Improving the precision of QCD coupling from hadronic tau decays

Diogo Boito
University of São Paulo
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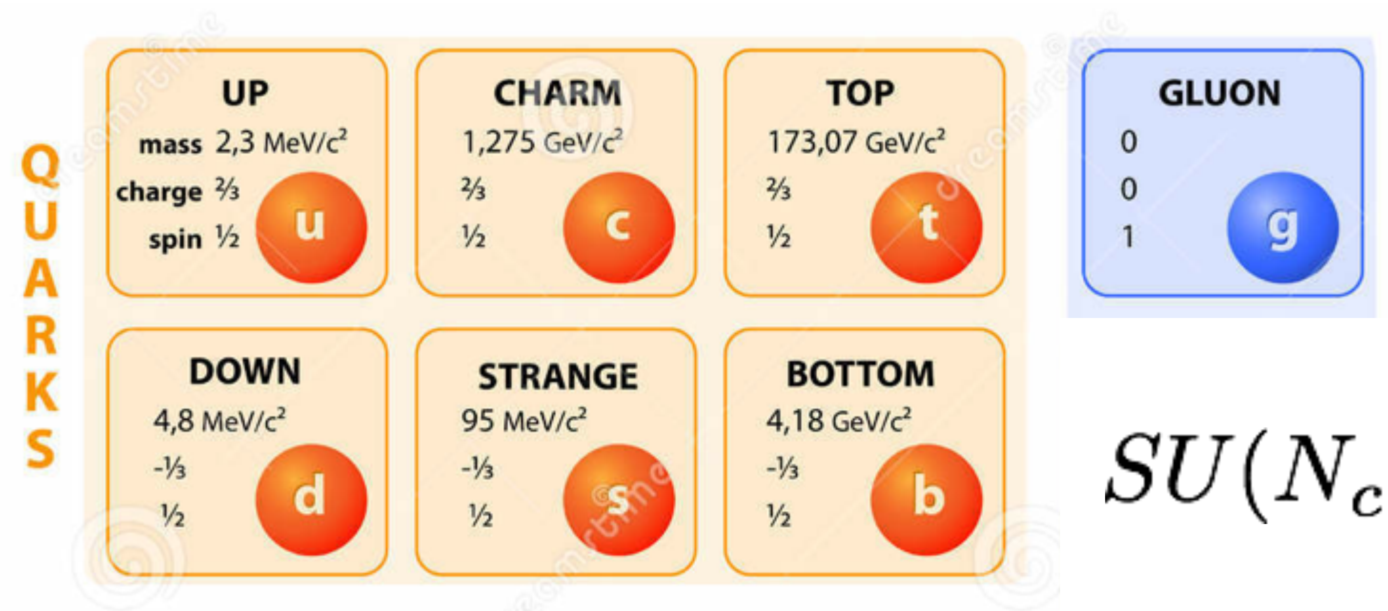
Benitez-Rathgeb, DB, Hoang, Jamin, 2202.10957.
DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, 2012.10440 PRD (2021)

March 14, 2022

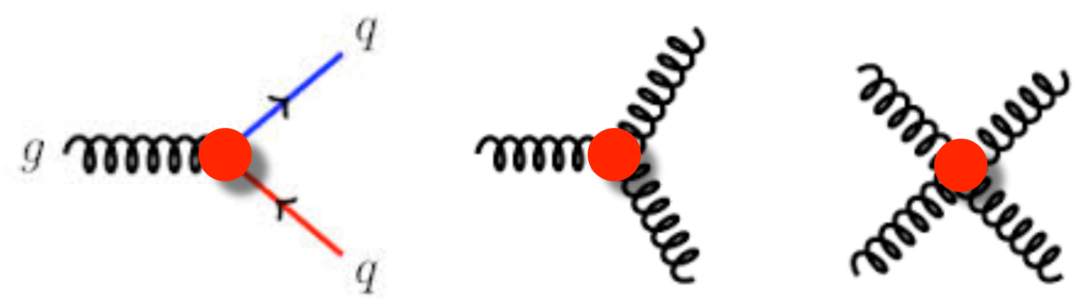
Università di Genova | Dipartimento di Fisica

QCD and its parameters

Six quark masses and the strong coupling (+ strong CP problem)

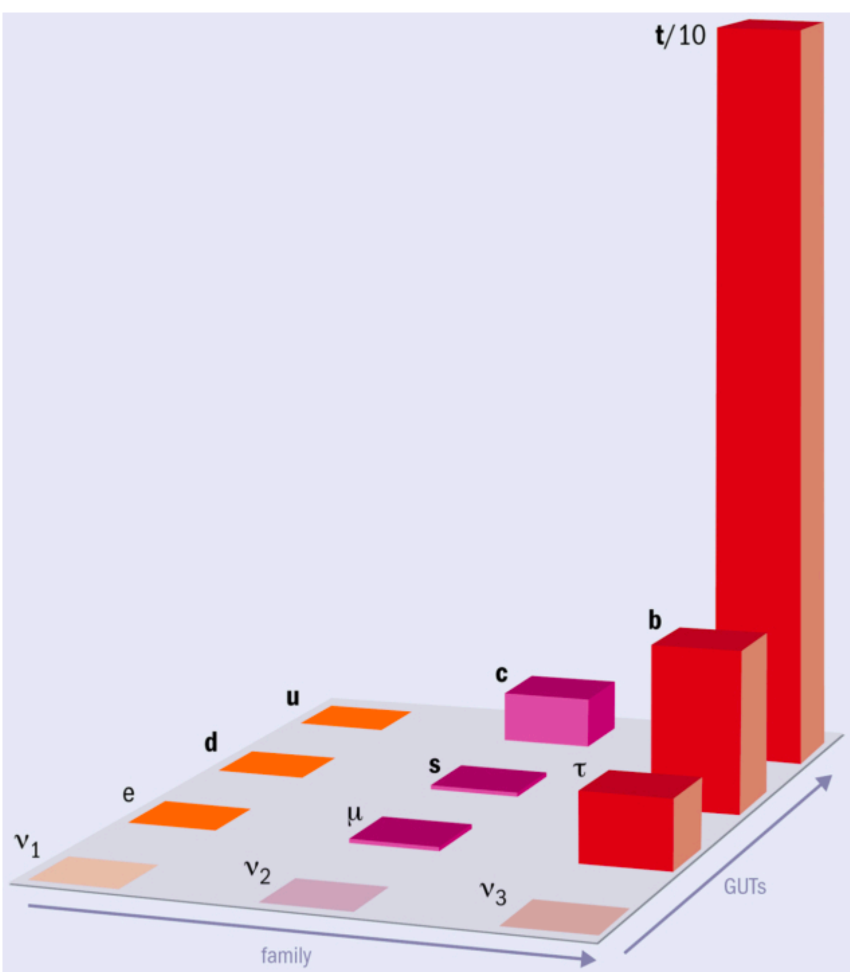


$SU(N_c)$



$\alpha_s(\mu)$

strong coupling (not constant)



CERN courrier

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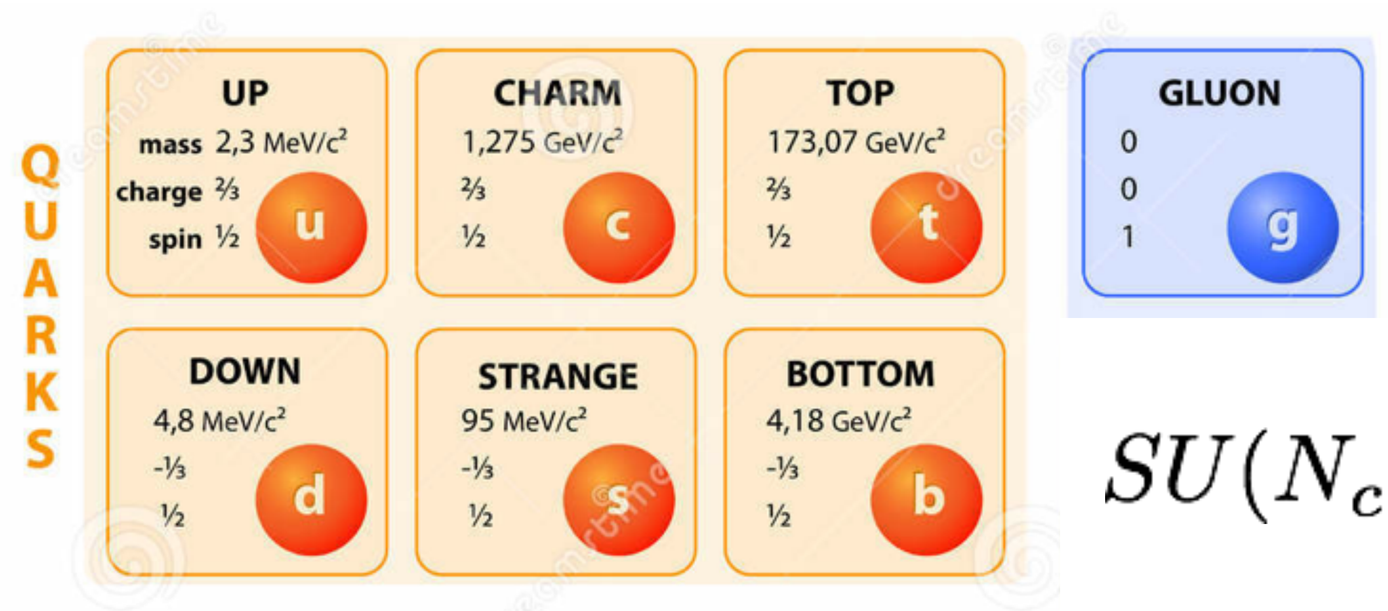

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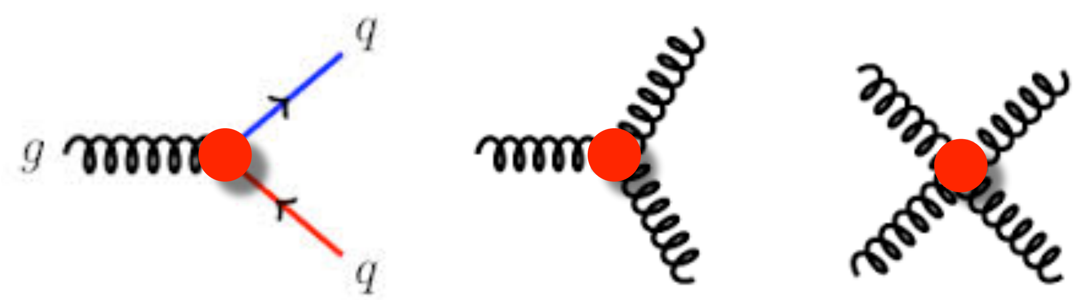

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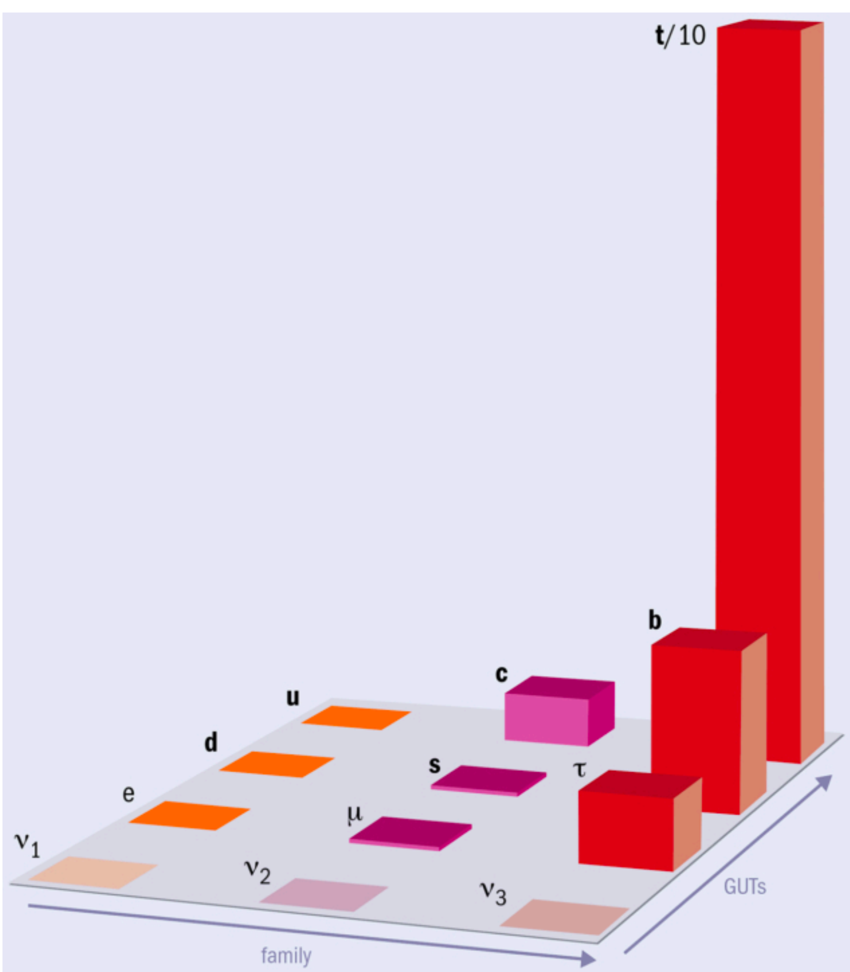

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Most important for the majority of SM tests are

$$\alpha_s, m_c, m_b, m_t$$

Crucial for the theoretical precision of many EW observables!

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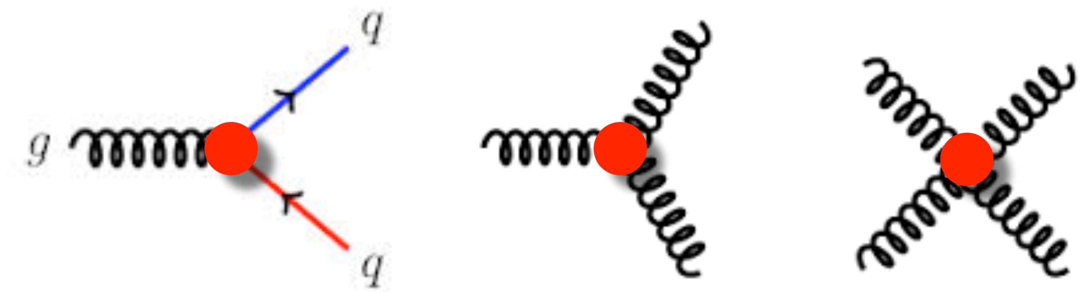

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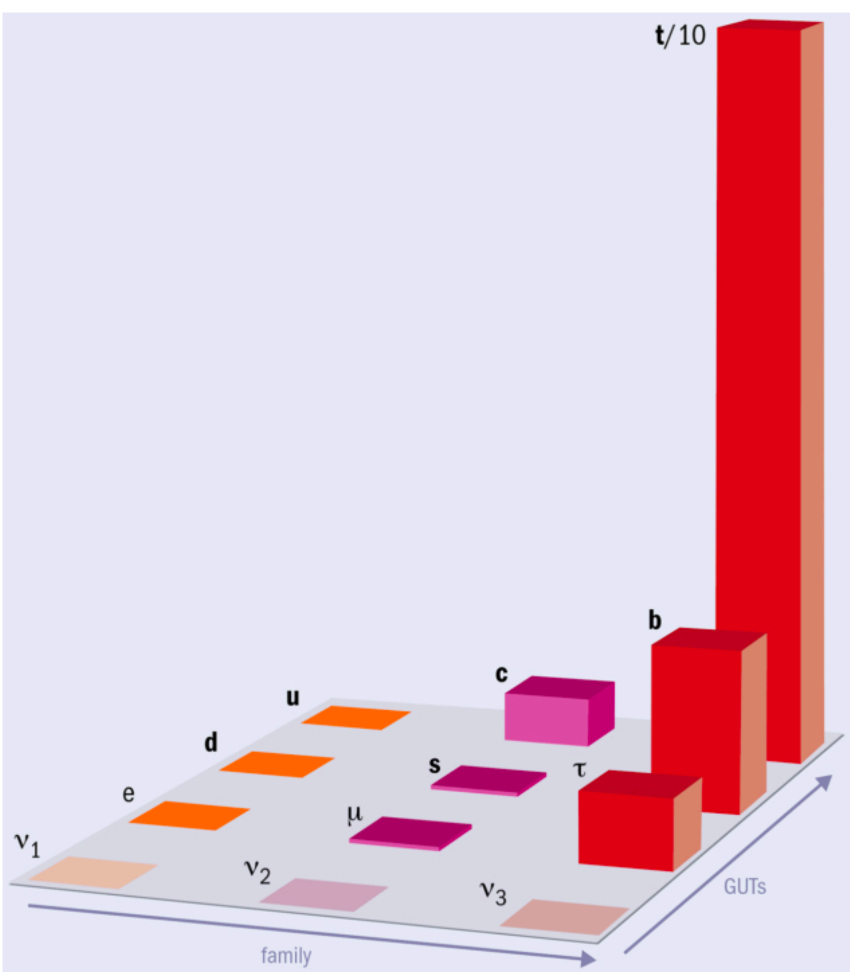

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Crucial for the theoretical precision of many EW observables!

$$a_s = \frac{\alpha_s}{\pi}$$

QCD and its parameters

Theory tells you how the parameters evolve with the energy but not their value

1 loop

$$\gamma(\alpha_s) \equiv \quad -\frac{\mu}{m_q}\frac{\mathrm{d}m_q}{\mathrm{d}\mu} = \quad \gamma_1 a_s + \cdots$$

$$\beta(\alpha_s) \equiv \quad -\frac{\mu}{\pi}\frac{\mathrm{d}\alpha_s}{\mathrm{d}\mu} = \quad \beta_1 a_s^2 + \cdots$$

1 loop

'73

asymptotic freedom

$$\beta_1 = -\frac{1}{2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$$

The Nobel Prize in Physics 2004

David J. Gross, H. David Politzer, Frank Wilczek

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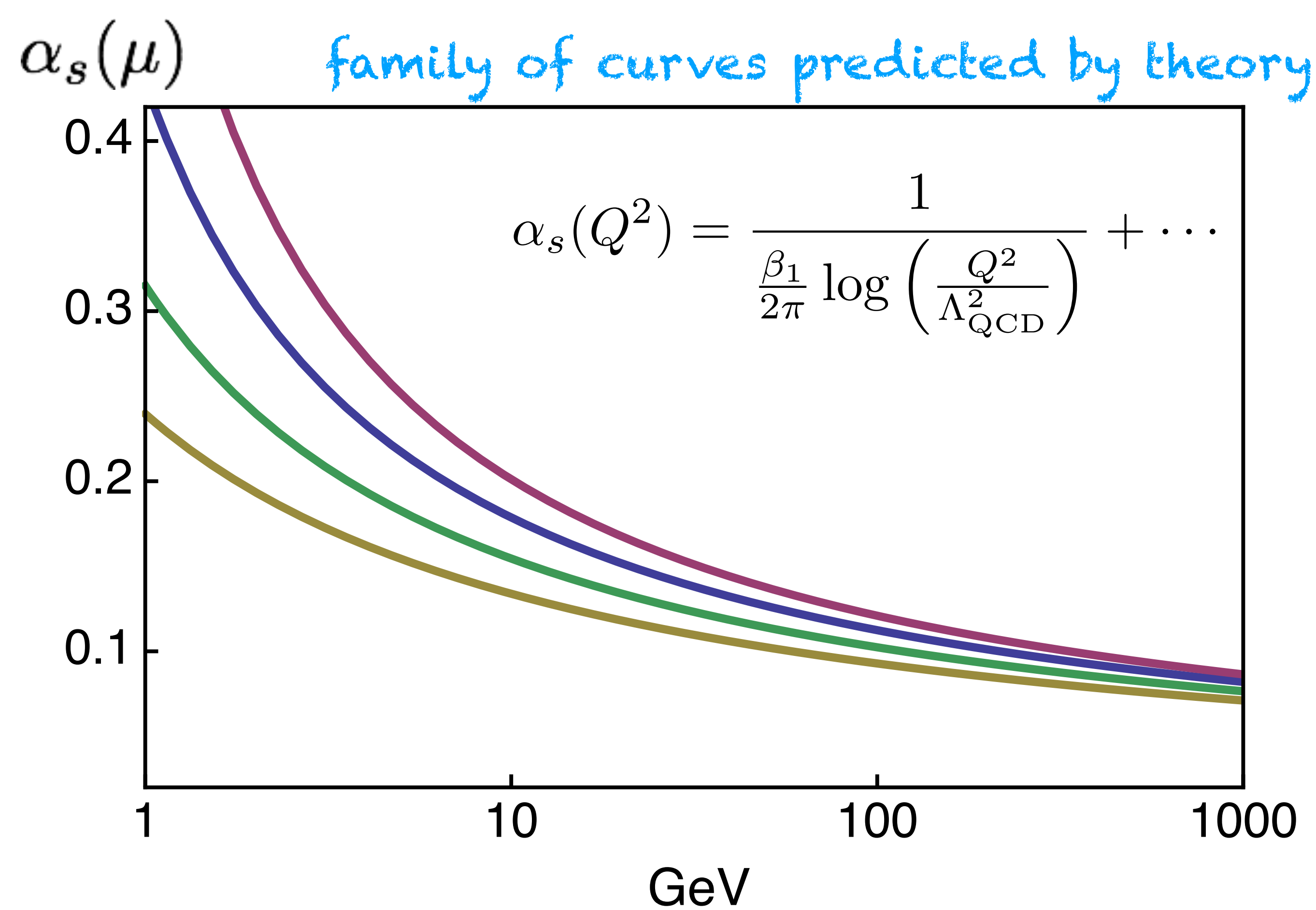
family of curves predicted by theory



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SU(3): Baikov, Chetyrkin, Kühn '14
SU(N): Luthe et al '16

	1 loop	2 loops ('81)	3 loop ('82)	4 loops ('97)	5 loops

$$\gamma(\alpha_s) \equiv -\frac{\mu}{m_q}\frac{\mathrm{d}m_q}{\mathrm{d}\mu} = \gamma_1 a_s + \gamma_2 a_s^2 + \gamma_3 a_s^3 + \gamma_4 a_s^4 + \gamma_5 a_s^5 \ldots$$

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	1 loop ('73)	2 loops ('74)	3 loop ('80)	4 loops ('97)	5 loops

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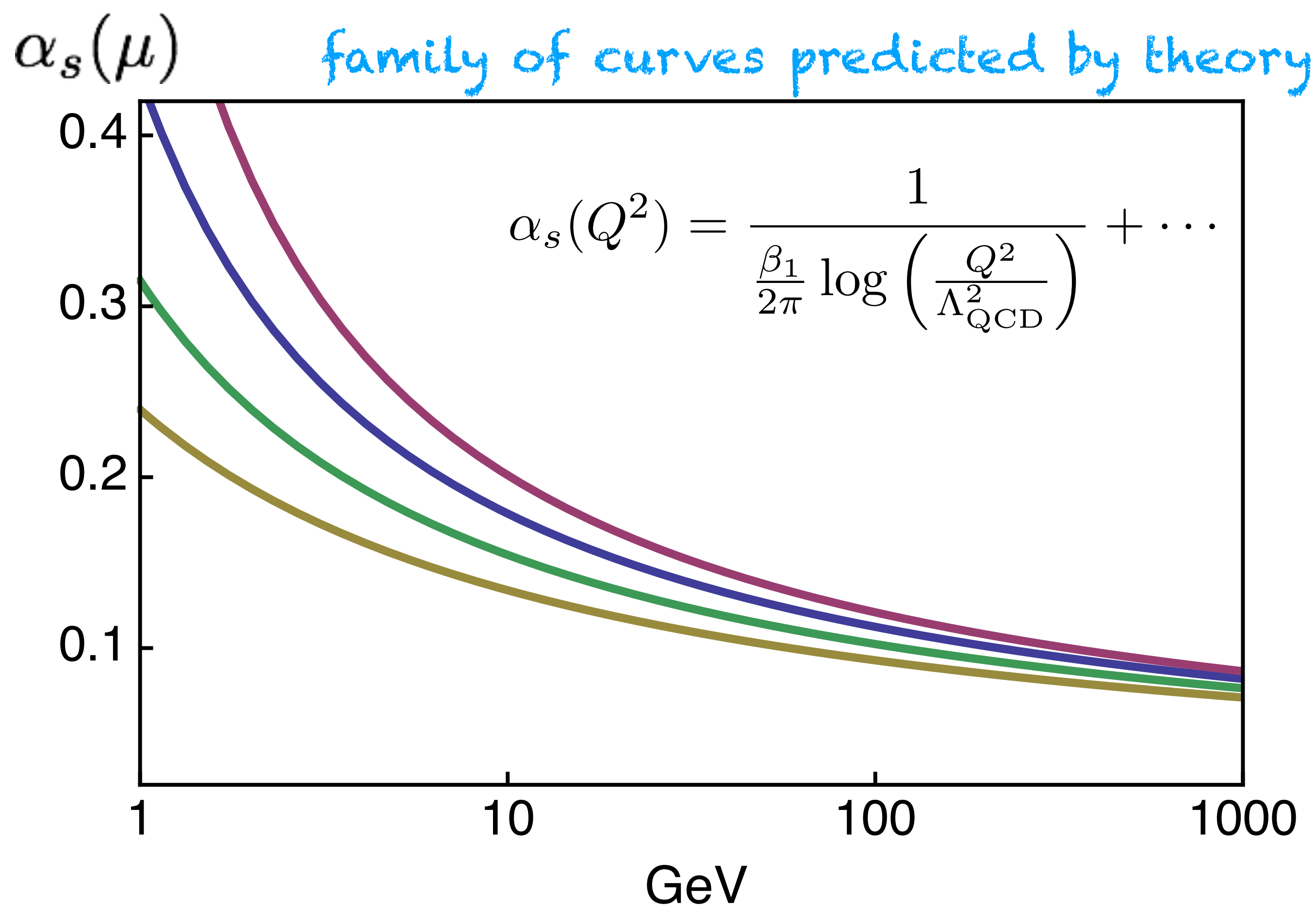

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QCD and its parameters

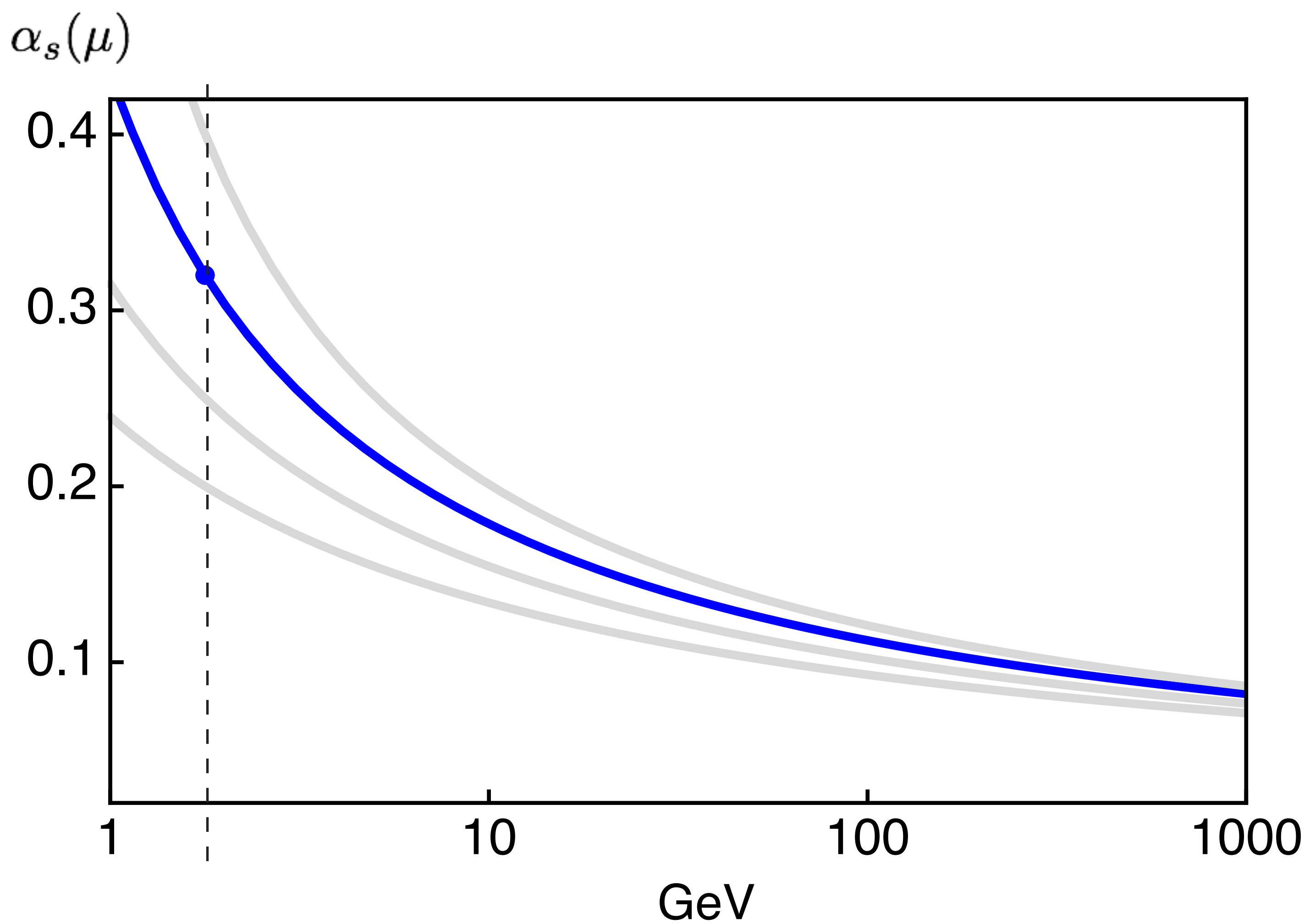
The coupling and masses are not physical observables (renormalization scheme)

Extracted indirectly: QCD calculation of some experimentally accessible quantity.

$$R_{\rm th}(\alpha_s) = \sum_n c_n \alpha_s^n + (\text{non} - \text{pert.}) = R_{\rm exp}$$

theory

experiment (or lattice)



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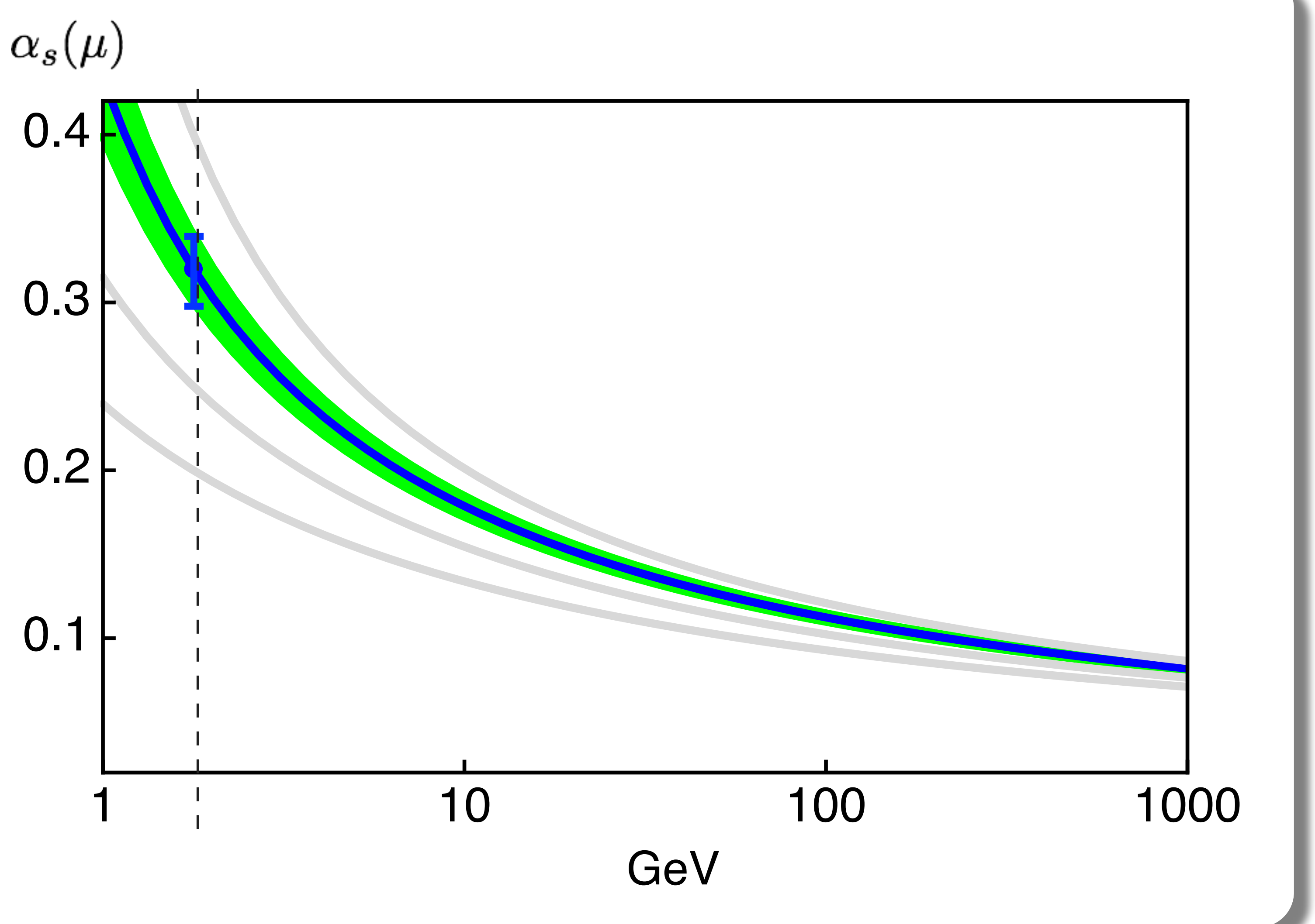

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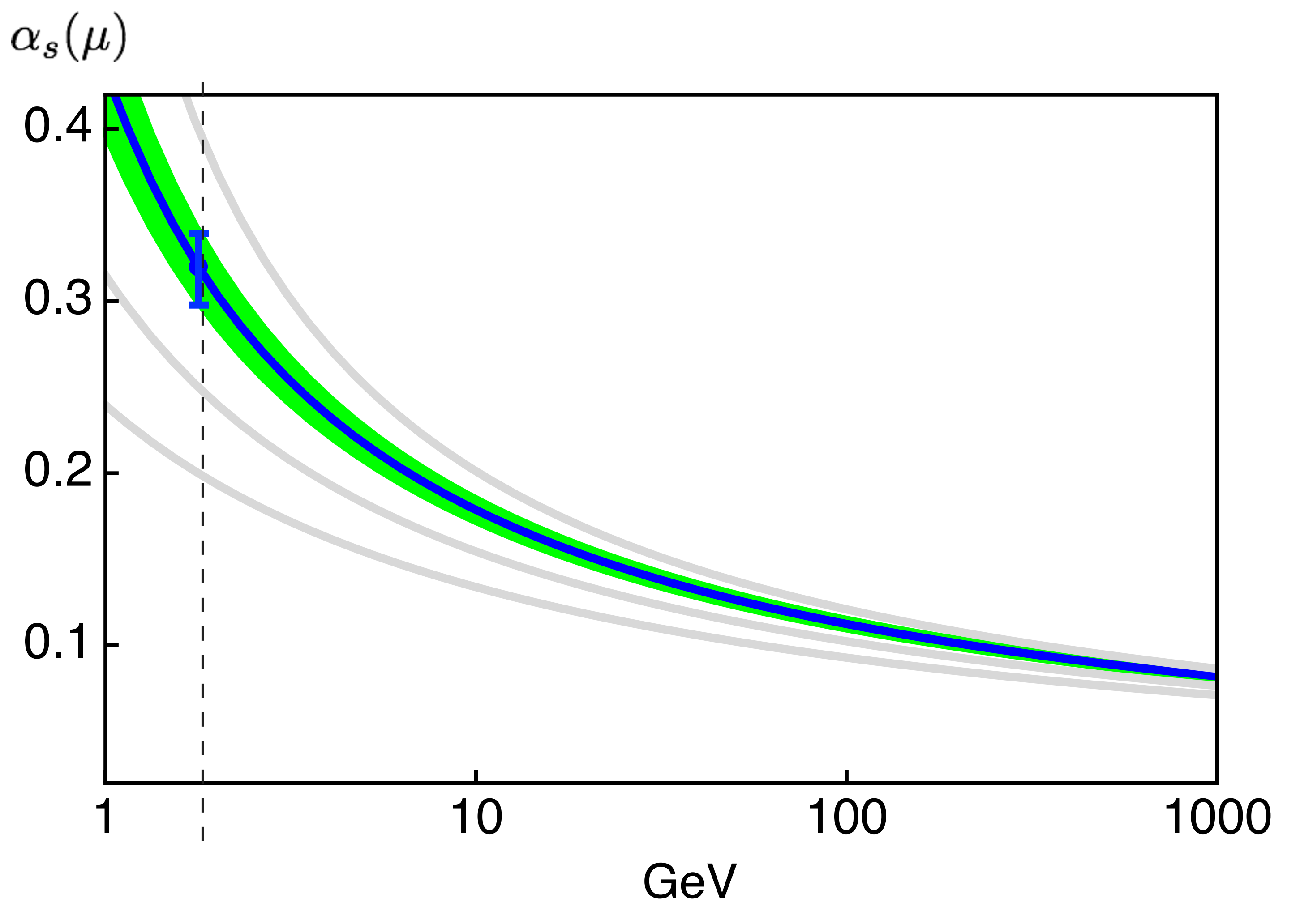
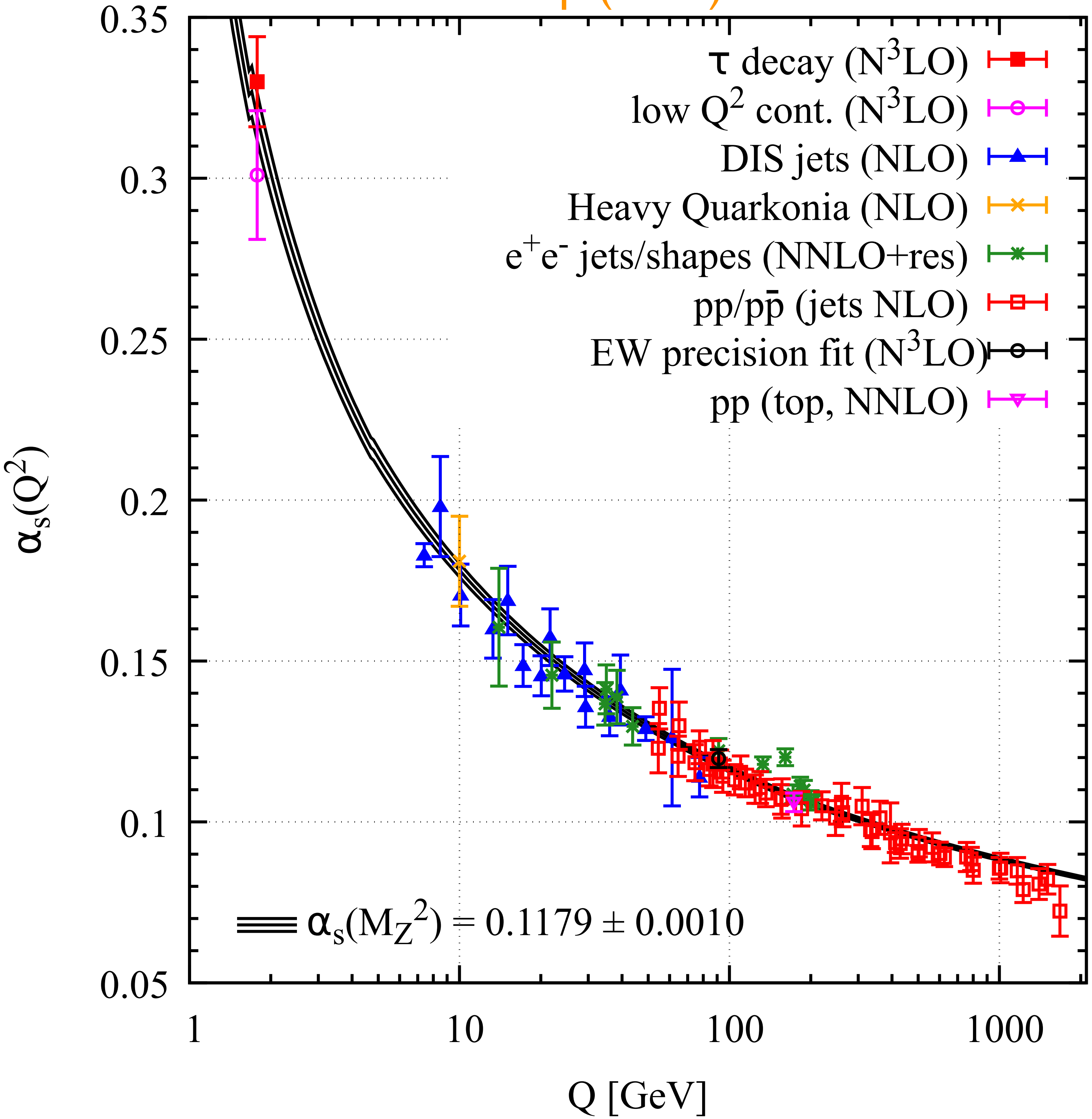


Different extractions compared at the same reference scale (mZ)

strong coupling: world average

Particle Data Group (PDG) 2021

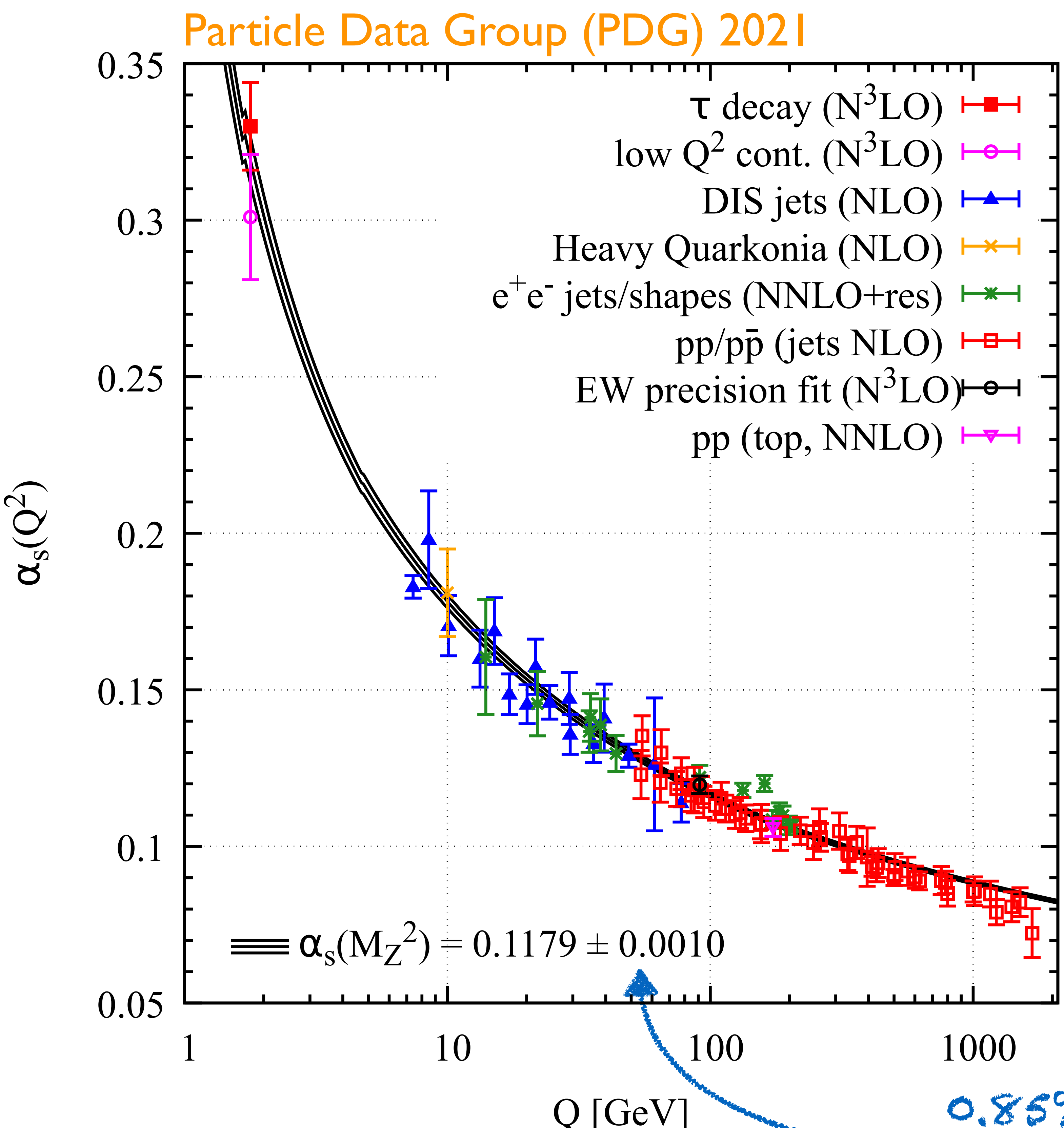


$\alpha_s(Q^2) \quad [\overline{\mathrm{MS}}, n_f]$

Overall picture is very consistent.

Discrepancies persist:
uncertainty has been *enlarged*.

strong coupling: world average



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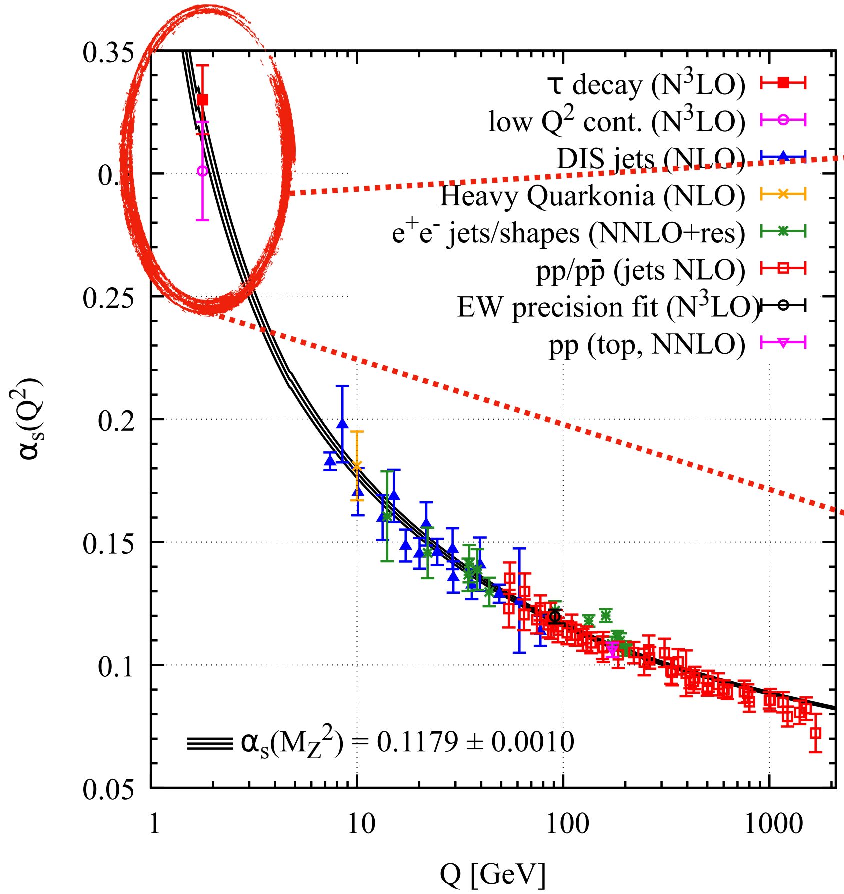
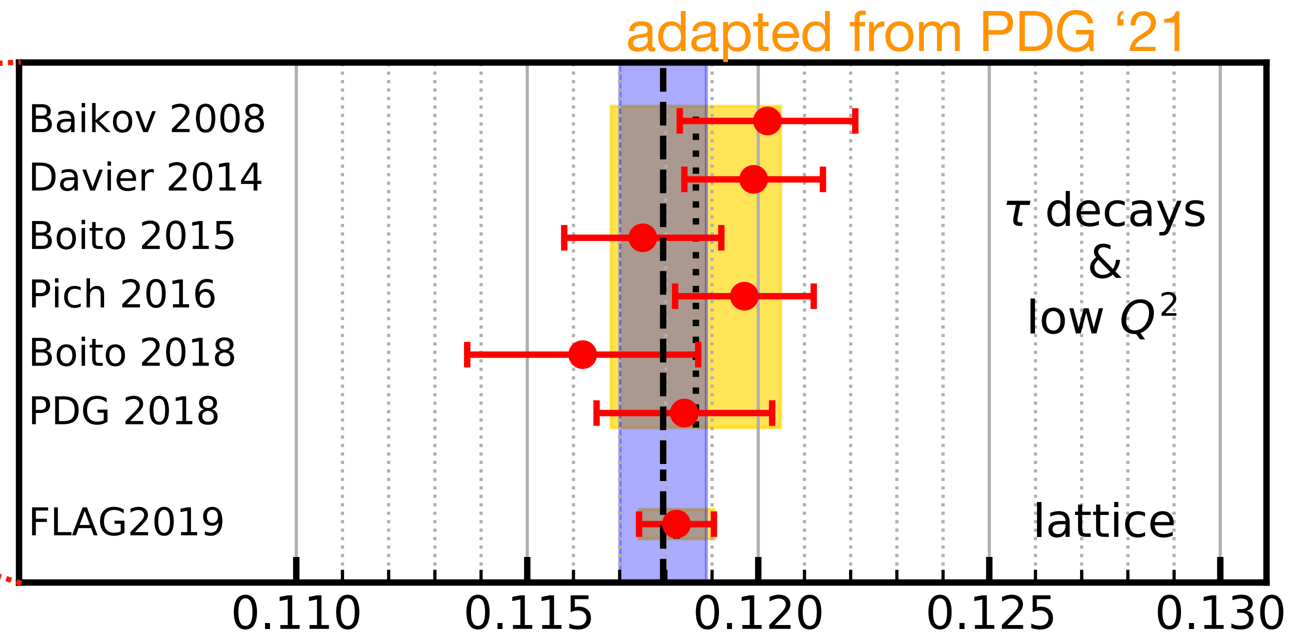
0.85% error. (The PDG uncertainty was +/- 0.0007 in 2014)

strong coupling: world average



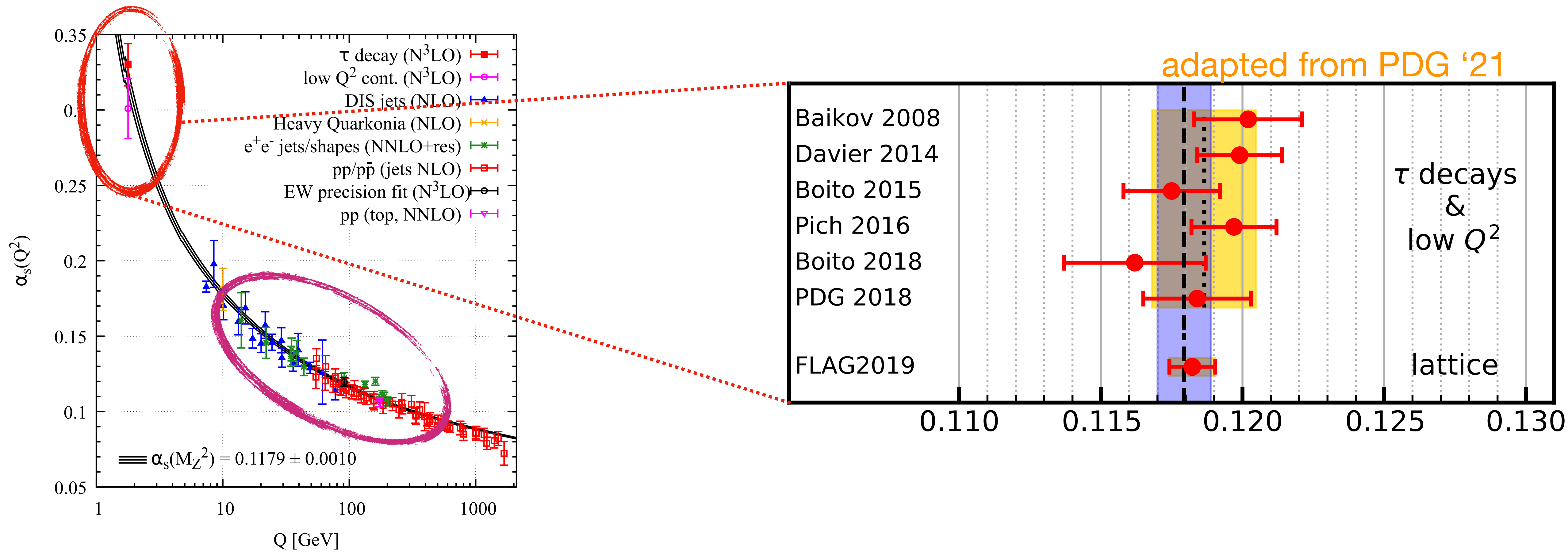


Lower energies

Larger coupling, more sensitivity to QCD corrections.

Larger non-perturbative physics (OPE, DVs), Problems with pt. theory (renormalons,...).

strong coupling: world average



Lower energies

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Larger non-perturbative physics (OPE, DVs),
Problems with pt. theory (renormalons,...).

Higher energies

Smaller coupling, less sensitive to QCD corrections, more precision required from exp.

Small contamination from non-perturbative physics,
pt. series is almost convergent

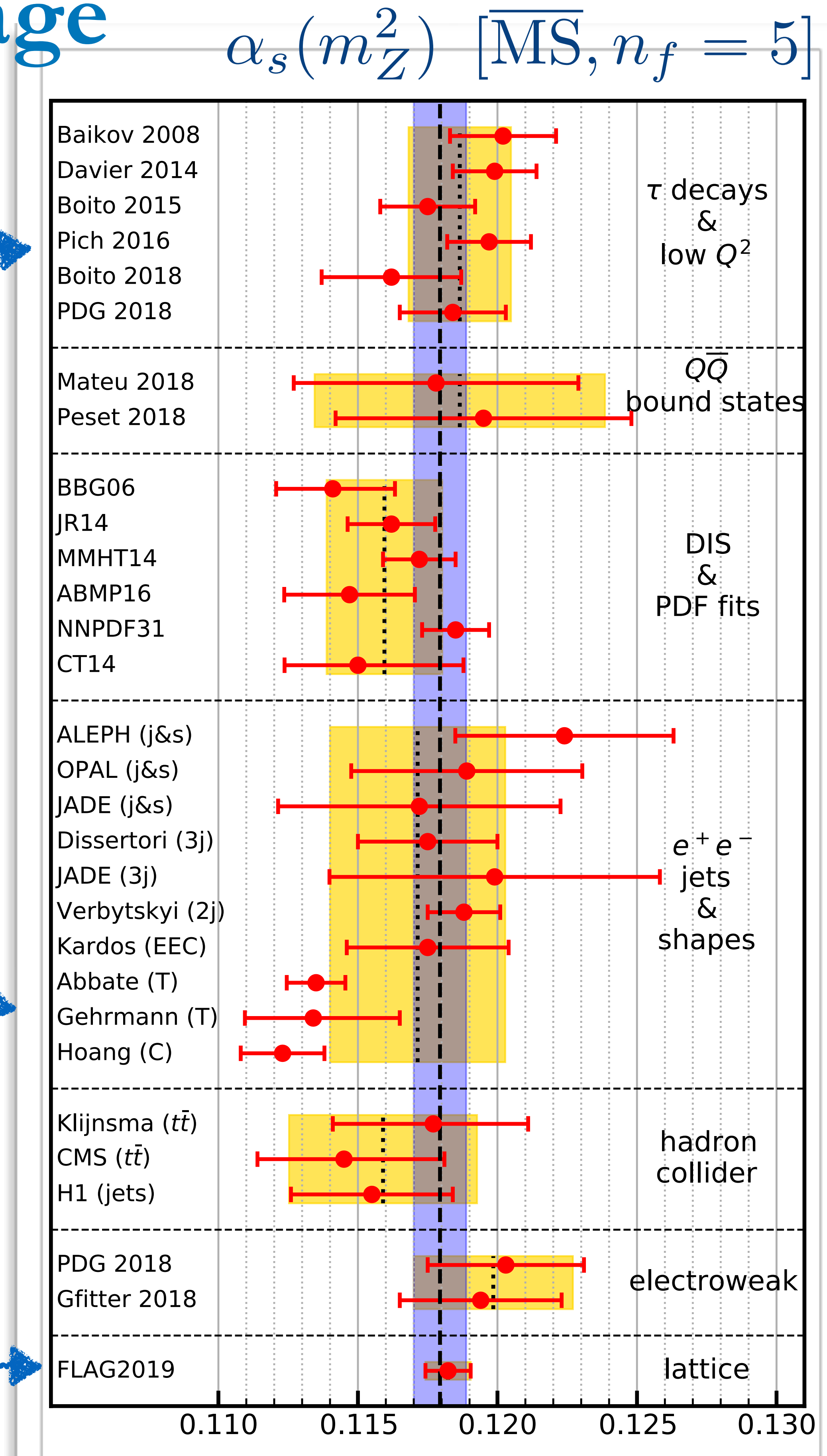
strong coupling: world average

Tensions in determinations from same data

Important to scrutinise uncertainties

Event shapes give systematically lower results

Starting to be dominated by lattice



$$\alpha_s^{(\rm PDG)}(m_Z) = 0.1179 \pm 0.0010$$

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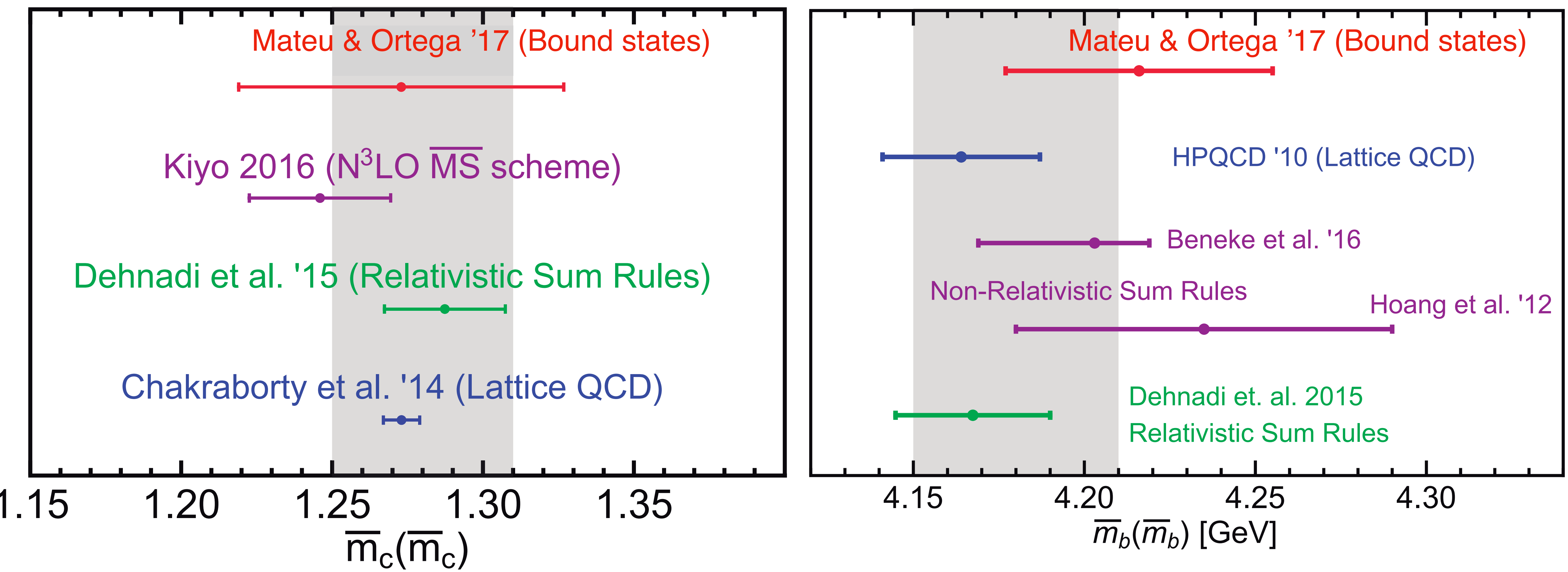
charm and bottom masses

$m_q(\mu)$ $[\overline{\text{MS}}]$

Convention: $m_q(m_q)$

Mateu & Ortega ’17



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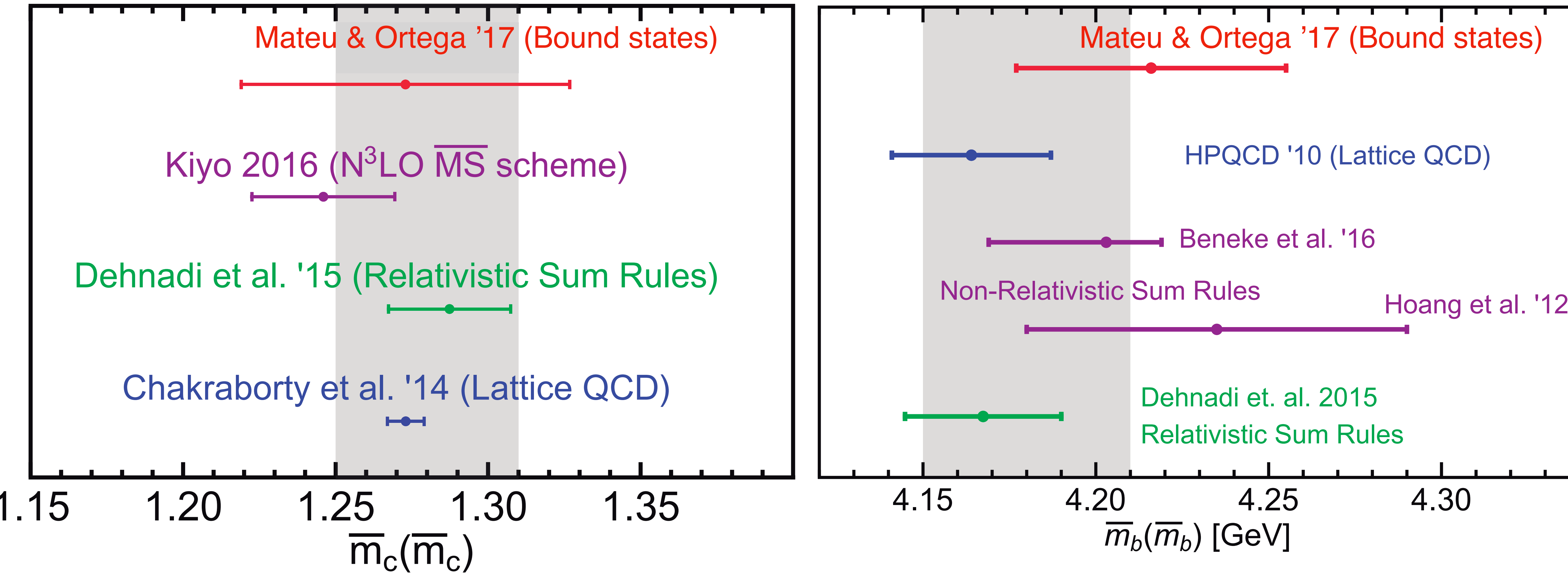


$$m_c(m_c) = 1.27 \pm 0.02 \text{ GeV}$$

$$m_b(m_b) = 4.18^{+0.03}_{-0.02} \text{ GeV}$$

PDG

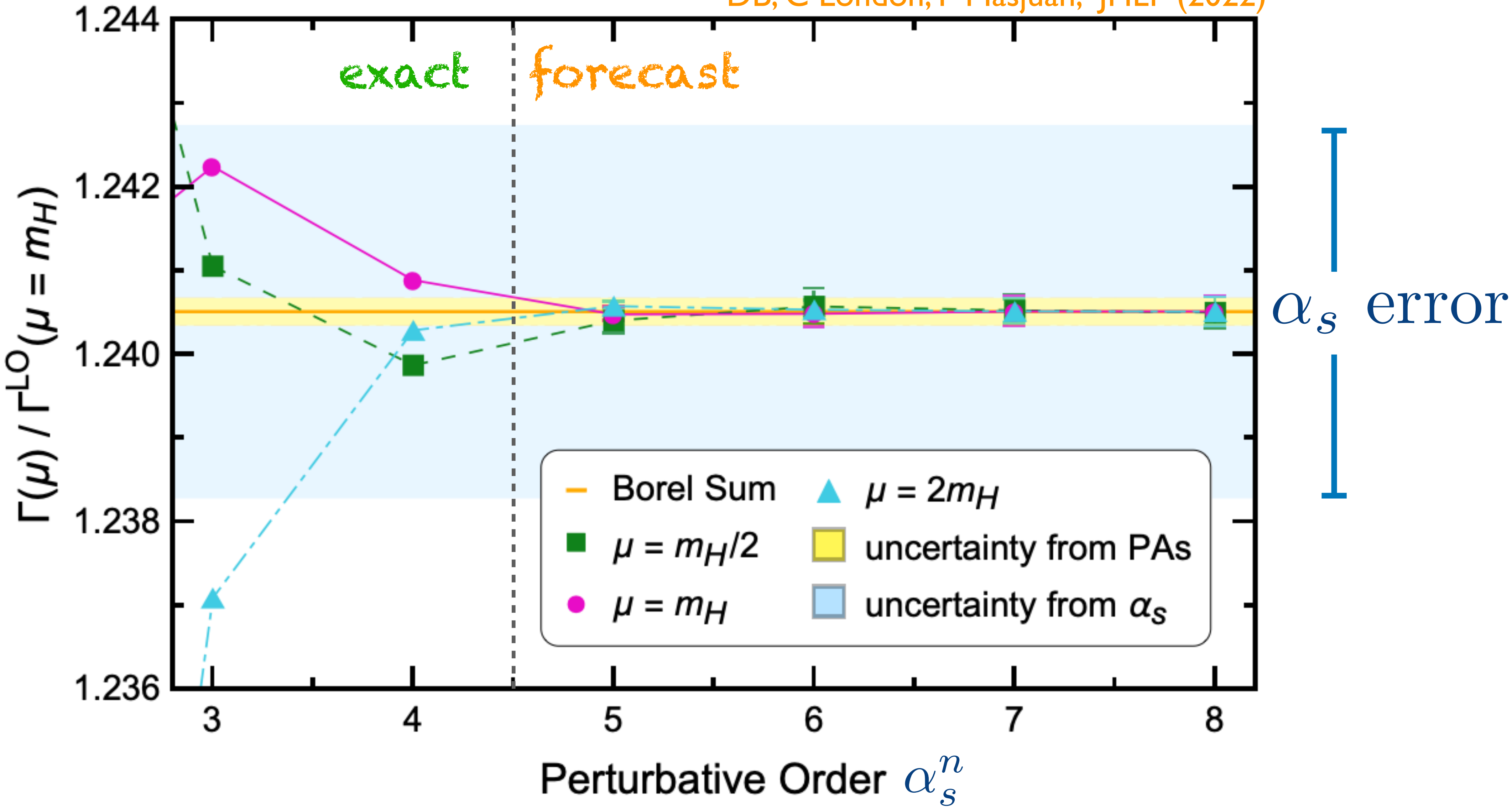
1.6% error

0.6% error

concrete example: $H \to b\bar{b}$

Normalized decay width (uncertainty from m_b not shown)

DB, C London, P Masjuan, JHEP (2022)



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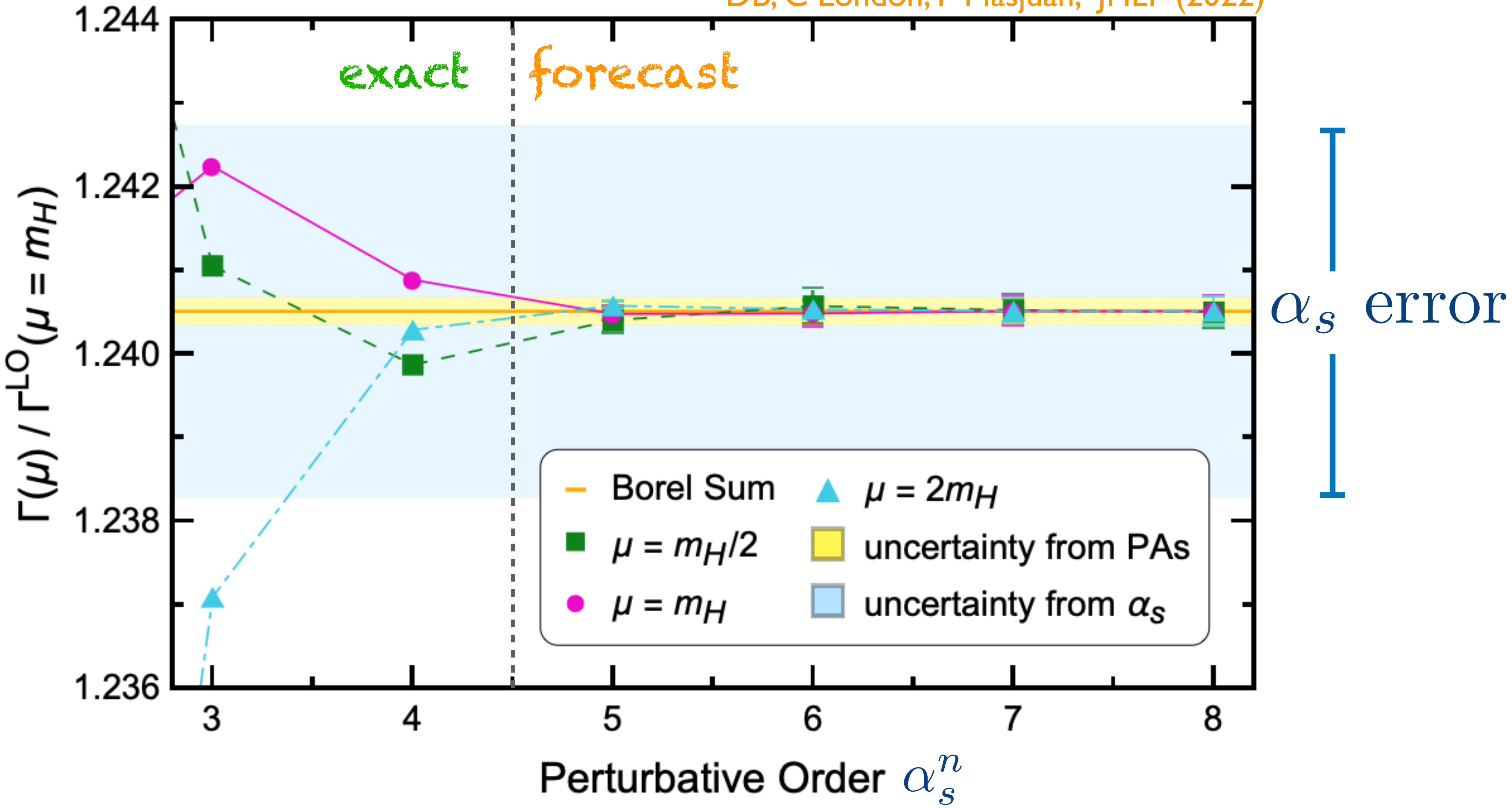


QCD parameters

$$\Gamma(H \to b\bar{b}) = 2.3806\binom{+0.041}{-0.027}_{m_b} \pm (0.0042)_{\alpha_s}$$
$$\pm (0.0032)_{m_H} \pm (0.0002)_{\mu^2} \pm (0.0003)_{\text{PAs}} \text{ MeV}.$$

missing higher orders in pt theory

Structure of Perturbative QCD

perturbation theory is divergent

Divergence of Perturbation Theory in Quantum Electrodynamics

F. J. DYSON
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received November 5, 1951)

An argument is presented which leads tentatively to the conclusion that all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge. The divergence in no way restricts the accuracy of practical calculations that can be made with the theory, but raises important questions of principle concerning the nature of the physical concepts upon which the theory is built.

$$R \sim \sum_{n}^{\infty} r_n \alpha_{(s)}^{n+1}$$

↓

?

in realistic QFTs we only know the expansion of R

divergent but (hopefully) asymptotic (seems to agree with exp.)

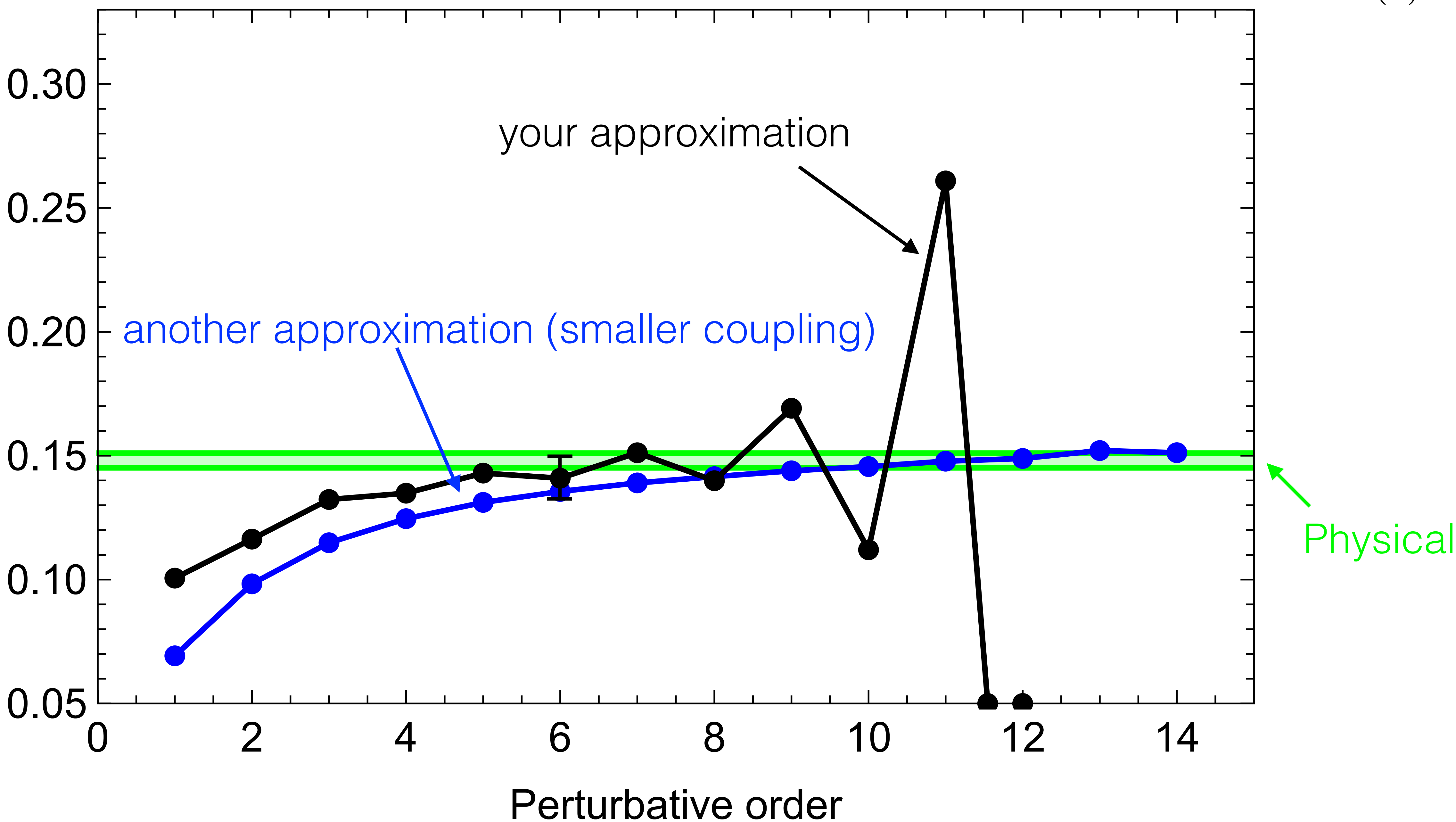
"What do we really know? What we have said appears to be compelling on physics grounds and is (probably) correct, but mathematical proofs are rare."

– M Beneke, "Renormalons" '99

toy asymptotic series

Perturbative expansions in QFTs are (at best) asymptotic

Optimal truncation depends on the magnitude of the coupling $N^* \sim \dfrac{1}{\alpha_{(s)}}$



not everything is lost

> *Divergent series converge faster than convergent series because they don't have to converge*
>
> — G. Carrier's rule

perturbative QCD

Perturbation theory is divergent (asymptotic series)

Dyson '52 $r_n \sim n!$

$$R \sim \sum_{n=0}^{n^*} r_n \alpha_s^{n+1} + e^{-p/\alpha_s}$$

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operator product expansion (OPE) cond.

$$R \sim \sum_{n=0}^{n^*} r_n \alpha_s^{n+1} + e^{-p/\alpha_s} \xrightarrow{\alpha_s(Q^2)} R \sim \sum_{n=0}^{n^*} r_n \alpha_s^{n+1} + \left(\frac{\Lambda^2}{Q^2}\right)^p$$

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$$R \sim \sum_{n}^{n^*} r_n \alpha_s^{n+1} + \sum_{k}^{k^*} \frac{C_{2k}}{Q^{2k}}$$

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The OPE is also only asymptotic

Duality violations

$$R \sim \sum_{n}^{n^*} r_n \alpha_s^{n+1} + \sum_{k}^{k^*} \frac{C_{2k}}{Q^{2k}} + e^{-\gamma q^2} \kappa \sin(\alpha + \beta q^2)$$

DB, I. Caprini, M. Golterman, K. Maltman, S. Peris '18

O. Catà, M. Golterman, S. Peris '05, '06, '08

Resurgent and trans-series: powers, exponentials and logs. Analytic continuation in the complex plane.

J Écalle, 80s. See works by G Dunne for (mathematical) physicist perspective

perturbative QCD: renormalons

Perturbation theory is divergent (asymptotic series)

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Borel transform method

$$B[R](u) = \sum_{n=0}^{\infty} \frac{r_n}{n!} \left(\frac{2\pi}{\beta_1}\right)^{n+1} u^n$$

"true" value $\Longrightarrow$

$$\tilde{R} \equiv \int_0^{\infty} du\, e^{-2u/(\beta_1 a_s)} B[R](u)$$

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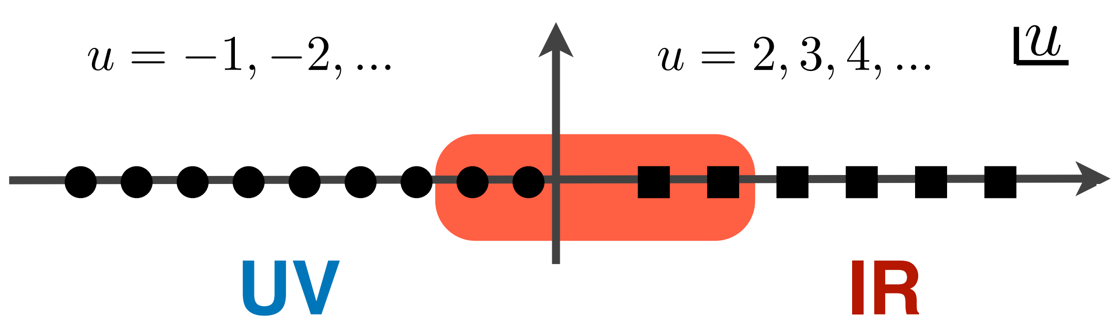
Singularities in the Borel plane: ***renormalons***

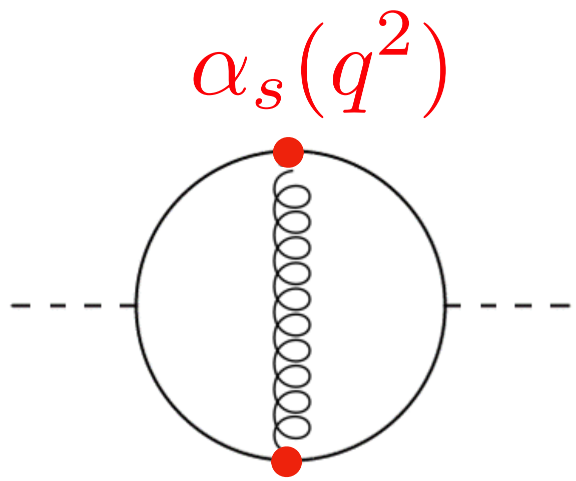
Beneke '99

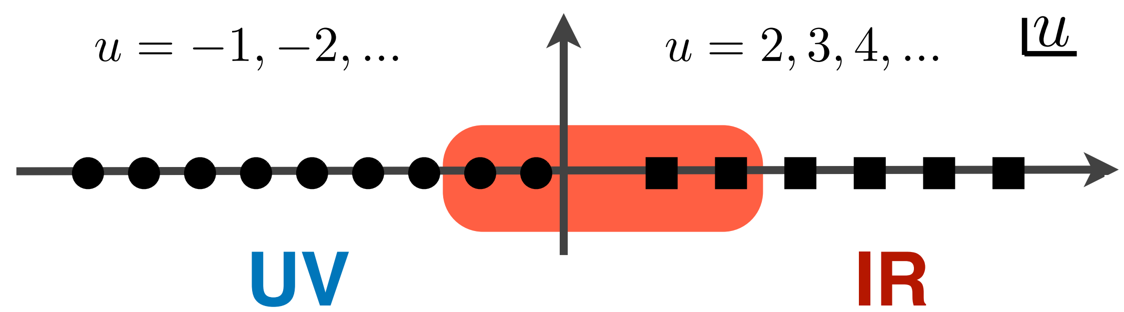


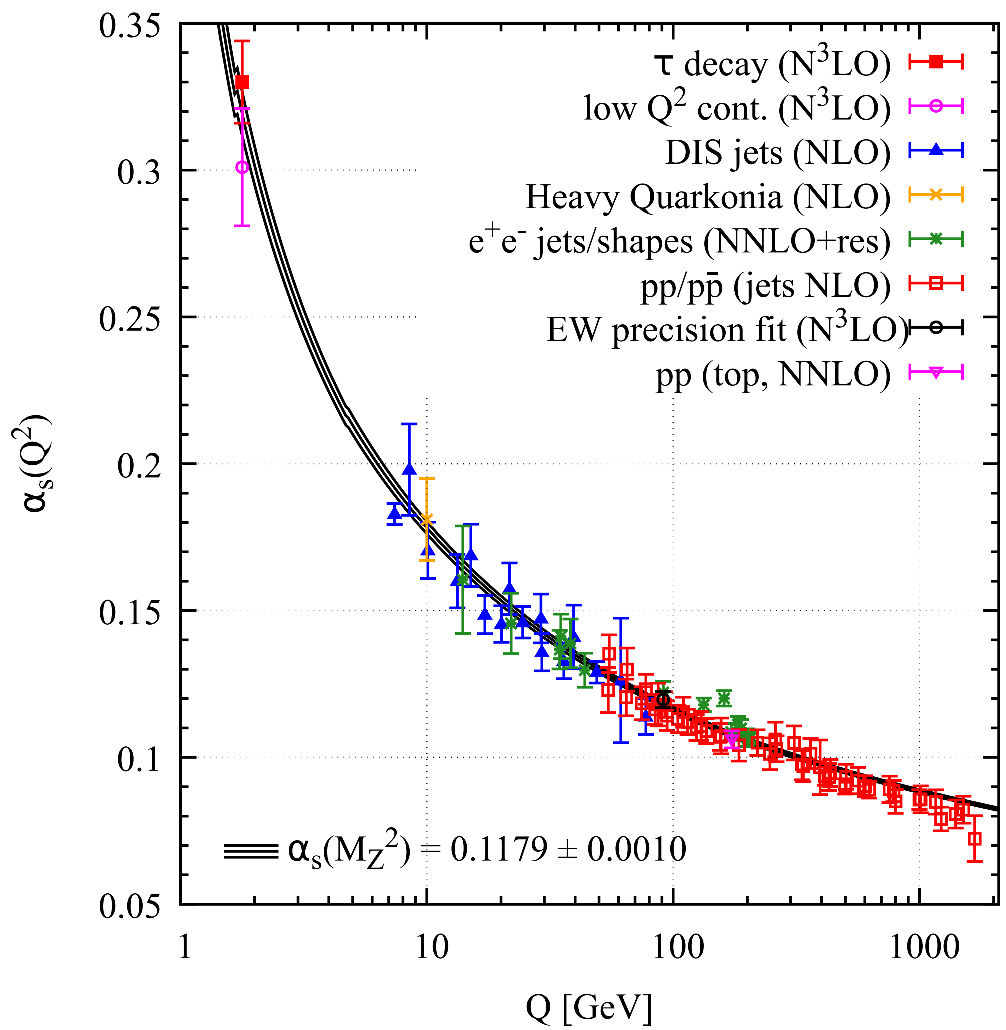
OPE

$$\frac{1}{(u-2)} \longleftrightarrow \left(\frac{\Lambda^2}{Q^2}\right)^2$$

perturbative QCD: renormalons







UV renormalons: strong coupling goes to zero, but not fast enough

IR renormalons: manifestation of the Landau pole

perturbative QCD

General structure of the perturbative series

perturbative expansion

"duality violations"

$$R \sim \sum_{n}^{n^*} r_n \alpha_s^{n+1} + \frac{C_4}{Q^4} + \frac{C_6}{Q^6} + \cdots + e^{-\gamma Q^2}(\cdots)$$

OPE condensates

non-perturbative contributions [np (!)]
smaller with higher energies

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OPE condensates

non-perturbative contributions [np (!)]
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our knowledge is always partial: theoretical uncertainty must be carefully estimated

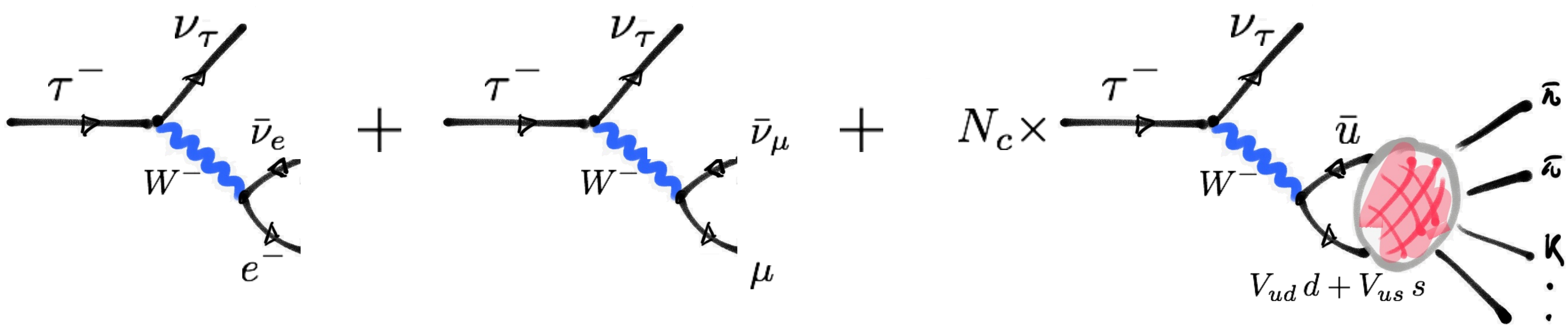
Strong coupling from

$$\tau \to (\text{hadrons}) + \nu_\tau$$

perturbative QCD

Universality of the weak charged-current interactions

$$\mathcal{L}_{CC}^{\rm SM} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_\ell \bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\ell + \bar{u}\gamma^\mu(1-\gamma_5)(V_{ud}\, d + V_{us}\, s) \right] + \text{h.c.}$$



Same branching ratios for leptonic decays (up to EW and mass corrections)

Naïve predictions

$$\text{Br}(\tau \to \nu_\tau\, e\, \bar{\nu}_e) = \frac{1}{5} = 20\%$$

$$\text{Br}(\tau \to \nu_\tau\, \mu\, \bar{\nu}_\mu) = \frac{1}{5} = 20\%$$

$$\frac{\Gamma(\tau \to \nu_\tau + \text{hadrons})}{\Gamma(\tau \to \nu_\tau\, e\, \bar{\nu}_e)} \approx N_c = 3$$

perturbative QCD

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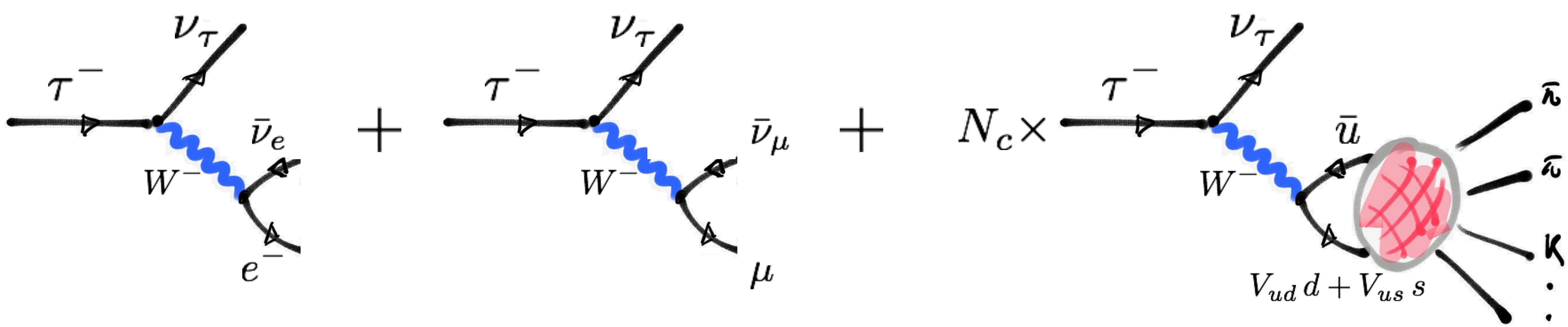

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Experiments

$$\text{Br}(\tau \to \nu_\tau\, e\bar{\nu}_e) = 17.818(41)\%$$

$$\text{Br}(\tau \to \nu_\tau\, \mu\bar{\nu}_\mu) = 17.392(40)\%$$

$$\frac{\Gamma(\tau \to \nu_\tau + \text{hadrons})}{\Gamma(\tau \to \nu_\tau\, e\, \bar{\nu}_e)} = 3.6280(94)$$

perturbative QCD

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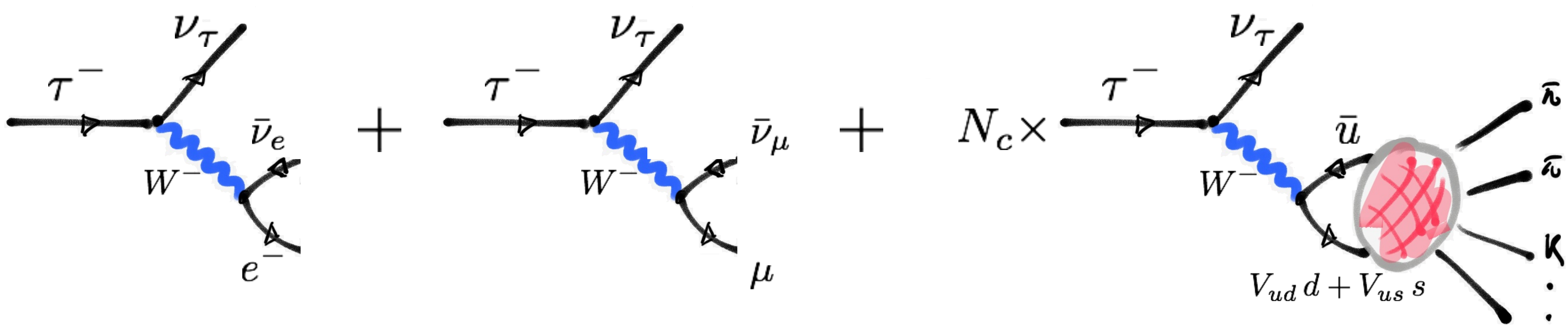

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~ 20% more than naïve

QCD

Experiments

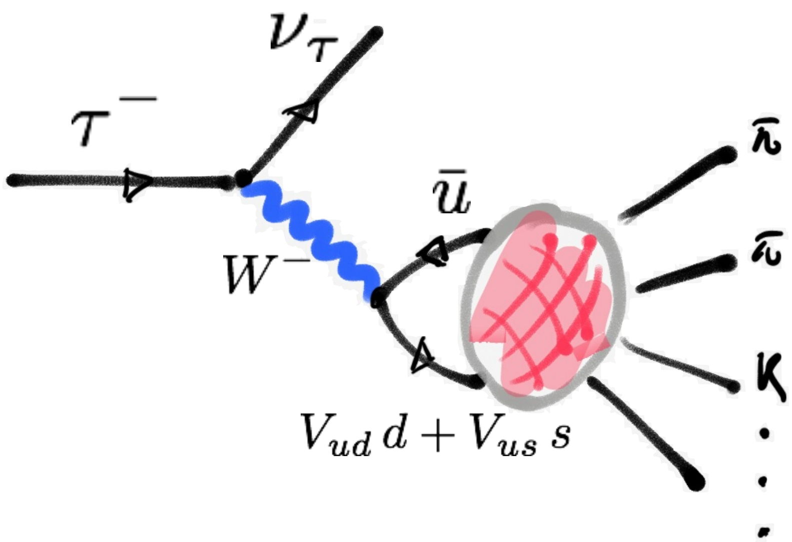
$$\text{Br}(\tau \to \nu_\tau\, e \bar{\nu}_e) = 17.818(41)\%$$

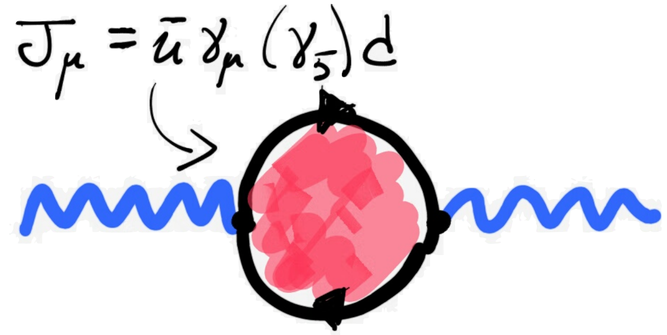
$$\text{Br}(\tau \to \nu_\tau\, \mu \bar{\nu}_\mu) = 17.392(40)\%$$

$$\frac{\Gamma(\tau \to \nu_\tau + \text{hadrons})}{\Gamma(\tau \to \nu_\tau\, e\, \bar{\nu}_e)} = 3.6280(94)$$

hadronic tau decays

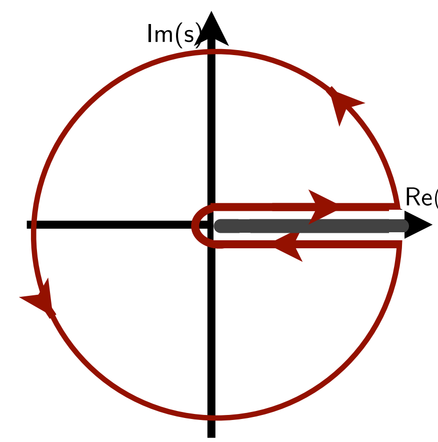
Inclusive quantity: quark hadron duality





$$\Pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle 0|T\{J_\mu(x)J_\nu(0)^\dagger\}|\rangle$$

Massless (V&A) correlators



Sum rules (using Cauchy's theorem)

$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s)\frac{1}{\pi}\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(z)\Pi(z)$$

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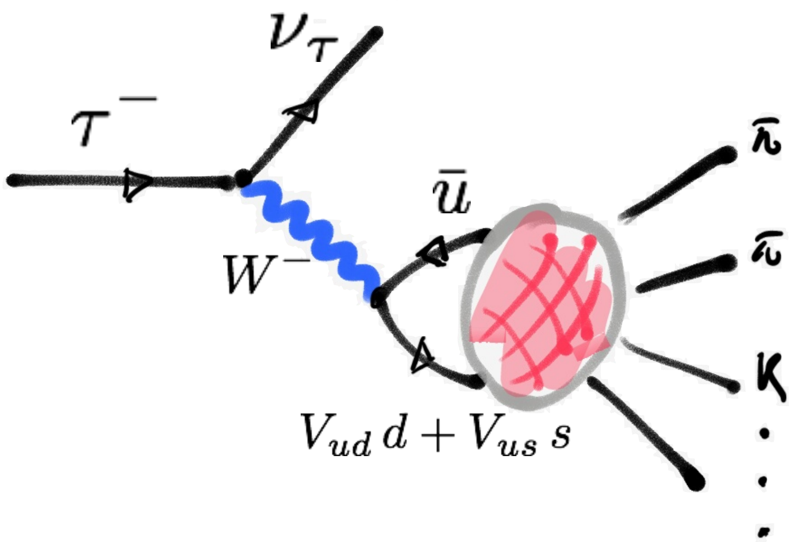

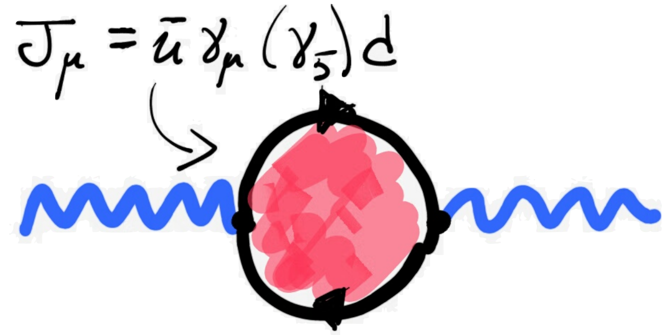

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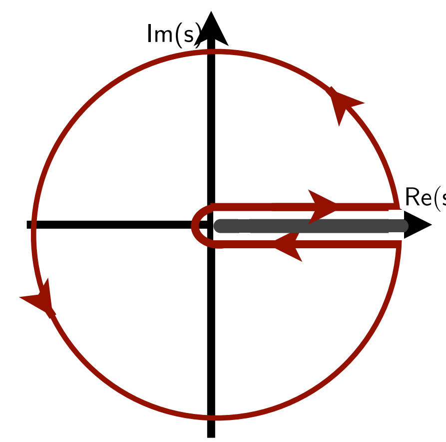

Sum rules (using Cauchy's theorem)

experiment

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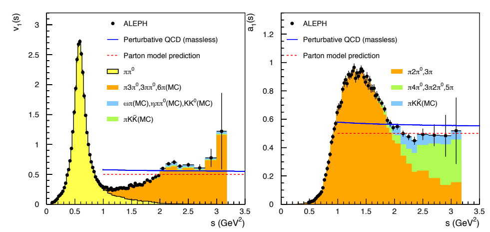
theory

hadronic tau decays

experiment

$$\frac{1}{s_0}\int_0^{s_0} ds\, w(s)\, \frac{1}{\pi}\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\, w(z)\Pi(z)$$

theory



Any analytic weight function gives rise to a valid sum rule

$x = s/s_0$

Kinematic moment: special case

$$w_\tau(s) = (1 - s/s_0)^2(1 + 2s/s_0) = (1-x)^2(1+2x) \underset{s_0 = m_\tau^2}{\Longrightarrow} R_\tau = \frac{\Gamma[\tau \to \text{hadrons}\, \nu_\tau]}{\Gamma[\tau \to e^- \bar{\nu}_e\, \nu_\tau]}$$

overview of QCD in hadronic tau decays

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\,\Pi(z) \approx S_{\rm EW}\, N_c(1+\delta^{(0)}+\delta_{\rm EW}+\delta_{\rm OPE}+\delta_{\rm DVs})$$

theory

Perturbation theory (OPE) ⟶ $$\sum_{n=0}^{4}\left(\frac{\alpha_s}{\pi}\right)^n \sum_{k=0}^{n+1} c_{n,k}\log^k\left(\frac{-s}{\mu^2}\right) + \frac{C_4}{Q^4}+\frac{C_6}{Q^6}+\frac{C_8}{Q^8}+\cdots$$

Gorishnii, Kataev, Larin '91
Surguladze&Samuel '91

Baikov, Chetyrkin, Kühn '08

α_s^1 α_s^2 α_s^3 α_s^4

$$\delta^{(0)}_{\rm FO} = 0.1012+0.0533+0.0273+0.0133 = 0.1952$$

(fixed order, more about that soon)

pt. correction is ~20%

overview of QCD in hadronic tau decays

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$$\delta^{(0)}_{\rm FO} = 0.1012 + 0.0533 + 0.0273 + 0.0133 \;=\; 0.1952$$

(fixed order, more about that soon)

pt. correction is ~20%

Duality Violations ⟶ $$\rho_{\rm DV}(s) = e^{-\delta-\gamma s}\sin(\alpha+\beta s)$$

Ansatz based on widely accepted assumptions about QCD: Regge behaviour and large-N_c. Leading corrections: logarithmic and powers of *1/s*.

DB, Caprini, Golterman, Maltman, Peris, PRD '18

Fixed Order vs Contour Improved

$x = s/s_0$

$$\delta^{(0)} = \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}W_i(x)\sum_{n=1}^{\infty} a_\mu^n \sum_{k=1}^{n} c_{n,k}\log^{k-1}\left(\frac{-s_0 x}{\mu^2}\right)$$

Fixed renormalization scale, strict Fixed Order expansion (Fixer Order Pt. Theory)

FOPT

$\mu^2 = s_0$

$$\delta^{(0)}_{\mathrm{FO},w_i} = \sum_{n=1}^{\infty} a(s_0)^n \sum_{k=1}^{n} k\, c_{n,k} J_{k-1}^{\mathrm{FO},w_i}$$

coefficients

expansion in powers of the coupling

$$J_n^{\mathrm{FO},w_i} \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}W_i(x)\log^n(-x)$$

integrals over logs

Running renormalization scale, no longer a strict power expansion in the coupling

CIPT

$\mu^2 = -s_0 x$

$$\delta^{(0)}_{\mathrm{CI},w_i} = \sum_{n=1}^{\infty} c_{n,1} J_n^{\mathrm{CI},w_i}(s_0)$$

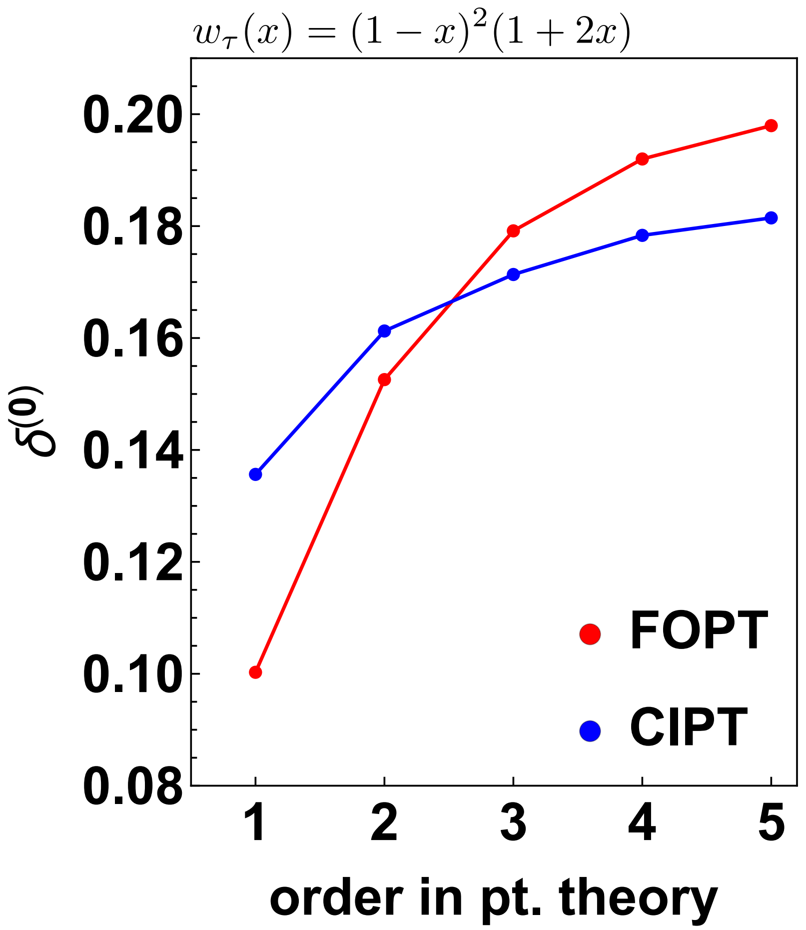
no obvious expansion parameter

$$J_n^{\mathrm{CI},w_i}(s_0) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}W_i(x)a^n(-s_0 x)$$

integral over the running coupling

Pich & Le Diberder '92
Pivovarov '92

Fixed Order vs Contour Improved



We use and estimate for the 6-loop result

$$c_{5,1} = 280 \pm 140$$

Beneke & Jamin '08
DB, Masjuan, Oliani '18
Caprini '19
Jamin '21

- CIPT leads in general to smaller perturbative contributions
- Strong coupling from CIPT larger than from FOPT
- OPE corrections assumed to be universal

$$\alpha_s(m_Z) = 0.1171 \pm 0.0010 \quad \text{FOPT}$$

$$\alpha_s(m_Z) = 0.1191 \pm 0.0012 \quad \text{CIPT}$$

discrepancy much larger than individual errors

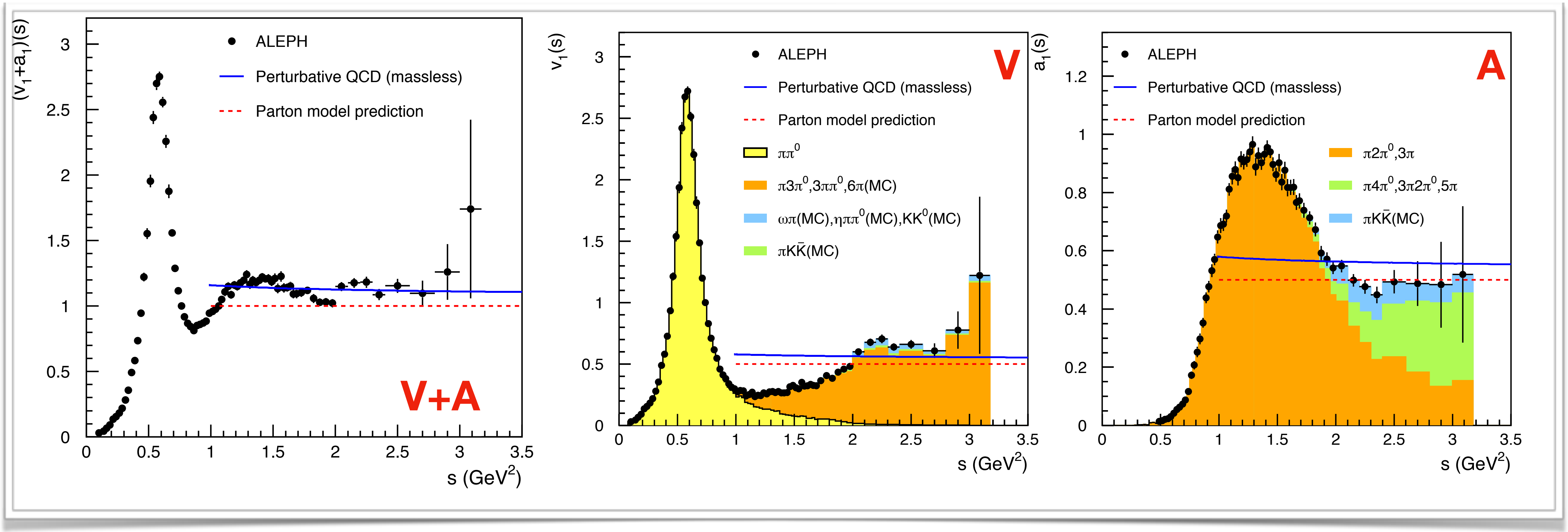
DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, 2012.10440, PRD `21

Strong coupling from $\tau \to (\text{hadrons}) + \nu_\tau$: data

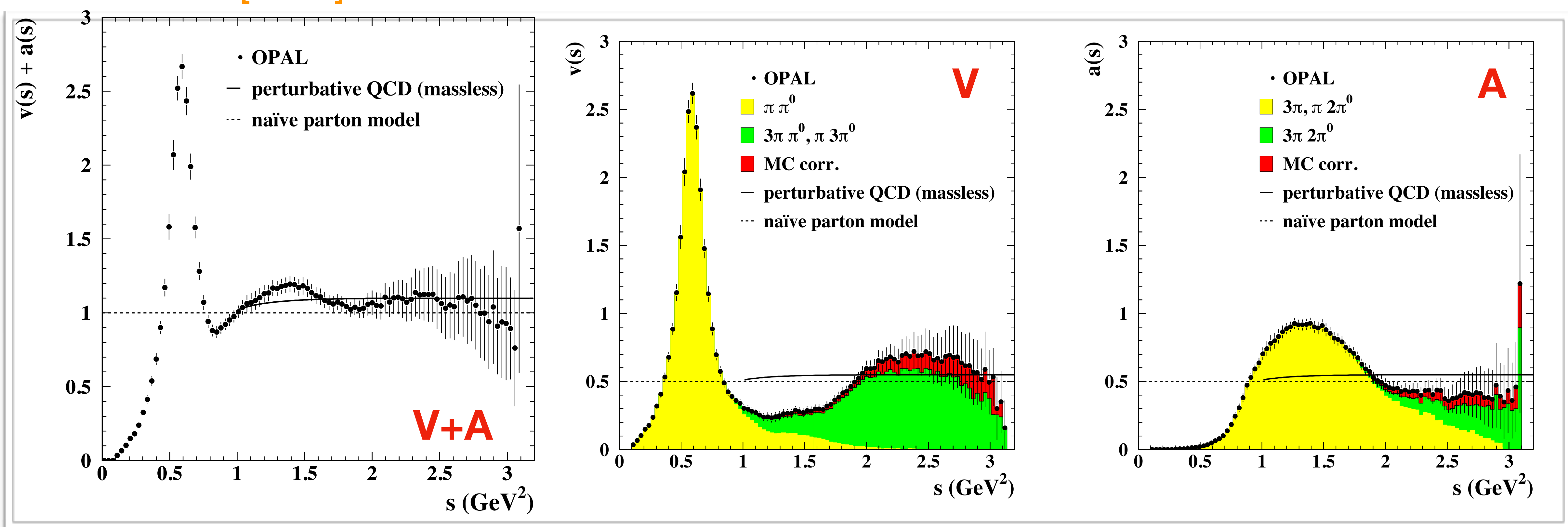
DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440 PRD (2021)

inclusive hadronic tau decay data

Davier et al [ALEPH] '14

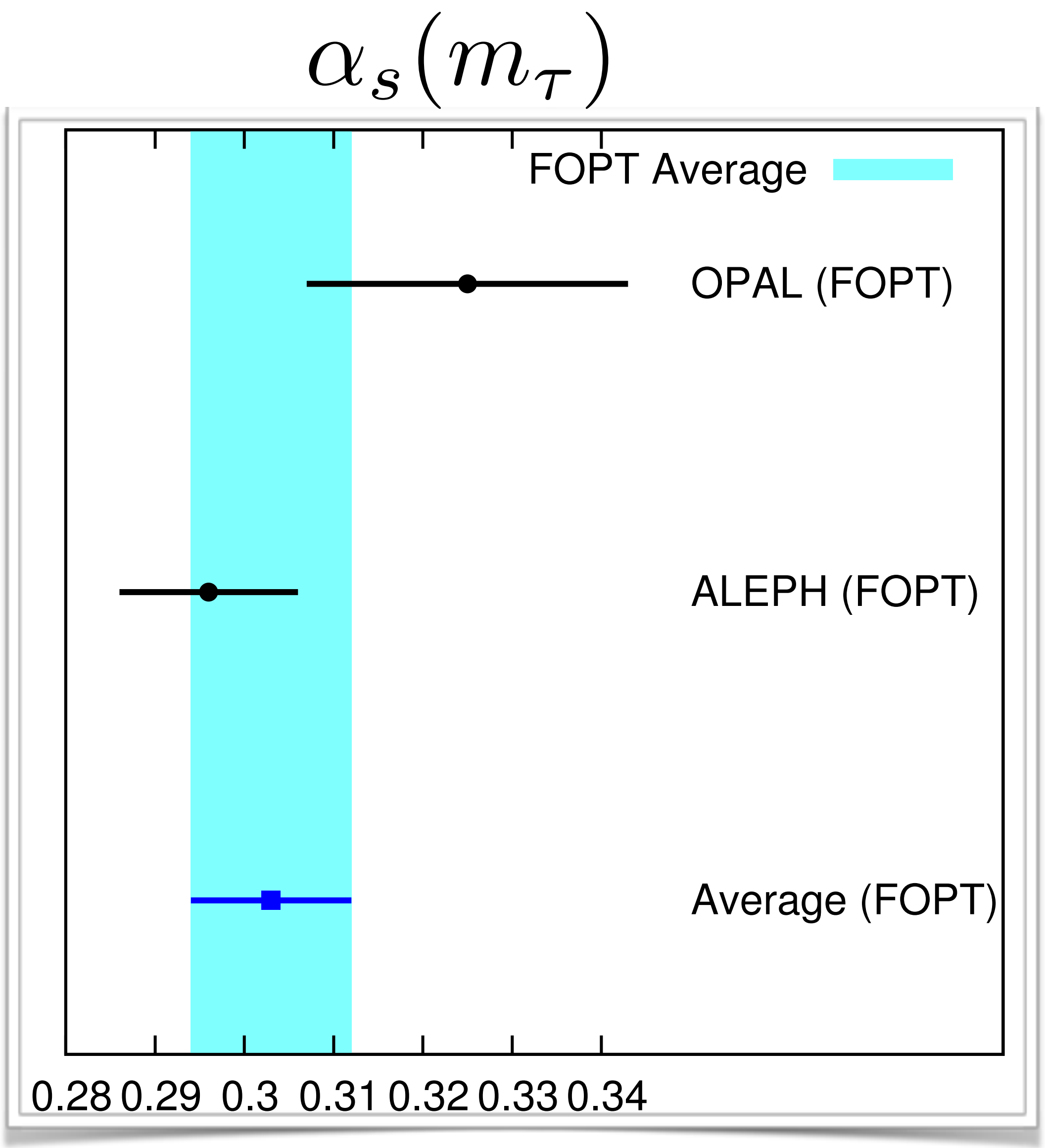


Ackerstaff et al [OPAL] '98 updated for recent values of branching fractions Boito et al '11, '21



tau decay data and the strong coupling

The two data sets lead to compatible values for $\alpha_s(m_\tau)$: average



DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, Peris, '12

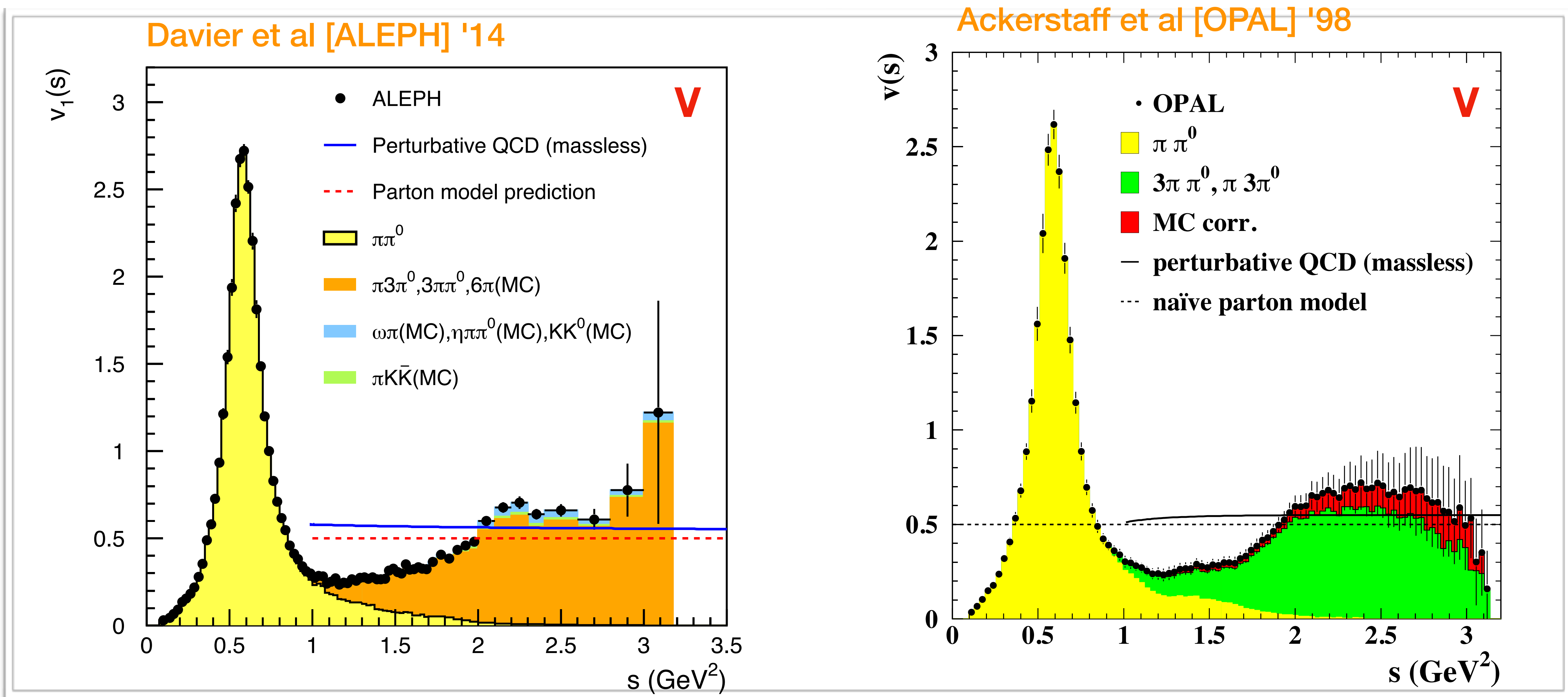
DB, Golterman, Maltman, Osborne, Peris, '15

Are the data sets compatible? Can we combine them?

Can the inclusive spectral functions be improved with recent data?

anatomy of the data sets

- V channel dominated by $\tau \to 2\pi + \nu_\tau$ and $\tau \to 4\pi + \nu_\tau$
- "Residual" channels subdominant (but important for α_s!)
- Monte Carlo (MC) inputs for several channels



Recently measured channels in e^+e^- can be used to improve the vector channel

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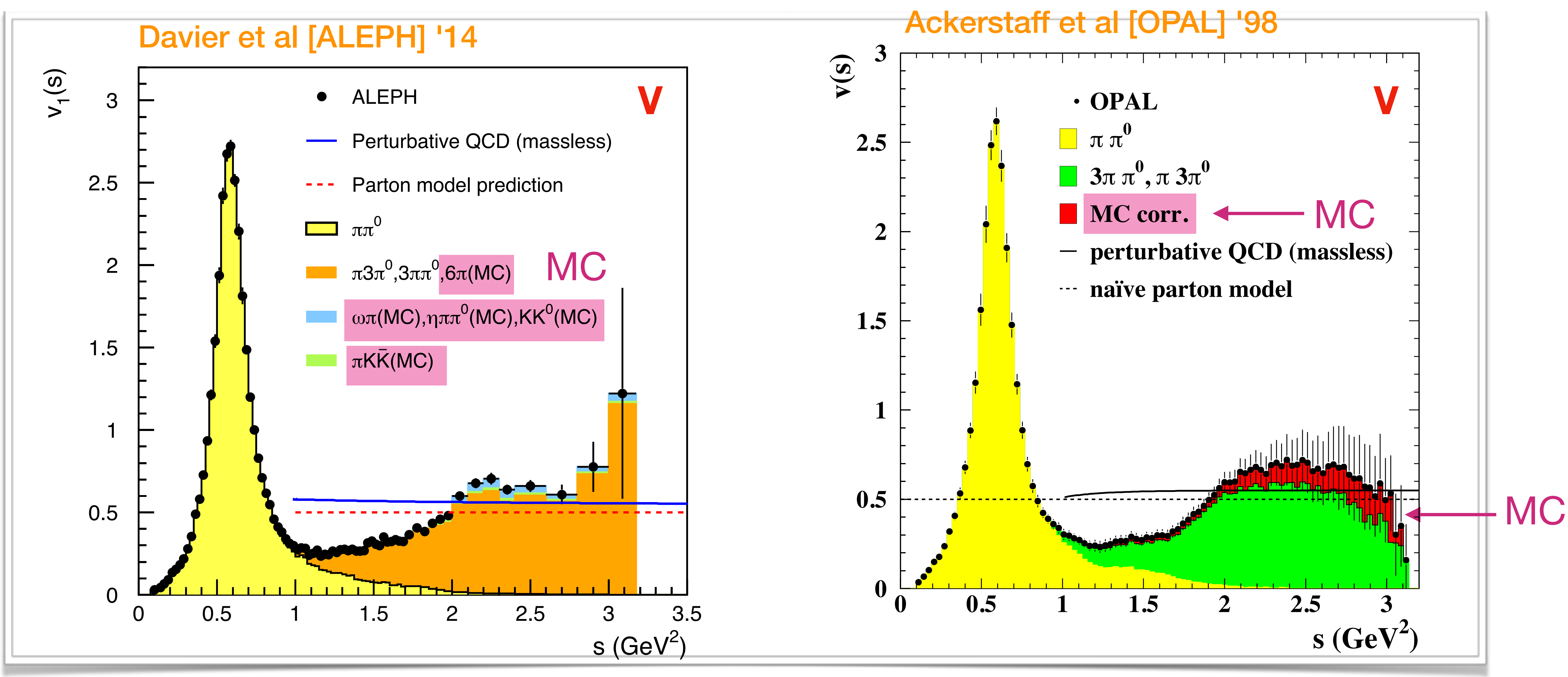


Recently measured channels in e^+e^- can be used to improve the vector channel

new vector isovector spectral function

- Combined data for 2π and 4π channels from ALEPH & OPAL

  Data combination: same algorithm used in R-data combination for muon g-2.

  Keshavarzi, Nomura, Teubner '18

- Exp. data only: 7 residual channels from e^+e^- using CVC (conserved vector current) and BaBar data for $\tau \to K K_S \nu_\tau$

  No Monte Carlo inputs; IB corrections to CVC negligible

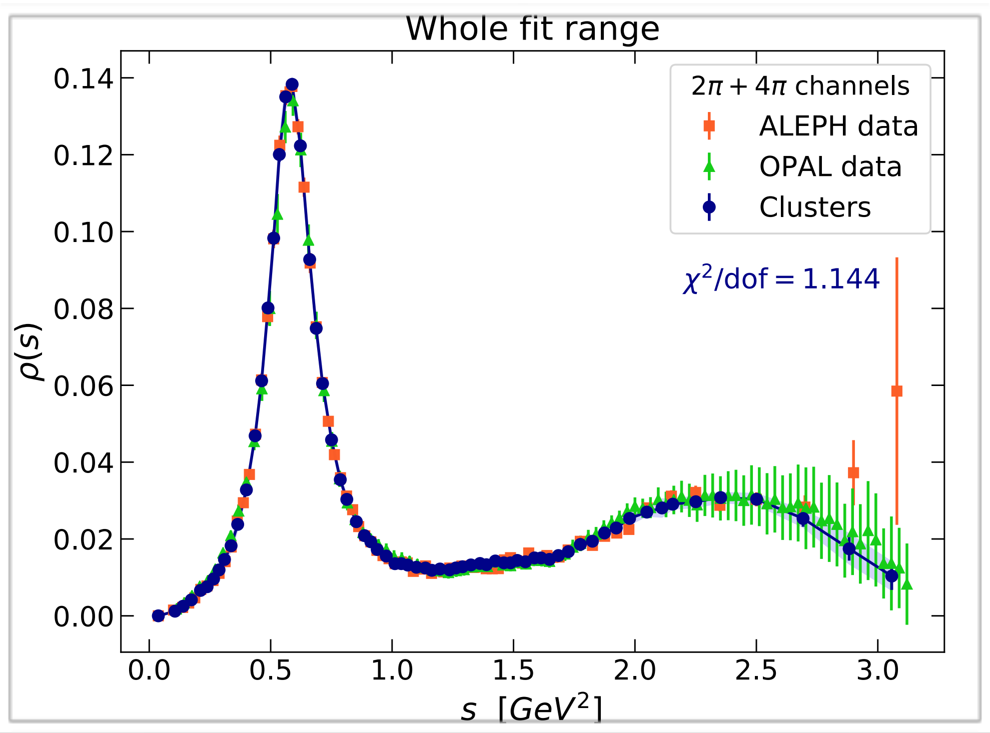
- Results updated for recent branching ratio measurements

new vector isovector spectral function

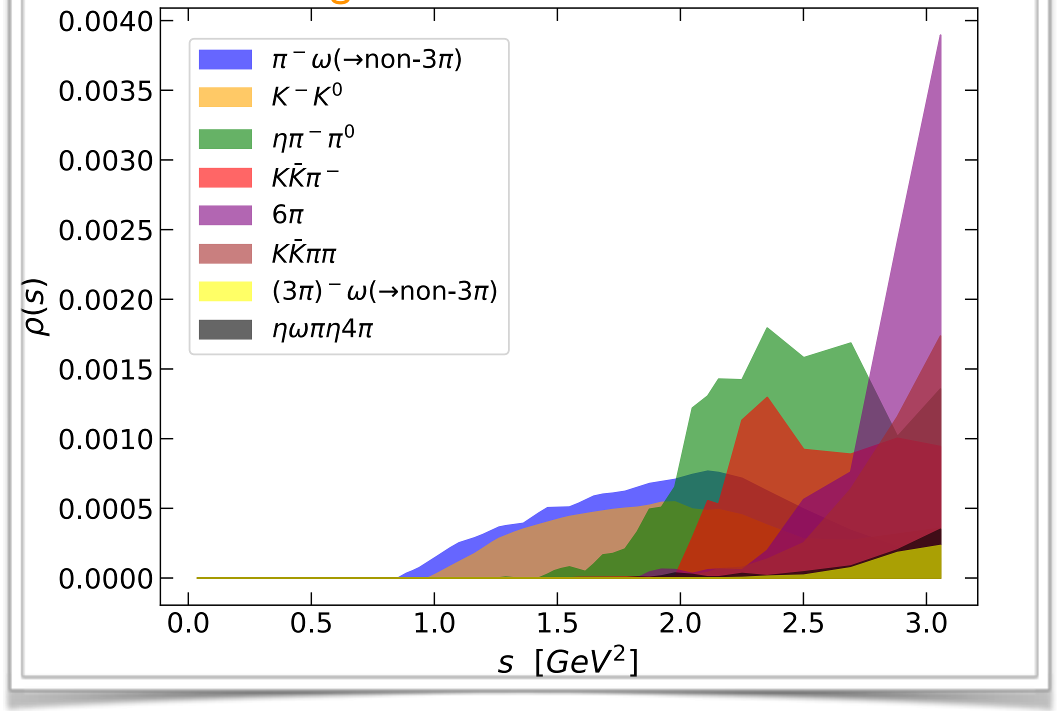
Combination of 2π $+4\pi$ channels

Good χ^2 both locally and globally, no χ^2 inflation needed

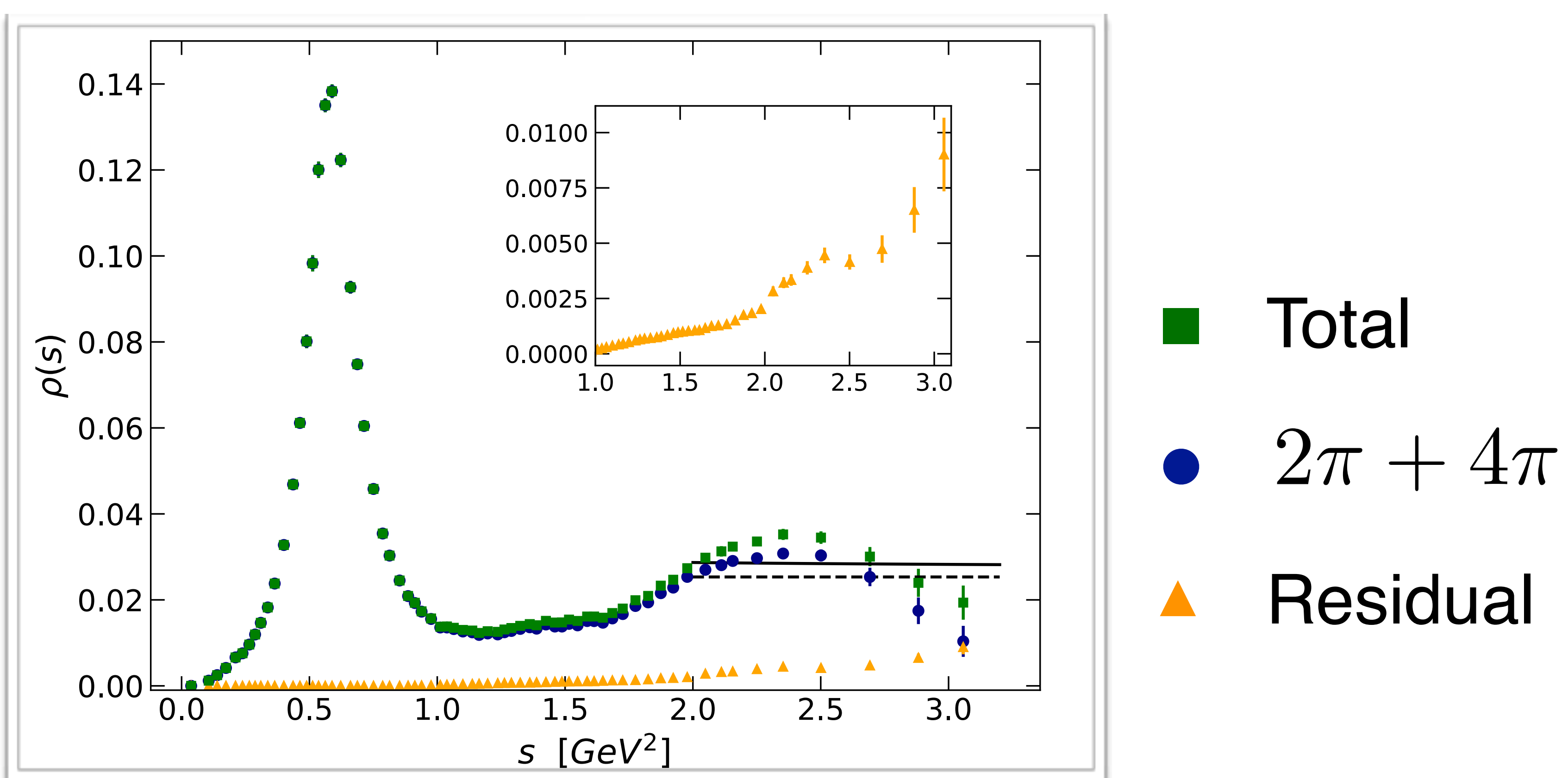
No Monte Carlo input



Original data sets from: BABAR, SND and CMD-3

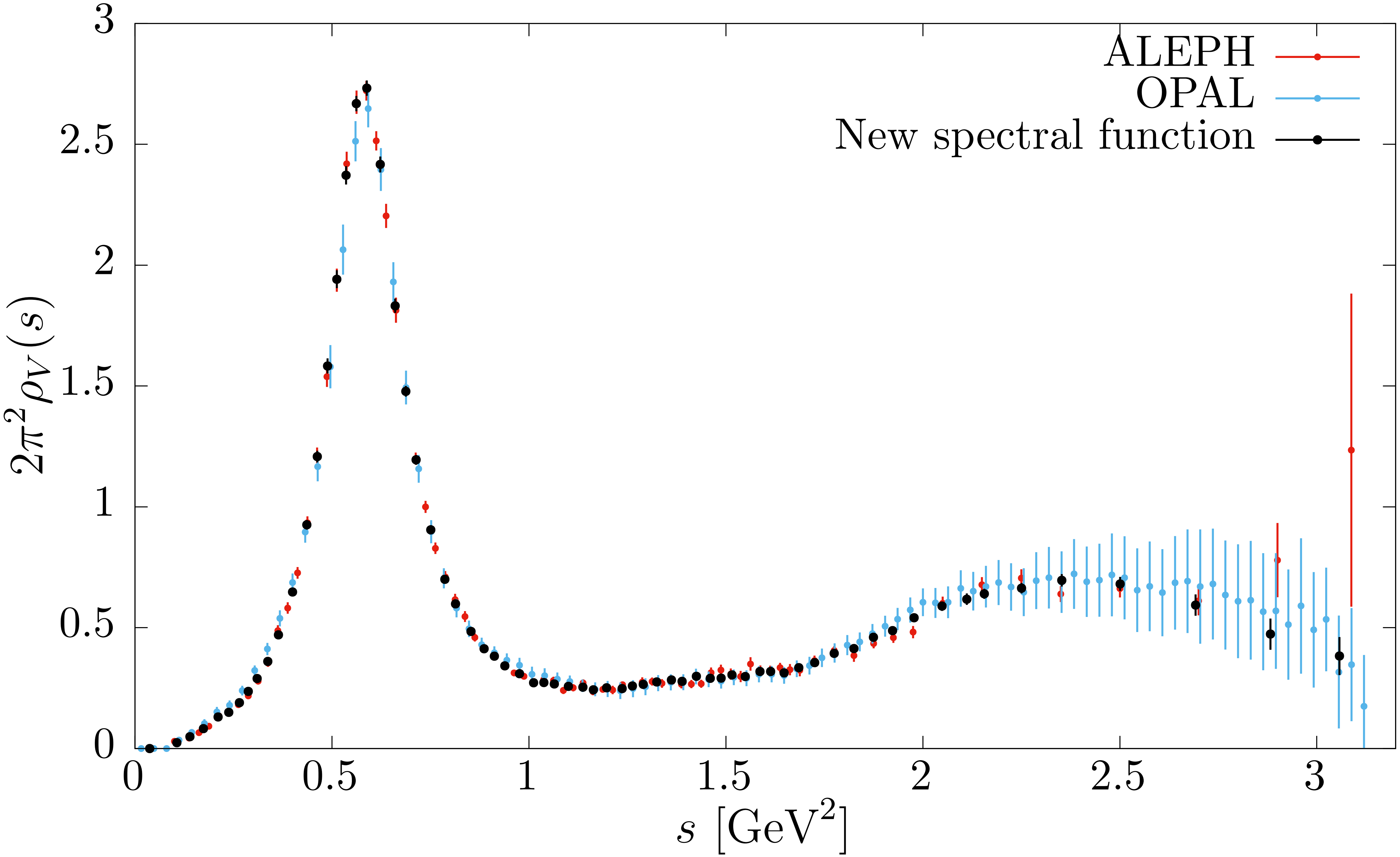


new vector-isovector spectral function



new vector isovector spectral function

New electroproduction data (after 2014) allow for an improvement of the vector spectral function



DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, 2012.10440, PRD `21

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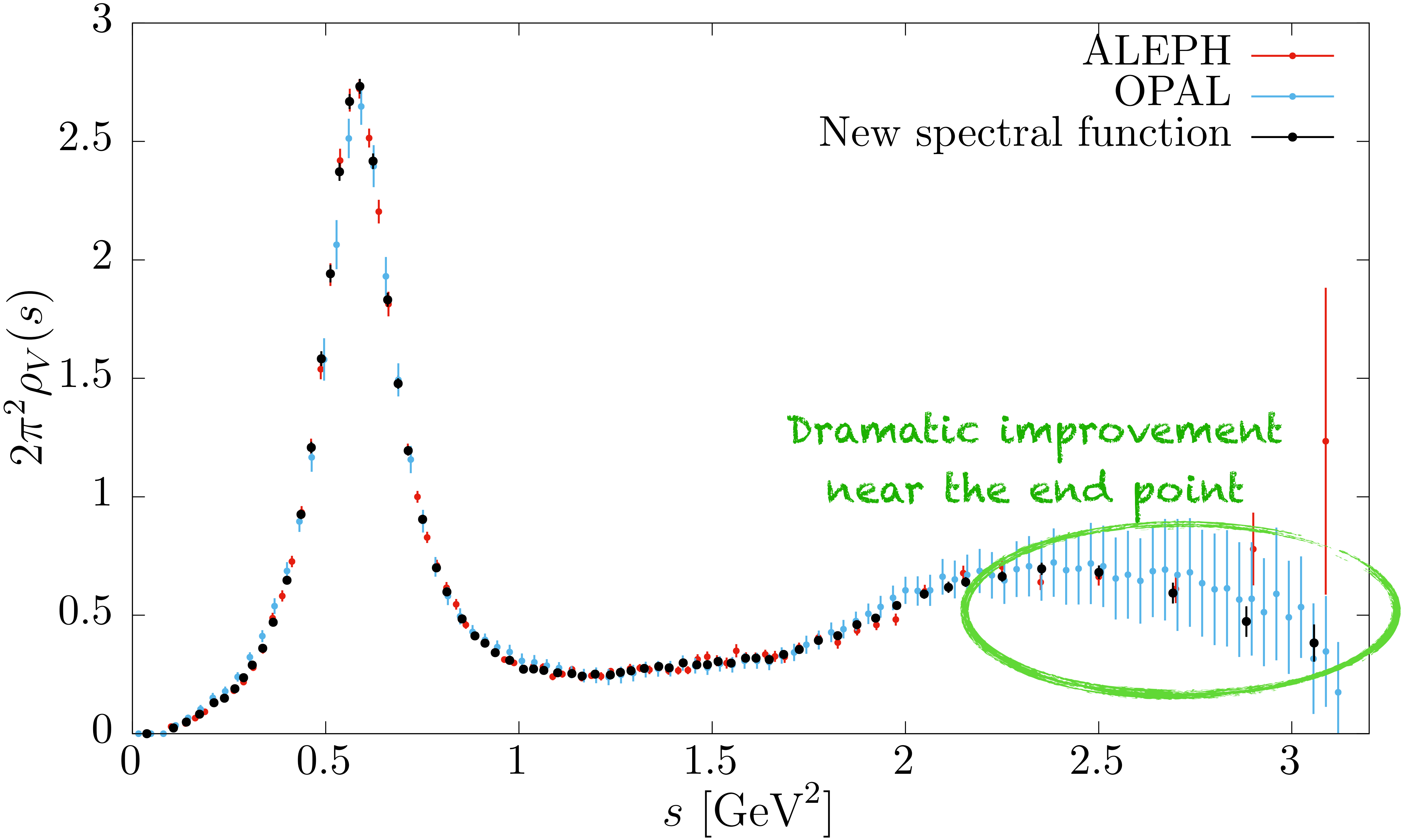

DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, 2012.10440, PRD `21

Results

analysis strategy

$$\frac{-1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi(z) \approx S_{\rm EW}\, N_c (1 + \delta^{(0)} + \delta_{\rm EW} + \delta_{\rm OPE} + \delta_{\rm DVs})$$

theory

Desired properties from the choice of weights

1. Good perturbative behaviour.
2. Small condensate contributions.
3. Suppression of DVs.

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theory

Desired properties from the choice of weights

1. Good perturbative behaviour.
2. Small condensate contributions.
3. Suppression of DVs.

Choice of weights

$w_0(y) = 1$	Tiny condensate contributions, sensitive to DVs
$w_2(y) = 1 - y^2$	Only D=6
$w_3(y) = (1-y)^2(1+2y)$	Only D=6 and 8 Tau kinematical Moment (R_τ)
$w_4(y) = (1-y^2)^2$	Only D=6 and 10

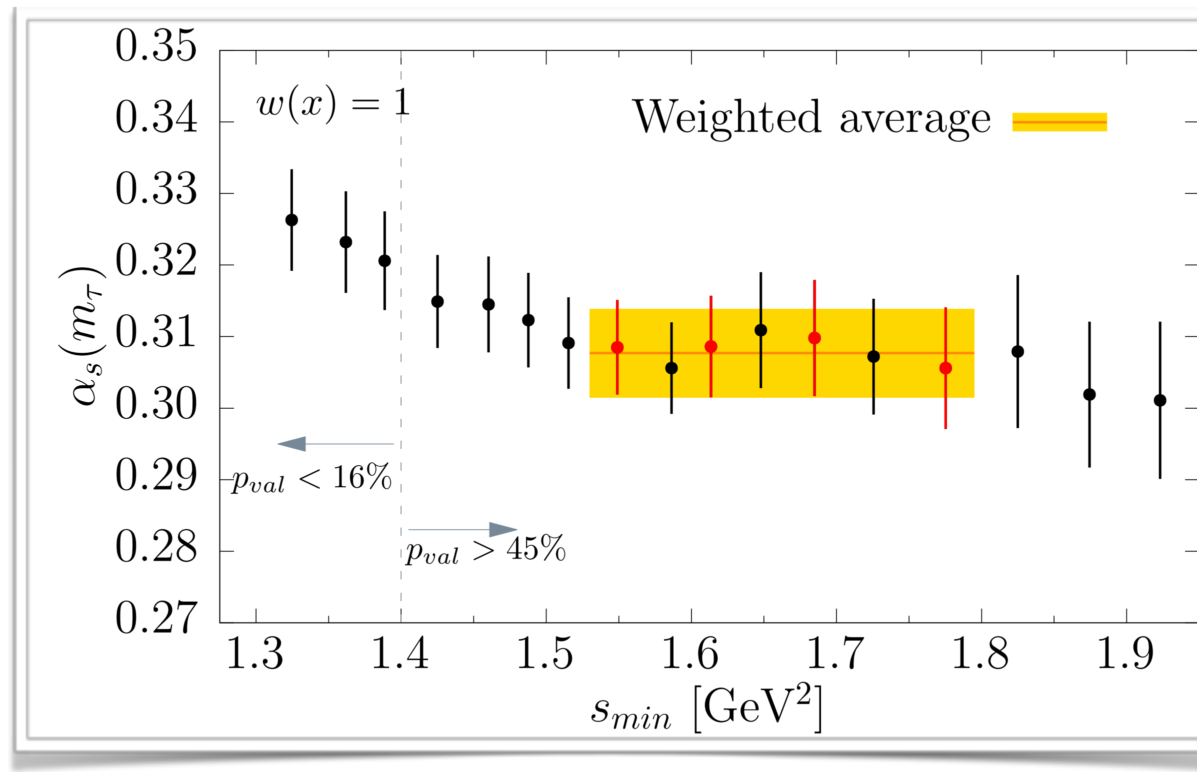
DB, Cata, Golterman, Jamin, Maltman '11, Beneke, DB, Jamin '12, DB, M. Golterman, K. Maltman, S. Peris '16
DB F Oliani '20

results from tau decays

$$\frac{1}{s_0}\int_0^{s_0} ds\,w(s)\frac{1}{\pi}\mathrm{Im}\Pi(s) = -\frac{1}{2\pi i s_0}\oint_{|z|=s_0} dz\,w(z)\Pi(z)$$

Fits with several moments:

$$w_0(y) = 1$$
$$w_2(y) = 1 - y^2$$
$$w_3(y) = (1-y)^2(1+2y)$$
$$w_4(y) = (1-y^2)^2$$



Final value

pt. series truncation, scale variation

$$\alpha_s(m_\tau) = 0.3077 \pm 0.0065_{\text{stat}} \pm 0.0038_{\text{pert}}$$
$$= 0.3077 \pm 0.0075 \qquad (n_f = 3,\ \text{FOPT})$$

Results evolved to m_Z

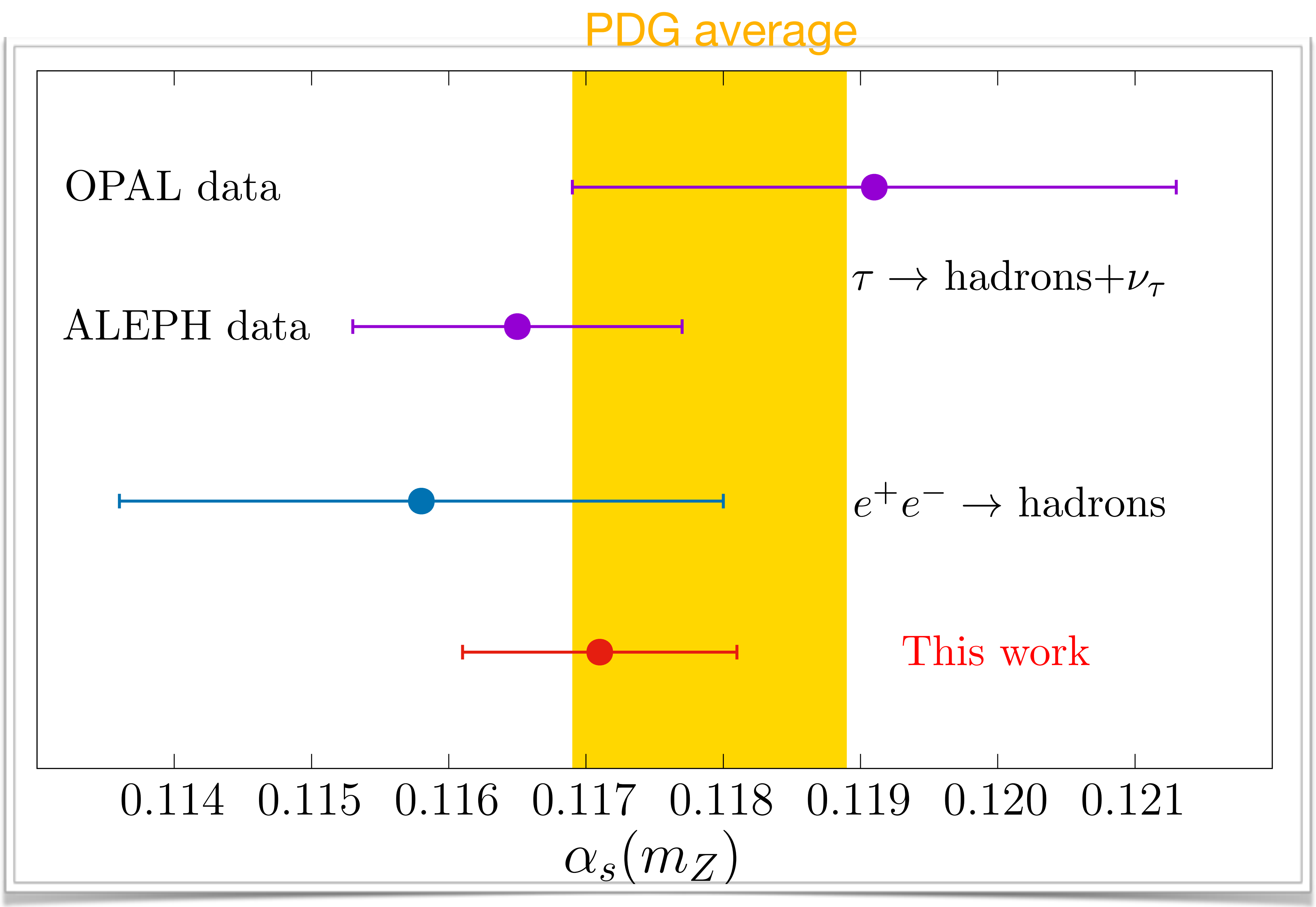
$$\alpha_s(m_Z) = 0.1171 \pm 0.0010$$

DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440, PRD 2021

Final result

Results evolved to m_Z $(\overline{\rm MS}, N_f = 5)$

$\alpha_s(m_Z) = 0.1171 \pm 0.0010$ FOPT



DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, Peris, ‘12

DB, Golterman, Maltman, Osborne, Peris, ‘15

DB, Golterman, Keshavarzi, Maltman, Nomura, Peris, Teubner ‘18

DB, Golterman, Maltman, Peris, Rodrigues, Schaaf, ’21

(FOPT results only)

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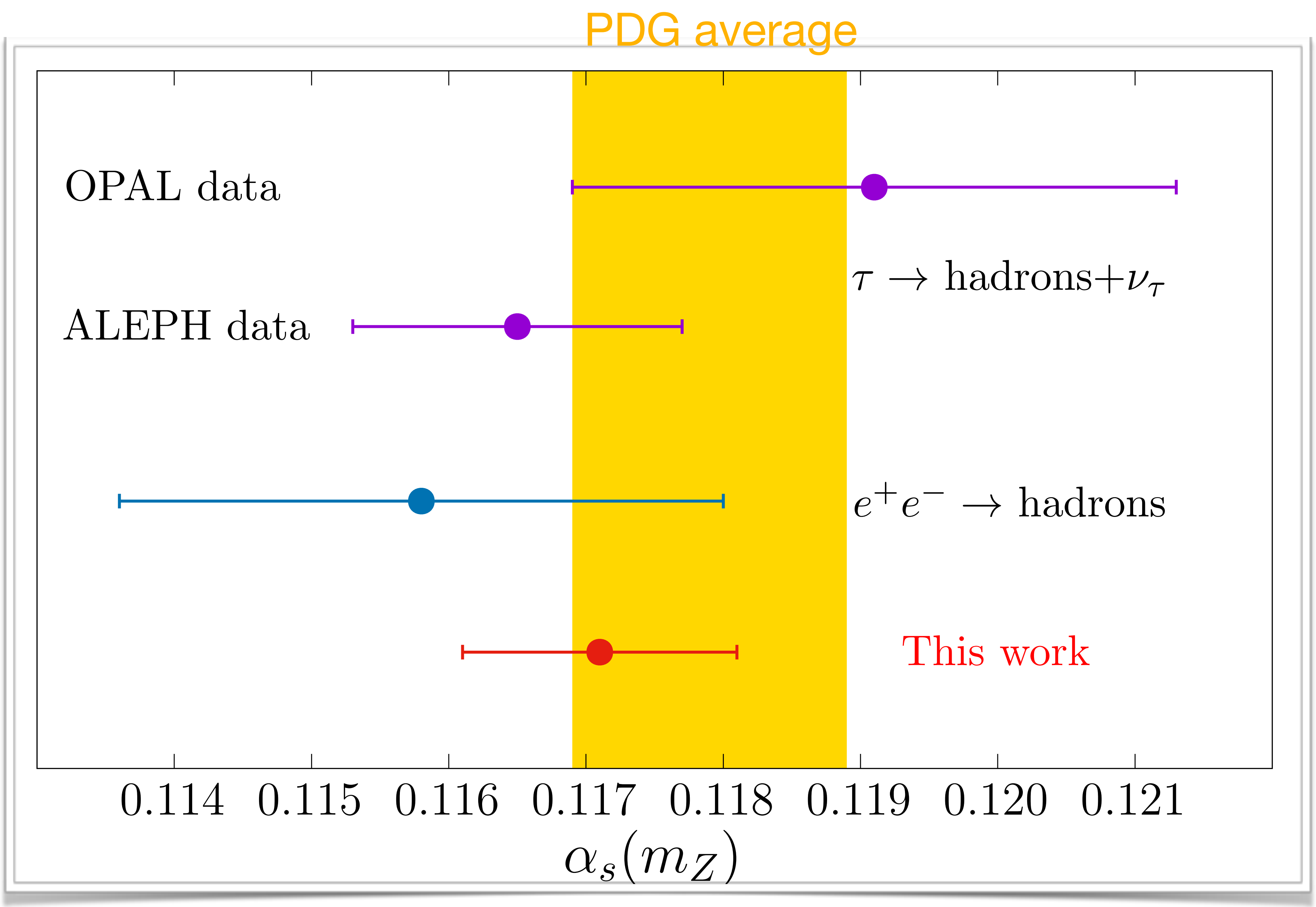

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DB, Golterman, Maltman, Peris, Rodrigues, Schaaf, ’21

(FOPT results only)

Resolution of the FOPT-CIPT discrepancy

Benitez-Rathgeb, DB, Hoang, Jamin, 2202.10957

Asymptotic separation

Starting point of the analysis: FOPT and CIPT have different Borel sums!

FOPT has the standard Borel representation (discussed before)

Hoang & Regner, '20 '21

$$\delta^{(0),\mathrm{FOPT}}_{W_i,\mathrm{Borel}}(s_0) = \mathrm{PV}\int_0^\infty \mathrm{d}u\,\frac{1}{2\pi i}\oint\limits_{|x|=1}\frac{\mathrm{d}x}{x}\,W_i(x)\,B[\hat{D}](u)\,e^{-\frac{4\pi u}{\beta_0\alpha_s(-xs_0)}}$$

$\beta_0 \equiv 2\beta_1$

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$$\beta_0 \equiv 2\beta_1$$

CIPT: no obvious expansion parameter

$$\delta^{(0),\mathrm{CIPT}}_{W_i}(s_0) = \frac{1}{2\pi i}\sum_{n=1}^{\infty} c_{n,1}\oint_{|x|=1}\frac{\mathrm{d}x}{x}\,W_i(x)\left(\frac{\alpha_s(-xs_0)}{\pi}\right)^n = \frac{1}{2\pi i}\sum_{n=1}^{\infty}\underbrace{\left(\frac{\alpha_s(s_0)}{\pi}\right)^n}_{\text{expansion variable}}\underbrace{c_{n,1}\oint_{|x|=1}\frac{\mathrm{d}x}{x}\,W_i(x)\left(\frac{\alpha_s(-xs_0)}{\alpha_s(s_0)}\right)^n}_{\text{coefficient}},$$

New result: CIPT Borel representation

$$\delta^{(0),\mathrm{CIPT}}_{W_i,\mathrm{Borel}}(s_0) = \int_0^\infty \mathrm{d}\bar{u}\,\frac{1}{2\pi i}\oint_{\mathcal{C}_x}\frac{\mathrm{d}x}{x}\,W_i(x)\left(\frac{\alpha_s(-xs_0)}{\alpha_s(s_0)}\right)B[\hat{D}]\left(\frac{\alpha_s(-xs_0)}{\alpha_s(s_0)}\bar{u}\right)e^{-\frac{4\pi\bar{u}}{\beta_0\alpha_s(s_0)}}$$

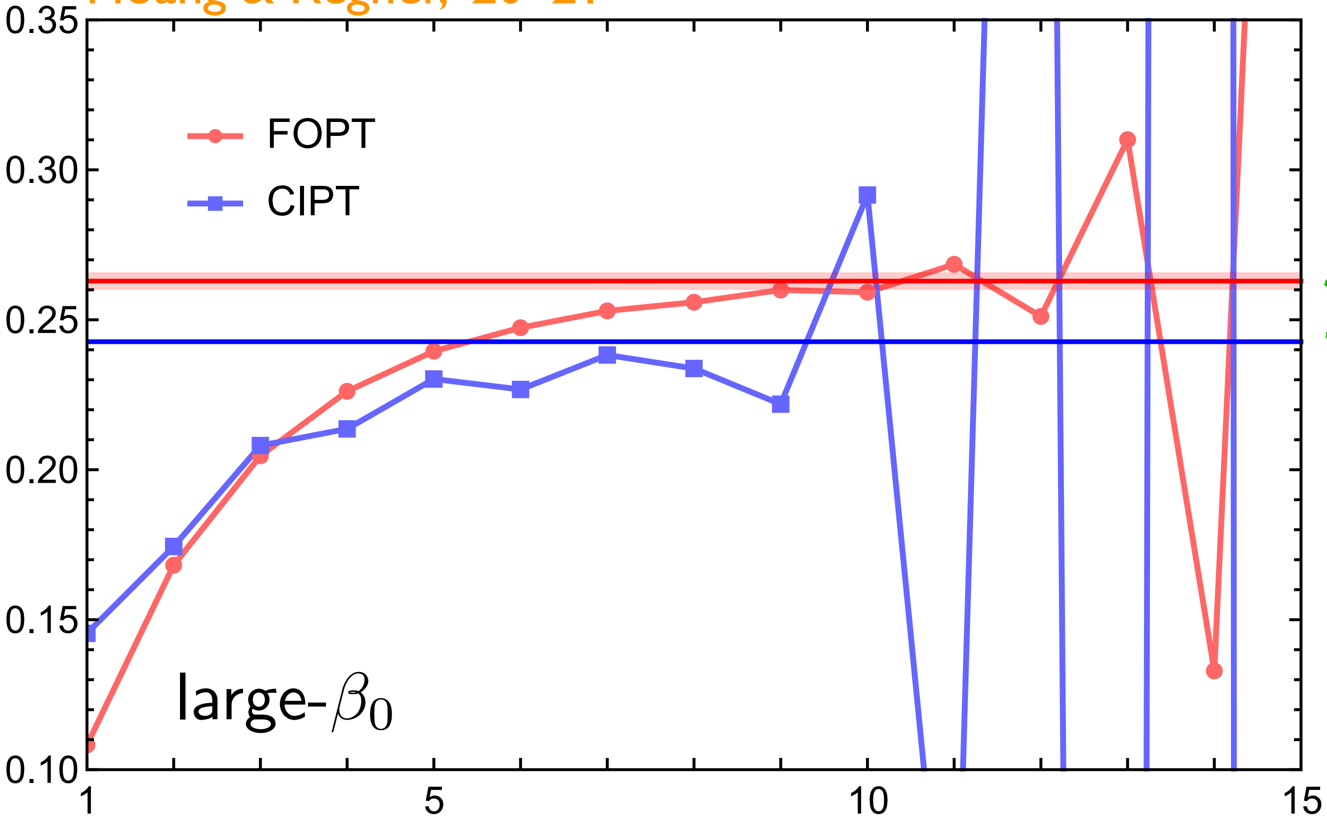
Contour needs to be deformed from |x|=1

Asymptotic separation

$$\Delta_W(s_0) \equiv \delta^{(0),\mathrm{CIPT}}_{W,\mathrm{Borel}}(s_0) - \delta^{(0),\mathrm{FOPT}}_{W,\mathrm{Borel}}(s_0)$$

$$\Delta_W(s_0) \sim \frac{\Lambda^d_{\mathrm{QCD}}}{s_0^{d/2}}$$

Hoang & Regner, '20 '21



Asymptotic separation (AS)

Assumption: the GC renormalon gives a sizeable contribution to the pt. series, i.e. it is not “unnaturally" suppressed.

Discrepancy is systematic, not accidental, and is not an effect due to missing higher orders

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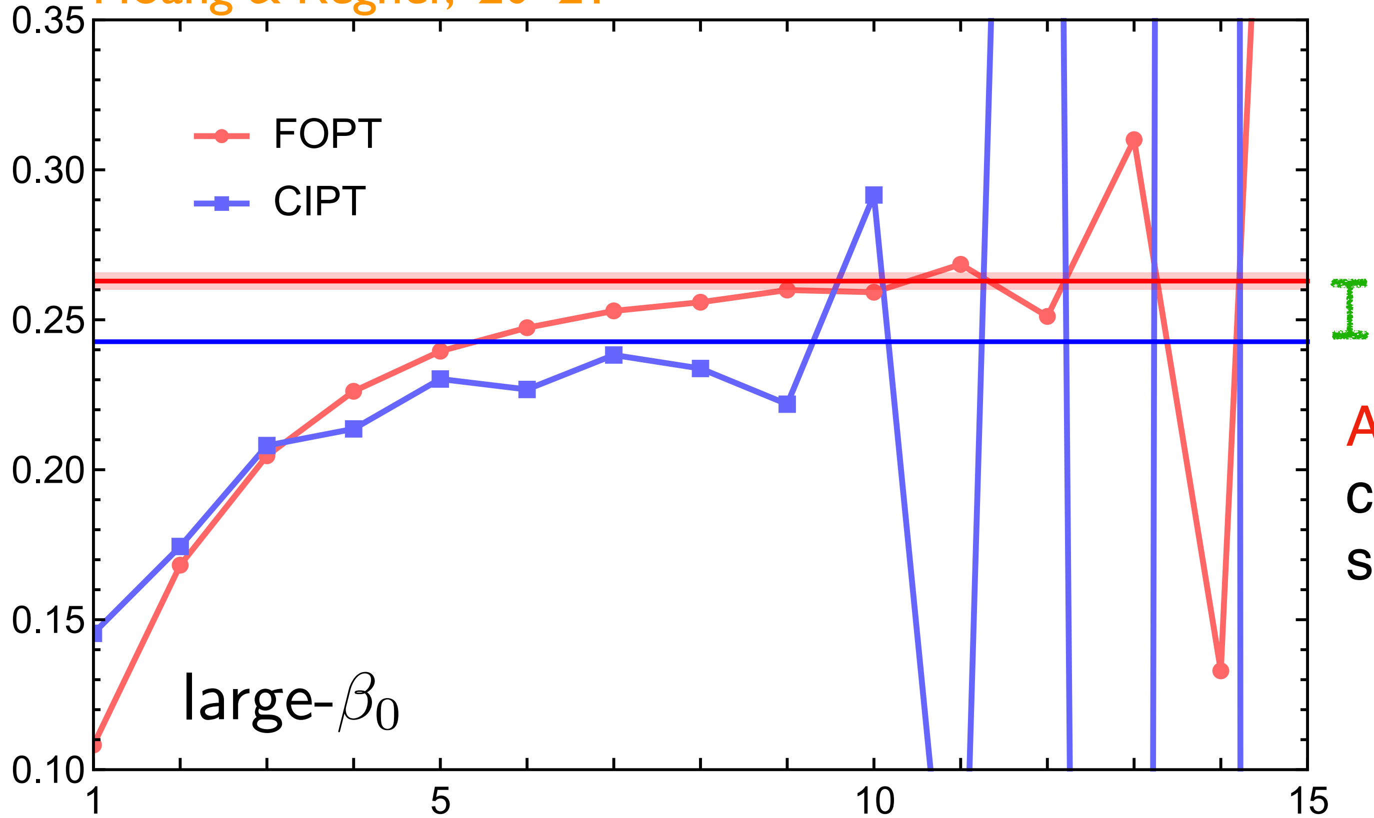
Hoang & Regner, '20 '21



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Conclusions from the asymptotic separation

- Asymptotic separation vanishes if IR renormalons are absent
- The leading IR renormalon dominates the asymptotic separation (> 99%)
- CIPT and FOPT should become consistent in an IR-subtracted scheme

Renormalon Free Gluon Condensate Scheme

General structure of the gluon condensate (GC) pole is known in QCD at NLO

$$\bar{a}_Q \equiv \frac{\beta_1}{2\pi}\alpha_s(Q)$$

normalization not determined by theory (app. known)

$$B_{4,0}(u) = \left[1 + \bar{c}^{(1)}_{4,0}\,\bar{a}_Q\right] \frac{N_{4,0}}{(2-u)^{1+4\hat{b}_1}} \longmapsto N_{4,0}\left(1 + \bar{c}^{(1)}_{4,0}\bar{a}_Q\right)\sum_{\ell=1}^{\infty} r^{(4,0)}_{\ell}\,\bar{a}^{\ell}_Q$$

$$r^{(4,0)}_{\ell} = \left(\frac{1}{2}\right)^{\ell+4\hat{b}_1} \frac{\Gamma(\ell+4\hat{b}_1)}{\Gamma(1+4\hat{b}_1)}$$

determined on general grounds from QCD

contribution of the GC singularity to the perturbative series

coefficients that diverge factorially are known

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Infrared-subtracted scheme for the GC condensate ("short distance scheme")

Benitez-Rathgeb, DB, Hoang, Jamin, 2202.10957

$$\langle \bar{G}^2\rangle^{(n)} \equiv \langle G^2\rangle(R^2) - R^4\sum_{\ell=1}^{n} N_g\, r_\ell^{(4,0)}\,\bar{a}_R^\ell$$

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coefficients that diverge factorially are known

Infrared-subtracted scheme for the GC condensate ("short distance scheme")

Benitez-Rathgeb, DB, Hoang, Jamin, 2202.10957

IR scale

to be expanded (coherently) in perturbation theory

$$\langle \bar{G}^2\rangle^{(n)} \equiv \langle G^2\rangle(R^2) - R^4\sum_{\ell=1}^{n} N_g\, r_\ell^{(4,0)}\,\bar{a}_R^\ell$$

Renormalon Free Gluon Condensate Scheme

General structure of the gluon condensate (GC) pole is known in QCD at NLO

$$\bar{a}_Q \equiv \frac{\beta_1}{2\pi}\alpha_s(Q)$$

normalization not determined by theory (app. known)

$$B_{4,0}(u) = \left[1 + \bar{c}^{(1)}_{4,0}\bar{a}_Q\right]\frac{N_{4,0}}{(2-u)^{1+4\hat{b}_1}} \longmapsto N_{4,0}\left(1+\bar{c}^{(1)}_{4,0}\bar{a}_Q\right)\sum_{\ell=1}^{\infty} r_\ell^{(4,0)}\bar{a}_Q^\ell$$

$$r_\ell^{(4,0)} = \left(\frac{1}{2}\right)^{\ell+4\hat{b}_1}\frac{\Gamma(\ell+4\hat{b}_1)}{\Gamma(1+4\hat{b}_1)}$$

determined on general grounds from QCD

contribution of the GC singularity to the perturbative series

coefficients that diverge factorially are known

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Its more convenient to work with scale invariant GC

"tree level" (unexpanded) contribution

$$\langle\bar{G}^2\rangle^{(n)} \equiv \langle G^2\rangle^{\rm RF} - R^4\sum_{\ell=1}^{n} N_g\, r_\ell^{(4,0)}\,\bar{a}_R^\ell + N_g\,\bar{c}_0(R^2)$$

$$\bar{c}_0(R^2) \equiv R^4\ \mathrm{PV}\int_0^\infty \frac{\mathrm{d}u\ e^{-\frac{u}{\bar{a}_R}}}{(2-u)^{1+4\hat{b}_1}}$$

$$\frac{d}{d\log R^2}\langle G^2\rangle^{\rm RF} = 0$$

scale invariant

Borel sum unchanged, for any value of the norm. Minimal scheme.

Diogo Boito

A technical comment

We use the *C*-scheme for the QCD coupling

$$\frac{\pi}{\bar{\alpha}_s(Q^2)} + \frac{\beta_1}{4\beta_0}\ln(\bar{\alpha}_s(Q^2)) = \frac{\pi}{\alpha_s(Q^2)} + \frac{\beta_1}{4\beta_0}\ln(\alpha_s(Q^2)) + \frac{\beta_0}{2}\int_0^{\alpha_s(Q^2)} d\tilde{\alpha}\left[\frac{1}{\beta(\tilde{\alpha})} + \frac{2\pi}{\beta_0\tilde{\alpha}^2} - \frac{\beta_1}{2\beta_0^2\tilde{\alpha}}\right]$$

new coupling in the C scheme

input coupling (here always $\overline{\mathrm{MS}}$)

DB, Jamin, Miravtllas, PRL '16

The beta function is exact in this scheme

$$\frac{d\bar{\alpha}_s(Q^2)}{d\ln Q} = \bar{\beta}(\bar{\alpha}_s(Q^2)) \equiv -2\,\bar{\alpha}_s(Q^2)\,\frac{\beta_0\,\bar{\alpha}_s(Q^2)}{4\pi - \frac{\beta_1}{\beta_0}\bar{\alpha}_s(Q^2)}$$

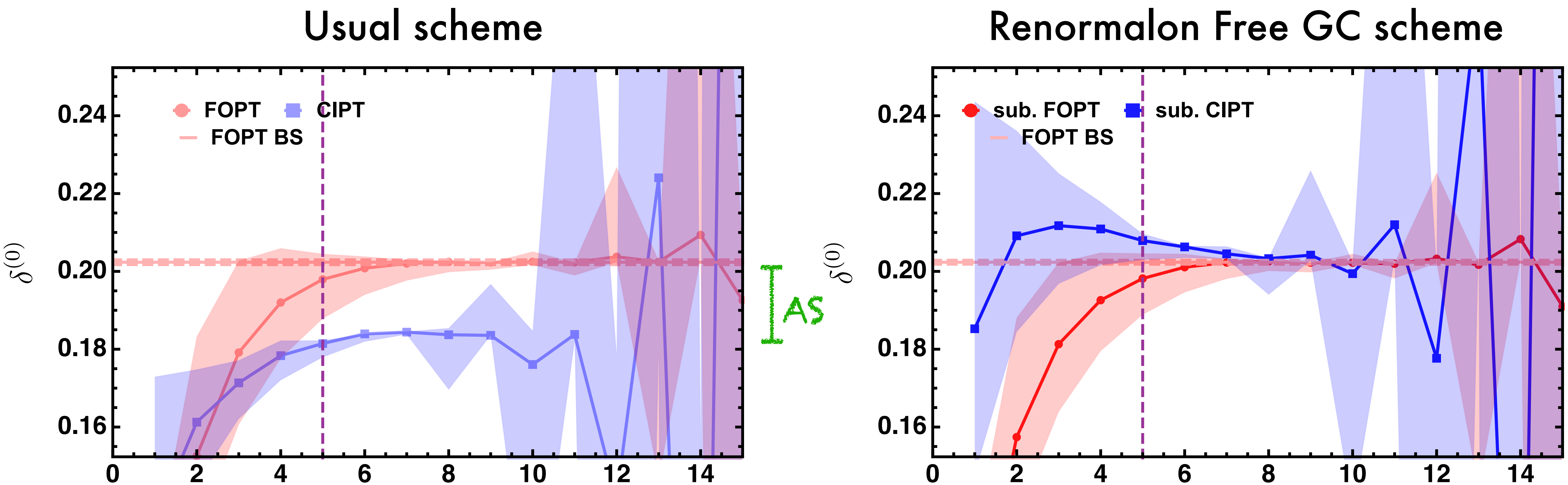
The C-scheme very convenient for the renormalon analysis

$$B_{4,0}(u) = \left[1 + \bar{c}_{4,0}^{(1)}\,\bar{a}_Q\right]\frac{N_{4,0}}{(2-u)^{1+4\hat{b}_1}}$$

this expression is exact in the C-scheme, no subleading corrections

Final results are re-expressed in the usual $\overline{\mathrm{MS}}$ scheme

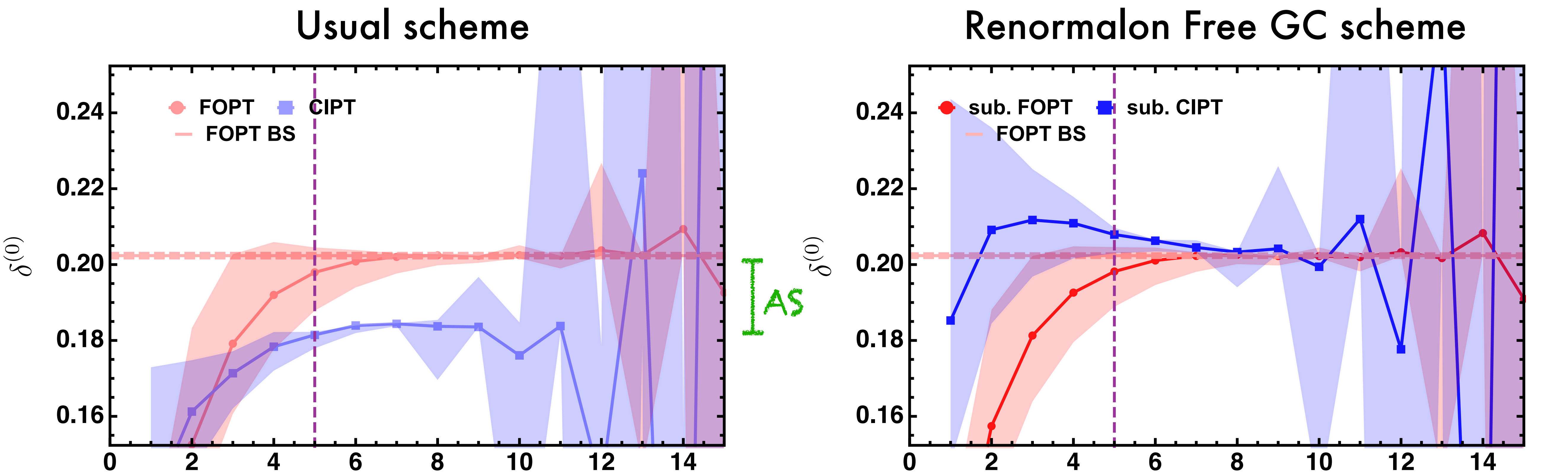
Renormalon Free Gluon Condensate Scheme



(Higher-orders from a multi-renormalon model with an estimated 6-loop result)

Beneke & Jamin '08

Renormalon Free Gluon Condensate Scheme



(Higher-orders from a multi-renormalon model with an estimated 6-loop result)

Beneke & Jamin '08

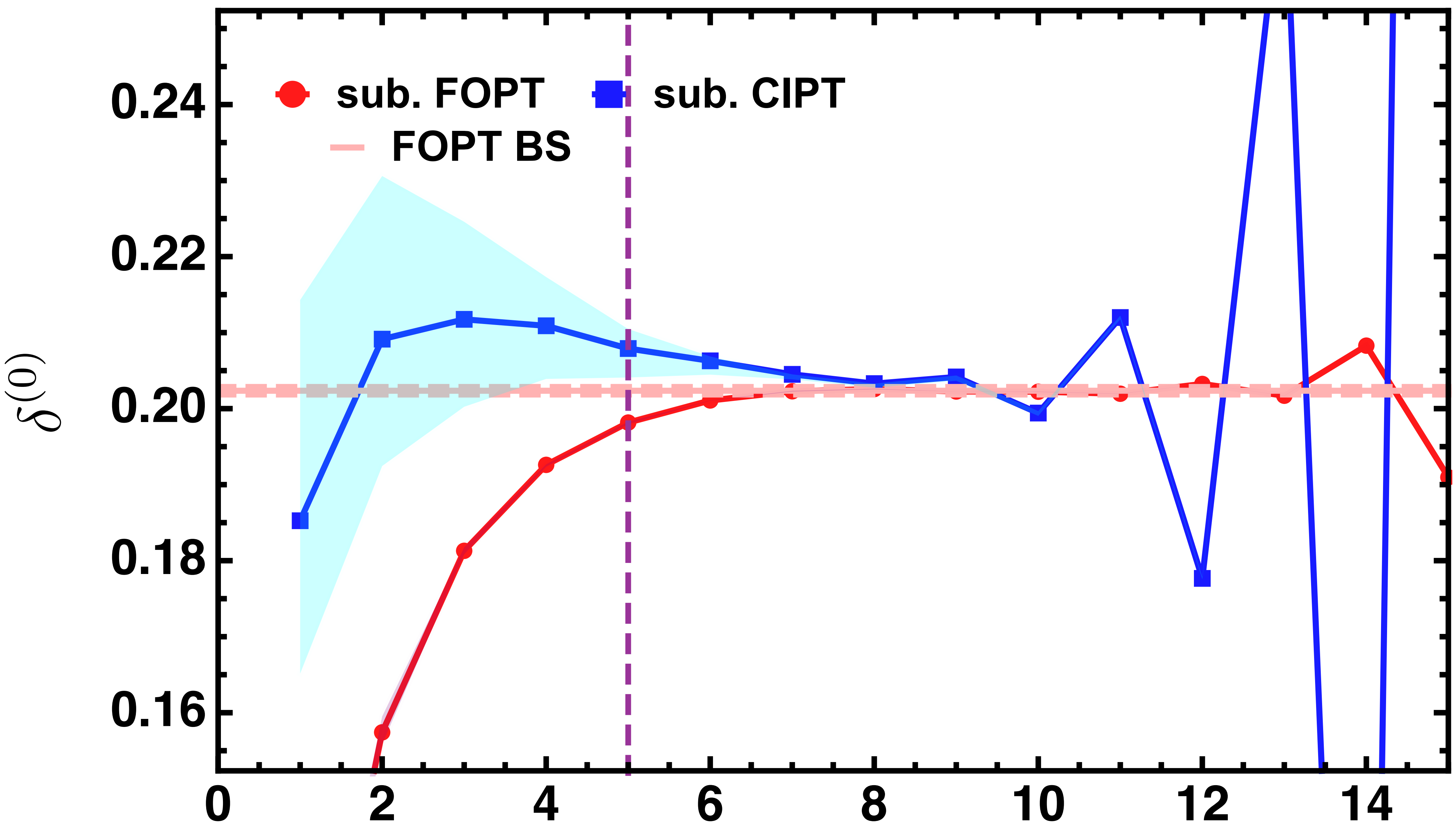
- Discrepancy between FOPT and CIPT is removed in the new RF scheme
- CIPT becomes consistent with FOPT (which is only slightly modified)
- Can lead to smaller theoretical uncertainties in $\alpha_s(m_\tau^2)$
- Additional uncertainty from the determination of N_g not very large

Benitez-Rathgeb, DB, Hoang, Jamin, in preparation

Renormalon Free Gluon Condensate Scheme

$$\langle \bar{G}^2 \rangle^{(n)} \equiv \langle G^2 \rangle^{\rm RF} - R^4 \sum_{\ell=1}^{n} N_g \, r_\ell^{(4,0)} \, \bar{a}_R^\ell + N_g \, \bar{c}_0(R^2)$$

Residual IR scale dependence becomes smaller at higher orders



Typical behaviour of a renormalization scale

Conclusions

Conclusions

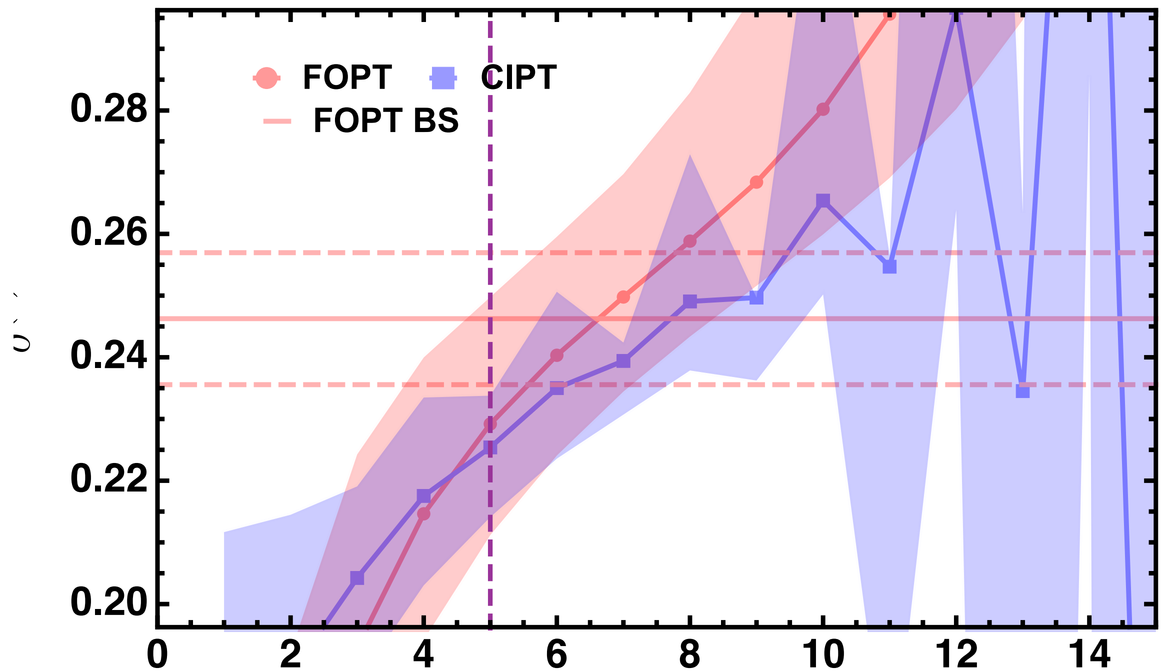
- Strong coupling determination from tau decays is one of the most precise from experiment
- Recent electroproducton can now be used to improve the precision of this determination
- Good prospects for new data for the dominant channels (Belle II?)
- Discrepancy between CIPT and FOPT understood: asymptotic separation
- Assumption: sizeable GC renormalon
  - Known four and five loop results consistent with this assumption
  - Everything indicates the IR origin of the discrepancy
  - Normalization of the GC can be extracted with sufficient precision (preliminary)

Benitez-Rathgeb, DB, Hoang, Jamin, in preparation

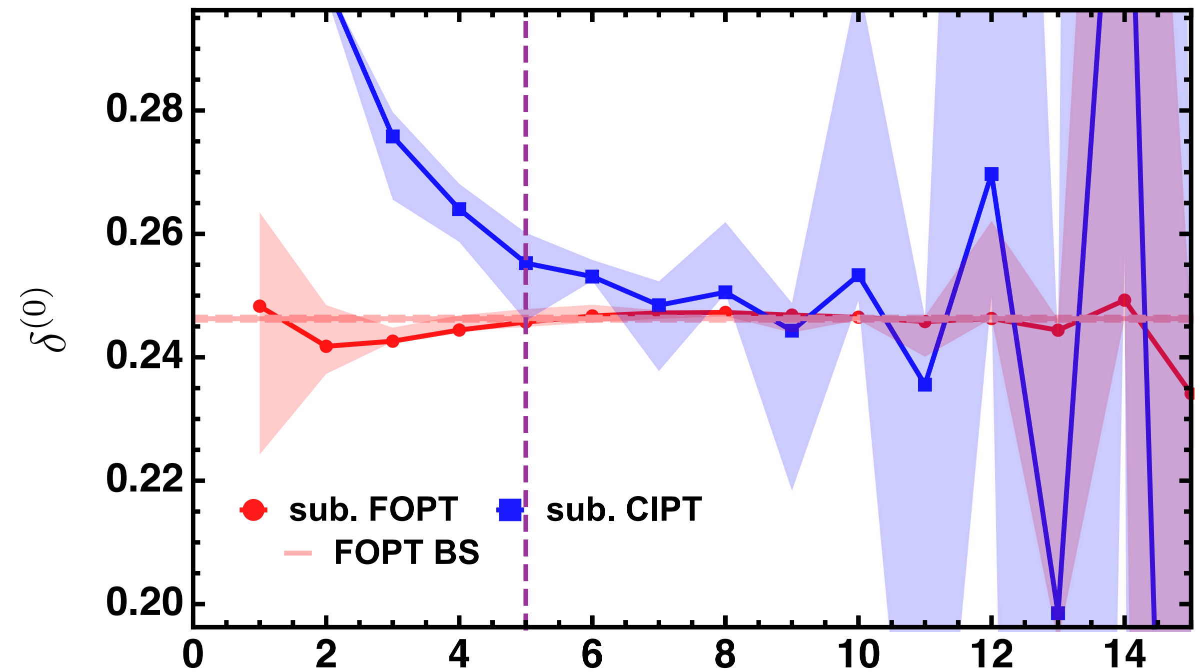
- New renormalon free GC scheme: discrepancy resolved
- The scheme is minimalistic and transparent
- Excellent prospects for reducing the theoretical uncertainty of $\alpha_s(m_\tau^2)$

Results for other moments

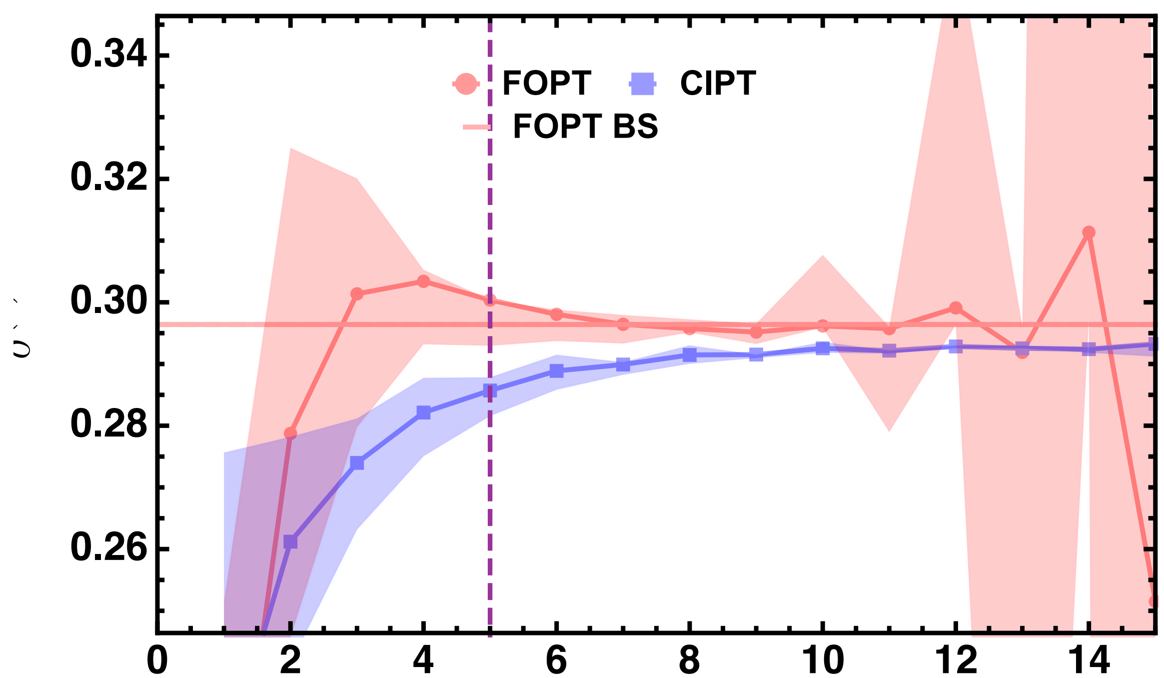
$\bar{c}^{(1)}_{4,0} = -22/81,\ W(x) = (1-x)^3$



$\bar{c}^{(1)}_{4,0} = -22/81,\ R = 0.8m_\tau,\ W(x) = (1-x)^3$



$\bar{c}^{(1)}_{4,0} = -22/81,\ W(x) = 2(1-x)$



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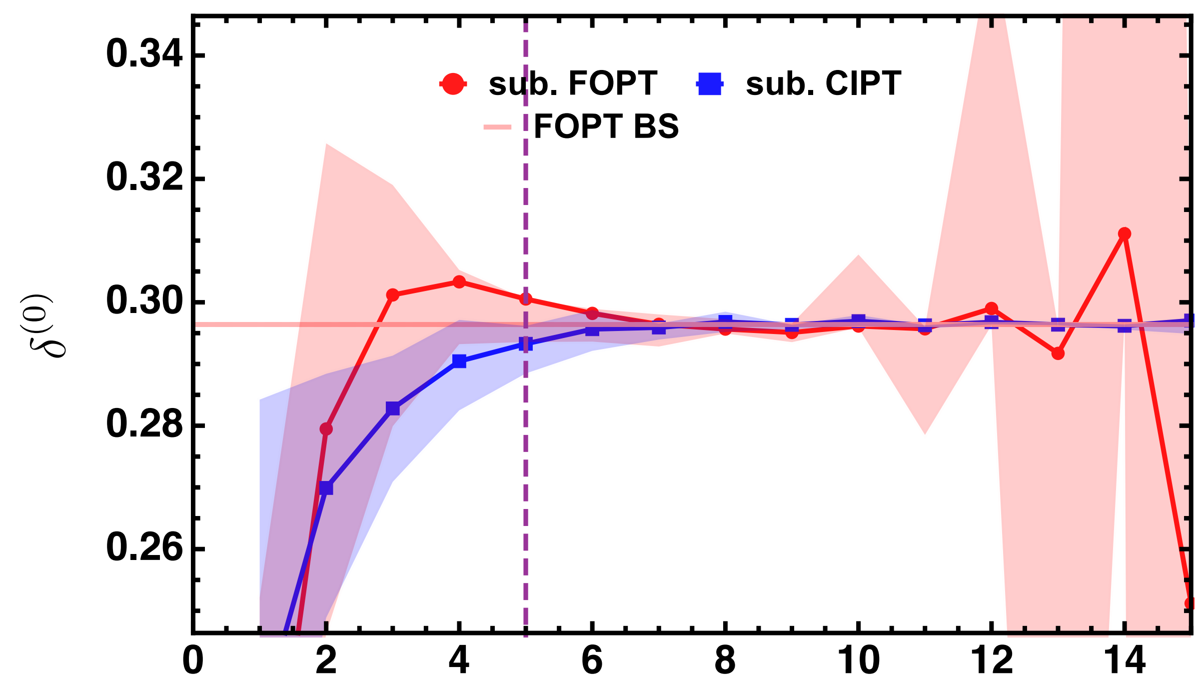

Results for other moments