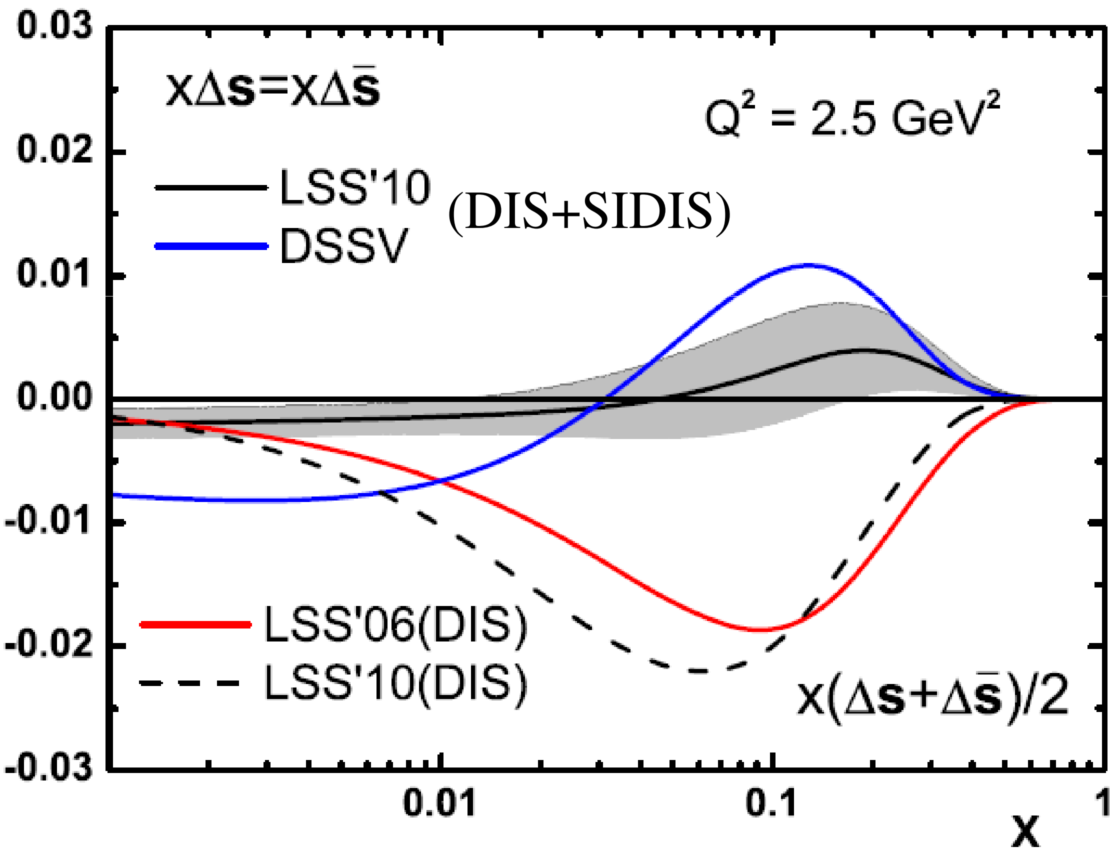
# Preasymptotic effects in extraction of strange quark polarization

**Aram Kotzinian**

*Uni&INFN, Torino & YerPhI, Armenia*

- Introduction
  - Flavor separation in SIDIS (polarized strangeness)
- Polarized hadronization
  - Generalized expression for polarized SIDIS
- Example (warning): application to HERMES (data from PhD thesis)
- Spin-dependent azimuthal asymmetries of SIDIS
- Discussion & Conclusions

# Strangeness PDF



# Christova & Leader (2000)

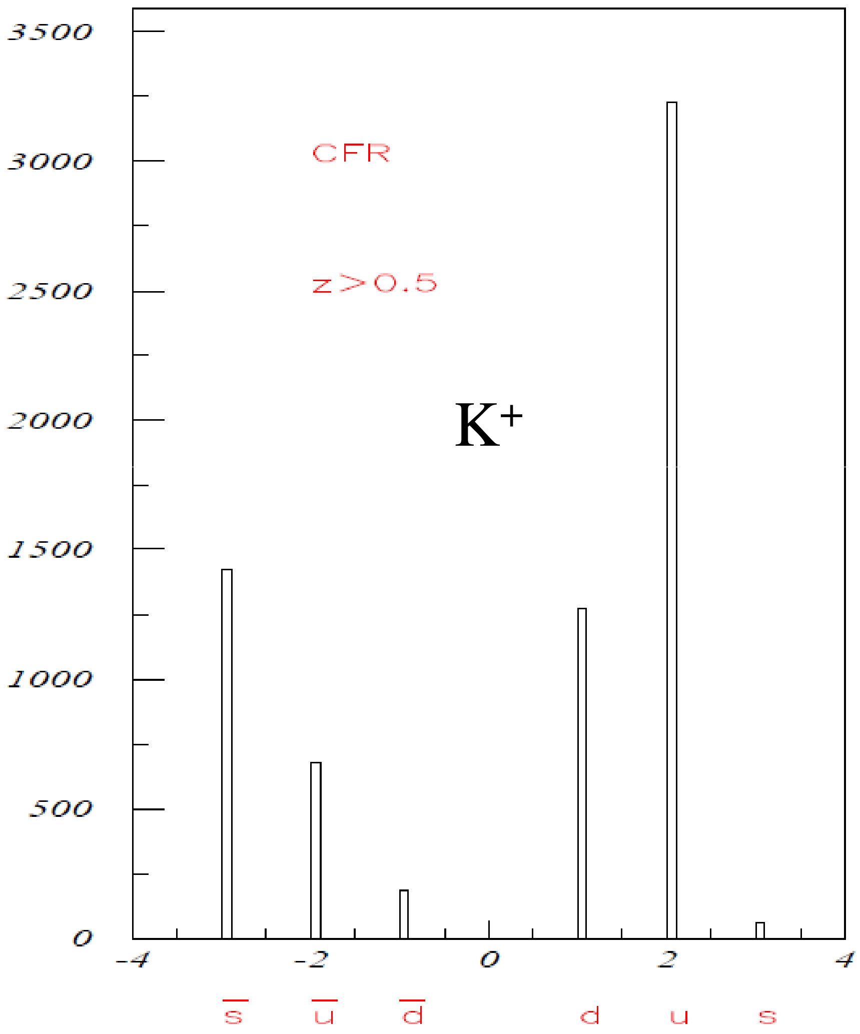
## A Strategy for the Analysis of Semi-Inclusive Deep Inelastic Scattering

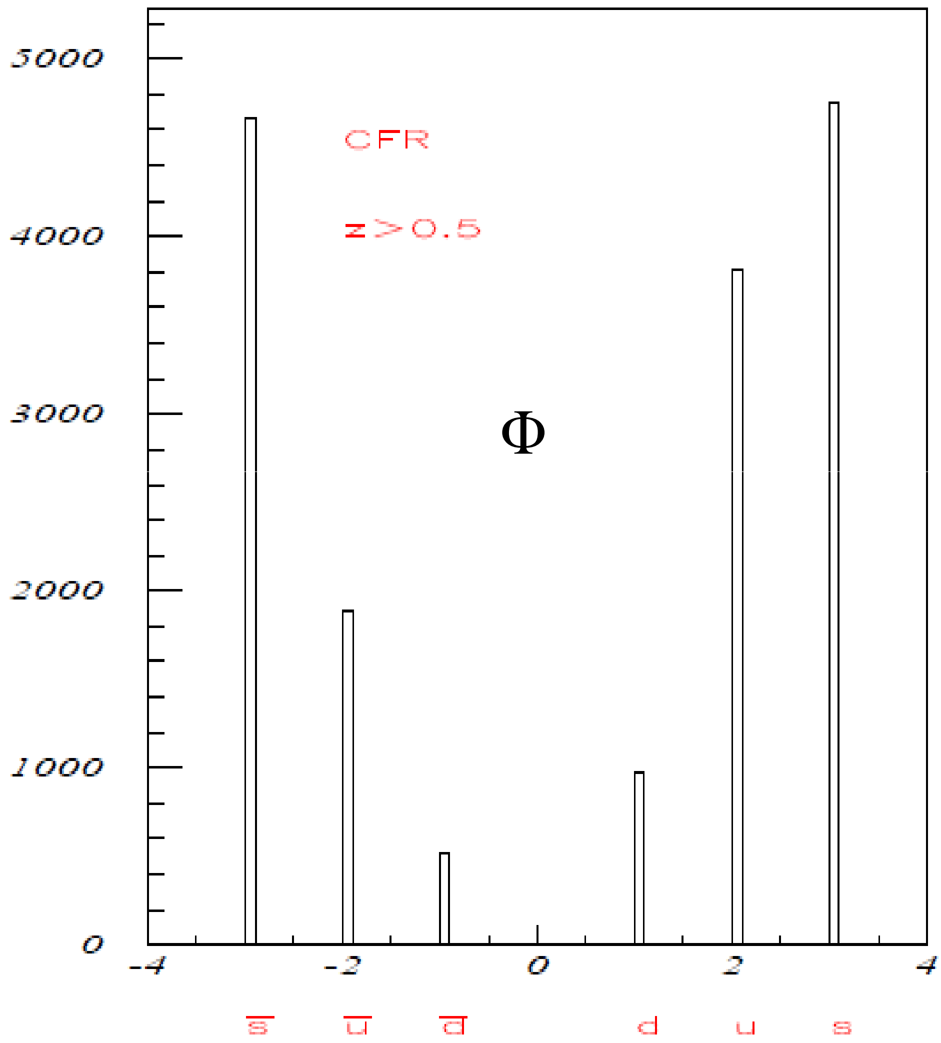
$$\frac{\Delta s + \Delta \bar{s}}{s + \bar{s}} = \frac{3\Delta q_3(\Delta\sigma_p^{\Phi} + \Delta\sigma_n^{\Phi}) - 5\Delta q_+(\Delta\sigma_p^{\Phi} - \Delta\sigma_n^{\Phi})}{D(y)[3q_3(\sigma_p^{\Phi} + \sigma_n^{\Phi}) - 5q_+(\sigma_p^{\Phi} - \sigma_n^{\Phi})]}$$

$$\Delta q_+ = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d}$$

$$\Delta q_3 = \Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d}$$

# LEPTO (160 GeV μ, $Q^2$>1GeV$^2$ , 0.2<y<09)



# $\Delta$s from $\Phi$ production asymmetry

AK, October 2000, COMPASS meeting in Dubna

$$D_u^\Phi(z) = D_{\bar{u}}^\Phi(z) = D_d^\Phi(z) = D_{\bar{d}}^\Phi(z), \quad r(z) \doteq \frac{D_u^\Phi(z)}{D_s^\Phi(z)} \ll 1 \quad \text{at high } z$$

$$S(x) \doteq s(x) + \bar{s}(x), \;\; \Delta S(x) \doteq \Delta s(x) + \Delta \bar{s}(x)$$

$$V(x) \doteq 4\left[u(x) + \bar{u}(x)\right] + d(x) + \bar{d}(x), \;\; \Delta V(x) \doteq 4\left[\Delta u(x) + \Delta \bar{u}(x)\right] + \Delta d(x) + \Delta \bar{d}(x)$$

$$\boxed{A_{1,p}^\Phi = \frac{\Delta S(x) + \Delta V(x) r(z)}{S(x) + V(x) r(z)}}$$

$r(z)$ from *unpolarized* $\Phi$ yields in different $x-$bins at fixed $z$:

$$\boxed{\frac{\sigma_p^\Phi(x_1, z)}{\sigma_p^\Phi(x_1, z)} = \frac{\left\langle \frac{1+(1-y)^2}{Q^2 xy} \right\rangle_1 \left[S(x_1) + V(x_1) r(z)\right]}{\left\langle \frac{1+(1-y)^2}{Q^2 xy} \right\rangle_2 \left[S(x_2) + V(x_2) r(z)\right]}}$$

# HERMES isoscalar method 1

DIS and $K^+$ & $K^-$ production in Deuterium target

$$Q(x) = u(x) + \overline{u}(x) + d(x) + \overline{d}(x) \qquad S(x) = s(x) + \overline{s}(x)$$

$$\Delta Q(x) = \Delta u(x) + \Delta\overline{u}(x) + \Delta d(x) + \Delta\overline{d}(x) \qquad \Delta S(x) = \Delta s(x) + \Delta\overline{s}(x)$$

$$A_1(x, Q^2) = \frac{5\Delta Q(x, Q^2) + 2\Delta S(x, Q^2)}{5Q(x, Q^2) + 2S(x, Q^2)}$$

$$A_1^K(x, Q^2) = \frac{\Delta Q(x)[4\int D_u^K(z)dz + \int D_d^K(z)dz] + 2\Delta S(x)\int D_s^K(z)dz}{Q(x)[4\int D_u^K(z)dz + \int D_d^K(z)dz] + 2S(x)\int D_s^K(z)dz}$$

Inputs from HERMES: Ahmed El Alaoui PhD thesis

$$= \frac{\Delta Q(x)\int D^K_{\text{non-étrange}}(z)dz + \Delta S(x)\int D^K_{\text{étrange}}(z)dz}{Q(x)\boxed{\int D^K_{\text{non-étrange}}(z)dz} + S(x)\boxed{\int D^K_{\text{étrange}}(z)dz}}$$

# HERMES isoscalar method 2

$$P_Q(x) = \frac{5Q(x)}{5Q(x) + 2S(x)}$$

$$P_S(x) = \frac{2S(x)}{5Q(x) + 2S(x)}$$

$$P_Q^K(x) = \frac{Q(x)\int \mathcal{D}^K_{\text{non-étrange}}(z)dz}{Q(x)\int \mathcal{D}^K_{\text{non-étrange}}(z)dz + S(x)\int \mathcal{D}^K_{\text{étrange}}(z)dz}$$

$$P_S^K(x) = \frac{S(x)\int \mathcal{D}^K_{\text{étrange}}(z)dz}{Q(x)\int \mathcal{D}^K_{\text{non-étrange}}dz + S(x)\int \mathcal{D}^K_{\text{étrange}}dz}$$

$$\begin{pmatrix} A_D(x) \\ A_D^K(x) \end{pmatrix} = C_R(x,Q^2) \begin{pmatrix} P_Q(x) & P_S(x) \\ P_Q^K(x) & P_S^K(x) \end{pmatrix} \begin{pmatrix} \frac{\Delta Q(x)}{Q(x)} \\ \frac{\Delta S(x)}{S(x)} \end{pmatrix}$$

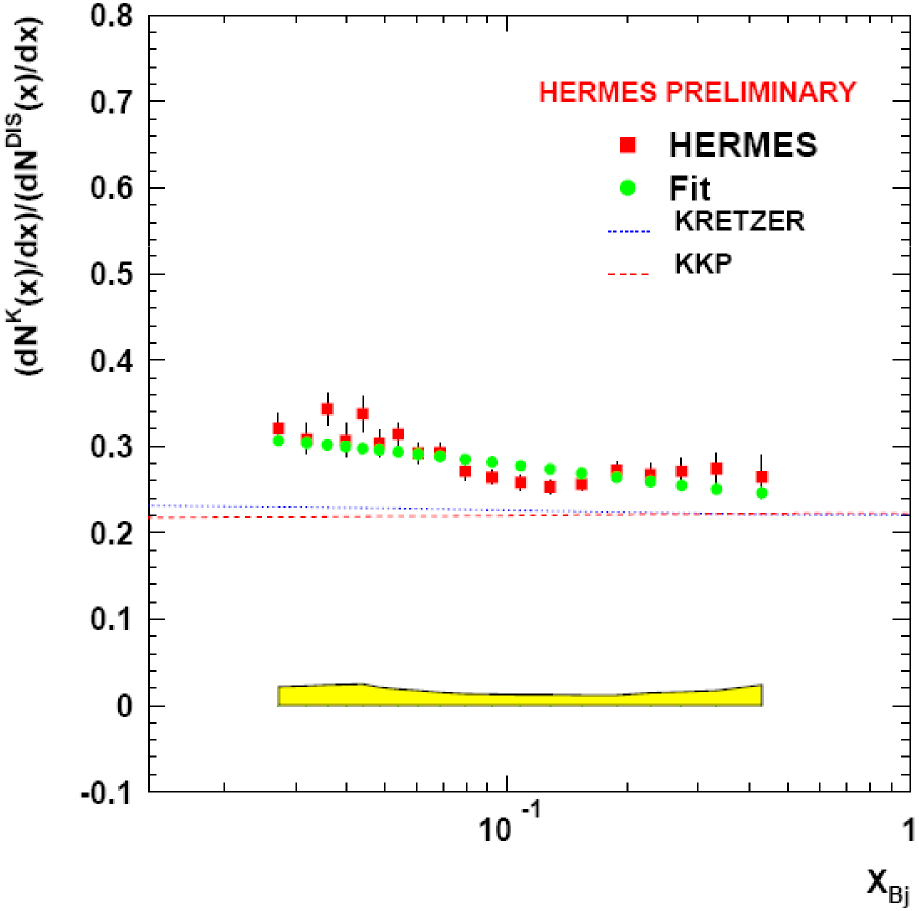
Two unknown fragmentation integrals -- from unpolarized data

# HERMES isoscalar method 3

$$\frac{dN^K(x)/dx}{dN^{DIS}(x)/dx} = \frac{Q(x)\boxed{\int \mathcal{D}^K_{nstrg}(z)dz} + S(x)\boxed{\int \mathcal{D}^K_{strg}(z)dz}}{5Q(x)+2S(x)}$$

Two unknowns, 19 x-bins

| | Ce Travail | Kretzer | KKP |
|---|---|---|---|
| $\int \mathcal{D}^K_{nstrg}(z)dz$ | $1.20 \pm 0.06 \pm 0.03$ | 1.103 | 1.111 |
| $\int \mathcal{D}^K_{strg}(z)dz$ | $1.43 \pm 0.36 \pm 0.15$ | 0.392 | 0.150 |




No checks for x-z factorization

(Extract unknowns in different subsets of x-bins)

# HERMES isoscalar method 4

| $<x>$ | $P_Q(x)$ | $P_S(x)$ | $P_Q^{K^++K^-}(x)$ | $P_S^{K^++K^-}(x)$ |
|---|---|---|---|---|
| 0.0330 | 0.9487 | 0.0513 | 0.7569 | 0.2431 |
| 0.0474 | 0.9546 | 0.0454 | 0.7799 | 0.2201 |
| 0.0647 | 0.9601 | 0.0399 | 0.8020 | 0.1980 |
| 0.0870 | 0.9657 | 0.0343 | 0.8258 | 0.1742 |
| 0.1181 | 0.9718 | 0.0282 | 0.8532 | 0.1468 |
| 0.1658 | 0.9790 | 0.0210 | 0.8871 | 0.1129 |
| 0.2391 | 0.9867 | 0.0133 | 0.9260 | 0.0740 |
| 0.3387 | 0.9932 | 0.0068 | 0.9609 | 0.0391 |
| 0.4469 | 0.9970 | 0.0030 | 0.9827 | 0.0173 |

TAB. 4.1 – Les valeurs des éléments de la matrice $\mathcal{P}$ calculées en fonction de $x$ à partir de la paramétrisation CTEQ6LO, évaluées pour un moment de transfert moyen $< Q_0^2 >= 2.5\,GeV^2$, et à partir des fonctions de fragmentation des kaons obtenues à partir des données d'HERMES.

# HERMES isoscalar: Results

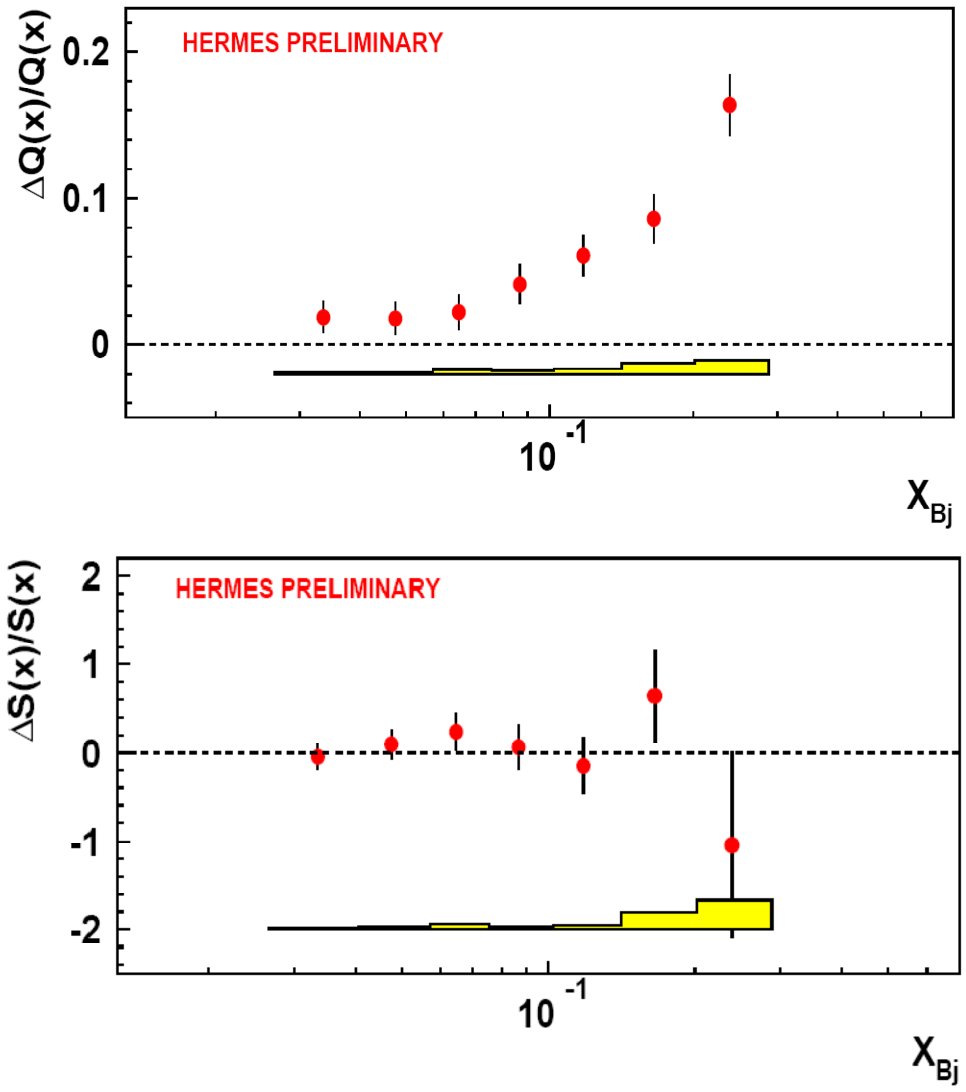
$$\int_{0.02}^{1} \Delta Q \, \mathrm{d}x = 0.286 \pm 0.026 \pm 0.011$$

$$\int_{0.02}^{1} \Delta S \, \mathrm{d}x = 0.006 \pm 0.029 \pm 0.007$$

COMPASS DIS

$$(\Delta s + \Delta \bar{s})_{Q^2 \to \infty} =$$

$$= -0.08 \pm 0.01 \,(\text{stat.}) \pm 0.02 \,(\text{syst.})$$



# Factorization theorem

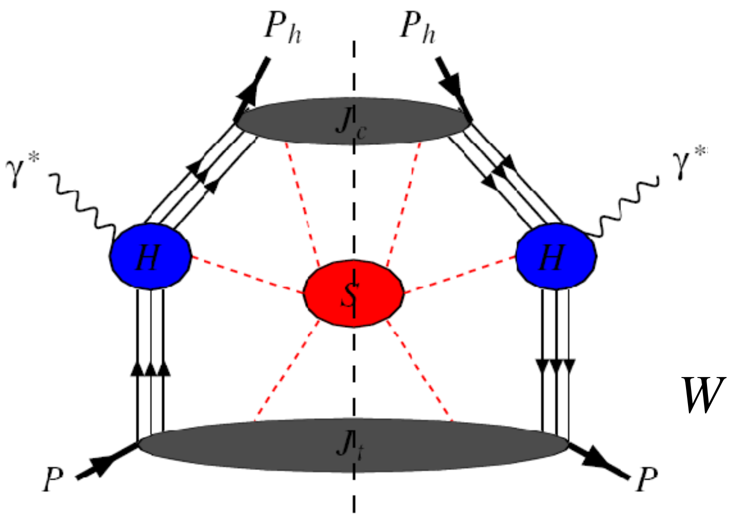



FIG. 6 (color online). A general reduced diagram for semi-inclusive DIS.

$$Q^2 \to \infty$$

$$W^2 = M^2 + \frac{Q^2(1-x)}{x}$$

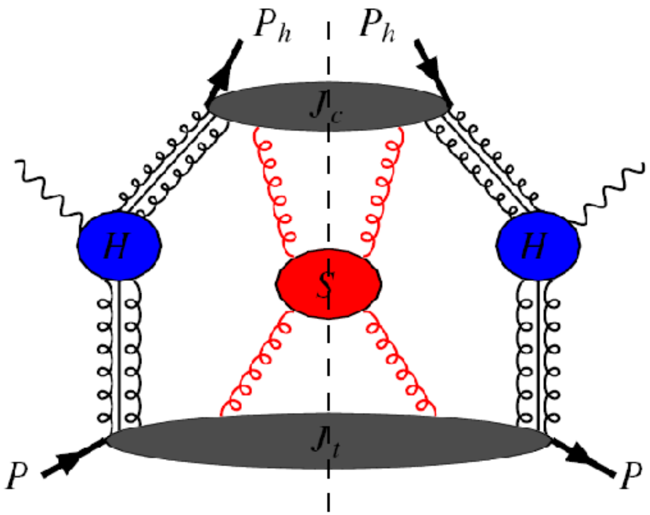



FIG. 7 (color online). The leading region for semi-inclusive DIS.

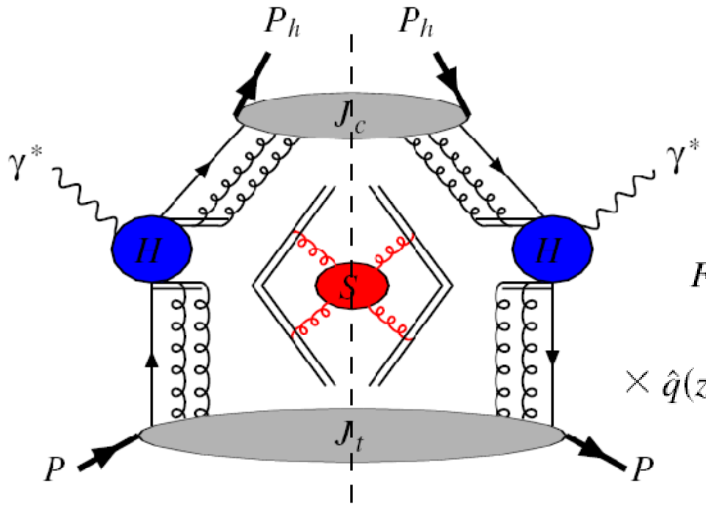


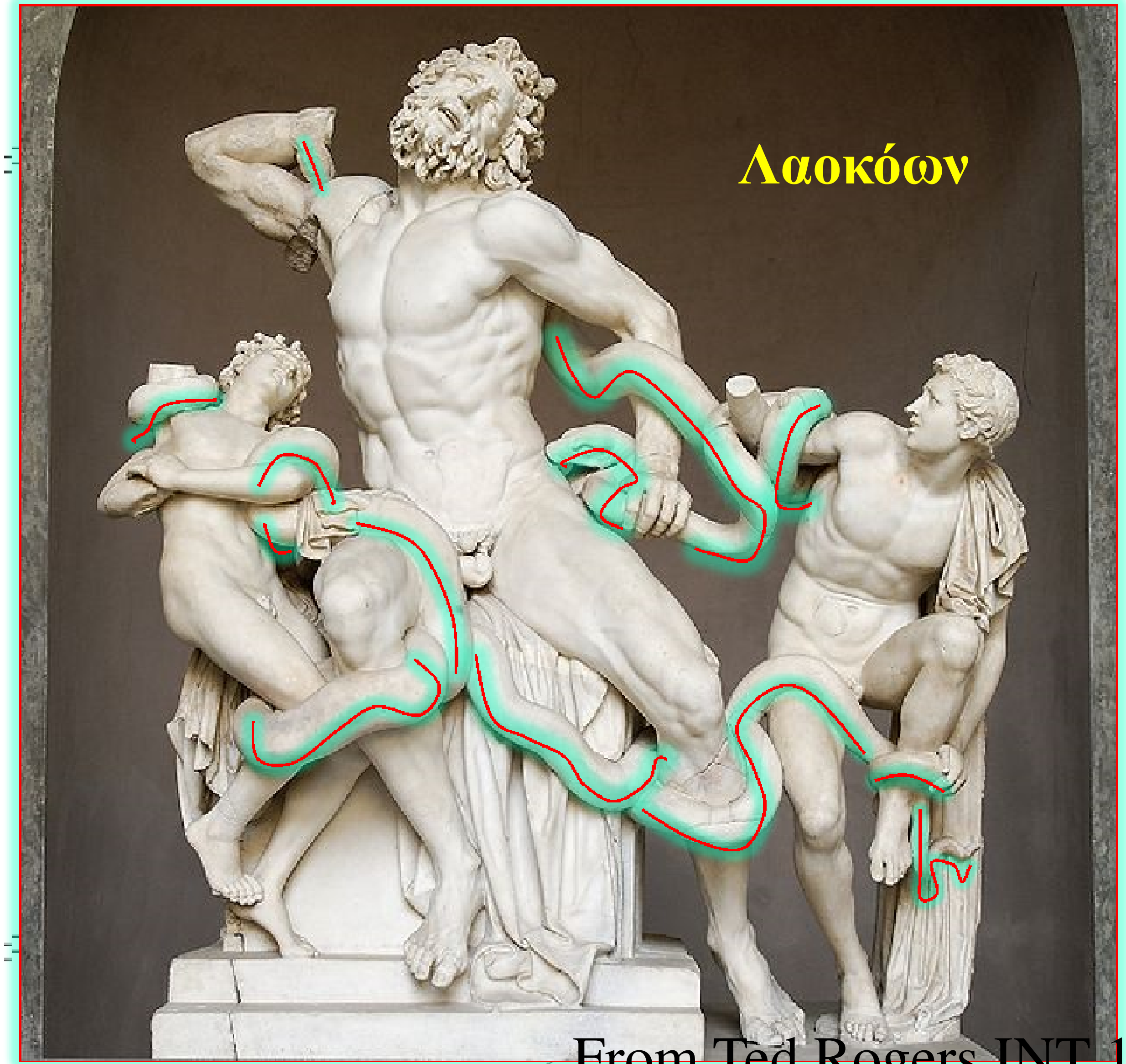

FIG. 8 (color online). The leading region for SIDIS after soft and collinear factorizations.

Ji, Ma & Yuan, PRD 71, 034005 (2005)

$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp q(x_B, k_\perp, \mu^2, x_B\zeta, \rho)$$
$$\times\, \hat{q}(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho) S(\vec{\ell}_\perp, \mu^2, \rho) H(Q^2, \mu^2, \rho)\delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp})$$

Open questions: At which $z$ independent fragmentation works? At which $Q^2$ one can neglect higher twist terms

# 2015 years ago
# Gauge link



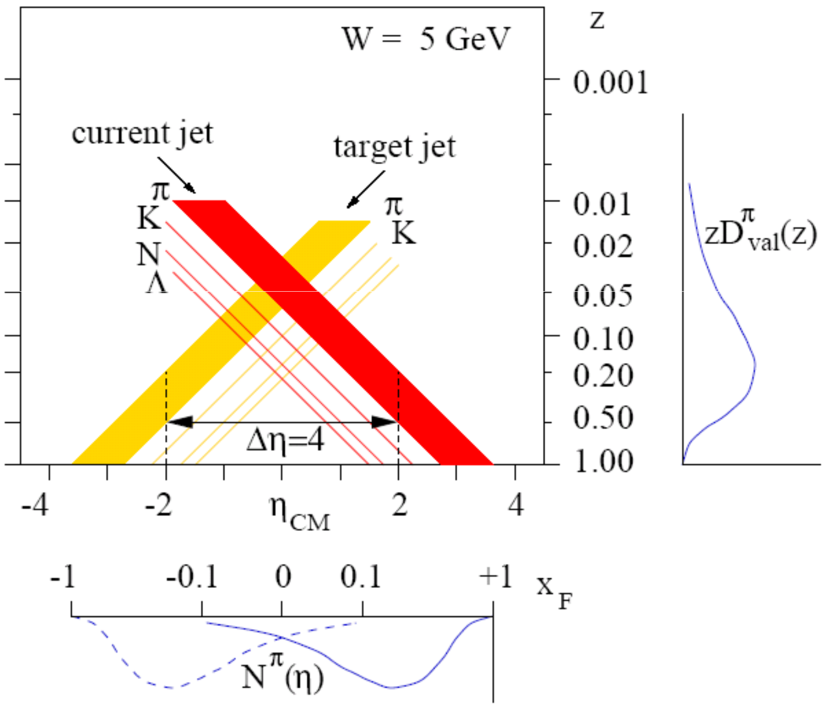

From Ted Rogers INT 10-3 talk

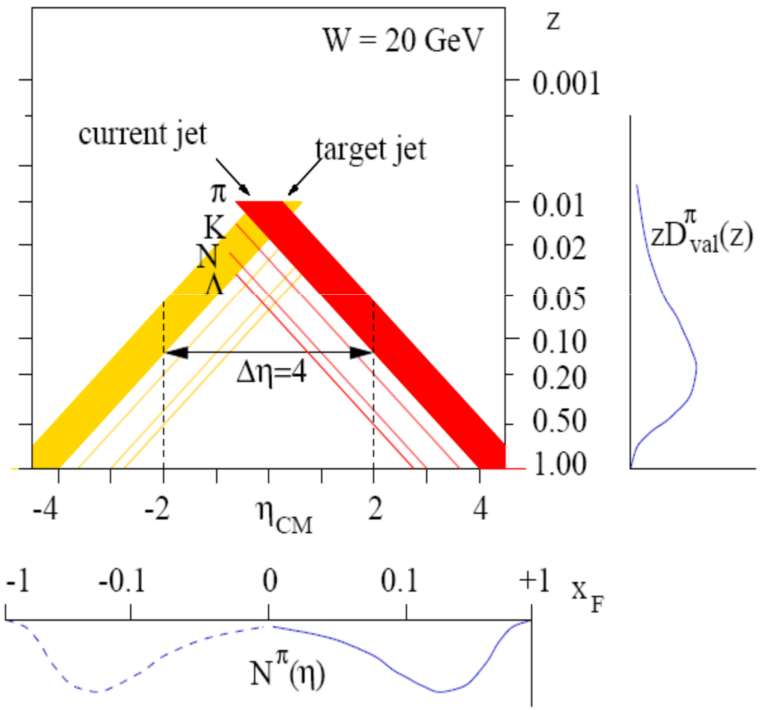
# Ed. Berger criterion for independent hadronization

The typical hadronic correlation length in rapidity is

$$\Delta y_h \simeq 2$$

Illustrations from P. Mulders:



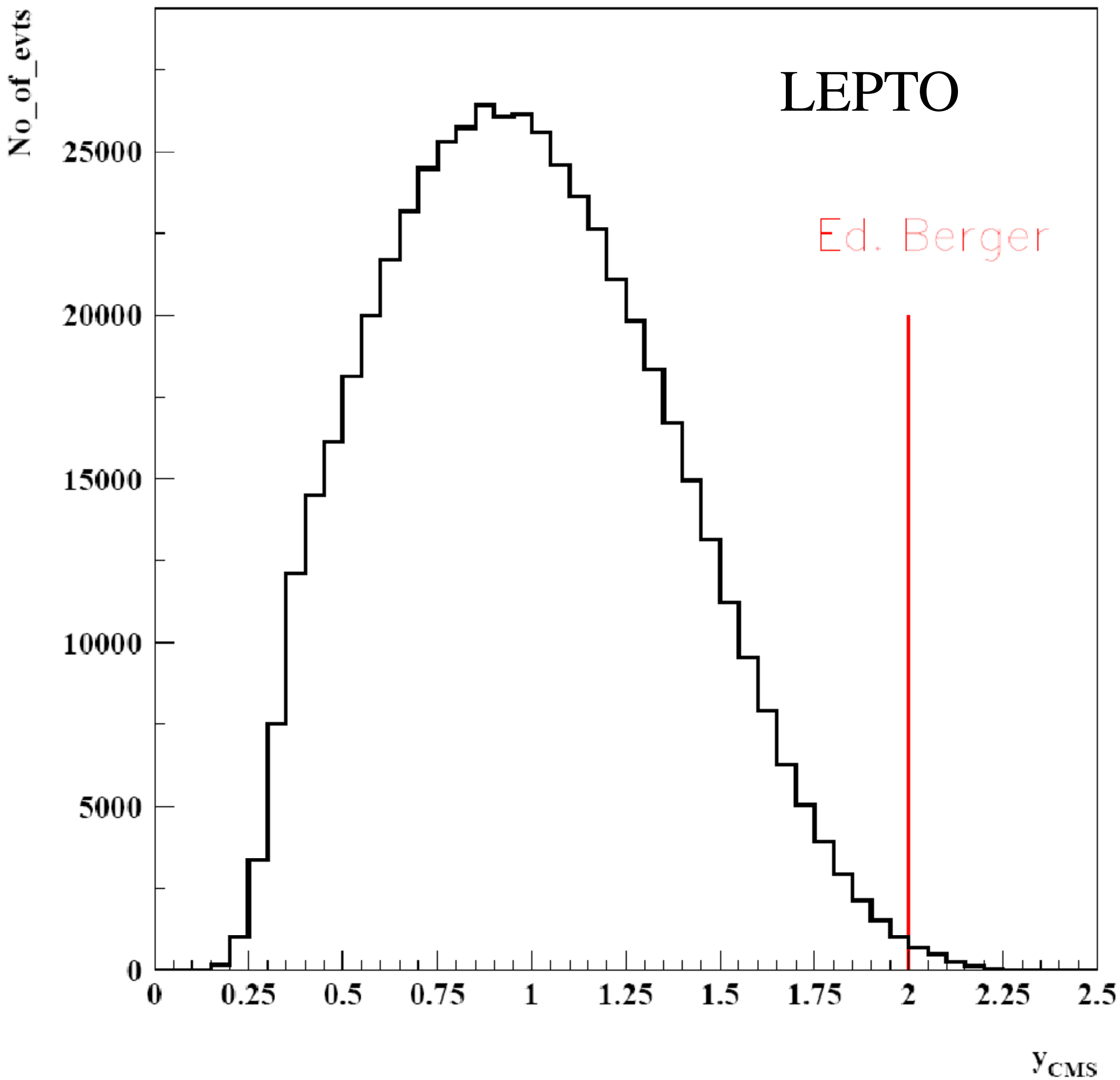




HERMES: ~4 GeV; COMPASS: ~11 GeV; EIC(5+50, y>0.4): ~24GeV

# Ed. Berger criterion

According to Ed. Berger criterion for independent quark fragmentation there is almost no room for kaons @ HERMES

$z > 0.2$ ???



# Higher twist: all contributions out of canonical factorized expression

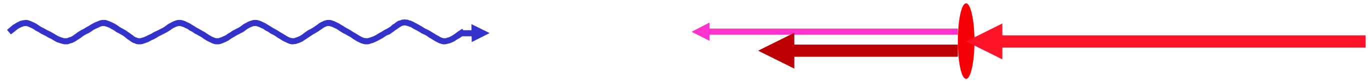
$$d\sigma^{lN\to l'X} = \sum_q f_q(x,\mathbf{k}_T)\otimes d\sigma^{lq\to l'q'}$$

$$d\sigma^{lN\to l'hX} = \sum_q f_q(x,\mathbf{k}_T)\otimes d\sigma^{lq\to l'q'}\otimes D_{q'}^{h}(z)$$

"I have no data yet. It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts."

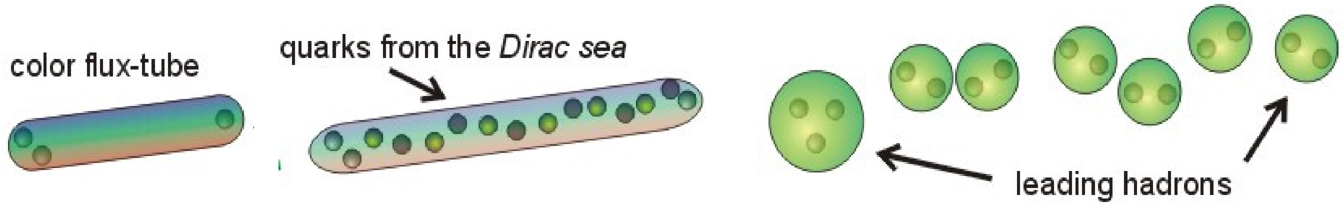
Sherlock Holmes in "**A Scandal in Bohemia** ", by Sir Arthur Conan Doyle, 1891

# LO SIDIS in LEPTO

- Before 

- After Target remnant  quark

-Example: valence struck quark




Does Lund hadronization exactly correspond to independent quark fragmentation in the CFR with z > 0.2?

The important property of FFs is universality:

1. Independence of Bjorken variable $x$
2. Target type independence
3. Process type independence

$$D_q^h(z,Q^2) = \frac{N_{q/N}^{h,SIDIS}(x,z,Q^2)}{N_{q/N}^{DIS}(x,z,Q^2)}$$

# LUND string fragmentation

The primary hadrons produced in string fragmentation come from the string as a whole, rather than from an individual parton.

**PYTHIA 6.2**

**Physics and Manual**

Torbjörn Sjöstrand,[1] Leif Lönnblad,[1]
Stephen Mrenna,[2] Peter Skands[1]

[1]Department of Theoretical Physics,
Lund University, Sölvegatan 14A,
S-223 62 LUND, SWEDEN

# Bjorken variable dependence of "FFs" in LEPTO

Cuts:

$Q^2 > 1 GeV^2$

$W^2 > 10 GeV^2$

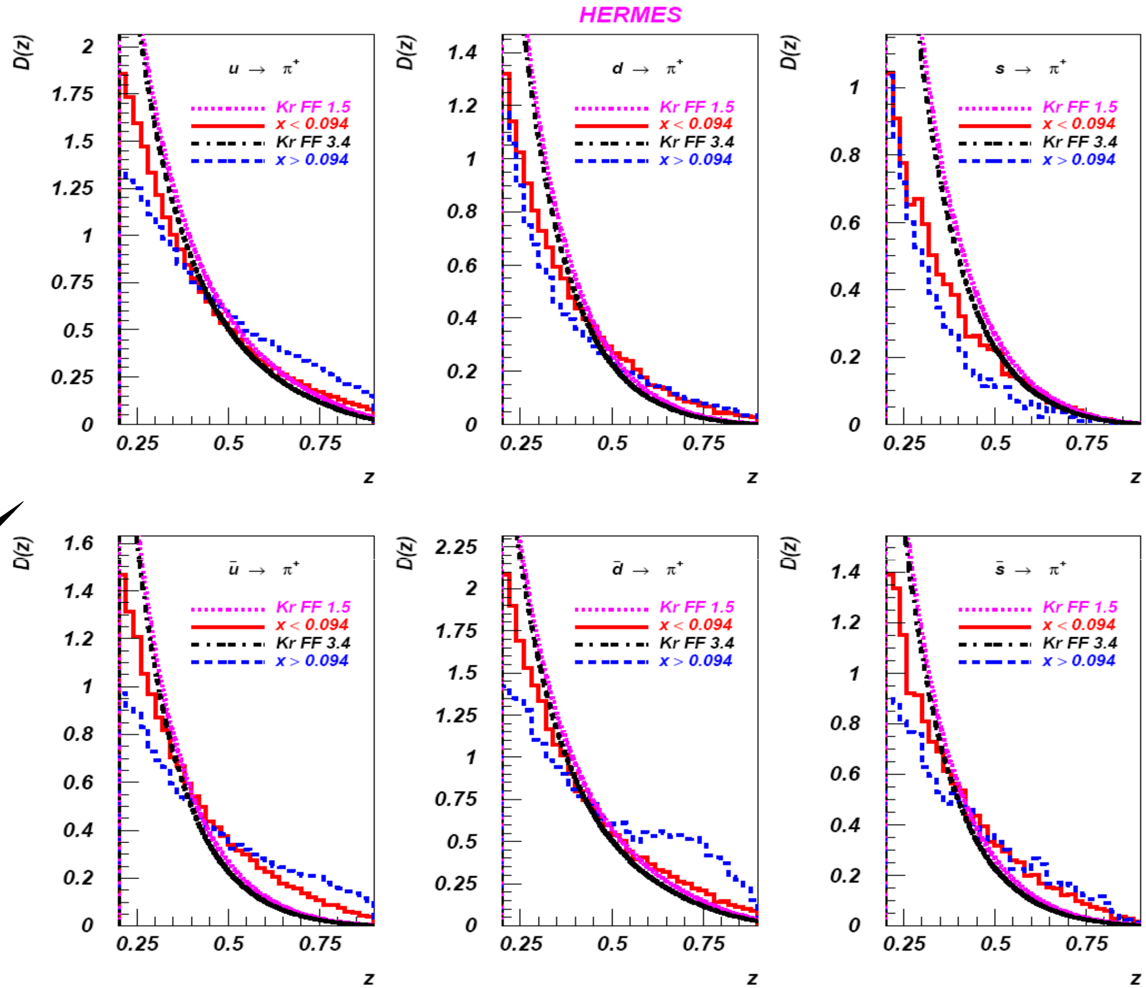
$y<0.85$;

$0.023<x<0.6$

$E' > 3.5 GeV$

$z > 0.2$

$x_F > 0.1$




$x < 0.1$

$Q^2 = 1.5 GeV^2$

$x > 0.1$

$Q^2 = 3.4 GeV^2$

The dependence of "FFs" on $\boldsymbol{x}$

cannot be attributed to $\boldsymbol{Q^2}$ evolution

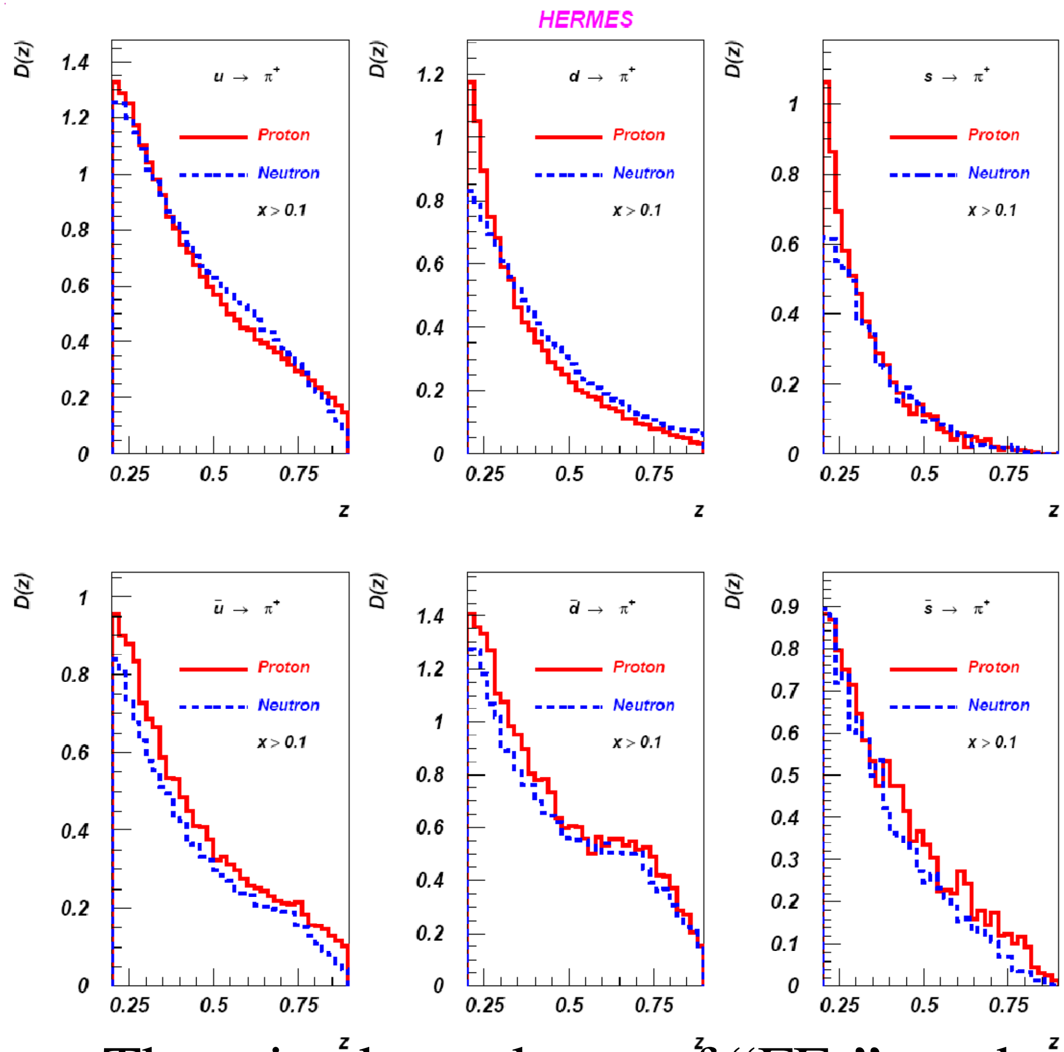
How this dependence can change HERMES purities?

# Target type dependence of “FFs” in LEPTO

Example of target remnant: removed valence ***u***-quark:

$$p-u \Rightarrow (ud)$$

$$n-u \Rightarrow (dd)$$




There is dependence of “FFs” on the target type at 10% level

# Target remnant in Polarized SIDIS

JETSET is based on SU(6) quark-diquark model

$$p^+ = \frac{1}{\sqrt{18}}\{u^+[3(ud)_{0,0} + (ud)_{1,0}] - \sqrt{2}u^-(ud)_{1,1} - \sqrt{2}d^+(uu)_{1,0} + 2d^-(uu)_{1,1}\}$$

$$n^+ = \frac{1}{\sqrt{18}}\{d^+[3(ud)_{0,0} + (ud)_{1,0}] - \sqrt{2}d^-(ud)_{1,1} - \sqrt{2}u^+(dd)_{1,0} + 2u^-(dd)_{1,1}\}$$

$$\Delta q(x) = q_+(x) - q_-(x)$$

$$u_+(x) \longrightarrow p^+ \ominus u^+ \Longrightarrow \begin{cases} \{(ud)_{0,0} \cdots\cdots u^+\}, & w = 0.9 \\ \{(ud)_{1,0} \cdots\cdots u^+\}, & w = 0.1 \end{cases}$$

90% scalar

$$u_-(x) \longrightarrow p^- \ominus u^+ \Longrightarrow \{(ud)_{1,-1} \cdots\cdots u^+\}, \qquad w = 1$$

100% vector

$$d_+(x) \longrightarrow n^+ \ominus u^+ \Longrightarrow \{(dd)_{1,0} \cdots\cdots u^+\}, \qquad w = 1$$

$$d_-(x) \longrightarrow n^- \ominus u^+ \Longrightarrow \{(dd)_{1,-1} \cdots\cdots u^+\}, \qquad w = 1$$

Probabilities of different string spin configurations depend on quark and target polarizations, target type and process type

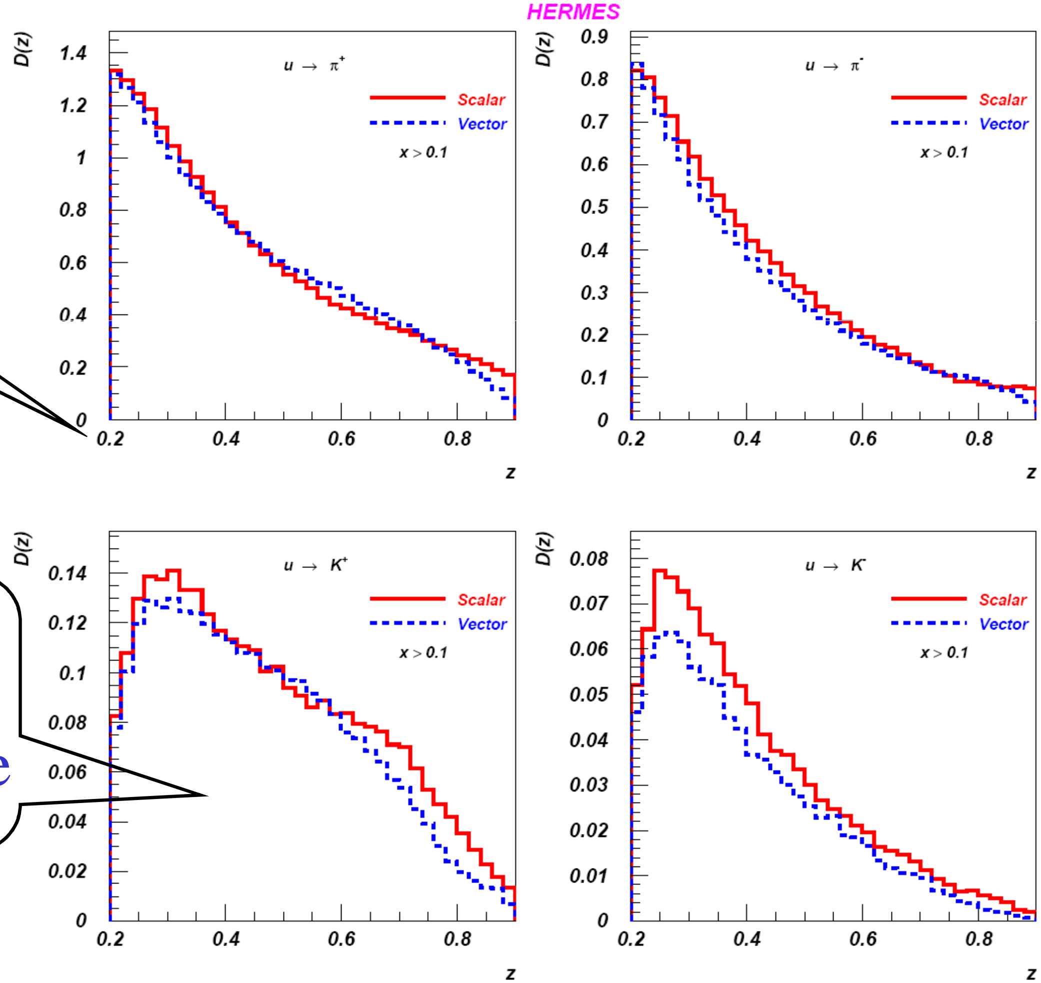
# Fragmentation functions in LEPTO
# Dependence on target remnant spin state

Example: valence ***u*-quark** is removed from proton. Default LEPTO: the remnant (***ud***) diquark is in 75% (25%) of cases scalar (vector)

$\{(ud)_0 \cdots\cdots u\}, \qquad w=1.$

$\{(ud)_1 \cdots\cdots u\}, \qquad w=1.$

Even in unpolarized LEPTO there is a dependence on target remnant spin state




***(ud)*$_0$: first rank $\Lambda$ is possible**

***(ud)*$_1$: first rank $\Lambda$ is impossible**

# Polarized SIDIS & HF

$\sigma^h_{N\,\lambda_l\lambda_N}$ and $H^h_{q/N\,\lambda_q\lambda_N}$ -- spin dependent cross section and HFs

$$\sigma^h_{N++} \propto \sum_q e_q^2 \{q^+ H^h_{q/N++} + (1-y)^2 q^- H^h_{q/N-+}\}$$
$$\sigma^h_{N+-} \propto \sum_q e_q^2 \{q^- H^h_{q/N+-} + (1-y)^2 q^+ H^h_{q/N--}\}$$

$$H^h_{q/N} = H^h_{q/N++} + H^h_{q/N+-}$$
$$\Delta H^h_{q/N} = H^h_{q/N++} - H^h_{q/N+-}$$

$$\boxed{\begin{aligned} \sigma^h_{N\lambda_l\lambda_N} \propto {} & [1+(1-y)^2]\sum_q e_q^2\{qH^h_{q/N} + \Delta q \Delta H^h_{q/N}\} + \\ & \lambda_l\lambda_N[1-(1-y)^2]\sum_q e_q^2\{\Delta q H^h_{q/N} + q\Delta H^h_{q/N}\}, \end{aligned}}$$

In contrast with FFs, HFs in addition to $z$ depend on ***x and target type***

$\Delta H^h_{q/N} \neq 0$ double spin effect, as in DFs.

# Possible source for strange ΔS

- x-z factorization was not checked
  - Extract unknown integrals of fragmentation functions in different subsets of x-bins and compare them
- Missing term in the (polarized) SIDIS equation related to polarization dependent hadronization

A.K. EPJ C44, 211 (2005)

Neglected

$$A_1^h(x,z,Q^2)=\frac{\sum_q e_q^2 q(x,Q^2)H^h_{q/N}(x,z,Q^2)\left(\frac{\Delta q(x,Q^2)}{q(x,Q^2)}+\boxed{\frac{\Delta H^h_{q/N}(x,z,Q^2)}{H^h_{q/N}(x,z,Q^2)}}\right)}{\sum_q e_q^2 q(x,Q^2)H^h_{q/N}(x,z,Q^2)\left(1+\boxed{\frac{\Delta q(x,Q^2)\Delta H^h_{q/N}(x,z,Q^2)}{q(x,Q^2)H^h_{q/N}(x,z,Q^2)}}\right)}$$

# Asymmetry

$$A_{1N}^{h,Exp}(x,z,Q^2)=\frac{\sum_q e_q^2\left(\Delta q(x,Q^2)H_{q/N}^{h}(x,z,Q^2)+q(x,Q^2)\Delta H_{q/N}^{h}(x,z,Q^2)\right)}{\sum_q e_q^2\left(q(x,Q^2)H_{q/N}^{h}(x,z,Q^2)+\Delta q(x,Q^2)\Delta H_{q/N}^{h}(x,z,Q^2)\right)}$$

$$\approx A_{1N}^{h,Std}(x,z,Q^2)+\frac{\sum_q e_q^2 q(x,Q^2)\Delta H_{q/N}^{h}(x,z,Q^2)}{\sum_q e_q^2 q(x,Q^2)H_{q/N}^{h}(x,z,Q^2)}$$

$$\cong A_{1N}^{h,Std}(x,z,Q^2)+\varepsilon(x,z,Q^2)$$

The standard expression for SIDIS asymmetry is obtained when

$$H_{q/N}^{h}(x,z,Q^2)\rightarrow D_q^h(z,Q^2) \qquad \Delta H_{q/N}^{h}(x,z,Q^2)\rightarrow 0$$

Only standard part of expression for asymmetry contains quark polarizations

$$A_{1N}^{h,Std}(x,z,Q^2)=A_{1N}^{h,Exp}(x,z,Q^2)-\varepsilon(x,z,Q^2)$$

# Modeling ε in LEPTO

$$\varepsilon(x,z,Q^2) = \frac{\sum_q e_q^2 q(x,Q^2)\Delta H_{q/N}^h(x,z,Q^2)}{\sum_q e_q^2 q(x,Q^2) H_{q/N}^h(x,z,Q^2)}$$

LEPTO: HERMES tuning

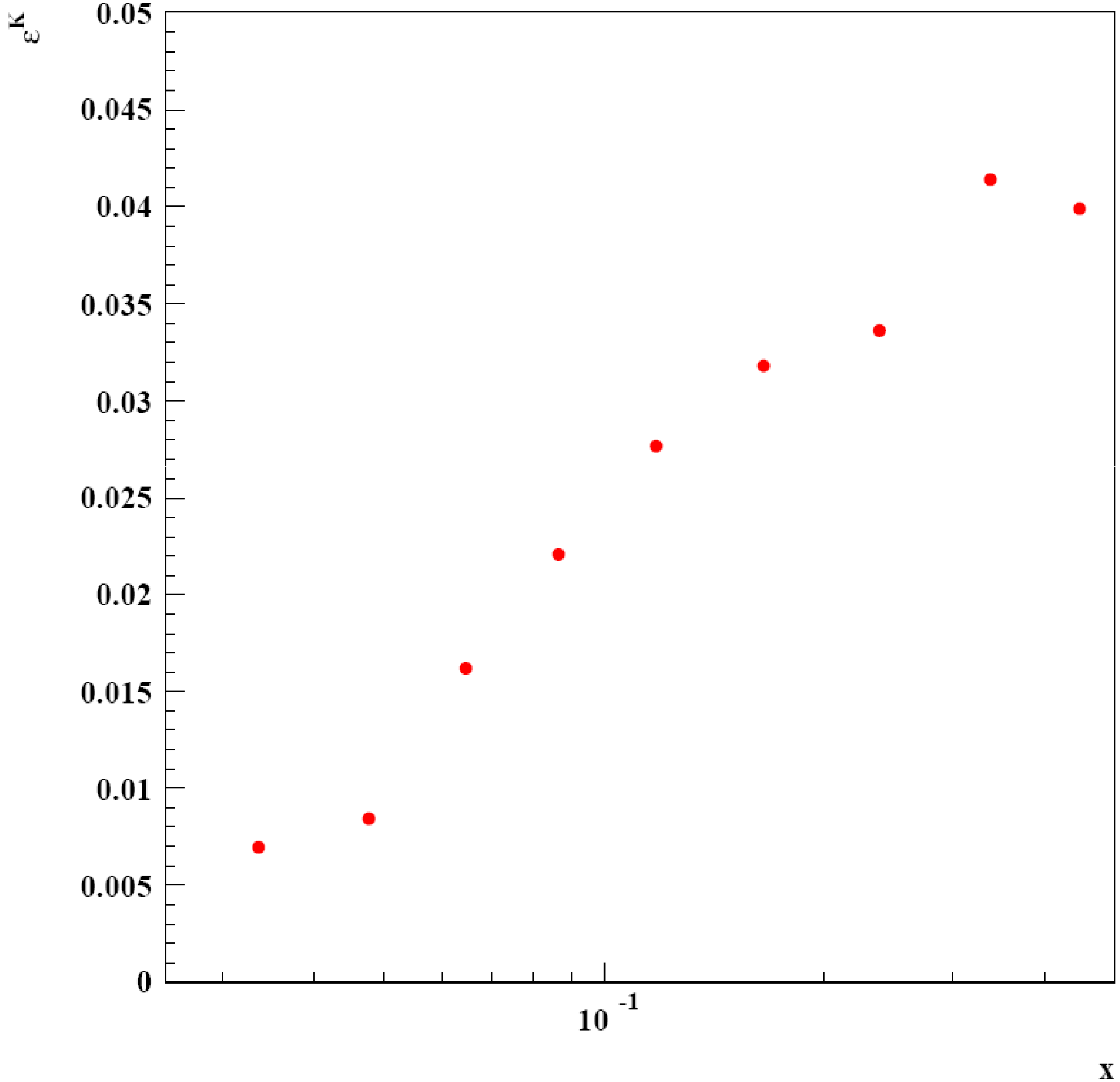
parl(4)=probability of scalar diquark

$$parl(4) = 0.9 \Rightarrow N_{++}^{K/N} \propto \left(1+(1-y)^2\right)\sum_q e_q^2 q(x) H_{++}^{K/N},$$

$$parl(4) = 0.0 \Rightarrow N_{+-}^{K/N} \propto \left(1+(1-y)^2\right)\sum_q e_q^2 q(x) H_{+-}^{K/N}$$

$$\varepsilon_d^K(x,z,Q^2) = \frac{N_{++}^{K/p} + N_{++}^{K/n} - N_{+-}^{K/p} - N_{+-}^{K/n}}{N_{++}^{K/p} + N_{++}^{K/n} + N_{+-}^{K/p} + N_{+-}^{K/n}}$$

# ε(x)




LEPTO with

HERMES

tuning and cuts

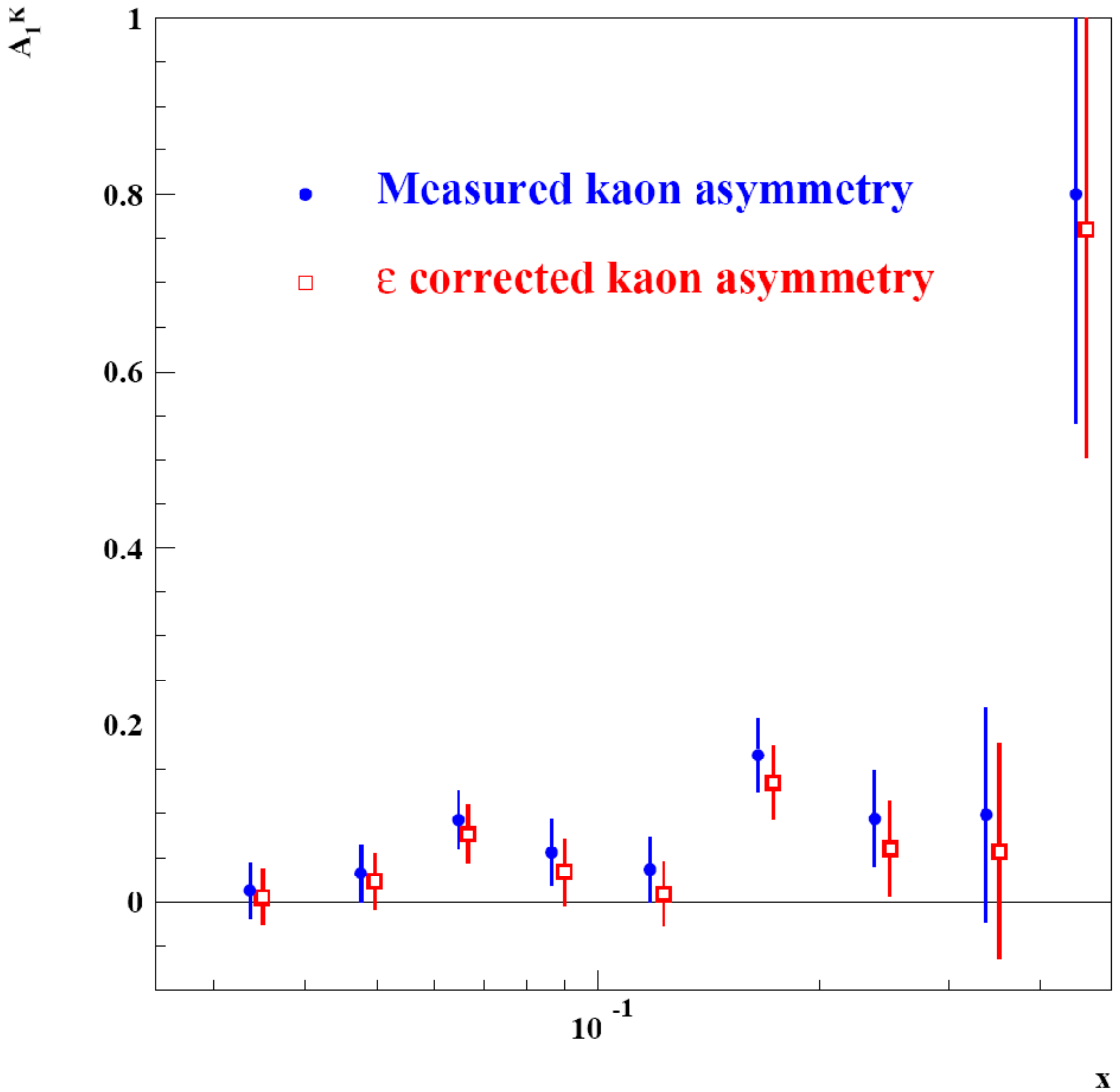
CTEQ6 LO

$K^+$, $K^-$ production

off deuterium target

# Kaon production asymmetry

Corrected asymmetry has to be used for quark polarization determination



$$p_q(x) = \frac{\Delta q(x)}{q(x)}$$

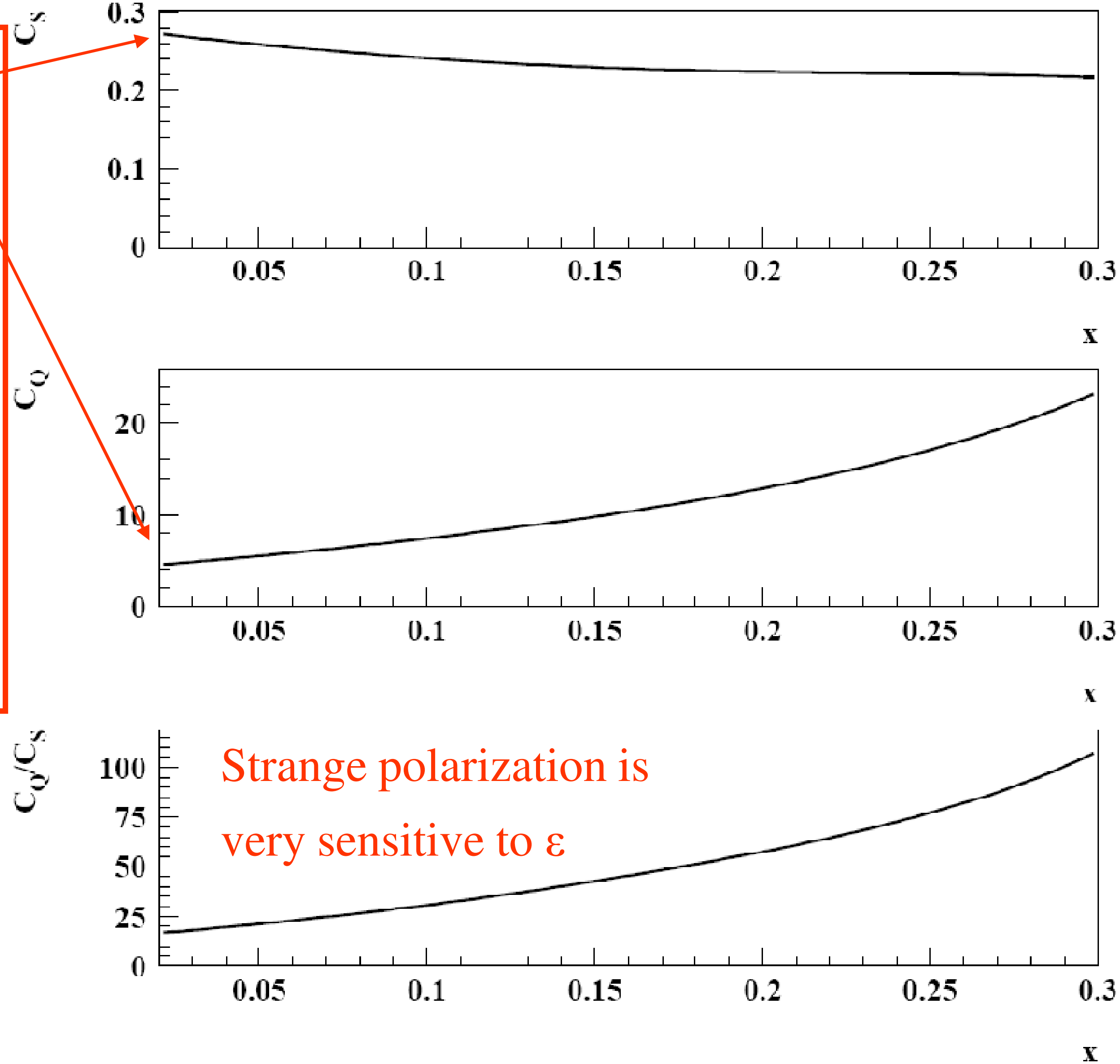
# Quark polarizations

$$p_Q = p_Q^{HERMES} + \varepsilon_d^K C_S$$

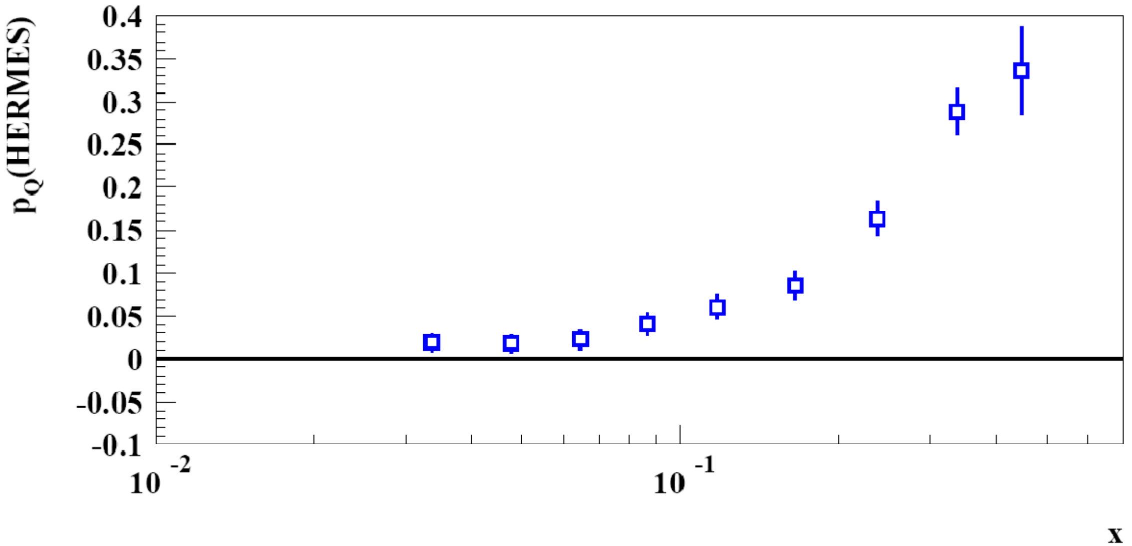
$$p_S = p_S^{HERMES} - \varepsilon_d^K C_Q$$

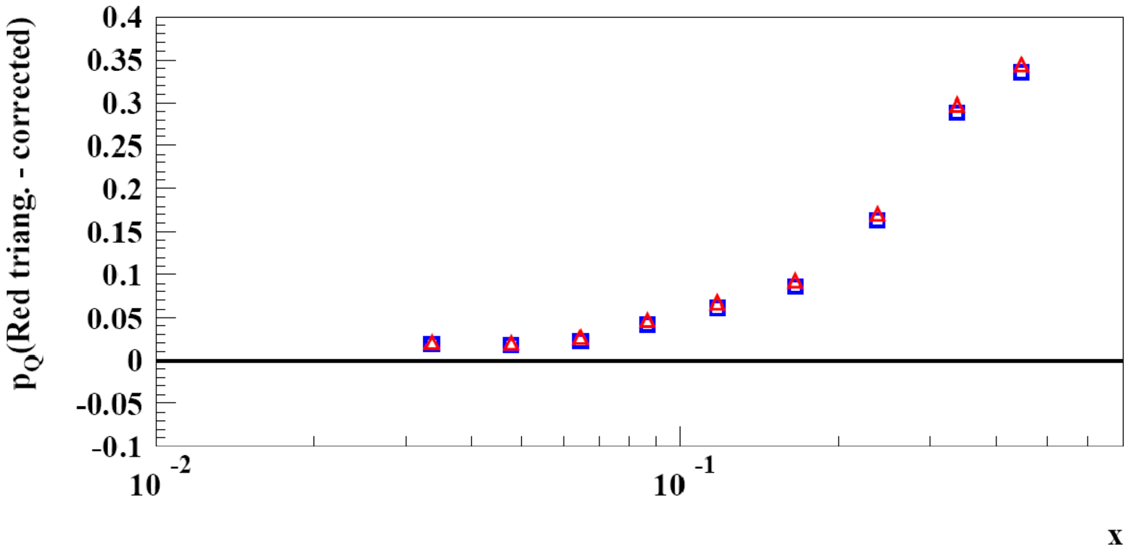
$$C_S = \frac{P_S}{P_Q P_S^K - P_S P_Q^K}$$

$$C_Q = \frac{P_Q}{P_Q P_S^K - P_S P_Q^K}$$



# Light quark polarization 1




Inputs from HERMES:

Ahmed El Alaoui

PhD thesis

# Light quarks polarization 2

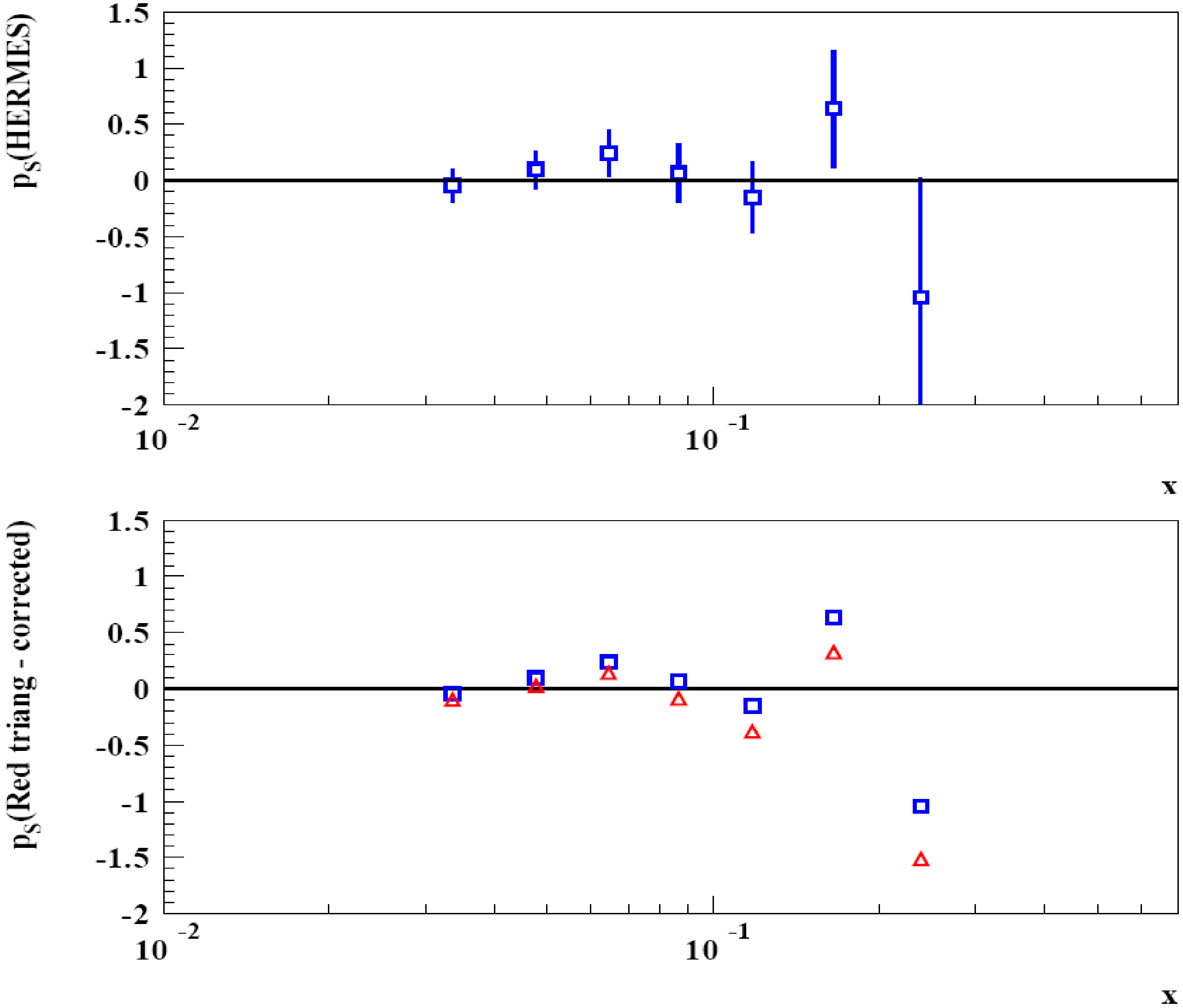
M. Ehrenfried, DIS 2006 proceedings

$$\int_{0.02}^{1} dx\Delta Q(x) = 0.286 \pm 0.026 \pm 0.011$$

$$\Delta Q_{HERMES} = \int_{0.02}^{0.6} dx\Delta Q(x) = \sum_{i=1}^{9} \frac{\Delta Q}{Q}(x_i)_{HERMES} \int_{x_i}^{x_{i+1}} dx Q(x) = 0.271$$

$$\Delta Q_{\varepsilon-corr} = \int_{0.02}^{0.6} dx\Delta Q(x) = \sum_{i=1}^{9} \frac{\Delta Q}{Q}(x_i)_{\varepsilon-corr} \int_{x_i}^{x_{i+1}} dx Q(x) = 0.288$$

# Strange quarks polarization 1



# Strange quarks polarization 2

$$\Delta S_{HERMES} = \int_{0.02}^{0.3} dx \Delta S(x) = \sum_{i=1}^{7} \frac{\Delta S}{S}(x_i)_{HERMES} \int_{x_i}^{x_{i+1}} dx S(x) = 0.0055$$

$$\Delta S_{\varepsilon-corr} = \int_{0.02}^{0.3} dx \Delta S(x) = \sum_{i=1}^{7} \frac{\Delta S}{S}(x_i)_{\varepsilon-corr} \int_{x_i}^{x_{i+1}} dx S(x) = -0.027$$

Compare:

HERMES SIDIS: $\int_{0.02}^{1} \Delta S \, \mathrm{d}x = 0.006 \pm 0.029 \pm 0.007$

COMPASS DIS: $-0.08 \pm 0.01 \, (\text{stat.}) \pm 0.02 \, (\text{syst.})$

New value is more than one standard deviation away

# Contributions to ε(x)

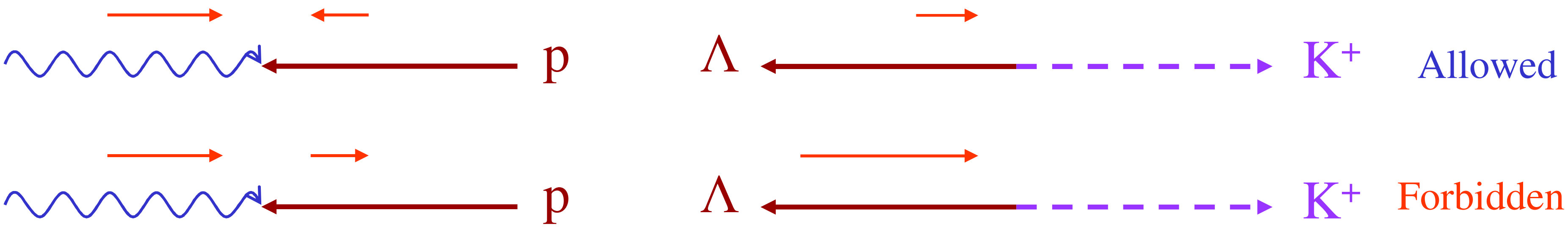
- To obtain ε-correction LEPTO was used
  - Only initial state of string was changed
  - Hadronization was not modified to include spin-dependent effects
    - JETSET takes care only for momentum and quantum numbers conservation.
    - Angular momentum conservation (spin and orbital momentum dynamics) is not modeled.

# Other contributions to ε. Spin dynamics

Extreme case – quasi elastic forward production of pseudo-scalar meson.

$K^+$ can be accompanied with many pions with orbital momentum zero

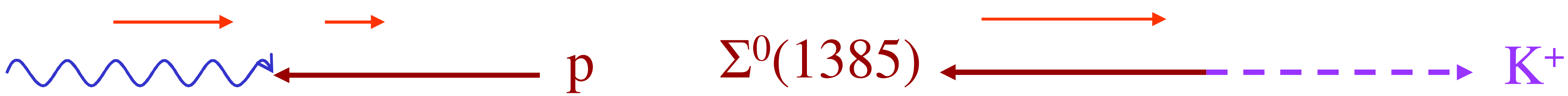
Initial state                Final state




$A_1$(“elastic”)=1; again positive contribution to ε

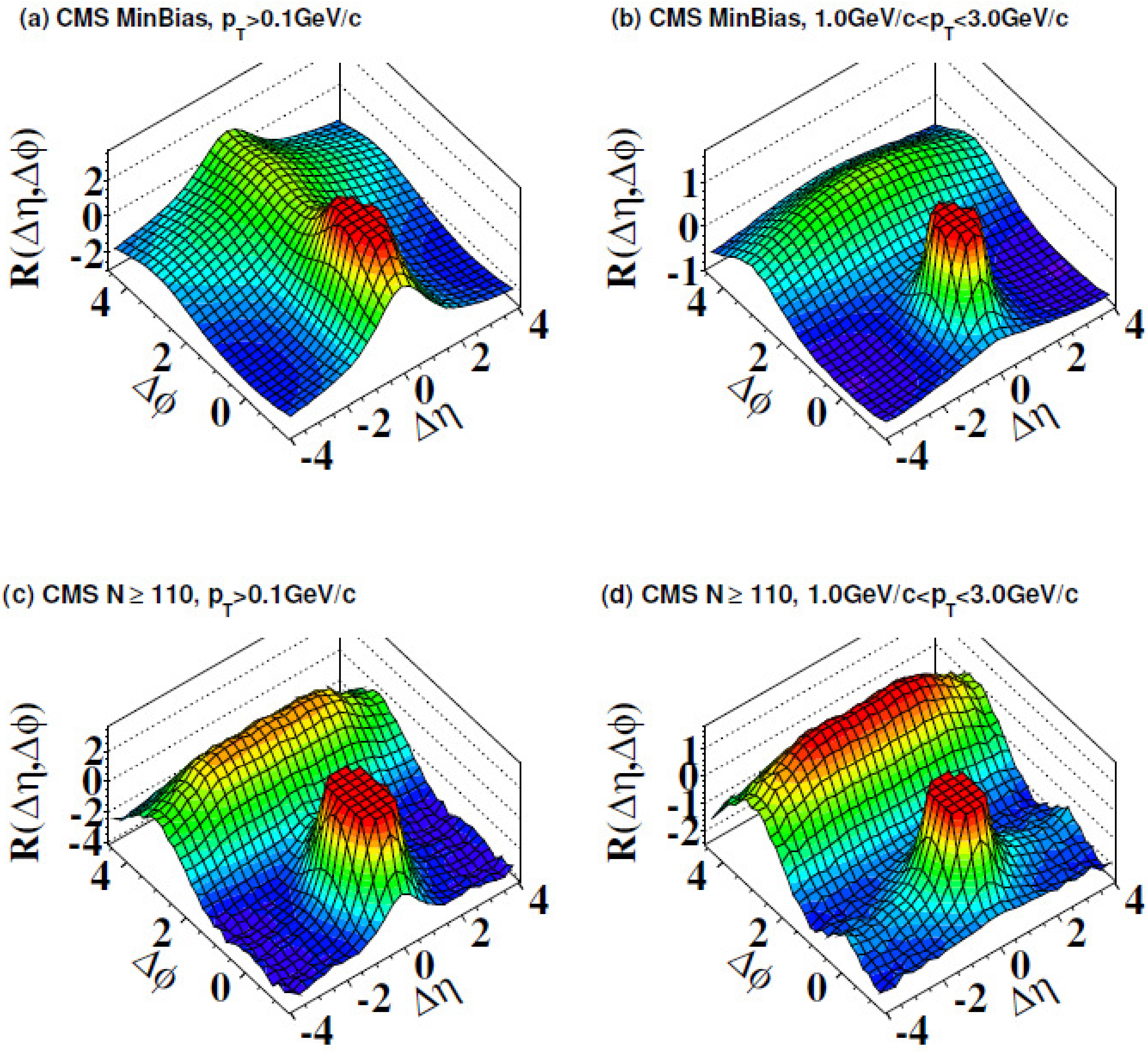
Dilutions: unobserved particles (orbital momentum), heavier hyperons…

Initial state                Final state




Ideas: Misha Rekalo, Egle Gustafson for ΔG from open charm

# Azimuthal correlations



# General expression for 1h production cross-section

$$
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =
$$

$$
\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}
$$

$$
+\varepsilon\boxed{\cos(2\phi_h)}F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}
$$

$$
+S_{\parallel}\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\boxed{\sin(2\phi_h)}F_{UL}^{\sin 2\phi_h}\right]
$$

$$
+S_{\parallel}\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]
$$

$$
+|\boldsymbol{S}_\perp|\Bigg[\boxed{\sin(\phi_h-\phi_S)}\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)
$$

$$
+\varepsilon\boxed{\sin(\phi_h+\phi_S)}F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\boxed{\sin(3\phi_h-\phi_S)}F_{UT}^{\sin(3\phi_h-\phi_S)}
$$

$$
+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]
$$

$$
+|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}
$$

$$
+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
$$

Valid for SIDIS CFR, TFR and exclusive reactions

$$
\boxed{\varepsilon=\frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}}
$$

# Parton model for SIDIS in CFR

$$d\sigma^{l+N\to l'+h+X} \propto DF \otimes d\sigma^{l+q\to l'+q'} \otimes FF$$

At twist-two

$$\mathcal{P}_N^q(x,\mathbf{k}_T) = f_1^q(x,k_T^2) + f_{1T}^{\perp q}(x,k_T^2)\frac{[\mathbf{k}_T \times \hat{\mathbf{P}}_N]\cdot \mathbf{S}_T^N}{M},$$

$$f_1^q(x,k_T^2)s_L^q(x,\mathbf{k}_T) = g_{1L}^q(x,k_T^2)\lambda_N + g_{1T}^{\perp q}(x,k_T^2)\frac{\mathbf{k}_T \cdot \mathbf{S}_T^N}{M},$$

$$f_1^q(x,k_T^2)\mathbf{s}_T^q(x,\mathbf{k}_T) = h_{1T}^q(x,k_T^2)\mathbf{S}_T^N + [h_{1L}^{\perp q}(x,k_T^2)\lambda_N + h_{1T}^{\perp q}(x,k_T^2)\frac{\mathbf{k}_T \cdot \mathbf{S}_T^N}{M}]\frac{\mathbf{k}_T}{M} + h_1^{\perp q}(x,k_T^2)\frac{[\mathbf{k}_T \times \hat{\mathbf{P}}_N]}{M}$$

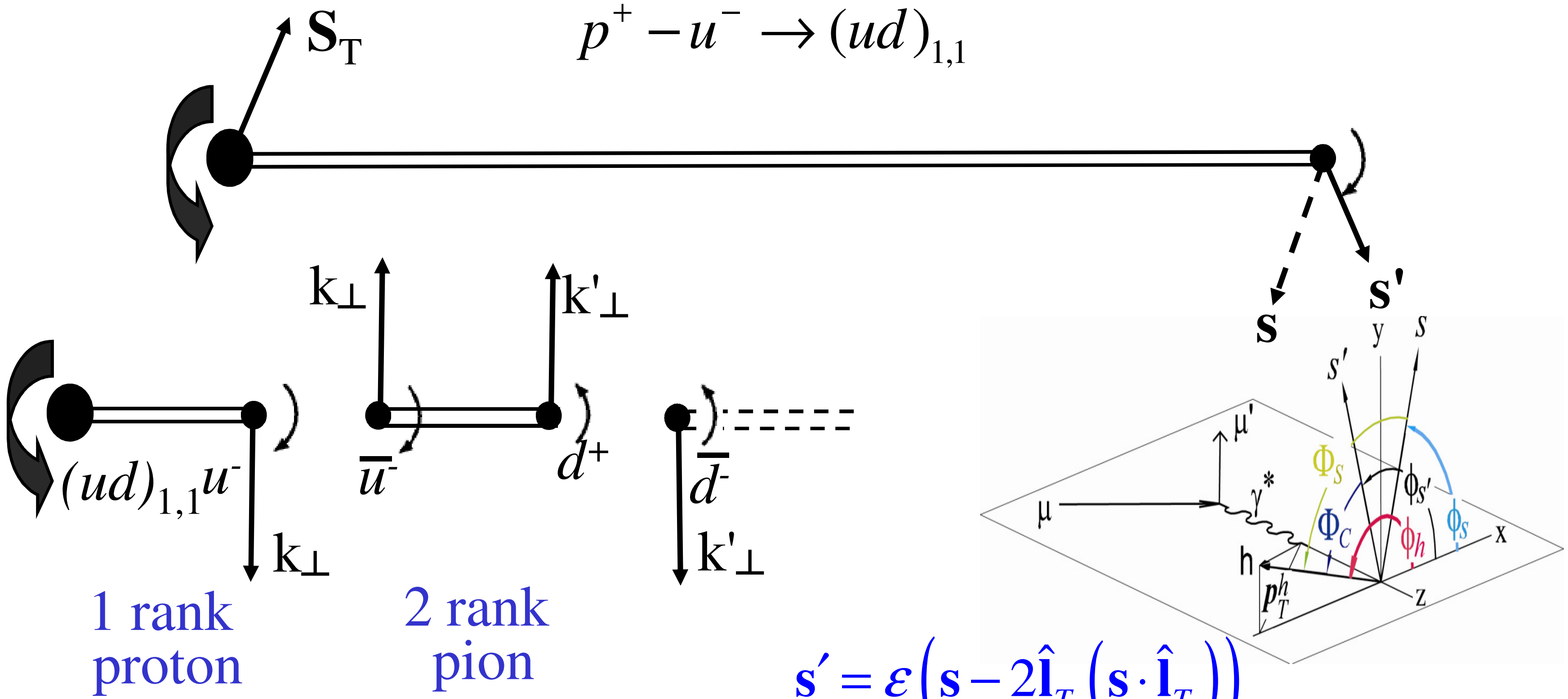
Often used: $$h_1^q(x,k_T^2) = h_{1T}^q(x,k_T^2) + \frac{k_T^2}{2M^2}h_{1T}^{\perp q}(x,k_T^2)$$

$$\mathcal{P}_{q\uparrow}^h(z,\mathbf{P}_{Tq}^h) = D_q^h(z,P_{Tq}^h) + H_q^h(z,P_{Tq}^h)\frac{[\mathbf{P}_{Tq}^h \times \hat{\mathbf{k}}']\cdot \mathbf{s}_T'}{M} = D_q^h(z,P_{Tq}^h) + s_T'\frac{P_{Tq}^h}{M}H_q^h(z,P_{Tq}^h)\sin(\phi_{Collins})$$

# Target remnant polarization state

JETSET is based on SU(6) quark-diquark model

$$p^+ = \frac{1}{\sqrt{18}}\{u^+[3(ud)_{0,0} + (ud)_{1,0}] - \sqrt{2}u^-(ud)_{1,1} - \sqrt{2}d^+(uu)_{1,0} + 2d^-(uu)_{1,1}\}$$
$$n^+ = \frac{1}{\sqrt{18}}\{d^+[3(ud)_{0,0} + (ud)_{1,0}] - \sqrt{2}d^-(ud)_{1,1} - \sqrt{2}u^+(dd)_{1,0} + 2u^-(dd)_{1,1}\}$$




$$\mathbf{s}' = \varepsilon\left(\mathbf{s} - 2\hat{\mathbf{l}}_T\left(\mathbf{s}\cdot\hat{\mathbf{l}}_T\right)\right)$$

# Transverse pseudo vectors

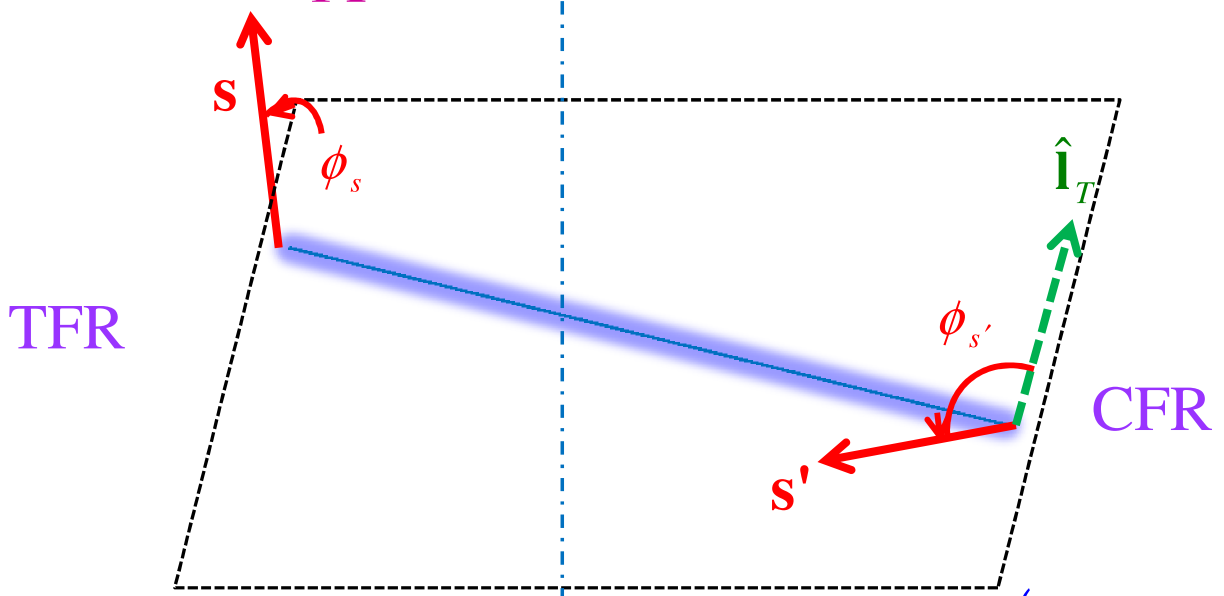
Unpolarizer target: $\mathbf{s} \propto [\mathbf{k}_\perp \times \hat{\mathbf{q}}]$

Transversely polarizer target: $\mathbf{s} \propto \mathbf{S}_T^N$

Transversely polarizer target: $\mathbf{s} \propto \left(\mathbf{S}_T^N \cdot \mathbf{k}_\perp\right)\mathbf{k}_\perp$

Longitudinally polarizer target: $\mathbf{s} \propto S_L^N \mathbf{k}_\perp$

# $h_{1T}$ contribution




$\mathbf{s} \propto \mathbf{S}_T^N$

$\mathbf{p}_\perp \approx \mathbf{P}_T - x_F \mathbf{k}_\perp$

$\mathbf{s}' = \varepsilon\left(\mathbf{s} - 2\hat{\mathbf{l}}_T\left(\mathbf{s}\cdot\hat{\mathbf{l}}_T\right)\right)$

$$\int d^2k_\perp \widehat{h}_{1T}^{dq}(x,k_\perp^2)[\mathbf{p}_\perp \times \mathbf{s}]\cdot\hat{\mathbf{q}}_T \widehat{H}_{dq}^h\left(x_F, p_\perp^2\right)$$
$$= P_T \cdot S_T^N \widehat{\Phi}_1''\left(x, x_F, P_T^2\right)\sin\left(\phi_h - \phi_s\right)$$

$$\int d^2k_\perp h_{1T}^q(x,k_\perp^2)[\mathbf{p}_\perp \times \mathbf{s}']\cdot\hat{\mathbf{q}}_T H_q^h\left(x_F, p_\perp^2\right)$$
$$= \varepsilon P_T \cdot S_T^N \Phi_1''\left(x, x_F, P_T^2\right)\sin\left(\phi_h - \phi_{s'}\right)$$
$$= -\varepsilon P_T \cdot S_T^N \Phi_1''\left(x, x_F, P_T^2\right)\sin\left(\phi_h + \phi_S\right)$$

# $h_{1T}^{\perp}$ contribution

TFR

$$\mathbf{s} \propto \left(\mathbf{k}_{\perp} \cdot \mathbf{S}_T^N\right)\mathbf{k}_{\perp}$$

$$\int d^2k_{\perp} \widehat{h}_{1T}^{\perp dq}(x, k_{\perp}^2)\left[\mathbf{p}_{\perp} \times \mathbf{s}\right] \cdot \hat{\mathbf{q}}_T \widehat{H}_{dq}^h\left(x_F, p_{\perp}^2\right)$$

$$= P_T \cdot S_T^N \widehat{\Phi}_1'''\left(x, x_F, P_T^2\right) \sin\left(\phi_h - \phi_s\right)$$

CFR

$$\mathbf{s}' = \varepsilon\left(\mathbf{s} - 2\hat{\mathbf{l}}_T\left(\mathbf{s} \cdot \hat{\mathbf{l}}_T\right)\right)$$

$$\int d^2k_{\perp} h_{1T}^{\perp q}(x, k_{\perp}^2)\left[\mathbf{p}_{\perp} \times \mathbf{s}'\right] \cdot \hat{\mathbf{q}}_T H_q^h\left(x_F, p_{\perp}^2\right)$$

$$= \varepsilon P_T \cdot S_T^N \Phi_1'''\left(x, x_F, P_T^2\right) \sin\left(\phi_h + \phi_{s'}\right)$$

$$+ \varepsilon P_T \cdot S_T^N \Phi_2'''\left(x, x_F, P_T^2\right) \sin\left(3\phi_h - \phi_S\right)$$

# $h_1^{\perp}$ contribution

TFR

$$\mathbf{s} \propto \left[\mathbf{k}_{\perp} \times \hat{\mathbf{P}}_N\right] = [\hat{\mathbf{q}} \times \mathbf{k}_{\perp}]$$

$$\int d^2k_{\perp} \widehat{h}_1^{\perp dq}(x, k_{\perp}^2)[\mathbf{p}_{\perp} \times \mathbf{s}] \cdot \hat{\mathbf{q}}_T \widehat{H}_q^h\left(x_F, p_{\perp}^2\right)$$

$$= P_T \cdot S_T^N \widehat{\Phi}_1'''\left(x, x_F, P_T^2\right) = Const(\phi_h)$$

CFR

$$\mathbf{s}' = \varepsilon\left(\mathbf{s} - 2\hat{\mathbf{l}}_T\left(\mathbf{s} \cdot \hat{\mathbf{l}}_T\right)\right)$$

$$\int d^2k_{\perp} h_1^q(x, k_{\perp}^2)[\mathbf{p}_{\perp} \times \mathbf{s}'] \cdot \hat{\mathbf{q}}_T H_q^h\left(x_F, p_{\perp}^2\right)$$

$$= \varepsilon S_L^N \Phi_2'''\left(x, x_F, P_T^2\right)\cos\left(2\phi_h\right)$$

# $h_{1L}^{\perp}$ contribution

TFR

$$\mathbf{s} \propto S_L \mathbf{k}_\perp$$

$$\int d^2k_\perp \widehat{h}_{1L}^{\perp dq}(x,k_\perp^2)[\mathbf{p}_\perp \times \mathbf{s}] \cdot \hat{\mathbf{q}}_T \widehat{H}_{dq}^{h}\left(x_F, p_\perp^2\right)$$

$$\equiv 0$$

CFR

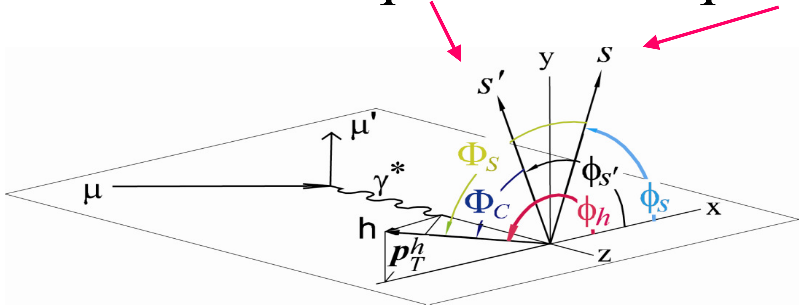
$$\mathbf{s}' = \varepsilon\left(\mathbf{s} - 2\hat{\mathbf{l}}_T\left(\mathbf{s}\cdot\hat{\mathbf{l}}_T\right)\right)$$

$$\int d^2k_\perp h_{1L}^{\perp q}(x,k_\perp^2)[\mathbf{p}_\perp \times \mathbf{s}'] \cdot \hat{\mathbf{q}}_T H_q^h\left(x_F, p_\perp^2\right)$$

$$= \varepsilon S_L^N \Phi_2''''\left(x, x_F, P_T^2\right)\sin\left(2\phi_h\right)$$

# Quark transverse spin effects in TFR of SIDIS

Spin quantization axis for final quark and diquark




$$\phi_{s'} = \pi - \phi_s$$

In contrast to CFR no sideway spin flip

1) no $D_{nn}(y) = \varepsilon = \dfrac{2(1-y)}{1+(1-y)^2}$ factor, which appears due to magnitude of **s'** in Collins fragmentation of scattered quark

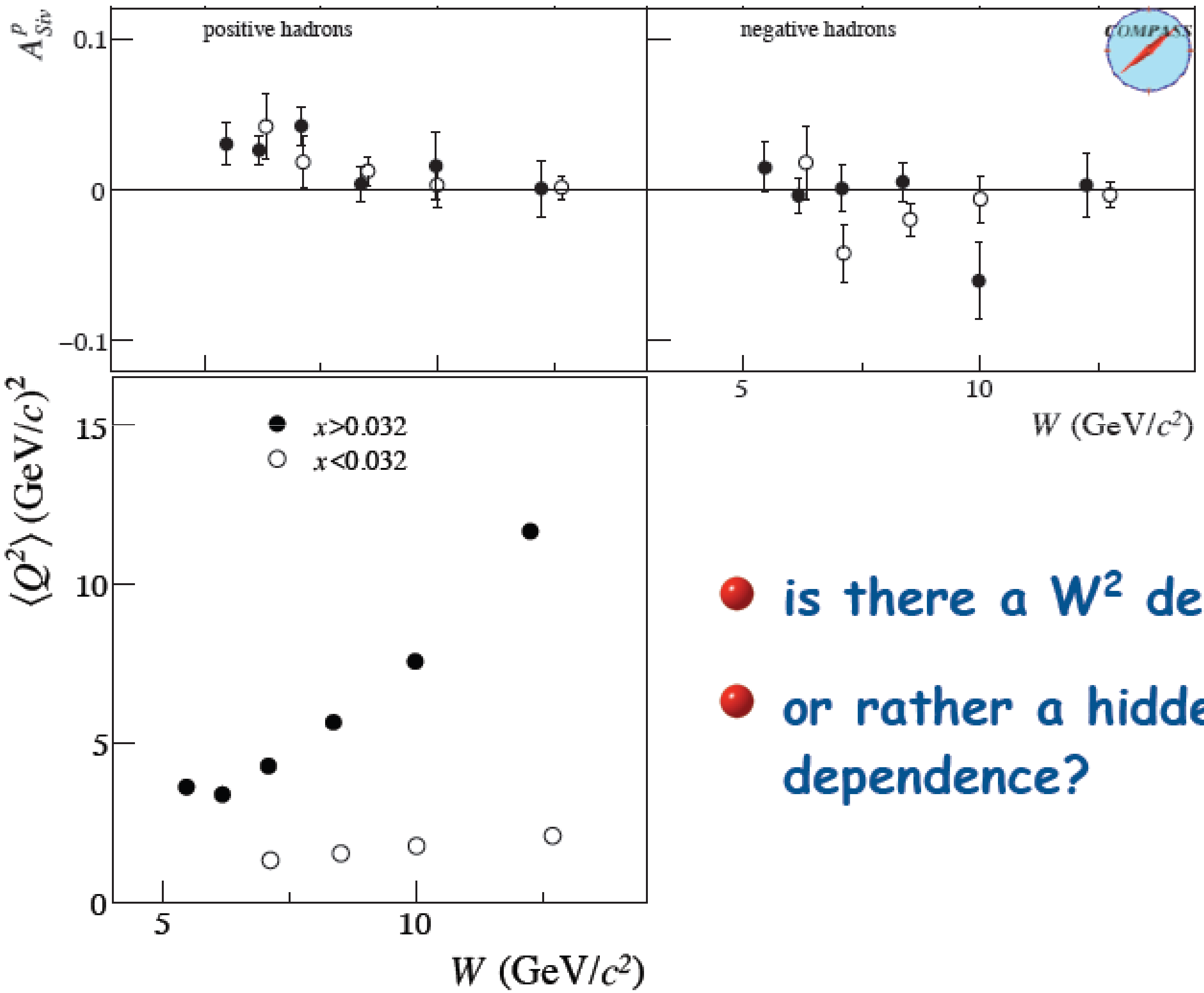
2) the only modulation induced by combined effect of target remnant transverse polarization and Collins fragmentation at leading order is $\sin(\phi_h - \phi_s)$

Same type of contribution as Sivers effect

# Discussion

- Hypothetic scenario: Sivers effect is equal to zero
  - $\sin(\Phi_h\text{-}\Phi_S)$ modulation in CFR can be reflection of "long"-range correlation from Collins effect in the target remnant at low $W$
- If other than $\sin(\Phi_h\text{-}\Phi_S)$ modulation will be observed in the TFR it will be indication on correlation between transverse momentum of hadrons in TFR with transverse polarization of struck quark

# COMPASS results




- is there a $W^2$ dependence?
- or rather a hidden $Q^2$ dependence?

# Conclusions

- To study the nucleon structure we need deeper understanding of hadronization process
  - is it spin independent?
- There is no mystery of strangeness polarization extracted from SIDIS and DIS analyses.
  - The simple example considered here prove that observed in canonized description of SIDIS discrepancy can be considered as a demonstration of the existence and importance of polarization effects in hadronization at intermediate energies.
- Further experimental checks of independent fragmentation are needed
- The future JLab 12 and EIC facilities with $4\pi$ acceptance detectors will help better understand the correlation effects in hadronization

# Thanks for hospitality @ Frascati