0.5	
$\sum_{PS*}$	3	3	
PS poles total	105	98	93.8(4.0)

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a_1	7.9	7	
$f_1 + f_1'$	19.7	18	
$\sum_{a_1^*}$	2.4	3	
$\sum_{f_1^{(\prime)*}}$	6	6	
AV+LSDC total	36	34	21(16)

(precision on integer values still in the works)

NB: m_s and m_{WV} reduce total AV contribution from 4 to 3.5 times a_1 sector

cp.: U(3)-symmetric (HW1m∼HW3m)$_{\mathrm{II}}$: $\mathbf{39.5(1.6)} \times 10^{-11}$

non-U(3)-symmetric $\frac{3.5}{4}$(HW1m∼HW3m)$_{\mathrm{II}}$: $\mathbf{34.6(1.4)} \times 10^{-11}$

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(precision on integer values still in the works)

CAVEATS:

- equivalent photon decay rate of f_1, f_1' higher than L3 data indicate
- only partially compensated in a_μ by AV and AV* masses too high

Expect results to be upper bounds, with downward errors of maximally 30%:

$$34^{+2?}_{-10?} \times 10^{-11}$$

Conclusions for $a_\mu^{\rm HLBL}$

- hQCD is not QCD, but sophisticated toy model that can give clues on
  - how short-distance constraints can be implemented at the hadronic level
    - important fundamental role of axial-vector mesons $\leftrightarrow$ anomaly
  - semi-quantitative estimates of the ballparks to be expected (HW1–HW2 brackets experimental results for pion TFF!)
    - pion contribution from hQCD in perfect agreement with data-driven approach
    - axial-vector contributions greater than estimated previously

$$a_\mu^{\rm AV}[L+T] = \mathbf{35(6)}\,[20(3)+15(3)]\times 10^{-11} \qquad \text{for HW1}\sim\text{HW2}$$

$$\to \mathbf{34^{+2?}_{-10?}} \times 10^{-11} \quad \text{preliminary HW1m+U(1)}_A$$

vs. WP: $a_\mu^{\rm SDC+axials} = \mathbf{21(16)}\,[15(10)+6(6)]\times 10^{-11}$

Conclusions for a_μ^{HLBL}

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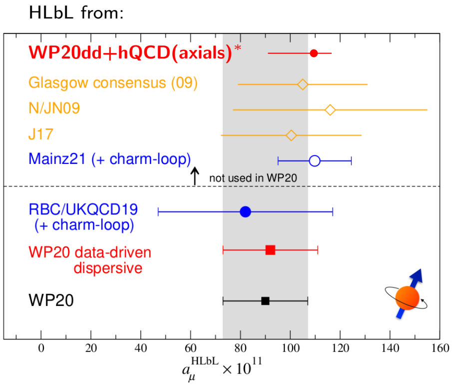
$$\rightarrow \mathbf{34}^{+\mathbf{2}?}_{-\mathbf{10}?}\times 10^{-11} \text{ preliminary HW1m+U(1)}_A$$

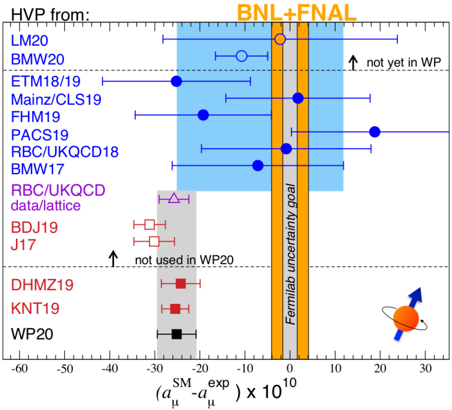
vs. WP: $a_\mu^{\mathrm{SDC+axials}} = \mathbf{21}(\mathbf{16})\,[15(10)+6(6)]\times 10^{-11}$

More experimental data on $\mathrm{A}\rightarrow\gamma\gamma^*,\ \rho\gamma,\ e^+e^-$ would be useful to check hQCD predictions for AV TFF

Conclusions for a_μ

- NB: a new complete lattice QCD calculation [Mainz21, 2104.02632] has $a_\mu^{\rm HLBL} = 106.8(14.7) \times 10^{-11}$ while WP20 disp. result is at $92(19) \times 10^{-11}$
- with hQCD results WP20 slightly closer to experimental result: $+17 \times 10^{-11}$ $\to 109^{+5?}_{-13?} \times 10^{-11}$ (including also excited PS)





* replacing axials and SDC of WP20 by our (preliminary) hQCD results (my addition to fig. of Snowmass paper 2203.15810)

Hadronic vacuum polarization in hQCD

[J. Leutgeb, AR, M. Stadlbauer, PRD105, 2203.16508]

LO-HVP contribution from lightest quarks ($N_f = 2$) in multiples of 10^{-10}:

holographic QCD:

	$a_{\mu(N_f=2)}^{\text{LO-HVP}}$	mismatch
HW1	476.9	0.86
HW2	773.9	1.39

mismatch = ratio of hQCD results over $a_{\mu(\pi\pi,\pi\pi\pi,\pi\gamma)}^{\text{LO-HVP}}$ in dispersive approach.

dispersive approach $a_{\mu(\pi\pi,\pi\pi\pi,\pi\gamma)}^{\text{LO-HVP}}$:

	DHMZ19	KNT19
$\pi^+\pi^-$	$508(3)$	$504(2)$
$\pi^+\pi^-\pi^0$	$46(1)$	$47(1)$
$\pi^0\gamma$	4.4(1)	4.6(1)
Sum	$558(4)$	$555(3)$

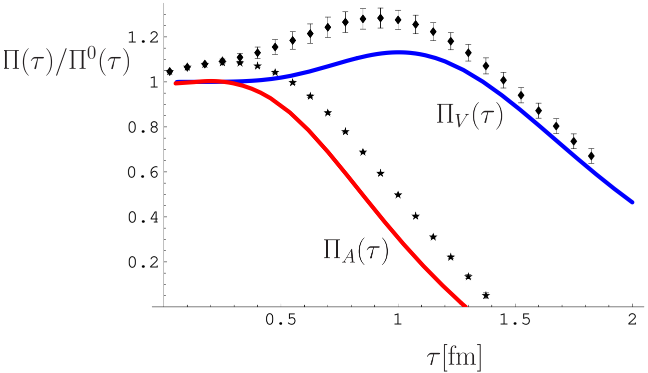
[Davier et al., 1908.00921]
[Keshavarzi et al., 1911.00367]

with g_5^2 *reduced by a factor 0.9* to better match large-Q^2 behavior of VV correlator and pion TFF simultaneously (and F_{ρ^0} even completely!): $a_{\mu(N_f=2)}^{\text{LO-HVP}} \to 530 \times 10^{-10}$ (HW1)

while $a_\mu^{\pi^0}$ decreased by factor 0.96 to $a_\mu^{\pi^0} = (6.17 \ldots 6.39) \times 10^{-10}$ in HWm models

s.t. in complete agreement with data driven result $6.30_{-21}^{+27} \times 10^{-10}$ [WP]

Vector current correlation functions in HW1 AdS/QCD



We show the ratio of the correlation function to the free correlator as a function of euclidean separation τ. The solid curves show the result in the holographic model. The data points are taken from an analysis of Aleph data on hadronic tau decays [from T. Schäfer, 0711.0236]

Vector meson spectra and decay constants

[J. Leutgeb, AR, M. Stadlbauer, 2203.16508]

Vector (rho) meson masses m_n and decay constants $F_n^{1/2}$ in MeV.

	HW1		HW2(IR|UV-fit)		SW		SHW		TC(fit1|2)	
n	m_n	$F_n^{1/2}$	m_n	$F_n^{1/2}$	m_n	$F_n^{1/2}$	m_n	$F_n^{1/2}$	m_n	$F_n^{1/2}$
1	*775*	329.1	*775*|987.2	371.4|419.2	*775*	260.0	*775*	314.0	765.4|803.9	313.2|329.0
2	1779	615.8	1779|2266	695.1|784.5	1096	309.2	*1465*	458.5	1382|1453	418.0|438.9
3	2789	863.3	2789|3553	974.4|1100	1342	342.2	1903	498.7	1806|1899	488.7|513.0
4	3800	1089	3800|4841	1229|1387	1550	367.7	2230	540.0	2158|2269	538.4|564.9
5	4812	1300	4812|6130	1467|1656	1733	388.8	2511	570.7	2466|2593	577.3|605.4
6	5824	1500	5824|7419	1693|1911	1898	406.9	2762	597.4	2744|2885	610.4|639.6
7	6836	1692	6836|8708	1909|2155	2050	422.9	2991	621.4	2999|3153	639.4|669.7
8	7848	1876	7848|9997	2118|2390	2192	437.2	3203	642.8	3236|3402	665.5|696.6

Experimental value: $m_{\rho^0} = 775.26(25)$ MeV,

$F_{\rho^0}^{1/2} = 348(1)$ MeV (fitted by HW1 with $g_5^2 = 0.89\,\frac{12\pi^2}{N_c}$)

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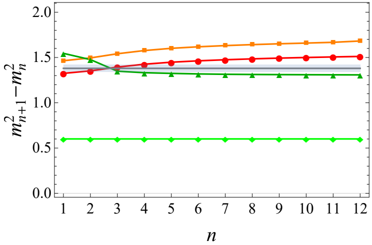
SW model with linear confinement and $m_n^2 \propto n$ has too small slope of radial Regge trajectory:



TC (fit 2)

TC (fit 1) [Tachyon cond. model, Iatrakis-Kiritsis-Paredes, 2007]

⟵ Experimental fit [Masjuan et al., 1203.4782]

Semi-hard wall [Kwee & Lebed, 2007]

Soft-wall model [Karch-Katz-Son-Stephanov, 2006]