Quantum correlations in neutrino mixing and oscillations

Massimo Blasone

Università di Salerno and INFN, Salerno, Italy

Summary

1. Entanglement in neutrino mixing & oscillations
2. Quantum correlations & nonlocality in neutrino oscillations
3. Complete Complementarity Relations in neutrino oscillations
4. Quantum Field Theory of neutrino mixing and oscillations
5. Chiral oscillations and lepton/antineutrino entanglement

Motivations

- Entanglement as an intrinsic property of neutrinos*;
- Recent interest in the study of quantumness of neutrino oscillations (entanglement, Leggett-Garg inequalities†, quantum coherence, quantum correlations);
- Neutrinos as a resource for quantum information;

---

*M.B. et al., Phys. Rev. D (2008); EPL (2009).
For an early review on entanglement in particle physics see R.A.Bertlmann, Lect.Not.Phys.(2006).

†J.A. Formaggio et al., Phys. Rev. Lett. (2016).

Motivations

- Access fundamental properties of (elementary) particles via quantum correlations[‡];

- Necessity for a treatment of entanglement in the context of Quantum Field Theory[§];

- Neutrino mixing and oscillations in Quantum Field Theory[¶].

[‡] For the case of $K^0\bar{K}^0$ system, see A.Di Domenico, 2208.06789 [hep-ph].

[§] M.O.Terra Cunha, J.A.Dunningham and V.Vedral, Proc. Royal Soc. A (2007).

[¶] M.B and G.Vitiello, Ann. Phys. (1995)

Entanglement in neutrino mixing & oscillations

Entanglement in neutrino mixing[‖]

– Flavor mixing (neutrinos)

$$|\nu_e\rangle = \quad \cos\theta\,|\nu_1\rangle \,+\, \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle \,+\, \cos\theta\,|\nu_2\rangle$$

• Correspondence with two-qubit states:

$$|\nu_1\rangle \equiv |1\rangle_1|0\rangle_2 \equiv |10\rangle, \qquad |\nu_2\rangle \equiv |0\rangle_1|1\rangle_2 \equiv |01\rangle,$$

$|\rangle_i$ denotes states in the Hilbert space for neutrinos with mass m_i.

$\Rightarrow$ flavor states are entangled superpositions of the mass eigenstates:

$$|\nu_e\rangle = \quad \cos\theta\,|10\rangle \,+\, \sin\theta\,|01\rangle.$$

‖ F.Dell'Anno, S.De Siena and F.Illuminati, EPL (2009).

Composite structure of Hilbert space for neutrinos

• Necessity of tensor-product structure of Hilbert space for two generations:

Orthogonality of Hilbert spaces for fields with different masses**

Example: two scalar fields with different masses

$$(\Box + \mu_1^2)\phi_1(x) = 0 \quad , \qquad (\Box + \mu_2^2)\phi_2(x) = 0$$

with boundary conditions $\phi_1(0, \mathbf{x}) = \phi_2(0, \mathbf{x})$ and $\dot{\phi}_1(0, \mathbf{x}) = \dot{\phi}_2(0, \mathbf{x})$

One obtains

$$_1\langle 0|0\rangle_2 \simeq \exp\left\{-\frac{V}{64\pi^2}\int_0^\infty dk\,\frac{(\mu_1^2 - \mu_2^2)^2}{k^2}\right\}$$

which vanishes in the infinite volume limit.

**G.Barton, Introduction to Advanced Field Theory, Intersc. Publ. (1963)

Entanglement - mathematical definition

- Given a bipartite system $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, a system is entangled, iff

$$\rho_{AB} \neq \sum_k p_k \, \rho_k^{(A)} \otimes \rho_k^{(B)}$$

with $0 \leq p_i \leq 1, \quad \sum_k p_k = 1.$

- For a generic pure state of the form:

$$|\psi\rangle_{AB} = \sum_{ij} c_{ij} |i\rangle_A \otimes |j\rangle_B$$

the condition for entanglement reads

$$|\psi\rangle_{AB} \neq |\phi\rangle_A \otimes |\chi\rangle_B$$

Single-particle entanglement*

- A state like $|\psi\rangle_{A,B} = |0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B$ is entangled;

– entanglement among field modes, rather than particles;

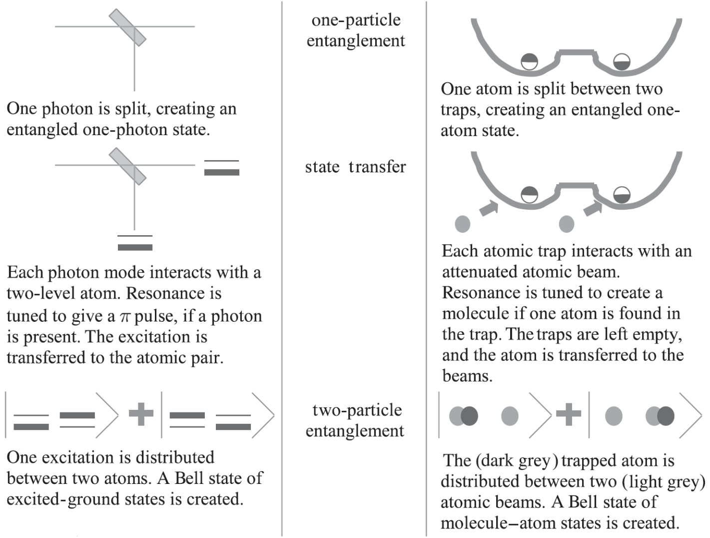
– entanglement is a property of composite systems, rather than of many-particle systems;

– entanglement and non-locality are not synonyms;

– single-particle entanglement is as good as two-particle entanglement for applications (quantum cryptography, teleportation, violation of Bell inequalities, etc..).

* J.van Enk, Phys. Rev. A (2005), (2006);
J.A.Dunningham and V.Vedral, Phys. Rev. Lett. (2007).

Protocols for extraction of single-particle entanglement †



† M.O.Terra Cunha, J.A.Dunningham and V.Vedral, Proc. Royal Soc. A (2007)

Multipartite entanglement in neutrino mixing[‡]

– Neutrino mixing (three flavors):

$$|\underline{\nu}_f\rangle \quad = \quad U(\tilde{\theta},\delta)\,|\underline{\nu}_m\rangle$$

with $|\underline{\nu}_f\rangle = (|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle)^T$ and $|\underline{\nu}_m\rangle = (|\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle)^T$.

– Mixing matrix (PMNS)

$$U(\tilde{\theta},\delta) \quad = \quad \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

where $(\tilde{\theta},\delta) \equiv (\theta_{12}, \theta_{13}, \theta_{23}; \delta)$, $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

- Correspondence with three-qubit states:

$$|\nu_1\rangle \equiv |1\rangle_1|0\rangle_2|0\rangle_3 \equiv |100\rangle, \qquad |\nu_2\rangle \equiv |0\rangle_1|1\rangle_2|0\rangle_3 \equiv |010\rangle,$$

$$|\nu_3\rangle \equiv |0\rangle_1|0\rangle_2|1\rangle_3 \equiv |001\rangle$$

[‡] M.B., F.Dell'Anno, S.De Siena, M.Di Mauro and F.Illuminati, PRD (2008).

(Flavor) Entanglement in neutrino oscillations[§]

– Two-flavor neutrino states

$$|\underline{\nu}^{(f)}\rangle \, = \, \mathbf{U}(\theta,\delta)\,|\underline{\nu}^{(m)}\rangle$$

where $|\underline{\nu}^{(f)}\rangle \, = \, (|\nu_e\rangle, |\nu_\mu\rangle)^T$ and $|\underline{\nu}^{(m)}\rangle \, = \, (|\nu_1\rangle, |\nu_2\rangle)^T$ and
$\mathbf{U}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

– Flavor states at time t:

$$|\underline{\nu}^{(f)}(t)\rangle \, = \, \mathbf{U}(\theta,\delta)\,\mathbf{U}_0(t)\,\mathbf{U}(\theta,\delta)^{-1}\,|\underline{\nu}^{(f)}\rangle \equiv \, \widetilde{\mathbf{U}}(t)|\underline{\nu}^{(f)}\rangle\,,$$

with $\mathbf{U}_0(t) = \begin{pmatrix} e^{-iE_1 t} & 0 \\ 0 & e^{-iE_2 t} \end{pmatrix}$.

§ M.B., F.Dell'Anno, S.De Siena and F.Illuminati, EPL (2009).

– Transition probability for $\nu_\alpha \to \nu_\beta$

$$P_{\nu_\alpha \to \nu_\beta}(t) \,=\, |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 \,=\, |\widetilde{\mathbf{U}}_{\alpha\beta}(t)|^2\,.$$

• We now take the flavor states at initial time as our qubits:

$$|\nu_e\rangle \equiv |1\rangle_e |0\rangle_\mu \equiv |10\rangle_f\,, \quad |\nu_\mu\rangle \equiv |0\rangle_e |1\rangle_\mu \equiv |01\rangle_f\,,$$

• Starting from $|10\rangle_f$ or $|01\rangle_f$, time evolution generates the (entangled) Bell-like states:

$$|\nu_\alpha(t)\rangle \,=\, \widetilde{\mathbf{U}}_{\alpha e}(t) |1\rangle_e |0\rangle_\mu + \widetilde{\mathbf{U}}_{\alpha\mu}(t) |0\rangle_e |1\rangle_\mu\,, \quad \alpha = e, \mu.$$

Entanglement measure: linear entropy

- Let $\rho = |\psi\rangle\langle\psi|$ be the density operator for a pure state $|\psi\rangle$

Bipartition of the N-partite system $S = \{S_1, S_2, \ldots, S_N\}$ in two subsystems S_{A_n}, $S_{B_{N-n}}$

- Reduced density matrix of S_{A_n} after tracing over $S_{B_{N-n}}$:

$$\rho_{A_n} \equiv \rho_{i_1,i_2,\ldots,i_n} = Tr_{B_{N-n}}[\rho] = Tr_{j_1,j_2,\ldots,j_{N-n}}[\rho]$$

- Linear entropy associated to such a bipartition:

$$S_L^{(A_n;B_{N-n})}(\rho) = \frac{d}{d-1}(1 - Tr_{A_n}[\rho_{A_n}^2]),$$

d is the Hilbert-space dimension.

Entanglement measures

It is necessary to distinguish the various entanglement measures for pure and mixed states (which may contain classical correlations).

- **Measures for pure states:**
  - von Neumann entropy
  - Geometric Entanglement
- **Measures for mixed states:**
  - Entanglement of Formation and Concurrence
  - Logarithmic negativity
  - Relative Entropy of Entanglement

Entanglement in neutrino oscillations: two-flavors

Consider the density matrix for the electron neutrino state $\rho^{(e)} = |\nu_e(t)\rangle\langle\nu_e(t)|$, and trace over mode $\mu \;\Rightarrow\; \rho_e^{(e)}$.

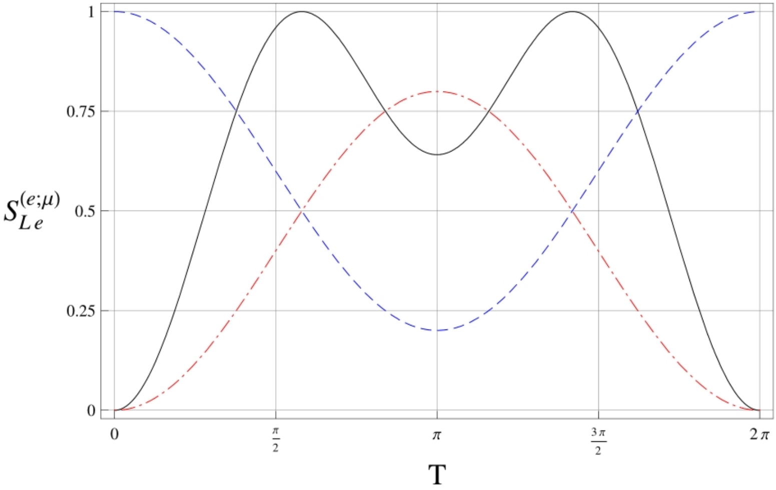
- The associated linear entropy is :

$$S_L^{(e;\mu)}(\rho^{(e)}) \,=\, 4\,|\widetilde{\mathbf{U}}_{e\mu}(t)|^2\,|\widetilde{\mathbf{U}}_{ee}(t)|^2 \,=\, 4\,P_{\nu_e\to\nu_e}(t)\,P_{\nu_e\to\nu_\mu}(t)$$

The linear entropy for the state $\rho^{(\alpha)}$ is:

$$\begin{aligned} S_{L\alpha}^{(e;\mu)} = S_{L\alpha}^{(\mu;e)} &= 4\,|\widetilde{\mathbf{U}}_{\alpha\mu}(t)|^2\,|\widetilde{\mathbf{U}}_{\alpha e}(t)|^2 \\ &= 4|\widetilde{\mathbf{U}}_{\alpha e}(t)|^2\,(1-|\widetilde{\mathbf{U}}_{\alpha e}(t)|^2) \\ &= 4|\widetilde{\mathbf{U}}_{\alpha\mu}(t)|^2\,(1-|\widetilde{\mathbf{U}}_{\alpha\mu}(t)|^2)\,. \end{aligned}$$

- Linear entropy given by product of transition probabilities



Linear entropy $S_{Le}^{(e;\mu)}$ (full) as a function of the scaled time $T = \frac{2Et}{\Delta m_{12}^2}$, with $\sin^2\theta = 0.314$. Transition probabilities $P_{\nu_e\to\nu_e}$ (dashed) and $P_{\nu_e\to\nu_\mu}$ (dot-dashed) are reported for comparison.

Other results

- Generalization to three flavors. Extension to wave packets;*
- Flavor entanglement in Quantum Field Theory.†
- ν–oscillations as a resource for quantum information - Experimental scheme for the transfer of the flavor entanglement of a neutrino beam into a single-particle system with *spatially separated modes*.

* M.B., F.Dell'Anno, S.De Siena and F.Illuminati, EPL (2015).

† M.B., F.Dell'Anno, S.De Siena and F.Illuminati, EPL (2014).

Quantum correlations & nonlocality in neutrino oscillations

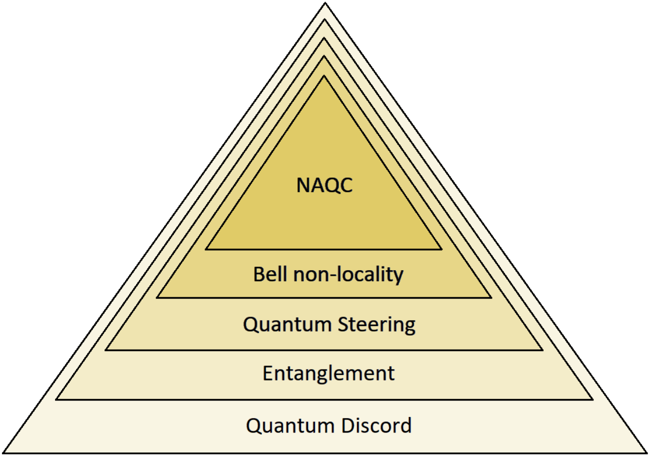
Quantum Correlations

Quantum systems exhibit properties that are beyond our understanding of reality. They show correlations that have no classical counterpart.

Entanglement is the most known of these correlations. But the terminology *quantum correlations* refers to a broader concept:

- **Quantum correlations related to entanglement:**
  - Bell non-locality
  - Entanglement
  - Quantum steering
- **Quantum correlations beyond entanglement:**
  - Quantum discord

Quantum Correlations[‡]



Hierarchy of quantum correlations

[‡] G. Adesso, T.R. Bromley and M. Cianciaruso, J. Phys. A (2016)

Quantum correlations in neutrino oscillations

- Recently, quantum correlations have been investigated in the context of high-energy particle physics;

Focus on neutrinos and mesons, which are candidates for quantum information applications beyond photons.

Quantum correlations in neutrino oscillations (partial list):

A.K. Alok et al., *Quantum correlations in terms of neutrino oscillation probabilities* Nuc. Phys. B (2016)

J.A. Formaggio et al., *Violation of the Leggett- Garg Inequality in Neutrino Oscillation* Phys. Rev. Lett. (2016).

X.-S. Song et al. *Quantifying quantum coherence in experimentally observed neutrino oscillations* Phys. Rev. A (2018)

J.Naikoo et al. *Leggett-Garg inequality in the context of three flavor neutrino oscillation* Phys. Rev. D (2019)

K. Dixit et al., *Study of coherence and mixedness in meson and neutrino systems* Eur. Phys. J. C (2019)

F. Ming et al. *Quantification of quantumness in neutrino oscillations* Eur. Phys. J. C (2020)

L.-J. Li et al. *Characterizing entanglement and measurement's uncertainty in neutrino oscillations* Eur. Phys. J. C (2021)

P.Kurashvili et al *Coherence and mixedness of neutrino oscillations in a magnetic field* Eur. Phys. J. C (2021)

S.Shafaq and P.Mehta *Enhanced violation of Leggett–Garg inequality in three flavour neutrino oscillations via non-standard interactions* J.Phys.G (2021)

K.Dixit, A.K.Alok *New physics effects on quantum coherence in neutrino oscillations* Eur. Phys. J. P (2021)

A.K.Jha, S.Mukherjee, B.A.Bambah *Tri-partite entanglement in neutrino oscillations* Mod. Phys. Lett. A (2021)

A.K.Jha, A. Chatla *Quantum studies of neutrinos on IBMQ processors* Eur. Phys. J. S. T.(2022)

B. Yadav, T.Sarkar, K.Dixit, A.K.Alok *Can NSI affect non-local correlations in neutrino oscillations?* Eur. Phys. J. C (2022)

Z. Askaripour Ravari et al. *Quantum coherence in neutrino oscillation in matter* Eur. Phys. J. P (2022)

Y.W.Li et al. *Genuine tripartite entanglement in three-flavor neutrino oscillations* arXiv preprint arXiv:2205.11058, 2022

A.K.Jha, A.Chatla, B.A.Bambah *Neutrinos as Qubits and Qutrits* arXiv:2203.13485 (2022)

M.B., S.De Siena and C.Matrella, *Wave packet approach to quantum correlations in neutrino oscillations*, Eur. Phys. J. C (2021)

M.B., S.De Siena and C.Matrella, *Complete complementarity relations for quantum correlations in neutrino oscillations*, Eur. Phys. J. C (2022)

M.B., F.Illuminati, L.Petruzziello and L.Smaldone, *Leggett-Garg inequalities in the quantum field theory of neutrino oscillations*, arXiv preprint arXiv:2111.09979

M.B., S.De Siena and C.Matrella, *Nonlocality and entropic uncertainty relations in neutrino oscillations*, arXiv preprint arXiv:2206.13218

Non-local Advantage of Quantum Coherence[†]

- A state is said to be coherent provided that there are non-zero non-diagonal elements in its matrix representation.

Coherence can be quantified by means of the l_1-norm of coherence:[*]

$$C_{l_1}(\rho) = \sum_{i \neq j} |\rho_{i,j}|$$

If the qubit is prepared in either spin up or down state along z, it is incoherent in z-basis ($C_{l_1}^{z} = 0$) and fully coherent in x- and y-basis ($C_{l_1}^{x(y)} = 1$).

Upper bound beyond which the effects of non-locality emerge:

$$\sum_{i=x,y,z} C_i^{l_1}(\rho) \leq C_{max}.$$

[*] T.Baumgratz, M.Cramer and M.B.Plenio, Phys. Rev. Lett. (2014).
[†] D. Mondal, T. Pramanik, A.K. Pati, Phys. Rev. A (2017).

Non-local Advantage of Quantum Coherence

Consider a bipartite system made of two spatially separated subsystems. Alice performs a measurement Π_i^b on σ_i eigenbasis with outcome $b = \{0, 1\}$ and probability $p_{\Pi_i^b} = \mathrm{Tr}[(\Pi_i^b \otimes \mathbf{1})\rho_{AB}]$.

Measured state for the two-qubit state is$\rho_{AB|\Pi_i^b} = (\Pi_i^b \otimes \mathbf{1})\rho_{AB}(\Pi_i^b \otimes \mathbf{1})/p_{\Pi_i^b}$ and the conditional state for qubit B is $\rho_{B|\Pi_i^b} = \mathrm{Tr}_A(\rho_{AB|\Pi_i^b})$.

Then Alice tells Bob her measurement choice and Bob has to measure the coherence of qubit B at random in the eigenbases of the other two Pauli matrices σ_j and σ_k.

If the above condition for locality is violated then we cannot have a single-system description of the coherence of subsytem B.

The criterion for achieving a NAQC of qubit B can be written as:

$$N_{l_1}(\rho_{AB}) = \frac{1}{2}\sum_{i,j,b} p(\rho_{B|\Pi^b_{j\neq i}}) C^{\sigma_i}_{l_1}(\rho_{B|\Pi^b_{j\neq i}}) > \sqrt{6}.$$

Quantification of quantumness in neutrino oscillations

- Recently[‡], quantumness in neutrino oscillations has been quantified through correlation measures such as Non-local Advantage of Quantum Coherence (NAQC), quantum steering and Bell non-locality.

– The criterion for NAQC is:

$$N^{l_1}(\rho_{AB}) = \frac{1}{2}\sum_{i,j,b} p(\rho^b_{\Pi_{j\neq i}}) C^{\sigma_i}_{l_1}(\rho_{B|\Pi_{j\neq i}}) > \sqrt{6}.$$

– Bell non-locality (violation of CHSH inequality):

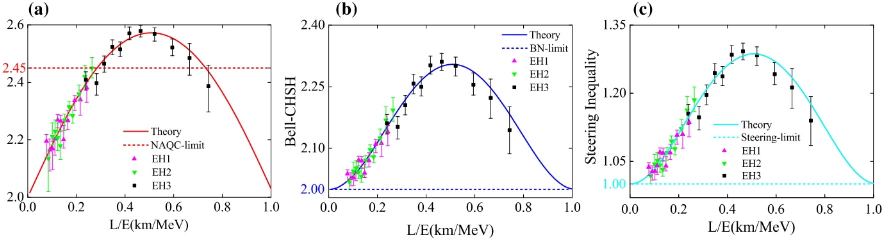
$$B(\rho_{AB}) = |\langle B_{CHSH}\rangle| \leq 2.$$

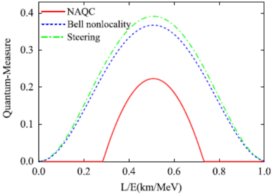
– Quantum steering:

$$F_n(\rho_{AB}, \varsigma) = \frac{1}{\sqrt{n}}\left|\sum_{i=1}^{n} Tr(\rho_{AB} A_i \otimes B_i)\right| \leq 1.$$

[‡]F. Ming, X-K. Song, D. Wang, Eur. Phys. J. C (2020)

Quantumness in neutrino oscillations (Daya Bay) §

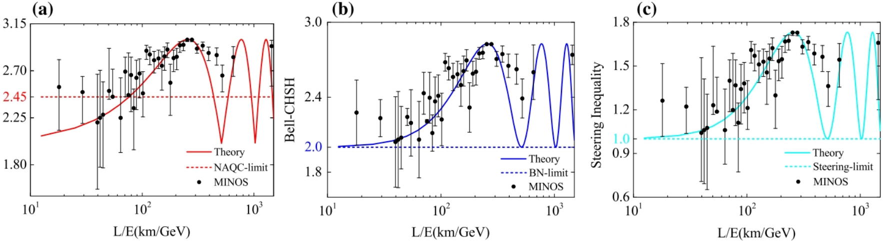


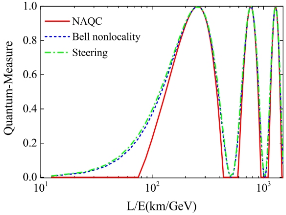


- Daya Bay: $\sin^2 2\theta_{13} = 0.084$ and $\Delta m^2_{ee} = 2.42 \times 10^{-3} eV^2$
- NAQC is a stronger nonclassical correlation than Bell non-locality and quantum steering.

§F. Ming, X-K. Song, D. Wang, Eur. Phys. J., (2020)

Quantumness in neutrino oscillations (MINOS) ¶





- MINOS: $\sin^2 2\theta_{23} = 0.95$ and $\Delta m^2_{32} = 2.32 \times 10^{-3} eV^2$.
- The NAQC is a stronger nonclassical correlation than Bell non-locality and quantum steering.

¶F. Ming, X-K. Song, D. Wang, Eur. Phys. J., (2020)

NAQC & Bell nonlocality in the wave packet approach**

• We have extended the studies on quantumness of neutrino oscillations through NAQC using the wave packet approach.‖

Neutrino with definite flavor:

$$|\nu_\alpha(x,t)\rangle = \sum_j U^*_{\alpha j}\psi_j(x,t)|\nu_j\rangle$$

where:

$$\psi_j(x,t) = \frac{1}{\sqrt{2\pi}}\int dp\ \psi_j(p)e^{ipx-iE_j(p)t}$$

with:

$$\psi_j(p) = \left(2\pi{\sigma_p^P}^2\right)^{-\frac{1}{4}}\exp-\frac{(p-p_j)^2}{4{\sigma_p^P}^2}$$

‖C. Giunti, C.W. Kim, *Fundamentals of Neutrino Physics and Astrophysics*, Oxford University Press (2007)

**M.B., S. De Siena and C. Matrella, Eur. Phys. J. C (2021)

Wave packet description of neutrino oscillations

Assume the condition $\sigma_p^P \ll E_j^2(p_j)/m_j$. Then we have:

$$E_j(p) \simeq E_j + v_j(p - p_j)$$

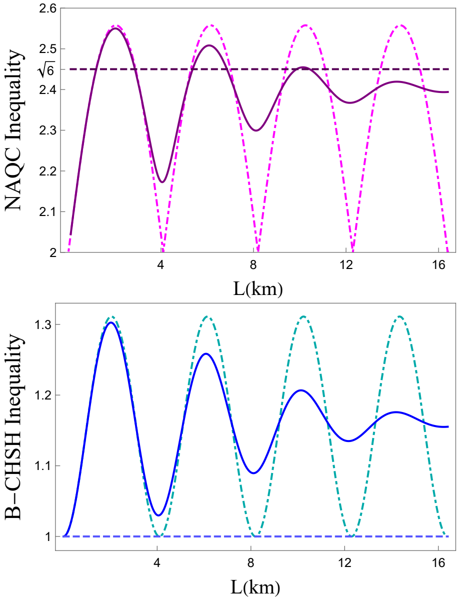
Integrating on p, one gets the wave packet in coordinate space:

$$\psi_j(x,t) = \left(2\pi {\sigma_x^P}^2\right)^{-\frac{1}{4}} \exp\left[-iE_j t + ip_j x - \frac{(x - v_j t)^2}{4{\sigma_x^P}^2}\right]$$

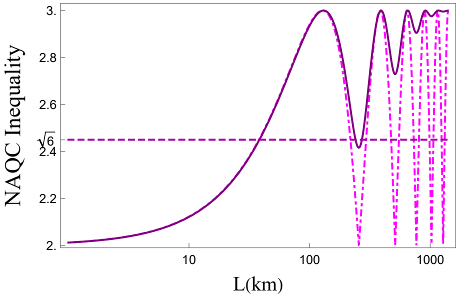
Write density matrix operator $\rho_\alpha(x,t) = |\nu_\alpha(x,t)\rangle\langle\nu_\alpha(x,t)|$. After time integration, one gets the oscillation formula in space

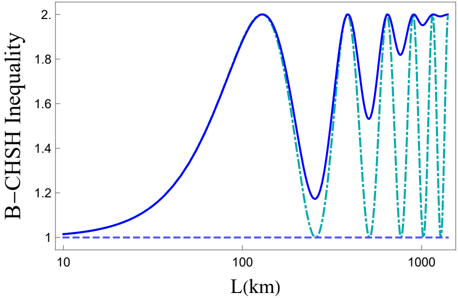
$$P_{\alpha\beta}(L) = \sum_{j,k} U_{\alpha j}^* U_{\alpha k} U_{\beta j}^* U_{\beta k} \exp\left[-2\pi i \frac{L}{L_{jk}^{osc}} - \left(\frac{L}{L_{jk}^{coh}}\right)^2 - 2\pi^2(1-\xi)^2\left(\frac{\sigma_x}{L_{jk}^{osc}}\right)^2\right]$$

NAQC in the wave packet approach (Daya Bay)



NAQC in the wave packet approach (MINOS)





Results and perspectives

- Our treatment based on wave packets leads to a better agreement with experimental data in the case of MINOS.*

- NAQC has a different long-distance behaviour for the two experiments, due to the different values of the mixing angle.

- Existence of a “critical” angle for which NAQC exceeds the bound.

* M.B., S. De Siena and C. Matrella, Eur. Phys. J. C (2021)

Complete Complementarity Relations in neutrino oscillations

Complete Complementarity Relations

To better understand the above results, we resort to the recently introduced concept of CCR.

- N.Bohr (1928): complementarity principle
- W.K.Wootters and W.H.Zurek, *Complementarity in the double-slit experiment: quantum nonseparability and a quantitative statement of Bohr's principle*, Phys. Rev. D (1979)
- M.Jakob and J.A.Bergou, *Quantitative complementarity relations in bipartite systems: entanglement as a physical reality*, Opt. Comm. (2010)
- M.L.W.Basso and J.Maziero, *Complete complementarity relations for multipartite pure states*, J. Phys. A (2020)

N.Bohr (1928)†: Complementarity Principle

• Complementarity: a quantum system may possess properties which are equally real but mutually exclusive.

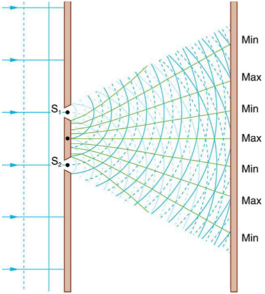
It is often associated with wave-particle duality, the complementarity aspect between propagation and detection.

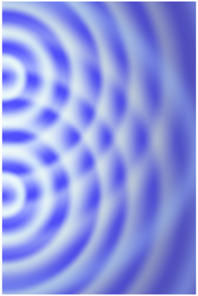
In the double-slit interferometer, the wave aspect is characterized by the *interference fringes visibility*, while the particle nature is given by the *which-way information* of the path along the interferometer.

† N. Bohr, *The quantum postulate and the recent development of atomic theory*, Nature (1928)

Double-slit

Usual view on complementarity: The complete knowledge of the path destroys the interference pattern visibility and vice-versa.





Quantitative wave-particle duality

• Wootters and Zurek *: first quantitative version of the wave-particle duality. A path-detecting device can give incomplete which-way information and a sharply interference pattern can still be retained.

Their work was then extended and formulated in terms of a complementarity relation†

$$P^2 + V^2 \leq 1$$

where P is the predictability and V is the visibility.

• A "**quanton**"‡ may behave partially as a wave or as a particle at the same time.

---

* W.K.Wootters and W.H.Zurek, Phys. Rev. D (1979)

† D.M.Greenberger and A.Yasin, Phys.Lett. A (1988); B.-G. Englert, PRL (1996).

‡ J.-M.Lévy-Leblond, Physica (1988)

Triality relation

- For bipartite systems a complete complementarity relation (CCR) can be obtained by including the correlations between A and B subsystems[§]:

$$V_k^2 + P_k^2 + C^2 = 1$$

  - V_k and P_k, $k = 1, 2$, generate *local* single-partite realities which can be related to wave-particle duality.
  - C is the entanglement measure **concurrence** which generate an exclusive bipartite *nonlocal* reality.

[§] M.Jakob and J.A.Bergou, Opt. Comm. (2010)

Concurrence¶

The concurrence for a generic qubit system described by the density matrix ρ is given by

$$C(\rho) \equiv \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

where the λ_i are the square root of the eigenvalues λ_i^2 of the operator $\rho\tilde{\rho}$ in decreasing order, with

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

¶S. A. Hill, W. K. Wootters, Phys. Rev. Lett. (1997)

Triality relation

Consider the most general bipartite state of two qubits:

$$|\Theta\rangle = a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle$$

One obtains:

$$C = |\langle\Theta|\tilde{\Theta}\rangle| = 2|ad - bc|$$

$$V_k = 2|\langle\Theta|\sigma_k^\dagger|\Theta\rangle| \rightarrow \begin{cases} V_1 = 2|ac^* + bd^*| \\ V_2 = 2|ab^* + cd^*| \end{cases}$$

$$P_k = |\langle\Theta|\sigma_{z,k}|\Theta\rangle| \rightarrow \begin{cases} P_1 = |(|c|^2 + |d|^2) - (|a|^2 + |b|^2)| \\ P_2 = |(|b|^2 + |d|^2) - (|a|^2 + |c|^2)| \end{cases}$$

where: $|\tilde{\Theta}\rangle = (\sigma_y \otimes \sigma_y)\,|\Theta^*\rangle, \quad \sigma_k^\dagger = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma_{z,k} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$

The complementarity relation is satisfied, since the left hand side is just the square norm of the general *pure* bipartite state $|\Theta\rangle$:
$(|a|^2 + |b|^2 + |c|^2 + |d|^2)^2 = 1.$

Examples

- Bell states (maximally entangled states)

$$\Phi^{\pm} = \frac{1}{\sqrt{2}} \left(|00\rangle \pm |11\rangle \right), \quad \Psi^{\pm} = \frac{1}{\sqrt{2}} \left(|01\rangle \pm |10\rangle \right)$$

We have $C = 1, \; V_1 = V_2 = P_1 = P_2 = 0.$

- Separable state

$$|\Theta_1\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |01\rangle \right) = \frac{1}{\sqrt{2}} |0\rangle \left(|0\rangle + |1\rangle \right)$$

In this case $C = 0, \; V_1 = P_2 = 0, \; V_2 = P_1 = 1.$

- Unbalanced state

$$|\Theta_2\rangle = \frac{1}{2} |00\rangle + \frac{\sqrt{3}}{2} |11\rangle.$$

In this case $C = \frac{\sqrt{3}}{2}, \; V_1 = V_2 = 0, \; P_1 = P_2 = \frac{1}{2}.$

Examples

- A separable state with all four terms

$$|\Theta_3\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) \;=\; \frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle).$$

We have $C = 0, \;\; V_1 = \; V_2 = 1, \;\; P_2 = P_2 = 0.$

- Unbalanced state with all four terms

$$|\Theta_4\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2\sqrt{2}}|10\rangle + \frac{\sqrt{3}}{2\sqrt{2}}|11\rangle.$$

In this case we have $C = \frac{\sqrt{3}-1}{2\sqrt{2}}, V_1 = \frac{\sqrt{3}+1}{2\sqrt{2}}, \;\; V_2 = \frac{\sqrt{3}+2}{4}, \;\; P_1 = 0, \;\; P_2 = \frac{1}{4}.$

Complete Complementarity Relation for pure states

Alternative form of CCR for multipartite states*.

Consider a bipartite pure state in the Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$:

$$\rho_{A,B} = \sum_{i,k=0}^{d_A-1} \sum_{j,l=0}^{d_B-1} \rho_{ij,kl} \left| i,j \right\rangle \left\langle k,l \right| .$$

If the state of subsystem A is mixed:

$$P_{hs}(\rho_A) + C_{hs}(\rho_A) < \frac{d_A - 1}{d_A}.$$

where $P_{hs}(\rho_A)$ and $C_{hs}(\rho_A)$ are the predictability and the Hilbert-Schmidt quantum coherence (generalization of the visibility†).

---

*M.L.W.Basso and J.Maziero, J. Phys. A (2020)

†T. Qureshi, Quanta (2019).

CCR for pure states

- The missing information about subsystem A is being shared via correlations with the subsystem B:

$$P_{hs}(\rho_A) + C_{hs}(\rho_A) + C^{nl}_{hs}(\rho_{A|B}) = \frac{d_A - 1}{d_A}$$

– Predictability

$$P_{hs}(\rho_A) \equiv \sum_{i=0}^{d_A-1} (\rho^A_{ii})^2 - \frac{1}{d_A}\,,$$

– Quantum coherence (visibility)

$$C_{hs}(\rho_A) \equiv \sum_{i \neq k}^{d_A-1} |\rho^A_{ik}|^2$$

– Non-local quantum coherence (entanglement)

$$C^{nl}_{hs}(\rho_{A|B}) = \sum_{i\neq k, j\neq l} |\rho_{ij,kl}|^2 - 2 \sum_{i \neq k, j<l} \Re(\rho_{ij,kj}\rho^*_{il,kl})$$

$C^{nl}_{hs}(\rho_{A|B})$ is equivalent to the linear entropy of subsystem A.

CCR for pure states - entropic formulation

- Another form of CCR can be obtained by defining the predictability and the coherence measures in terms of the von Neumann entropy:

$$C_{\mathrm{re}}(\rho_A) + P_{vn}(\rho_A) + S_{vn}(\rho_A) = \log_2 d_A$$

where

$$C_{\mathrm{re}}(\rho_A) = S_{vn}(\rho_{A\mathrm{diag}}) - S_{vn}(\rho_A)$$

$$P_{vn}(\rho_A) \equiv \log_2 d_A - S_{vn}(\rho_{A\mathrm{diag}})$$

For pure states $S_{vn}(\rho_A) = -\mathrm{Tr}\,(\rho_A \log_2 \rho_A)$ is a measure of entanglement between A and B.

CCR for mixed states*

• For mixed states, $S_{vn}(\rho_A)$ does not quantify entanglement, but it is just a measure of mixedness of A. CCR have to be modified:

$$P_{vn}(\rho_A) + C_{re}(\rho_A) + I_{A:B}(\rho_{AB}) + S_{A|B}(\rho_{AB}) = \log_2 d_A,$$

where:

- $P_{vn}(\rho_A) \equiv \ln d_A - S_{vn}(\rho_{Adiag})$ is the **predictability**;
- $C_{re}(\rho_A) = S_{vn}(\rho_{Adiag}) - S_{vn}(\rho_A)$ is the **relative entropy of coherence**;
- $I_{A:B}(\rho_{AB}) = S_{vn}(\rho_A) + S_{vn}(\rho_B) - S_{vn}(\rho_{AB})$ is the **mutual information** of A and B;
- $S_{A|B}(\rho_{AB}) = S_{vn}(\rho_{AB}) - S_{vn}(\rho_B)$ is the **conditional entropy**: It tells how much it is convenient knowing about subsystem A with respect to the whole system.

* M.L.W.Basso and J.Maziero, EPL (2021)

CCR for oscillating neutrinos†

• We now consider the CCR for neutrino oscillations, both for pure and mixed states.

Let us consider a two-flavor neutrino state:

$$|\nu_\alpha(t)\rangle = a_{\alpha\alpha}(t)\,|\nu_\alpha\rangle + a_{\alpha\beta}(t)\,|\nu_\beta\rangle$$

We can use the following correspondence:

$$|\nu_\alpha\rangle = |1\rangle_\alpha \otimes |0\rangle_\beta = |10\rangle$$
$$|\nu_\beta\rangle = |0\rangle_\alpha \otimes |1\rangle_\beta = |01\rangle$$

For an initial electronic neutrino, we have:

$$|\nu_e(t)\rangle = a_{ee}\,|10\rangle + a_{e\mu}\,|01\rangle$$

† V.Bittencourt, M.B., S.De Siena and C.Matrella, Eur. Phys. J. C. (2022)

CCR for oscillating neutrinos

The corresponding density matrix is:

$$\rho_{e\mu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |a_{e\mu}|^2 & a_{ee}a_{e\mu}^* & 0 \\ 0 & a_{e\mu}a_{ee}^* & |a_{ee}|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The state of subsystems e and μ are:

$$\rho_e = \begin{pmatrix} |a_{ee}|^2 & 0 \\ 0 & |a_{e\mu}|^2 \end{pmatrix}; \qquad \rho_\mu = \begin{pmatrix} |a_{e\mu}|^2 & 0 \\ 0 & |a_{ee}|^2 \end{pmatrix}$$

CCR for oscillating neutrinos

We verify that the CCRs for pure states are verified in the case of neutrino. We find:[‡]

$$
\begin{aligned}
P_{hs}(\rho_e) &= P_{ee}^2 + P_{e\mu}^2 - \frac{1}{2} \\
C_{hs}(\rho_e) &= 0 \\
C_{hs}^{nl}(\rho_{e\mu}) &= 2P_{ee}P_{e\mu}
\end{aligned}
$$

where $|a_{ee}|^2 = P_{ee}$, $|a_{e\mu}|^2 = P_{e\mu}$ and $P_{ee} + P_{e\mu} = 1$.

Thus:

$$P_{hs}(\rho_e) + C_{hs}(\rho_e) + C_{hs}^{nl}(\rho_{e\mu}) = \frac{1}{2}$$

as expected.

[‡] V.Bittencourt, M.B., S.De Siena and C.Matrella, Eur. Phys. J. C. (2022)

CCR for oscillating neutrinos

Analogously:

$$
\begin{aligned}
P_{vn}(\rho_e) &= 1 + |a_{ee}|^2 \log_2 |a_{ee}|^2 + |a_{e\mu}|^2 \log_2 |a_{e\mu}|^2 \\
C_{re}(\rho_e) &= 0 \\
S_{vn}(\rho_e) &= -|a_{ee}|^2 \log_2 |a_{ee}|^2 - |a_{e\mu}|^2 \log_2 |a_{e\mu}|^2
\end{aligned}
$$

and the CCR is verified:

$$P_{vn}(\rho_e) + C_{re}(\rho_e) + S_{vn}(\rho_e) = 1$$

CCR for neutrino mixed state

In a wave-packet description of neutrino oscillations, one starts with a pure state $\rho_\alpha(x,t)$ which become mixed after time integration:

$$\rho_\alpha(x) = \sum_{k,j} U_{\alpha k} U^*_{\alpha j} f_{jk}(x) \left|\nu_j\right\rangle \left\langle\nu_k\right| ,$$

where:

$$f_{jk}(x) = \exp\left[-i \frac{\Delta m^2_{jk} x}{2E} - \left(\frac{\Delta m^2_{jk} x}{4\sqrt{2} E^2 \sigma_x} \right)^2 \right]$$

By considering:

$$\left|\nu_i\right\rangle = \sum_{\alpha} U_{\alpha i} \left|\nu_\alpha\right\rangle , \qquad \left|\nu_\alpha\right\rangle = \left|\delta_{\alpha e}\right\rangle_e \left|\delta_{\alpha \mu}\right\rangle_\mu \left|\delta_{\alpha \tau}\right\rangle_\tau$$

we can write:

$$\rho_\alpha(x) = \sum_{\beta\gamma} F^\alpha_{\beta\gamma}(x) \left|\delta_{\beta e}\delta_{\beta\mu}\delta_{\beta\tau}\right\rangle \left\langle\delta_{\gamma e}\delta_{\gamma\mu}\delta_{\gamma\tau}\right|$$

where:

$$F^\alpha_{\beta\gamma}(x) = \sum_{kj} U^*_{\alpha j} U_{\alpha k} f_{jk}(x) U_{\beta j} U^*_{\gamma k}$$

CCR for neutrino mixed state

- We consider the CCR in the case of a two-flavor neutrino oscillation, for an initial electron neutrino

$$P_{vn}(\rho_e) + C_{re}(\rho_e) + I_{A:B}(\rho_{e\mu}) + S_{e|\mu}(\rho_{e\mu}) = \log_2 d_e,$$

where:

$$\begin{aligned} P_{vn}(\rho_e) &= \log_{d_e} - S_{vn}(\rho_{e_{diag}}) \\ C_{re}(\rho_e) &= S_{vn}(\rho_{e_{diag}}) - S_{vn}(\rho_e) \\ I_{A:B}(\rho_{e\mu}) &= S_{vn}(\rho_e) + S_{vn}(\rho_\mu) - S_{vn}(\rho_{e\mu}) \\ S_{e|\mu}(\rho_{e\mu}) &= S_{vn}(\rho_{e\mu}) - S_{vn}(\rho_\mu) \end{aligned}$$

For a generic matrix ρ, the von Neumann entropy is defined as $S_{vn}(\rho) = -\sum_i \lambda_i \log_2 \lambda_i$, where λ_i are the eigenvalues of ρ.

CCR for neutrino mixed state

The starting density matrix is:

$$\rho_{e\mu}(x) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & F^e_{ee} & F^e_{e\mu} & 0 \\ 0 & F^e_{\mu e} & F^e_{\mu\mu} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and the reduced density matrices are:

$$\rho_e(x) = \begin{pmatrix} F^e_{ee} & 0 \\ 0 & F^e_{\mu\mu} \end{pmatrix} \qquad \rho_\mu(x) = \begin{pmatrix} F^e_{\mu\mu} & 0 \\ 0 & F^e_{ee} \end{pmatrix}$$

By evaluating the eigenvalues of these matrices, we obtain:

$$P_{vn}(\rho_e) = 1 + F^e_{ee}\log_2 F^e_{ee} + F^e_{\mu\mu}\log_2 F^e_{\mu\mu}$$
$$C_{re}(\rho_e) = 0$$
$$I_{e:\mu}(\rho_{e\mu}) + S_{e|\mu}(\rho_{e\mu}) = -F^e_{ee}\log_2 F^e_{ee} - F^e_{\mu\mu}\log_2 F^e_{\mu\mu}$$

By adding all the terms we find that the CCR for mixed states is satisfied for a neutrino state.

CCR for neutrino mixed state

The sum of the non-local terms of the CCR is equal to the Quantum Discord, defined as:

$$QD(\rho_{AB}) = I(\rho_{AB}) - CC(\rho_{AB}),$$

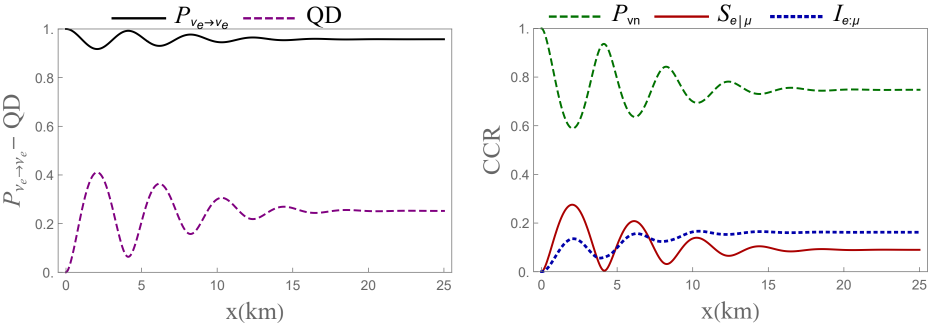
where $I(\rho_{AB})$ is the total correlations between the subsystems A and B; and $CC(\rho_{AB})$ quantifies the classical correlations. We have

$$QD(\rho_{AB}) = S_{\mathrm{vn}}(\rho_A) - S_{\mathrm{vn}}(\rho_{AB}) + \min_{\{\Pi_i^b\}} S_{\mathrm{vn},\{\Pi_i^b\}}(\rho_{A|B})$$

that, for the neutrino density matrix under consideration, gives

$$QD(\rho_{e\mu}) = -F_{ee}^e \log_2 F_{ee}^e - F_{\mu\mu}^e \log_2 F_{\mu\mu}^e$$

CCR for neutrino oscillations* - DAYA BAY

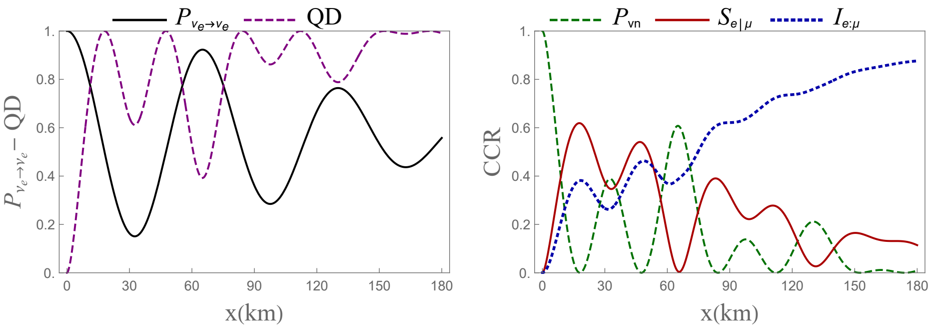


(a) *DAYA BAY* $(L \in [364m, 1912m])$

$$\Delta m^2_{ee} = 2.42 \times 10^{-3} eV^2,\ \sin^2 2\theta_{13} = 0.084,\ E = 4MeV$$

*V.Bittencourt, M.B., S.De Siena and C.Matrella, Eur. Phys. J. C. (2022)

CCR for neutrino oscillations[†] - KamLAND

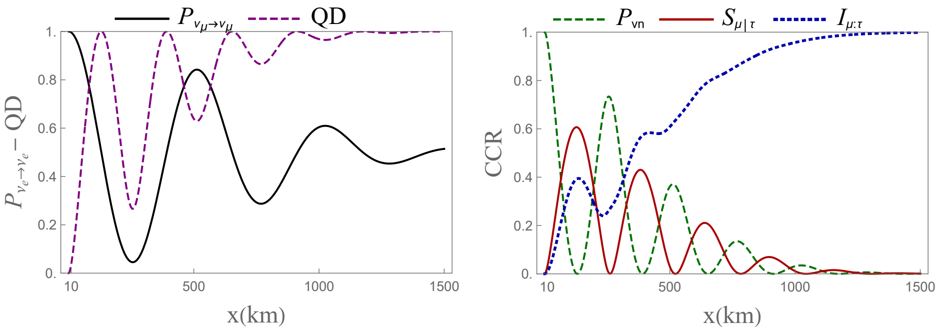


(b) *KamLAND (L = 180 Km)*

$$\Delta m^2_{12} = 7.49 \times 10^{-5} eV^2,\ \tan^2 2\theta_{12} = 0.47,\ E = 2MeV$$

[†] V.Bittencourt, M.B., S.De Siena and C.Matrella, Eur. Phys. J. C. (2022)

CCR for neutrino oscillations[‡] - MINOS



(c) *MINOS* $(L = 735\ km)$

$$\Delta m^2_{32} = 2.32 \times 10^{-3} eV^2,\ \sin^2 2\theta_{23} = 0.95,\ E = 0.5 GeV$$

‡V.Bittencourt, M.B., S.De Siena and C.Matrella, Eur. Phys. J. C. (2022)

Results and perspectives

- We have studied CCR for the oscillating neutrino systems, both in the pure and in the mixed case.
- Complete characterization of quantum correlations in neutrino oscillations.
- Interesting long-distance behaviour of the correlations, depending on the mixing angle.
- To be done: Extension to three flavors, multipartite entanglement.

Quantum Field Theory of neutrino mixing and oscillations

Neutrino mixing in QFT

- Mixing relations for two Dirac fields

$$\nu_e(x) = \cos\theta\,\nu_1(x) + \sin\theta\,\nu_2(x)$$
$$\nu_\mu(x) = -\sin\theta\,\nu_1(x) + \cos\theta\,\nu_2(x)$$

can be written as*

$$\nu_e^\alpha(x) = G_\theta^{-1}(t)\,\nu_1^\alpha(x)\,G_\theta(t)$$
$$\nu_\mu^\alpha(x) = G_\theta^{-1}(t)\,\nu_2^\alpha(x)\,G_\theta(t)$$

– Mixing generator:

$$G_\theta(t) = \exp\left[\theta \int d^3\mathbf{x}\,\left(\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)\right)\right]$$

For ν_e, we get $\frac{d^2}{d\theta^2}\,\nu_e^\alpha = -\nu_e^\alpha$ with i.c. $\nu_e^\alpha\big|_{\theta=0} = \nu_1^\alpha$, $\frac{d}{d\theta}\nu_e^\alpha\big|_{\theta=0} = \nu_2^\alpha$.

---

*M.B. and G.Vitiello, Annals Phys. (1995)

• The vacuum $|0\rangle_{1,2}$ is not invariant under the action of $G_\theta(t)$:

$$|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t)\, |0\rangle_{1,2}$$

• Relation between $|0\rangle_{1,2}$ and $|0(t)\rangle_{e,\mu}$: orthogonality! (for $V \to \infty$)

$$\lim_{V\to\infty} {}_{1,2}\langle 0|0(t)\rangle_{e,\mu} = \lim_{V\to\infty} e^{V\int \frac{d^3\mathbf{k}}{(2\pi)^3}\, \ln\left(1-\sin^2\theta\, |V_\mathbf{k}|^2\right)^2} = 0$$

with

$$|V_\mathbf{k}|^2 \;\; \equiv \;\; \sum_{r,s} |\, v^{r\dagger}_{-\mathbf{k},1} u^s_{\mathbf{k},2}\, |^2 \neq 0 \quad for \quad m_1 \neq m_2$$

.

- The "flavor vacuum" $|0(t)\rangle_{e,\mu}$ is a $SU(2)$ generalized coherent state[†]:

$$|0\rangle_{e,\mu} = \prod_{\mathbf{k},r} \Big[(1-\sin^2\theta\,|V_{\mathbf{k}}|^2) - \epsilon^r \sin\theta\,\cos\theta\,|V_{\mathbf{k}}|\,(\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2} + \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1})$$

$$+\,\epsilon^r \sin^2\theta\,|V_{\mathbf{k}}||U_{\mathbf{k}}|\,(\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},1} - \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},2}) + \sin^2\theta\,|V_{\mathbf{k}}|^2\,\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2}\alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1}\Big]\,|0\rangle_{1,2}$$

- Condensation density:

$${}_{e,\mu}\langle 0(t)|\alpha^{r\dagger}_{\mathbf{k},i}\alpha^{r}_{\mathbf{k},i}|0(t)\rangle_{e,\mu} \;=\; {}_{e,\mu}\langle 0(t)|\beta^{r\dagger}_{\mathbf{k},i}\beta^{r}_{\mathbf{k},i}|0(t)\rangle_{e,\mu} \;=\; \sin^2\theta\,|V_{\mathbf{k}}|^2$$

vanishing for $m_1 = m_2$ and/or $\theta = 0$ (in both cases no mixing).

– Condensate structure as in systems with SSB (e.g. superconductors)

– Exotic condensates: mixed pairs

– Note that $|0\rangle_{e\,\mu} \neq |a\rangle_1 \otimes |b\rangle_2 \Rightarrow$ entanglement.

[†] A. Perelomov, *Generalized Coherent States,* (Springer V., 1986)

• Structure of the annihilation operators for $|0(t)\rangle_{e,\mu}$:

$$\alpha^r_{\mathbf{k},e}(t) = \cos\theta\,\alpha^r_{\mathbf{k},1} + \sin\theta\left(U^*_{\mathbf{k}}(t)\,\alpha^r_{\mathbf{k},2} + \epsilon^r V_{\mathbf{k}}(t)\,\beta^{r\dagger}_{-\mathbf{k},2}\right)$$

$$\alpha^r_{\mathbf{k},\mu}(t) = \cos\theta\,\alpha^r_{\mathbf{k},2} - \sin\theta\left(U_{\mathbf{k}}(t)\,\alpha^r_{\mathbf{k},1} - \epsilon^r V_{\mathbf{k}}(t)\,\beta^{r\dagger}_{-\mathbf{k},1}\right)$$

$$\beta^r_{-\mathbf{k},e}(t) = \cos\theta\,\beta^r_{-\mathbf{k},1} + \sin\theta\left(U^*_{\mathbf{k}}(t)\,\beta^r_{-\mathbf{k},2} - \epsilon^r V_{\mathbf{k}}(t)\,\alpha^{r\dagger}_{\mathbf{k},2}\right)$$

$$\beta^r_{-\mathbf{k},\mu}(t) = \cos\theta\,\beta^r_{-\mathbf{k},2} - \sin\theta\left(U_{\mathbf{k}}(t)\,\beta^r_{-\mathbf{k},1} + \epsilon^r V_{\mathbf{k}}(t)\,\alpha^{r\dagger}_{\mathbf{k},1}\right)$$

• Mixing transformation = Rotation + Bogoliubov transformation .

– Bogoliubov coefficients:

$$U_{\mathbf{k}}(t) \;=\; u^{r\dagger}_{\mathbf{k},2} u^r_{\mathbf{k},1}\, e^{i(\omega_{k,2}-\omega_{k,1})t} \quad ; \qquad V_{\mathbf{k}}(t) = \epsilon^r\, u^{r\dagger}_{\mathbf{k},1} v^r_{-\mathbf{k},2}\, e^{i(\omega_{k,2}+\omega_{k,1})t}$$

$$|U_{\mathbf{k}}|^2 + |V_{\mathbf{k}}|^2 = 1$$

Decomposition of mixing generator *

The mixing generator can be expressed in terms of a rotation and a Bogoliubov transformation. Define:

$$R(\theta) \equiv \exp\left\{\theta \sum_{\mathbf{k},r} \left[\left(\alpha^{r\dagger}_{\mathbf{k},1}\alpha^{r}_{\mathbf{k},2} + \beta^{r\dagger}_{\mathbf{k},1}\beta^{r}_{\mathbf{k},2}\right)e^{i\psi_k} - \left(\alpha^{r\dagger}_{\mathbf{k},2}\alpha^{r}_{\mathbf{k},1} + \beta^{r\dagger}_{\mathbf{k},2}\beta^{r}_{\mathbf{k},1}\right)e^{-i\psi_k}\right]\right\},$$

$$B_i(\Theta_i) \equiv \exp\left\{\sum_{\mathbf{k},r} \Theta_{\mathbf{k},i}\ \epsilon^r \left[\alpha^{r}_{\mathbf{k},i}\beta^{r}_{-\mathbf{k},i}e^{-i\phi_{k,i}} - \beta^{r\dagger}_{-\mathbf{k},i}\alpha^{r\dagger}_{\mathbf{k},i}e^{i\phi_{k,i}}\right]\right\}, \qquad i = 1, 2$$

Since $[B_1, B_2] = 0$ we put $B(\Theta_1, \Theta_2) \equiv B_1(\Theta_1)\, B_2(\Theta_2)$.

- We find:

$$G_\theta \ =\ B(\Theta_1, \Theta_2)\ \ R(\theta)\ \ B^{-1}(\Theta_1, \Theta_2)$$

which is realized when the $\Theta_{\mathbf{k},i}$ are chosen as:

$$U_{\mathbf{k}} = e^{-i\psi_k}\ \cos(\Theta_{\mathbf{k},1} - \Theta_{\mathbf{k},2})\,; \qquad V_{\mathbf{k}} = e^{\frac{(\phi_{k,1}+\phi_{k,2})}{2}}\ \sin(\Theta_{\mathbf{k},1} - \Theta_{\mathbf{k},2})$$

---

* M.B., M.V.Gargiulo and G.Vitiello, Phys. Lett. B (2017)

Bogoliubov vs Pontecorvo

- Bogoliubov and Pontecorvo do not commute!

[ , ] ≠ 0

As a result, flavor vacuum gets a non-trivial term:

$$|0\rangle_{e,\mu} \equiv G_\theta^{-1}|0\rangle_{1,2} \;=\; |0\rangle_{1,2} \;+\; \Big[B(m_1, m_2)\;,\; R^{-1}(\theta)\Big]\;\, |\widetilde{0}\rangle_{1,2}$$

- Non-diagonal Bogoliubov transformation

$$|0\rangle_{e,\mu} \cong \left[\mathbb{1} + \theta\, a \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}}\; \tilde{V}_{\mathbf{k}} \sum_r \epsilon^r \left(\alpha^{r\dagger}_{\mathbf{k},1}\beta^{r\dagger}_{-\mathbf{k},2} + \alpha^{r\dagger}_{\mathbf{k},2}\beta^{r\dagger}_{-\mathbf{k},1}\right)\right]|0\rangle_{1,2}\,,$$

with $a \equiv \frac{(m_2-m_1)^2}{m_1 m_2}$.

Neutrino oscillation formula (QFT)

– We have, for an electron neutrino state:

$$\begin{aligned}\mathcal{Q}_{\mathbf{k},\sigma}(t) &\equiv \langle\nu^r_{\mathbf{k},e}| :: Q_\sigma(t) :: |\nu^r_{\mathbf{k},e}\rangle \\ &= \left|\left\{\alpha^r_{\mathbf{k},\sigma}(t), \alpha^{r\dagger}_{\mathbf{k},e}(0)\right\}\right|^2 + \left|\left\{\beta^{r\dagger}_{-\mathbf{k},\sigma}(t), \alpha^{r\dagger}_{\mathbf{k},e}(0)\right\}\right|^2\end{aligned}$$

with $Q_\sigma(t) \equiv \int d^3\mathbf{x}\, \nu^\dagger_\sigma(x)\, \nu_\sigma(x)$.

- Neutrino oscillation formula (exact result)*:

$$\mathcal{Q}_{\mathbf{k},e}(t) = 1 - |U_\mathbf{k}|^2 \sin^2(2\theta)\, \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}\, t\right) - |V_\mathbf{k}|^2 \sin^2(2\theta)\, \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}\, t\right)$$

- For $k \gg \sqrt{m_1 m_2}$, $|U_\mathbf{k}|^2 \to 1$ and $|V_\mathbf{k}|^2 \to 0 \Rightarrow$ Pontecorvo formula is recovered.

---

*M.B., P.Henning and G.Vitiello, Phys. Lett. **B** (1999).

Neutrino Ontology: flavor or mass?

- In view of the unitary inequivalence of mass and flavor representations, we have the problem of the fundamental (ontological) nature of neutrino.

Flavor or mass, that is the question...

Neutrino ontology: research directions

- How to verify the fundamental nature of neutrino states?

Two directions:

- Investigate the phenomenology of flavor neutrinos, with corrections expected in the non-relativistic regime: oscillations, beta decay endpoint, quantum correlations, ...
- Use the formal consistency of QFT, by comparing neutrino processes in two different frames (inertial and comoving) for accelerated particle: Unruh effect.*

---

*M.B., G.Lambiase and G.Luciano, Phys.Rev.D(2017); M.B., G.Lambiase, G.Luciano and L.Petruzziello, Phys.Rev.D(2018); Phys.Lett.B(2020); EPJC(2020).

Chiral oscillations and lepton/antineutrino entanglement

Chiral oscillations

- Taking into account (bi)spinorial nature of neutrinos and chiral nature of weak interaction, one naturally gets chiral oscillations [*]
- They occur even with one flavor; interplay with flavor oscillations in the non-relativistic region[†]
- For neutrinos from CνB, chiral oscillations reduce detection by a factor of 2.[‡]
- Application: lepton-antineutrino entanglement and chiral oscillations in pion decay.[§]

[*] A. Bernardini and S. De Leo, Phys. Rev. D (2005)

[†] V.A.Bittencourt, A.Bernardini and M.B.,Eur.Phys.J.C(2021);EPL Persp.(2022);

[‡] S.-F. Ge and P.Pasquini, Phys. Lett. B (2020)

[§] V.A.Bittencourt, A.Bernardini and M.B., Universe (2021)

Chiral oscillations

Chiral representation of the Dirac matrices

$$\hat{\alpha}_i = \begin{bmatrix} \hat{\sigma}_i & 0 \\ 0 & -\hat{\sigma}_i \end{bmatrix}, \qquad \hat{\beta} = \begin{bmatrix} 0 & \hat{I}_2 \\ \hat{I}_2 & 0 \end{bmatrix},$$

Any bispinor $|\xi\rangle$ can be written in this representation as

$$|\xi\rangle = \begin{bmatrix} |\xi_R\rangle \\ |\xi_L\rangle \end{bmatrix},$$

The Dirac equation $\hat{H}_D\,|\xi\rangle = i|\dot{\xi}\rangle$ can then be written as

$$\hat{\mathbf{p}} \cdot \hat{\sigma}\,|\xi_R\rangle + m\,|\xi_L\rangle = i\partial_t\,|\xi_R\rangle\,,$$
$$-\hat{\mathbf{p}} \cdot \hat{\sigma}\,|\xi_L\rangle + m\,|\xi_R\rangle = i\partial_t\,|\xi_L\rangle\,,$$

• Evolution under the free Dirac Hamiltonian $\hat{H}_D$ induces left-right chiral oscillations.

Take initial state $|\psi(0)\rangle = [0,\ 0,\ 0,\ 1]^T$ which has negative helicity and negative chirality: $\hat{\gamma}_5 |\psi(0)\rangle = -|\psi(0)\rangle$.

The time evolved state $|\psi_m(t)\rangle = e^{-i\hat{H}_D t} |\psi(0)\rangle$ is given by

$$
\begin{aligned}
|\psi_m(t)\rangle \quad = \quad & \sqrt{\frac{E_{p,m}+m}{4E_{p,m}}} \left[\left(1 + \frac{p}{E_{p,m}+m}\right) e^{-iE_{p,m}t} |u_-(p,m)\rangle \right. \\
& \left. - \left(1 - \frac{p}{E_{p,m}+m}\right) e^{iE_{p,m}t} |v_-(-p,m)\rangle \right],
\end{aligned}
$$

with (for one-dimensional propagation along the $\mathbf{e}_z$ direction)

$$
|u_\pm(p,m)\rangle = \sqrt{\frac{E_{p,m}+m}{4E_{p,m}}} \begin{bmatrix} \left(1 \pm \frac{p}{E_{p,m}+m}\right) |\pm\rangle \\ \left(1 \mp \frac{p}{E_{p,m}+m}\right) |\pm\rangle \end{bmatrix},
$$

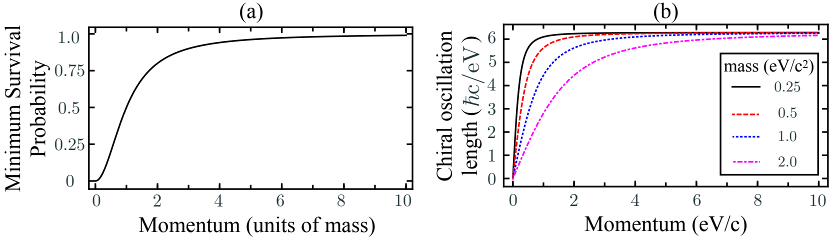
$$
|v_\pm(p,m)\rangle = \sqrt{\frac{E_{p,m}+m}{4E_{p,m}}} \begin{bmatrix} \left(1 \pm \frac{p}{E_{p,m}+m}\right) |\pm\rangle \\ -\left(1 \mp \frac{p}{E_{p,m}+m}\right) |\pm\rangle \end{bmatrix},
$$

- Survival probability of initial left-handed state

$$\mathcal{P}(t) = |\langle\psi_m(0)|\psi_m(t)\rangle|^2 = 1 - \frac{m^2}{E_{p,m}^2}\sin^2(E_{p,m}t)\,,$$

Average value of the chiral operator $\langle\hat{\gamma}_5\rangle(t)$

$$\langle\hat{\gamma}_5\rangle(t) = \langle\psi_m(t)|\hat{\gamma}_5|\psi_m(t)\rangle = -1 + \frac{2m^2}{E_{p,m}^2}\sin^2(E_{p,m}t)\,.$$



Chiral and flavor oscillations

- State of a neutrino of flavor α at a given t:

$$|\nu_\alpha(t)\rangle = \sum_i U_{\alpha,i} \, |\psi_{m_i}(t)\rangle \otimes |\nu_i\rangle \,,$$

where $|\psi_{m_i}(t)\rangle$ are bispinors.

- The state at $t = 0$ reads

$$|\nu_\alpha(0)\rangle = |\psi(0)\rangle \otimes \sum_i U_{\alpha,i} \, |\nu_i\rangle = |\psi(0)\rangle \otimes |\nu_\alpha\rangle \,,$$

where $|\psi(0)\rangle$ is a left handed bispinor.

- Survival probability:

$$\mathcal{P}_{\alpha\to\alpha} = |\,\langle\nu_\alpha(0)|\nu_\alpha(t)\rangle\,|^2 = |\sum_i |U_{\alpha,i}|^2 \, \langle\psi(0)|\psi_{m_i}(t)\rangle\,|^2.$$

Two flavor mixing:

$$
\begin{aligned}
|\nu_e(t)\rangle &= \left[\cos^2\theta\,|\psi_{m_1}(t)\rangle + \sin^2\theta\,|\psi_{m_2}(t)\rangle\right] \otimes |\nu_e\rangle \\
&\qquad + \sin\theta\cos\theta\left[|\psi_{m_1}(t)\rangle - |\psi_{m_2}(t)\rangle\right] \otimes |\nu_\mu\rangle\,,
\end{aligned}
$$

- The survival probability can be decomposed as

$$
\mathcal{P}_{e\to e}(t) = \mathcal{P}^S_{e\to e}(t) + \mathcal{A}_e(t) + \mathcal{B}_e(t).
$$

$\mathcal{P}^S_{e\to e}(t)$ is the standard flavor oscillation formula

$$
\mathcal{P}^S_{e\to e}(t) = 1 - \sin^2 2\theta \sin^2\left(\frac{E_{p,m_2} - E_{p,m_1}}{2}t\right)
$$

and

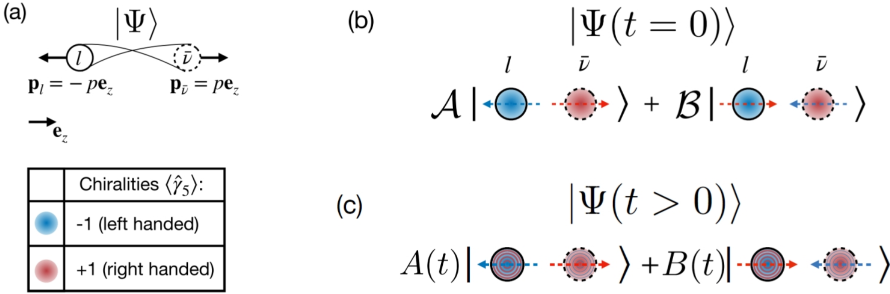
$$
\begin{aligned}
\mathcal{A}_e(t) &= -\left[\frac{m_1}{E_{p,m_1}}\cos^2\theta\sin\left(E_{p,m_1}t\right) + \frac{m_2}{E_{p,m_2}}\sin^2\theta\sin\left(E_{p,m_2}t\right)\right]^2, \\
\mathcal{B}_e(t) &= \frac{1}{2}\sin^2 2\theta\sin(E_{p,m_1}t)\sin(E_{p,m_2}t)\left(\frac{p^2 + m_1 m_2}{E_{p,m_1}E_{p,m_2}} - 1\right),
\end{aligned}
$$

are correction terms due to the bispinorial structure.

- Agreement with the QFT formula.

Lepton-antineutrino entanglement and chiral oscillations*

- As an application of chiral oscillations, we consider induced spin correlations in pion decay products ($\pi \to l + \bar{\nu}$)



*V.A.Bittencourt, A.Bernardini and M.B., Universe (2021)

- For the creation process, we assume the following superposition

$$|\Phi\rangle = \frac{|v_{\uparrow}(p, m_{\bar{\nu}})\rangle \otimes |u_{\downarrow}(-p, m_l)\rangle - |v_{\downarrow}(p, m_{\bar{\nu}})\rangle \otimes |u_{\uparrow}(-p, m_l)\rangle}{\sqrt{2}}$$

$$|u_{\uparrow}(p, m)\rangle = N_{p,m}\begin{bmatrix} f_+(p, m)|\uparrow\rangle \\ f_-(p, m)|\uparrow\rangle \end{bmatrix}, \qquad |u_{\downarrow}(p, m)\rangle = N_{p,m}\begin{bmatrix} f_-(p, m)|\downarrow\rangle \\ f_+(p, m)|\downarrow\rangle \end{bmatrix},$$

$$|v_{\uparrow}(p, m)\rangle = N_{p,m}\begin{bmatrix} f_+(p, m)|\uparrow\rangle \\ -f_-(p, m)|\uparrow\rangle \end{bmatrix}, \qquad |v_{\downarrow}(p, m)\rangle = N_{p,m}\begin{bmatrix} f_-(p, m)|\downarrow\rangle \\ -f_+(p, m)|\downarrow\rangle \end{bmatrix},$$

with

$$N_{p,m} = \sqrt{\frac{E_{p,m} + m}{4E_{p,m}}}, \quad f_{\pm}(p, m) = 1 \pm \frac{p}{E_{p,m} + m},$$

- At $t = 0$ the state is given by

$$|\Psi(0)\rangle = \frac{\hat{\Pi}_R^{(\bar{\nu})} \otimes \hat{\Pi}_L^{(l)} |\Phi\rangle}{\langle \Phi | \hat{\Pi}_R^{(\bar{\nu})} \otimes \hat{\Pi}_L^{(l)} |\Phi\rangle}.$$

Chirality projectors $\hat{\Pi}_{R(L)}^{(A)} = (\hat{I}^{(A)} + (-)\hat{\gamma}_5^{(A)})/2$, with $\hat{\gamma}_5 = \mathrm{diag}\left[\hat{I}_2, -\hat{I}_2\right]$ and $A = \bar{\nu}$, l, such that

$$|\Psi(0)\rangle = \mathcal{A}(p, m_l, m_{\bar{\nu}}) |\bar{\nu}_\uparrow(0)\rangle \otimes |l_\downarrow(0)\rangle - \mathcal{B}(p, m_l, m_{\bar{\nu}}) |\bar{\nu}_\downarrow(0)\rangle \otimes |l_\uparrow(0)\rangle.$$

The chirality projected states at $t = 0$ are

$$|\bar{\nu}_{\uparrow(\downarrow)}(0)\rangle = \begin{bmatrix} |\uparrow(\downarrow)\rangle \\ 0 \end{bmatrix}, \quad |l_{\uparrow(\downarrow)}(0)\rangle = \begin{bmatrix} 0 \\ |\uparrow(\downarrow)\rangle \end{bmatrix},$$

and the coefficients of the superposition are given by

$$\mathcal{A}(p, m_l, m_{\bar{\nu}}) = N_{p,m_l} N_{p,m_{\bar{\nu}}} f_+(p, m_{\bar{\nu}}) f_-(p, m_l) \left[\frac{1}{2} - \frac{p^2}{2E_{p,m_l} E_{p,m_{\bar{\nu}}}}\right]^{-\frac{1}{2}},$$

$$\mathcal{B}(p, m_l, m_{\bar{\nu}}) = N_{p,m_l} N_{p,m_{\bar{\nu}}} f_-(p, m_{\bar{\nu}}) f_+(p, m_l) \left[\frac{1}{2} - \frac{p^2}{2E_{p,m_l} E_{p,m_{\bar{\nu}}}}\right]^{-\frac{1}{2}}.$$

Spin entanglement at $t = 0$

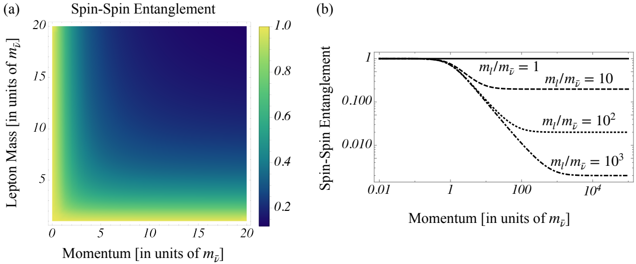
- The state of the lepton-antineutrino pair is then described in the composite Hilbert space $\mathcal{H}_{C_{\bar{\nu}}} \otimes \mathcal{H}_{S_{\bar{\nu}}} \otimes \mathcal{H}_{C_l} \otimes \mathcal{H}_{S_l}$.

- It is a 4-qubit entangled state.

- We can write $|\Psi(0)\rangle = |+_{C_{\bar{\nu}}}\rangle \otimes |-_{C_l}\rangle \otimes |\Psi_{S_{\bar{\nu}},S_l}\rangle$, with $|\pm_A\rangle$ denoting the positive (negative) chirality of $A = C_{\bar{\nu},l}$, and

$$|\Psi_{S_{\bar{\nu}},S_l}\rangle = \mathcal{A}(p, m_l, m_{\bar{\nu}})|\uparrow_{S_{\bar{\nu}}}\rangle \otimes |\downarrow_{S_l}\rangle - \mathcal{B}(p, m_l, m_{\bar{\nu}})|\downarrow_{S_{\bar{\nu}}}\rangle \otimes |\uparrow_{S_l}\rangle$$

is the joint spin state at $t = 0$.



Spin entanglement at $t = 0$

- Reduced density matrix

$$\rho_{S_{\bar{\nu}},S_l}(0) = \mathrm{Tr}_{C_l,C_{\bar{\nu}}}\Big[|\Psi(0)\rangle\langle\Psi(0)|\Big] = |\Psi_{S_{\bar{\nu}},S_l}\rangle\langle\Psi_{S_{\bar{\nu}},S_l}|$$
$$= \mathcal{A}^2(p,m_l,m_{\bar{\nu}})|\uparrow_{\bar{\nu}}\downarrow_l\rangle\langle\uparrow_{\bar{\nu}}\downarrow_l| + \mathcal{B}^2(p,m_l,m_{\bar{\nu}})|\downarrow_{\bar{\nu}}\uparrow_l\rangle\langle\downarrow_{\bar{\nu}}\uparrow_l|$$
$$- \mathcal{A}(p,m_l,m_{\bar{\nu}})\mathcal{B}(p,m_l,m_{\bar{\nu}})\Big[|\uparrow_{\bar{\nu}}\downarrow_l\rangle\langle\downarrow_{\bar{\nu}}\uparrow_l| + |\downarrow_{\bar{\nu}}\uparrow_l\rangle\langle\uparrow_{\bar{\nu}}\downarrow_l|\Big].$$

- Partial transposition yields $\rho^T_{S_{\bar{\nu}},S_l}$ from which we obtain the spin-spin negativity for the state at $t = 0$

$$\mathcal{N}_{S_{\bar{\nu}},S_l}(0) \equiv \mathcal{N}[\rho_{S_{\bar{\nu}},S_l}(0)] = 2|\mathcal{A}(p,m_l,m_{\bar{\nu}})\mathcal{B}(p,m_l,m_{\bar{\nu}})|.$$

Spin entanglement at $t \neq 0$

- For the joint lepton-antineutrino state we get

$$|\Psi(t)\rangle = \mathcal{A}(p, m_l, m_{\bar{\nu}})|\bar{\nu}_\uparrow(t)\rangle \otimes |l_\downarrow(t)\rangle - \mathcal{B}(p, m_l, m_{\bar{\nu}})|\bar{\nu}_\downarrow(t)\rangle \otimes |l_\uparrow(t)\rangle,$$

– antineutrino components:

$$|\bar{\nu}_\uparrow(t)\rangle = \mathcal{N}_{p,m_{\bar{\nu}}}\left[e^{-iE_{p,m_{\bar{\nu}}}t} f_+(p, m_{\bar{\nu}})|u_\uparrow(p, m_{\bar{\nu}})\rangle + e^{iE_{p,m_{\bar{\nu}}}t} f_-(p, m_{\bar{\nu}})|v_\uparrow(-p, m_{\bar{\nu}})\rangle\right],$$

$$|\bar{\nu}_\downarrow(t)\rangle = \mathcal{N}_{p,m_{\bar{\nu}}}\left[e^{-iE_{p,m_{\bar{\nu}}}t} f_-(p, m_{\bar{\nu}})|u_\downarrow(p, m_{\bar{\nu}})\rangle + e^{iE_{p,m_{\bar{\nu}}}t} f_+(p, m_{\bar{\nu}})|v_\downarrow(-p, m_{\bar{\nu}})\rangle\right],$$

– lepton components:

$$|l_\uparrow(t)\rangle = \mathcal{N}_{p,m_l}\left[e^{-iE_{p,m_l}t} f_+(p, m_l)|u_\uparrow(-p, m_l)\rangle - e^{iE_{p,m_l}t} f_-(p, m_l)|v_\uparrow(p, m_l)\rangle\right],$$

$$|l_\downarrow(t)\rangle = \mathcal{N}_{p,m_l}\left[e^{-iE_{p,m_l}t} f_-(p, m_l)|u_\downarrow(-p, m_l)\rangle - e^{iE_{p,m_l}t} f_+(p, m_l)|v_\downarrow(p, m_l)\rangle\right].$$

Spin entanglement at $t \neq 0$

- The reduced matrix $\rho_{S_{\bar{\nu}},S_l}(t) = \mathrm{Tr}_{\mathrm{Chirality}}\left[|\Psi(t)\rangle\langle\Psi(t)|\right]$ describes a mixed state with entanglement affected by chiral oscillations.
- Entanglement between the spins at time t

$$\mathcal{N}_{S_{\bar{\nu}},S_l}(t) \equiv \mathcal{N}[\rho_{S_{\bar{\nu}},S_l}(t)] = ||\rho^T_{S_{\bar{\nu}},S_l}(t)|| - 1 = \mathcal{N}_{S_{\bar{\nu}},S_l}(0)\Gamma(t)$$

with

$$\Gamma(t) = \prod_{j=\bar{\nu},l}\left[1 - \frac{p^2}{m_j^2}\left(\langle\hat{\gamma}_5\rangle_j(t) - 1\right)^2\right]^{\frac{1}{2}}.$$

The average chiralities are given by $\langle\hat{\gamma}_5\rangle_A(t) = \mathrm{Tr}_A[\rho_A(t)]$ with $A = \bar{\nu}, l$:

$$\begin{aligned}
\langle\hat{\gamma}_5\rangle_{\bar{\nu}}(t) &= 1 - \frac{m_{\bar{\nu}}^2}{E_{p,m_{\bar{\nu}}}^2}\left[1 - \cos\left(2E_{p,m_{\bar{\nu}}}t\right)\right], \\
\langle\hat{\gamma}_5\rangle_l(t) &= -1 + \frac{m_l^2}{E_{p,m_l}^2}\left[1 - \cos\left(2E_{p,m_l}t\right)\right].
\end{aligned}$$

Spin entanglement at $t \neq 0$

- Degree of mixedeness of the spin density matrix:

$$\mathrm{Tr}[\rho^2_{S_{\bar{\nu}},S_l}(t)] = 1 - \frac{\mathcal{N}^2_{S_{\bar{\nu}},S_l}(0)(1-|\Gamma(t)|^2)}{2},$$

quantifies entanglement in the bipartition $(S_{\bar{\nu}}, S_l);(C_{\bar{\nu}}, C_l)$, i.e. between spins and chiralities.

- $\mathrm{Tr}[\rho^2_{S_{\bar{\nu}},S_l}(t)] < 1 \Rightarrow$ entanglement initially encoded only in the spins redistributes into spin-chirality entanglement.

- Entanglement encoded in the bipartition $(C_{\bar{\nu}}, S_{\bar{\nu}});(C_l, S_l)$ is conserved:

$$\begin{aligned}\mathrm{Tr}[\rho^2_{\bar{\nu}}(t)] = \mathrm{Tr}[\rho^2_l(t)] &= \mathcal{A}^4(p, m_l, m_{\bar{\nu}}) + \mathcal{B}^4(p, m_l, m_{\bar{\nu}}) \\ &= 1 - \frac{\mathcal{N}^2_{S_{\bar{\nu}},S_l}(0)}{2} < 1.\end{aligned}$$

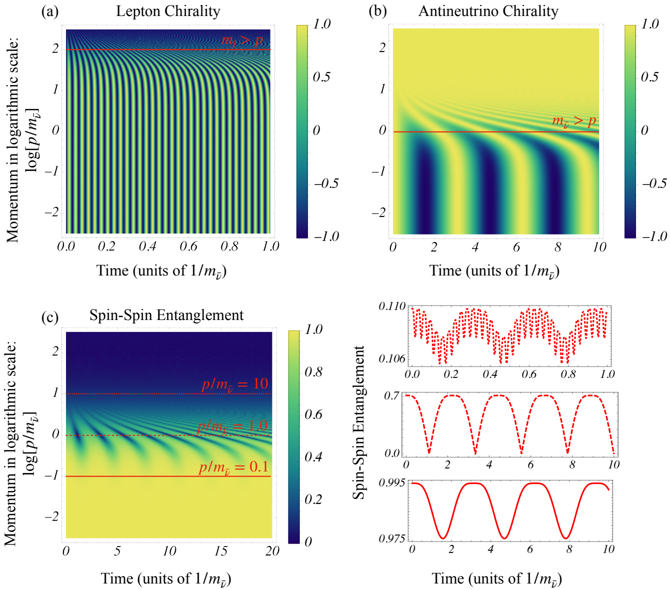


Figure 1: (a) Average lepton chirality, (b) average antineutrino chirality and (c) spin-spin entanglement as a function of the momentum and of time.

Chiral oscillations and Bell spin correlations

- The quantity

$$B[\rho(t)] = |\langle \hat{S}_{\bar{\nu},1} \otimes \hat{S}_{l,1} \rangle + \langle \hat{S}_{\bar{\nu},1} \otimes \hat{S}_{l,2} \rangle + \langle \hat{S}_{\bar{\nu},2} \otimes \hat{S}_{l,1} \rangle - \langle \hat{S}_{\bar{\nu},2} \otimes \hat{S}_{l,2} \rangle|,$$

is the Bell observable used to investigate non-local correlations[†].

For pure states, $B[\rho] > 2$ indicates that the correlations shared between the spins are non-local and that the state is entangled.

[†] N.Brunner et al., Rev. Mod. Phys. (2014)

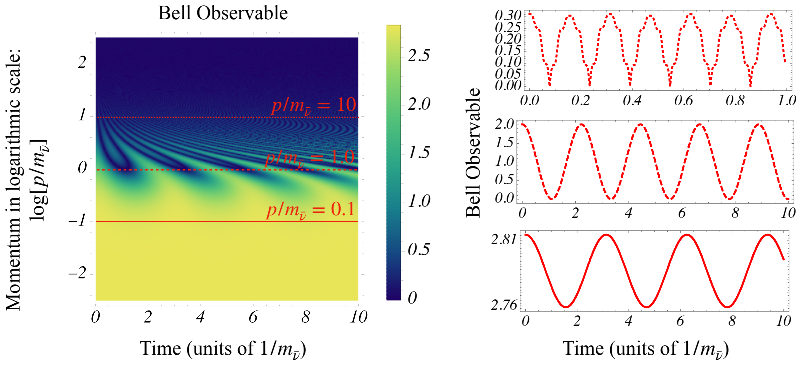


Figure 2: Bell observable as a function of the momentum (in units of the antineutrino mass and in log scale) and of time for $m_l/m_{\bar{\nu}} = 10^2$.

Results & Perspectives

- We find that chiral oscillations do affect spin-spin correlations for the entangled lepton–antineutrino couple.

- Resonance of oscillation amplitude at neutrino mass: possibility of extracting fundamental information via quantum correlations

- We plan to study Leggett-Garg inequalities for the reduced system involving only leptonic d.o.f.

- Inclusion of flavor oscillations[‡].

[‡]M.B, V.Bittencourt and G.Zanfardino, work in progress