Neutrino magnetic moments and solar electron aniteneutrinos

Evgeny Akhmedov

Max-Planck Institute für Kernphysik, Heidelberg

In collaboration with Pablo Martínez-Miravé

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Transition magnetic moments and SFP

Neutrino magnetic dipole moments of Majorana neutrinos:

$$\mathcal{L}_\mu = \frac{1}{2}\mu_{\alpha\beta}\nu_{\alpha L}^T \mathcal{C}\sigma_{\mu\nu}\nu_{\beta L}F^{\mu\nu} + h.c. \qquad (\mu^T = -\mu)$$

In the presence of an external magnetic field $B_\perp$ (interaction with $B_\parallel$ suppressed by $\gamma \gg 1$): Simultaneous flip of neutrino chirality and flavour (spin-flavour precession). E.g.

$$\diamondsuit \quad \nu_{eL} \leftrightarrow \bar{\nu}_{\mu R}$$

(Schechter & Valle, 1981; Voloshin, Vysotsky & Okun, 1986)

Can be resonantly enhanced in matter (similarly to the MSW effect) – RSFP (Lim & Marciano, 1988; EA, 1988)

RSFP in the solar magn. field: For some time was considered an alternative explanation of the observed solar neutrino deficit.

Ruled out as the main mechanism of solar ν conversion by KamLAND in 2002.

Still may be present at a subdominant level!

How can this be tested? Effects on the usually detected solar ν_e (and also NC-detected ν_μ and ν_τ) are very small.

<u>But:</u> Production of solar $\bar{\nu}_e$ is possible (qualitatively new effect).

For Majorana neutrinos $\mu_{\alpha\alpha} = 0$ – no direct $\nu_{eL} \to \bar{\nu}_{eR}$ conversion possible.

But: may proceed as a two-step process through joint action of SFP and usual flavour transformations. E.g.:

$$\nu_{eL} \xrightarrow{\text{osc.}} \nu_{\mu L} \xrightarrow{\text{SFP}} \bar{\nu}_{eR} \tag{1}$$

$$\nu_{eL} \xrightarrow{\text{SFP}} \bar{\nu}_{\mu R} \xrightarrow{\text{osc.}} \bar{\nu}_{eR} \tag{2}$$

Amplitudes of (1) and (2) are of opposite sign (due to $\mu^T = -\mu$);
in the Sun nearly cancel each other $\Rightarrow$
SFP and oscillations should occur in separate spatial regions.
This singles out (2).

N.B.: $\bar{\nu}_e$ has clear experimental signature. Large cross section for detection through IBD.

Two-flavour analysis, basis $(\nu_{eL},\, \nu_{\mu L},\, \bar{\nu}_{eR},\, \bar{\nu}_{\mu R})$ (EA & Pulido, 2002). SFP treated perturbatively; simple analytical expression for $P(\nu_e \to \bar{\nu}_e)$ obtained.

Used by KamLAND, Borexino and Super-Kamiokande to analyze their data on searches of $\bar{\nu}_e$ of astrophysical origin; upper bounds on $\mu B_\perp$ derived.

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Our goals:

- Extend the results to full 3f framework
  [basis: $(\nu_{eL},\, \nu_{\mu L},\, \nu_{\tau L},\, \bar{\nu}_{eR},\, \bar{\nu}_{\mu R},\, \bar{\nu}_{\tau R})$].
- Use more recent solar models.
- Revisit and update the limits on $\mu B_\perp$ found by KamLAND, Borexino and Super-Kamiokande.

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One question: In 2f case – just one μ ($\mu_{e\mu} = \mu_{12}$). In 3f case: $\mu_{12},\ \mu_{13}$ and μ_{23}. Which combination governs $\nu_e \to \bar{\nu}_e$ transitions?

Evolution equation in the rotated basis

A convenient (primed) basis for considering flavour transitions in the Sun:

$$\nu_{flL} = O_{23}\Gamma_\delta O_{13}\nu'_L$$

with $\nu'_L \equiv (\nu'_{eL}\ \nu'_{\mu L}\ \nu'_{\tau L})^T$. The evolution equation:

$$i\frac{d}{dr}\begin{pmatrix}\nu'_L\\ \bar{\nu}'_R\end{pmatrix} = \begin{pmatrix}H' & \mathcal{B}'\\ \mathcal{B}'^\dagger & \bar{H}'\end{pmatrix}\begin{pmatrix}\nu'_L\\ \bar{\nu}'_R\end{pmatrix}$$

Here $\bar{\nu}'_R = (\nu'_L)^c$,

$$\mathcal{B}' = \begin{pmatrix} 0 & \mu_{e'\mu'} & \mu_{e'\tau'}\\ -\mu_{e'\mu'} & 0 & \mu_{\mu'\tau'}\\ -\mu_{e'\tau'} & -\mu_{\mu'\tau'} & 0\end{pmatrix} B_\perp(r)e^{i\phi(r)} \equiv \upmu' \cdot B_\perp(r)e^{i\phi(r)}\,.$$

$\phi(r)$ is the azimuthal angle defining the direction of $\vec{B}_\perp$ in the transverse plane ($d\phi/dr$ is the “twist” of the magnetic field), Approximations used in deriving analytical expressions:

- Leading order perturbation theory in $\mu B_\perp$
- $\Delta m_{21}^2 \ll \Delta m_{31}^2$

Also: approximate calculation of the resulting integrals of fast oscill. functions

We also performed full numerical integration of the evolution equations.

Analytical expressions for $P(\nu_{eL} \to \bar{\nu}_{eR})$

With only $|\mu_{e'\mu'}| = |\mu_{12}| \neq 0$ (contributions of μ_{13} and μ_{23} are $\sim s_{13}(\delta/\Delta)$):

$$P(\nu_{eL} \to \bar{\nu}_{eR}) = \frac{1}{2} c_{13}^4 \sin^2 2\theta_{12} B_\perp^2(r_0) |\mu_{e\mu}|^2 \left(\frac{\cos^2 \tilde{\theta}(r_0)}{g_1'(r_0)} + \frac{\sin^2 \tilde{\theta}(r_0)}{g_2'(r_0)} \right)^2$$

r_0: the coordinate of the neutrino production point in the Sun,

$$g_{1,2}(r) = \phi + \int_{r_0}^{r} \left[E_{1,2} - \left(2c_{12}^2 \delta - V_n \right) \right] dr' \qquad (\delta \equiv \frac{\Delta m_{21}^2}{4E})$$

$$E_{1,2} \equiv \delta + c_{13}^2 \frac{V_e}{2} + V_n \mp \sqrt{\left(\delta \cos 2\theta_{12} - c_{13}^2 V_e/2 \right)^2 + \delta^2 \sin^2 2\theta_{12}}$$

$\tilde{\theta}(r)$ is eff. mixing angle in matter:

$$\cos 2\tilde{\theta}(r) = \frac{\cos 2\theta_{12} - c_{13}^2 V_e/2\delta}{\sqrt{\left(\cos 2\theta_{12} - c_{13}^2 V_e/2\delta \right)^2 + \sin^2 2\theta_{12}}}$$

Comparison with results of EA & Pulido 2002

Simplified expression (neglecting $\cos^2\tilde{\theta}(r_0)$ term, justified for $E \gtrsim 5\,\mathrm{MeV}$):

$$\diamondsuit \quad P(\nu_{eL} \to \bar{\nu}_{eR})_{\mathrm{simpl.}} = \frac{1}{2} c_{13}^4 \sin^2 2\theta_{12} B_\perp^2(r_0) |\mu_{e\mu}|^2 \left(\frac{\sin^2\tilde{\theta}(r_0)}{g_2'(r_0)} \right)^2$$

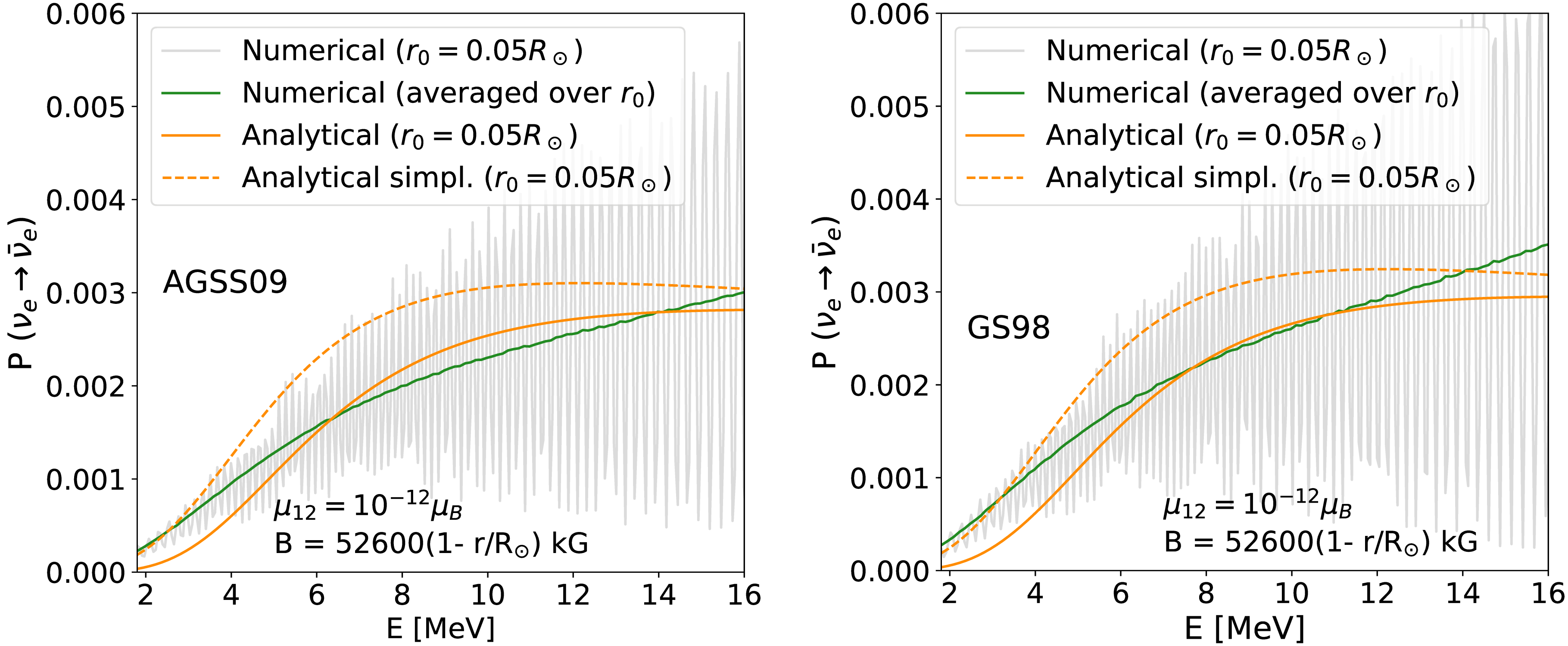
Numerically for $r_0 = 0.05 R_\odot$, $E = 12$ MeV:

$$\diamondsuit \quad P(\nu_{eL} \to \bar{\nu}_{eR}) \simeq 1.1 \times 10^{-10} \left(\frac{\mu_{12} B_\perp(r_0)}{10^{-12}\mu_B \cdot 10\,\mathrm{kG}} \right)^2$$

Num. coeff.: factor of 1.4 smaller than in 2f approach (EA & Pulido 2002).
Due to 3f effects, using updated neutrino mixing parameters and solar model and because of the approximation $\cos\tilde{\theta}(r_0) \ll \sin\tilde{\theta}(r_0)$ used in 2f analysis.

Not suitable for experiments sensitive to pp, pep or ^{7}Be neutrinos!

Analyt. vs. numer. calculations for ^{8}B νs

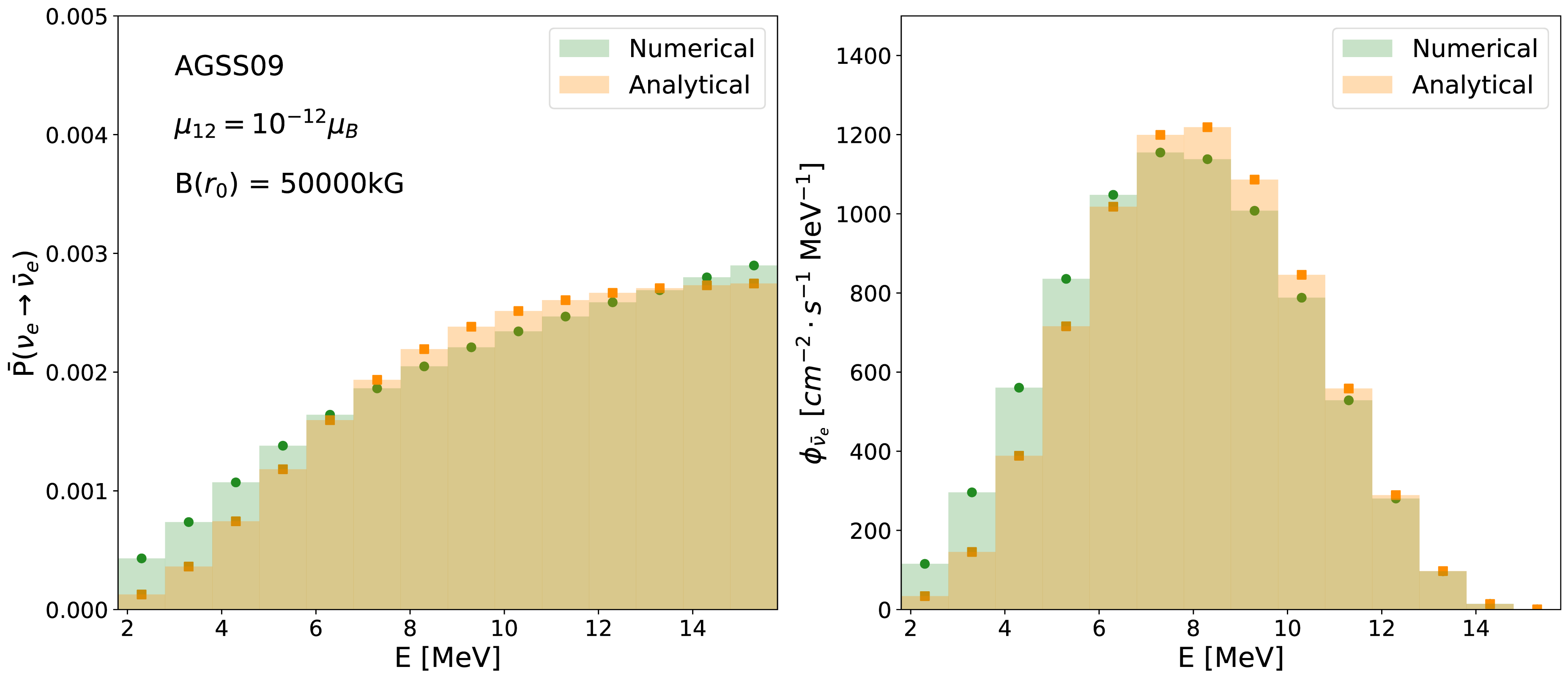


Numerical: solution of the full system of 6 coupled evolution equations.

Good general agreement between numer. and analyt. results, esp. for $E \gtrsim 5$ MeV. Larger discrepancy for smaller E, where $P(\nu_e \to \bar{\nu}_e)$ is relatively small.

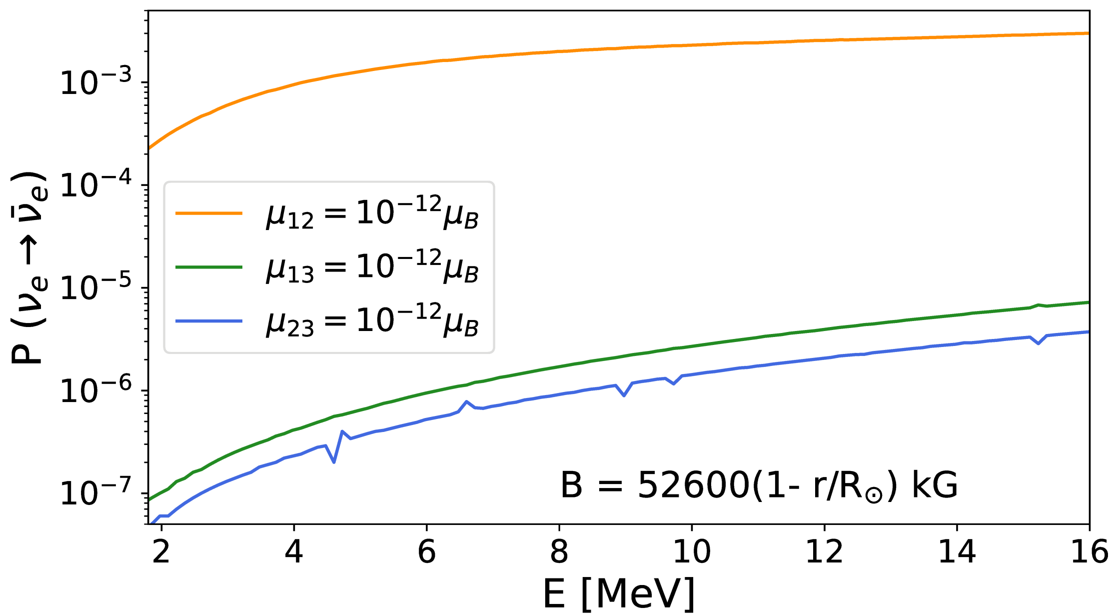
Analyt. vs. numer. calculations for ^{8}B νs

With averaging over 1 MeV energy bins:



Roles of various magnetic moments

Assuming only one non-zero transtion magnetic moment at a time:



Electron antineutrino appearance probability on the Earth as a function of neutrino energy for nonzero μ_{12}, μ_{13} and μ_{23}, for the AGSS09 solar model. Evolution equations were solved numerically.

Revisiting KL, BX and SK limits

KamLAND (2021) data, our results (90% C.L.)

$$(\mu_{12}B_\perp(r_0))_{\rm AGSS09} < (4.9 - 5.1) \times 10^{-9}\mu_B\,{\rm kG}$$
$$(\mu_{12}B_\perp(r_0))_{\rm GS98} < (4.7 - 4.8) \times 10^{-9}\mu_B\,{\rm kG}$$

(Lower numbers: our analyt. approximation, higher numbers: full numerical calculation).

KamLAND's own analysis (based on results of EA & Pulido 2002):

$$\mu B_\perp(r_0) < 4.9 \times 10^{-9}\mu_B\,{\rm kG}$$

Revisiting KL, BX and SK limits

Borexino (2019) data, our results (90% C.L.)

$$(\mu_{12}B_\perp(r_0))_{\mathrm{AGSS09}} < (1.8-1.9)\times 10^{-8}\mu_B\,\mathrm{kG}\,,$$
$$(\mu_{12}B_\perp(r_0))_{\mathrm{GS98}} < (1.7-1.8)\times 10^{-8}\mu_B\,\mathrm{kG}\,,$$

Borexino's own analysis (for high-metallicity GS98 SSM):

$$\mu B_\perp(r_0) < 6.9\times 10^{-9}\mu_B\,\mathrm{kG}.$$

Factor ~ 2.6 discrepancy (more stringent limit) probably related to Borexino's using the simplified energy-independent formula from EA & Pulido (derived for $E\sim 5-10$ MeV) for neutrinos of smaller energies (outside its range of validity).

Revisiting KL, BX and SK limits

Super-Kamiokande (2020) data, our results:

$$(\mu_{12} B_\perp(r_0))_{\rm AGSS09} < (7.1 - 7.3) \times 10^{-9} \mu_B \, {\rm kG}$$

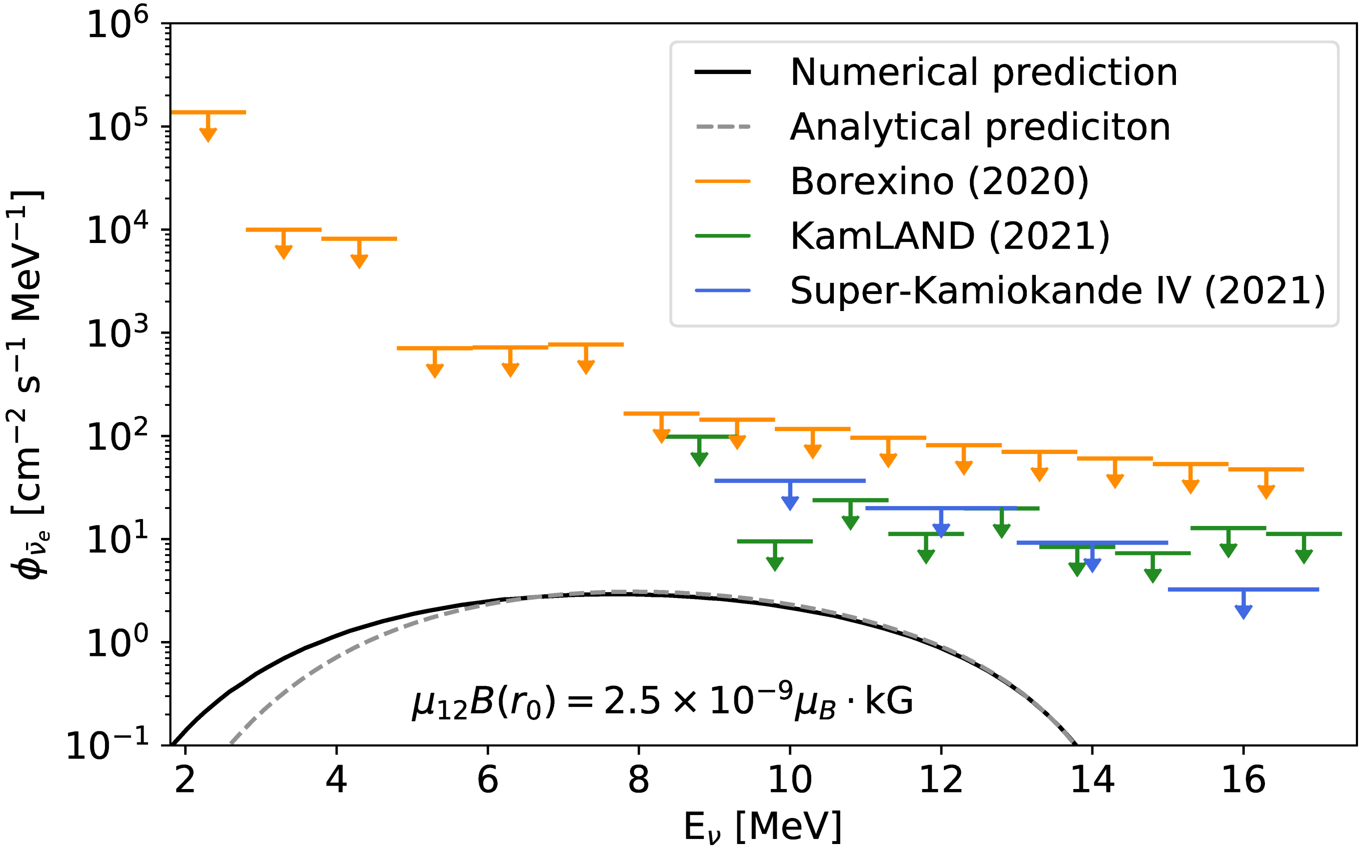
$$(\mu_{12} B_\perp(r_0))_{\rm GS98} < (6.8 - 6.9) \times 10^{-9} \mu_B \, {\rm kG}$$

Super-Kamiokande's own analysis:

$$\mu B_\perp(r_0) < 1.5 \times 10^{-8} \mu_B \, {\rm kG}$$

Factor ~ 2 weaker than our limit. Probably because SK looked for $\bar{\nu}_e$s in the energy range 9.3 to 17.3 MeV but used in the analysis the same simplified energy-independent $\bar{\nu}_e$ appearance probability derived in EA & Pulido 2002 for smaller energies.

Experimental upper limits vs. our predictions



Model-independent limits on astrophysical $\bar{\nu}_e$ flux from KamLAND, Borexino and Super-Kamiokande. Black curves: expected solar $\bar{\nu}_e$ flux for $\mu_{12}B_\perp(r_0) = 2.5 \times 10^{-9}\mu_B$ kG for AGSS09 SSM (our calculations).

Other constraints on $\mu_{\nu\mathrm{eff}}$

Bounds on $\mu_{\rm eff}$ (SBL accel. & reactor exps.)

Experiment	Limit	Reference	Method
LAMPF	$\mu_{\nu_e} < 1.08 \times 10^{-9} \mu_B$ at 90%C.L.	[34]	Accelerator $\nu_e e^-$
LSND	$\mu_{\nu_e} < 1.1 \times 10^{-9} \mu_B$ at 90%C.L.	[35]	Accelerator $\nu_e e^-$
Krasnoyarsk	$\mu_{\nu_e} < 1.4 \times 10^{-10} \mu_B$ at 90% C.L.	[36]	Reactor $\bar{\nu}_e e^-$
ROVNO	$\mu_{\nu_e} < 1.9 \times 10^{-10} \mu_B$ at 95% C.L.	[37]	Reactor $\bar{\nu}_e e^-$
MUNU	$\mu_{\nu_e} < 9 \times 10^{-11} \mu_B$ at 90% C.L.	[38]	Reactor $\bar{\nu}_e e^-$
TEXONO	$\mu_{\nu_e} < 7.4 \times 10^{-11} \mu_B$ at 90% C.L.	[39]	Reactor $\bar{\nu}_e e^-$
GEMMA	$\mu_{\nu_e} < 2.9 \times 10^{-11} \mu_B$ at 90% C.L.	[40]	Reactor $\bar{\nu}_e e^-$
CONUS	$\mu_{\nu_e} < 7.5 \times 10^{-11} \mu_B$ at 90% C.L	[41]	Reactor CEνNS
Dresden-II	$\mu_{\nu_e} < 2.2 \times 10^{-10} \mu_B$ at 90% C.L	[42,43]	Reactor CEνNS
LAMPF	$\mu_{\nu_\mu} < 7.4 \times 10^{-10} \mu_B$ at 90%C.L.	[34]	Accelerator $\nu_\mu e^-$
BNL-E-0734	$\mu_{\nu_\mu} < 8.5 \times 10^{-10} \mu_B$ at 90%C.L.	[44]	Accelerator $\nu_\mu e^-$
LSND	$\mu_{\nu_\mu} < 6.8 \times 10^{-10} \mu_B$ at 90%C.L.	[35]	Accelerator $\nu_\mu e^-$
DONUT	$\mu_{\nu_\tau} < 3.9 \times 10^{-7} \mu_B$ at 90%C.L.	[45]	Accelerator $\nu_\tau e^-$

Limits on $\mu_{\nu \rm eff} = \mu_{\nu \rm SOLAR}$ moment from elastic scattering of solar ν on electrons.

Experiment	Limit at 90%C.L.	Reference	Energy range
Borexino	$\mu_{\nu \rm SOLAR} < 2.8 \times 10^{-11} \mu_B$	[46,47]	0.19 MeV – 2.93 MeV
Super-Kamiokande	$\mu_{\nu \rm SOLAR} < 1.1 \times 10^{-10} \mu_B$	[48]	5 MeV – 20 MeV
LUX-ZEPLIN	$\mu_{\nu \rm SOLAR} < 6.2 \times 10^{-12} \mu_B$	[7]	E $\leq$ 2 MeV
XENONnT	$\mu_{\nu \rm SOLAR} < 6.3 \times 10^{-12} \mu_B$	[5]	E $\leq$ 1 MeV

Limits on $\mu_{\nu \rm PLASMON} \equiv \sum_{i,j} |\mu_{i,j}|^2$ from plasmon decays in stars

Limit	Reference	Method
$\mu_{\nu \rm PLASMON} < 1.2 \times 10^{-12} \mu_B$ at 95%C.L.	[54]	Tip of red-giant branch
$\mu_{\nu \rm PLASMON} < 1.0 \times 10^{-11} \mu_B$ at 95%C.L.	[55]	Pulsating white dwarfs
$\mu_{\nu \rm PLASMON} < 2.2 \times 10^{-12} \mu_B$ at 95%C.L.	[57]	Luminosity
$\mu_{\nu \rm PLASMON} < 2.2 \times 10^{-12} \mu_B$ at 95%C.L.	[58]	Luminosity

Summary

- $\nu_e \to \bar{\nu}_e$ conversion of solar neutrinos due to joint action of SFP and flavour conversion considered in 3f framework
- Simple analytical expression for $\bar{\nu}_e$ appearance probability found and compared with results of full numerical integration of evolution equations
- Roles of different neutrino magn. moments clarified; μ_{12} gives by far the largest contribution
- Constraints on $\mu_{12}B_{\perp}$ obtained by KamLAND, Borexino and SK from non-observation of astrophysical $\bar{\nu}_e$ revisited
- In 2207.04516 also other constraints on neutrino magn. moments coming from astrophyics, cosmology and lab experiments (reactor, accelerator, DM searches) summarized, with special emphasis on the involved $\mu_{\rm eff}$

Backup slides

$\mu_{\nu\text{eff}}$ probed in various experiments

I. SBL experiments (reactor, acclerator)

$$\mu^2_{\nu_\alpha \mathrm{SB}} = \sum_\beta |\mu_{\alpha\beta}|^2\,, \quad \alpha, \beta = e, \mu, \tau \qquad \text{(Dirac, Majorana)}$$

Expressions through elements μ_{ij} in mass eig. basis more complicated.

II. Solar neutrinos ($\mu_{\nu\text{eff}}$ probed through νe scattering and CE$\nu_\odot$NS)

$$\mu^2_{\nu\mathrm{SOLAR}} = (|\mu_{11}|^2 + |\mu_{21}|^2 + |\mu_{31}|^2) c^2_{13} \cos^2\tilde{\theta}$$
$$+ (|\mu_{12}|^2 + |\mu_{22}|^2 + |\mu_{32}|^2) c^2_{13} \sin^2\tilde{\theta}$$
$$+ (|\mu_{13}|^2 + |\mu_{23}|^2 + |\mu_{33}|^2) s^2_{13} \qquad \text{(Dirac)}$$

$$\mu^2_{\nu\mathrm{SOLAR}} = |\mu_{12}|^2 c^2_{13} + |\mu_{13}|^2 (c^2_{13} \cos^2\tilde{\theta} + s^2_{13})$$
$$+ |\mu_{23}|^2 (c^2_{13} \sin^2\tilde{\theta} + s^2_{13}) \qquad \text{(Majorana)}$$

For $E \lesssim 1$ MeV: $\tilde{\theta} \simeq \theta_{12}$. For $E \gtrsim 5-7$ MeV: $\tilde{\theta} \simeq \pi/2$. In general energy dependence of $\tilde{\theta}$ should be taken into account.

$\mu_{\nu\mathrm{eff}}$ probed in various experiments

III. Plasmon decay (white dwarfs, red giants)

$$\mu^2_{\nu\mathrm{PLASMON}} = \sum_{i,j} |\mu_{ij}|^2 \qquad \text{(Dirac, Majorana)}$$

Also governs other processes contributing to stellar cooling: $\gamma e^- \to e^- \bar{\nu}\nu$, $e^+e^- \to \bar{\nu}\nu$, $e^-(Ze) \to (Ze)e^-\bar{\nu}\nu$.

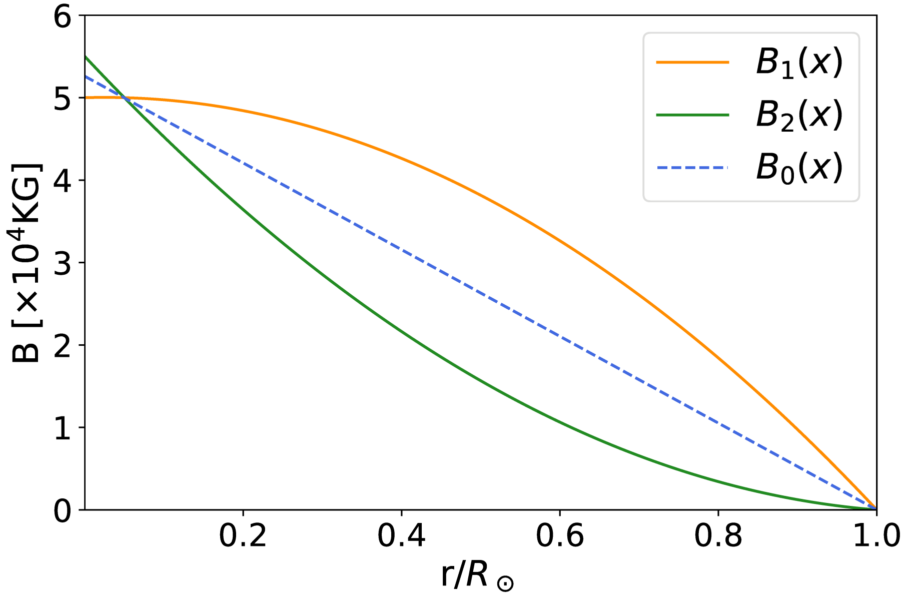
IV. SN 1987A constraints (for Dirac neutrinos, from $\nu_\alpha \to \nu_s$ transition via μ_ν). Model dependent, no simple expression for $\mu_{\nu\mathrm{eff}}$.

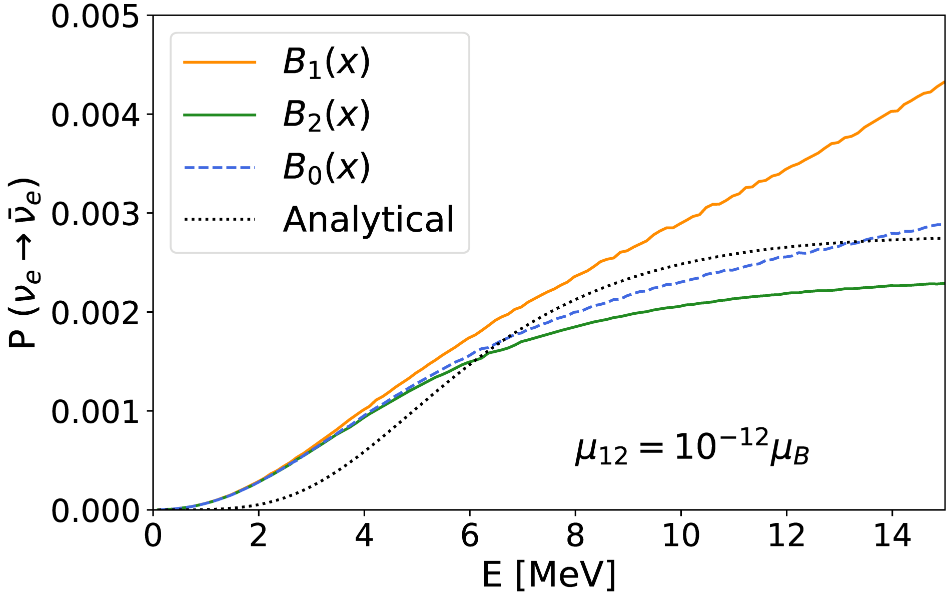
V. $\nu_e \to \bar{\nu}_e$ conversion of supernova PNB ν_es (Majorana neutrinos)

$$\mu_{\nu\mathrm{eff}} = \mu_{12}c_{13}e^{-i\lambda_2} + (\mu_{13}s_{12} - \mu_{23}c_{12}e^{-i\lambda_2})s_{13}e^{i(\delta_{\mathrm{CP}}-\lambda_3)} \qquad \text{(NO)}$$

$$\mu_{\nu\mathrm{eff}} = (\mu_{13}c_{12} + \mu_{23}s_{12}e^{-i\lambda_2})e^{-i\lambda_3} \qquad \text{(IO)}$$

For different magnetic field profiles $B_\perp$





$$B_0(r) \equiv 52600(1 - r/R_\odot)\,\text{kG}\,,$$

$$B_1(r) = 50000 + 2632(r/R_\odot) - 52632(r/R_\odot)^2\;\text{kG} \qquad (3)$$

$$B_2(r) = 55000 - 102368(r/R_\odot) + 47368(r/R_\odot)^2\;\text{kG} \qquad (4)$$

Stationary phase points

$dg_i/dr = 0$ $(i = 1, ..., 5)$. In the Sun can only be satisfied for $i = 1, 2$ and only for nonzero twist velocity ϕ'. Correspond to the resonance of SFP.

$$\frac{d\phi_{1,2}}{dr} = 0 \qquad \Rightarrow \qquad \frac{d\phi}{dr} = 2c_{12}^2\delta - V_n - E_{1,2}$$

Can be reduced to

$$d\phi/dr + 2V_n + c_{13}^2 V_e - 2\delta\cos 2\theta_{12} = \frac{\delta^2 \sin^2 2\theta_{12}}{d\phi/dr + 2V_n}$$

Has a solution if

$$1 + \sin^2 2\theta_{12}\,\frac{c_{13}^2 V_e}{d\phi/dr + 2V_n} \geq 0\,.$$

This condition is satisfied if

$$\frac{d\phi}{dr} > 2|V_n| \quad \text{or} \quad -\frac{d\phi}{dr} \geq c_{13}^2 V_e \left(\sin^2 2\theta_{12} - \frac{1-Y_e}{c_{13}^2 Y_e}\right),$$

Y_e varies between 0.67 and 0.88 in the Sun $\Rightarrow$ expression in the brackets is > 0 and on the order of 0.3 – 0.7 $\Rightarrow$ for non-twisting magn. fields stationary phase condition cannot be fulfilled.

Stat. phase condition requires $|d\phi/dr| \sim V_e,\ |V_n|$ (vary from $\sim 7\times10^{-12}\,\mathrm{eV}$ in neutrino production region to ~ 0 at $r = R_\odot$).

Can be fulfilled e.g. for magn. fields with const. twist $|d\phi/dr| \sim 10/R_\odot \sim 3\times10^{-15}\,\mathrm{eV}$.

If stat. phase points exist, their contributions dominate over contributions of the endpoints in the integrals defining $\bar\nu'_\mu$ and $\bar\nu'_\tau$.

$\Rightarrow$ Neglecting possible twist of $B_\perp$ yields more conservative constraints on $\mu B_\perp$.

Evoluton equations, etc.

Primed basis: rotate away θ_{23}, θ_{13} and $\delta_{\rm CP}$

The evolution equation:

$$i\frac{d}{dr}\begin{pmatrix}\nu'_L\\ \bar{\nu}'_R\end{pmatrix} = \begin{pmatrix}H' & \mathcal{B}'\\ \mathcal{B}'^{\dagger} & \bar{H}'\end{pmatrix}\begin{pmatrix}\nu'_L\\ \bar{\nu}'_R\end{pmatrix}$$

$\bar{\nu}'_R = (\nu'_L)^c$.

$$H' = \begin{pmatrix} 2\delta s_{12}^2 + c_{13}^2 V_e + V_n & 2\delta s_{12}c_{12} & s_{13}c_{13}V_e \\ 2\delta s_{12}c_{12} & 2\delta c_{12}^2 + V_n & 0 \\ s_{13}c_{13}V_e & 0 & 2\Delta + s_{13}^2 V_e + V_n \end{pmatrix}$$

$V_e = \sqrt{2}G_F N_e(r)$ and $V_n = -\sqrt{2}G_F N_n(r)/2\,.$

$\bar{H}'$ is obtained from H' by substituting $\delta_{\rm CP} \to -\delta_{\rm CP}$, $V_e \to -V_e$ and $V_n \to -V_n$.

$$\mathcal{B}' = \begin{pmatrix} 0 & \mu_{e'\mu'} & \mu_{e'\tau'} \\ -\mu_{e'\mu'} & 0 & \mu_{\mu'\tau'} \\ -\mu_{e'\tau'} & -\mu_{\mu'\tau'} & 0 \end{pmatrix} B_\perp(r) e^{i\phi(r)} \equiv \mu' \cdot B_\perp(r) e^{i\phi(r)}$$

In terms of the more fundamental magnetic moments μ_m in the neutrino mass eigenstate basis:

$$\mu' = \Gamma_\delta O_{12} K^* \mu_m K^* O_{12}^T \Gamma_\delta \,.$$

$\Gamma = \mathrm{diag}(1,\, 1,\, e^{i\delta_{\rm CP}})$, $\qquad K = \mathrm{diag}(1,\, e^{i\lambda_2},\, e^{i\lambda_3})$.

Nonzero matrix elements of μ':

$$\mu_{e'\mu'} = \mu_{12} e^{-i\lambda_2} \,,$$

$$\mu_{e'\tau'} = \left(\mu_{13} c_{12} + \mu_{23} s_{12} e^{-i\lambda_2}\right) e^{-i(\lambda_3 - \delta_{\rm CP})} \,,$$

$$\mu_{\mu'\tau'} = \left(\mu_{23} c_{12} e^{-i\lambda_2} - \mu_{13} s_{12}\right) e^{-i(\lambda_3 - \delta_{\rm CP})} \,.$$

The evolution equation in more detailed form:

$$
\begin{aligned}
i\frac{d}{dr}\nu'_{eL} &= H_{e'e'}\nu'_{eL} + H_{e'\mu'}\nu'_{\mu L} + H_{e'\tau'}\nu'_{\tau L} + \mathcal{B}_{e'\mu'}\bar{\nu}'_{\mu R} + \mathcal{B}_{e'\tau'}\bar{\nu}'_{\tau R}\,, \\
i\frac{d}{dr}\nu'_{\mu L} &= H_{\mu'e'}\nu'_{eL} + H_{\mu'\mu'}\nu'_{\mu L} + \mathcal{B}_{\mu'e'}\bar{\nu}'_{eR} + \mathcal{B}_{\mu'\tau'}\bar{\nu}'_{\tau R}\,, \\
i\frac{d}{dr}\nu'_{\tau L} &= H_{\tau'e'}\nu'_{eL} + H_{\tau'\tau'}\nu'_{\tau' L} + \mathcal{B}_{\tau'e'}\bar{\nu}'_{eR} + \mathcal{B}_{\tau'\mu'}\bar{\nu}'_{\mu R}\,, \\
i\frac{d}{dr}\bar{\nu}'_{eR} &= \bar{H}_{e'e'}\bar{\nu}'_{eR} + \bar{H}_{e'\mu'}\bar{\nu}'_{\mu R} + \bar{H}_{e'\tau'}\bar{\nu}'_{\tau R} - \mathcal{B}^{*}_{e'\mu'}\nu'_{\mu L} - \mathcal{B}^{*}_{e'\tau'}\nu'_{\tau L}\,, \\
i\frac{d}{dr}\bar{\nu}'_{\mu R} &= \bar{H}_{\mu'e'}\bar{\nu}'_{eR} + \bar{H}_{\mu'\mu'}\bar{\nu}'_{\mu R} - \mathcal{B}^{*}_{\mu'e'}\nu'_{eL} - \mathcal{B}^{*}_{\mu'\tau'}\nu'_{\tau L}\,, \\
i\frac{d}{dr}\bar{\nu}'_{\tau R} &= \bar{H}_{\tau'e'}\bar{\nu}'_{eR} + \bar{H}_{\tau'\tau'}\bar{\nu}'_{\tau R} - \mathcal{B}^{*}_{\tau'e'}\nu'_{eL} - \mathcal{B}^{*}_{\tau'\mu'}\nu'_{\mu L}\,.
\end{aligned}
$$

Here we have taken into account that the diagonal elements of the matrix $\mathcal{B}'$ vanish and that $H_{\mu'\tau'} = H_{\tau'\mu'} = \bar{H}_{\mu'\tau'} = \bar{H}_{\tau'\mu'} = 0$.

Simplified system of evolution equations

Neglect: $\bar{\nu}'_{eR}$, terms containing $H_{e'\tau'} = H_{\tau'e'}$ and $\bar{H}_{e'\tau'} = \bar{H}_{\tau'e'}$ ($\ll H_{\tau'\tau'}$ and $\bar{H}_{\tau'\tau'}$); work to leading order in PT in $\mu B_\perp$. Need to find $\bar{\nu}'_{\mu R}$ and $\bar{\nu}'_{\tau R}$.

$$i\frac{d}{dr}\nu'_{eL} = \left(2\delta s_{12}^2 + c_{13}^2 V_e + V_n\right)\nu'_{eL} + 2\delta s_{12}c_{12}\nu'_{\mu L}\,,$$

$$i\frac{d}{dr}\nu'_{\mu L} = 2\delta s_{12}c_{12}\nu'_{eL} + \left(2\delta c_{12}^2 + V_n\right)\nu'_{\mu L}\,,$$

$$i\frac{d}{dr}\nu'_{\tau L} = \left(2\Delta + s_{13}^2 V_e + V_n\right)\nu'_{\tau L}\,,$$

$$i\frac{d}{dr}\bar{\nu}'_{\mu R} = \left(2\delta c_{12}^2 - V_n\right)\bar{\nu}'_{\mu R} + \mu^*_{e'\bar{\mu}'}B_\perp e^{-i\phi}\nu'_{eL} - \mu^*_{\mu'\bar{\tau}'}B_\perp e^{-i\phi}\nu'_{\tau L}\,,$$

$$i\frac{d}{dr}\bar{\nu}'_{\tau R} = \left(2\Delta - s_{13}^2 V_e - V_n\right)\bar{\nu}'_{\tau R} + \mu^*_{e'\bar{\tau}'}B_\perp e^{-i\phi}\nu'_{eL} + \mu^*_{\mu'\bar{\tau}'}B_\perp e^{-i\phi}\nu'_{\mu L}\,.$$

Simplified system of evolution equations

Neglect: $\bar\nu'_{eR}$, terms containing $H_{e'\tau'} = H_{\tau' e'}$ and $\bar H_{e'\tau'} = \bar H_{\tau' e'}$ ($\ll H_{\tau'\tau'}$ and $\bar H_{\tau'\tau'}$); work to leading order in PT in $\mu B_\perp$. Need to find $\bar\nu'_{\mu R}$ and $\bar\nu'_{\tau R}$.

$$i\frac{d}{dr}\nu'_{eL} = \left(2\delta s_{12}^2 + c_{13}^2 V_e + V_n\right)\nu'_{eL} + 2\delta s_{12}c_{12}\nu'_{\mu L}\,,$$

$$i\frac{d}{dr}\nu'_{\mu L} = 2\delta s_{12}c_{12}\nu'_{eL} + \left(2\delta c_{12}^2 + V_n\right)\nu'_{\mu L}\,,$$

$$i\frac{d}{dr}\nu'_{\tau L} = \left(2\Delta + s_{13}^2 V_e + V_n\right)\nu'_{\tau L}\,,$$

$$i\frac{d}{dr}\bar\nu'_{\mu R} = \left(2\delta c_{12}^2 - V_n\right)\bar\nu'_{\mu R} + \mu^*_{e'\bar\mu'}B_\perp e^{-i\phi}\nu'_{eL} - \mu^*_{\mu'\bar\tau'}B_\perp e^{-i\phi}\nu'_{\tau L}\,,$$

$$i\frac{d}{dr}\bar\nu'_{\tau R} = \left(2\Delta - s_{13}^2 V_e - V_n\right)\bar\nu'_{\tau R} + \mu^*_{e'\bar\tau'}B_\perp e^{-i\phi}\nu'_{eL} + \mu^*_{\mu'\bar\tau'}B_\perp e^{-i\phi}\nu'_{\mu L}\,.$$

Simplified system of evolution equations

Neglect: $\bar{\nu}'_{eR}$, terms containing $H_{e'\tau'} = H_{\tau' e'}$ and $\bar{H}_{e'\tau'} = \bar{H}_{\tau' e'}$ ($\ll H_{\tau'\tau'}$ and $\bar{H}_{\tau'\tau'}$); work to leading order in PT in $\mu B_\perp$. Need to find $\bar{\nu}'_{\mu R}$ and $\bar{\nu}'_{\tau R}$.

$$i\frac{d}{dr}\nu'_{eL} = \left(2\delta s_{12}^2 + c_{13}^2 V_e + V_n\right)\nu'_{eL} + 2\delta s_{12} c_{12} \nu'_{\mu L}\,,$$

$$i\frac{d}{dr}\nu'_{\mu L} = 2\delta s_{12} c_{12} \nu'_{eL} + \left(2\delta c_{12}^2 + V_n\right)\nu'_{\mu L}\,,$$

$$i\frac{d}{dr}\nu'_{\tau L} = \left(2\Delta + s_{13}^2 V_e + V_n\right)\nu'_{\tau L}\,,$$

$$i\frac{d}{dr}\bar{\nu}'_{\mu R} = \left(2\delta c_{12}^2 - V_n\right)\bar{\nu}'_{\mu R} + \mu^*_{e'\bar{\mu}'} B_\perp e^{-i\phi}\nu'_{eL} - \mu^*_{\mu'\bar{\tau}'} B_\perp e^{-i\phi}\nu'_{\tau L}\,,$$

$$i\frac{d}{dr}\bar{\nu}'_{\tau R} = \left(2\Delta - s_{13}^2 V_e - V_n\right)\bar{\nu}'_{\tau R} + \mu^*_{e'\bar{\tau}'} B_\perp e^{-i\phi}\nu'_{eL} + \mu^*_{\mu'\bar{\tau}'} B_\perp e^{-i\phi}\nu'_{\mu L}\,.$$

The evolution equations for ν'_{eL} and $\nu'_{\mu L}$ decouple from the rest of the system. In the adiabatic approximation:

$$\nu'_{eL}(r) = c_{13}\left[\cos\tilde{\theta}(r_0)\cos\tilde{\theta}(r)e^{-i\int_{r_0}^{r}E_1 dr'} + \sin\tilde{\theta}(r_0)\sin\tilde{\theta}(r)e^{-i\int_{r_0}^{r}E_2 dr'}\right]$$

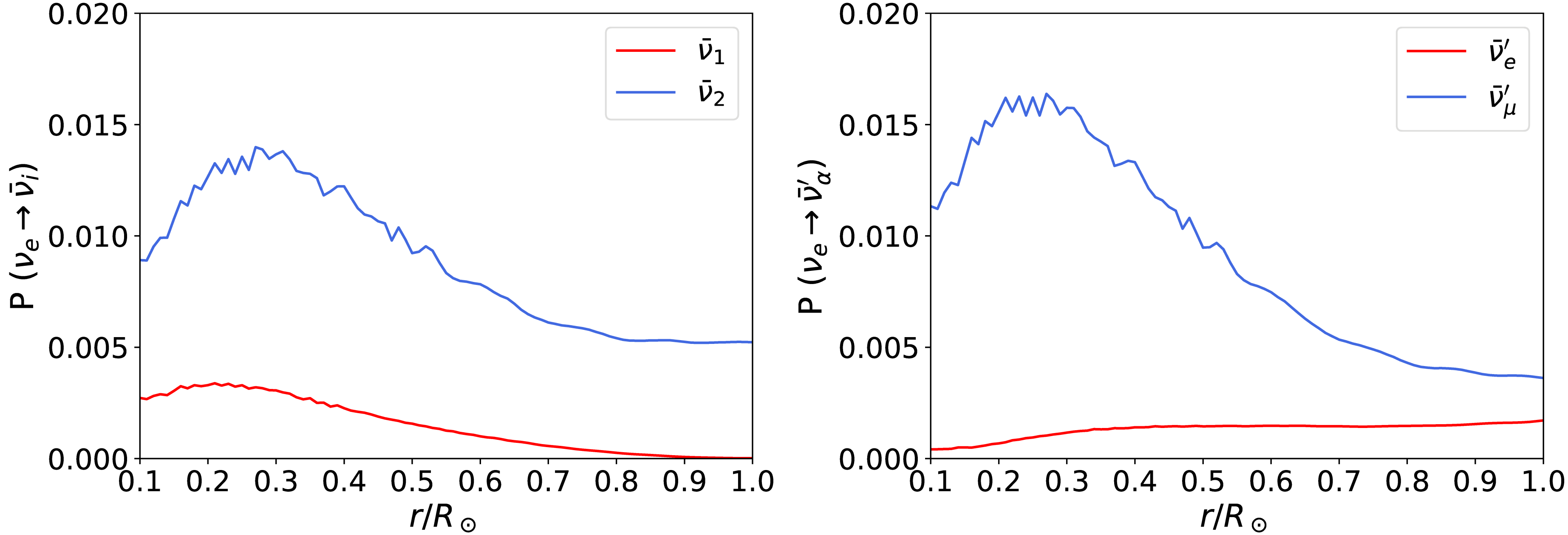
$$\nu'_{\mu L}(r) = c_{13}\left[-\cos\tilde{\theta}(r_0)\sin\tilde{\theta}(r)e^{-i\int_{r_0}^{r}E_1 dr'} + \sin\tilde{\theta}(r_0)\cos\tilde{\theta}(r)e^{-i\int_{r_0}^{r}E_2 dr'}\right]$$

$\tilde{\theta}(r)$ is the effective mixing angle in matter:

$$\cos 2\tilde{\theta}(r) = \frac{\cos 2\theta_{12} - c_{13}^2 V_e/2\delta}{\sqrt{\left(\cos 2\theta_{12} - c_{13}^2 V_e/2\delta\right)^2 + \sin^2 2\theta_{12}}}\,,$$

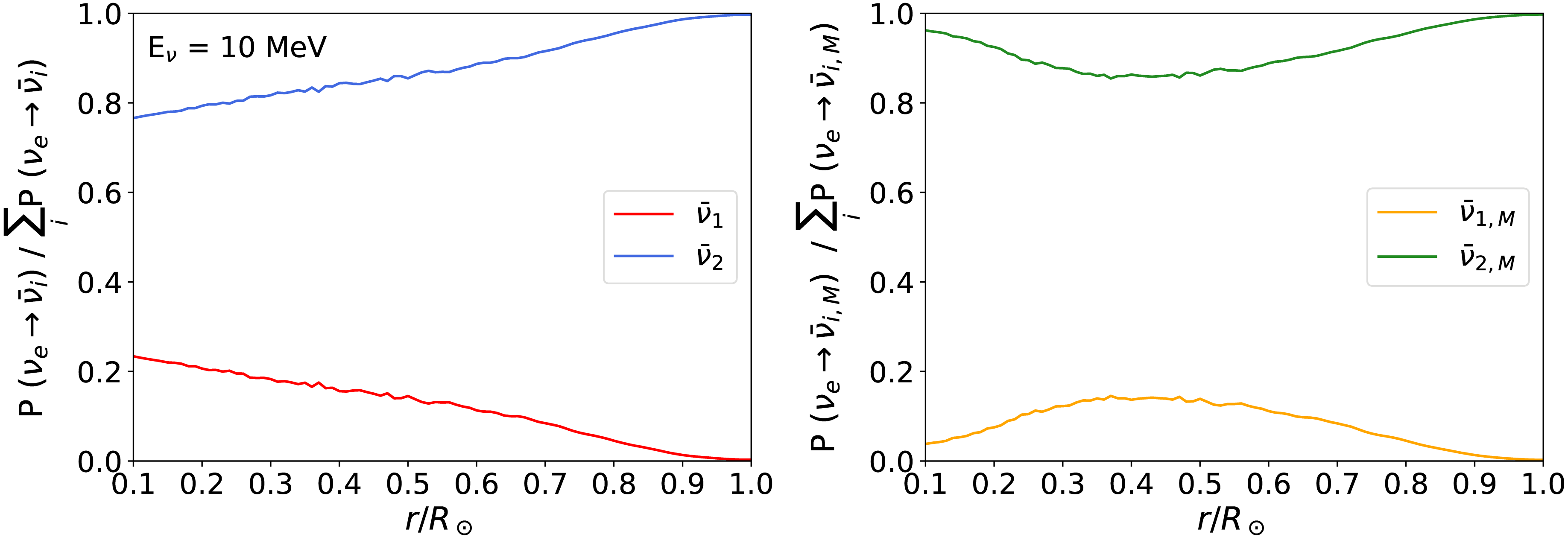
$$E_{1,2} \equiv \delta + c_{13}^2\frac{V_e}{2} + V_n \mp \sqrt{\left(\delta\cos 2\theta_{12} - c_{13}^2 V_e/2\right)^2 + \delta^2\sin^2 2\theta_{12}}\,.$$

Inside the Sun



2f antineutrino appearance probabilities for mass eigenstates (left) and states in the primed basis (right).

$E = 10$ MeV, $\mu_{12} = 10^{-12}\mu_B$, $\mu_{13} = \mu_{23} = 0$, $r_0 = 0.05 R_\odot$.



2f evolution of antineutrino appearance probabilities inside the Sun for mass eigenstates (left) and for the matter eigenstates (right), normalised to the unit total antineutrino appearance probability.

$E = 10$ MeV, $\mu_{12} = 10^{-12}\mu_B$, $\mu_{13} = \mu_{23} = 0$, $r_0 = 0.05 R_\odot$.