

New SSM vs helioseismic and solar neutrino data

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(*) based on work done in collaboration with **Aldo Serenelli (ICE and IEEC, BCN)**

The Sun observed with:

neutrinos

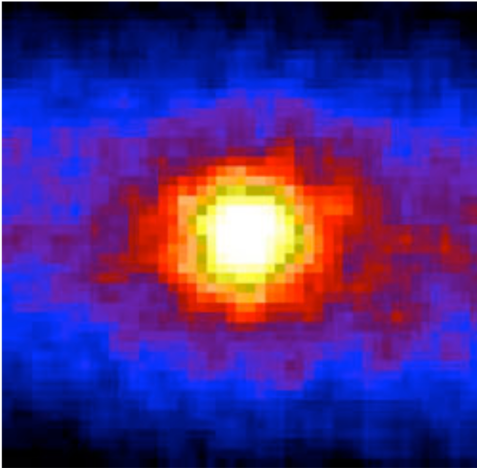
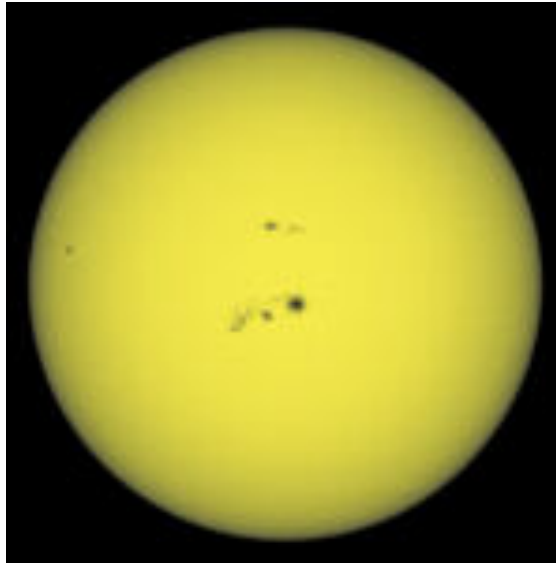
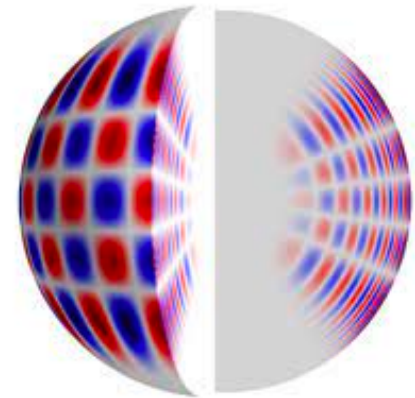


Image credits: Super-Kamiokande Coll.

photons



Helioseismology



*The Sun provides the **benchmark** for stellar evolution and a laboratory for fundamental physics.*

The Standard Solar Model (SSM)

Our comprehension of the Sun is based on the **Standard Solar Model (SSM)**.

This implies:

[Bahcall et al. 1969]

- ✓ Stellar structure equations;
(α = mixing length)
- ✓ Chemical evolution paradigm:
ZAMS homogenous model (Y_{ini} , Z_{ini})
Nuclear reactions + elemental diffusion
- ✓ Knowledge of the properties of solar plasma
(i.e. opacity, equation of state, nuc. cross sections);

No free parameters

The unknown quantities

- α , Y_{ini} , Z_{ini} ,

are fixed by requiring that the present Sun ($t_{\text{sun}}=4.57$ Gyr) reproduces its observational properties

- R_{sun} , L_{sun} , $(Z/X)_{\text{surf}}$

The Standard Solar Model (SSM)

Note that:

Given the calibration procedure, the observed luminosity, radius and surface composition of the Sun provide no test for solar models

The predictions of SSMs can be, however, **falsified** by other observations. e.g.:

- *Solar neutrinos:*

Hydrogen fusion in the solar core produce a huge amount of neutrinos that can be measured in suitable detectors (Davis 1964, Bahcall 1964)



Solar Neutrino Problem

Nuclear energy generation (cross sections, etc.)

- *Helioseismology:*

Solar oscillations originally discovered by Leighton et al. 1962 and interpreted as standing acoustic waves

Elemental Diffusion

Opacity, EoS, ...

Constant improvement in SSM constitutive physics was triggered during last decades by solar neutrino and helioseismic data

Helioseismology

Helioseismology

The Sun is a non radial oscillator. The observed oscillation frequencies can be used to determine the properties of the Sun. Linearizing around a known solar model:

$$\frac{\delta\nu_{nl}}{\nu_{nl}} = \int_0^R dr K_{u,Y}^{nl}(r) \frac{\delta u}{u}(r) + \int_0^R dr K_{Y,u}^{nl}(r) \delta Y + \frac{F(\nu_{nl})}{\nu_{nl}}$$

squared isothermal sound speed

Related to temperature stratification in the sun

surface helium abundance

See Basu & Antia 07
for a review

Impressive agreement with SSM predictions ...

Surface helium abundance

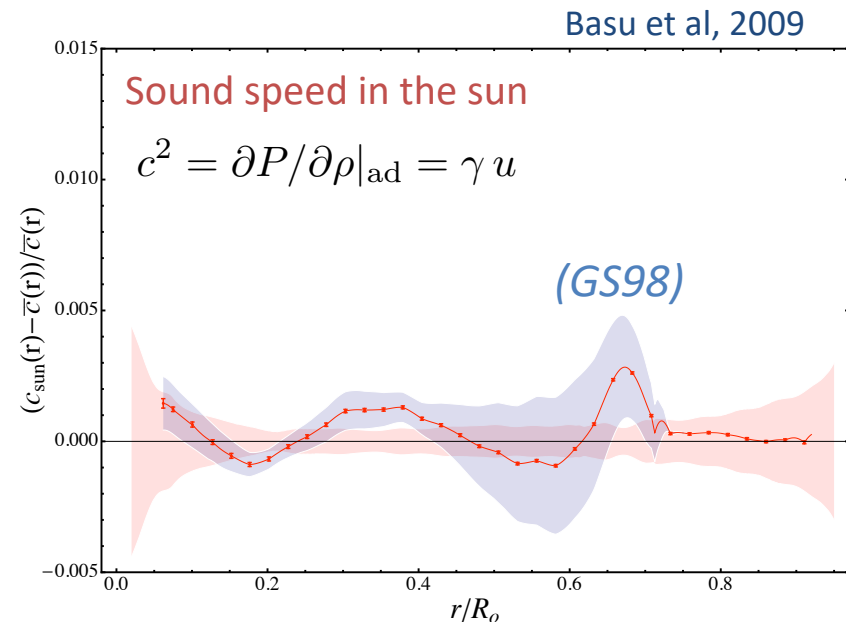
$$Y_b = 0.2485 \pm 0.0035$$

$$Y_b = 0.243 \quad (\text{GS98})$$

Inner radius of the solar convective envelope

$$R_b/R_\odot = 0.713 \pm 0.001$$

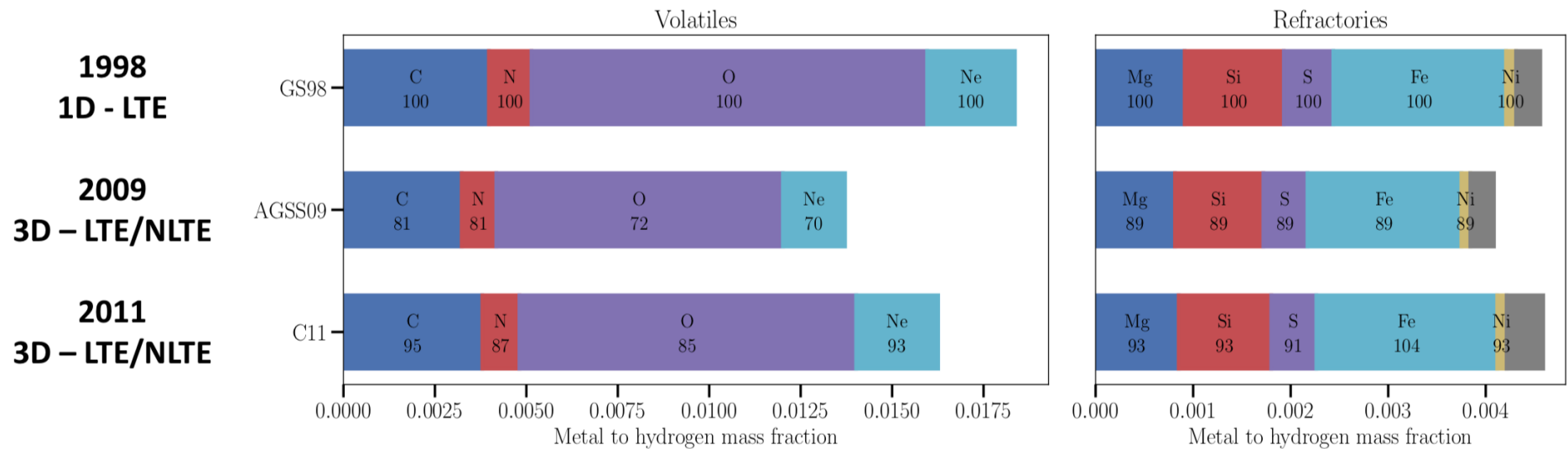
$$R_b/R_\odot = 0.712 \quad (\text{GS98})$$



... till few years ago

Downward revision of solar surface abundances

Solar surface composition is a fundamental input for SSMs → determined with spectroscopic techniques (3D models of solar atmosphere, NLTE corrections, ...)



Orebi Gann et al. 2021/2022

GS98: Grevesse & Sauval 1998 – AGSS09: Asplund, Grevesse et al. 2009 – C11: Caffau et al. 2011

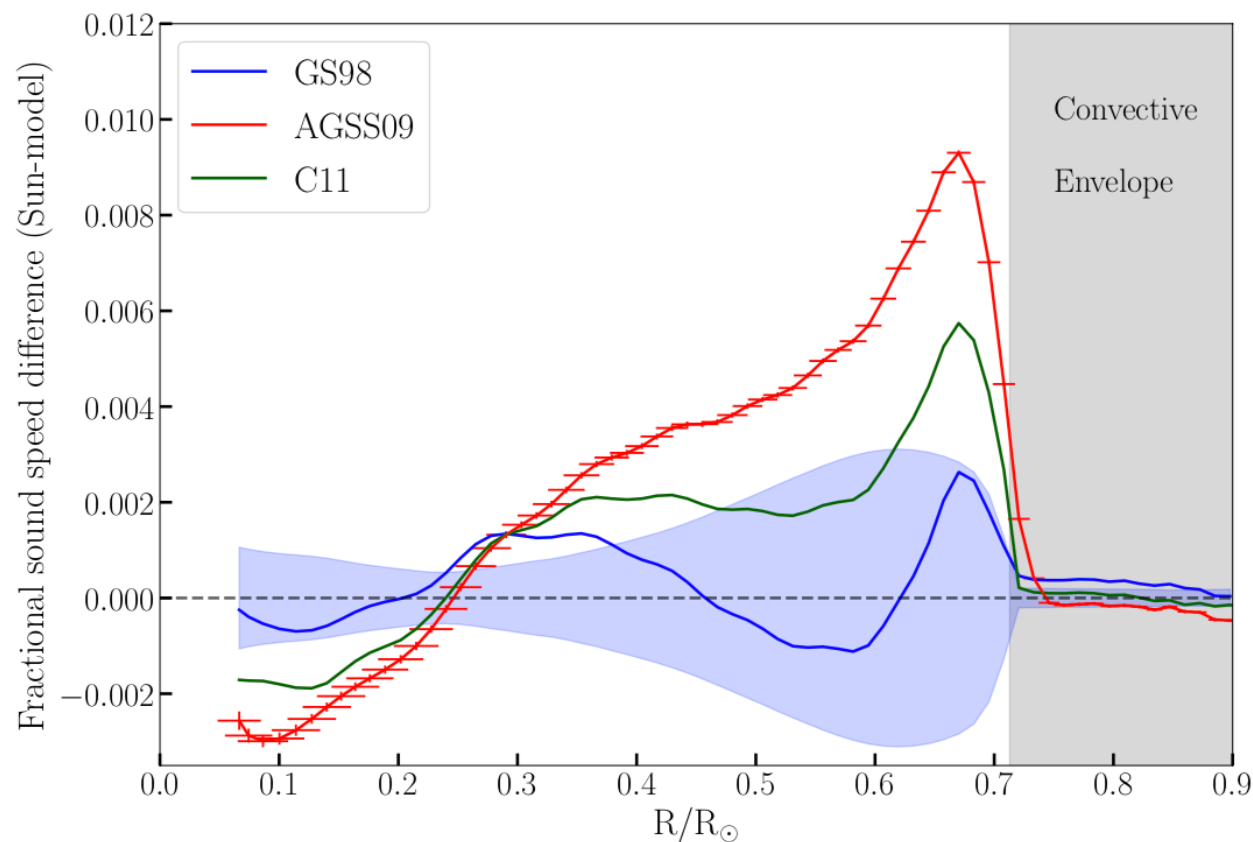
The solar abundance problem

Model	R_{CZ}/R_{\odot}	Y_s
MB22-phot	0.7123	0.2439
MB22-met	0.7120	0.2442
AAG21	0.7197	0.2343
AGSS09-met	0.7231	0.2316
GS98	0.7122	0.2425
C11	0.7162	0.2366

Helioseismic determinations

$$R_b/R_{\odot} = 0.713 \pm 0.001$$

$$Y_b = 0.2485 \pm 0.0035$$



Magg et al. 2022

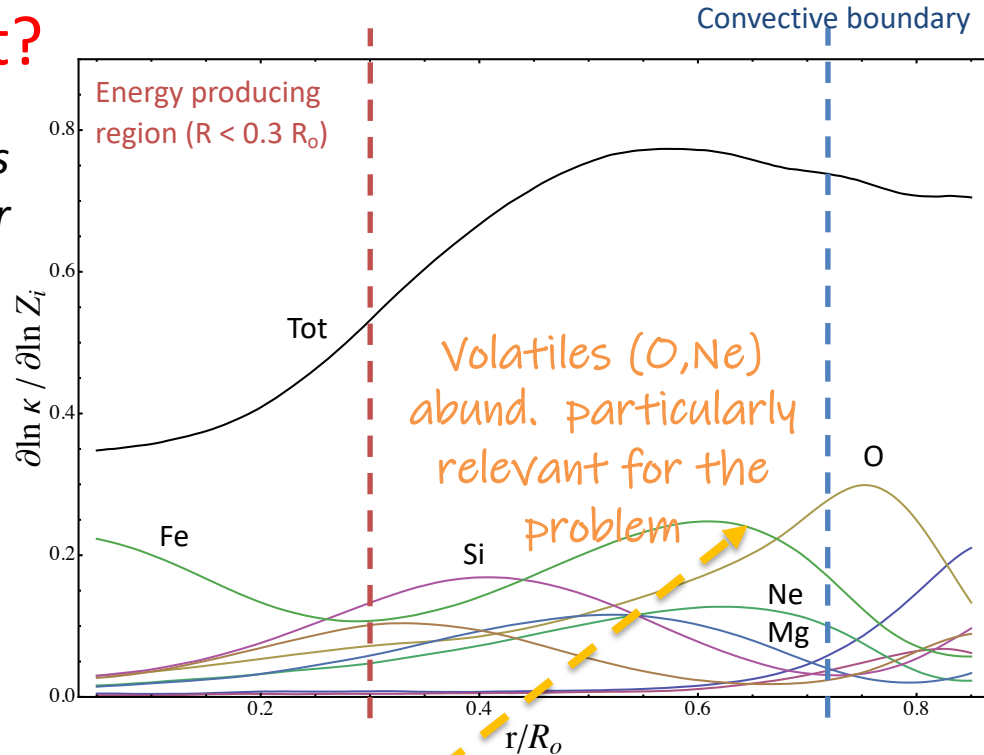
Why metals are so important?

A change of the solar composition affects the efficiency of radiative energy transfer in the core of the Sun

Composition opacity change:

$$\delta\kappa(r) = \sum_j \frac{\partial \ln \kappa(r)}{\partial \ln Z_j} \delta z_j$$

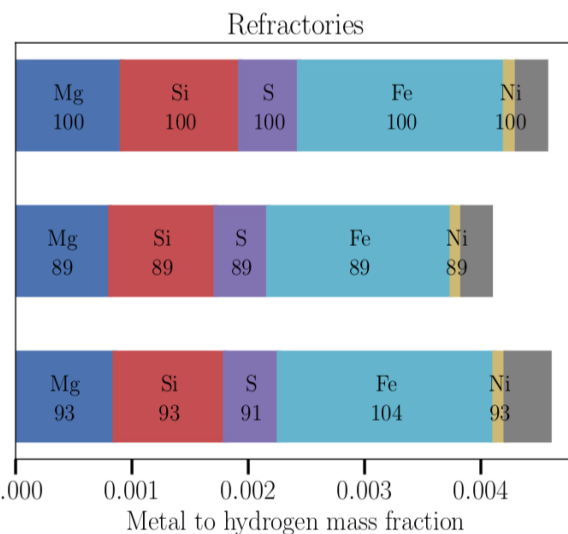
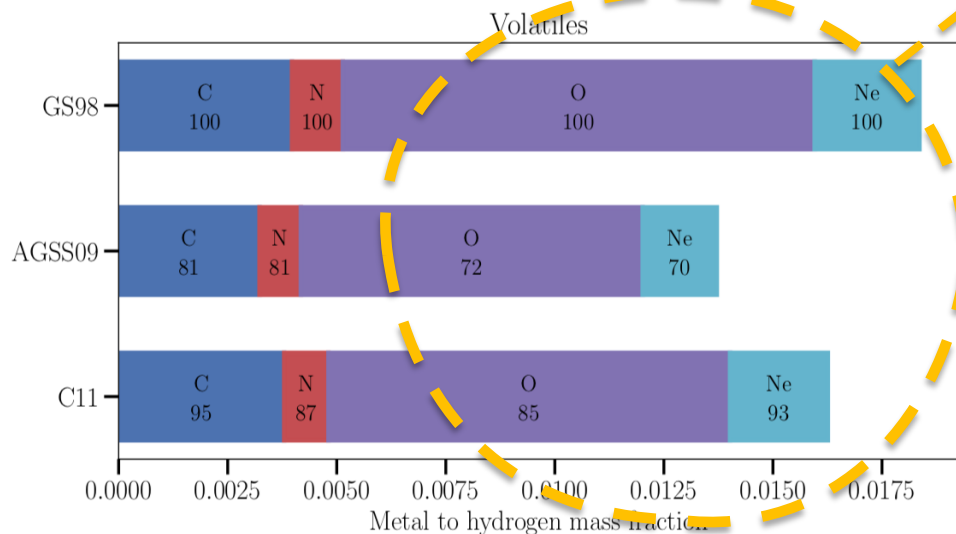
Different temperature stratification



1998
1D - LTE

2009
3D - LTE/NLTE

2011
3D - LTE/NLTE

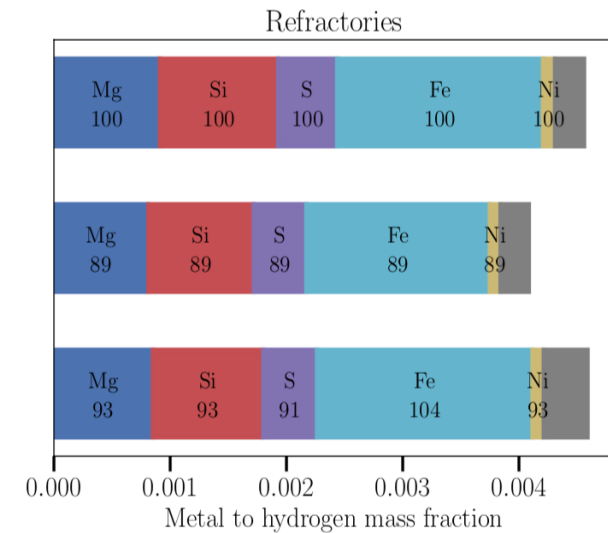
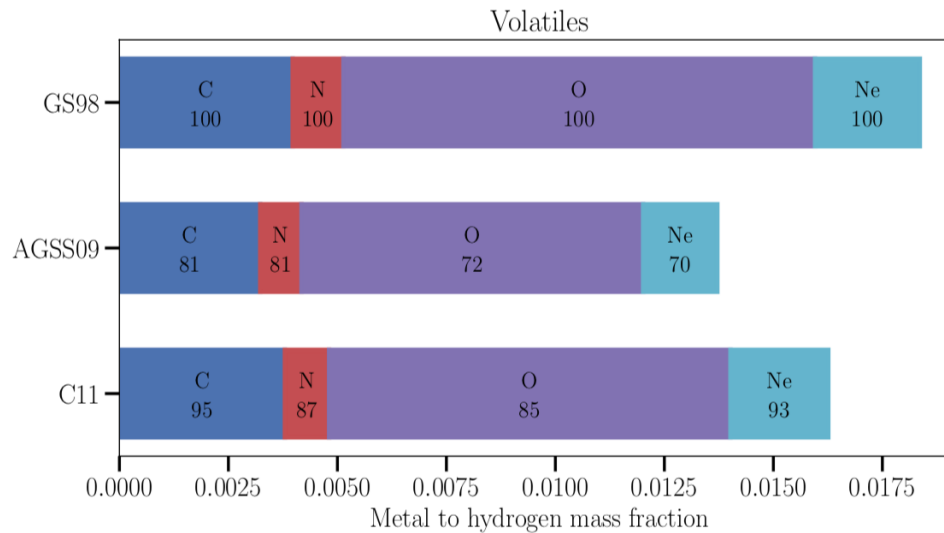


Updates in solar abundances

1998
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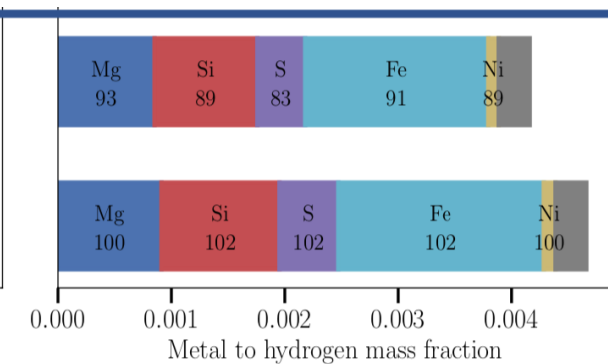
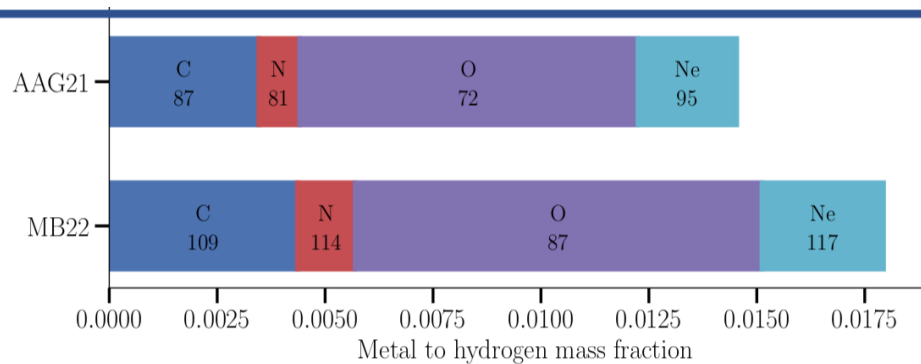
2011
3D - LTE/NLTE



Orebi Gann et al. 2021/2022

2021
3D - NLTE

2022
3D - NLTE

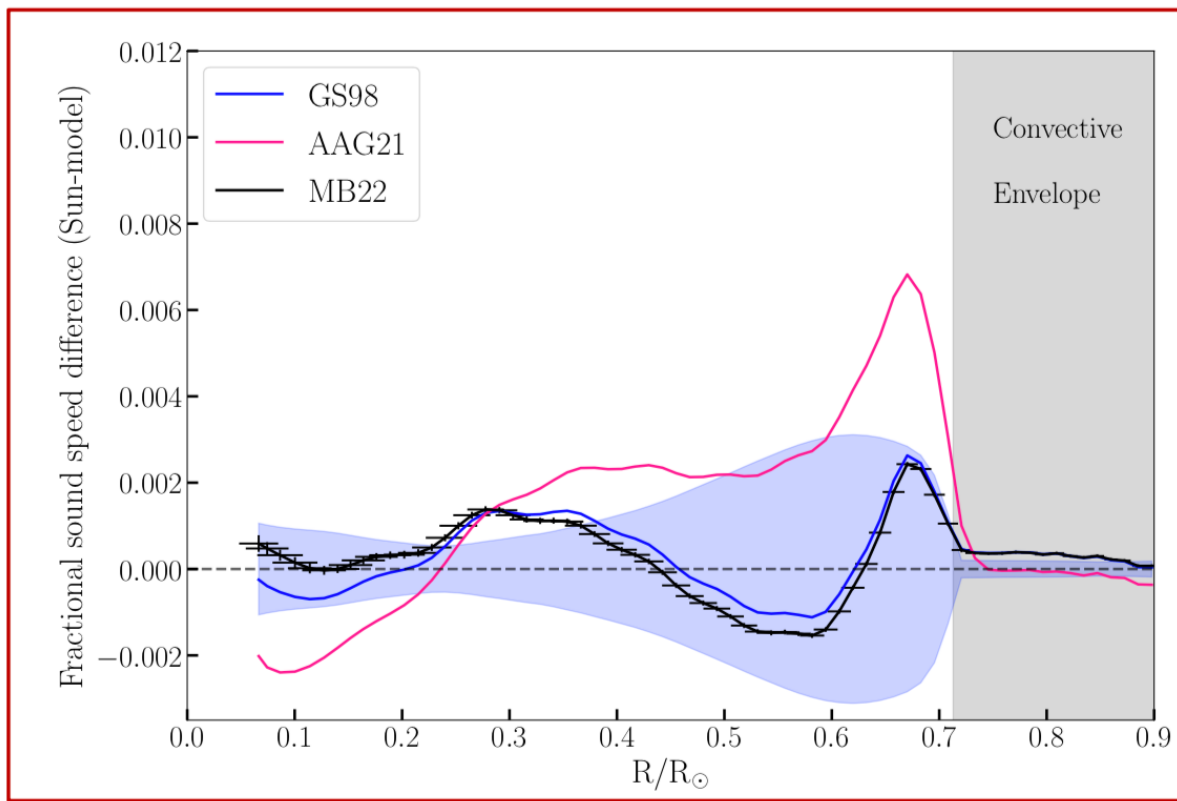


AAG21: Asplund, Amarsi & Grevesse 2021 – MB22: Magg, Bergemann et al. 2022

Helioseismic results

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Situation in 2022



Helioseismic determinations

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$$Y_b = 0.2485 \pm 0.0035$$


Magg et al. 2022


Solar composition “dichotomy” still persists but now based on 3D NLTE abundances

Can we conclude that LZ abundances are wrong?

The interpretation is complicated by the **opacity-composition degeneracy**.

$$\delta\kappa(r) = \delta\kappa_{\text{I}}(r) + \sum_j \frac{\partial \ln \kappa(r)}{\partial \ln Z_j} \delta z_j$$

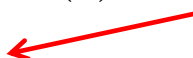
Intrinsic opacity change
(e.g. opacity table “errors”) 


 *Composition opacity change*

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Intrinsic opacity change
(e.g. opacity table “errors”) 

 *Composition opacity change*

Opacity uncertainty in SSMs is parameterized as:

$$\delta\kappa(T) = \kappa_a + (\kappa_b/\Delta) \ln(T/T_c)$$

κ_a, κ_b = random variables
(means equal to 0 and variances $\sigma_a = 0.02$ and $\sigma_b = 0.067$)

This prescription is motivated by:

- Opacity calculations more accurate at the solar core (~2%) than at the base of the convective envelope (~7%);
- It avoids underestimating the opacity error contribution to sound speed and convective radius (sensitive to tilt and not to scale of opacity)

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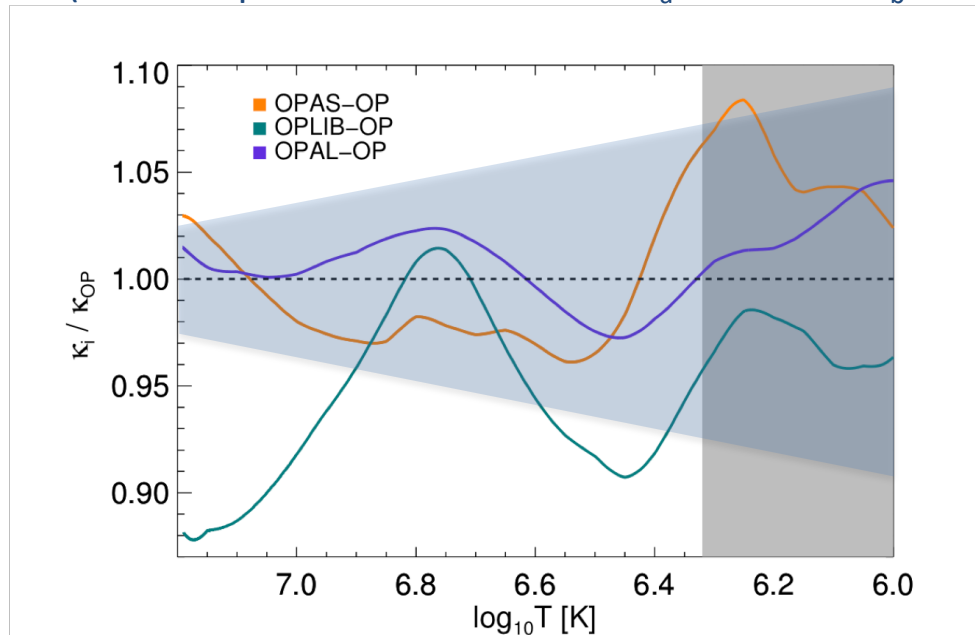
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... but **it still remains** a very simplified description of the real situation



Neutrinos

Hydrogen Burning: PP chain and CNO cycle

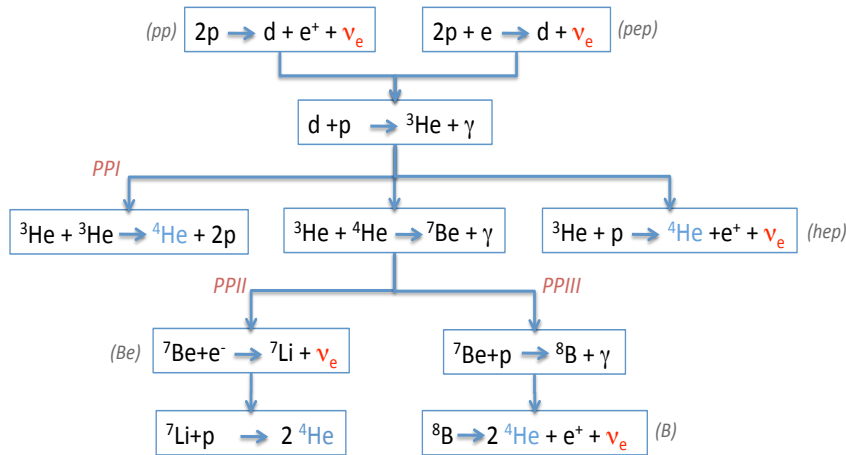
The Sun is powered by nuclear reactions that transform H into ${}^4\text{He}$:



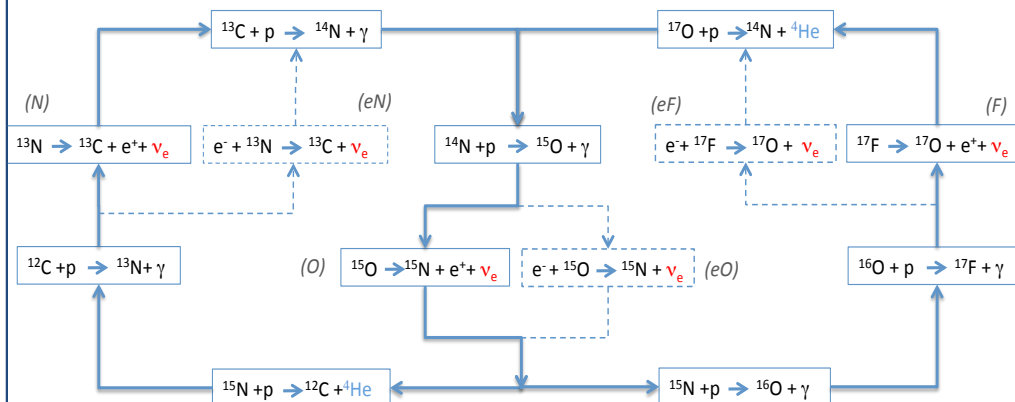
$Q = 26,7 \text{ MeV}$ (globally)

Free stream – 8 minutes to reach the earth
Direct information on the energy producing region.

The PP-chain



The CN-NO (bi-)cycle

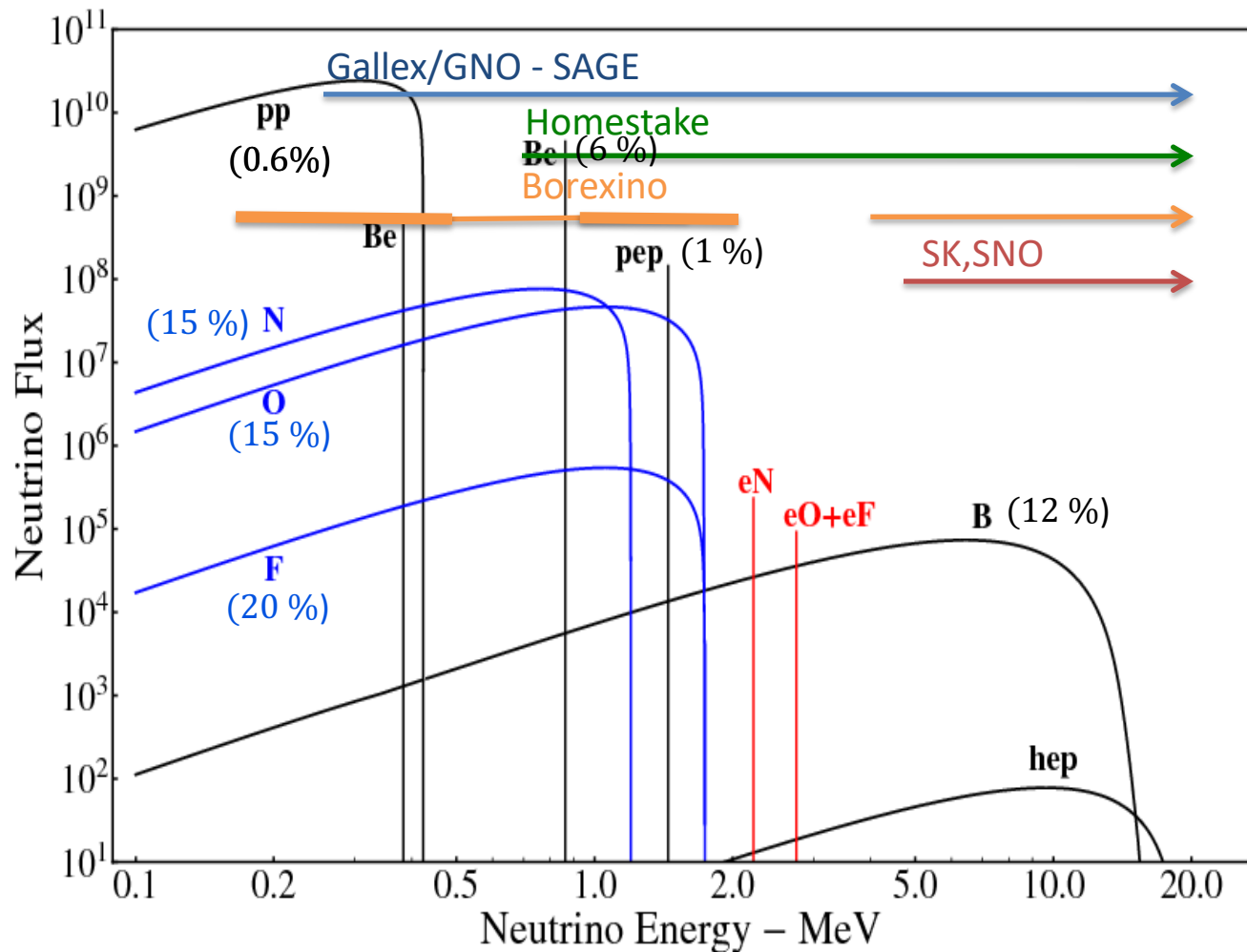


The **pp chain** is responsible for about 99% of the total energy (and neutrino) production.

C, N and O nuclei are used as catalysts for hydrogen fusion.

CNO (bi-)cycle is responsible for about 1% of the total neutrino (and energy) budget. Important for more advanced evolutionary stages

The solar neutrino spectrum



The different comp. of the solar neutrinos flux have been **directly** determined with accuracy level:

pp: ~ 10%
 pep: ~ 10%
 ^7Be : ~ 3 %
 ^8B : ~ 2 % CNO: ~ 30%

Recent Milestones from **Borexino** [Next talk, N. Rossi]:

- ^7Be (and ^8B) neutrino direct detection [PRL 2008]
- pp (and pep) neutrinos direct detection [Nature 2014, 2018]
- CNO neutrinos signal identification [Nature 2020]

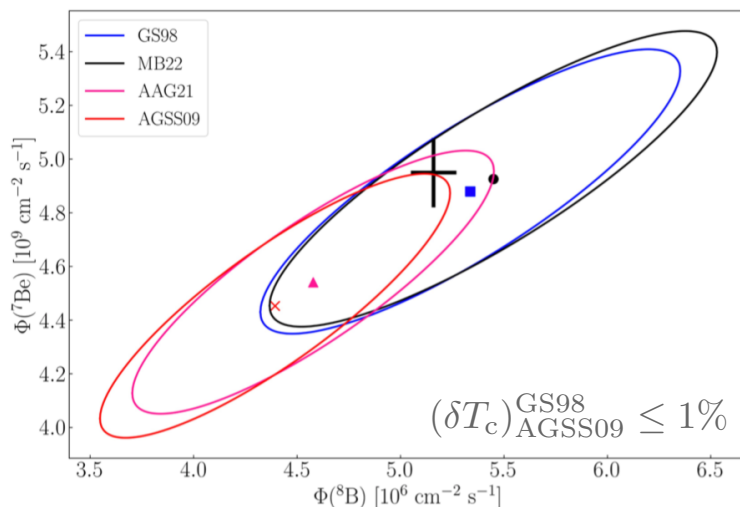
Neutrino fluxes

- Neutrino fluxes from the pp-chain (e.g. ${}^7\text{Be}$ and ${}^8\text{B}$ neutrinos) depend on the core temperature T_c and on the cross sections that control the branching of different pp-chain terminations

$$\begin{aligned}\delta\Phi({}^7\text{Be}) &= \delta S_{34} + \frac{1}{2}(\delta S_{11} - \delta S_{33}) + \beta_{\text{Be}} \delta T_c \\ \delta\Phi({}^8\text{B}) &= (\delta S_{17} - \delta S_{e7}) + \delta S_{34} + \frac{1}{2}(\delta S_{11} - \delta S_{33}) + \beta_{\text{B}} \delta T_c\end{aligned}\quad \left\{ \begin{array}{l} \beta_{\text{Be}} = \gamma_{34} + (\gamma_{11} - \gamma_{33})/2 \sim 11 \\ \beta_{\text{B}} = \beta_{\text{Be}} + \gamma_{17} + 1/2 \simeq 24 \end{array} \right.$$

N.B. The core temperature is an implicit function of surface composition and enviromental parameters (including opacity)

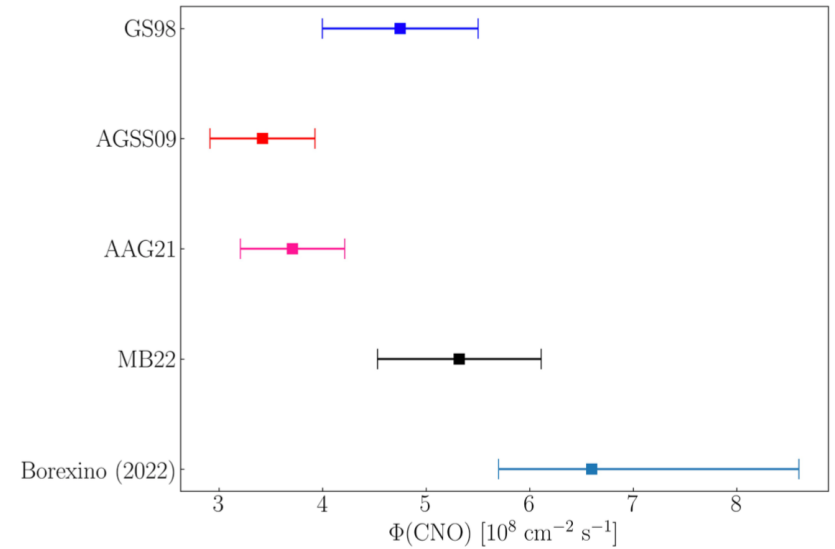
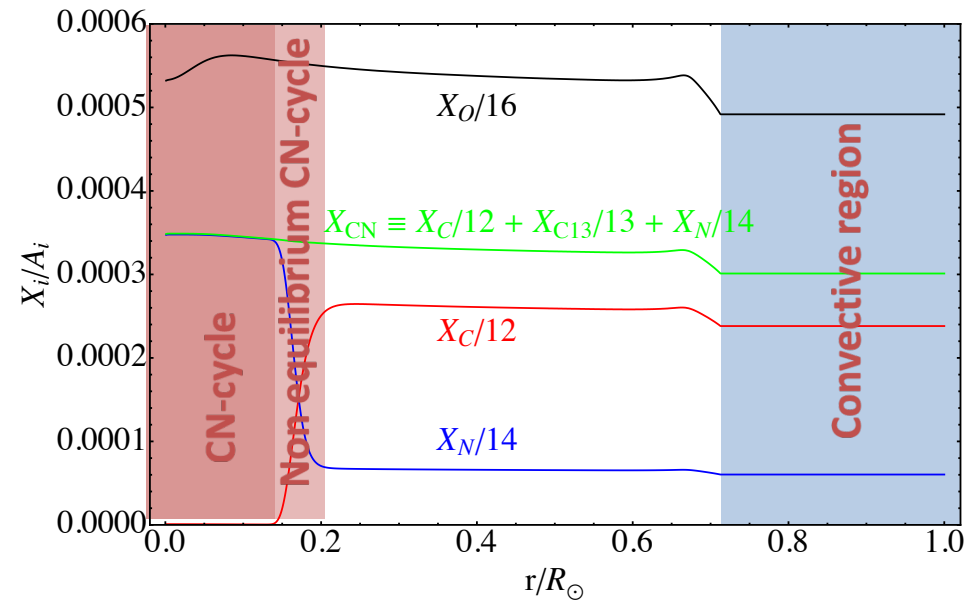
$$\delta T_c = f(\delta X_i, \delta(\text{opa}), \delta(\text{diffu}), \dots)$$



At the moment, **${}^7\text{Be}$ and ${}^8\text{B}$ neutrinos:**

- constrain the core temperature at < 1% level
- do not determine the core composition with suff. accuracy (degenerate with opacity)

CNO neutrino fluxes



- CNO neutrino fluxes also directly depend on the carbon+nitrogen in the core of the Sun (X_{CN})

Assuming equal C and N fractional variations
(i.e. $\delta X_{\text{N}}^{\text{core}} = \delta X_{\text{C}}^{\text{core}} \equiv \delta X_{\text{CN}}^{\text{core}}$):

$$\delta \Phi(^{15}\text{O}) = \delta X_{\text{CN}}^{\text{core}} + \beta_{\text{O}} \delta T_{\text{c}} + \delta S_{114}$$

$$\delta \Phi(^{13}\text{N}) = \delta X_{\text{CN}}^{\text{core}} + \beta_{\text{N}} \delta T_{\text{c}} + f \delta S_{114}$$

$$\beta_{\text{O}} = 20$$

$$\beta_{\text{N}} = f \beta_{\text{O}} = 15$$

$$f \simeq 0.7$$

Removing composition-opacity degeneracy

The combined measurement of pp-chain and CNO-cycle neutrinos can be used to directly **infer the solar core composition**. *Indeed:*

- The (strong) dependence on T_c (and opacity) can be eliminated by using **^8B -neutrinos as solar thermometer**;
- The additional dependence of CNO-neutrinos on X_{CN} can be used to infer core composition

In practical terms, one can form a weighted ratio of e.g. ^8B and ^{15}O neutrino fluxes that is:

- Essentially independent on environmental parameters (including opacity);
- **Directly proportional to Carbon+Nitrogen abundance in the solar core**

Serenelli et al., PRD 2013

See also (application to BX obs. rate):

Agostini et al, EPJ 2021

Villante & Serenelli, Frontiers 2021

$$\delta\Phi(^{15}\text{O}) - x \delta\Phi(^8\text{B}) \simeq \delta X_{\text{CN}}^{\text{core}} + \delta S_{114} - x \left(\delta S_{17} - \delta S_{e7} + \delta S_{34} + \frac{\delta S_{11}}{2} - \frac{\delta S_{33}}{2} \right)$$

$$x = \frac{\beta_{\text{O}}}{\beta_{\text{B}}} \sim 0.8$$

Probing solar composition with neutrinos

By considering

$$\frac{R_{\text{CNO}}^{\text{Bx}}}{R_{\text{CNO}}^{\text{SSM}}} = \frac{R_{15\text{O}}^{\text{Bx}}}{R_{15\text{O}}^{\text{SSM}}} = \frac{\Phi_{15\text{O}}^{\text{Bx}}}{\Phi_{15\text{O}}^{\text{SSM}}} = 1.35^{+0.30}_{-0.14}$$

Borexino CNO neutrino signal
(scaled to GS98 prediction)

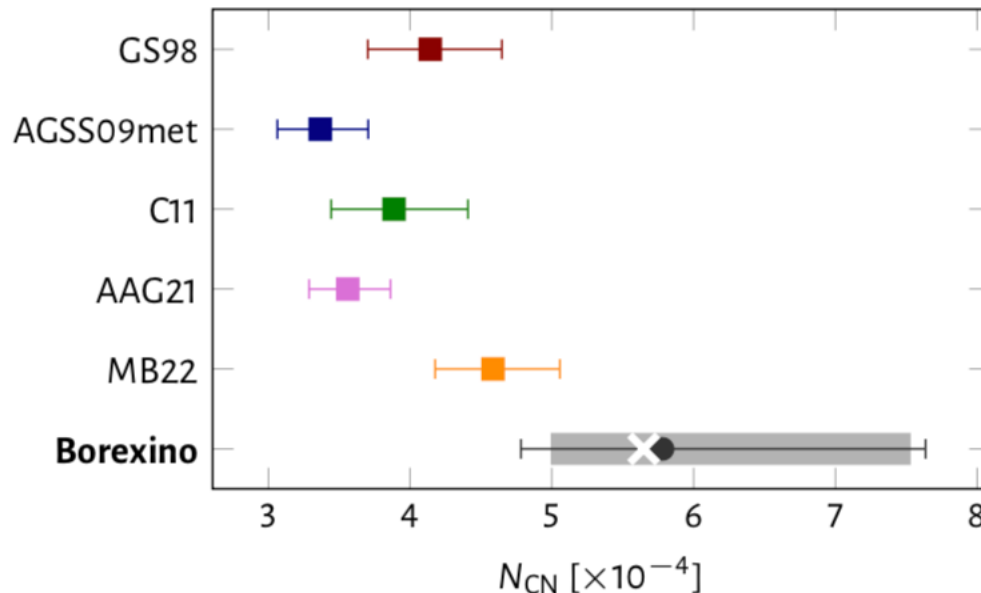
$$\frac{\Phi_{8\text{B}}^{\text{global}}}{\Phi_{8\text{B}}^{\text{SSM}}} = 0.96 \pm 0.027$$

^8B flux determined from global analysis
(scaled to GS98 prediction)

One obtains:

$$\frac{(N_{\text{C}} + N_{\text{N}})/N_{\text{H}}}{[(N_{\text{C}} + N_{\text{N}})/N_{\text{H}}]^{\text{SSM}}} = 1.35 \times (0.96)^{-0.769} \times$$

$$\times \left[1 \pm \left({}^{+0.303}_{-0.136}(\text{CNO}) \pm 0.097(\text{nucl}) \pm 0.023(^8\text{B}) \pm 0.005(\text{env}) \pm 0.027(\text{diff}) \pm 0.022(\text{O/N}) \right) \right]$$



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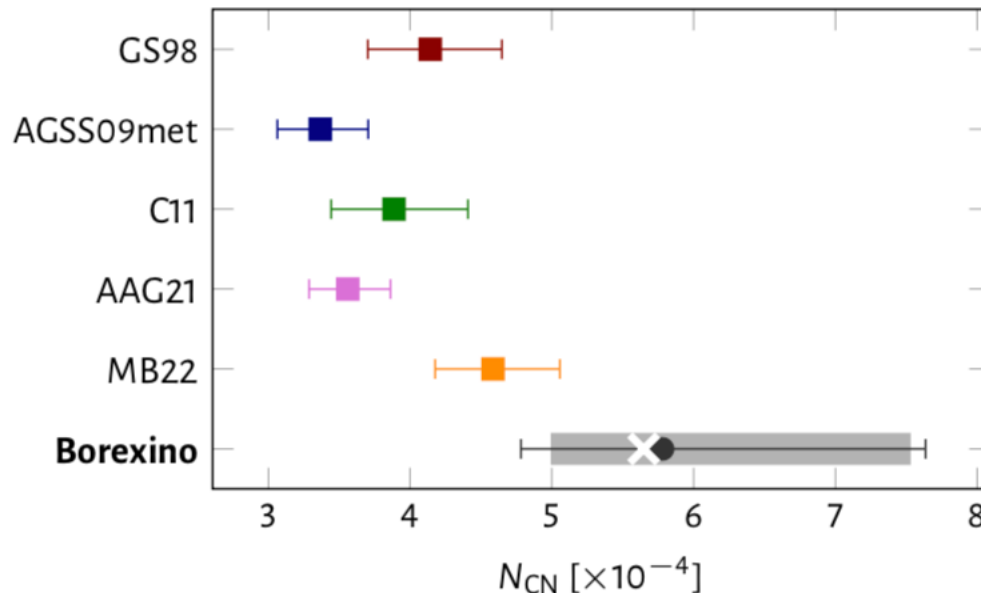
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N.B.

This determination is robust wrt to environmental parameters variations (including opacity).

Only limited by nuclear reaction uncertainties:

$$S_{114} \rightarrow 7.6 \%$$

$$S_{17} \rightarrow 3.5 \%$$

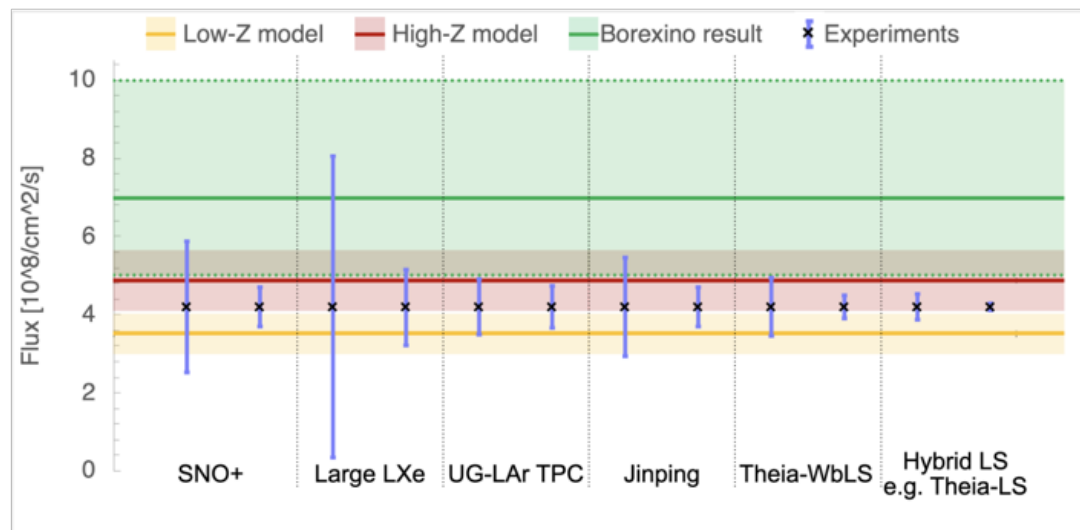
$$S_{34} \rightarrow 3.4 \%$$

Future perspectives

Borexino has opened the way to CNO neutrino detection

Improvements on the experimental side will be provided in the future by planned detectors, e.g.:

- SNO+
- JUNO
- Jinping
- Hyper-Kamiokande
- THEIA
- DUNE
- Dark Matter experiments
-



ARNP – Orebi Gann et al. in press

Note that: some minor components (hep and ecCNO) of the solar neutrino flux are still undetected

- **ecCNO neutrinos:** A challenge for gigantic ultra-pure LS detectors (Villante, PLB 2015)
Expt. requirements: *as clean (and deep) as Borexino; as large as JUNO*

Conclusions

- Solar neutrino physics entered the precision era.
- Borexino has opened the way to CNO neutrino detection
- Some unsolved puzzles could be addressed → (Present and future) CNO neutrino measurements, combined with precise determinations of ^8B and ^7Be fluxes, can shed light on the [solar abundance problem](#)
- To exploit the full potential of future measurements → improvements in the SSM constitutive physics are needed [[nuclear cross sections and radiative opacities](#)]

[Solar Fusion Cross Sections III \(INT-22-82W\)](#)

July 2022, UC Berkeley, Berkeley, CA, USA

Thank you

Standard Solar Models

Stellar structure equations are solved, starting from a ZAMS model to present solar age (we neglect rotation, magnetic fields, etc.):

$$\begin{aligned}\frac{\partial m}{\partial r} &= 4\pi r^2 \rho \\ \frac{\partial P}{\partial r} &= -\frac{G_N m}{r^2} \rho \\ P &= P(\rho, T, X_i) \\ \frac{\partial l}{\partial r} &= 4\pi r^2 \rho \epsilon(\rho, T, X_i) \\ \frac{\partial T}{\partial r} &= -\frac{G_N m T \rho}{r^2 P} \nabla\end{aligned}\quad \nabla = \text{Min}(\nabla_{\text{rad}}, \nabla_{\text{ad}}) \rightarrow \begin{cases} \nabla_{\text{rad}} = \frac{3}{16\pi ac G_N} \frac{\kappa(\rho, T, X_i) l P}{m T^4} \\ \nabla_{\text{ad}} = (d \ln T / d \ln P)_s \simeq 0.4 \end{cases}$$

Chemical evolution driven by nuclear reaction, diffusion and gravitational settling, convection

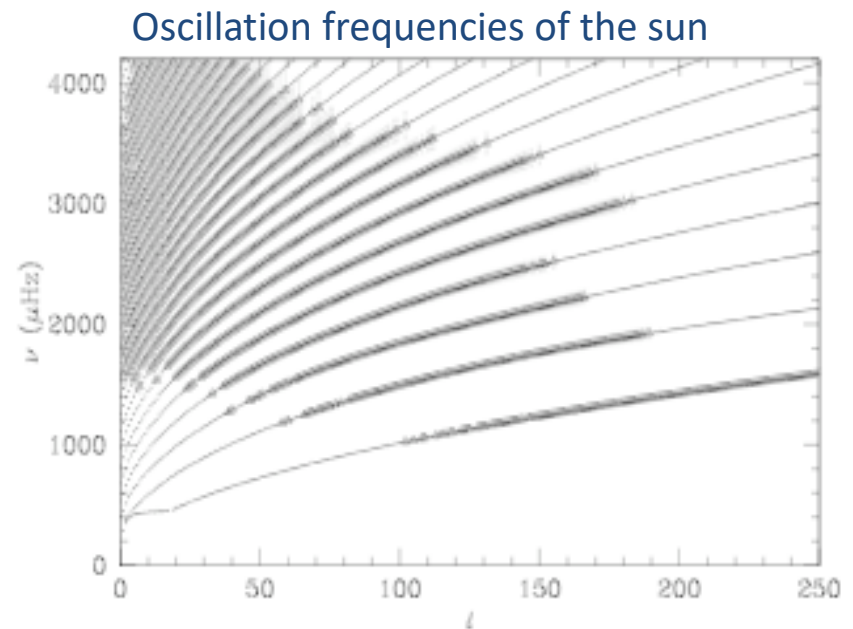
Standard input physics for equation of states, nuclear reaction rates, opacity, etc.

Free-parameters (**mixing length**, Y_{ini} , Z_{ini}) adjusted to match the observed properties of the Sun (**radius**, **luminosity**, Z/X).

Note that equations are non-linear \rightarrow Iterative method to determine mixing length, Y_{ini} , Z_{ini}

The solar abundance problem

The **downward revision** of heavy elements photospheric abundances leads to SSMs which **do not correctly reproduce helioseismic observables**



The Sun is a non radial oscillator. The observed oscillation frequencies can be used to determine the properties of the Sun. Linearizing around a known solar model:

$$\frac{\delta\nu_{nl}}{\nu_{nl}} = \int_0^R dr K_{u,Y}^{nl}(r) \frac{\delta u}{u}(r) + \int_0^R dr K_{Y,u}^{nl}(r) \delta Y + \frac{F(\nu_{nl})}{\nu_{nl}}$$

squared isothermal sound speed

Related to temperature stratification in the sun

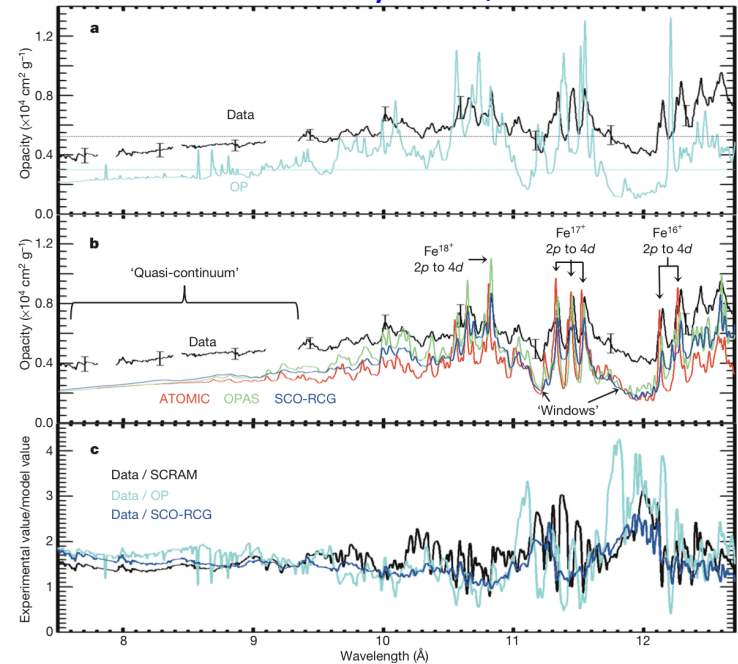
surface helium abundance

See Basu & Antia 07
for a review

Wrong opacity?

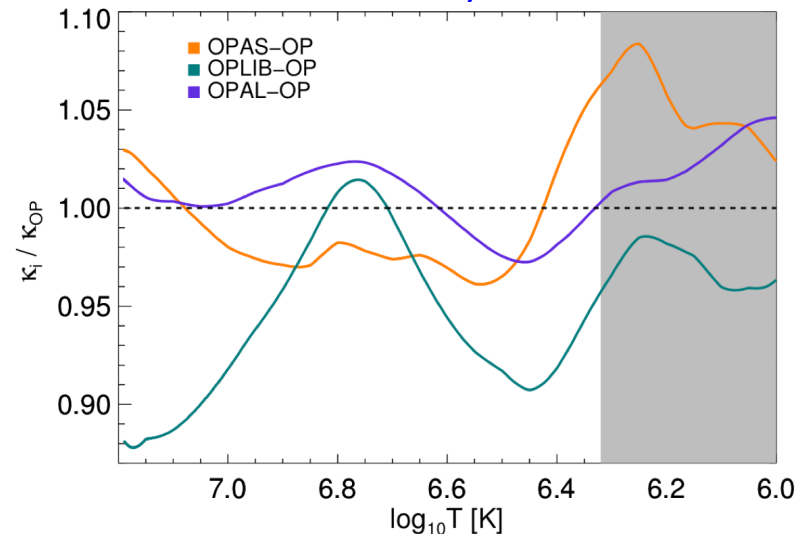
- Opacity is being measured at stellar interiors conditions (Bailey et al., Nature 2015);
- Monochromatic opacity is higher than expected for iron (up to a factor 2);
- Total opacity (integrated over the wavelength and summed over the composition) is increased by about 7%

Bailey et al., Nature 2015



- Different opacity tables may differ “locally” by a large amount (up to 10%) and with a complicated pattern

Vinyoles et al., 2017



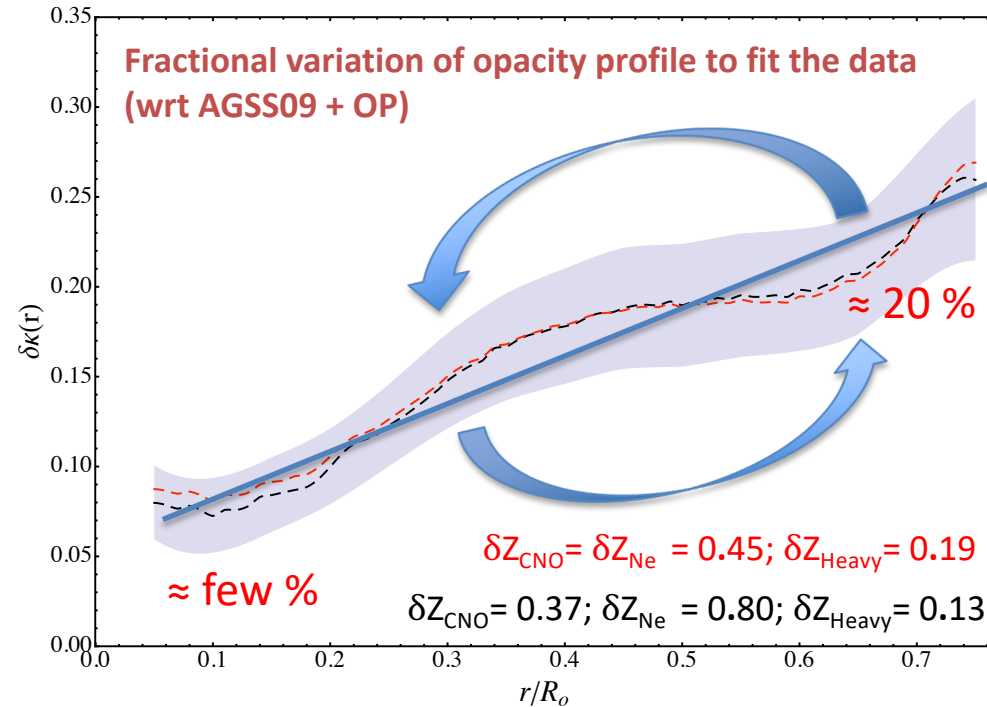
The solar opacity profile

F.L. Villante and B. Ricci - *Astrophys.J.* 714:944-959,2010
 F.L. Villante – *Astrophys.J.* 724:98-110,2010
 F.L. Villante, A. Serenelli et al., *Astrophys.J.* 787 (2014) 13

The “**optimal**” opacity profile of the Sun can be determined from obs. data

Note that:

- The sound speed and the convective radius determine **the tilt** of $\delta\kappa(r)$ (but not **the scale**)
- The surface helium and the neutrino fluxes determine **the scale** for $\delta\kappa(r)$



The interpretation is however complicated by the **opacity-composition degeneracy**. Which fraction of the required $\delta\kappa(r)$ has to be ascribed to **intrinsic** ($\delta\kappa_I(r)$) and/or **composition** opacity changes?

$$\delta\kappa(r) = \delta\kappa_I(r) + \sum_j \frac{\partial \ln \kappa(r)}{\partial \ln Z_j} \delta z_j$$

Opacity table “errors”
 Non standard effects (WIMPs in solar core)
 ...

different admixtures $\{\delta z_j\}$ can do equally well the job

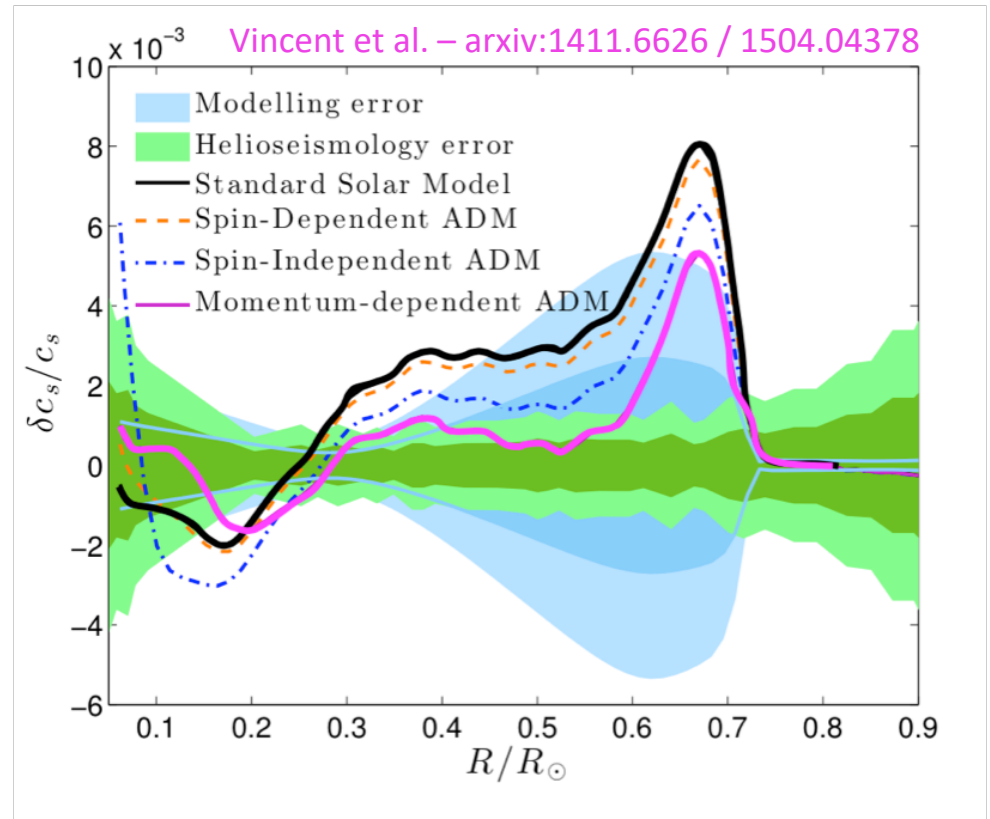
Asymmetric DM

DM accumulation in the solar core:

- Additional energy transport;
- **Reduction** of the “effective opacity”;
- Modification of temperature profile;

Agreement with helioseismic data can be improved. However:

- DM accumulation do not provide the optimal opacity profile;
- Potential tension with neutrino fluxes and surface helium;
- **Caveat:** DM evaporation not accounted for (relevant for few GeV masses)



$$\sigma = \sigma_0 \left(\frac{q}{q_0} \right)^2 \quad \begin{cases} m_\chi &= 3 \text{ GeV} \\ \sigma_0 &= 10^{-37} \text{ cm}^2 \\ q_0 &= 40 \text{ MeV} \end{cases}$$

Wrong chemical evolution?

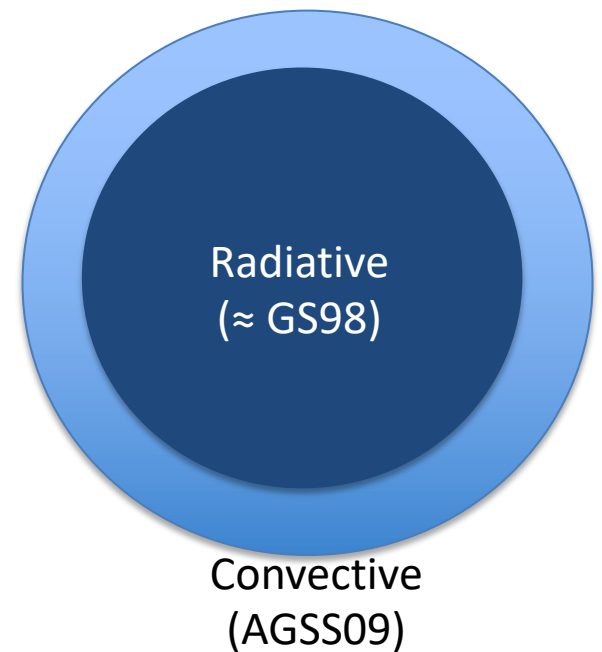
Helioseismic observables and neutrino fluxes are sensitive to **the metallicity of the radiative interior of the Sun.**

The observations determine **the chemical composition of the convective envelope** (2-3% of the solar mass).

Difference between AGSS09 and GS98 correspond to $\approx 40M_{\oplus}$ of metal, when integrated over the Sun's convective zone.

Could this difference be accounted in non standard chemical evolution scenarios (e.g. by accretion of material with non standard composition)?

See A. Serenelli et al. – ApJ 2011

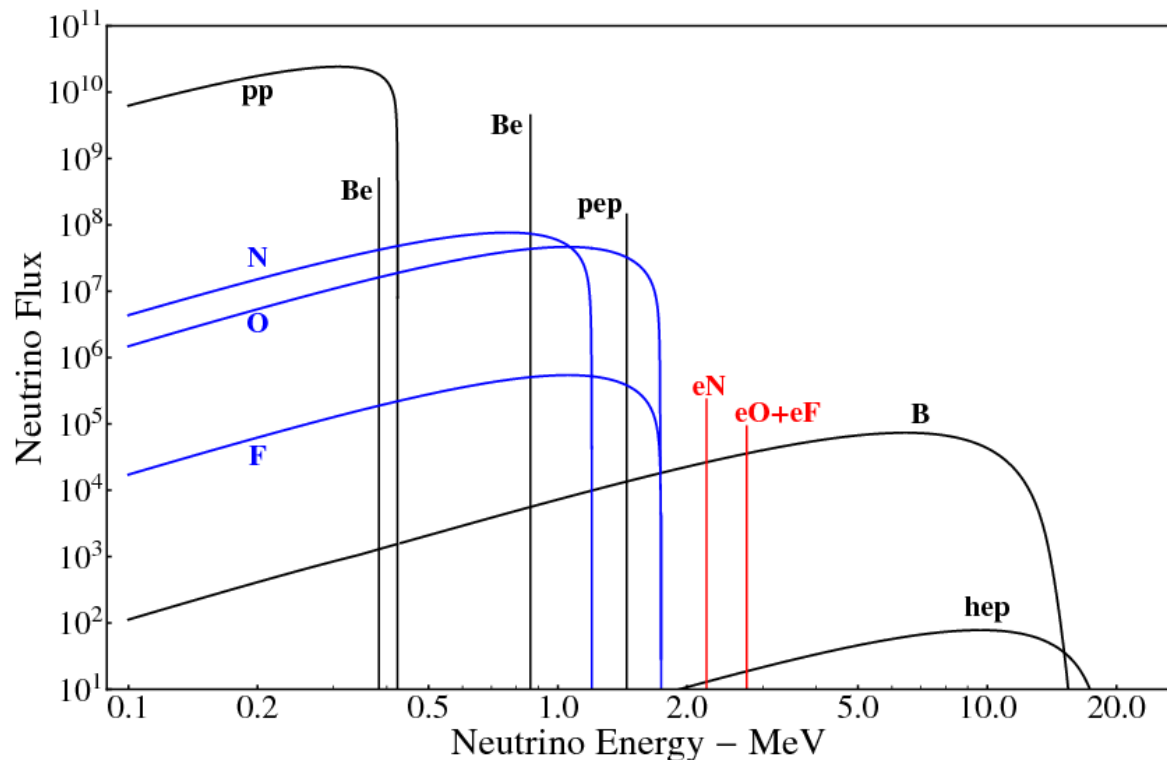


This is a well posed and extremely important question but ...

... no satisfactory solutions have been proposed up to now, in my opinion

ecCNO neutrinos

In the CN-NO cycle, besides the conventional CNO neutrinos (blue lines), **monochromatic ecCNO neutrinos (red lines)** are also produced by **electron capture** reactions:



F.L. Villante, PLB 742 (2015) 279-284

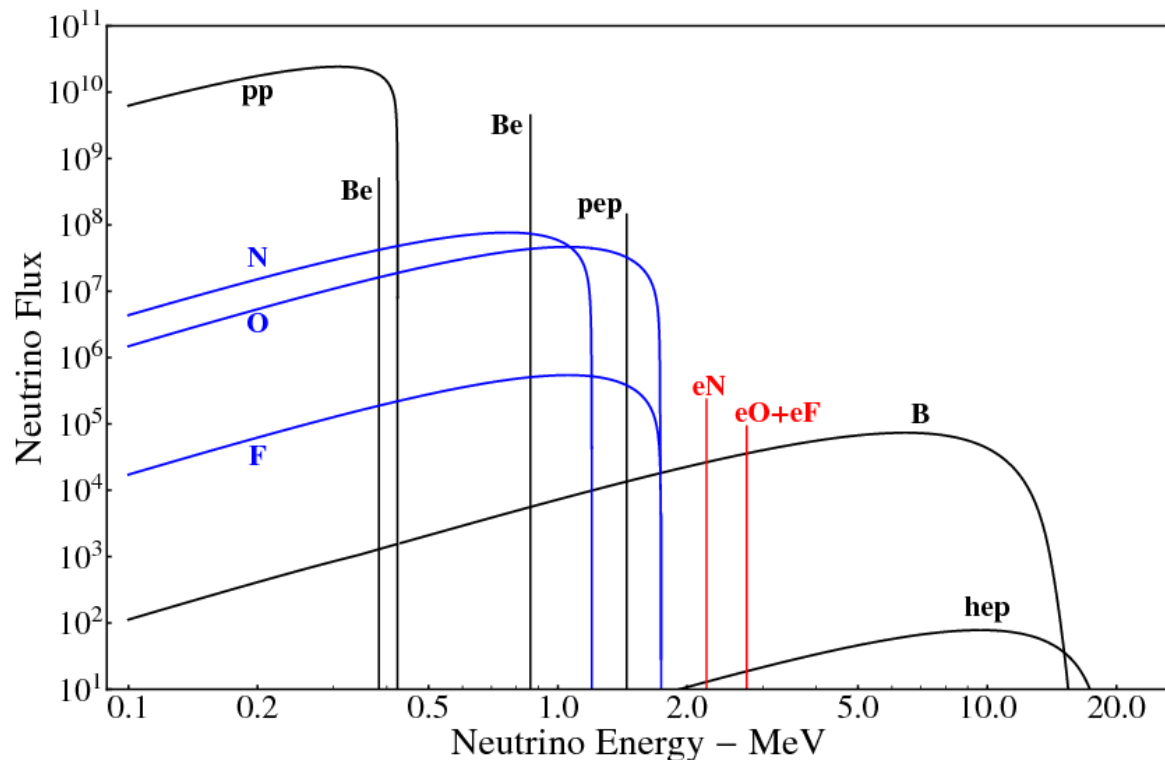
L.C. Stonehill et al, PRC 69, 015801 (2004)

J.N. Bahcall, PRD 41, 2964 (1990).

ecCNO neutrinos

The ecCNO fluxes are extremely low: $\Phi_{\text{ecCNO}} \approx (1/20) \Phi_{\text{B}}$. Detection is extremely difficult but could be rewarding. Indeed:

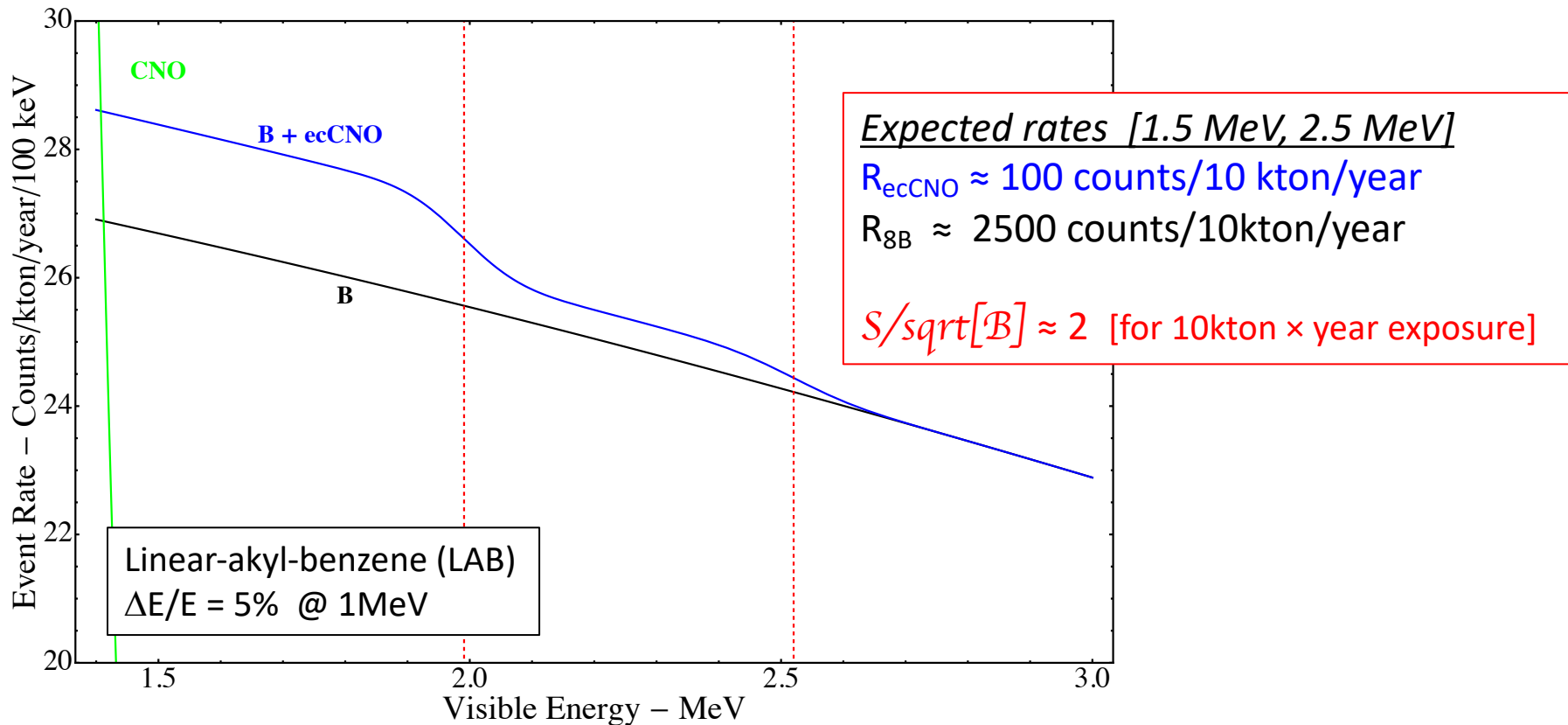
- ecCNO neutrinos are sensitive to the **metallic content of the solar core** (same infos as CNO neutrinos);
- Being monochromatic, they probe the solar neutrino **survival probability** at specific energies ($E_{\nu} \approx 2.5 \text{ MeV}$) exactly **in the transition region**.



*F.L. Villante, PLB 742 (2015) 279-284
L.C. Stonehill et al, PRC 69, 015801 (2004)
J.N. Bahcall, PRD 41, 2964 (1990).*

Expected rates in Liquid Scintillators

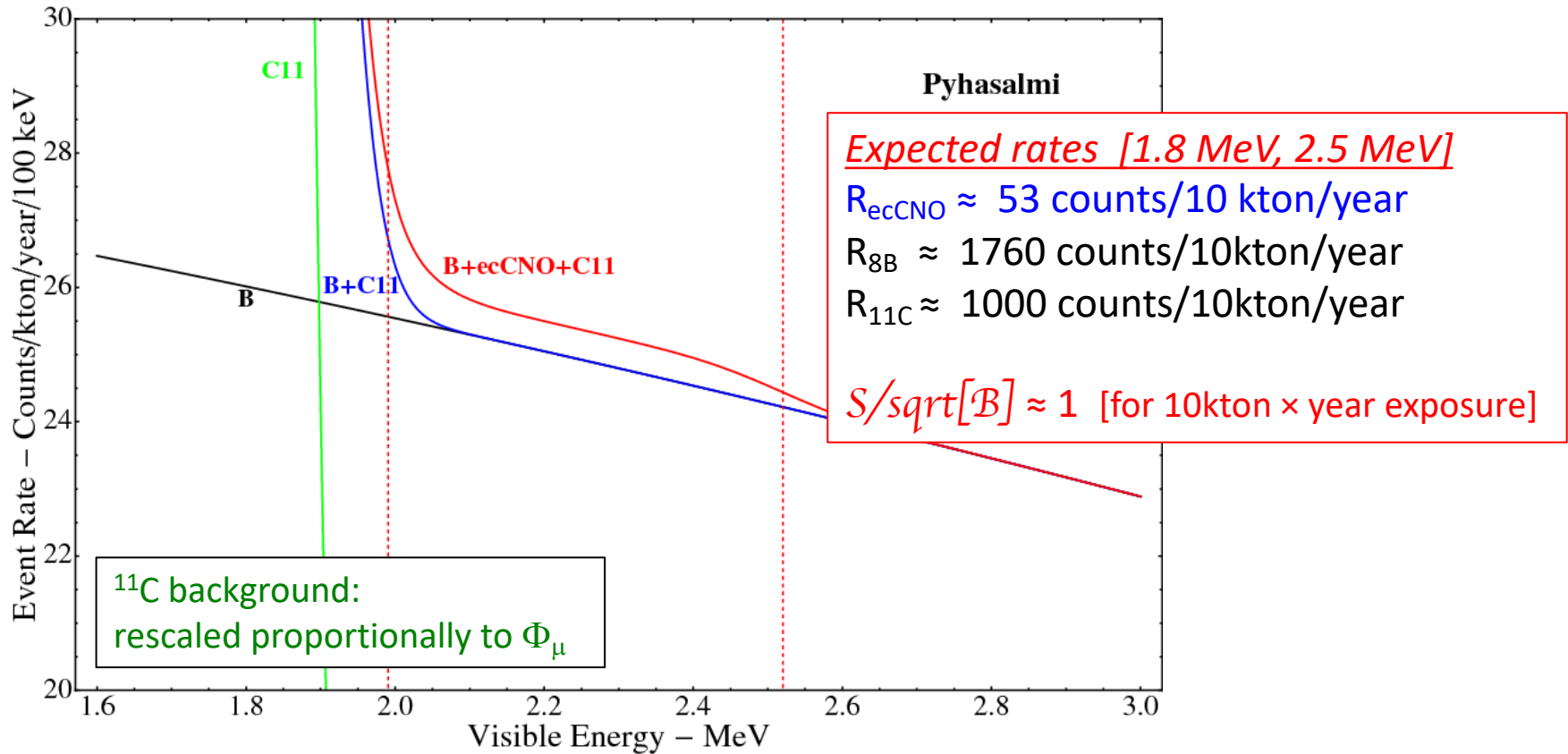
- ν – e elastic scattering of ecCNO neutrinos produces Compton shoulders (smeared by energy resolution) at 2.0 and 2.5 MeV;
- ecCNO neutrino signal has to be extracted statistically from the (irreducible) ^8B neutrino background.



Expected rates in Liquid Scintillators

Additional background sources:

- **Intrinsic:** negligible/tagged (with Borexino Phase-I radio-purity levels);
- **External:** reduced by self-shielding (Fid. mass reduced from 50 to ≈ 20 kton in LENA);
- **Cosmogenic:** ^{11}C overlap with the observation window.



Signal comparable to stat. fluctuations for exposures 10 kton \times year or larger.

100 counts / year above 1.8 MeV in 20 kton detector $\rightarrow 3\sigma$ detection in 5 year in LENA

Removing composition-opacity degeneracy

The combined measurement of pp-chain and CNO-cycle neutrinos can be used to directly **infer the solar core composition**. *Indeed:*

- The (strong) dependence on T_c (and opacity) can be eliminated by using **^8B -neutrinos as solar thermometer**;
- The additional dependence of CNO-neutrinos on X_{CN} can be used to infer core composition

In practical terms, one can form a weighted ratio of e.g. ^8B and ^{15}O neutrino fluxes that is:

- Essentially independent on environmental parameters (including opacity);
- **Directly proportional to Carbon+Nitrogen abundance in the solar core**

Serenelli et al., PRD 2013

See also (application to BX obs. rate):

Agostini et al, EPJ 2021

Villante & Serenelli, Frontiers 2021

$$\begin{aligned}
 \varphi_{^{15}\text{O}} / \varphi_{^8\text{B}}^{0.769} &= X_{\text{C}}^{0.802} X_{\text{N}}^{0.204} X_{\text{D}}^{0.181} \\
 &\times \left[X_{\text{S}_{11}}^{-0.866} X_{\text{S}_{33}}^{0.345} X_{\text{S}_{34}}^{-0.689} X_{\text{S}_{e7}}^{0.769} X_{\text{S}_{17}}^{-0.791} X_{\text{S}_{\text{hep}}}^{0.000} X_{\text{S}_{114}}^{1.046} X_{\text{S}_{116}}^{0.001} \right] \quad (\text{nucl}) \\
 &\times \left[X_{\text{Age}}^{0.313} X_{\text{L}_{\odot}}^{0.602} X_{\kappa_a}^{0.018} X_{\kappa_b}^{-0.050} \right] \quad (\text{solar}) \\
 &\times \left[X_{\text{O}}^{0.006} X_{\text{Ne}}^{-0.003} X_{\text{Mg}}^{-0.003} X_{\text{Si}}^{0.001} X_{\text{S}}^{0.001} X_{\text{Ar}}^{0.001} X_{\text{Fe}}^{0.005} \right] \quad (\text{met})
 \end{aligned}$$

Probing solar composition with neutrinos

$$\frac{(N_C + N_N)/N_H}{[(N_C + N_N)/N_H]^{SSM}} = 1.35 \times (0.96)^{-0.769} \times$$

$$\times \left[1 \pm \left({}^{+0.303}_{-0.136}(\text{CNO}) \pm 0.097(\text{nucl}) \pm 0.023(^8\text{B}) \pm 0.005(\text{env}) \pm 0.027(\text{diff}) \pm 0.022(\text{O/N}) \right) \right]$$

Error contributions

