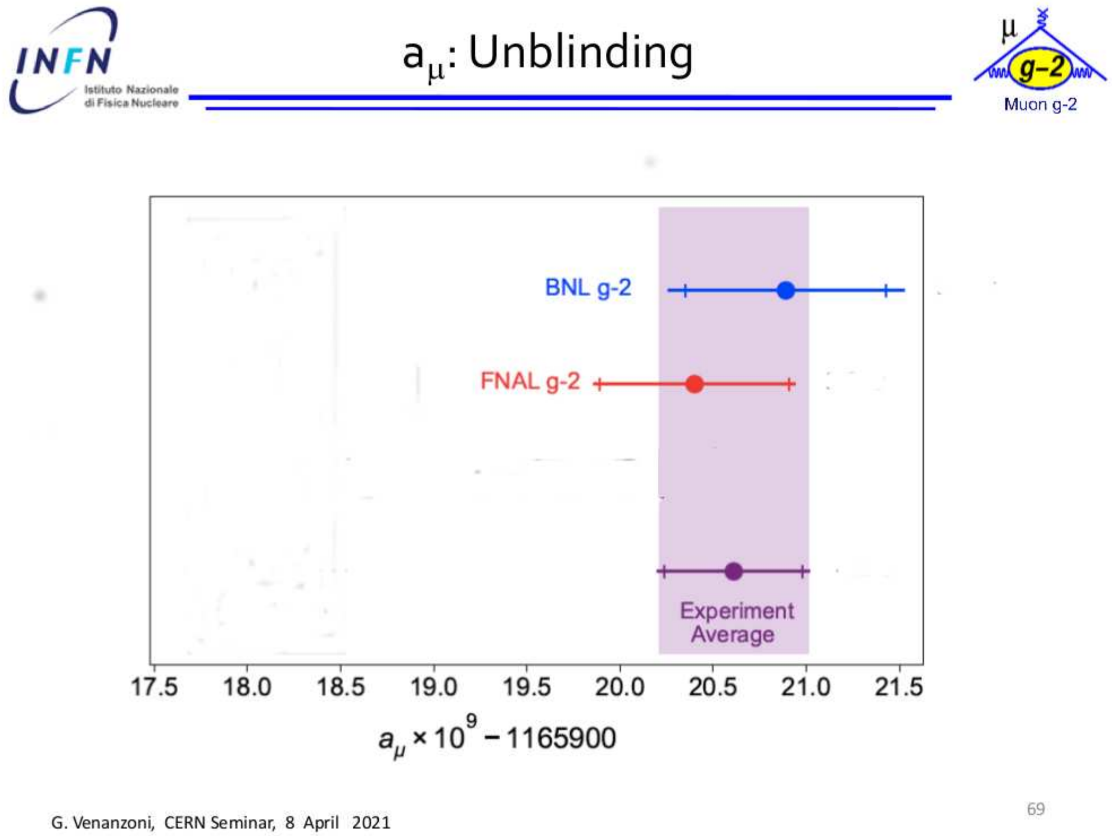
The Standard Model Prediction of the muon g-2: A Critical Review

Marc Knecht

Centre de Physique Théorique UMR7332,
CNRS Luminy Case 907, 13288 Marseille cedex 09 - France
knecht@cpt.univ-mrs.fr

5th Workshop on flavour changing and conserving processes (FCCP2022) – Anacapri, Sept. 22 - 24, 2022

Present situation



New measurement by FNAL-E989

$$a_{\mu^+}^{\mathrm{E989}} = 116\,592\,040(54) \cdot 10^{-11} \quad [0.46\mathrm{ppm}]$$

B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

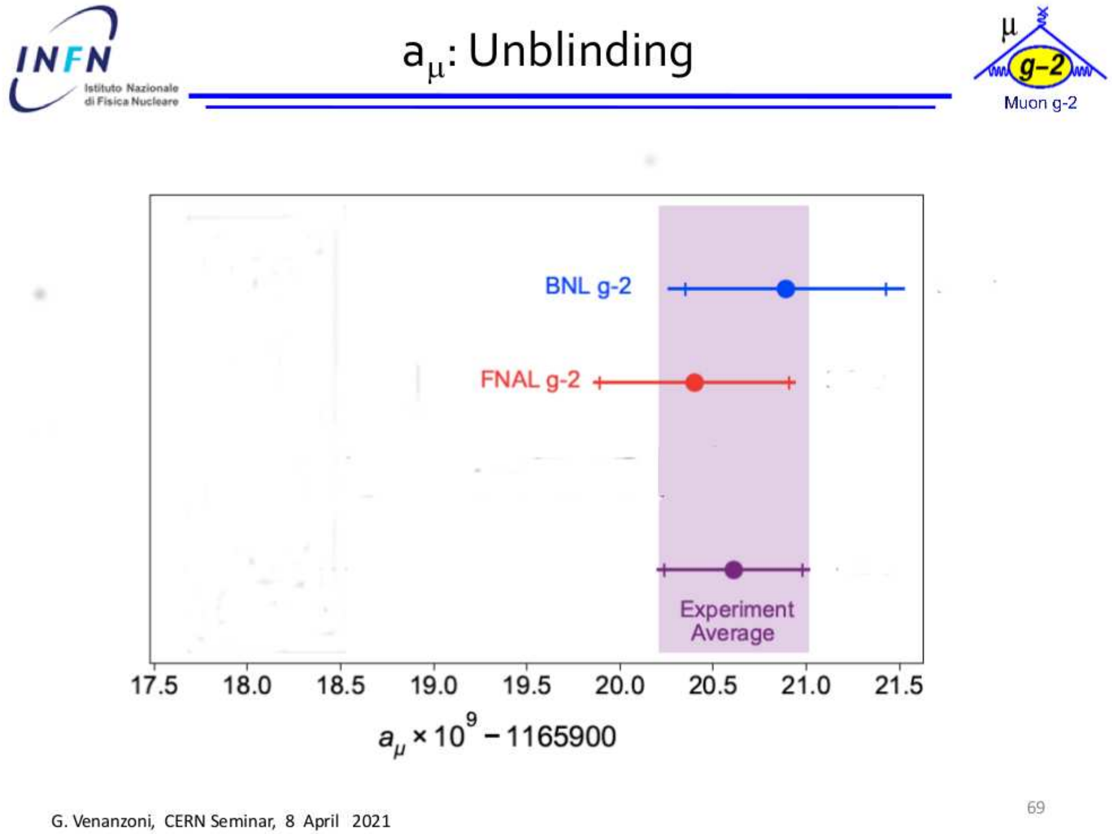
In line with previous BNL-E821 measurement ~ 20 years ago

$$a_{\mu}^{\mathrm{E821}} = 116\,592\,089(63) \cdot 10^{-11} \quad [0.54\mathrm{ppm}]$$

G. W. Bennett et al. [Muon g-2 Coll.], PRD 73, 072003 (2006) and in line with decades of muon storage ring experiments

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
BNL E821	1999	μ^+	11 659 202(15)	1.3
BNL E821	2000	μ^+	11 659 204(9)	0.73
BNL E821	2001	μ^-	11 659 214(9)	0.72
FNAL E989	2021	μ^+	11 659 2040(54)	0.46
FNAL E989	2023?	μ^+	???	$\sim$ 0.14
[J-PARC E34	2027?	μ^+	???	$\sim$ 0.45]

Present experimental situation $\longrightarrow$ quite satisfactory



$$a_\mu^{\mathrm{exp;WA}} = 116\,592\,061(41) \cdot 10^{-11} \;\; [0.35\mathrm{ppm}]$$

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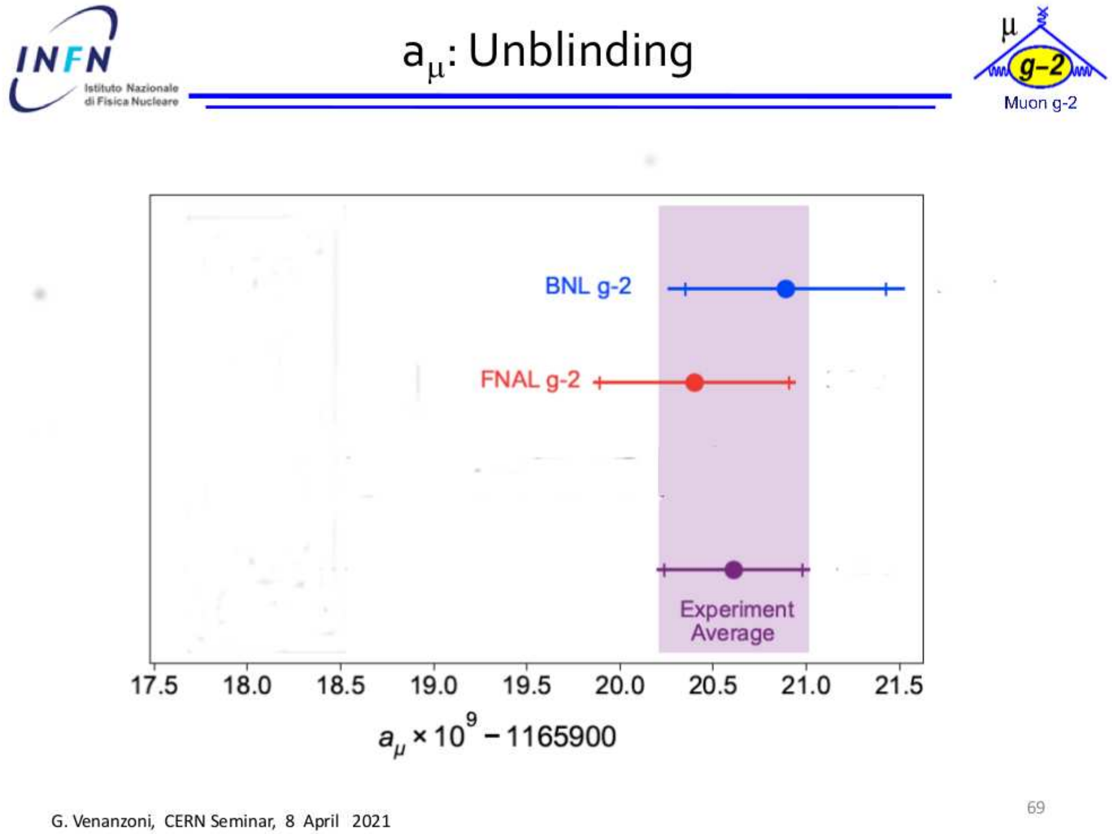

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Theory situation?

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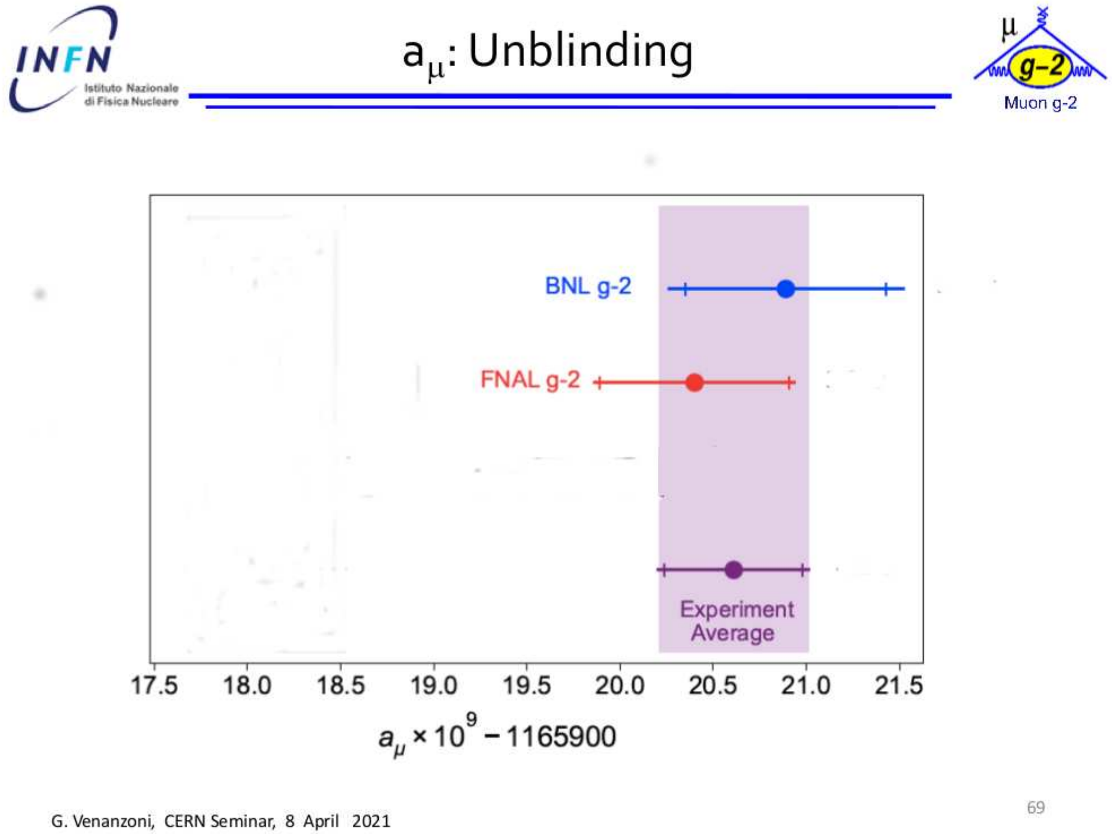

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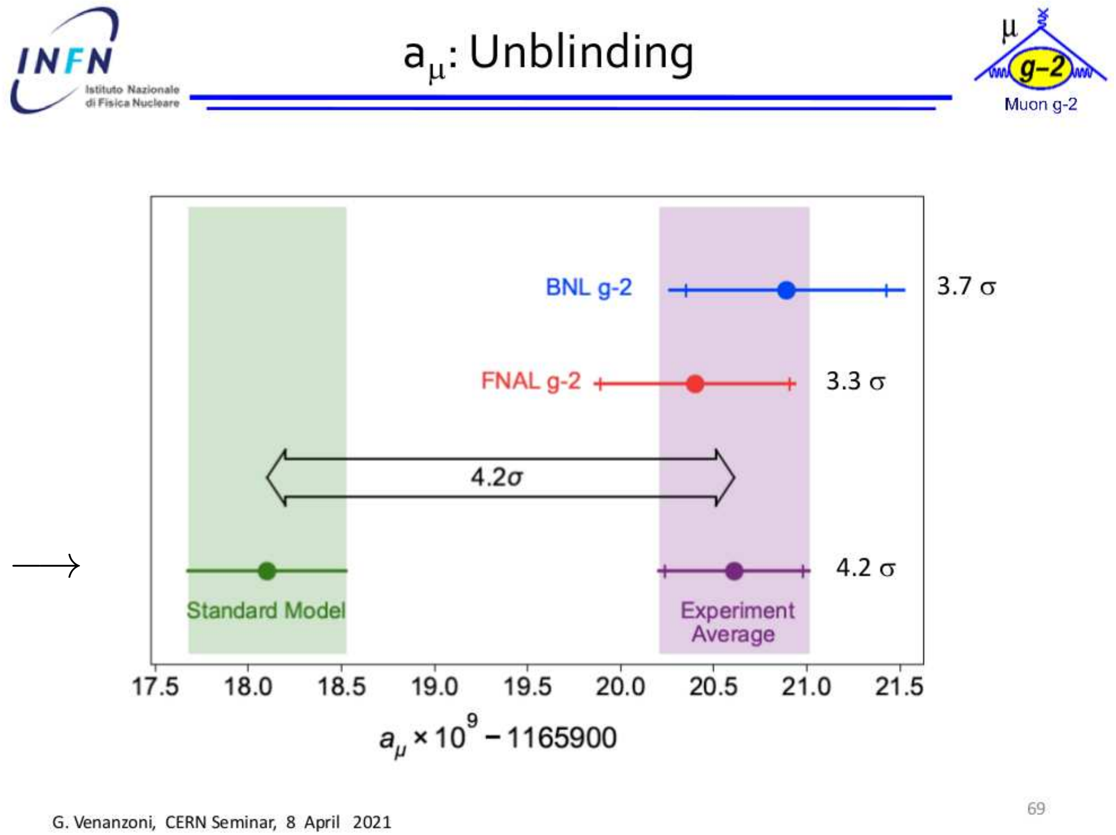
Theory situation?

Full and detailed account [up to June 15, 2020] given in the White Paper

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

Present situation

White Paper ⟶



$$a_\mu^{\text{th;WP}} = 116\,591\,810(43) \cdot 10^{-11} \quad [0.35\text{ppm}]$$

Leads to a discrepancy th. vs. exp. at the level of 4.2σ

$$a_\mu^{\text{exp;WA}} - a_\mu^{\text{th;WP}} = 251(59) \cdot 10^{-11} \quad [4.2\sigma]$$

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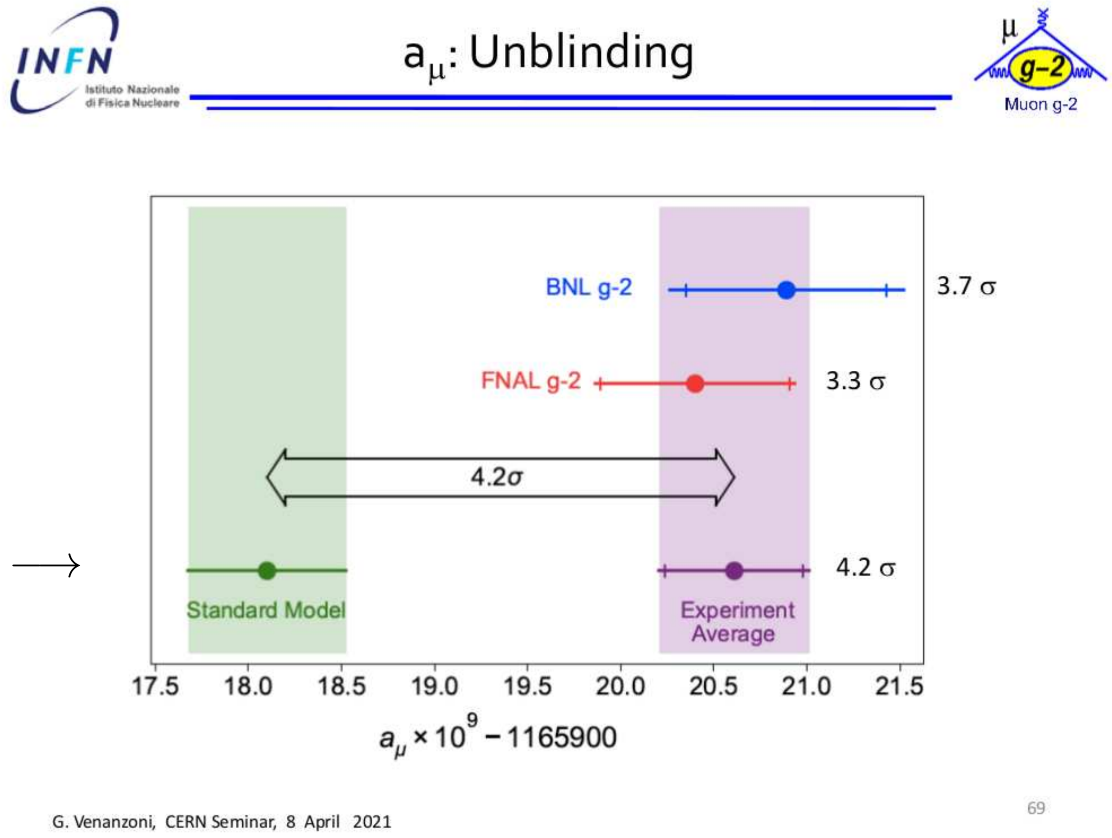

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Does $a_\mu^{\mathrm{th;WP}} = a_\mu^{\mathrm{th;SM}}$ still hold today?

Contribution from QED

QED contribution :

$\longrightarrow$ loops with only photons and leptons

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$$(\alpha/\pi)^4 = 2.91\ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76\ldots \cdot 10^{-14}$$

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$\longrightarrow$ becomes technically challenging $(1, 6, 72, 891, 12\,672, \ldots)$

$$a_\mu{}^{\text{QED}} \;=\; C_\mu^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\mu^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\mu^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\mu^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\mu^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

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$\longrightarrow$ multiflavour QED

$$C_\mu^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\mu/m_e) + A_2^{(2n)}(m_\mu/m_\tau) + A_3^{(2n)}(m_\mu/m_e, m_\mu/m_\tau)$$

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$$\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$$

A few comments about the QED contributions

- expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)

A. Petermann, Phys. Rev. 105, 1931 (1955)

H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

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- $A_1^{(8)}$ has also been evaluated! ($\longrightarrow a_e$) S. Laporta, Phys. Lett. B 772, 232 (2017)
- Mass dependent parts $A_2^{(8)}(m_\mu/m_e)$, $A_2^{(8)}(m_\mu/m_\tau)$, $A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau)$ evaluated numerically

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 91, 033006 (2015)

and crossed-checked by independent QFT methods A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

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A. Kurz et al., Nucl. Phys. B 879, 1 (2014); Phys. Rev. D 92, 073019 (2015)

- $(\alpha/\pi)^5$: 6 classes, 32 gauge invariant subsets

Five of these subsets are known analytically S. Laporta, Phys. Lett. B 328, 522 (1994)

J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results available

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

An independent numerical evaluation of $A_1^{(10)}$ is in progress

S. Volkov, Phys. Rev. D 98, 076018 (2018); arXiv:1905.08007; Phys.Rev.D 100, 096004 (2019)

$\longrightarrow$ discrepancy [4.8σ] found in the contribution of graphs without fermion loops ($\longrightarrow a_e$)

QED contribution :

$C_\mu^{(2)}$	0.5
$C_\mu^{(4)}$	$0.765\,857\,425(17)$
$C_\mu^{(6)}$	$24.050\,509\,96(32)$
$C_\mu^{(8)}$	$130.878\,0(61)$
$C_\mu^{(10)}$	$750.72(93)$

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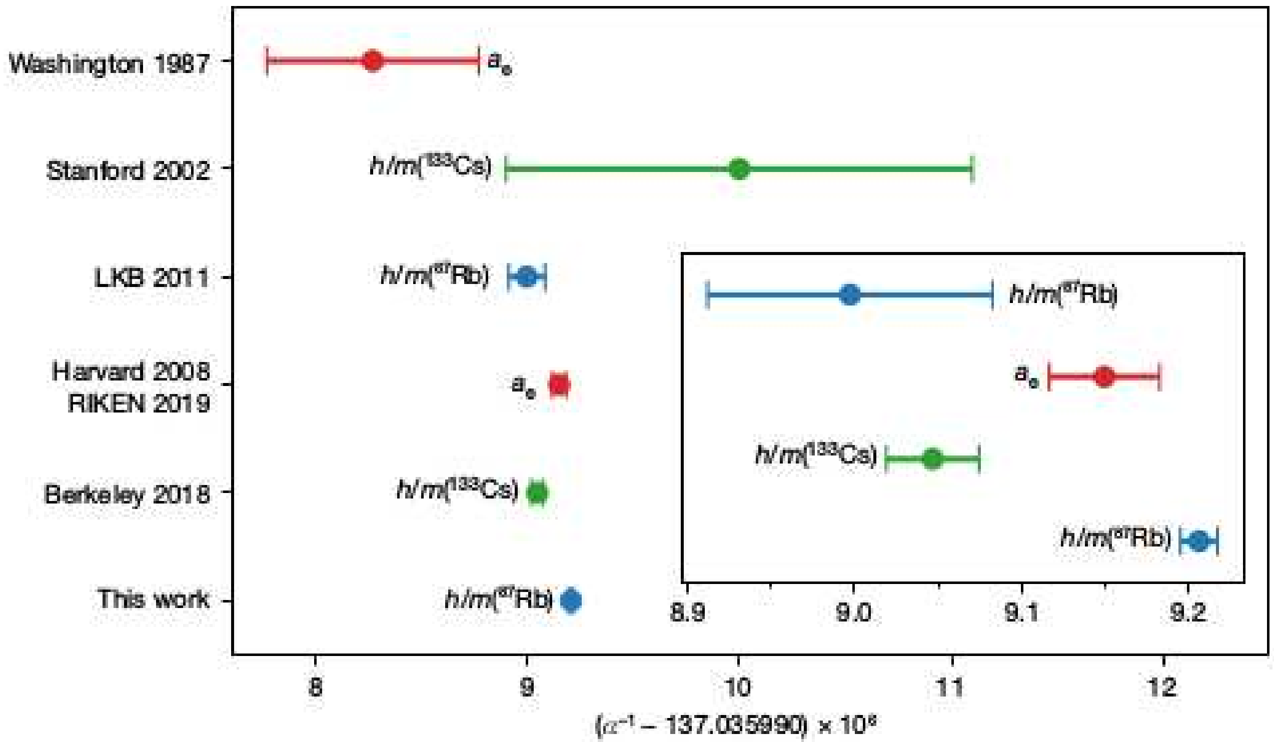
Requires an experimental determination of α with

$$\frac{\Delta\alpha}{\alpha} \sim \frac{\Delta a_\mu}{a_\mu} \sim 0.14\text{ppm}$$



$\longrightarrow$ existing tension/discrepancy between $\alpha(Cs18)$ and $\alpha(Rb20)$ (but also between $\alpha(Rb11)$ and $\alpha(Rb20)$) of no concern for a_μ

$\longrightarrow$ for a_μ the value of α could be provided by the qH effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \quad [19.7\text{ppb}]$$

P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)

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- $a_\mu^{\text{QED}}(Cs19) = 116\,584\,718.931(7)_{\text{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs19)} \cdot 10^{-11}$

- $$a_\mu^{\text{exp;WA}} - a_\mu^{\text{QED}}(Cs19) = 7342(41) \cdot 10^{-11}$$

- QED provides more than 99.99% of the total value, without uncertainties at this level of experimental precision

- The missing part has to be provided by weak and strong interactions (or else, new physics...)

A few comments about the QED contributions

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13} \qquad \Delta a_\mu^{\text{exp}} = 41 \cdot 10^{-11}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \longrightarrow \quad \sim 14 \cdot 10^{-11}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 380 \cdot 10^{-11} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 5 \cdot 10^{-11}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \mathrm{LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 5.4 \cdot 10^3 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 0.08 \cdot 10^{-11}$$

- No sign of substantial contribution to a_μ from higher order QED

$$\left(\frac{\alpha}{\pi}\right)^6 = 1.56\ldots \cdot 10^{-16} \qquad C_\mu^{(12)} \sim 10^6?!$$

Contribution from the electroweak sector

Weak contributions : W, Z,... loops

One-loop contributions

$$a_\mu^{\text{weak(1)}} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3} \left(1 - 4\sin^2\theta_W\right)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_Z^2} \log \frac{M_Z^2}{m_\mu^2} \right) + \mathcal{O}\left(\frac{m_\mu^2}{M_H^2} \log \frac{M_H^2}{m_\mu^2} \right) \right]$$

$$= 19.48 \times 10^{-10}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

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Two-loop bosonic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)
M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)
A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

Complete three-loop short-distance leading logarithms

G. Degrassi and G. F. Giudice, Phys. Rev. D 58, 053007 (1998)

Updated a few years ago: $a_\mu^{\text{weak}} = 15.36(10) \cdot 10^{-10}$
C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

Recent numerical evaluation: $a_\mu^{\text{weak}} = (152.9 \pm 1.0) \cdot 10^{-11}$
T. Ishikawa, N. Nakazawa and Y. Yasui, Phys. Rev. D 99, 073004 (2019)

Contribution from the strong interactions

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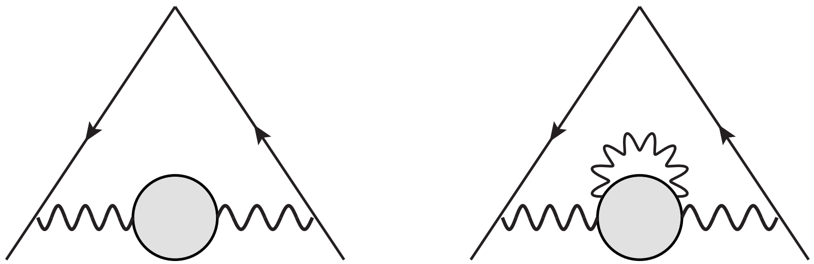
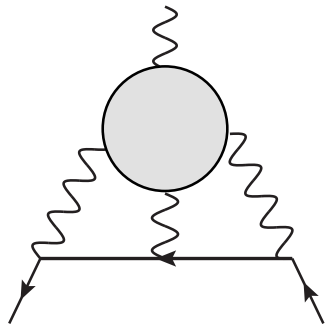
- hadronic vacuum polarization



- (virtual) hadronic light-by-light (HLxL



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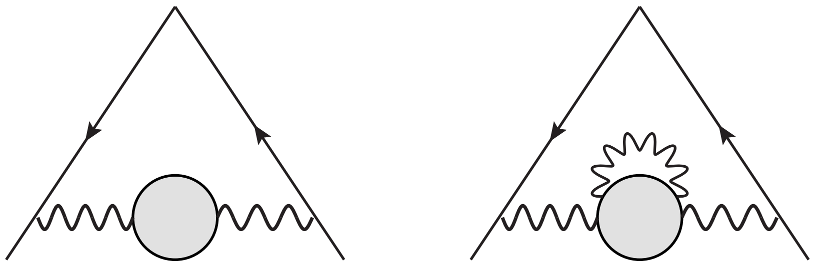

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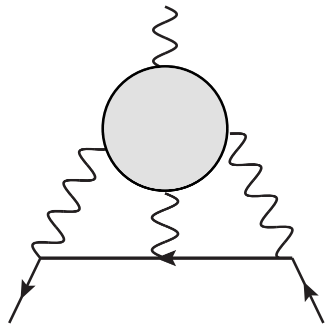
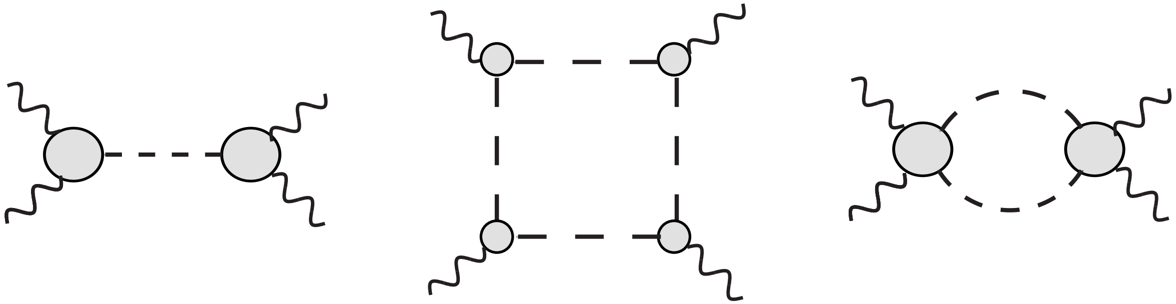


$\longrightarrow$ non-perturbative regime of QCD

Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$
- Involves the 4th-rank vacuum polarization tensor $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4), \sum q_i = 0$
- Used to be estimated via phenomenological models (pion pole,...)
- More recently: evaluation improved through dispersive approach



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015)

Needs input from data (transition form factors,...)

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)

A. Nyffeler, arXiv:1602.03398 [hep-ph]

- Short-distance constraints have been worked out...

K. Melnikov, A. Vainshtein, Phys.Rev.D 70, 113006 (2004)

J. Bijnens, N. Hermansson-Truedsson, A. Rodríguez-Sánchez, Phys.Lett. B 798, 134994 (2019)

J. Bijnens, N. Hermansson-Truedsson, L. Laub and A. Rodríguez-Sánchez, JHEP 10, 203 (2020); JHEP 04, 240 (2021)

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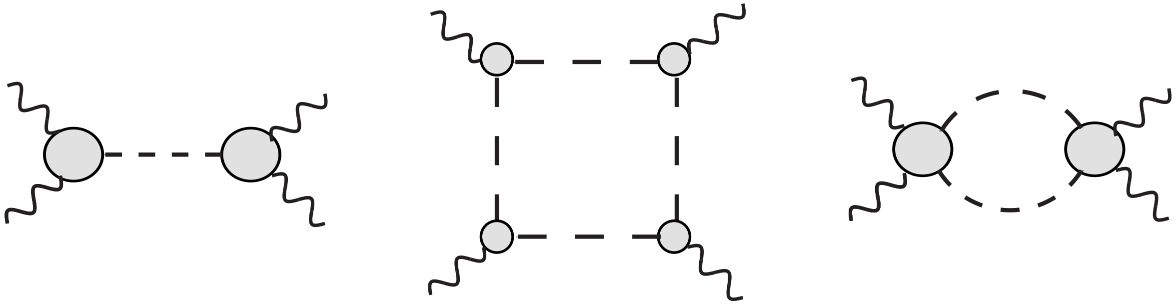

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- Short-distance constraints have been worked out...
- ... and implemented

G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub, P. Stoffer, JHEP03, 101 (2020)

J. Lüdtke, M. Procura, Eur. Phys. J. C 80, 1108 (2020)

J. G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub and P. Stoffer, Eur. Phys. J. C 81, 702 (2021)

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- More recently: evaluation improved through dispersive approach
- More recently: lattice QCD evaluations

T. Blum et al., Phys. Rev. Lett. 124, 132002 (2020)

T. Blum et al., Phys. Rev. D 93, 014503 (2016); Phys. Rev. Lett 118, 022005 (2017)

N. Asmussen et al., arXiv:1609.08454 [hep-lat]; arXiv:1510.08384 [hep-lat]

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White Paper summary

$$a_\mu^{\mathrm{HLxL}} = 92(19) \cdot 10^{-11}$$

Hadronic light-by-light

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- Involves the 4th-rank vacuum polarization tensor $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3, q_4)$, $\sum q_i = 0$
- Used to be estimated via phenomenological models (pion pole,...)
- More recently: evaluation improved through dispersive approach
- More recently: lattice QCD evaluations
- Post-WP: New lattice QCD result for HLxL at 15% accuracy

$$a_\mu^{\mathrm{HVP;LO}} = 107.4(11.3)(9.2) \cdot 10^{-11}$$

E.-H. Chao et al., Eur. Phys. J. C 81, 651 (2021)

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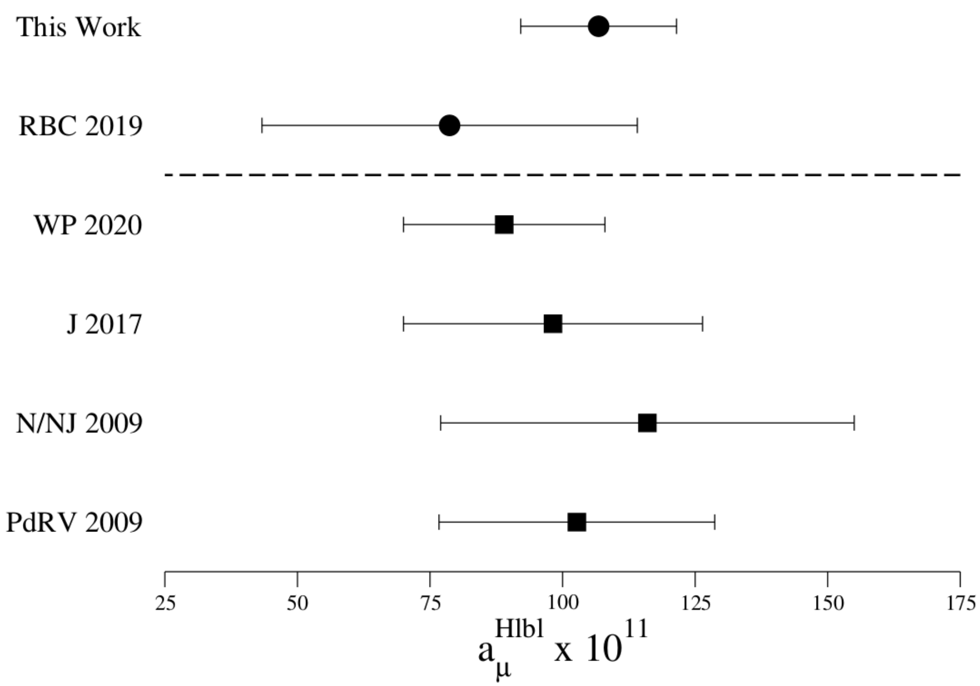


∼10% accuracy goal seems within reach

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

- Can be expressed as (optical theorem)

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{ds}{s} K(s) R^{\text{had}}(s) \quad K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\ell^2}}$$

C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)

L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)

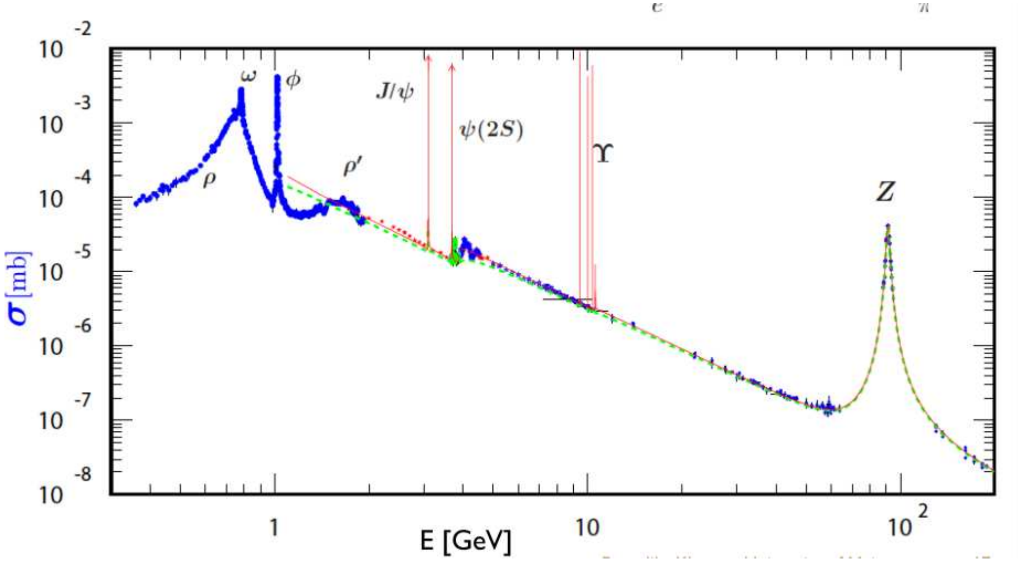
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

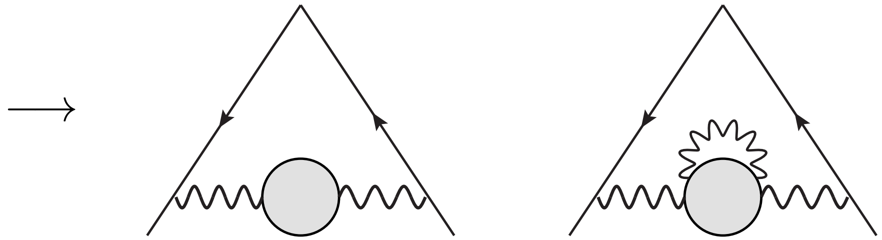
- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$

- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

Hadronic vacuum polarization

- Can be evaluated using available experimental data





- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

Hadronic vacuum polarization

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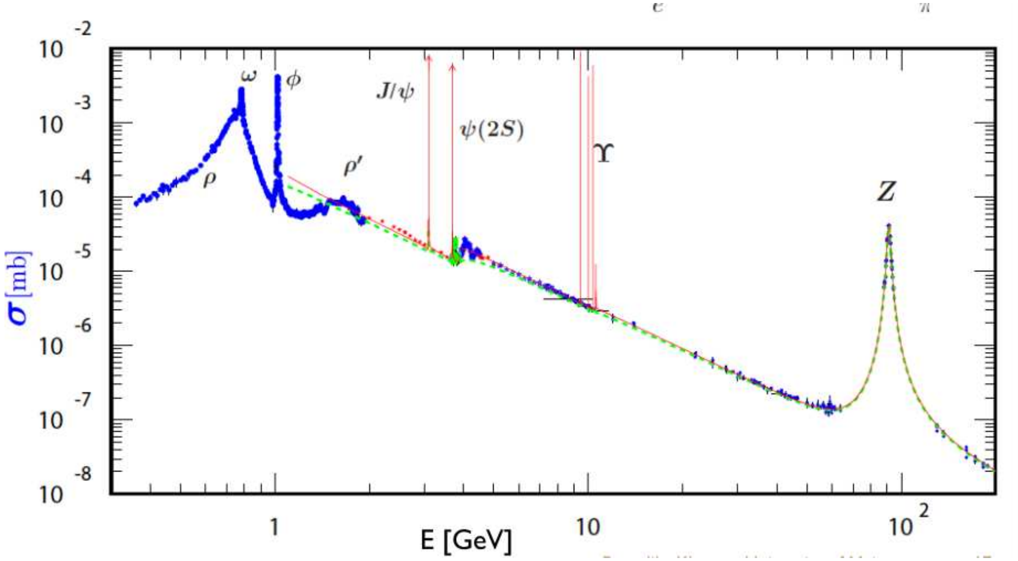
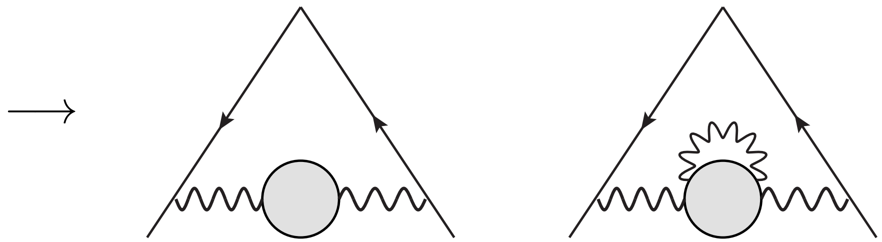


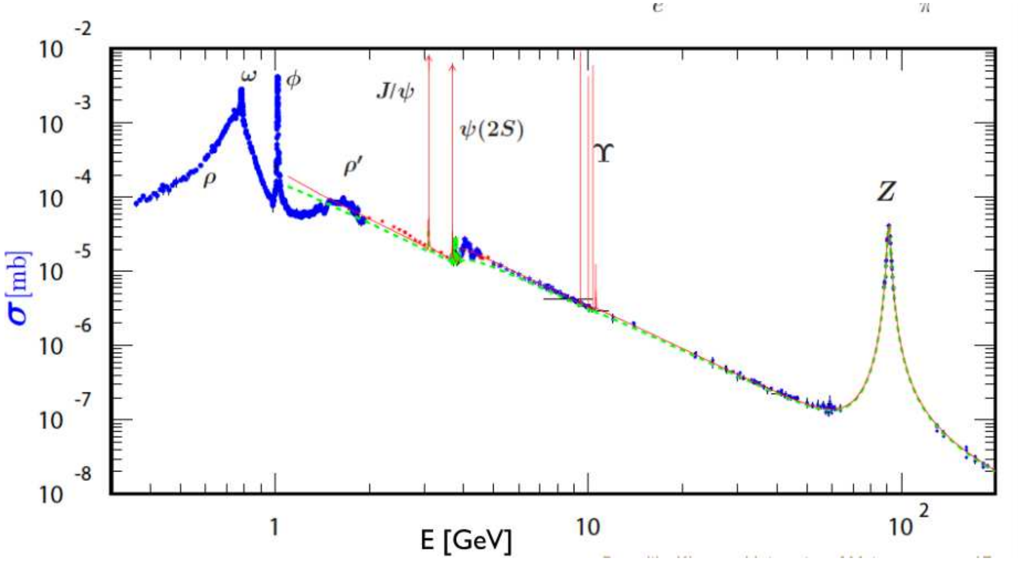


- Some tension, for instance, in the region of the ρ resonance

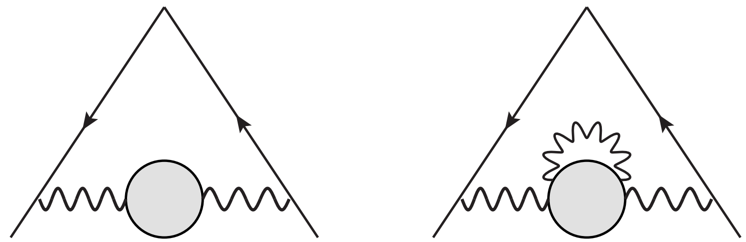
Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600 - 900\ \mathrm{MeV})$
KLOE 08	368.9(0.4)(2.3)(2.2)
KLOE 10	366.1(0.9)(2.3)(2.2)
KLOE 12	366.7(1.2)(2.4)(0.8)
KLOE comb.	366.9(2.1)
BaBar 09	376.7(2.0)(1.9)
BESIII 16	368.2(2.5)(3.3)
SND 04	371.7(5.0)
CMD-2 03,06	372.4 (3.0)

Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$



- Dispersive analyses of the $\pi\pi$ and $\pi\pi\pi$ channels provide cross-checks

$$a_\mu^{\pi\pi}|_{\leq 1GeV} = 495.0(1.5)(2.1) \cdot 10^{-10}$$

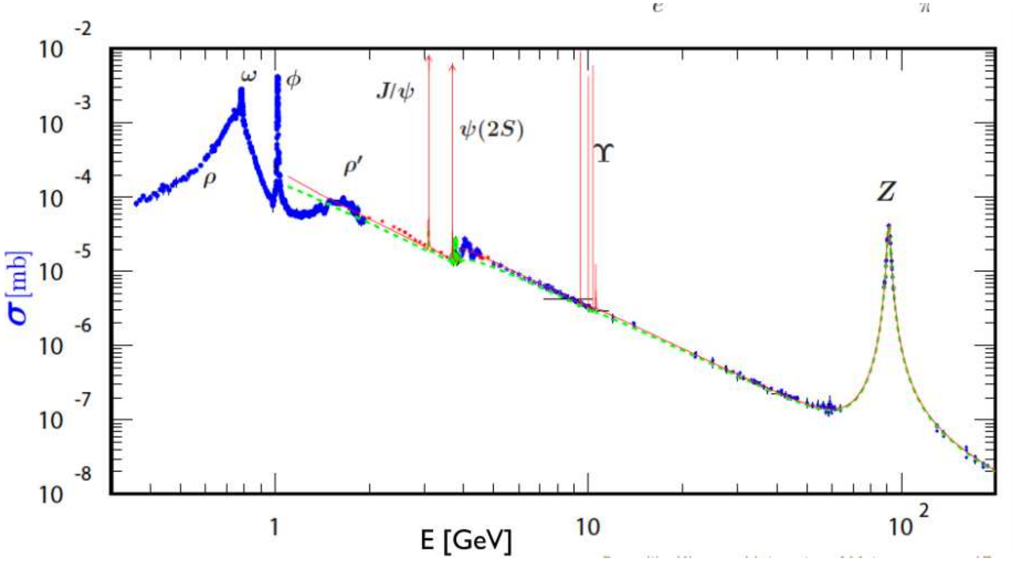
G. Colangelo, M. Hoferichter, P. Stoffer, JHEP 1902,006 (2019)

$$a_\mu^{\pi\pi\pi}|_{\leq 1.8GeV} = 46.2(6)(6) \cdot 10^{-10} \qquad a_\mu^{\text{HVP-LO}} = 692.3(3.3) \cdot 10^{-10}$$

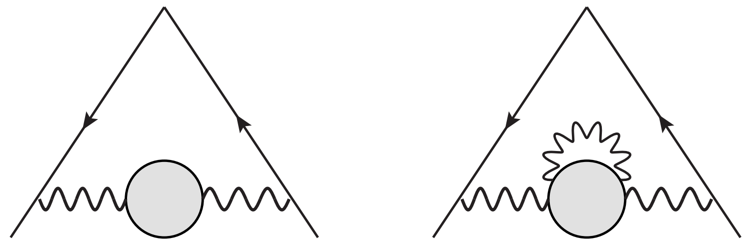
M. Hoferichter, B.-L. Hoid, B. Kubis, JHEP 08, 137 (2019)

Hadronic vacuum polarization

- Can be evaluated using available experimental data



$\longrightarrow$



- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

- More is to come

$\longrightarrow$ SND, CMD-3 (ongoing analysis of the $\pi\pi$ channel)

$\longrightarrow$ BaBar, BES III, BELLE II

$\longrightarrow$ Possiblity to measure HVP in the space-like region from μe scattering

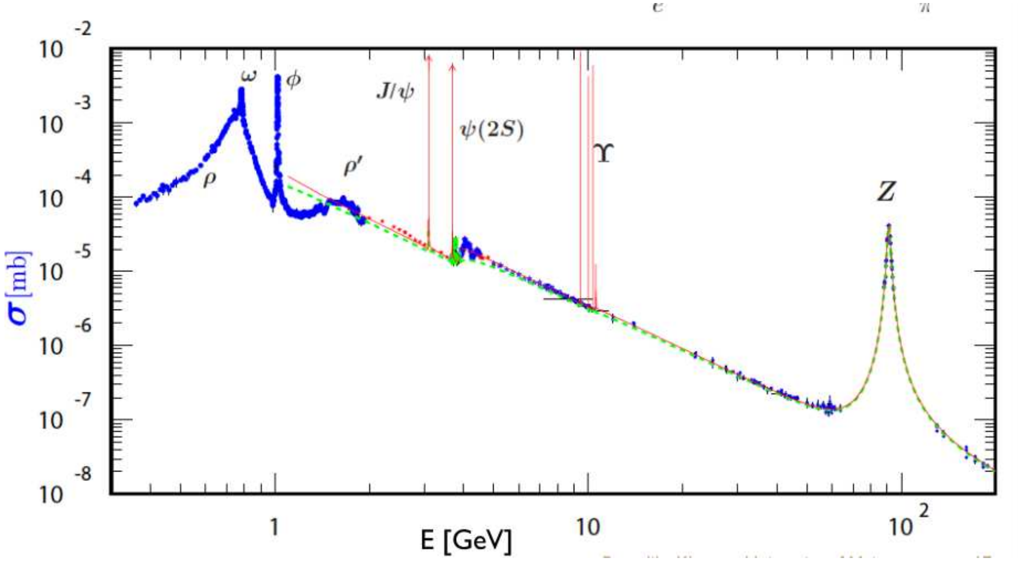
C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

(more "lattice-friendly", test run delayed $\longrightarrow$ 2023)

G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

Hadronic vacuum polarization

- Can be evaluated using available experimental data



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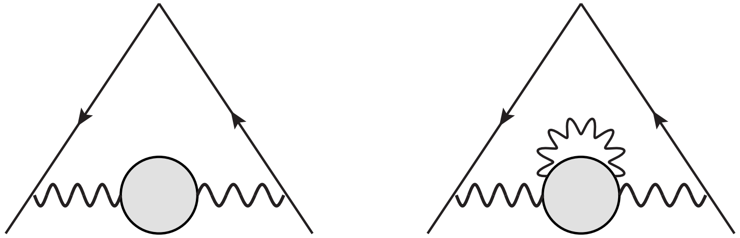


- Combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

- Several lattice results (for the time being, stick to WP)

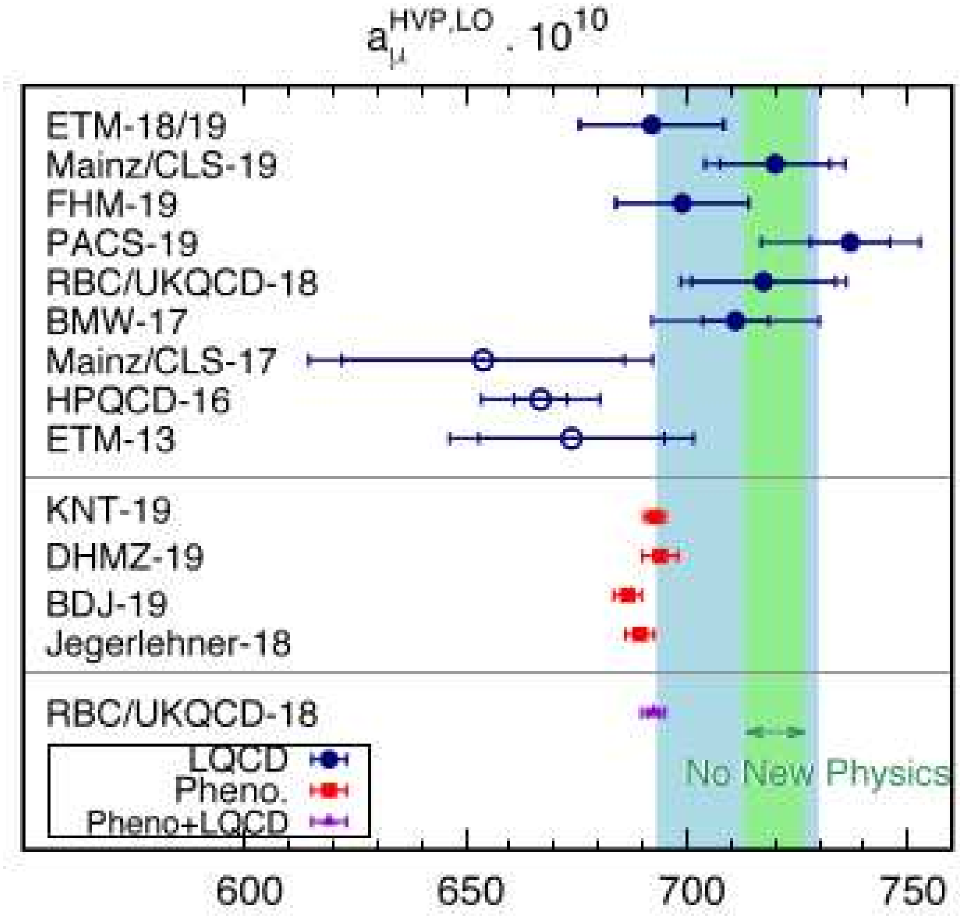
A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)
C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]
E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]
D. Giusti *et al.*, Phys. Rev. D 99, 114502 (2019)
T. Blum *et al.*, Phys. Rev. Lett. 121, 022003 (2018)
S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)
M. Della Morte *et al.*, JHEP 10, 020 (2017)

White Paper summary

- Data evaluation:

$$a_\mu^{\mathrm{HVP;LO}} = 6931(40)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NLO}} = -98.3(7)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NNLO}} = 12.4(1)\cdot 10^{-11}$$

- Lattice WA: $a_\mu^{\mathrm{HVP;LO}} = 7043(150)\cdot 10^{-11}$



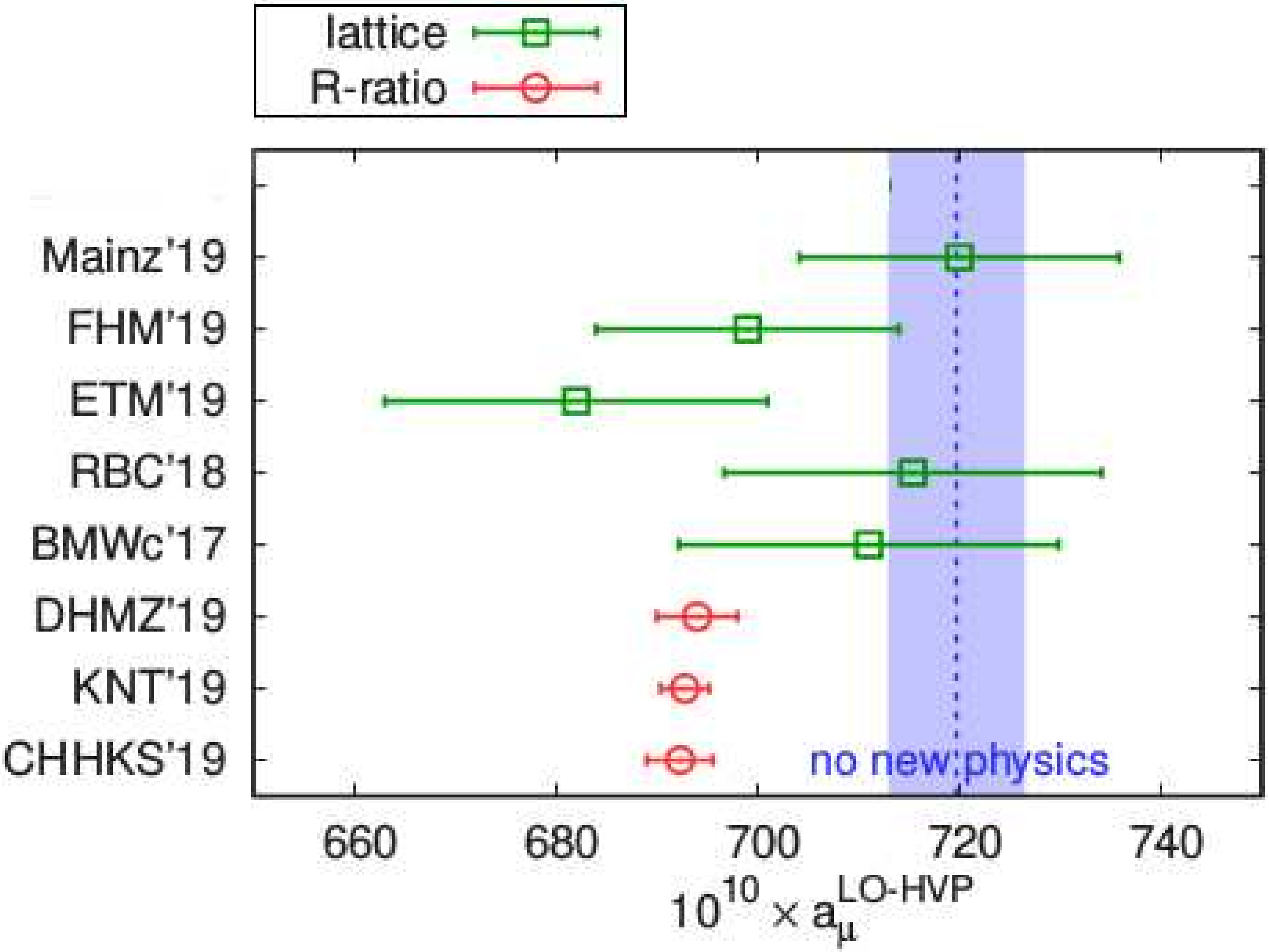
White Paper summary

$$a_\mu^{\mathrm{th;WP}} = 116\,591\,810(43) \cdot 10^{-11}$$

The situation today

$a_\mu^{\mathrm{th;WP}}$ based on

$$a_\mu^{\mathrm{HVP-LO;WP}} = 6931(40) \cdot 10^{-11}$$



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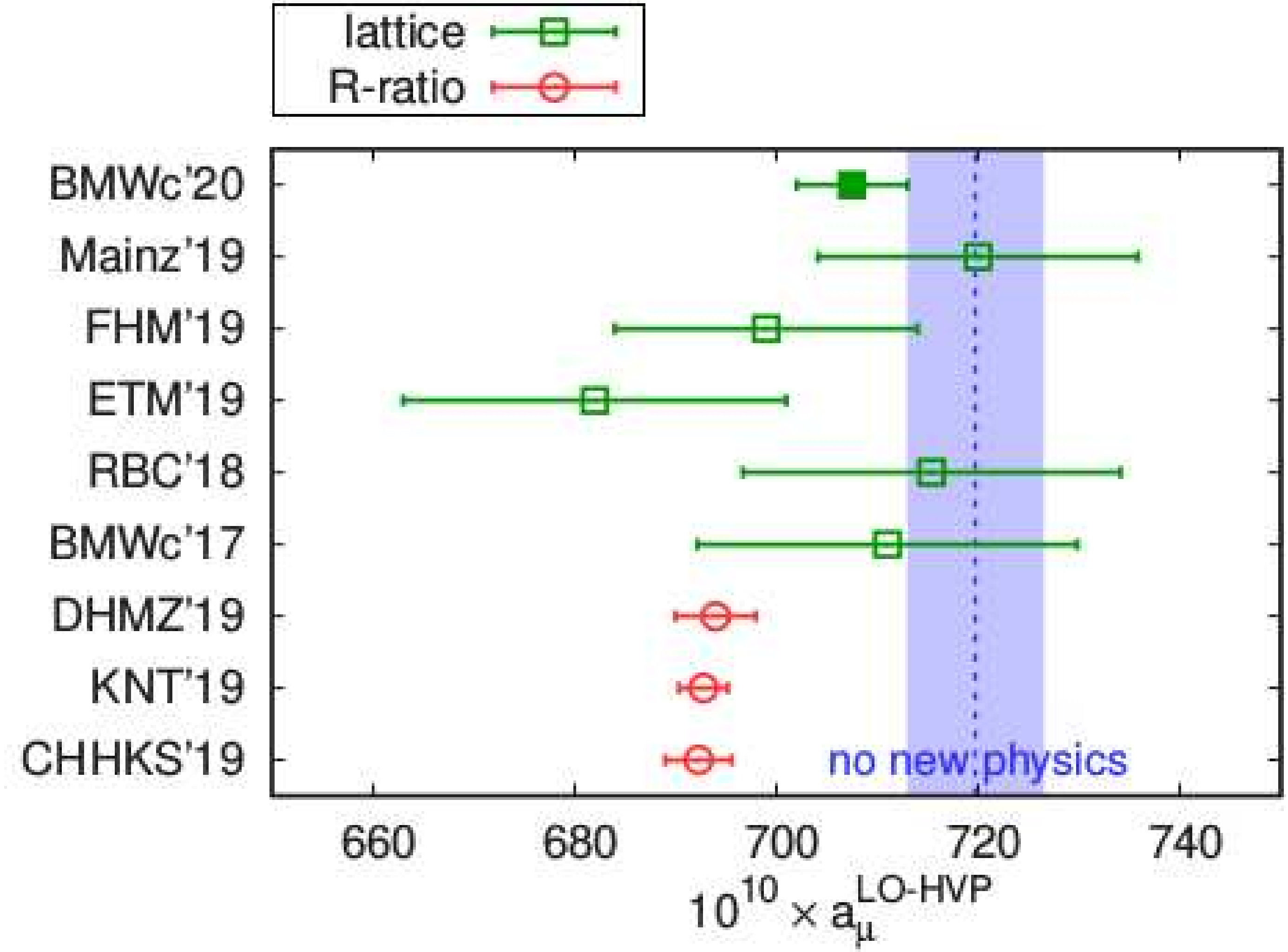
New lattice QCD result for HVP with 0.8% accuracy

$$a_\mu^{\mathrm{HVP;LO}} = 7075(55) \cdot 10^{-11}$$

S. Borsanyi et al., Nature 593, 7857 (2021)



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Systematic effets (finite size, discretization,...) need to be scrutinized

Requires independent confirmation

Post-WP results

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S. Borsanyi et al., Nature 593, 7857 (2021)

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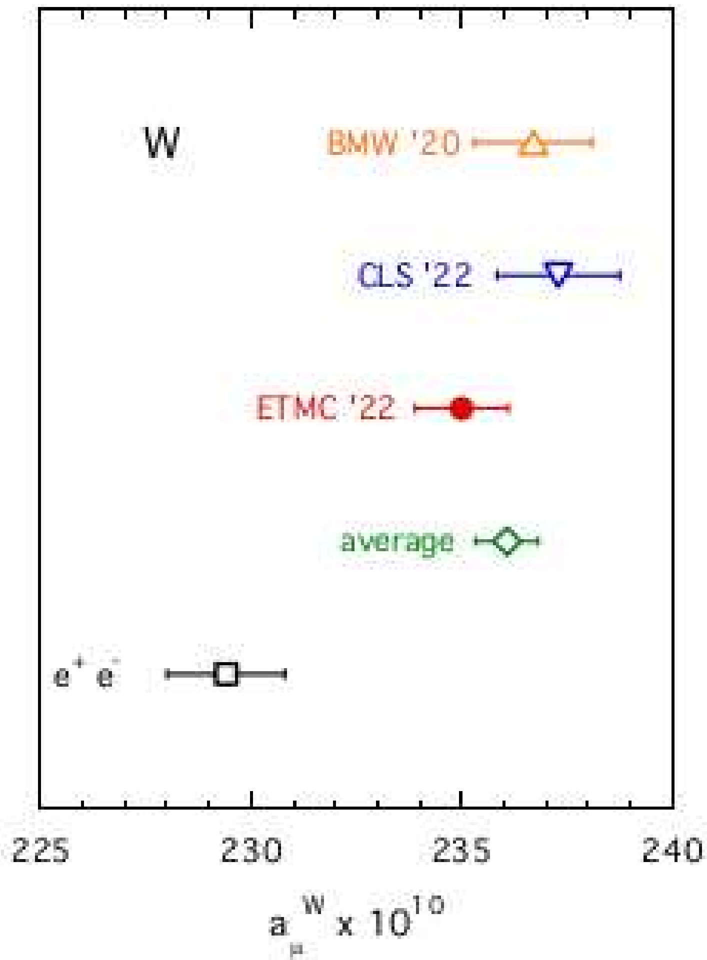
S. Borsanyi et al., Nature 593, 7857 (2021)

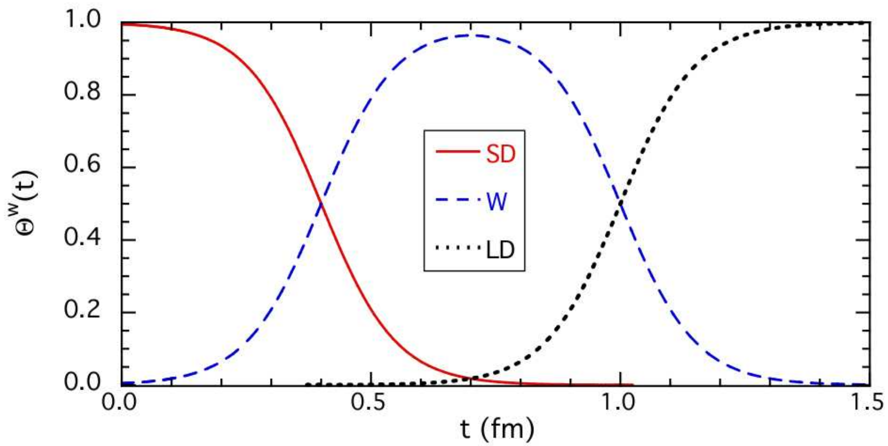
M. Cè et al., arXiv:2206.06582 [hep-lat]

C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

Several independent cross-checks for the intermediate window

a_μ^{IW}: $0.4\mathrm{fm} \leq t_{\mathrm{E}} \leq 1.0\mathrm{fm}$





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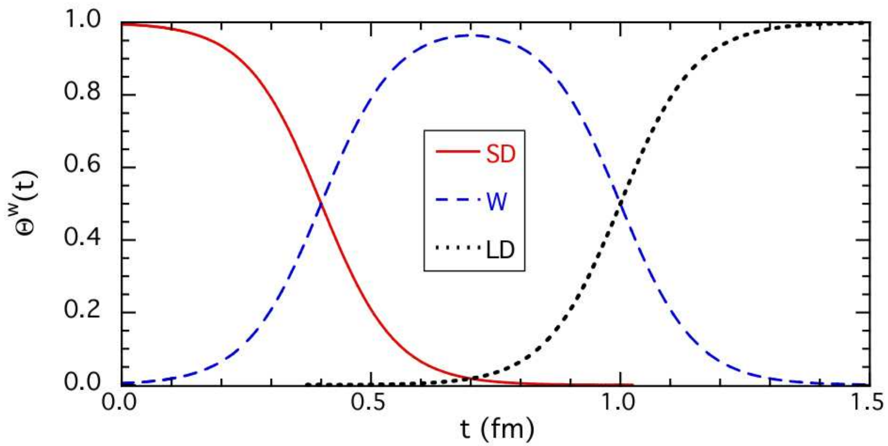

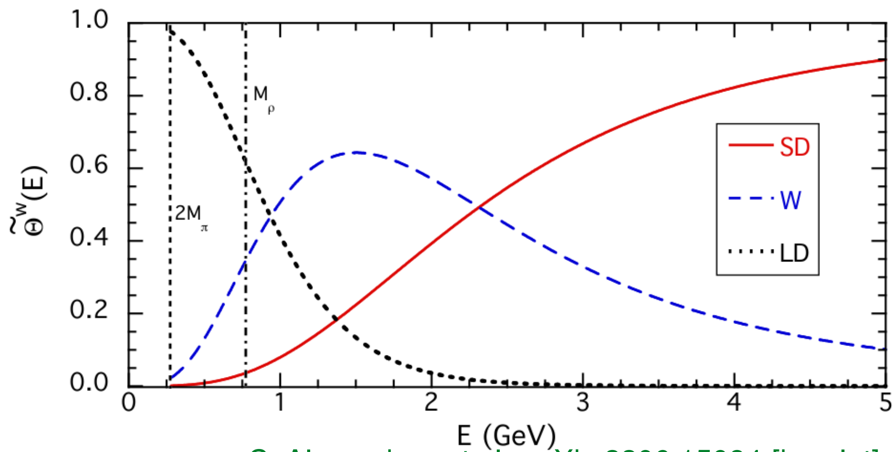

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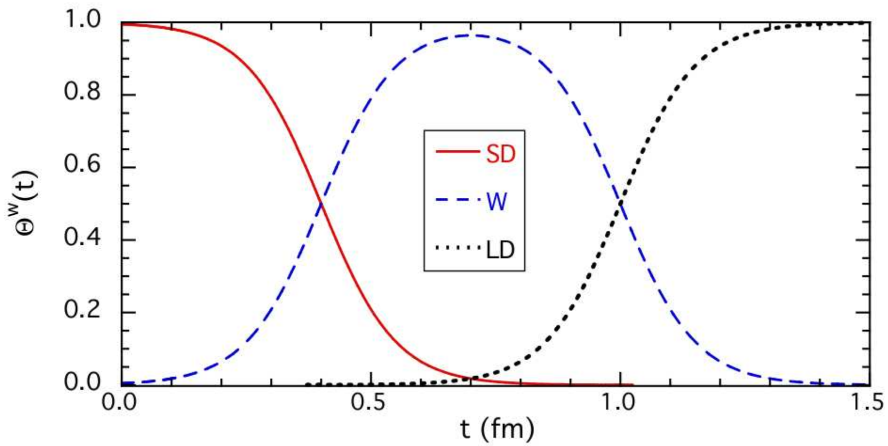

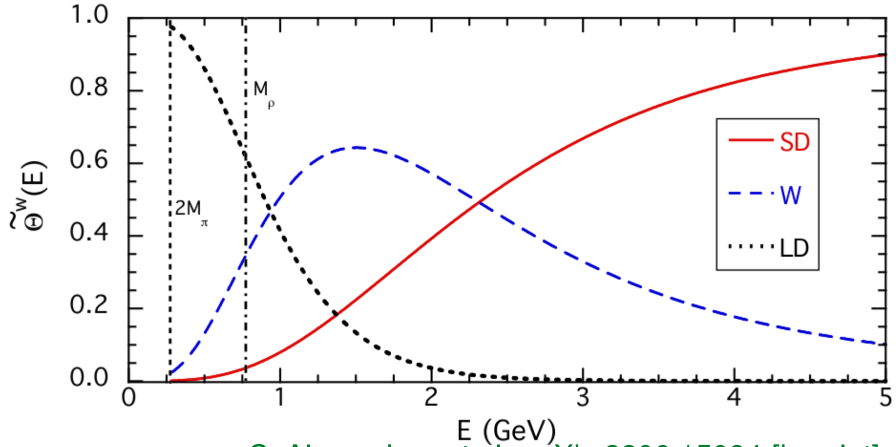

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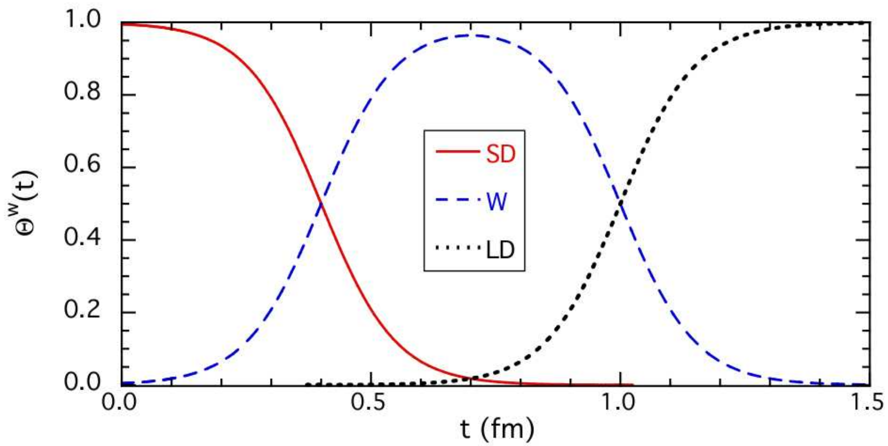

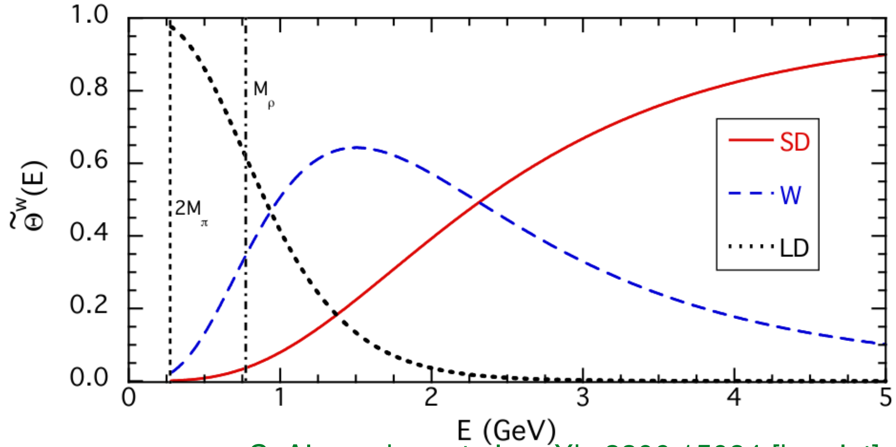

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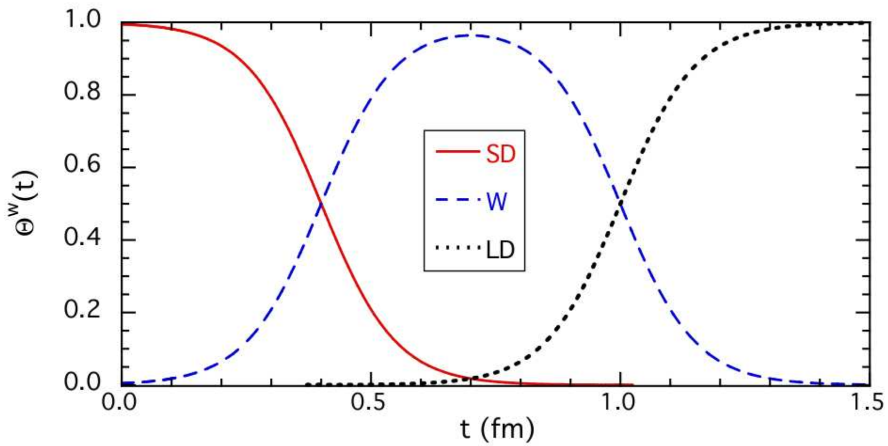
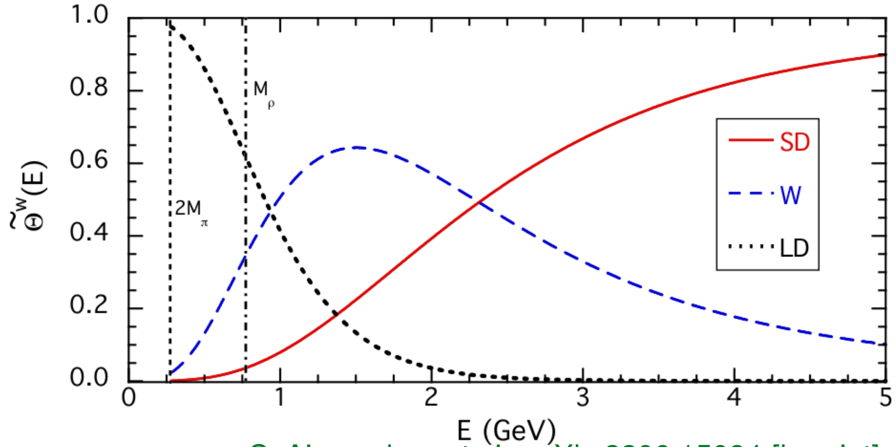
More windows can allow for finer studies of the discrepancy between LQCD and data-based determination (correlations matter)

G. Colangelo, A. X. El-Khadra, M. Hoferichter, A. Keshavarzi, C. Lehner, P. Stoffer, T. Teubner, Phys. Lett. B 833, 137313 (2022)

Work in progress





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So far, no complete LQCD cross-check of BMWc result available

Conclusion

FNAL-E989 running fine, next announcement eagerly awaited

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If $a_\mu^{\text{th;WP}} = a_\mu^{\text{th;SM}}$, discrepancy with $a_\mu^{\text{exp;WA}}$ will be difficult to explain within SM (many cross-checks)

$$a_\mu^{\text{exp}} - a_\mu^{\text{th;WP}} \sim \begin{cases} a_\mu^{\text{QED}}(\alpha^4) \\ 60 \cdot a_\mu^{\text{QED}}(\alpha^5) \\ 5 \cdot a_\mu^{\text{weak(2)}} \\ 3 \cdot a_\mu^{\text{HLxL}} \end{cases}$$

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Do we really understand $a_\mu^{\text{HVP}}|_{\text{data}}$ and $a_\mu^{\text{HVP}}|_{\text{LQCD}}$?

(at the level of precision that is claimed)

Thanks for your attention!