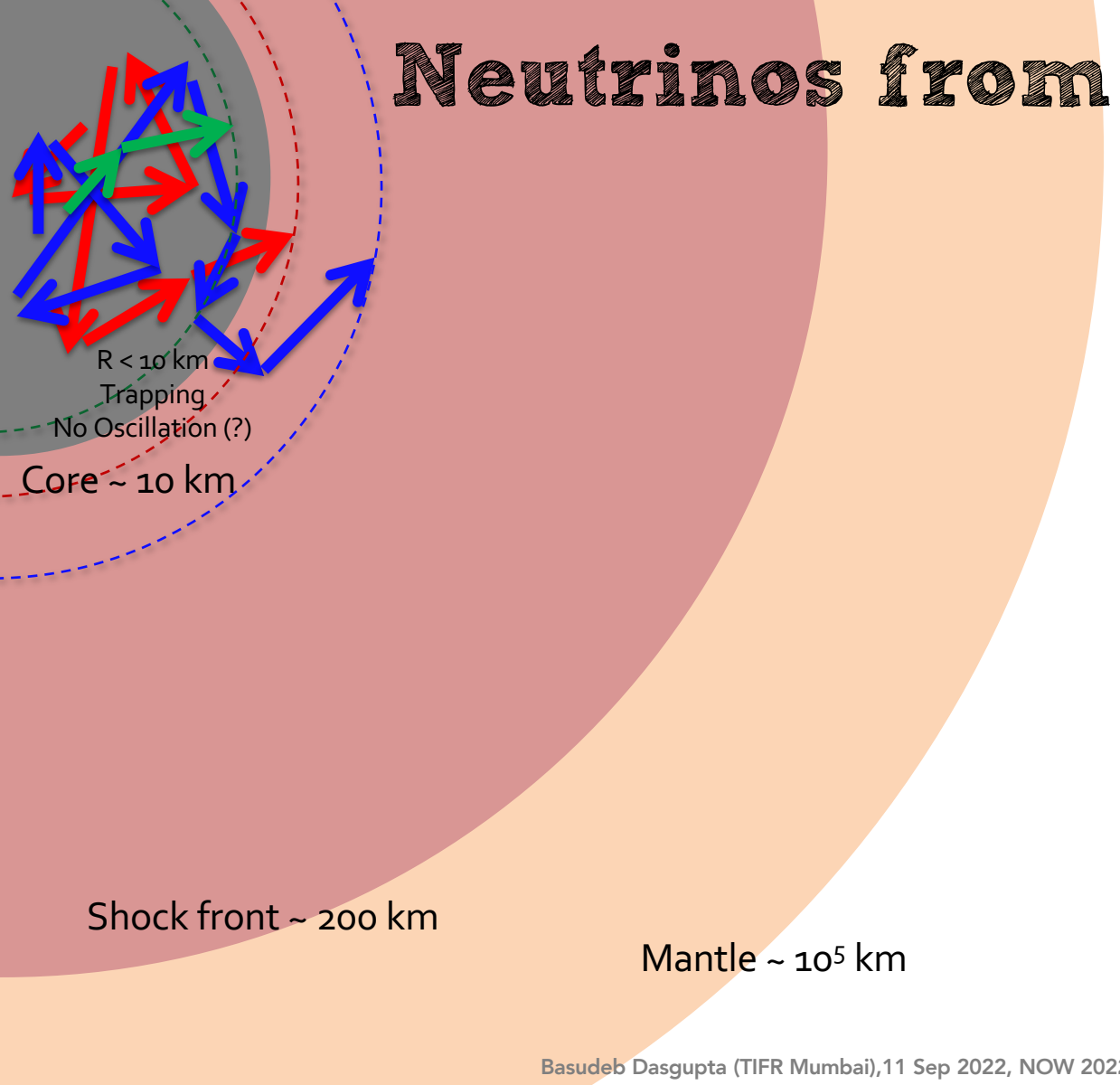
NOW 2022, Rosa Marina Ostuni, 11 Sep 2022

Supernovae and Neutrino Oscillations

www.bdasgupta.com

Basudeb Dasgupta (TIFR Mumbai)

Neutrinos from a SN



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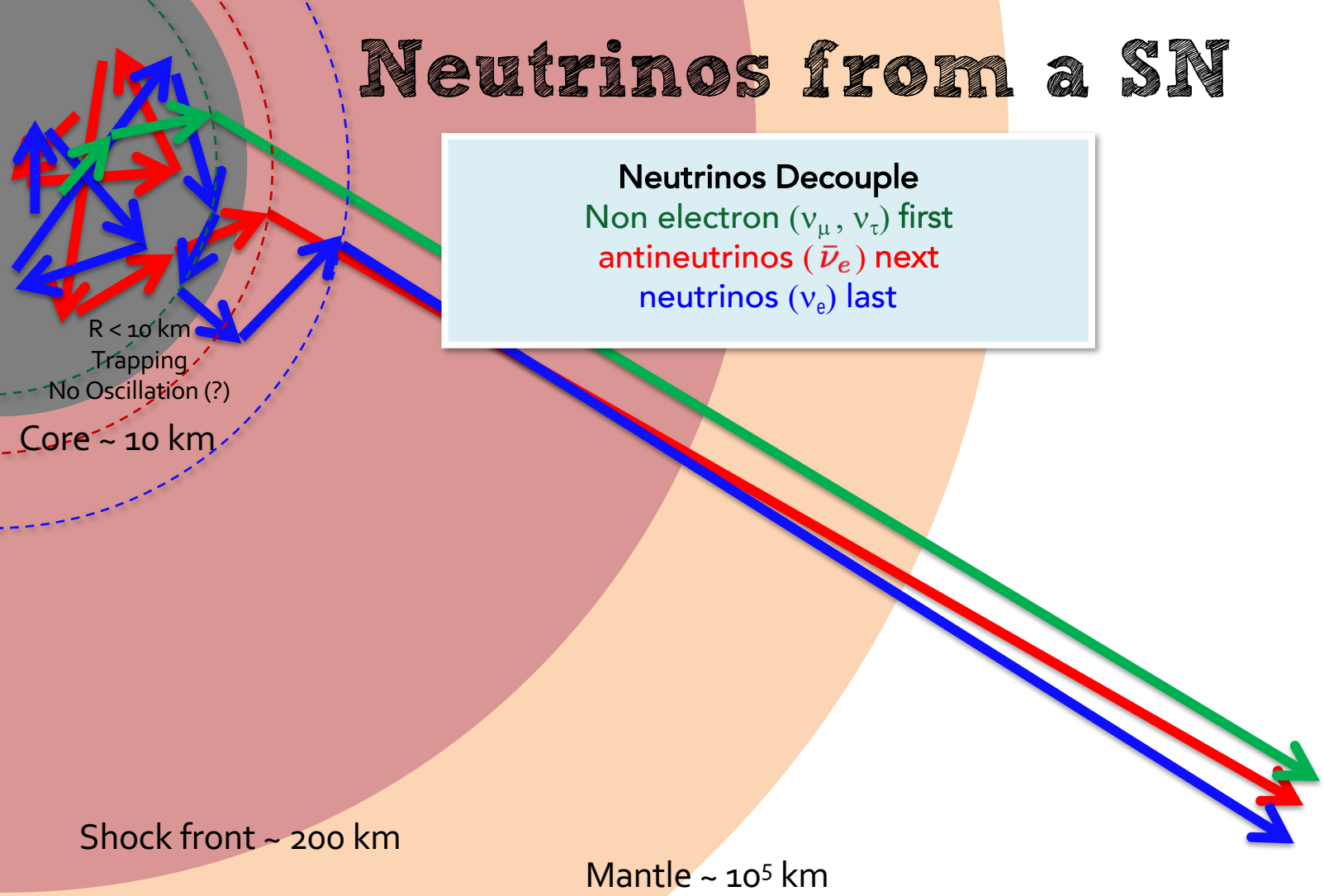


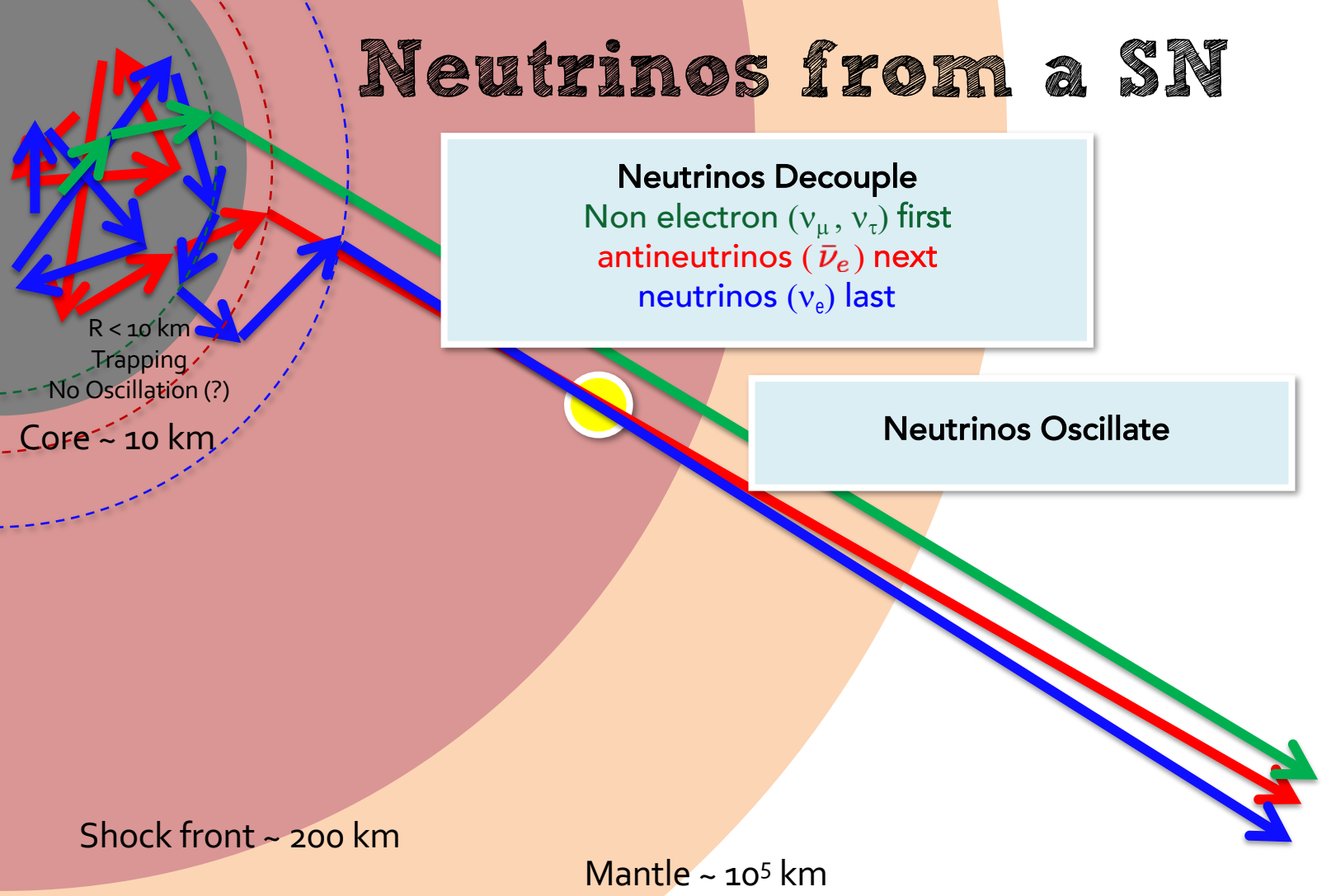
Neutrinos Decouple

Non electron (ν_μ , ν_τ) first

antineutrinos ($\bar{\nu}_e$) next

neutrinos (ν_e) last

Neutrinos from a SN



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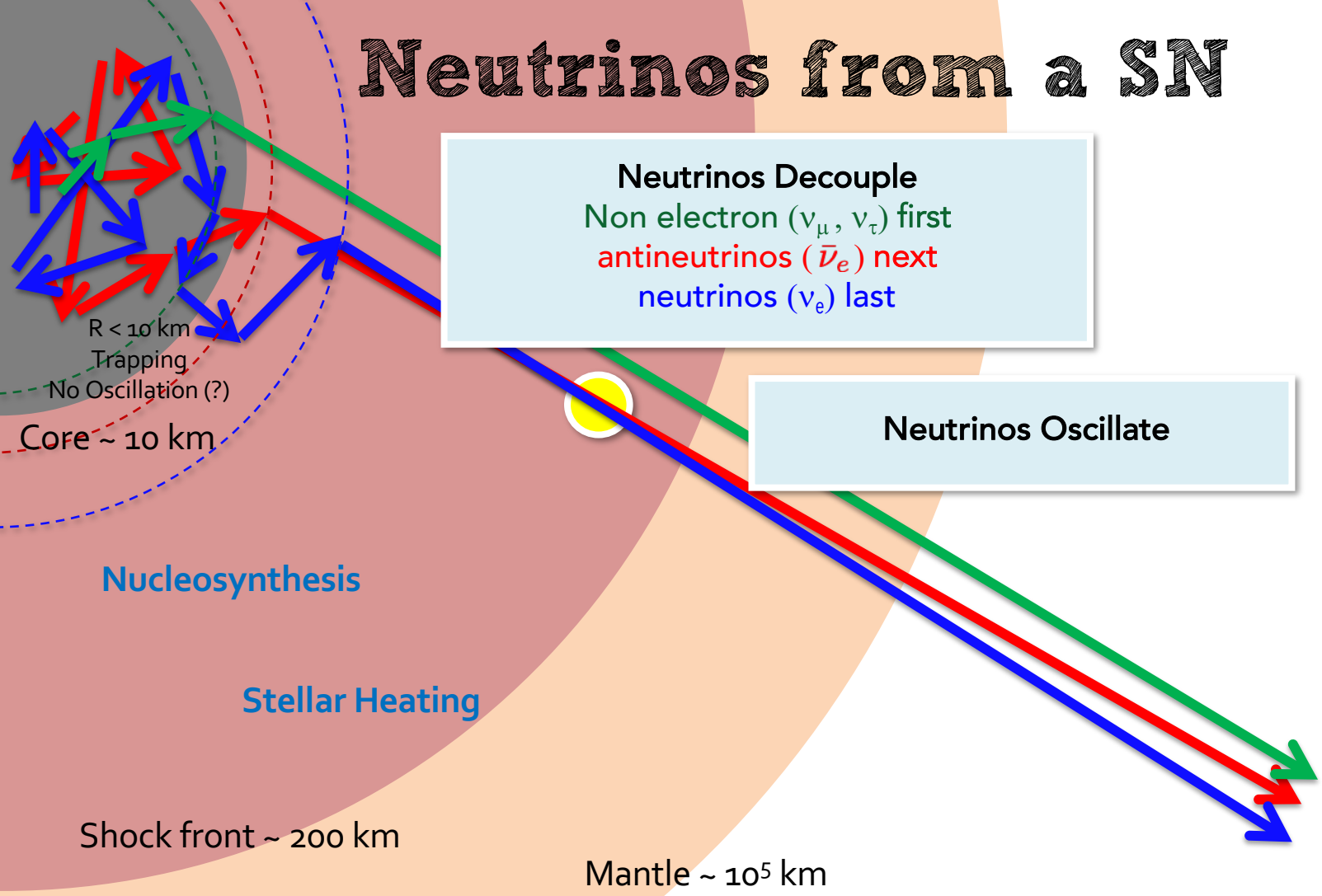

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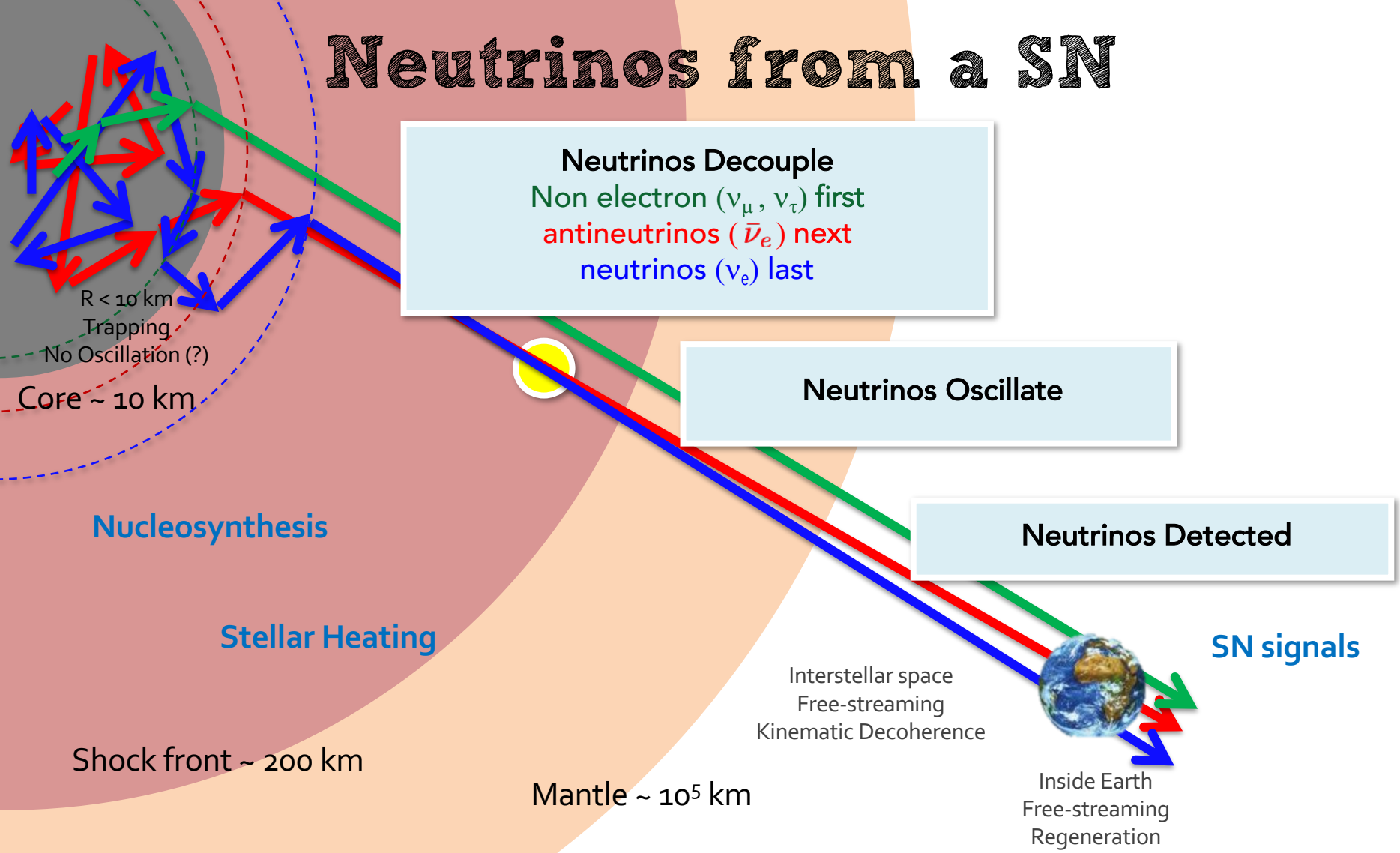

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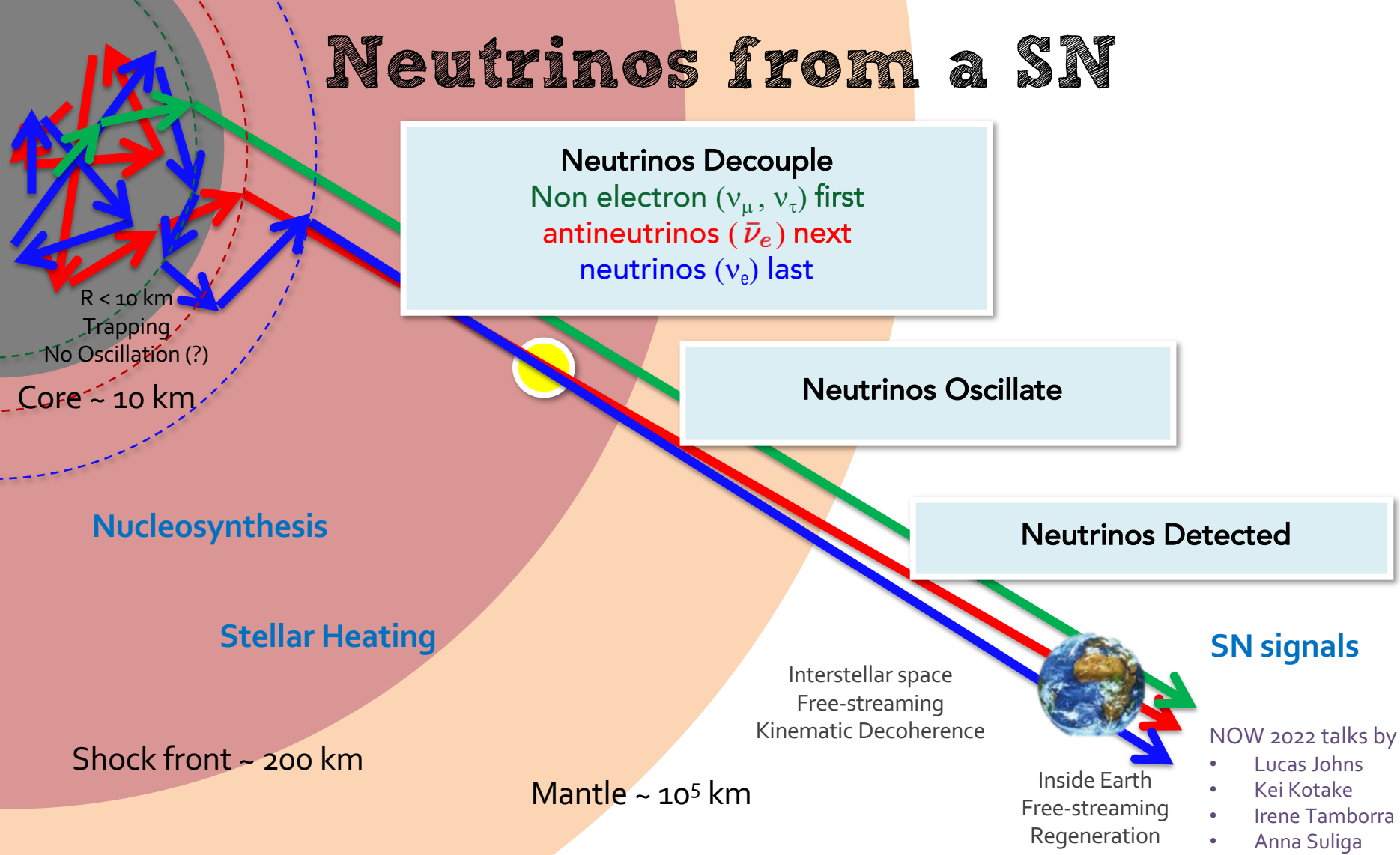
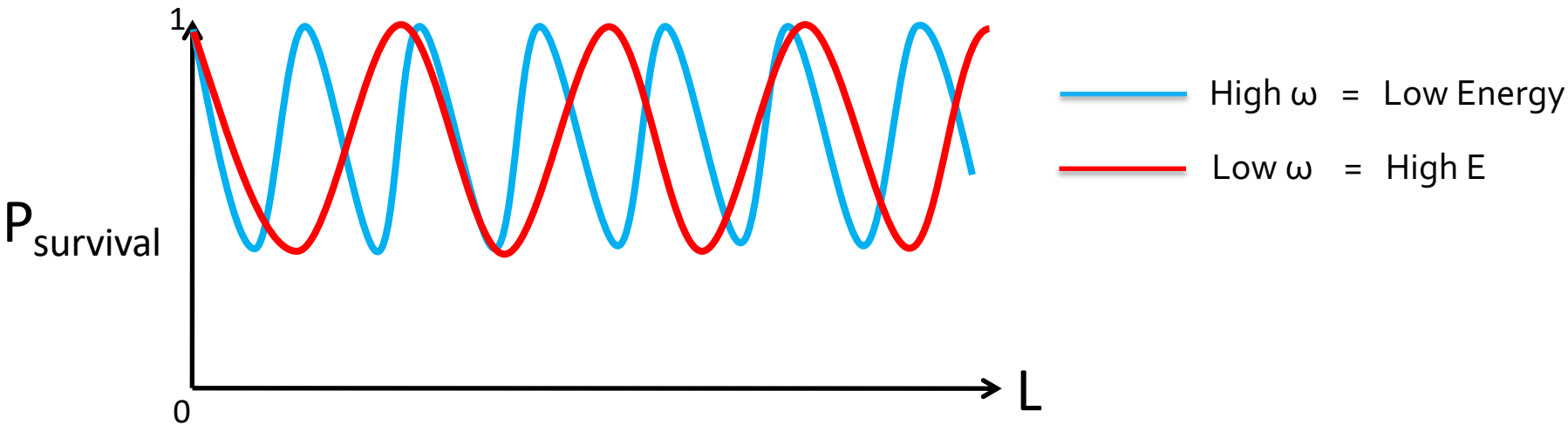


NOW 2022 talks by

- Lucas Johns
- Kei Kotake
- Irene Tamborra
- Anna Suliga
- Manibrata Sen

Neutrino Oscillations



$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \frac{\Delta m^2}{4E}\begin{pmatrix}+\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

Neutrinos change their flavor at a rate inversely proportional to their energy
The amplitude and rate of oscillation is affected by the medium

Neutrino Oscillations

$$P_{\rm surv} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} t\right)$$

At the beginning, for small θ

$$P_{\rm surv} = 1 - \theta^2 \left(\frac{\Delta m^2}{2E} t\right)^2$$

At late times

$$P_{\rm surv} = 1 - \frac{1}{2}\sin^2 2\theta$$

Talk by Alexei Smirnov at NOW 2022: coherence and decoherence

Matter Effect

Wolfenstein (1978), Mikheev & Smirnov (1984)

- In a medium, the oscillation frequency and mixing angle are density-dependent

$$\frac{\Delta m_m^2}{2E} = \sqrt{\left(\underbrace{\frac{\Delta m_0^2}{2E}}_{\omega_{\rm osc}} \cos^2 2\theta_0 + \underbrace{\sqrt{2} G_F n_e}_{V_{\rm MSW}}\right)^2 + \left(\frac{\Delta m_0^2}{2E} \sin^2 2\theta_0\right)^2}$$

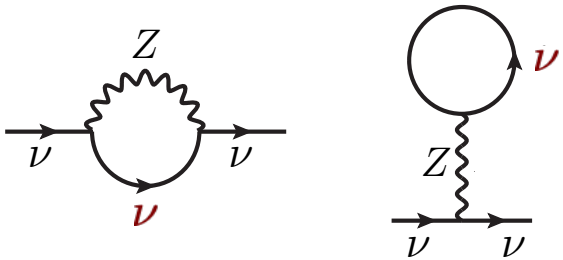
$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{\left(\cos 2\theta_0 + \frac{2E}{\Delta m^2}\sqrt{2} G_F n_e\right)^2 + \sin^2 2\theta_0}$$

- At large density
  - Oscillation frequency grows proportionally with density, i.e., as ~ N
  - Note however that the mixing becomes small as ~ 1/N^2

- Large mixing (with small theta) if there is a resonance where the first two terms cancel

Neutrino Matter Effect

- In a medium, neutrinos can scatter and coherence is destroyed
- However, the effect of "forward scattering" is subtle



Forward scattering on background matter changes flavor oscillations

Pantaleone (1992)

- If neutrinos *forward scatter* (same momentum, thus no change in quantum state), it can interfere with free propagation, and that adds a potential to the Hamiltonian, changing the probability

$$|\mathcal{A}_{\rm tot}|^2 = |\mathcal{A}_{\rm free} + \mathcal{A}_{\rm fwd\text{-}scatt}|^2$$
$$\approx |\mathcal{A}_{\rm free}|^2 + 2{\rm Re}(\mathcal{A}^*_{\rm free}\mathcal{A}_{\rm fwd\text{-}scatt})$$

Neutrino Matter Effect

- In a medium, neutrinos can scatter and coherence is destroyed
- However, the effect of "forward scattering" is subtle

Can we just add to the MSW potential an extra potential like $\sqrt{2}G_F(n_\nu - n_{\bar{\nu}})$?

...ne (1992)

...mentum, thus no change in quantum state), it ... free propagation, and that adds a potential to the Hamiltonian, changing the probability

$$|\mathcal{A}_{\text{tot}}|^2 = |\mathcal{A}_{\text{free}} + \mathcal{A}_{\text{fwd-scatt}}|^2$$
$$\approx |\mathcal{A}_{\text{free}}|^2 + 2\text{Re}(\mathcal{A}^*_{\text{free}}\mathcal{A}_{\text{fwd-scatt}})$$

Neutrino Matter Effect

- In a medium, neutrinos can scatter and coherence is destroyed
- However, the effect of "forward scattering" is subtle

V-A form of weak interaction means … $\sqrt{2}G_F \int d^3q\,(1-\hat{p}\cdot\hat{q})\left(\frac{d^3n_\nu}{d^3q}-\frac{d^3n_{\bar{\nu}}}{d^3q}\right)$

…momentum, thus no change in quantum state), it … free propagation, and that adds a potential to the Hamiltonian, changing the probability

$$|\mathcal{A}_{\rm tot}|^2 = |\mathcal{A}_{\rm free} + \mathcal{A}_{\rm fwd\text{-}scatt}|^2$$
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Neutrino Matter Effect

- In a medium, neutrinos can scatter and coherence is destroyed
- However, the effect of "forward scattering" is subtle

But neutrinos are mixed so it must be a matrix $\sqrt{2}G_F \int d^3q\,(1-\hat{p}\cdot\hat{q})\,(\varrho_q - \bar{\varrho}_q)$

... free propagation, and that adds a potential to the Hamiltonian, changing the probability

$$|\mathcal{A}_{\rm tot}|^2 = |\mathcal{A}_{\rm free} + \mathcal{A}_{\rm fwd\text{-}scatt}|^2$$
$$\approx |\mathcal{A}_{\rm free}|^2 + 2{\rm Re}(\mathcal{A}^*_{\rm free}\mathcal{A}_{\rm fwd\text{-}scatt})$$

Describing SN Neutrinos

$$i\left(\partial_t + \mathbf{v_p}\cdot\partial_{\mathbf{p}}\right)\rho_{\mathbf{p}} = +\left[\frac{\mathrm{M}^2}{2E},\,\rho_{\mathbf{p}}\right] + \sqrt{2}G_{\mathrm{F}}\left[\mathrm{L},\rho_{\mathbf{p}}\right] + \sqrt{2}G_{\mathrm{F}}\int\frac{d^3\mathbf{q}}{(2\pi)^3}(1-\hat{\mathbf{p}}\cdot\hat{\mathbf{q}})\left[\rho_{\mathbf{q}}-\bar{\rho}_{\mathbf{q}},\rho_{\mathbf{p}}\right]$$

Vacuum oscillations depend on neutrino mass squared. Minus-sign for antineutrinos

MSW effect depends on ordinary matter density L, i.e. mainly electron density

Collective effects depends on the neutrino density

$$\omega = \frac{\Delta m^2}{2E}$$

$$\lambda = \sqrt{2}G_F\, n_e$$

$$\mu = \sqrt{2}G_F\, n_\nu$$

In general, a 7 dimensional partial integro-differential eqn.
3 momentum (E, θ_p, ϕ_p) + 3 space (r, θ, ϕ) + 1 time (t)

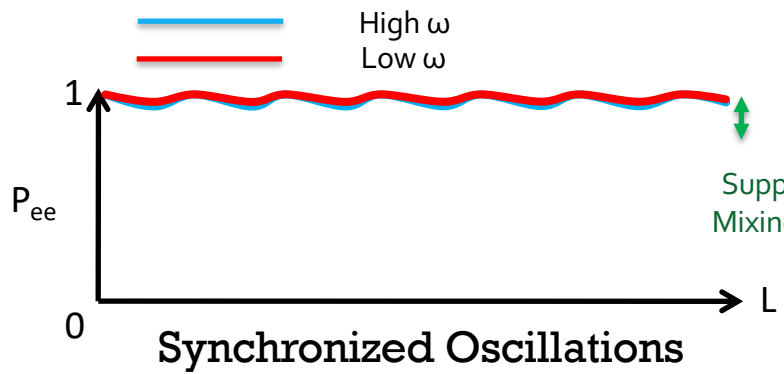
Similar equation (with/out "matrix" but with "collisions") used for SN simulations or neutrinos in early Universe
See talk by Kei Kotake and by I.M. Oldengott at NOW 2022

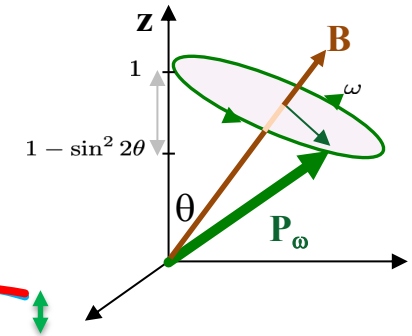
Collective Instability

When density is high

$$\mu = G_F n_\nu \gg \frac{\Delta m^2}{2E} = \omega$$

Oscillations are Collective





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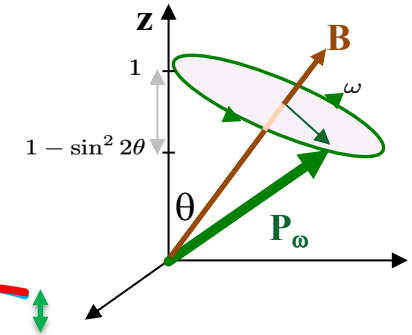

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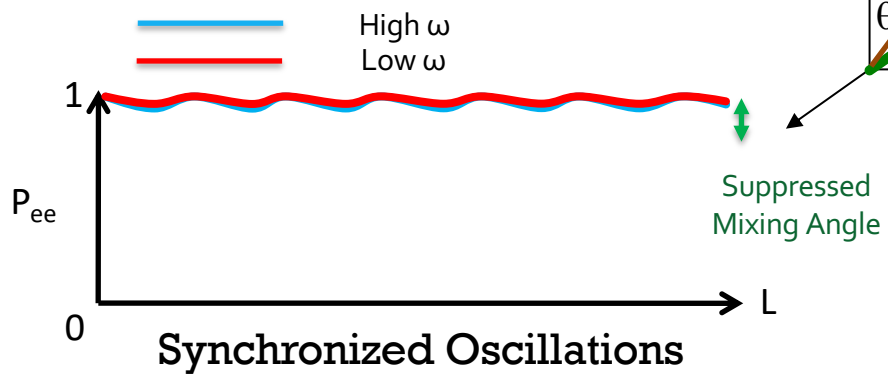
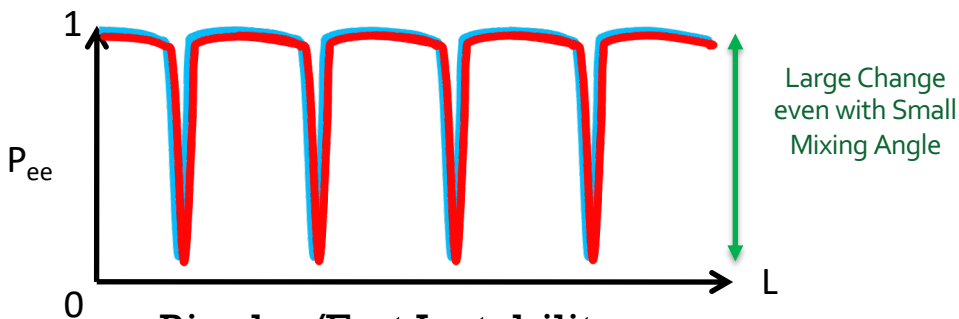


Synchronized Oscillations

Under some extra conditions, even for vanishing mixing, conversion grows at rate sqrt(ωμ) … (slow) or proportional to μ … (fast)



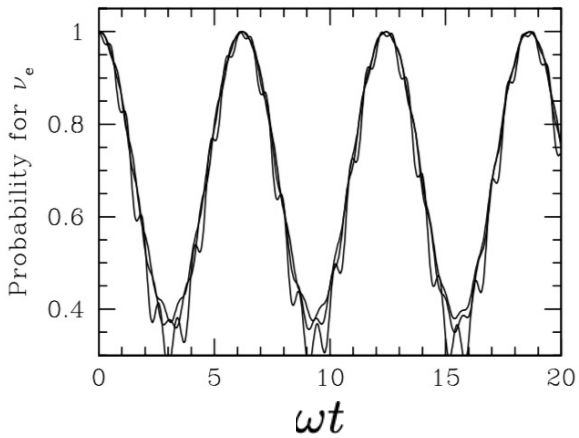
Bipolar/Fast Instability

Pioneering work by Pantaleone, Kostelecky, Samuel in the 90s
Second-wave in 2005-06 by Duan, Fuller, Carlson, Qian + others
Fast Oscillations by Sawyer (2015)
Chakraborty, Hansen, Izaguirre, Raffelt (2016)
Dasgupta, Mirizzi, Sen (2016)

Sync, Slow, and Fast

$$\omega = \frac{\Delta m^2}{2E}$$

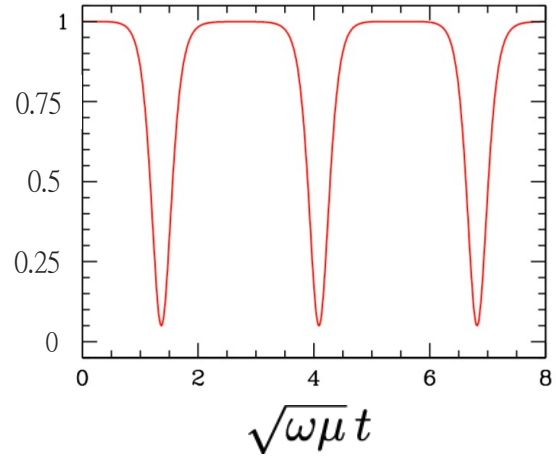
$$\mu = \sqrt{2} G_F \, n_\nu$$



Pastor, Raffelt, Semikoz (2001)

L ~ 1 km

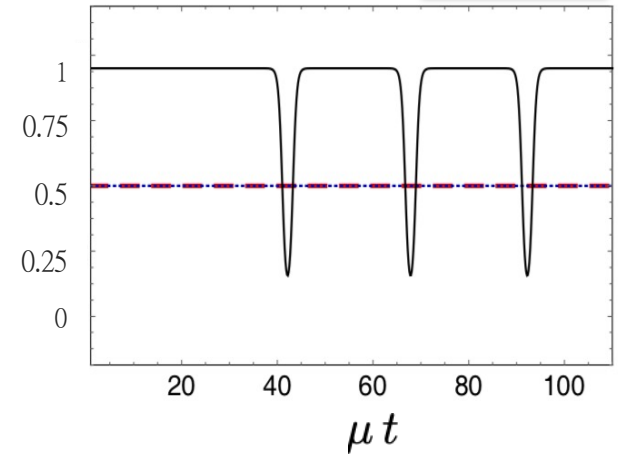
Relevant at ~500+ km



Hannestad, Raffelt, Sigl, Wong (2006)

L ~ 1 m

Relevant at ~50+ km



Dasgupta and Sen (2017)

L ~ 1 mm

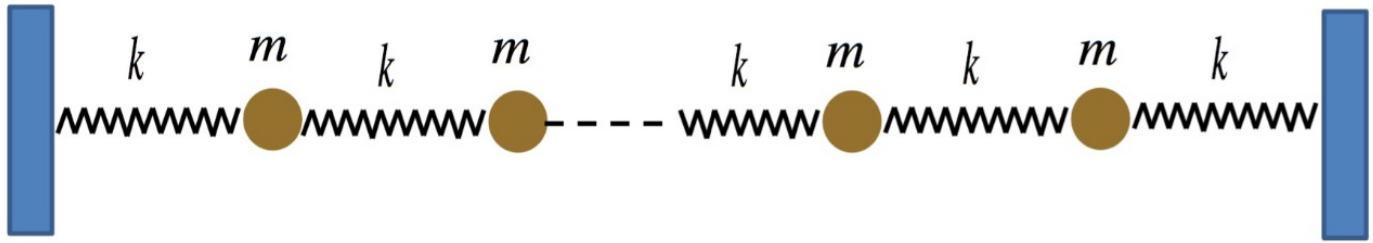
Relevant at ~5+ km

Synchronized oscillations for all energies but amplitude is small if matter is dense

Slow instabilities are collective but also unstable, i.e. large amplitude for small mixing

Fast instabilities are collective and unstable and depend on neutrino masses very weakly

Compare with Classical Oscillators



Normal frequencies of N coupled oscillators

$$\omega_n = 2\omega_0 \sin\left(\frac{n\pi}{2(N+1)}\right)$$

Note that this does NOT increase with N

Collective Instability

i.e., Large Flavor Oscillations with Vanishing Mixing Angle and/or Vanishing Mass-Sq-Difference

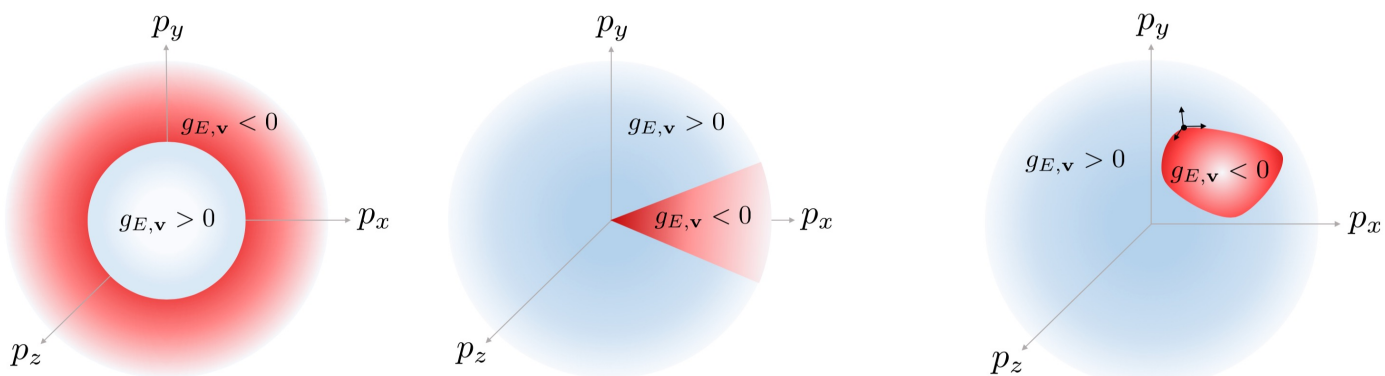
What causes it?

Instability Condition

Collective instability occurs *only if* momentum distributions of two flavors *cross each other* around some momentum

Proof by contradiction using the negative-definiteness of the imaginary part of the polarization tensor

Morinaga (2103.15267)
Dasgupta (PRL 2022)



$$g_\Gamma = \sqrt{2} G_F \begin{cases} f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} & \text{for } E > 0, \\ f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} & \text{for } E < 0, \end{cases}$$

Sketch of the Proof

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5. Show that "Non-positivity" requires that difference of phase space distribution does not have same sign everywhere

Sketch of the Proof

1. Linearize the eqn. of motion
2. Check if any Fourier mode grows exponentially
3. This requires a certain matrix to become non-positive
4. This matrix has elements that are integrals of the difference of phase space distributions of two flavors
5. Show that "Non-positivity" requires that difference of phase space distribution does not have same sign everywhere
6. Done!

Linearized EoM

Linearize a la Banerjee, Dighe, Raffelt (2010)
Notation of Izaguirre, Raffelt, Tamborra (2016)
and Airen et al. (2018)

- Equation of motion is $v^\alpha \partial_\alpha \rho_{\mathbf{p}} = -i\,[\mathsf{H}_{\mathbf{p}}, \rho_{\mathbf{p}}] + \mathsf{C}_{\mathbf{p}}$
- Write density matrix as $\frac{f_{\nu_e,\mathbf{p}} + f_{\nu_\mu,\mathbf{p}}}{2}\mathbb{1} + \frac{f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}}}{2}\begin{pmatrix} s_{\mathbf{p}} & S_{\mathbf{p}} \\ S^*_{\mathbf{p}} & -s_{\mathbf{p}} \end{pmatrix}$
- Collision term (in damping approx.) $\mathsf{C}^{e\mu}_{\mathbf{p}} = -|\Delta^{e\mu}_{\mathbf{p}}|\rho^{e\mu}_{\mathbf{p}}$
- EoM for off-diagonal becomes $i\,v^\alpha \partial_\alpha S_{\mathbf{p}} = (\omega_E + v^\alpha \Lambda_\alpha - i|\Delta_{\mathbf{p}}|)\,S_{\mathbf{p}}$

$$-v^\alpha \int d\mathbf{p}' v'_\alpha \left(S_{\mathbf{p}'} g_{\mathbf{p}'} - \bar{S}_{\mathbf{p}'} \bar{g}_{\mathbf{p}'}\right)$$

- In the above, the "spectrum" is given by

$$g_\Gamma = \sqrt{2} G_{\mathrm{F}} \begin{cases} f_{\nu_e,\mathbf{p}} - f_{\nu_\mu,\mathbf{p}} & \text{for } E > 0 \\ f_{\bar{\nu}_\mu,\mathbf{p}} - f_{\bar{\nu}_e,\mathbf{p}} & \text{for } E < 0 \end{cases}$$

- We will prove that *S* can grow exponentially *only if* *g* changes sign

Dispersion Relation

- Linearized EoM $\left(v^\alpha\,(i\,\partial_\alpha - \Lambda_\alpha) - \omega_E + i|\Delta_\Gamma|\right) S_\Gamma = -v^\alpha \int d\Gamma'\, v'_\alpha\, g_{\Gamma'}\, S_{\Gamma'}$
- Use Fourier expansion $S_{\Gamma,r} = \sum_K Q_{\Gamma,K}\, e^{-i(K_0 t - \mathbf{K}\cdot\mathbf{r})}$
- Get an eigenvalue equation $(v_\alpha k^\alpha - \omega_E + i|\Delta_\Gamma|)\, Q_{\Gamma,k} = v_\alpha A^\alpha_k$

  where $A^\alpha_k \;=\; -\int d\Gamma\, v^\alpha\, g_\Gamma\, Q_{\Gamma,k}$
- Ansatz for eigenvector $Q_{\Gamma,k} = \dfrac{v_\alpha A^\alpha_k}{v_\gamma k^\gamma - \omega_E + i|\Delta_\Gamma|}$
- Eigen-equation becomes $v_\alpha A^\alpha_k = -v^\alpha A^\beta_k \int d\Gamma'\, g_{\Gamma'}\, \dfrac{v'_\alpha v'_\beta}{v'_\gamma k^\gamma - \omega_{E'} + i|\Delta_{\Gamma'}|}$
- Nontrivial solutions require $\mathcal{D}(k) \equiv \det \Pi^{\alpha\beta}_k = 0$

  where $\Pi^{\alpha\beta}_k = h^{\alpha\beta} + \int d\Gamma\, g_\Gamma\, \dfrac{v^\alpha v^\beta}{v_\gamma k^\gamma - \omega_E + i|\Delta_\Gamma|}$

Proof by Contradiction

- Let us assume *g* has same sign everywhere BUT there exists $k_0 = \kappa + i\sigma$ with the imaginary part nonzero and real **k**, that solves $\mathcal{D}(k) \equiv \det \Pi_k^{\alpha\beta} = 0$

- We write $\Pi^{\alpha\beta} = M^{\alpha\beta} - iN^{\alpha\beta}$

  where
$$M^{\alpha\beta} = h^{\alpha\beta} + \int d\Gamma\, g_\Gamma \frac{(\kappa - \mathbf{v}\cdot\mathbf{k} - \omega_E)\, v^\alpha v^\beta}{(\kappa - \mathbf{v}\cdot\mathbf{k} - \omega_E)^2 + (\sigma + |\Delta_\Gamma|)^2}$$
$$N^{\alpha\beta} = \int d\Gamma\, g_\Gamma \frac{(\sigma + |\Delta_\Gamma|)\, v^\alpha v^\beta}{(\kappa - \mathbf{v}\cdot\mathbf{k} - \omega_E)^2 + (\sigma + |\Delta_\Gamma|)^2}$$

- Note that *N* is real-symmetric and thus orthogonally diagonalizable to
$$O^\alpha_\mu O^\beta_\nu N^{\mu\nu} = D^{\alpha\beta} \qquad \text{with} \qquad D^{\alpha\alpha} = \int d\Gamma\, g_\Gamma \frac{(\sigma + |\Delta_\Gamma|)\,(O^\alpha_\mu v^\mu)^2}{(\kappa - \mathbf{v}\cdot\mathbf{k} - \omega_E)^2 + (\sigma + |\Delta_\Gamma|)^2}.$$

- The dispersion relation $\mathcal{D}(k) \equiv \det \Pi_k^{\alpha\beta} = 0$ implies $\tilde{M}^{\alpha\beta} A_\beta = +iD^{\alpha\beta} A_\beta$
  which gives on multiplying by A^* gives $\tilde{M}^{\alpha\beta} A^*_\alpha A_\beta = -iD^{\alpha\beta} A^*_\alpha A_\beta$
  and on complex-conj. and *M*-symmetry gives $\tilde{M}^{\alpha\beta} A^*_\alpha A_\beta = +iD^{\alpha\beta} A^*_\alpha A_\beta$

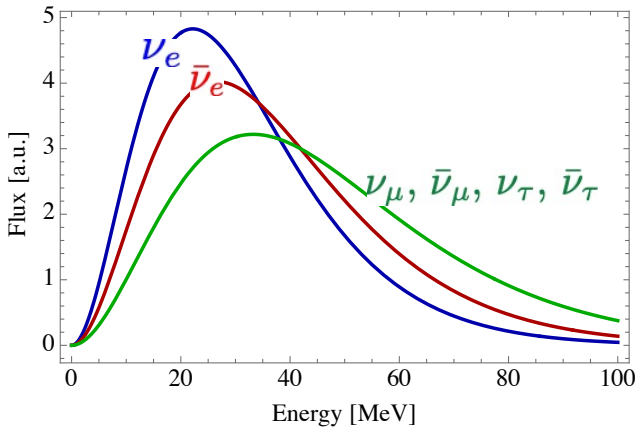
- However, then one finds that $\sum_\alpha D^{\alpha\alpha} |A_\alpha|^2 = 0$ which is impossible □

Morinaga (2103.15267)
Dasgupta (PRL, 2022)

Crossing Conditions

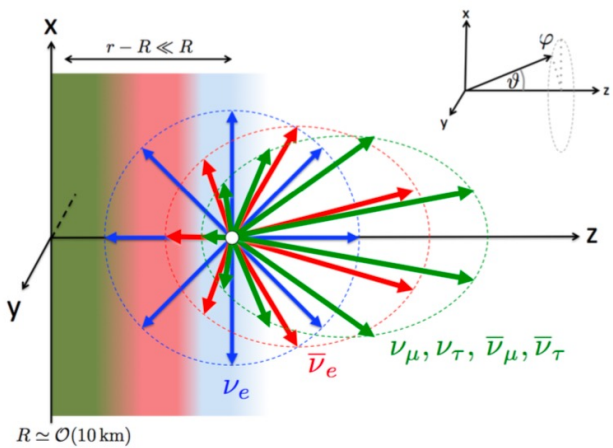
$$\omega = \frac{\Delta m^2}{2E}$$

$$\mu = \sqrt{2} G_F \, n_\nu$$



Different flavors have different energy spectrum

Crossing leads to (slow) instability i.e., exponential growth at a rate proportional to sqrt-ωμ



Different flavors have different angular distribution

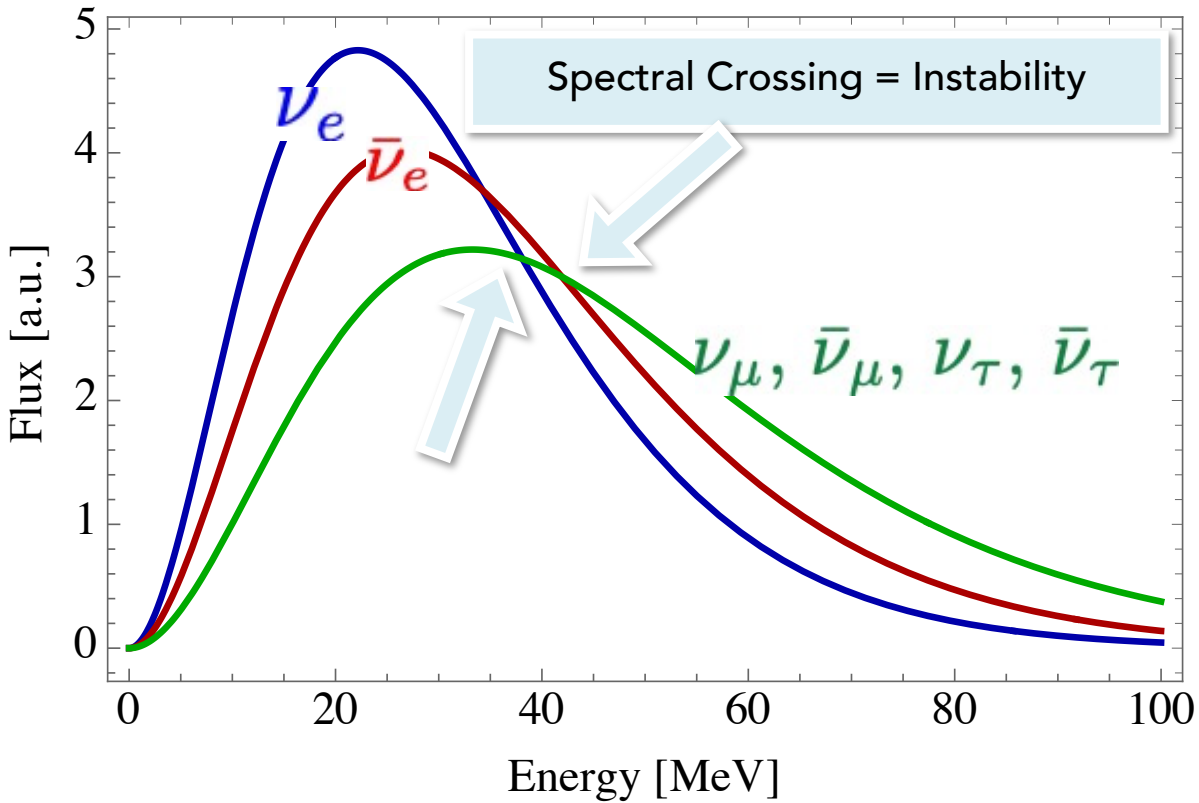
Crossing leads to (fast) instability i.e., exponential growth at a rate proportional to μ

Collective Instability

i.e., Large Flavor Oscillations with Vanishing Mixing Angle and/or Vanishing Mass-Sq-Difference and Exponential Growth in Initial Stages!

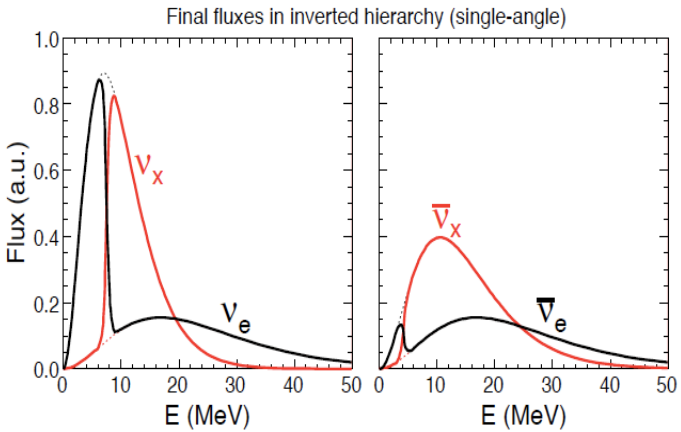
What does it do?

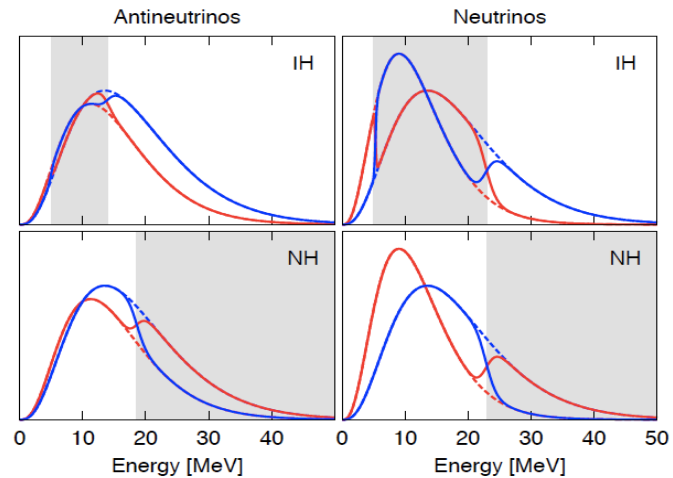
Slow Conversions



Dasgupta, Dighe, Raffelt, Smirnov (PRL, 2009)

Slow Spectral Swaps





Portions of the energy spectra get exchanged

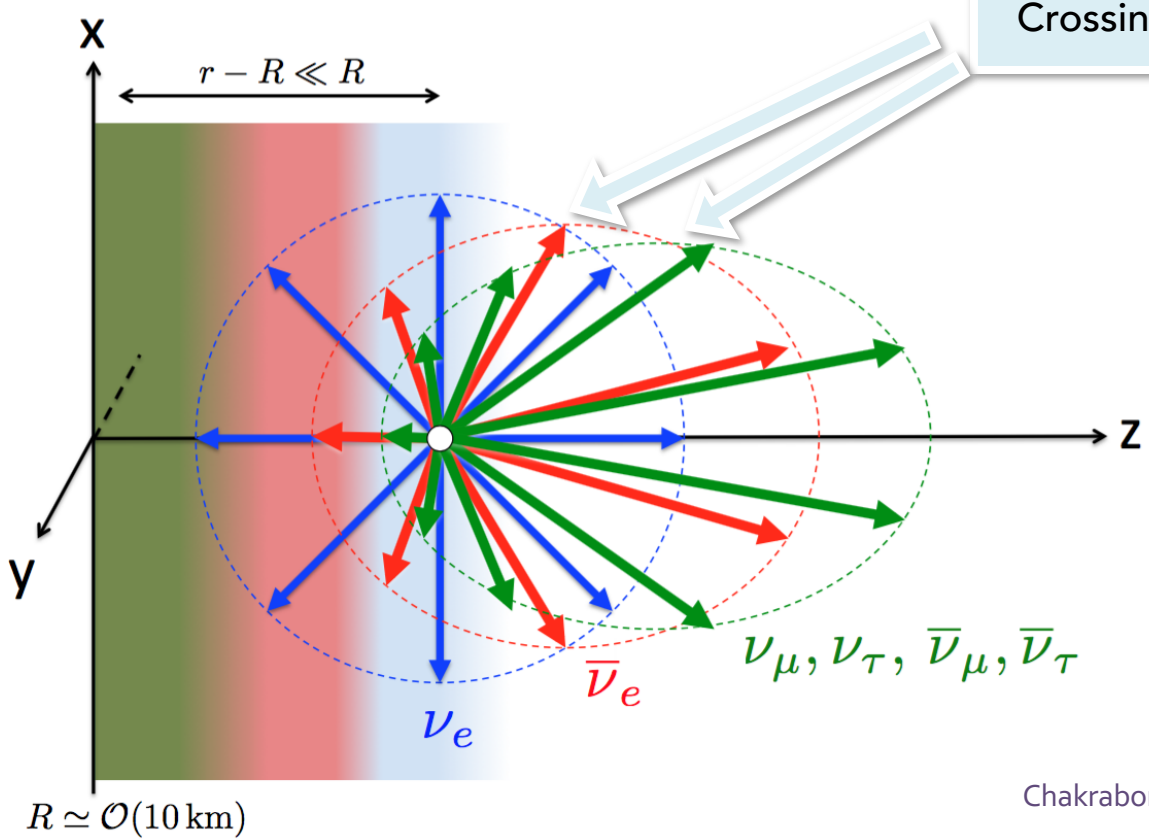
Initially thought to occur for Inverted Mass Ordering

Later realized that this occurs for any Mass Ordering and there can be Multiple Spectral Splits

Duan, Fuller, Carlson, Qian (2005, 2006, 2007)
Hannestad, Raffelt, Sigl, Wong (2006)
Raffelt and Smirnov (2007, 2007)
Fogli, Lisi, Marrone, Mirizzi (2008)

Dasgupta, Dighe, Raffelt, Smirnov (2009)
Friedland (2010)
Raffelt, Sarikas, Seixas (2013)

Fast Conversions

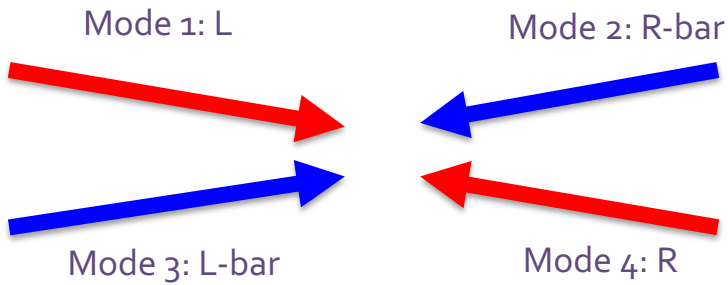


Crossing = Fast Instability

Instability grows at rate proportional to neutrino density

Sawyer (2005, 2015)
Chakraborty, Hansen, Izaguirre, Raffelt (2016)
Dasgupta, Mirizzi, Sen (2016)

Usually no analytical solution



$$\mathbf{Q} \equiv \mathbf{P}_L + \mathbf{P}_R + \overline{\mathbf{P}}_L + \overline{\mathbf{P}}_R - \frac{2\omega}{\mu(3-c)}\mathbf{B}$$
$$\mathbf{D} \equiv \mathbf{P}_L + \mathbf{P}_R - \overline{\mathbf{P}}_L - \overline{\mathbf{P}}_R\,,$$
$$\mathbf{X} \equiv \mathbf{P}_L - \mathbf{P}_R + \overline{\mathbf{P}}_L - \overline{\mathbf{P}}_R\,,$$
$$\mathbf{Y} \equiv \mathbf{P}_L - \mathbf{P}_R - \overline{\mathbf{P}}_L + \overline{\mathbf{P}}_R\,,$$

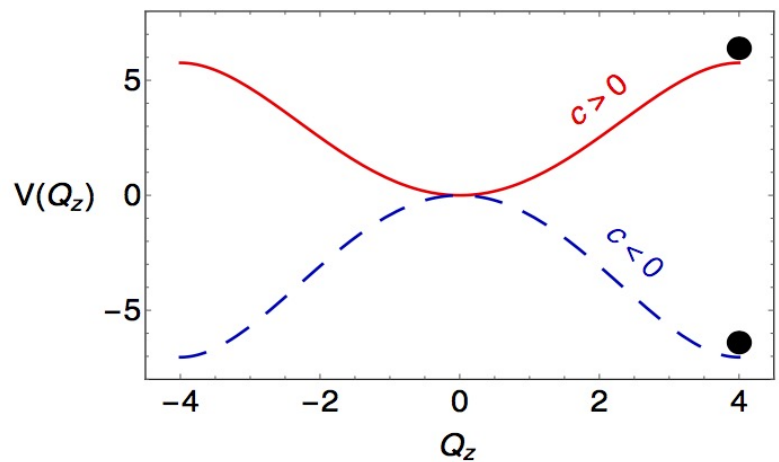
Ignore terms of order ω/μ or smaller

Working in limit $\mu >> \omega$

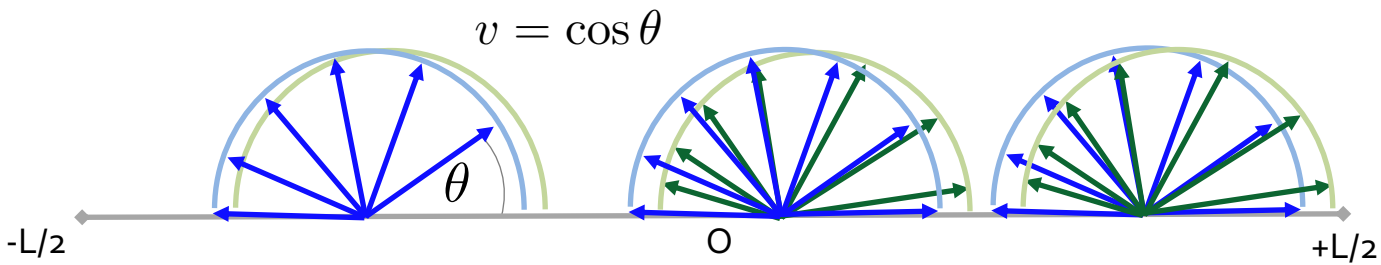
$$\ddot{\mathbf{Q}} = -\mu^2\, c\,(1-c)\left[|\mathbf{Q}_0|^2 - \mathbf{Q}\cdot\mathbf{Q}\right]\mathbf{Q}$$

$$\mathcal{L}_{\mathbf{Q}} = \frac{1}{2}|\dot{\mathbf{Q}}|^2 - \mu^2\, c\,(1-c)\left[|\mathbf{Q}_0|^2 - \frac{\mathbf{Q}\cdot\mathbf{Q}}{2}\right]\frac{\mathbf{Q}\cdot\mathbf{Q}}{2}$$

Dasgupta and Sen (PRD, 2017)



1 + 1 dimensional numerics



1 dimensional box of length L with periodic boundary conditions

Neutrinos are emitted in the electron or muon state from each point in the box with velocity v along the box-length.

Net-emission at velocity v is

$$G_v = f_e(v) - f_{muon}(v)$$

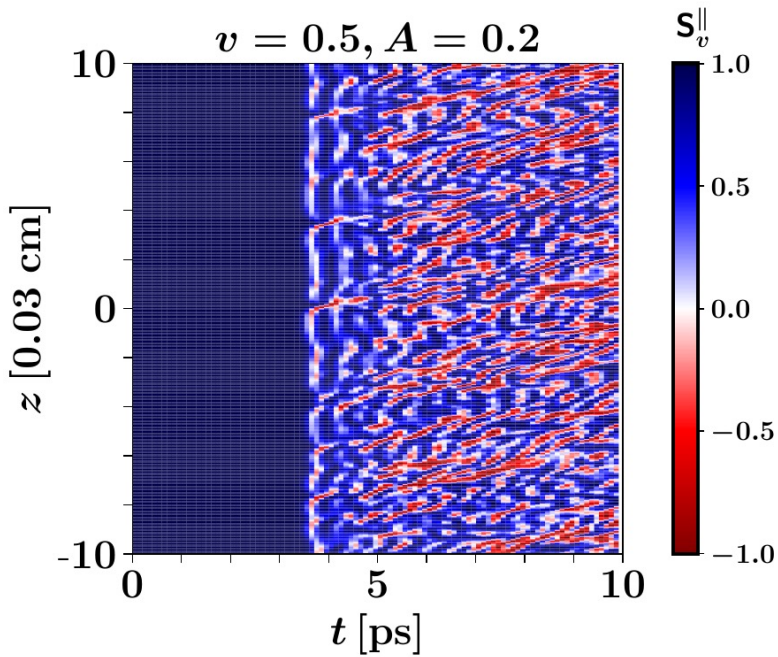
which is the difference of phase space distributions of the two flavors at each v.

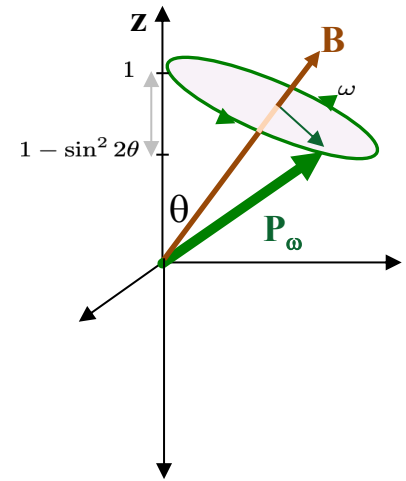
Solve this equation: $$(\partial_t + v\partial_z)\mathsf{S}_v = \mu_0 \int_{-1}^{+1} dv' G_{v'} \left(1 - vv'\right) \mathsf{S}_{v'} \times \mathsf{S}_v$$

1+1+1d calculation Bhattacharyya and Dasgupta (PRD 2020 ; PRL 2021, 2205.05129)

From beginning … to the end.

Numerical solution





Survival Probability starts at 1

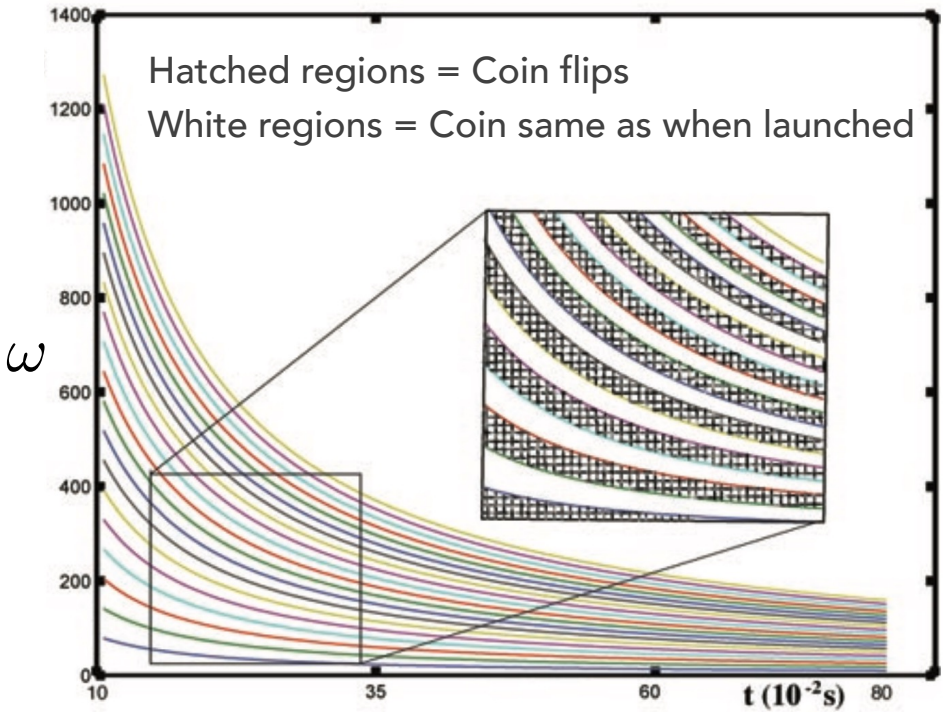
… oscillates coherently a few times

… and then decoheres

1+1+1d calculation Bhattacharyya and Dasgupta (PRD 2020 ; PRL 2021, 2205.05129)

Randomness from Coarsening

Tossed Coin



ω

$\sim v_{\rm ini}/2g$

Coarse-grained measurements lead to strong averaging over deterministic possibilities

Coarse-graining can lead to apparent irreversibility even if the evolution is reversible

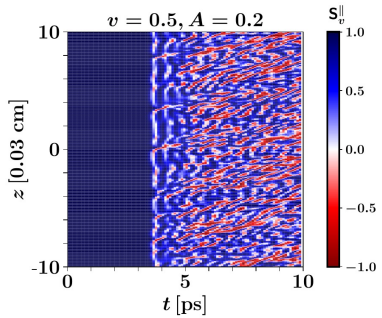
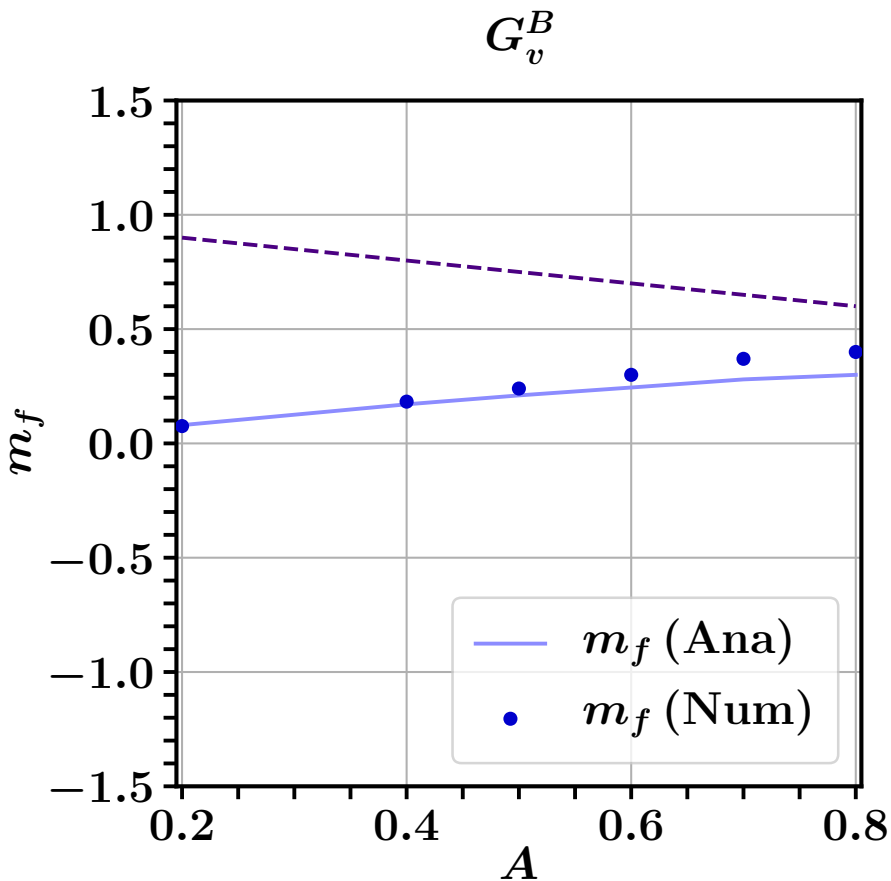


Fig. from Diaconis, Holmes, Mongomery (2007)

Predictable on Average?



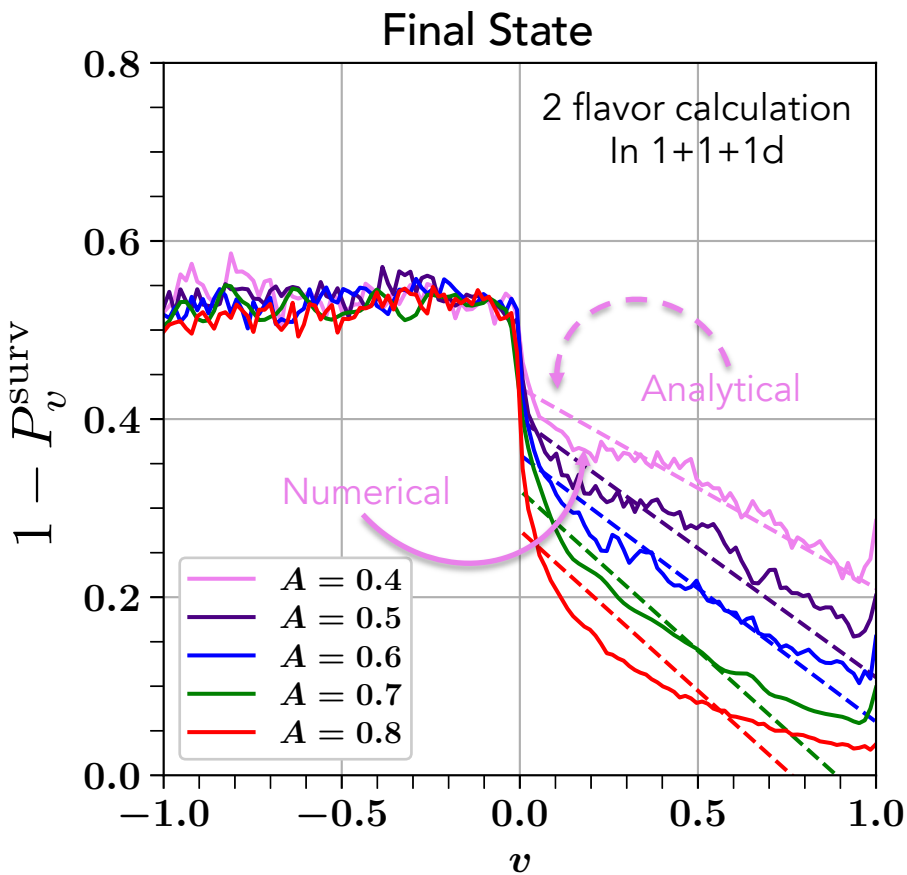
The final resting point of the average flavor-spin can be predicted analytically

$$m_f^{+-} = \frac{-bu_i^2 + \sqrt{b^2u_i^4 - 4\left(2b^2 - 4ku_i\right)bu_i^2 m_i}}{4b^2 - 8ku_i}$$

b and k are constants of motion, known from initial conditions, and m_i and u_i are initial conditions on the 1st and 2nd moment

Bhattacharyya and Dasgupta (2205.05129)

Fast Depolarization

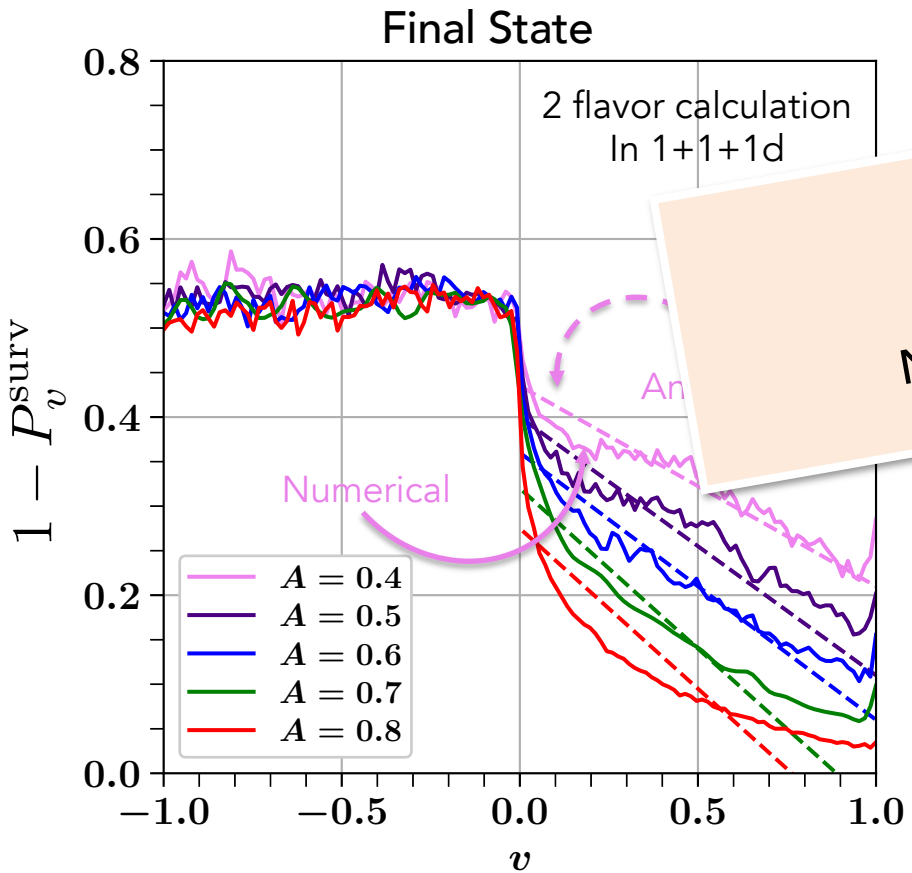


Several other groups have since obtained similar results

- Richers et al. @ Berkeley
- Meng-Ru Wu et al. @ Taiwan
- Sigl @ Hamburg
- Duan, Martin, et al. @ UNM

Bhattacharyya and Dasgupta
2020 PRD
2021 PRL
See 2205.05129 for more references

Fast Depolarization



$$P_{\rm surv} \neq 1 - \frac{1}{2}\sin^2 2\theta$$

Not 1:1 mixing either

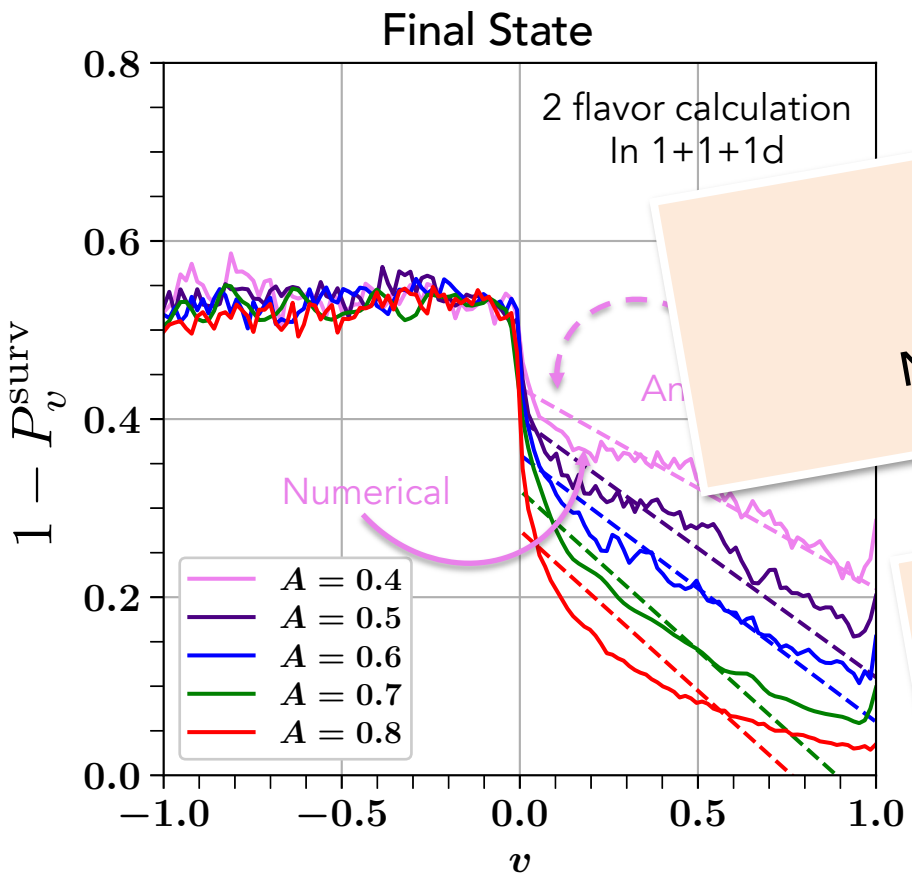
ince

an

Duan, Martin, et al. @ UNM

Bhattacharyya and Dasgupta
2020 PRD
2021 PRL
See 2205.05129 for more references

Fast Depolarization



ince

an

Duan, Martin, et al. @ UNM

$$P_{\rm surv} \neq 1 - \frac{1}{2}\sin^2 2\theta$$

Not 1:1 mixing either

Multi-scale (ω, μ, L, …) problem
… expect it to be hard in general

Bhattacharyya and Dasgupta
2020 PRD
2021 PRL
See 2205.05129 for more references

Ordinary vs. Collective

$$P_{\rm surv} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} t\right)$$

At the beginning, for small θ

$$P_{\rm surv} = 1 - \theta^2 \left(\frac{\Delta m^2}{2E} t\right)^2$$

At late times

$$P_{\rm surv} = 1 - \frac{1}{2}\sin^2 2\theta$$

No analytical solution in general, but

- it can rise exponentially in time !
- weak dependence on mixing angle !
- fast instability has ~no Δm^2 dependence?!

No analytical solution in general, so

- can't easily predict the degree of mixing
- 1:1:1 mixing is not quite correct

But why?








For Fun and Profit !

Why do we care ?

This is a lot of fun!

Instead of the usual $$i\partial_t|\nu_i\rangle \sim \omega_{\rm osc}|\nu_i\rangle$$

One has …

$$i\partial_t|\nu_i\rangle \sim \left[\omega_{\rm osc} + \frac{G_F}{V}\sum_{j=1}^{N}(1-\hat{p}_i\cdot\hat{p}_j)|\nu_j\rangle\langle\nu_j|\right]|\nu_i\rangle$$

Why do we care ?

This is a lot of fun!

Instead of the usual $$i\partial_t|\nu_i\rangle \sim \omega_{\rm osc}|\nu_i\rangle$$

One has …

$$i\partial_t|\nu_i\rangle \sim \left[\omega_{\rm osc} + \boxed{\text{Neutrino-Neutrino Forward Scattering of N neutrinos in a box}}\right]|\nu_i\rangle$$

- Can the oscillation frequency be proportional to N ?!
- What is required to get this truly unusual behavior?
- What is its late time behavior?

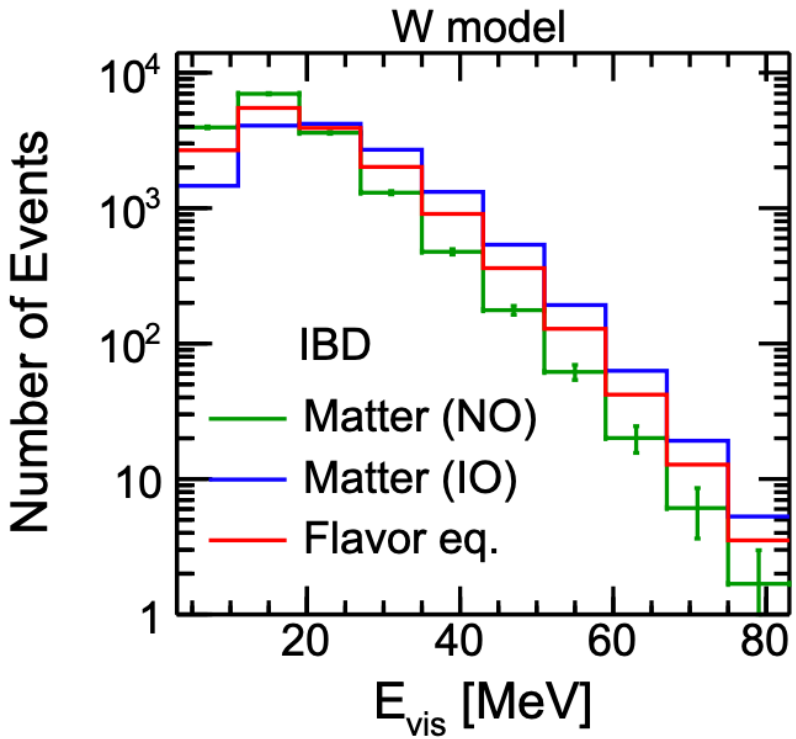
Why do we care ?

Neutrinos affect astrophysics

- The observable neutrino flux can be better decoded to understand neutrino properties
- Heating of the star depends on how energy is apportioned between the various flavors
- Formation of chemical elements depends on the electron anti/neutrino abundance

Expected Signals at Detector

Example: Super-K can distinguish fast decoherence from ordinary evolution



Capozzi, Dasgupta, Mirizzi (PRD, 2018)

Flavor Dependent Heating

Heating is due to electron (anti)-neutrinos

$$Q_+^{\text{before}} \sim \frac{L_{\nu_e}}{4\pi r^2}\sigma_0 \left(\frac{E_{\nu_e}}{E_0}\right)^2$$

But these flavors have the least average energy

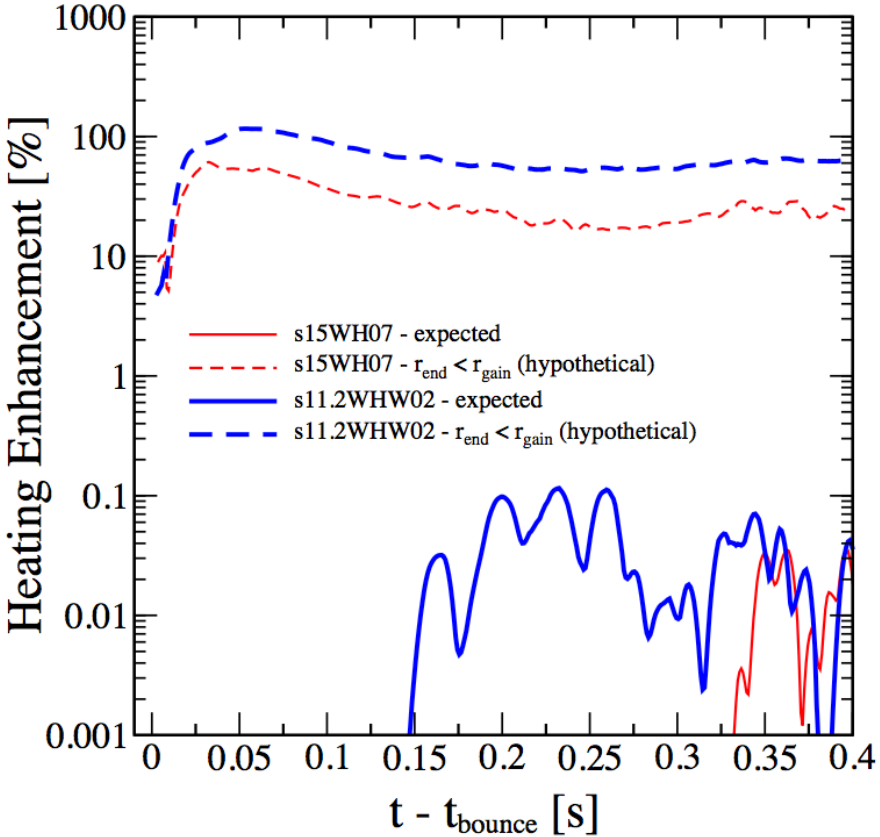
$$\langle E_{\nu_e}\rangle \ll \langle E_{\nu_x}\rangle$$

With a swap of the flavors, i.e., $\nu_e \leftrightarrow \nu_x$

Significant increase in heating rate is possible

$$Q_+^{\text{after}} \sim \frac{L_{\nu_x}}{4\pi r^2}\sigma_0 \left(\frac{E_{\nu_x}}{E_0}\right)^2 \gg Q_+^{\text{before}}$$

Conversion = Heating?



Dasgupta, O'Connor, Ott (PRD, 2011)

May be possible
if conversions occur
near/below gain radius!

Talk on Supernova Explosion Modelling
by Kei Kotake at NOW 2022 for other aspects

Nucleosynthesis

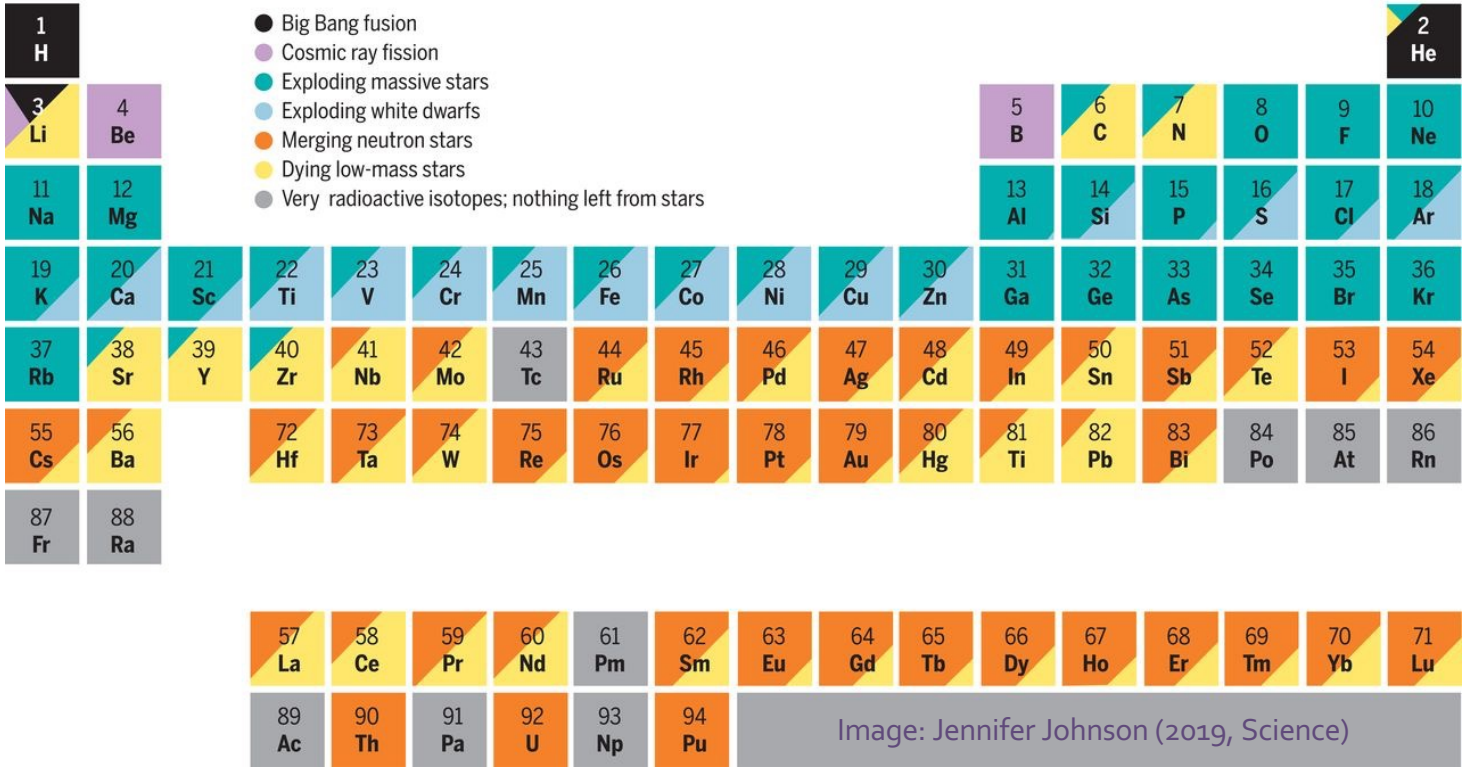


Image: Jennifer Johnson (2019, Science)

Vital elements for human existence are created in these stellar crucibles

Flavor & the r-Process

New elements created via (rapid) neutron capture

$$X(A,Z)+n \rightleftharpoons X(A+1,Z)+\gamma+Q_n(A,Z),$$

Depends on neutron abundance

$$\log\frac{n(A+1,Z)}{n(A,Z)}=\log n_n-34.07-\tfrac{3}{2}\log T_9+\frac{5.04}{T_9}Q_n$$

Burbidge, Burbidge, Fowler, Hoyle (RMP, 1957)

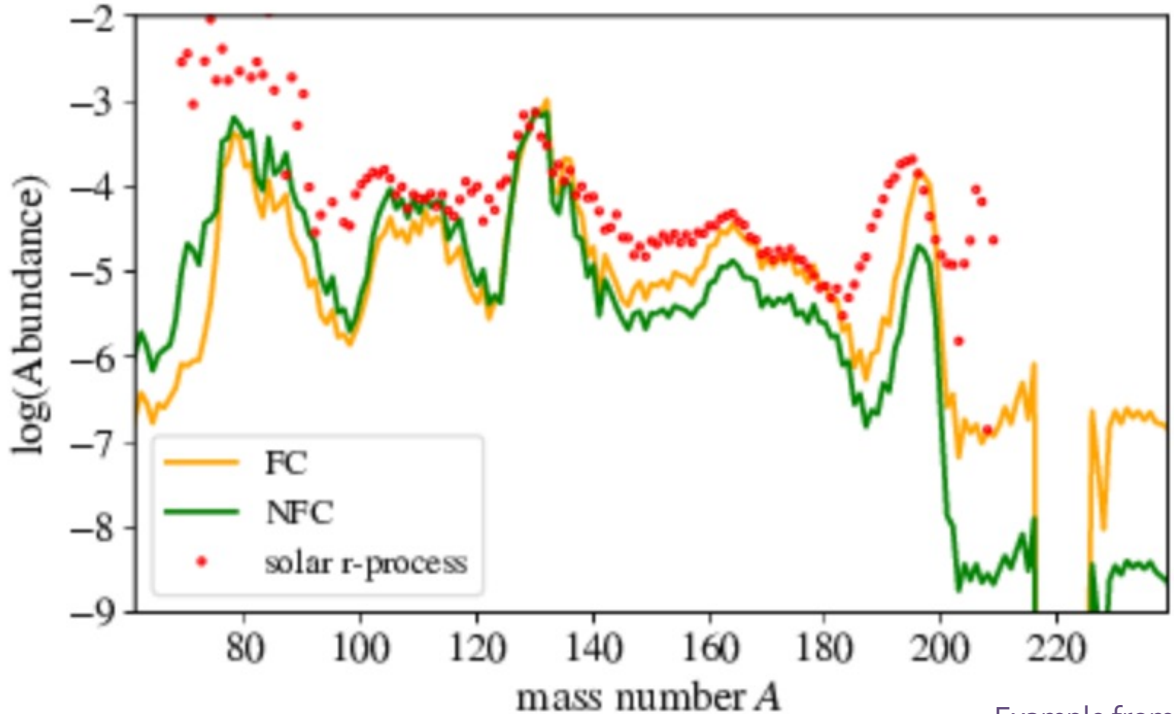
Q_n=Binding Energy in MeV
T_9=Temperature in 10^9 degrees

But electron (anti) neutrinos shift the n/p ratio

$$\bar{\nu}_e + p \rightleftharpoons n + e^+$$

$$\nu_e + n \rightleftharpoons p + e^-$$

Nucleosynthesis with Fast Conversions in Neutron Stars?



Example from Li and Siegel (PRL, 2021)

R process nucleosynthesis of A~140 onwards boosted by Fast Conversions

What about Collisions?

What about Collisions?

- **Relaxation approximation vs. improved implementation in neutrino oscillation calculations?**

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- Self-consistent inclusion in SN simulations?

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- New ideas?

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- New ideas?

NOW 2022 talks by
- Lucas Johns
- Irene Tamborra
- Kei Kotake

The Big Picture

- Neutrino Theory
  - What are the different modes of flavor conversion?
  - When/Where/How do they occur?
  - What are the final flavor dependent fluxes?
- SN Theory, Nuclear Astrophysics
  - How do conversions affect SN heating? Nucleosynthesis?
  - Are neutrino oscillations + collisions important?
- Neutrino Phenomenology
  - What are the signatures and signals at detectors?
  - How can we glean important physics and astrophysics from these observations?

See my Neutrino 2018 review for more phenomenology
https://zenodo.org/record/1287067

In a Nutshell …

$$i\partial_t|\nu\rangle = H|\nu\rangle$$

- Neutrinos usually change flavor at a rate ~$\Delta m^2/(2E)$, which is about ~1 per km for neutrinos with MeV energy

$$i\partial_t|\nu_i\rangle \sim \left[\omega_{\rm osc} + \frac{G_F}{V}\sum_{j=1}^{N}(1-\hat{p}_i\cdot\hat{p}_j)|\nu_j\rangle\langle\nu_j|\right]|\nu_i\rangle$$

- In a SN, neutrinos "influence" each other via forward scattering, when their density is high
- This allows "fast" (~1 per mm) exponential flavor change
  - Q1. When is this possible? Ans: Crossing of momentum distributions of the two flavors
  - Q2. What impact does it have? Ans: Large flavor mixing and depolarization. May affects heating of the star, creation of elements, and observable signals