$B_{d,s} \to \mu^+ \mu^- \gamma$ phenomenology
– overview –

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  ↳ *For light leptons: enhancement w.r.t. purely leptonic mode*
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  $B_s \to \mu\mu$ dataset. First LHCb analysis completed

- *With Run 3 (⇨ hopefully comparable e and μ efficiencies),*
  $B_s \to \mu\mu\,\gamma\,/\,B_s \to ee\,\gamma$ no more science fiction

$B_s \to \mu\mu\ \gamma$ from $B_s \to \mu\mu$

$B_s \to \mu\mu\ \gamma$: "indirect" method

[Dettori, DG, Reboud, 2017]

Basic Idea

Extract $B_s \to \mu\mu\ \gamma$ *from* $B_s \to \mu\mu$ *event sample,*
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- *Exploits rich and ever increasing* $B_s \to \mu\mu$ *dataset*

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- *Exploits rich and ever increasing $B_s \to \mu\mu$ dataset*
- *… to access $B_s \to \mu\mu\gamma$, that probes flavour anomalies more thoroughly*

[thanks F. Dettori]

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- No need to reconstruct the γ (factor-of-20 loss in efficiency)

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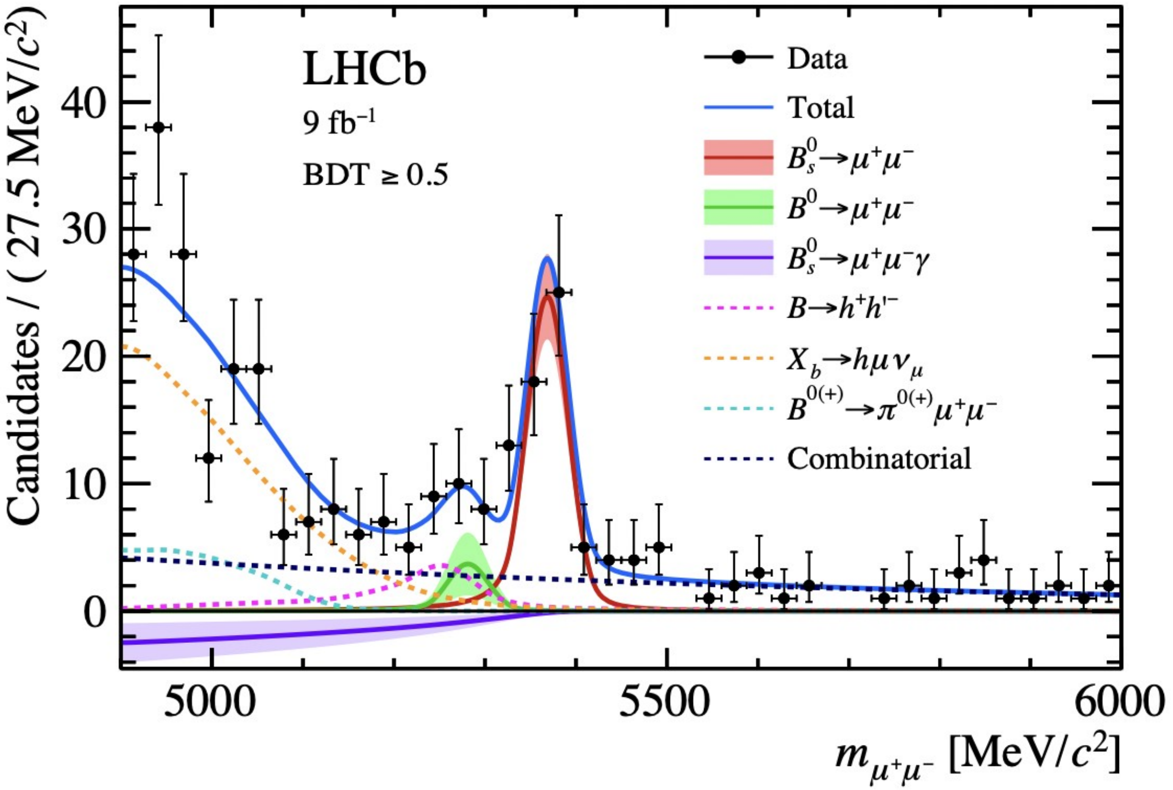
- Relatively (but not too) small q^2 range. Below (4.2 GeV)^2, $c\bar{c}$ pollution
- Trigger efficiency and reco somewhat below $B_s \to \mu\mu$
  But better than full γ reco
- Mass resolution, O(50 MeV), crucial: could be more challenging at ATLAS / CMS
- Calibration not trivial – no “analogous” channel

Results

[thanks F. Dettori]



[LHCb-PAPER-2021-007] [LHCb-PAPER-2021-008]

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \left(3.09\,{}^{+\,0.46}_{-\,0.43}\,{}^{+\,0.15}_{-\,0.11}\right) \times 10^{-9}$$

$$\mathcal{B}(B^0 \to \mu^+\mu^-) = \left(1.2\,{}^{+\,0.8}_{-\,0.7} \pm 0.1\right) \times 10^{-10} < 2.6 \times 10^{-10}$$

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-\gamma)_{m_{\mu\mu} > 4.9\,\mathrm{GeV}} = (-2.5 \pm 1.4 \pm 0.8) \times 10^{-9} < 2.0 \times 10^{-9}$$

No significant signal for $B^0 \to \mu^+\mu^-$ and $B_s^0 \to \mu^+\mu^-\gamma$, upper limits at 95%
First world limit on $B_s^0 \to \mu^+\mu^-\gamma$ decay

BRγ[4.0 GeV, m_{Bs}]$_{KMN}$
$\simeq$ 0.9 $\cdot$ 10^{-9}

The elephant in the room (f.f.’s)

Radiative leptonic f. f.'s in LQCD

Small $\boldsymbol{E_\gamma}$ [RM123, *'15*] [1st application ($K_{\ell 2}$), RM123, *'17*]

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$$\Gamma\left(E_\gamma^{\max}\right) = \Gamma_0 + \Gamma_1\left(E_\gamma^{\max}\right)$$

Total width
w/ either 0 or 1 γ

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IR-safe ***IR-safe***

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Requirement

E_γ^{max} small enough
to justify scalar-QED
approach

f.f.’s at low q^2

within factorization

$B_s \to \mu\mu\ \gamma$ with energetic γ

[Beneke-Bobeth-Wang, *'20*]

- *For low $q^2 \leq (6\ \mathrm{GeV})^2$, $B_s \to \gamma^*$ f.f.'s can be calculated in a systematic expansion in $1/m_b$, $1/E_\gamma$*

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  - *local NLP*
  - *non-local NLP* ●
    - ○ *actually dominant contribution by far*
    - ○ *escapes first-principle description*

similar to $B_u \to \ell\, \nu\, \gamma$

resonance paramet'n

Amplitude structure

[Beneke-Bobeth-Wang, *'20*]

- *Take the weak operators as* $O_i \equiv J_i^{(l)} \cdot J_i^{(q)}$

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FSR: only $S_\nu^{(10)} \neq 0$ $(\propto m_\ell)$ ⇨ *tiny*

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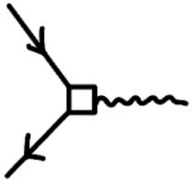
Main object to calculate

$$T_i^{\mu\nu} \propto \mathrm{FT}_x \langle 0| \, T\{J^\mu_{\mathrm{em}}(x), J_i^{(q)\,\nu}(0)\}|B\rangle$$

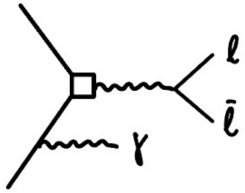
Notes on structure

[Beneke-Bobeth-Wang, *'20*]

- O_7 :



$T^{\mu\nu}_{7A}$:



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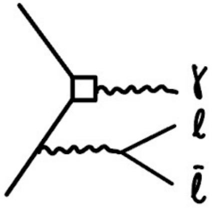

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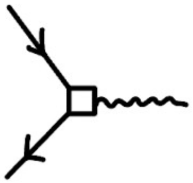

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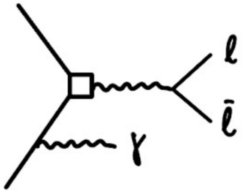

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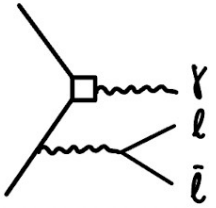

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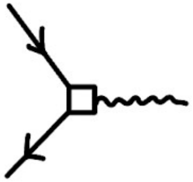

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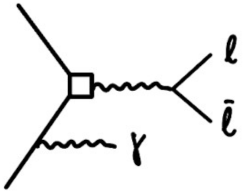

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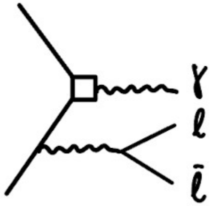

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- For $E_\gamma \gg \Lambda_{\text{QCD}}$ $$F_R^{(i)} \sim \frac{\Lambda_{\text{QCD}}}{E_\gamma} F_L^{(i)} \Rightarrow F_A^{(i)} \approx F_V^{(i)}$$

Two-step matching onto SCET

[Beneke-Bobeth-Wang, *'20*]

- *Decoupling of h modes O(m_b^2) in QCD → SCET$_I$ matching*

$$\sum_i^9 \eta_i C_i \, T_i^{\mu\nu} = \sum_i^9 C_i H_i(q^2) \cdot$$

$$\cdot \, \mathrm{FT}_x \langle 0| \, T\{J^\mu_{\mathrm{em,SCET_I}}(x), \left[\bar{q}_{\mathrm{hc}} \gamma_L^{\nu\perp} h_v\right](0)\} |B\rangle$$

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- *Decoupling of hc modes $O(E_\gamma\, \Lambda_{QCD}\, ;\, m_b\, \Lambda_{QCD})$ in $SCET_I$ → $SCET_{II}$*

Resonances

[Beneke-Bobeth-Wang, *'20*]

- $T_{7B}^{\mu\nu}$ *leads to* $\overline{A}_{res}$
  - *standard spectral repr. (à la BW)*
  - *formally power-suppressed*

    hence inclusion won't lead to double counting
    of some short-distance contributions

Concluding comments

[Beneke-Bobeth-Wang, *'20*]

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- *Large NLP + little phase space available + large λ_B dependence challenge a precise $B_s \to \mu\mu\,\gamma$ prediction at low q^2*

- *Prediction*

$$\langle \mathcal{B} \rangle_{[4m_\mu^2,\,6.0]} = \left(12.51^{+3.83}_{-1.93}\right) \cdot 10^{-9}, \qquad \langle \mathcal{B} \rangle_{[2.0,\,6.0]} = \left(0.30^{+0.25}_{-0.14}\right) \cdot 10^{-9}$$

i.e. ϕ region gives 97.6% of the BR

f.f.'s within LCSRs

[Janowski, Pullin, Zwicky, *'21*]

see also [Pullin, Zwicky, *'21;* Albrecht *et al.*, *19*]

- *Calculation includes: NLO at twist 1&2; LO at twist 3; partial twist 4*

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- *f.f.'s fitted to a z-expansion ansatz*

$$F_n^{\bar{B}\to\gamma}(q^2) = \frac{1}{1 - q^2/m_R^2}\left(\alpha_{n0} + \sum_{k=1}^{N} \alpha_{nk}(z(q^2) - z(0))^k\right)$$

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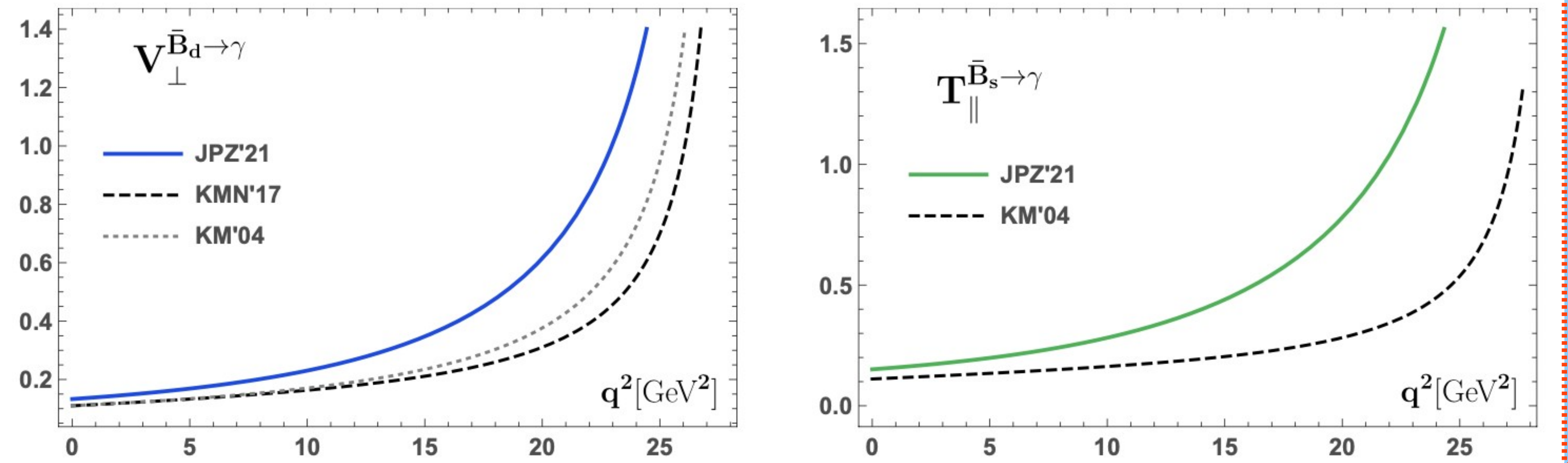
- *Comparison with the quark-model f.f. parameterizations in*

[Melikhov, Nikitin, '04; Kozachuk, Melikhov, Nitikin, '17]



Some specific observables

Guidelines

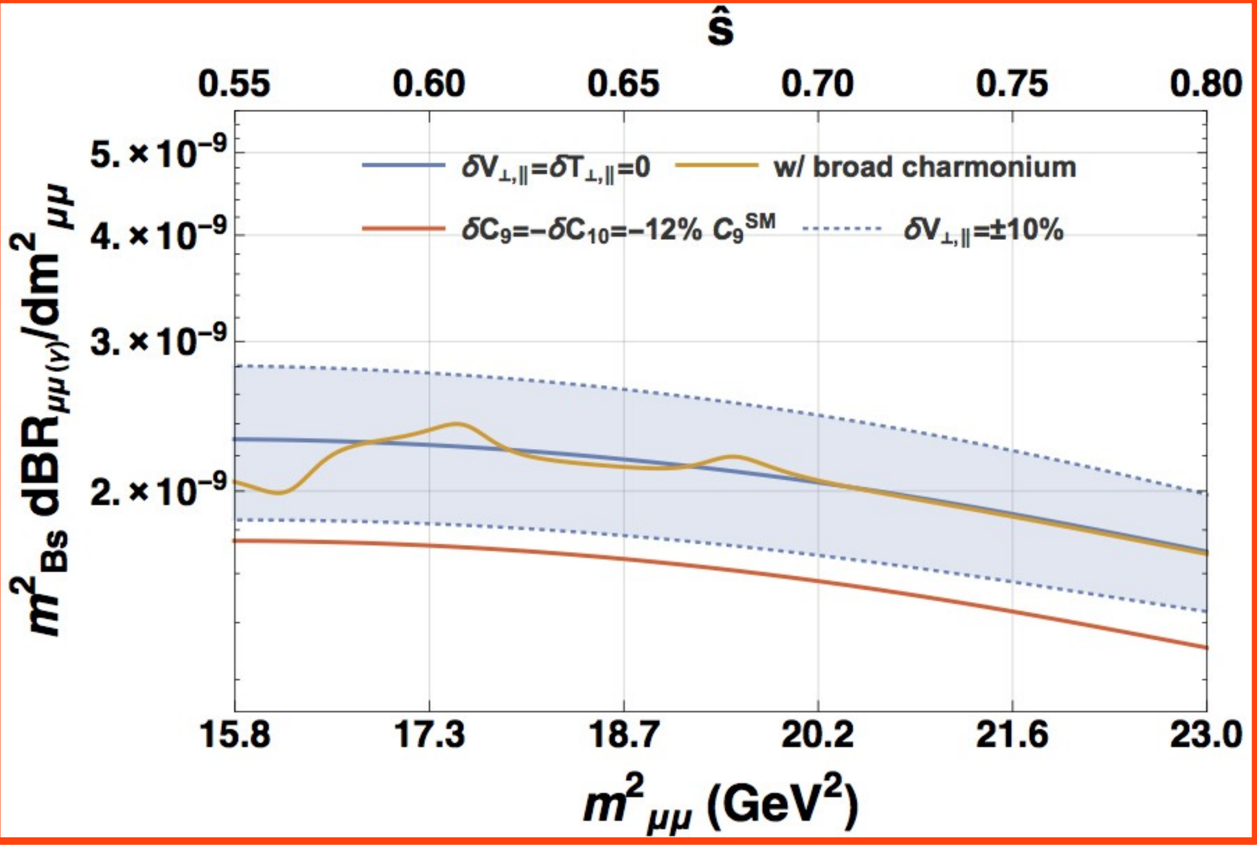
- *focus on high q^2*
- *minimise dependence on LD physics*

$B_s \to \mu\mu\gamma$ spectrum

- *In [DG, Reboud, Zwicky, '17] resonant ansatz used to rewrite low-q^2 BR in terms of the measured BR($B_s \to \phi\gamma$)*

$B_s \rightarrow \mu\mu\gamma$ spectrum

- *In [DG, Reboud, Zwicky, '17] resonant ansatz used to rewrite low-q^2 BR in terms of the measured BR($B_s \rightarrow \phi\gamma$)*
- *Then main focus on large-q^2 region, above narrow charmonium. Broad-charmonium pollution estimated with similar resonant ansatz*



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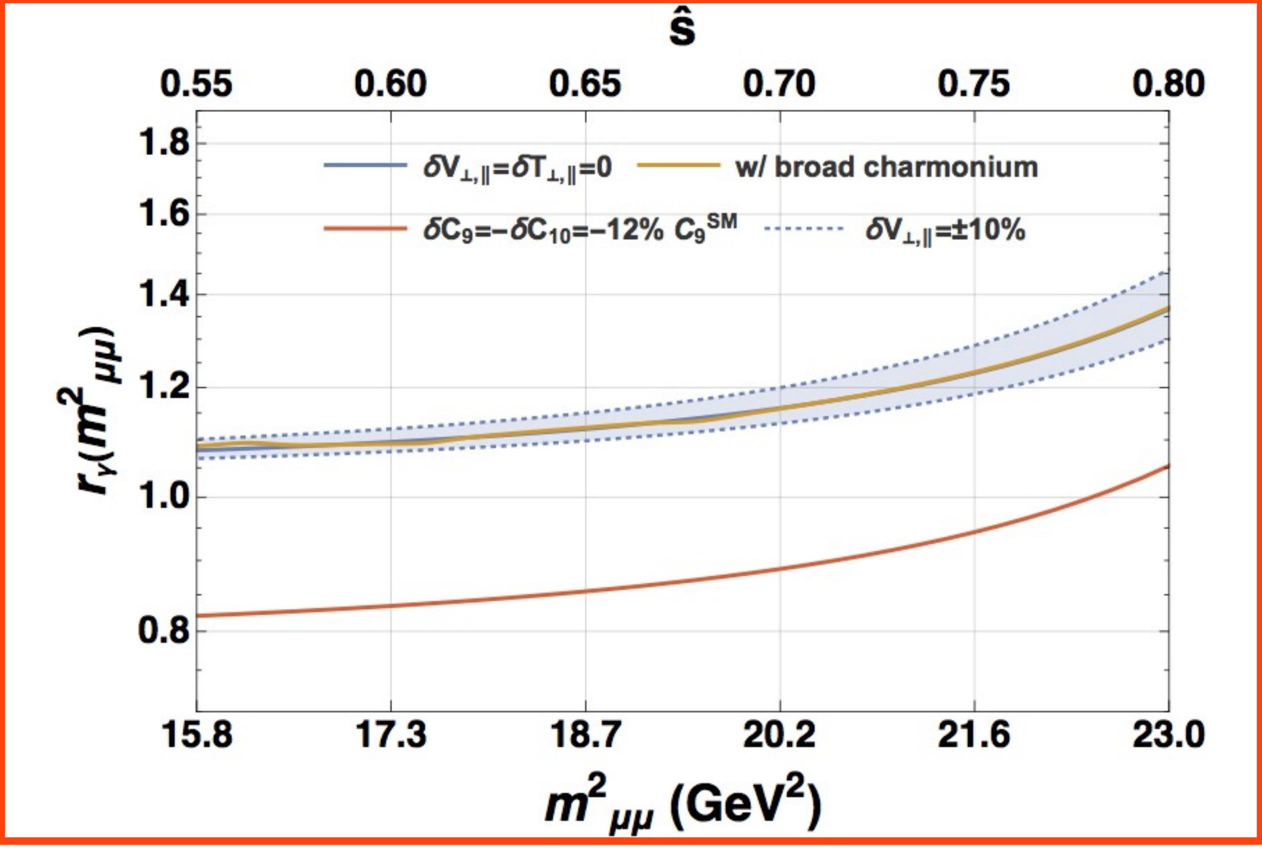
- Then main focus on large-q^2 region, above narrow charmonium. Pollution substantially tamed in suitable ratio observable



$$r_\gamma \equiv \frac{d\,BR(B_s \to \mu\mu\gamma)/dq^2}{d\,BR(B_s \to e\,e\,\gamma)/dq^2}$$

$B_s \to \mu\mu\gamma$ effective lifetime

[de Bruyn et al., *'12*]

- *Natural exp observable: untagged rate*

$$\langle \Gamma(B_s(t) \to f) \rangle \;\equiv\; \Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f)$$

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Recalling the time dependence of the respective |amplitudes|²

$$\begin{aligned} |\overset{(-)}{\mathcal{A}}_f(t)|^2 \;&=\; \frac{e^{-\Gamma_s t}}{2}\Big[\Big(|\mathcal{A}_f|^2 + |q/p|^2|\bar{\mathcal{A}}_f|^2\Big)\cosh(\Delta\Gamma_s\, t/2) \pm \Big(|\mathcal{A}_f|^2 - |q/p|^2|\bar{\mathcal{A}}_f|^2\Big)\cos(\Delta M_s\, t) \\ &- \;2\,\mathrm{Re}\left(q/p\,\bar{\mathcal{A}}_f\mathcal{A}_f^*\right)\sinh(\Delta\Gamma_s\, t/2) \mp 2\,\mathrm{Im}\left(q/p\,\bar{\mathcal{A}}_f\mathcal{A}_f^*\right)\sin(\Delta M_s\, t) \end{aligned}$$

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yields the following quantity sensitive to new CPV

$$A^f_{\Delta\Gamma_s} = \frac{-2\int_{\mathrm{PS}}\mathrm{Re}\left(q/p\,\bar{\mathcal{A}}_f\mathcal{A}_f^*\right)}{\int_{\mathrm{PS}}\left(|\mathcal{A}_f|^2 + |q/p|^2|\bar{\mathcal{A}}_f|^2\right)}$$

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- *$A_{\Delta\Gamma}$ can be extracted from (an accurate measurement of) the effective lifetime*

Motivation

[Carvunis et al., '21]

- $A_{\Delta\Gamma}$ looks like a natural "ratio-of-amplitudes-squared" observable

  With some luck, new CP phases may sizeably "misalign" numerator/denominator w.r.t. SM

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- *NP with non-standard CPV less constrained than NP with CKM CPV*

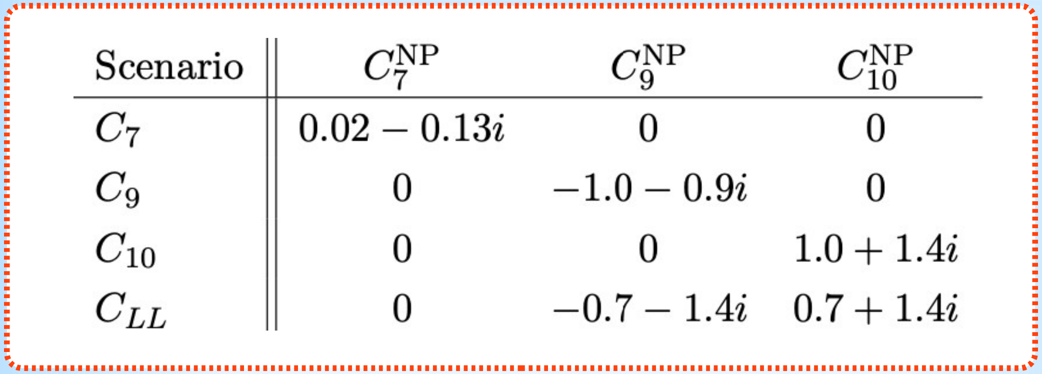
  (For NP with non-standard CPV, also constraints on Re(WCs) get looser)

Strategy

[Carvunis et al., '21]

- *Identify NP scenarios (within WET) accounting for the anomalies & with large CPV on top*

  (Wealth of b → s data still under-constraining for WC shifts w/ large non-CKM weak phases.)



Scenario	C_7^{NP}	C_9^{NP}	C_{10}^{NP}
C_7	$0.02 - 0.13i$	0	0
C_9	0	$-1.0 - 0.9i$	0
C_{10}	0	0	$1.0 + 1.4i$
C_{LL}	0	$-0.7 - 1.4i$	$0.7 + 1.4i$

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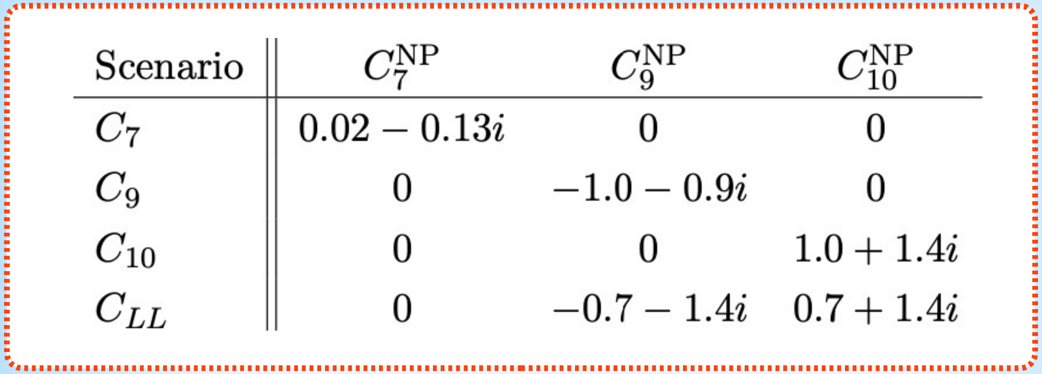

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- *Survey* $A_{\Delta\Gamma}$ *sensitivity to these scenarios*
  - *for both low and high* q^2
  - *taking into account f.f. & resonance-modelling errors*

$A_{\Delta\Gamma}$ at high q^2

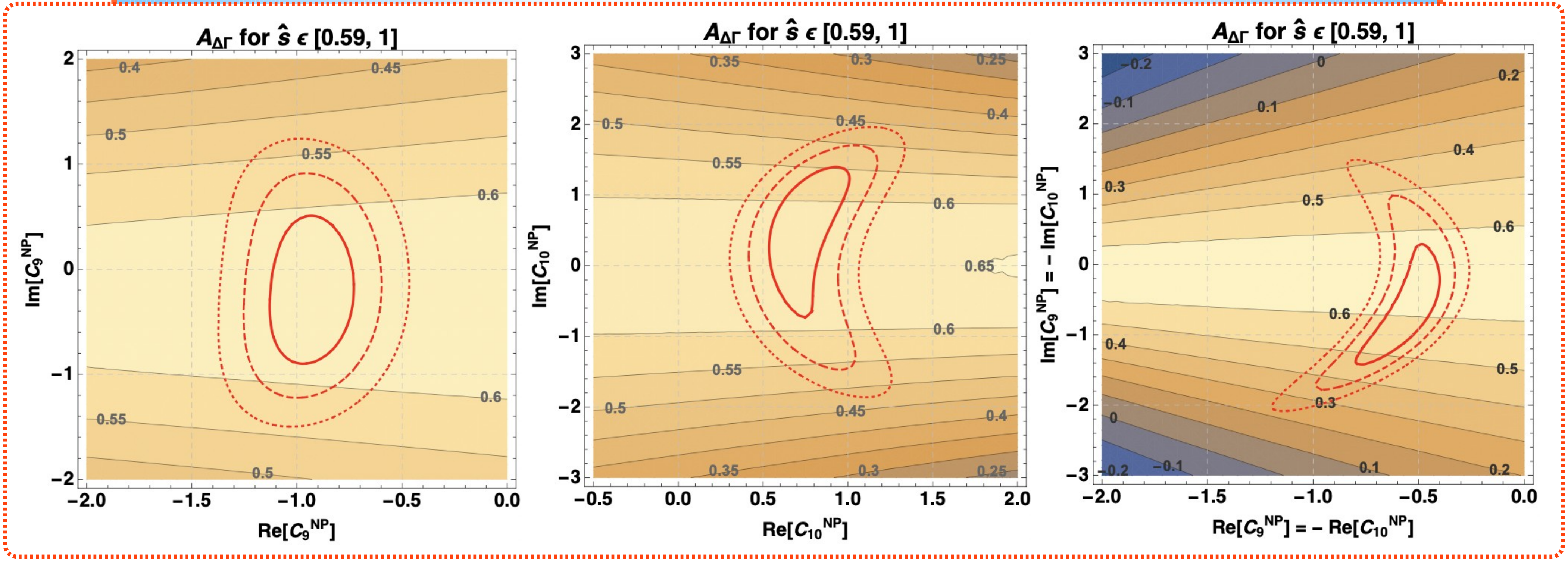
[Carvunis et al., '21]

- *Consider the range* $s \in [(4.1\ \text{GeV})^2\ ,\ m_{B_s}^2] = [0.59, 1]\ m_{B_s}^2$

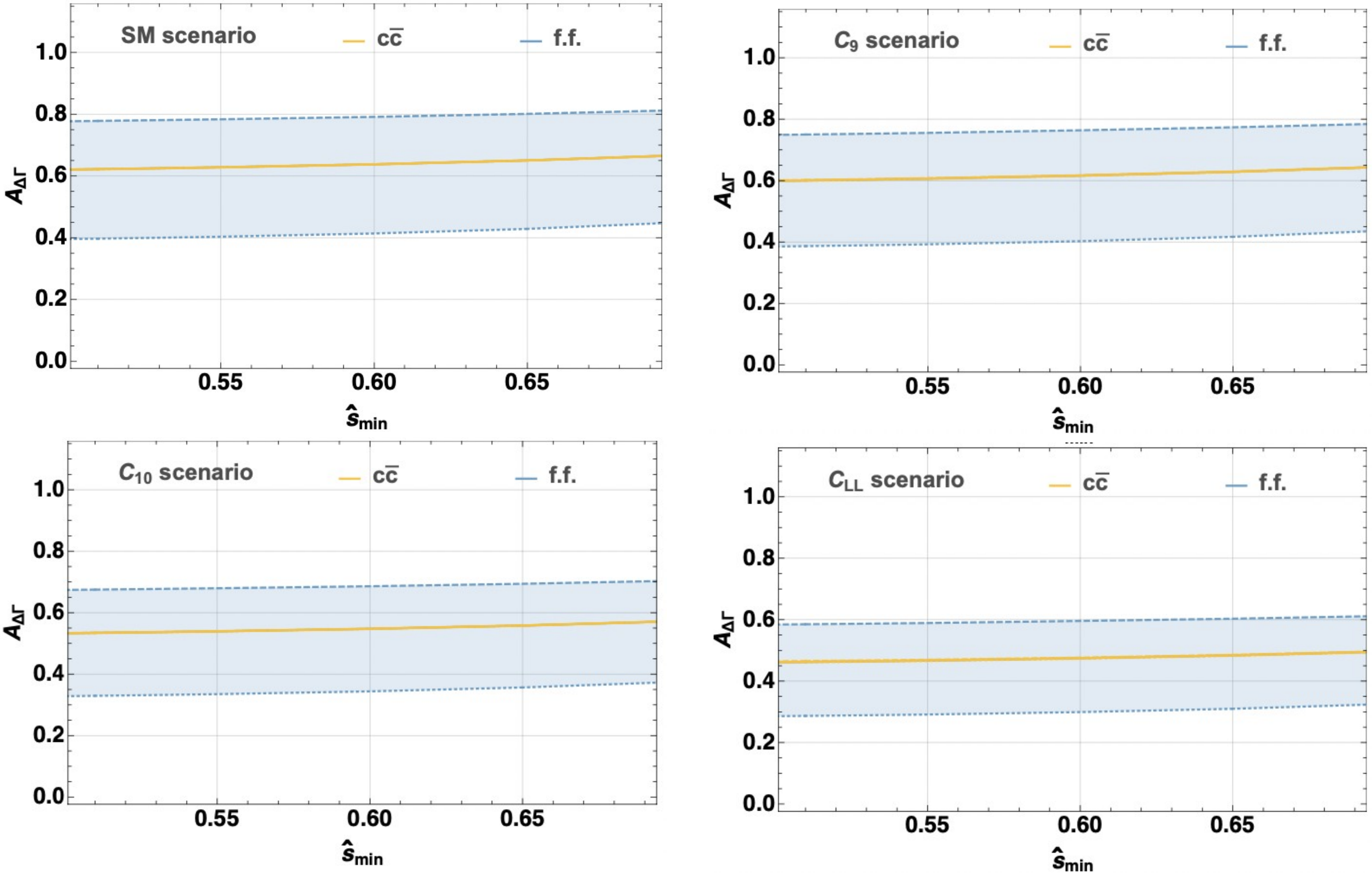
  We set FSR to 0.

  We keep ISR-FSR interference (not subtracted by PHOTOS, but small)

- *Size of effects* $\lesssim 30\%$ *(mostly* C_9, C_{10}, C_{LL}*)*



Impact of broad $c\bar{c}$



Impact of broad $c\bar{c}$

[Carvunis et al., ‘21]

- Bottom line: broad $c\bar{c}$ has surprisingly small impact on $A_{\Delta\Gamma}$

  But broad-$c\bar{c}$ shift to C_9 typically O(5%) – and with random phase

↳ Far from obvious why such a small impact on $A_{\Delta\Gamma}$

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  ⮡ Far from obvious why such a small impact on $A_{\Delta\Gamma}$

- Closer look (App. D for an analytic understanding)

  Cancellation is a conspiracy between

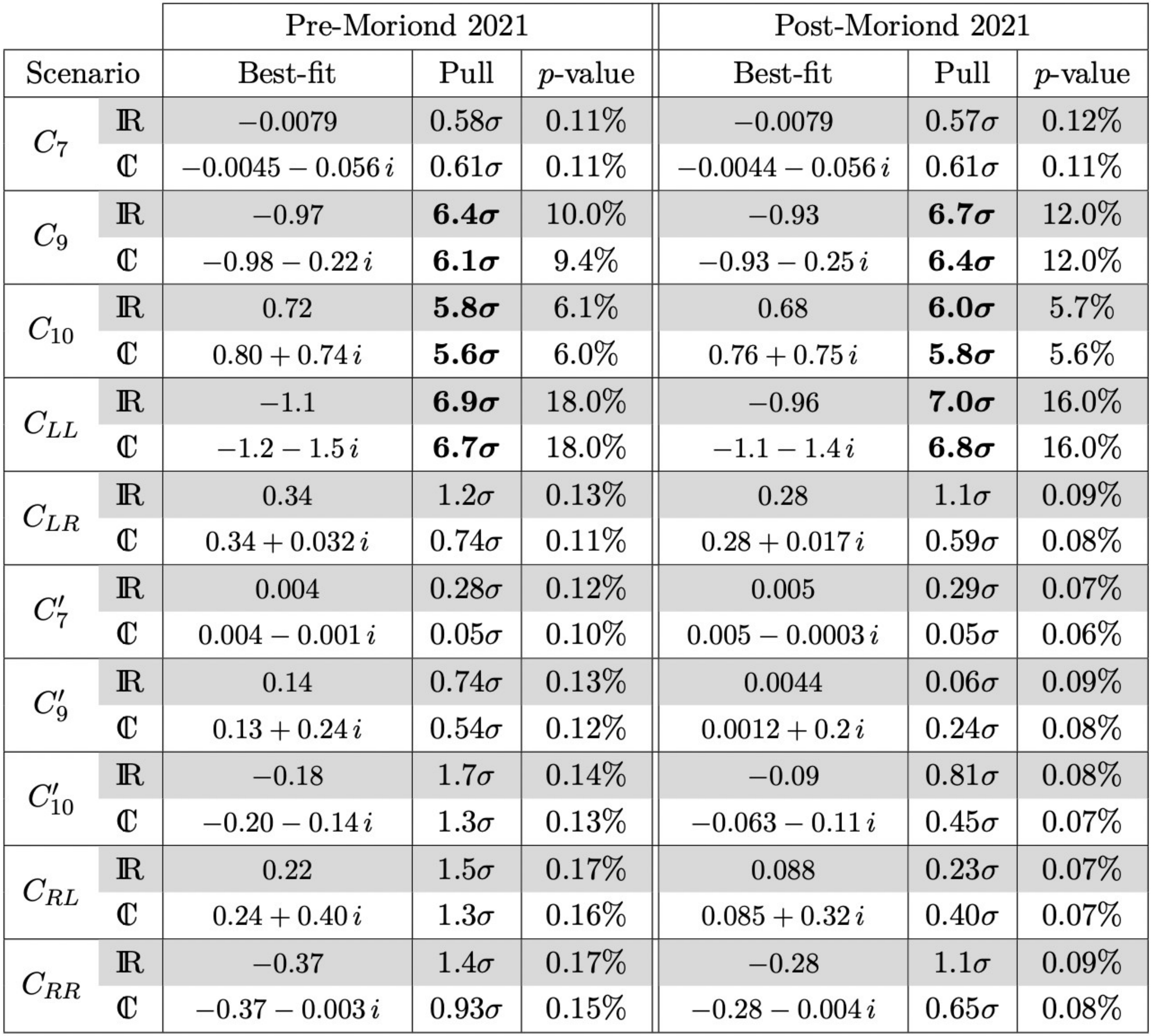
  - Complete dominance of contributions quadratic in C_9 and C_{10}
  - Multiplying f.f.’s F_V, F_A $\in \mathbb{R}$
  - Broad $c\bar{c}$ can be treated as small modif. of (numerically large) C_9

  ⮡ Ease cancellations between num & den in $A_{\Delta\Gamma}$

- Low impact of broad $c\bar{c}$ encouraging, given that this systematics inherently escapes a rigorous description
- f.f. uncertainty, even if still large, in principle “reducible”
- Maybe worthwhile to look for more observables with such properties

Spares

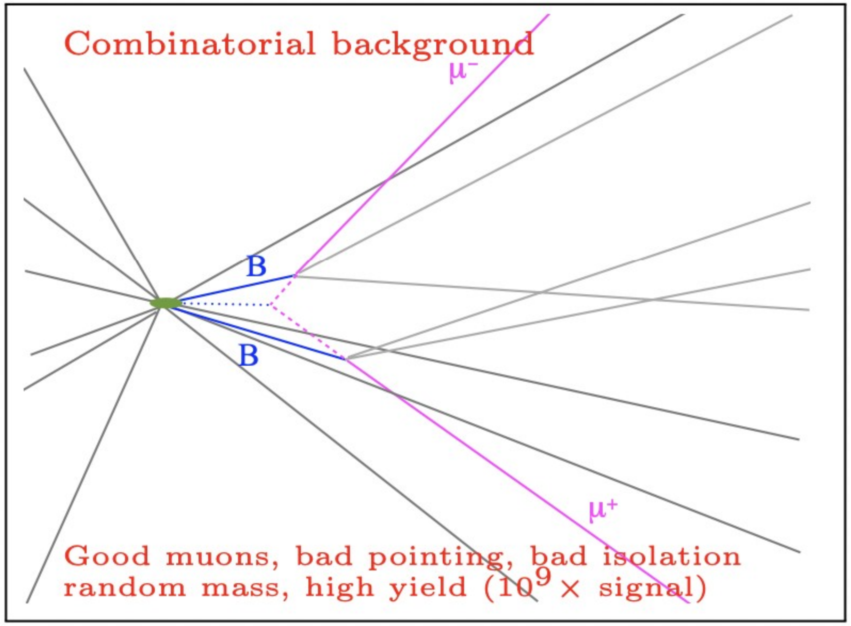
Im shifts to WCs: how large?



		Pre-Moriond 2021			Post-Moriond 2021		
Scenario		Best-fit	Pull	p-value	Best-fit	Pull	p-value
C_7	$\mathbb{R}$	-0.0079	0.58σ	0.11%	-0.0079	0.57σ	0.12%
	$\mathbb{C}$	$-0.0045 - 0.056\,i$	0.61σ	0.11%	$-0.0044 - 0.056\,i$	0.61σ	0.11%
C_9	$\mathbb{R}$	-0.97	$\boldsymbol{6.4\sigma}$	10.0%	-0.93	$\boldsymbol{6.7\sigma}$	12.0%
	$\mathbb{C}$	$-0.98 - 0.22\,i$	$\boldsymbol{6.1\sigma}$	9.4%	$-0.93 - 0.25\,i$	$\boldsymbol{6.4\sigma}$	12.0%
C_{10}	$\mathbb{R}$	0.72	$\boldsymbol{5.8\sigma}$	6.1%	0.68	$\boldsymbol{6.0\sigma}$	5.7%
	$\mathbb{C}$	$0.80 + 0.74\,i$	$\boldsymbol{5.6\sigma}$	6.0%	$0.76 + 0.75\,i$	$\boldsymbol{5.8\sigma}$	5.6%
C_{LL}	$\mathbb{R}$	-1.1	$\boldsymbol{6.9\sigma}$	18.0%	-0.96	$\boldsymbol{7.0\sigma}$	16.0%
	$\mathbb{C}$	$-1.2 - 1.5\,i$	$\boldsymbol{6.7\sigma}$	18.0%	$-1.1 - 1.4\,i$	$\boldsymbol{6.8\sigma}$	16.0%
C_{LR}	$\mathbb{R}$	0.34	1.2σ	0.13%	0.28	1.1σ	0.09%
	$\mathbb{C}$	$0.34 + 0.032\,i$	0.74σ	0.11%	$0.28 + 0.017\,i$	0.59σ	0.08%
C'_7	$\mathbb{R}$	0.004	0.28σ	0.12%	0.005	0.29σ	0.07%
	$\mathbb{C}$	$0.004 - 0.001\,i$	0.05σ	0.10%	$0.005 - 0.0003\,i$	0.05σ	0.06%
C'_9	$\mathbb{R}$	0.14	0.74σ	0.13%	0.0044	0.06σ	0.09%
	$\mathbb{C}$	$0.13 + 0.24\,i$	0.54σ	0.12%	$0.0012 + 0.2\,i$	0.24σ	0.08%
C'_{10}	$\mathbb{R}$	-0.18	1.7σ	0.14%	-0.09	0.81σ	0.08%
	$\mathbb{C}$	$-0.20 - 0.14\,i$	1.3σ	0.13%	$-0.063 - 0.11\,i$	0.45σ	0.07%
C_{RL}	$\mathbb{R}$	0.22	1.5σ	0.17%	0.088	0.23σ	0.07%
	$\mathbb{C}$	$0.24 + 0.40\,i$	1.3σ	0.16%	$0.085 + 0.32\,i$	0.40σ	0.07%
C_{RR}	$\mathbb{R}$	-0.37	1.4σ	0.17%	-0.28	1.1σ	0.09%
	$\mathbb{C}$	$-0.37 - 0.003\,i$	0.93σ	0.15%	$-0.28 - 0.004\,i$	0.65σ	0.08%

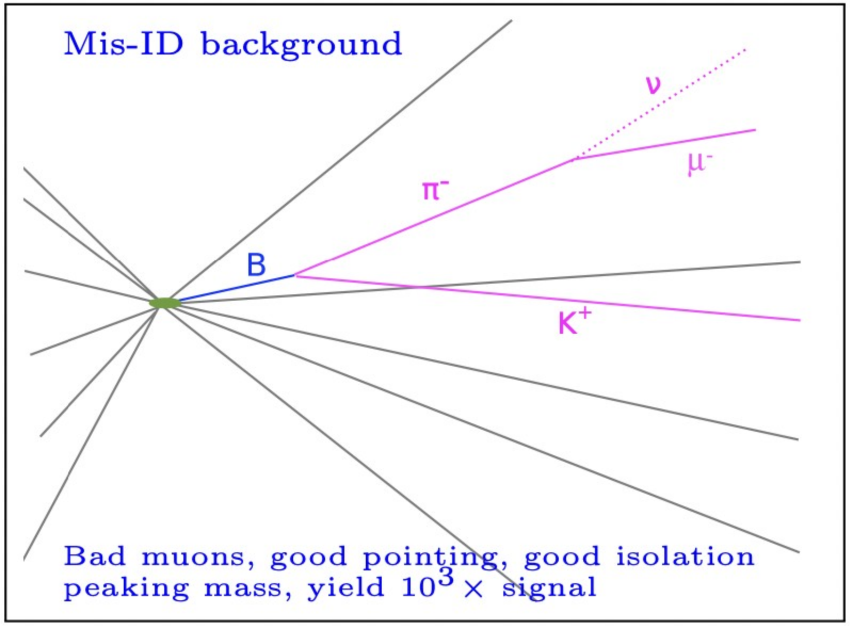
Backgrounds

[thanks F. Dettori]



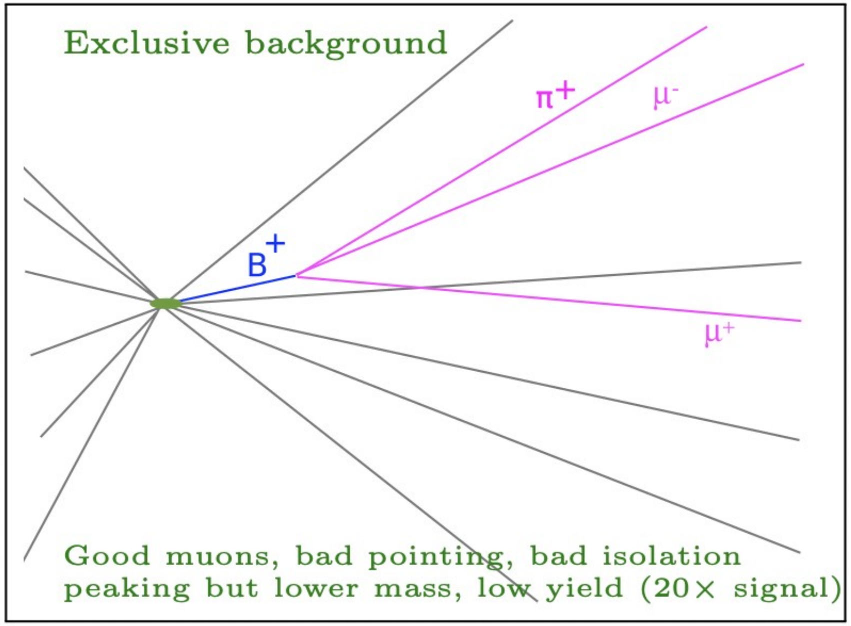
Combinatorial background

Good muons, bad pointing, bad isolation random mass, high yield ($10^9\times$ signal)



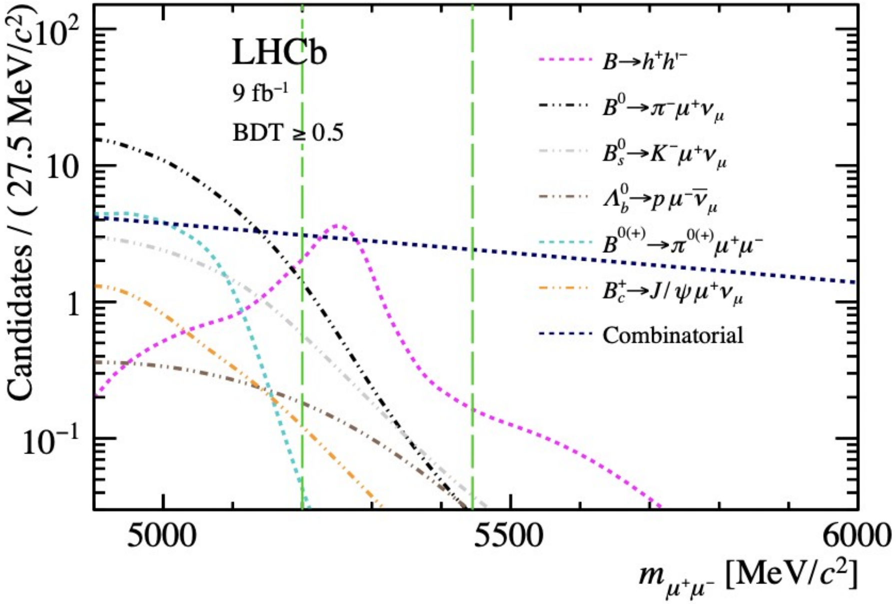
Mis-ID background

Bad muons, good pointing, good isolation peaking mass, yield $10^3\times$ signal



Exclusive background

Good muons, bad pointing, bad isolation peaking but lower mass, low yield ($20\times$ signal)



[LHCb-PAPER-2021-007] [LHCb-PAPER-2021-008]

Radiative leptonic f. f.'s in LQCD

Large E_γ

- *The required correlator (weak & e.m. current insertion between a B and the vac) has always the desired large-Euclidean-t behavior*

  [Kane, Lehner, Meinel, Soni, *'19*]

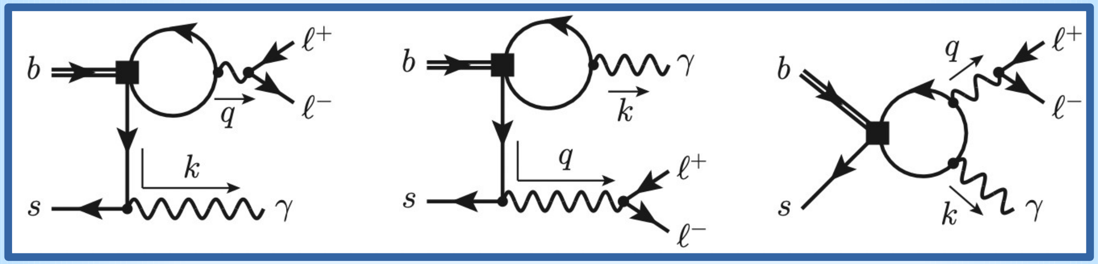
  Note that this is non-trivial – e.g. it doesn't seem to hold if there are hadronic final states along with the γ

- *However, the low-*q^2 *spectrum of* $B_s \to \mu\mu\ \gamma$ *is dominated by resonant contributions (~98% of the BR), that LQCD is unable to capture*

NLP

[Beneke-Bobeth-Wang, ‘20]

- *Three sources*
  - *coupling of γ to b quark*
  - *power corr’s to SCET$_I$ correlator at tree level*
  - *annihilation-type insertions of 4q operators* ⇨ *local*



- *Two soft f.f.’s*
  - $\xi(E_\gamma)$: *computable as in* $B_u \to \ell\, \nu\, \gamma$ [Beneke-Rohrwild, ‘11]
  - *For B-type contributions:* $\tilde{\xi}(E_\gamma)$
    Its Im develops resonances, thus escaping a factorization description

Impact of broad $c\bar{c}$

[Carvunis et al., '21]

- Parameterize the effect most generally (e.g. discussion in [Lyon, Zwicky, *'14*])

$$C_9 \to C_9 - \frac{9\pi}{\alpha^2}\bar{C}\sum_V |\eta_V| e^{i\delta_V} \frac{\hat{m}_V\,\mathcal{B}(V \to \mu^+\mu^-)\,\hat{\Gamma}^V_{\rm tot}}{\hat{q}^2 - \hat{m}_V^2 + i\hat{m}_V\hat{\Gamma}^V_{\rm tot}}$$

  - $|\eta_V| \in [1, 3]$ & $\delta_V \in [0, 2\pi)$ (uniformly and independently for the 5 resonances)
  - for $s_{min} \in [0.5, 0.7]\ m_{Bs}^2$
  - for all TH scenarios

$s_{\psi(2S),\ \psi(3770),\ \psi(4040),\ \psi(4160),\ \psi(4415)} = \{0.47, 0.49, 0.57, 0.61, 0.68\}$

Impact of f.f. error

[Carvunis et al., '21]

- We vary (JPZ) f.f.'s with uncorrelated normal distrib's around their errors

  Resulting f.f. error by far dominant w.r.t. $c\bar{c}$

  - Broad $c\bar{c}$ only shifts C_9 ⇨ efficient cancellations possible
  - f.f.'s enter in different ways (all numerically relevant) for the different WC combinations

- In short

  - f.f. error still too important to resolve between TH scenarios
  - Yet, dominance of jointly C_9 & C_{10} implies high sensitivity to C_{LL}

    ⇨ could be resolvable with ~ half the current f.f. error