Istituto Nazionale di Fisica Nucleare

NOW 2022
Neutrino Oscillation Workshop

Theoretical Models on the Structure of the Neutrino Mixing Matrix

Arsenii Titov

Departament de Física Teòrica, Universitat de València
IFIC, Universitat de València & CSIC

Universitat de València

IFIC Institut de Física Corpuscular

CSIC Consejo Superior de Investigaciones Científicas

Outline

- 3-neutrino mixing
- Flavour puzzle
- Discrete flavour symmetries
- Modular flavour symmetries
- Conclusions

3-neutrino mixing

Charged current weak interactions

$$-\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \overline{\ell_L}(x)\,\gamma_\alpha\,\nu_{\ell L}(x)\,W^{\alpha\dagger}(x) + \text{h.c.}$$

Mismatch between the interaction and mass eigenstates

$$\nu_{\ell L}(x) = \sum_{j=1}^{3} U_{\ell j}\,\nu_{jL}(x)$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix

The standard parameterisation (adopted by the PDG)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

Atmospheric angle θ_{23}

Reactor angle θ_{13}
Dirac phase δ

Solar angle θ_{12}

Majorana phases
α_{21} and α_{31}
(if ν are Majorana)

Global fit to neutrino oscillation data

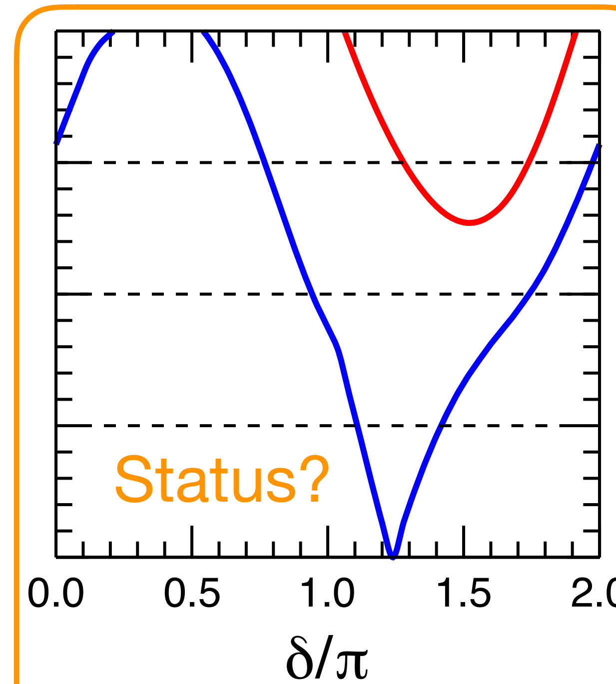
$\sin^2\theta_{12} = 0.303\ (4.5\%)$ [b.f.v. (relative 1σ)]

$\sin^2\theta_{13} = 0.0223\ (3\%)$

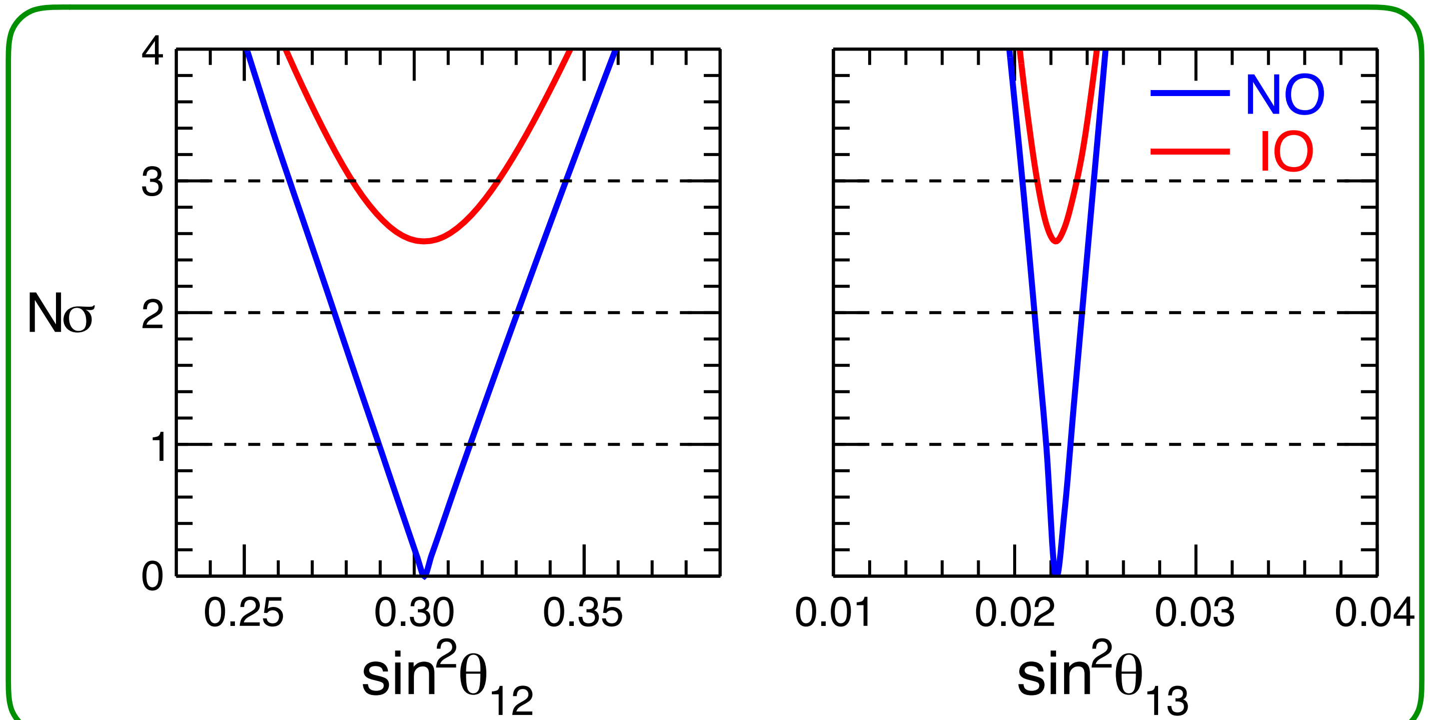
$\sin^2\theta_{23} = 0.455\ (6.7\%)$ or $0.569\ (5.5\%)$

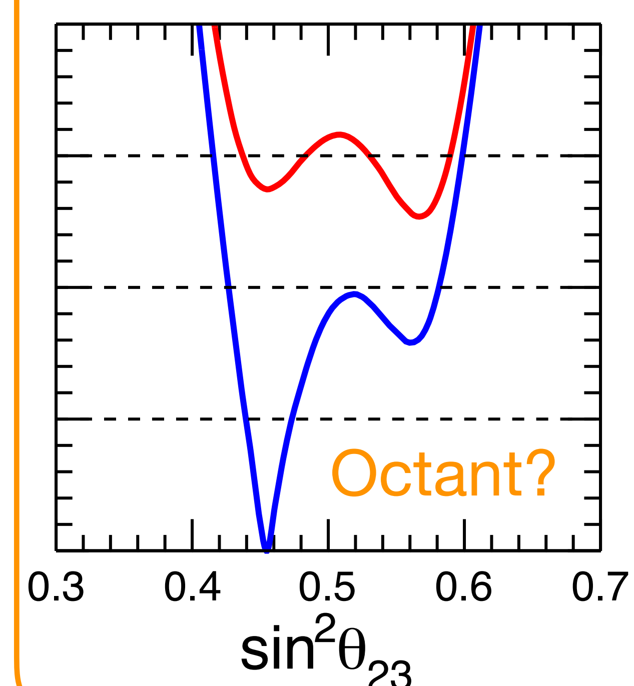
$\delta/\pi = 1.25\ (16\%)$ or $1.52\ (9\%)$

Unknown



Known





Capozzi et al., 2107.00532; see also Esteban et al., 2007.14792 and de Salas et al., 2006.11237

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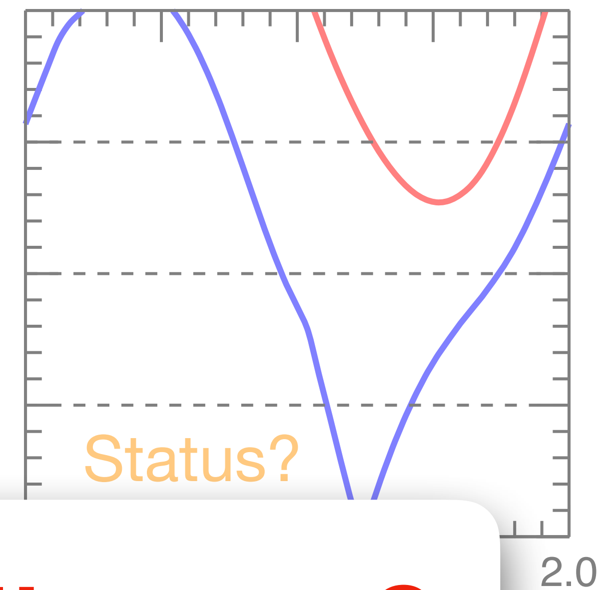
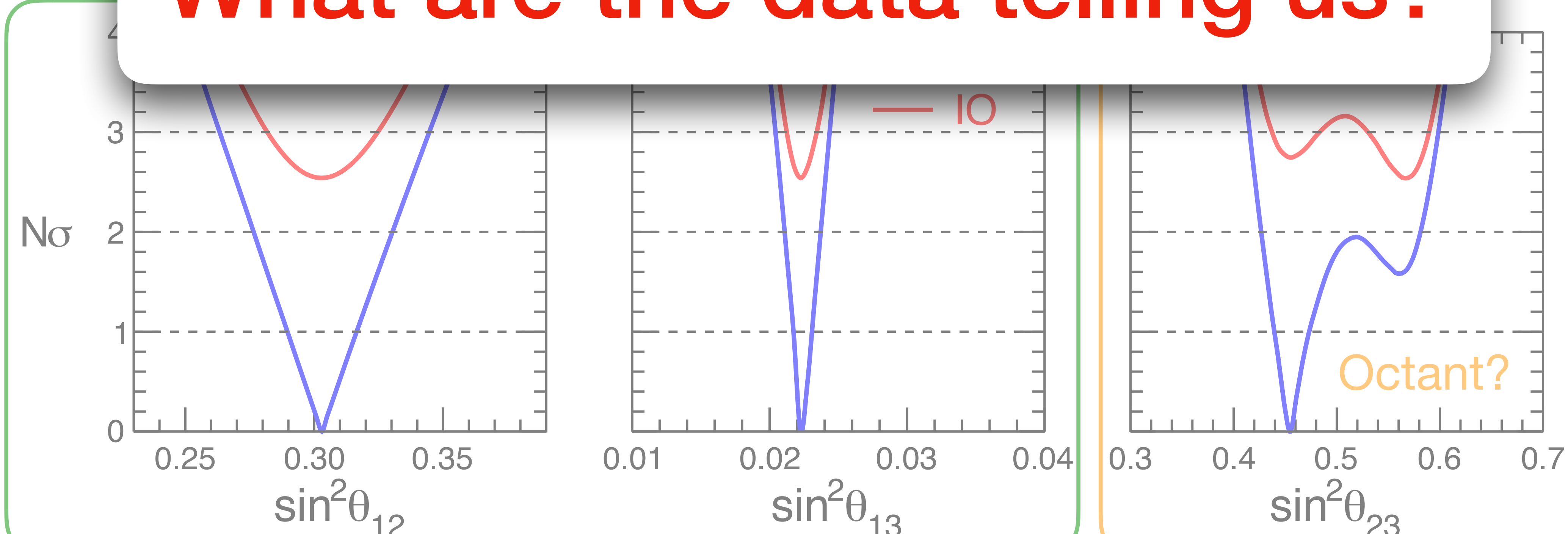


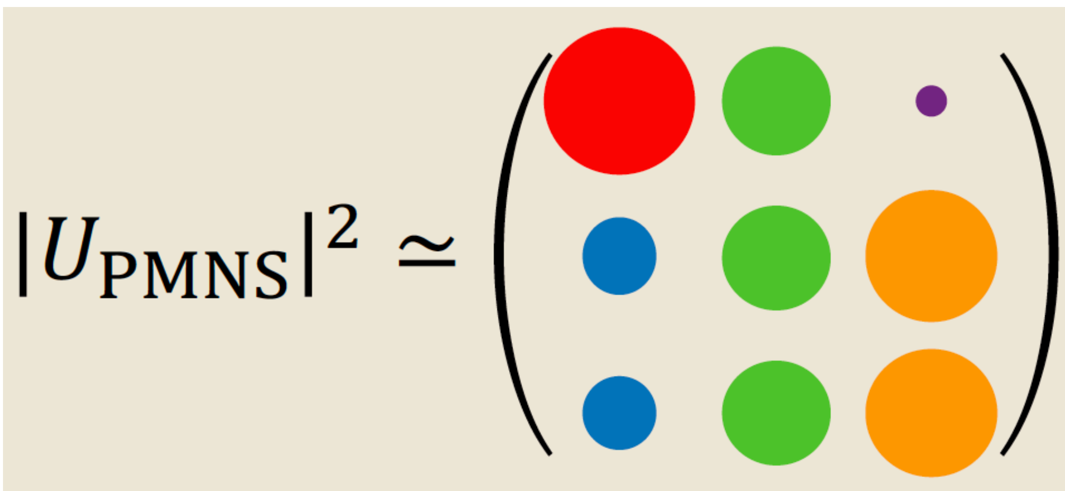
What are the data telling us?

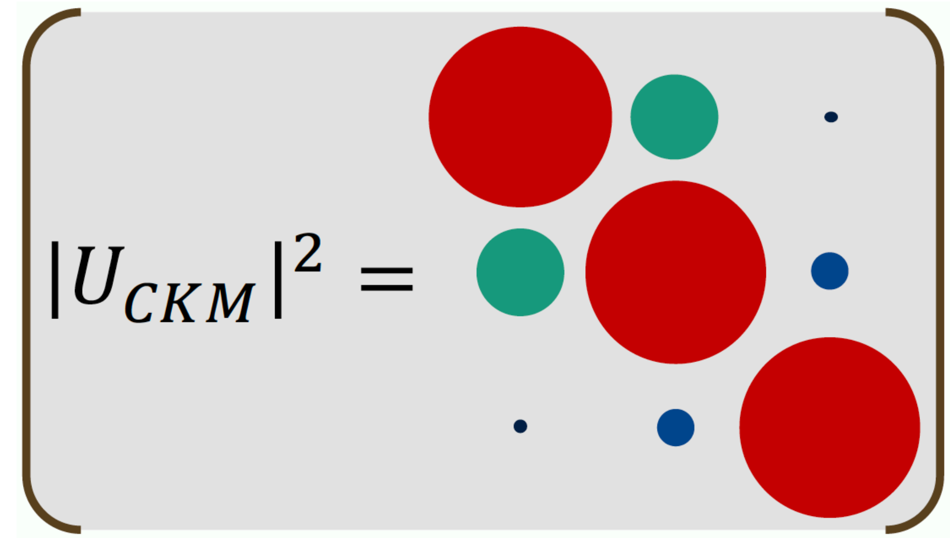


Capozzi et al., 2107.00532; see also Esteban et al., 2007.14792 and de Salas et al., 2006.11237

Flavour puzzle

2 large and 1 small (but non-zero) mixing angles -> very different from quarks





Images: Phill Litchfield

$$(s_{12}^2,\ s_{23}^2,\ s_{13}^2) \sim (0.3,\ 0.5,\ 0.022) \quad \text{vs} \quad (0.05,\ 1.8\times 10^{-3},\ 1.4\times 10^{-5})$$

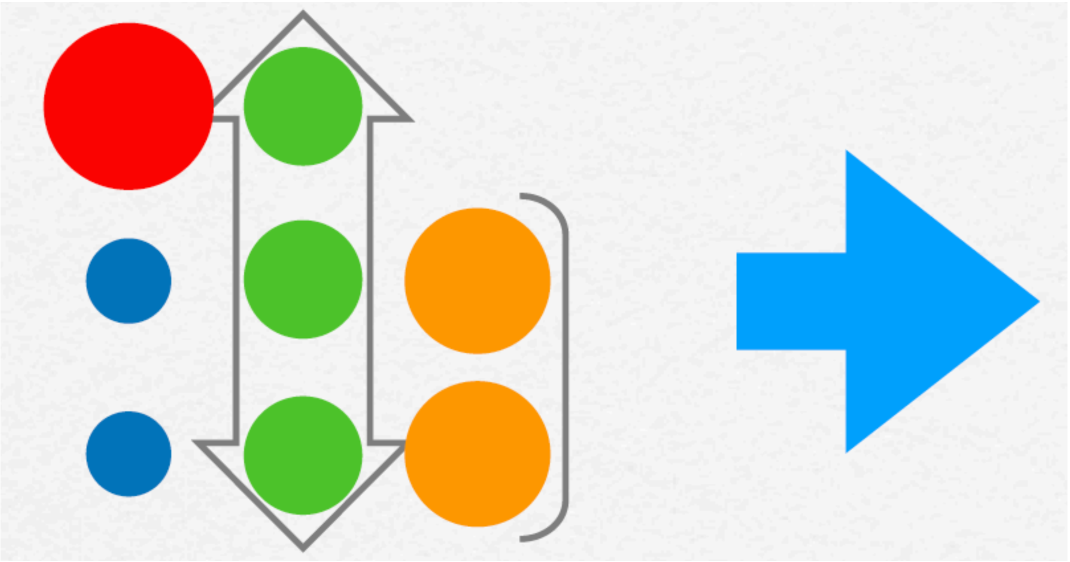
- Why are these mixing patterns so different?
- Is there any symmetry behind them or they follow anarchy?

These questions are integral part of the **flavour puzzle**:

- Why are there 3 generations of fermions?
- What is the absolute neutrino mass scale?
- What governs the fermion mass hierarchies?

Tri-bimaximal (TBM) mixing

Harrison, Perkins, Scott, hep-ph/0202074



$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13} e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sin^2\theta_{23} = \frac{1}{2} \qquad \sin^2\theta_{13} = 0 \qquad \sin^2\theta_{12} = \frac{1}{3}$$

allowed at 2σ — excluded at many σ — allowed at 2σ

Flavour symmetry

At high energies, the theory is invariant under

$$\varphi(x) \to \rho(g)\,\varphi(x)\,, \quad g \in G_f \qquad \text{e.g.} \qquad \varphi = \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix}$$

representation of G_f

flavour symmetry group

At low energies, flavour symmetry has to be broken

Clever way of breaking:

$$G_f$$

$G_e \subset G_f$ residual symmetries $G_\nu \subset G_f$

$$\rho(g_e)^\dagger M_e M_e^\dagger \rho(g_e) = M_e M_e^\dagger \qquad \rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

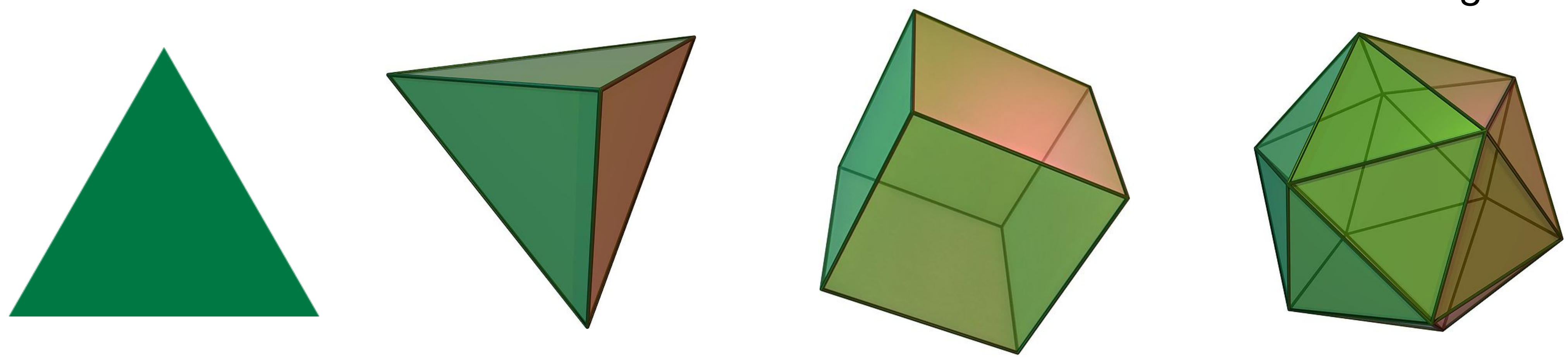
$$U_e^\dagger M_e M_e^\dagger U_e = \mathrm{diag}\left(m_e^2, m_\mu^2, m_\tau^2\right) \qquad U_\nu^T M_\nu U_\nu = \mathrm{diag}\left(m_1, m_2, m_3\right)$$

$$U_e^\dagger \rho(g_e) U_e = \rho(g_e)^{\mathrm{diag}} \qquad U_\nu^\dagger \rho(g_\nu) U_\nu = \rho(g_\nu)^{\mathrm{diag}}$$

$$U = U_e^\dagger U_\nu$$

Non-Abelian discrete symmetries

Images: WIKIPEDIA



S_3 (6) A_4 (12) S_4 (24) A_5 (60)

Generated by two elements S and T

$$\langle S,\, T \mid S^2 = (ST)^3 = T^N = I \rangle \,, \quad N = 2\,,\, 3\,,\, 4\,,\, 5$$

Another convenient presentation for S_4

$$\langle S,\, T,\, U \mid S^2 = T^3 = U^2 = (ST)^3 = (SU)^2 = (TU)^2 = (STU)^4 = I \rangle$$

A_4 , S_4 , and A_5 admit a 3-dimensional irrep (unification of families)

Reviews: Altarelli, Feruglio, 1002.0211; Ishimori et al., 1003.3552; King, Luhn, 1301.1340; Petcov, 1711.10806; Feruglio, Romanino, 1912.06028

TBM mixing from S4

<u>Example</u>

$$G_f = S_4$$

$$G_e = Z_3^T \qquad\qquad G_\nu = Z_2^S \times Z_2^U$$

$$\rho(T) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \qquad \rho(S) = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad \rho(U) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

$U_e = \mathbb{1}$

diagonalised by $U_\nu = U_{\rm TBM}$

$$U_{\rm PMNS} = U_e^\dagger U_\nu = U_{\rm TBM}$$

In concrete models, flavour symmetry breaking occurs spontaneously when flavons (scalar fields not charged under the SM) acquire VEVs

$\langle \phi^e \rangle \propto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ preserves T and $\langle \phi^\nu \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ preserves S and U

Reconciling TBM mixing with data

Break T: charged lepton corrections

U_e free and $U_\nu = U_{\rm TBM}$

Different ansatzes: $U_e^\dagger = U_{ij}\left(\theta^e_{ij}, \delta^e_{ij}\right)$, $U_e^\dagger = U_{ij}\left(\theta^e_{ij}, \delta^e_{ij}\right) U_{kl}\left(\theta^e_{kl}, \delta^e_{kl}\right)$, …

$$U_{13}\left(\theta^e, \delta^e\right) = \begin{pmatrix} \cos\theta^e & 0 & \sin\theta^e e^{-i\delta^e} \\ 0 & 1 & 0 \\ -\sin\theta^e e^{i\delta^e} & 0 & \cos\theta^e \end{pmatrix}$$

θ^e and δ^e are free parameters

Example: $U_e^\dagger = U_{12}\left(\theta^e, \delta^e\right)$

$$\sin\theta_{13} = \frac{1}{\sqrt{2}} \sin\theta^e$$

$$\sin^2\theta_{23} = \frac{1 - 2\sin^2\theta_{13}}{2(1-\sin^2\theta_{13})} \approx 0.489$$

$$\cos\delta = \frac{(1-2\sin^2\theta_{13})^{\frac{1}{2}}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\frac{1}{3} + \left(\sin^2\theta_{12} - \frac{2}{3}\right)\frac{1-3\sin^2\theta_{13}}{1-2\sin^2\theta_{13}}\right] \approx -0.156 \Rightarrow \delta/\pi = 1.45$$

neutrino mixing sum rules

Marzocca, Petcov, Romanino, Sevilla, 1302.0423

Petcov, 1405.6006

Girardi, Petcov, **AT**, 1410.8056, 1504.00658

Girardi, Petcov, Stuart, **AT**, 1509.02502

Reconciling TBM mixing with data

Break U: $G_\nu = Z_2^S$ (instead of $Z_2^S \times Z_2^U$)

$$U_{\rm PMNS} = U_{\rm TBM} U_{13}(\theta^\nu, \delta^\nu) = \begin{pmatrix} * & \sqrt{\frac{1}{3}} & * \\ * & \sqrt{\frac{1}{3}} & * \\ * & \sqrt{\frac{1}{3}} & * \end{pmatrix}$$

trimaximal mixing 2 (TM2)

Grimus, Lavoura, 0809.0226

$$\sin^2\theta_{12} = \frac{1}{3\left(1-\sin^2\theta_{13}\right)} > \frac{1}{3} \qquad \cos\delta = \frac{\left(1-2\sin^2\theta_{13}\right)\cot 2\theta_{23}}{\sin\theta_{13}\sqrt{2-3\sin^2\theta_{13}}}$$

Break S and U, preserving SU: $G_\nu = Z_2^{SU}$

$$U_{\rm PMNS} = U_{\rm TBM} U_{23}(\theta^\nu, \delta^\nu) = \begin{pmatrix} \sqrt{\frac{2}{3}} & * & * \\ -\sqrt{\frac{1}{6}} & * & * \\ -\sqrt{\frac{1}{6}} & * & * \end{pmatrix}$$

trimaximal mixing 1 (TM1)

Albright, Rodejohann, 0812.0436

$$\sin^2\theta_{12} = \frac{1-3\sin^2\theta_{13}}{3\left(1-\sin^2\theta_{13}\right)} < \frac{1}{3} \qquad \cos\delta = -\frac{\left(1-5\sin^2\theta_{13}\right)\cot 2\theta_{23}}{2\sqrt{2}\sin\theta_{13}\sqrt{1-3\sin^2\theta_{13}}}$$

Flavour models with CP

$S_4 \rtimes CP$

$G_e = Z_3$ and $G_\nu = Z_2 \times CP$

Case	I	II	IV	V
$\sin^2\theta_{13}$	$\frac{2}{3}\sin^2\theta$	$\frac{2}{3}\sin^2\theta$	$\frac{1}{3}\sin^2\theta$	$\frac{1}{3}\sin^2\theta$
$\sin^2\theta_{12}$	$\frac{1}{2+\cos 2\theta}$	$\frac{1}{2+\cos 2\theta}$	$\frac{\cos^2\theta}{2+\cos^2\theta}$	$\frac{\cos^2\theta}{2+\cos^2\theta}$
$\sin^2\theta_{23}$	$\frac{1}{2}$	$\frac{1}{2}\left(1-\frac{\sqrt{3}\sin 2\theta}{2+\cos 2\theta}\right)$	$\frac{1}{2}$	$\frac{1}{2}\left(1-\frac{2\sqrt{6}\sin 2\theta}{5+\cos 2\theta}\right)$
$\lvert\sin\delta_{CP}\rvert$	1	0	1	0

TM2 $(\sin^2\theta_{13} > 1/3)$ TM1 $(\sin^2\theta_{13} < 1/3)$

Feruglio, Hagedorn, Ziegler, 1211.5560

$A_5 \rtimes CP$

$G_e = Z_5$ $G_e = Z_3$ $G_e = Z_2 \times Z_2$ $G_\nu = Z_2 \times CP$

Case	II	III	IV	V	VII-a	VII-b
$\sin^2\theta_{13}$	$\frac{3-\varphi}{5}\sin^2\theta$	$\frac{\varphi}{\sqrt{5}}\sin^2\theta$	$\frac{\varphi}{\sqrt{5}}\sin^2\theta$	$\frac{1-\sin 2\theta}{3}$	$\frac{(\cos\theta-\varphi\sin\theta)^2}{4\varphi^2}$	
$\sin^2\theta_{12}$	$\frac{2\cos^2\theta}{3+2\varphi+\cos 2\theta}$	$\frac{4-2\varphi}{5-2\varphi+\cos 2\theta}$	$\frac{4-2\varphi}{5-2\varphi+\cos 2\theta}$	$\frac{1}{2+\sin 2\theta}$	$\frac{(\varphi\cos\theta+\sin\theta)^2}{4\varphi^2-(\cos\theta-\varphi\sin\theta)^2}$	
$\sin^2\theta_{23}$	$\frac{1}{2}$	$\frac{1}{2}-\frac{\sqrt{3-\varphi}\sin 2\theta}{3\varphi-2+\varphi\cos 2\theta}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{(\varphi^2\cos\theta-\sin\theta)^2}{4\varphi^2-(\cos\theta-\varphi\sin\theta)^2}$	$\frac{\varphi^2(\cos\theta+\varphi\sin\theta)^2}{4\varphi^2-(\cos\theta-\varphi\sin\theta)^2}$
$\lvert\sin\delta_{CP}\rvert$	1	0	1	1	0	

$\varphi = \left(1+\sqrt{5}\right)/2$

Li, Ding, 1503.03711

Compatibility with global data

$$\chi^2(\theta) = \left[\frac{\sin^2\theta_{12}(\theta) - \sin^2\theta_{12}}{\sigma(\sin^2\theta_{12})}\right]^2 + \left[\frac{\sin^2\theta_{13}(\theta) - \sin^2\theta_{13}}{\sigma(\sin^2\theta_{13})}\right]^2 + \left[\frac{\sin^2\theta_{23}(\theta) - \sin^2\theta_{23}}{\sigma(\sin^2\theta_{23})}\right]^2$$

1-parameter models compatible with global data at 3σ

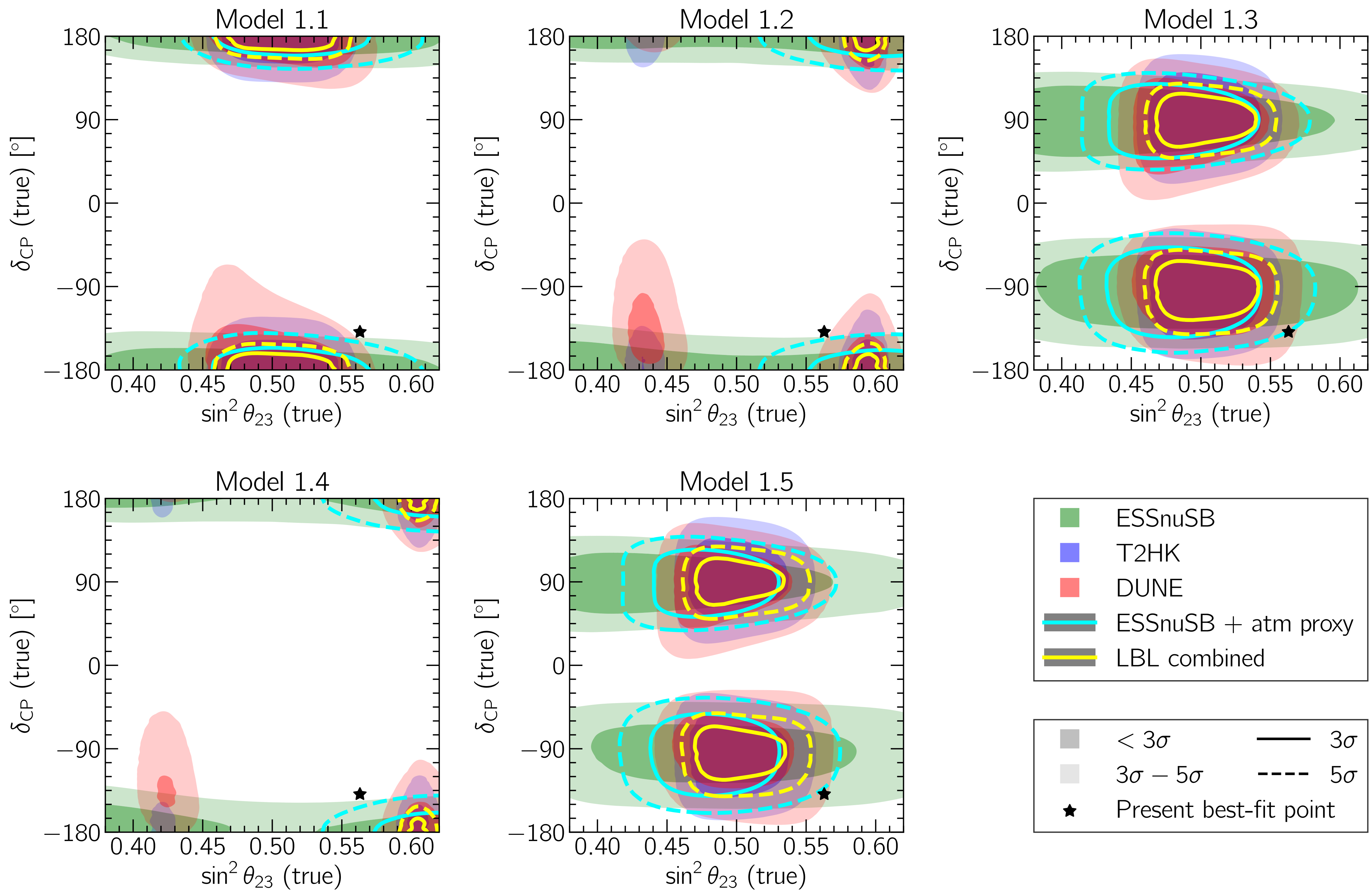
Model	$\theta_{\rm bf}$	$\theta_{3\sigma}$
1.1	17.0°	$(16.3^\circ, 17.7^\circ)$
1.2	169.9°	$(169.4^\circ, 170.4^\circ)$
1.3	15.0°	$(14.3^\circ, 15.7^\circ)$
	165.0°	$(164.3^\circ, 165.7^\circ)$
1.4	169.5°	$(169.0^\circ, 170.0^\circ)$
1.5	10.1°	$(9.6^\circ, 10.6^\circ)$
	169.9°	$(169.4^\circ, 170.4^\circ)$

Model	Case [Ref.]	Group	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\delta_{\rm CP}$	$\chi^2_{\rm min}$
1.1	VII-b [25]	$A_5 \rtimes \mathrm{CP}$	0.331	0.523	180°	5.37
1.2	III [25]	$A_5 \rtimes \mathrm{CP}$	0.283	0.593	180°	5.97
1.3	IV [24]	$S_4 \rtimes \mathrm{CP}$	0.318	1/2	$\pm 90^\circ$	7.28
1.4	II [24]	$S_4 \rtimes \mathrm{CP}$	0.341	0.606	180°	8.91
1.5	IV [25]	$A_5 \rtimes \mathrm{CP}$	0.283	1/2	$\pm 90^\circ$	11.3

Blennow, Ghosh, Ohlsson, **AT**, 2004.00017

[24] Feruglio, Hagedorn, Ziegler, 1211.5560
[25] Li, Ding, 1503.03711

Potential of future LBL experiments



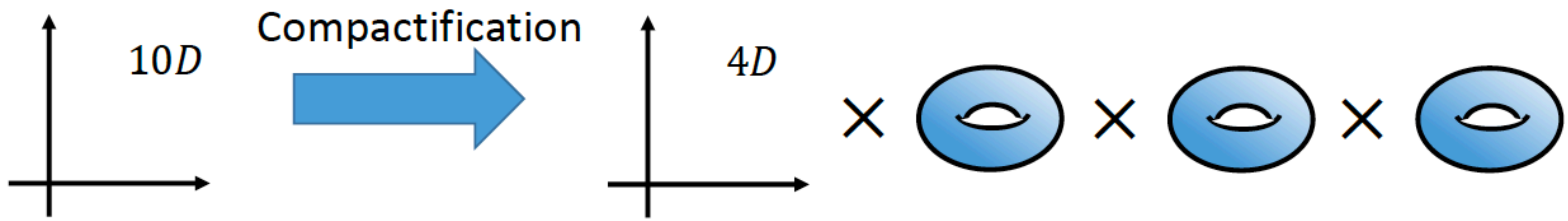
Blennow, Ghosh, Ohlsson, **AT,** 2005.12277

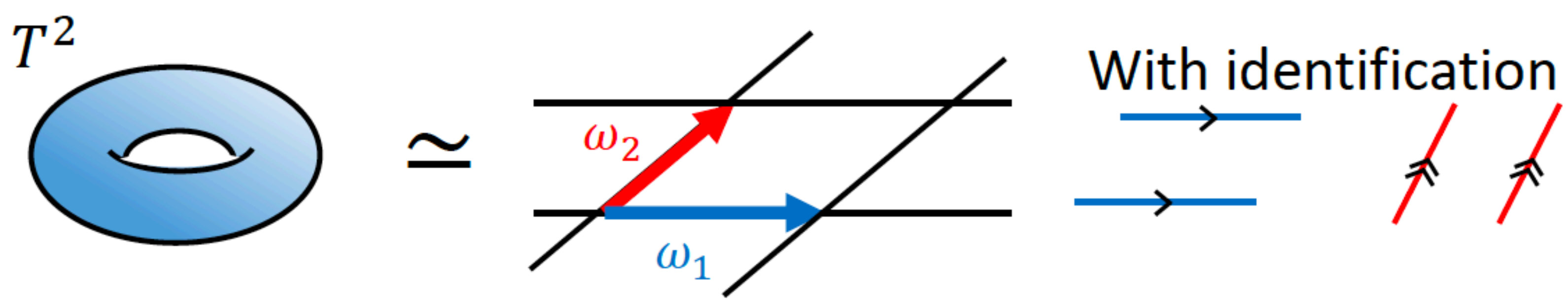
Challenges

- Many symmetry groups, many models, which one is correct (if any)?
- Symmetry breaking typically relies on numerous flavons
- Elaborated potentials to get desirable vacuum alignment
- Higher-dimensional operators can spoil leading-order predictions
- Mainly mixing, and not masses
- What is the origin of discrete flavour symmetries?

Modular invariance

Images: Takuya H. Tatsuishi





$$\tau = \frac{\omega_2}{\omega_1}, \quad \mathrm{Im}\,(\tau) > 0$$

modulus

Lattice left invariant by modular transformations

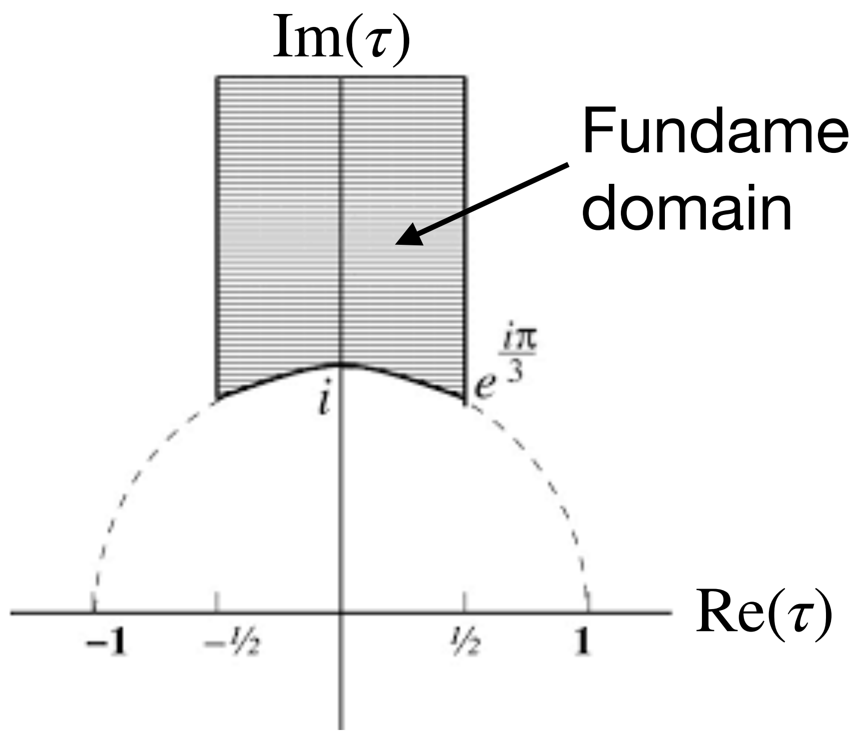
$$\tau \to \gamma\tau \equiv \frac{a\tau + b}{c\tau + d} \qquad a, b, c, d \in \mathbb{Z} \qquad ad - bc = 1$$

Proposal to apply modular invariance to flavour physics: Feruglio, 1706.08749

Modular symmetry

Modular group

$$\overline{\Gamma} = \langle S, T \mid S^2 = (ST)^3 = I \rangle \cong PSL(2,\mathbb{Z})$$



$$\tau \xrightarrow{S} -\frac{1}{\tau} \qquad \tau \xrightarrow{T} \tau + 1$$

duality — discrete shift symmetry

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Infinite normal subgroups of $SL(2,\mathbb{Z})$, $N = 2\,,\,3\,,\,4\,,\ldots$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})\,, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

Principal congruence subgroups of the modular group

$$\overline{\Gamma}(2) \equiv \Gamma(2)/\{I, -I\} \qquad \overline{\Gamma}(N) \equiv \Gamma(N)\,, \quad N > 2$$

Finite modular groups $\quad \Gamma_N \equiv \overline{\Gamma}/\overline{\Gamma}(N)$

$$\Gamma_2 \cong S_3 \qquad \Gamma_3 \cong A_4 \qquad \Gamma_4 \cong S_4 \qquad \Gamma_5 \cong A_5$$

Modular-invariant SUSY theories

$\mathcal{N} = 1$ rigid SUSY matter action

$$\mathcal{S} = \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \mathrm{d}^2\overline{\theta}\; K(\tau, \overline{\tau}, \psi, \overline{\psi}) + \int \mathrm{d}^4x\, \mathrm{d}^2\theta\; W(\tau, \psi) + \int \mathrm{d}^4x\, \mathrm{d}^2\overline{\theta}\; \overline{W}(\overline{\tau}, \overline{\psi})$$

Ferrara, Lust, Shapere, Theisen, PLB **225** (1989) 363

Ferrara, Lust, Theisen, PLB **233** (1989) 147

$$\begin{cases} \tau \to \dfrac{a\tau + b}{c\tau + d} \\ \psi_i \to \left(c\tau + d\right)^{-k_i} \rho_i(\tilde{\gamma})\, \psi_i \end{cases} \Rightarrow \begin{cases} W(\tau, \psi) \to W(\tau, \psi) \\ K(\tau, \overline{\tau}, \psi, \overline{\psi}) \to K(\tau, \overline{\tau}, \psi, \overline{\psi}) + f_K(\tau, \psi) + \overline{f_K}(\overline{\tau}, \overline{\psi}) \end{cases}$$

unitary representation of Γ_N

Feruglio, 1706.08749

$$W(\tau, \psi) = \sum_n \sum_{\{i_1, \ldots, i_n\}} \sum_s g_{i_1 \ldots i_n, s} \left(Y_{i_1 \ldots i_n, s}(\tau)\, \psi_{i_1} \ldots \psi_{i_n} \right)_{\mathbf{1}, s}$$

$$Y(\tau) \xrightarrow{\gamma} (c\tau + d)^{k_Y} \rho_Y(\tilde{\gamma})\, Y(\tau)$$

$$k_Y = k_{i_1} + \ldots + k_{i_n}$$

$$\rho_Y \otimes \rho_{i_1} \otimes \ldots \otimes \rho_{i_n} \supset \mathbf{1}$$

Yukawa couplings are modular forms!

Feruglio's modular A4 model

$\Gamma_3 \cong A_4$ (level $N = 3$)

Feruglio, 1706.08749

3 independent modular forms $Y_i(\tau)$ of weight $k = 2$ form a triplet of A_4

$$Y(\tau) = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} \to (c\tau + d)^2 \rho(\gamma) \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} \qquad L = \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \to (c\tau + d)^{-1} \rho(\gamma) \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix}$$

$$W_\nu \supset \frac{1}{\Lambda} \left(Y(\tau)\, LL \right)_{\mathbf{1}} H_u H_u \qquad \Rightarrow \qquad M_\nu(\tau) = \frac{\mathrm{v}_u^2}{\Lambda} \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix}$$

M_ν depends on 3 real parameters: $\mathrm{Re}(\tau)$, $\mathrm{Im}(\tau)$ and the overall scale v_u^2/Λ

$$\langle \tau \rangle = 0.0111 + 0.9946\, i$$

$\sin^2\theta_{12} = 0.295$ $\quad \sin^2\theta_{13} = 0.0447$ $\quad \sin^2\theta_{23} = 0.651$

$\delta/\pi = 1.55$ $\quad \alpha_{21}/\pi = 0.22$ $\quad \alpha_{31}/\pi = 1.80$

$m_1 = 0.0500$ eV $\quad m_2 = 0.0507$ eV $\quad m_3 = 0.0007$ eV (IO)

Minimal modular S4 model (with CP)

$\Gamma_4 \cong S_4$ (level $N = 4$)

Novichkov, Penedo, Petcov, **AT**, 1905.11970

5 independent modular forms of weight $k = 2$ form a doublet and a triplet of A_4

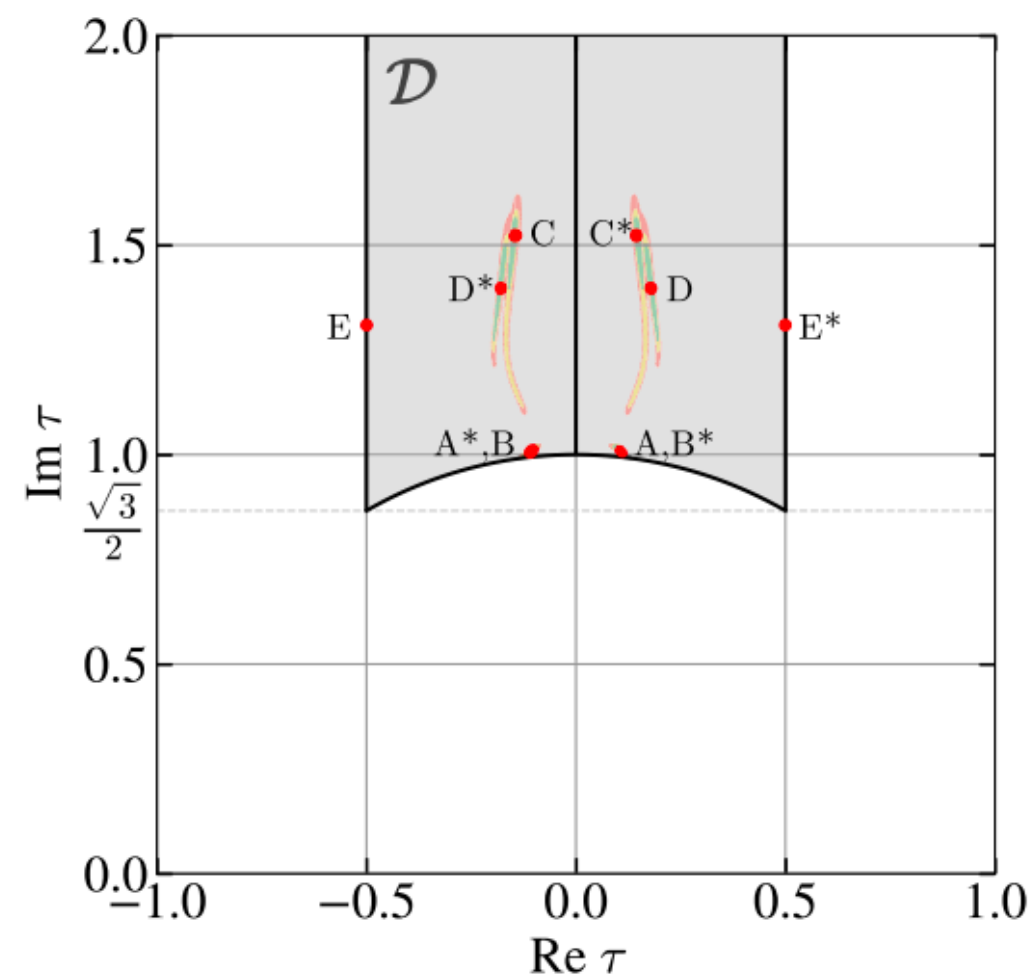
$$W = \alpha \left(E_1^c L\, Y_{\mathbf{3'}}^{(2)}\right)_{\mathbf{1}} H_d + \beta \left(E_2^c L\, Y_{\mathbf{3}}^{(4)}\right)_{\mathbf{1}} H_d + \gamma \left(E_3^c L\, Y_{\mathbf{3'}}^{(4)}\right)_{\mathbf{1}} H_d$$
$$+ g \left(N^c L\, Y_{\mathbf{2}}^{(2)}\right)_{\mathbf{1}} H_u + g' \left(N^c L\, Y_{\mathbf{3'}}^{(2)}\right)_{\mathbf{1}} H_u + \Lambda \left(N^c N^c\right)_{\mathbf{1}}$$

Solutions A and A*

Input parameters	
$\mathrm{Re}\,\tau$	± 0.0992
$\mathrm{Im}\,\tau$	1.0160
β/α	9.348
γ/α	0.0022
g'/g	-0.0209
$v_d \alpha$ [MeV]	53.61
$v_u^2 g_1^2/\Lambda$ [eV]	0.0135

Observables	
m_e/m_μ	0.0048
m_μ/m_τ	0.0576
r	0.0298
$\sin^2\theta_{12}$	0.305
$\sin^2\theta_{13}$	0.0214
$\sin^2\theta_{23}$	0.486
δm^2 [10^{-5} eV^2]	7.33
$\|\Delta m^2\|$ [10^{-3} eV^2]	2.457
$N\sigma$	1.01

Predictions	
m_1 [eV]	0.012
m_2 [eV]	0.015
m_3 [eV]	0.051
δ/π	± 1.64
α_{21}/π	± 0.35
α_{31}/π	± 1.25
$\|m_{ee}\|$ [eV]	0.012
$\sum_i m_i$ [eV]	0.078
Ordering	NO



7 (4) parameters vs 12 (9) observables

Modular vs conventional discrete

Advantages

✓ Numerous scalar fields (flavons) → (single) modulus

✓ Complicated scalar potential → moduli space

✓ Yukawa couplings → modular forms (known functions of τ)

✓ A_4, S_4, A_5 arise as quotient groups of the modular group

✓ Both mixing parameters and masses are predicted/constrained

Challenges

- What determines the level, weights and representations, (N, k_i, ρ_i) tuple?
- Kinetic terms are not constrained in the bottom-up approach
- Dynamical selection of the vacuum $\langle\tau\rangle$
- Extension to the quark sector
- Is SUSY necessary?

Conclusions

- We are still far from the Theory of Flavour
- Symmetries remain the best tool to approach the flavour puzzle
- Many viable models based on non-Abelian discrete symmetries broken to residual symmetries of the charged lepton and neutrino mass matrices
- The number of viable models will be reduced by future, more precise measurements of the neutrino mixing parameters, including δ_{CP} (DUNE, T2HK, ESSnuSB, JUNO are instrumental in this respect)
- Modular invariance has a number of advantages over conventional discrete flavour symmetries
- More effort is needed towards deciphering the nature of flavour

Backup slides

Discrete symmetry and CP

Generalised CP (GCP) transformation

$$\varphi(x) \xrightarrow{CP} X\varphi^*(x_P) \qquad x = (t, \mathbf{x}) \qquad x_P = (t, -\mathbf{x})$$

unitary matrix

Consistency condition (X is constrained by G_f)

$$X\rho^*(g)X^{-1} = \rho(g') \qquad g, g' \in G_f$$

Feruglio, Hagedorn, Ziegler, 1211.5560
Holthausen, Lindner, Schmidt, 1211.6953

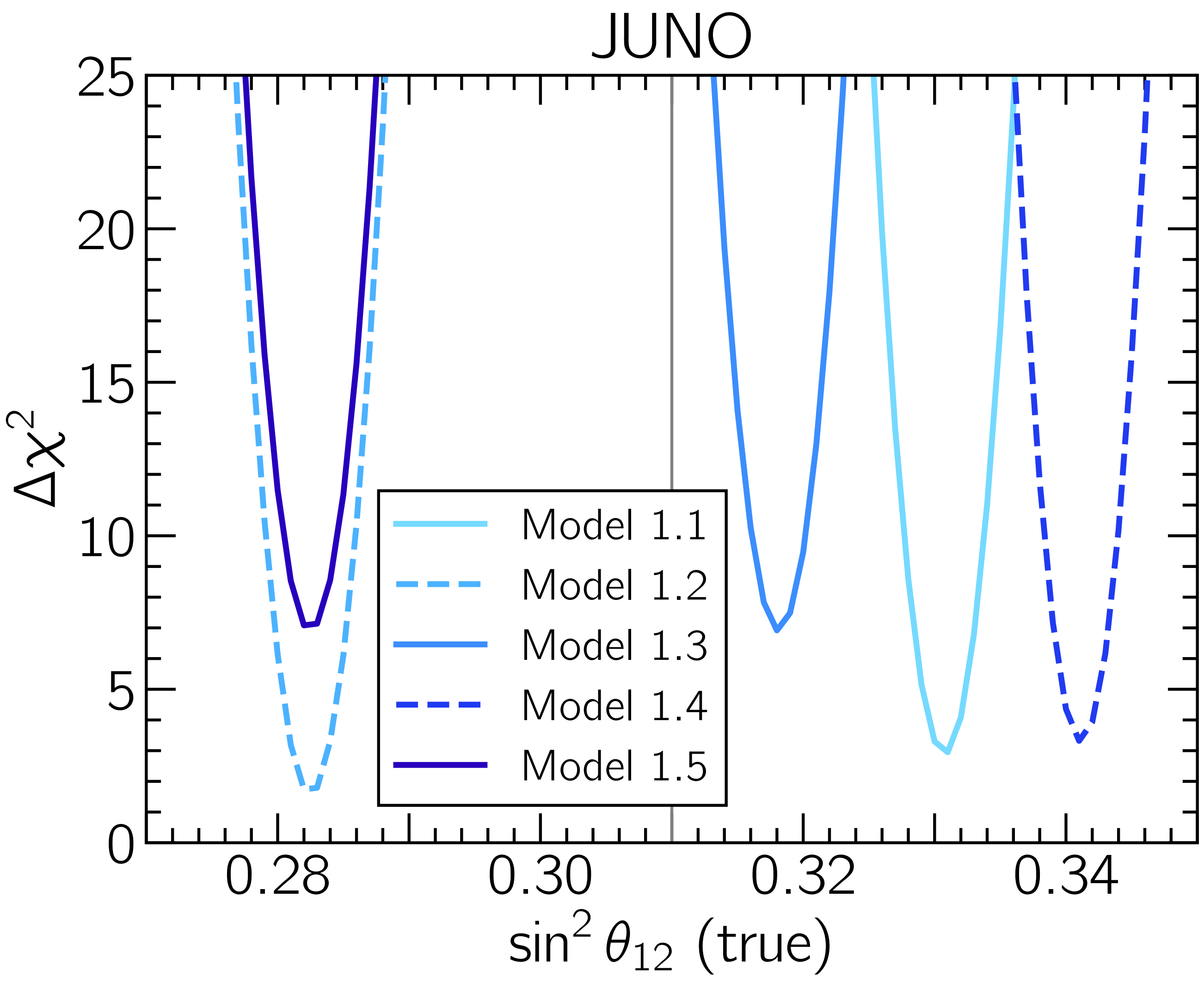
If $G_e > Z_2$ and $G_\nu = Z_2 \times CP$, the mixing matrix is defined up to a real rotation

$$U_{\rm PMNS} = U_{\rm fixed}\, R_{ij}(\theta) \qquad R_{ij}(\theta) = U_{ij}(\theta, 0) \qquad \theta \text{ is a free real angle}$$

- 1 free parameter => higher predictive power
- Predictions for the Majorana phases

JUNO potential

Blennow, Ghosh, Ohlsson, **AT,** 2005.12277



Modular forms

Holomorphic functions transforming under $\overline{\Gamma}(N)$ as

$$f(\gamma\tau) = (c\tau + d)^k f(\tau)\,, \quad \gamma \in \overline{\Gamma}(N)$$

k is weight
non-negative even integer

N is level
natural number

Modular forms of weight k and level N form a linear space $\mathcal{M}_k\left(\overline{\Gamma}(N)\right)$ of finite dimension. We can choose a basis in this space s.t. $F(\tau) \equiv \left(f_1(\tau)\,,\, f_2(\tau)\,,\, \ldots\right)^T$ transforms as

$$F(\gamma\tau) = (c\tau + d)^k \rho(\tilde{\gamma}) F(\tau)\,, \quad \gamma \in \overline{\Gamma}$$

ρ is a unitary representation of Γ_N

$\tilde{\gamma}$ represents the equivalence class of γ in Γ_N

Feruglio, 1706.08749

Dimension of linear space of modular forms

N	g	$d_{2k}(\Gamma(N))$	μ_N	Γ_N
2	0	$k+1$	6	S_3
3	0	$2k+1$	12	A_4
4	0	$4k+1$	24	S_4
5	0	$10k+1$	60	A_5
6	1	$12k$	72	
7	3	$28k-2$	168	

$k(\text{this presentation}) \equiv 2\,k(\text{this table})$

Feruglio, 1706.08749

Modular forms of level 2 and weight 2

Level *N* = 2 $\left(\Gamma_2 \simeq S_3 : \quad S^2 = (ST)^3 = T^2 = I\right)$

N\k	0	2	4	6
2	1	**2**	3	4

S (up to multiplicative factors)

Seed functions: $$\eta\left(\frac{\tau}{2}\right), \quad \eta\left(\frac{\tau+1}{2}\right), \quad \eta\left(2\tau\right)$$

T

$$\eta(\tau) \equiv q^{1/24}\prod_{n=1}^{\infty}\left(1-q^n\right), \quad q = e^{2\pi i \tau},$$ is the Dedekind eta function

$$\eta(\tau+1) = e^{i\pi/12}\,\eta(\tau) \qquad \eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau)$$

Kobayashi, Tanaka, Tatsuishi, 1803.10391

Modular forms of level 2 and weight 2

Level *N* = 2 $\left(\Gamma_2 \simeq S_3 : \quad S^2 = (ST)^3 = T^2 = I\right)$

N\k	0	2	4	6
2	1	**2**	3	4

$$Y(a_1, a_2, a_3 \,|\, \tau) \equiv \sum_{i=1}^{3} a_i \frac{\mathrm{d}}{\mathrm{d}\tau} \log \eta_i(\tau)\,, \qquad \sum_{i=1}^{3} a_i = 0$$

$$Y_{\mathbf{2}}(-1/\tau) = \tau^2 \rho(S)\; Y_{\mathbf{2}}(\tau) \qquad Y_{\mathbf{2}}(\tau+1) = \rho(T)\; Y_{\mathbf{2}}(\tau)$$

$$\rho(S) = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix} \qquad \rho(T) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Y_{\mathbf{2}}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix} = c \begin{pmatrix} Y(1,1,-2 \,|\, \tau) \\ Y(\sqrt{3}, -\sqrt{3}, 0 \,|\, \tau) \end{pmatrix}$$

S_3 doublet of weight 2 modular forms

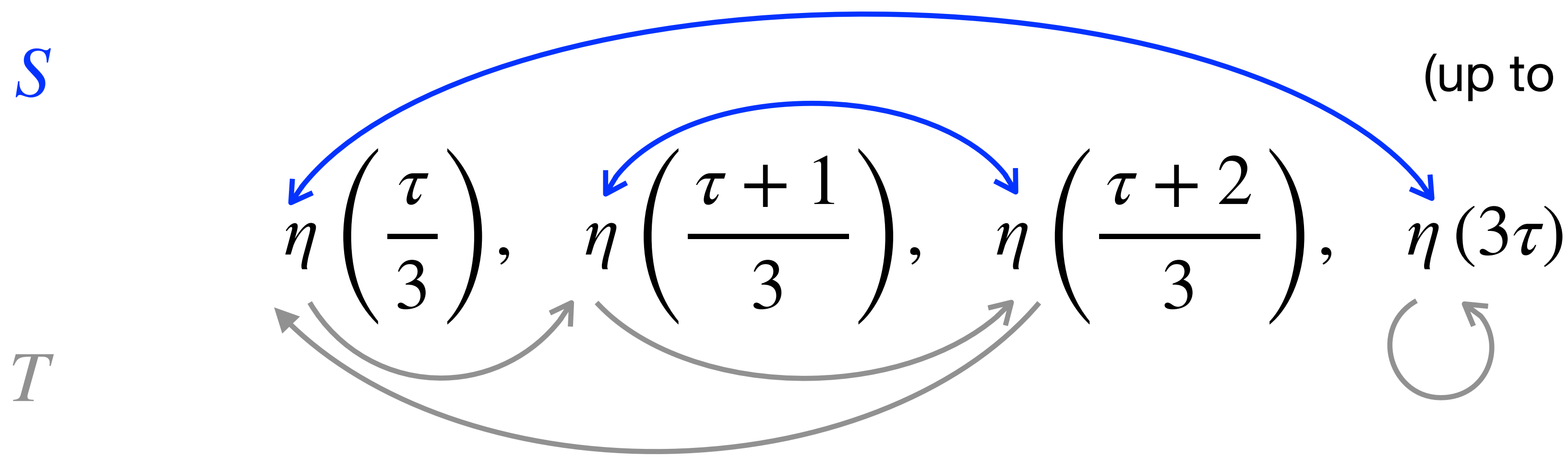
Kobayashi, Tanaka, Tatsuishi, 1803.10391

Modular forms of level 3 and weight 2

Level *N* = 3 $\left(\Gamma_3 \simeq A_4 : \quad S^2 = (ST)^3 = T^3 = I\right)$

N\k	0	2	4	6
3	1	**3**	5	7

(up to multiplicative factors)



$$Y_{\mathbf{3}}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = c \begin{pmatrix} Y(1,1,1,-3\,|\,\tau) \\ -2\,Y(1,\omega^2,\omega,0\,|\,\tau) \\ -2\,Y(1,\omega,\omega^2,0\,|\,\tau) \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

A_4 triplet of weight 2 modular forms

Feruglio, 1706.08749

Modular forms of level 4 and weight 2

Level *N* = 4 $\left(\Gamma_4 \simeq S_4 : \quad S^2 = (ST)^3 = T^4 = I\right)$

N\k	0	2	4	6
4	1	**5**	9	13

S

$$\eta\left(\tau + \frac{1}{2}\right),\quad \eta(4\tau),\quad \eta\left(\frac{\tau}{4}\right),\quad \eta\left(\frac{\tau+1}{4}\right),\quad \eta\left(\frac{\tau+2}{4}\right),\quad \eta\left(\frac{\tau+3}{4}\right)$$

T

$$Y_{\mathbf{2}}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix} = c \begin{pmatrix} Y(1,1,\omega,\omega^2,\omega,\omega^2\,|\,\tau) \\ Y(1,1,\omega^2,\omega,\omega^2,\omega\,|\,\tau) \end{pmatrix}$$

$$Y_{\mathbf{3'}}(\tau) = \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix} = c \begin{pmatrix} Y(1,-1,-1,-1,1,1\,|\,\tau) \\ Y(1,-1,-\omega^2,-\omega,\omega^2,\omega\,|\,\tau) \\ Y(1,-1,-\omega,-\omega^2,\omega,\omega^2\,|\,\tau) \end{pmatrix}$$

S_4 doublet and triplet $(\mathbf{3'})$ of weight 2 modular forms

Penedo, Petcov, 1806.11040