

Parametric scan of plasma parameters for optimization of the avalanche process in p - ^{11}B fusion

By:

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Outlook

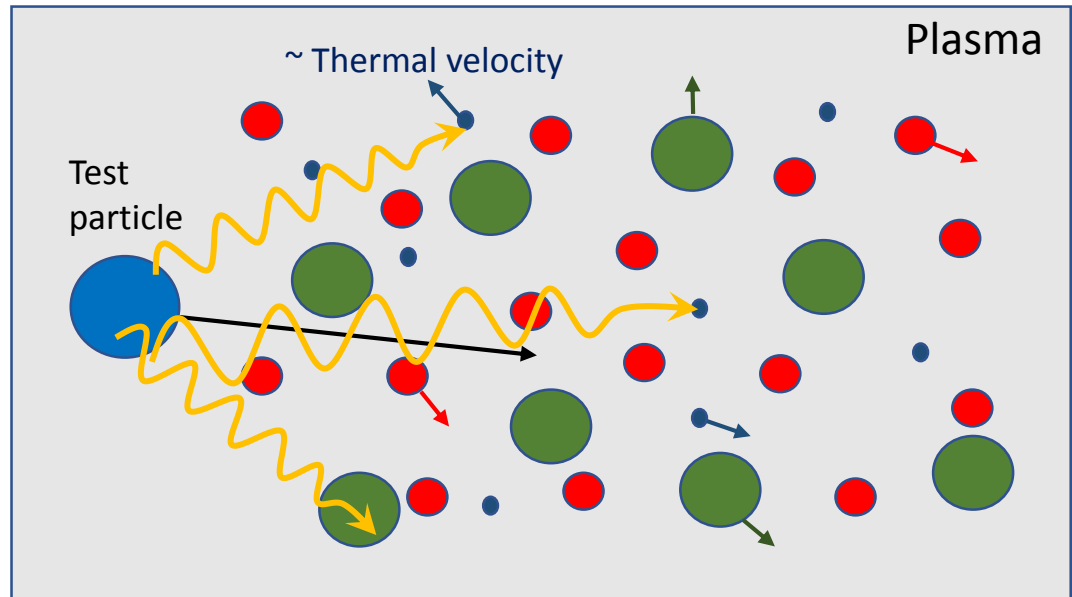
1. Terminology
2. Introduction - Our non-LTE scenario
3. The fusion probability dependance on temperature and electron density
4. Multiplication calculations
5. Conclusion

Terminology 1 – Microscopic interactions

Stopping power

Averaged elastic collisions with many particles =>

After the collision, the particles energy remain close to thermal

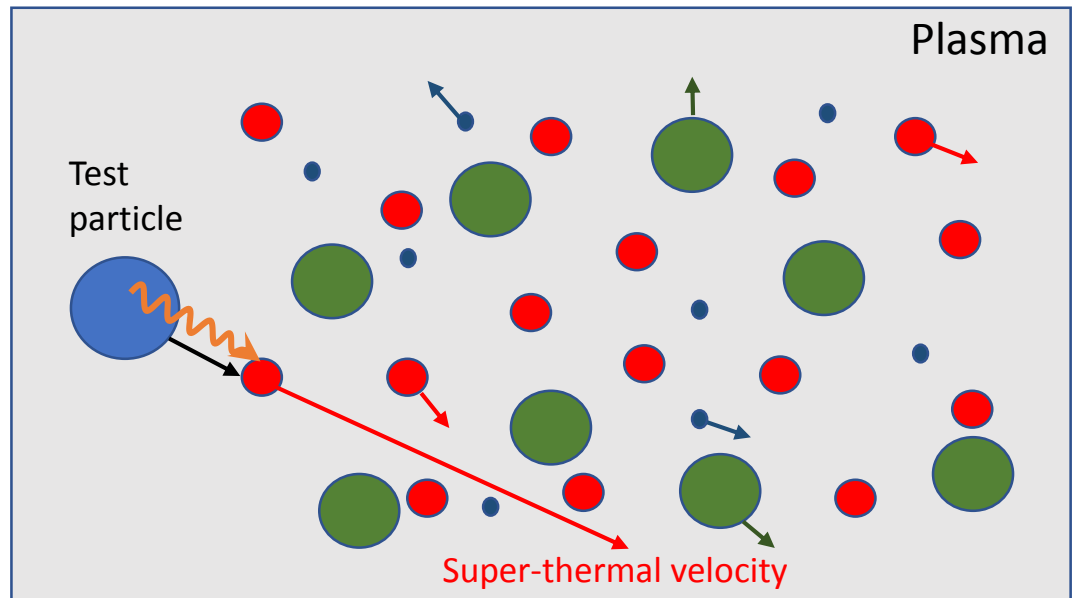


Elastic collision

Elastic collisions with one specific particle =>

=>

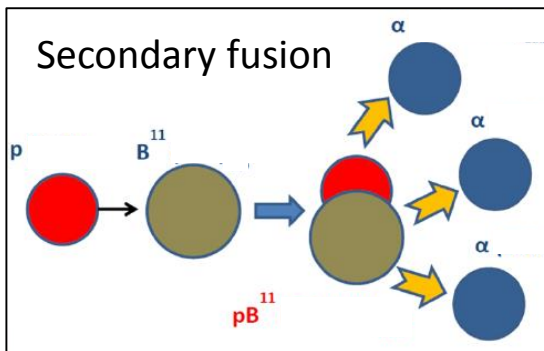
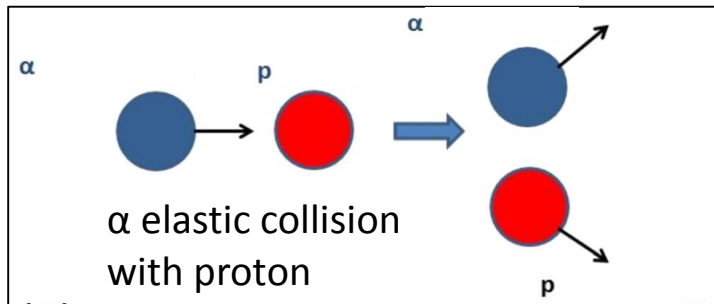
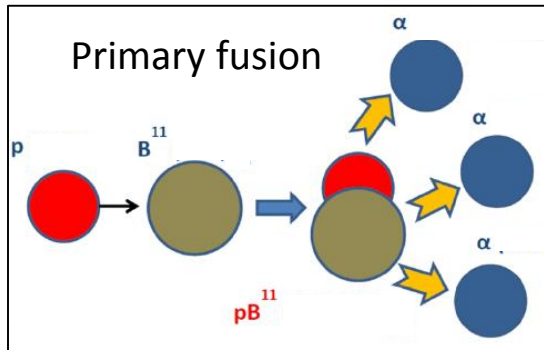
After the collision, the particles energy is super-thermal



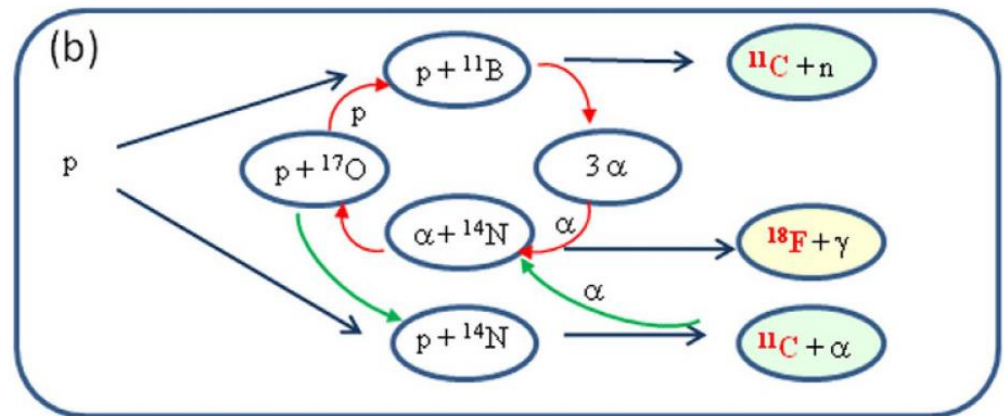
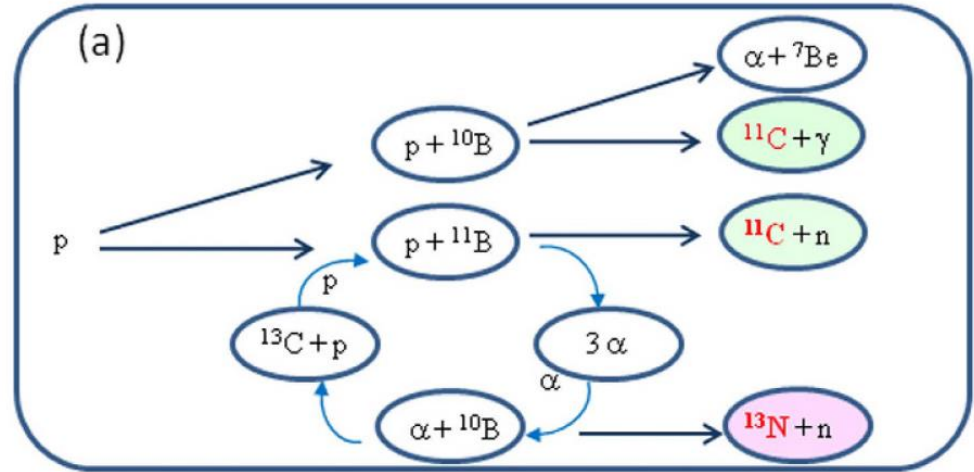
Both interactions may exist simultaneously

Terminology 2 – Microscopic interactions

Elastic chain reaction



Nuclear chain reaction



C. Labaune et al., Scientific Reports, 6, 21202 (2016).

Terminology 3 – Avalanche effect -Macroscopic effect

Can happen in LTE or non-LTE

$$dN = N \cdot \alpha \cdot dt \quad \longrightarrow \quad M = \frac{N}{N_0} = e^{\int_0^{t'} \alpha dt}$$

M – Multiplication factor



Non-LTE p-B fusion case [an over simplified view]

The fusion probability in a time interval:

$$\Delta P_f(t, t + dt) = n_B \sigma_f(v_{p0}) v_{p0} dt$$

The number of fusion reactions in a time interval:

$$dN_f = N_p(t) \cdot \Delta P_f(t, t + dt) = N_p(t) n_B \sigma_f(v_{p0}) v_{p0} dt$$

$$dN_\alpha = 3N_p n_B \sigma_f(v_{p0}) v_{p0} dt \quad \text{The number alphas generated in a time interval}$$

$$dN_p = dN_\alpha \cdot P_{\alpha p} = P_{\alpha p} 3N_p(t) n_B \sigma_f(v_{p0}) v_{p0} dt \quad \text{The number protons is determined by the elastic collision probability.}$$

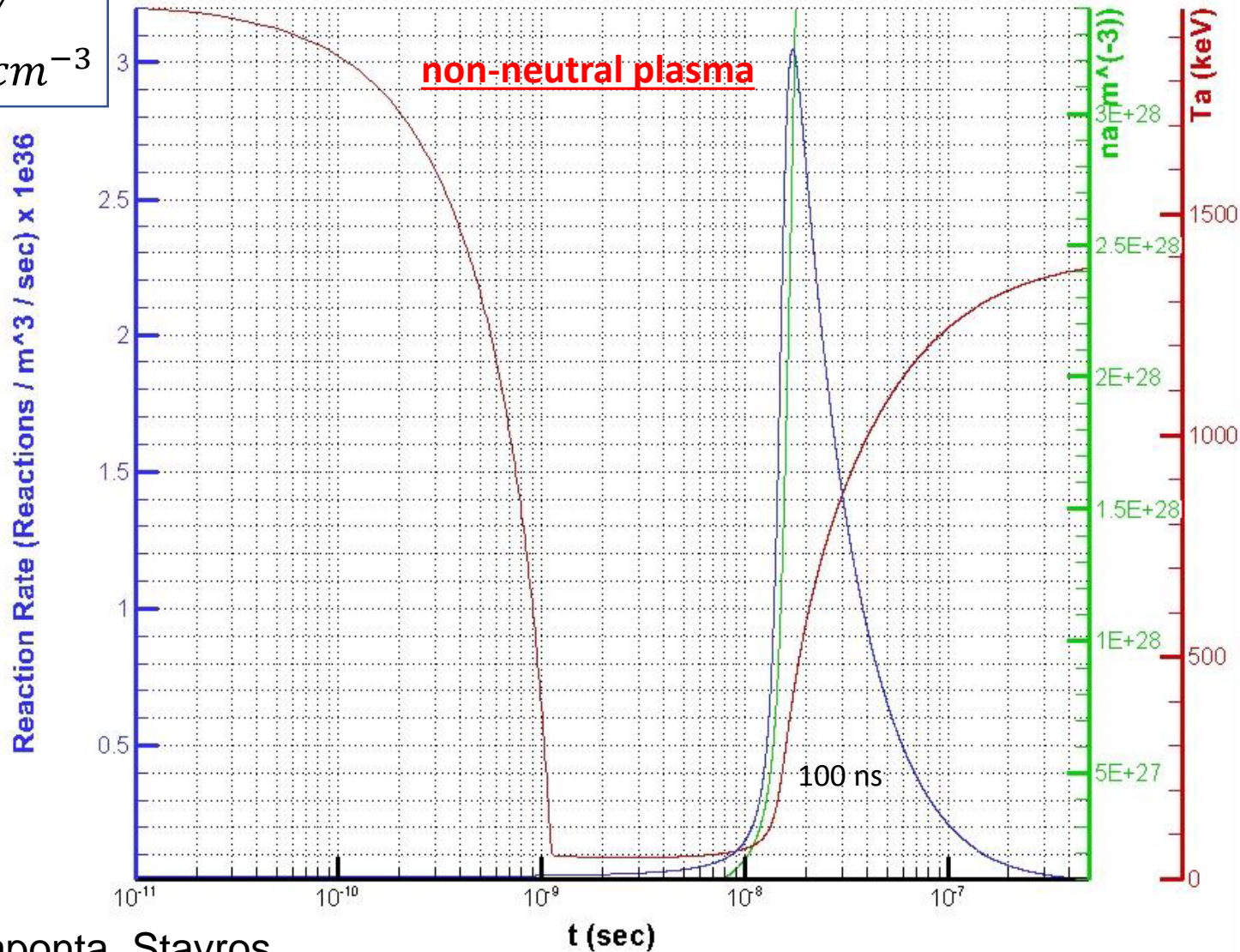


$$\frac{dN_p}{N_p(t)} = P_{\alpha p} 3n_B \sigma_f(v_{p0}) v_{p0} dt \quad \longrightarrow \quad M = \frac{N_{pf}}{N_{p0}} = e^{\int_0^{t'} P_{\alpha p} 3n_B \sigma_f(v_{p0}) v_{p0} dt}$$

Terminology 4

LTE case - Simulation of p-B fusion with 4 thermal fluids (B,p,e, α)

$$T_p = T_B = 40 \text{ keV}$$
$$n_B = n_p = 10^{23} \text{ cm}^{-3}$$

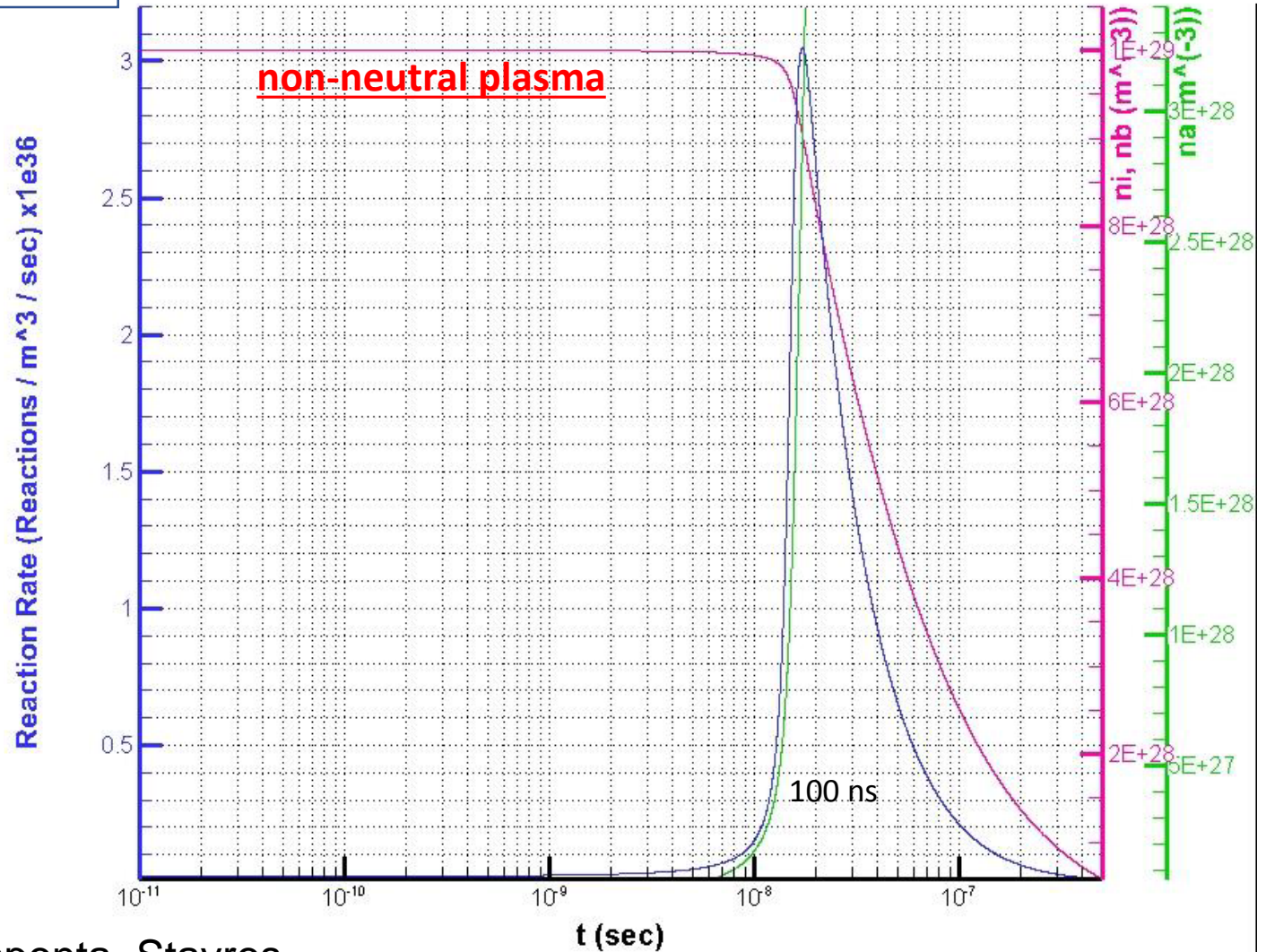


Simulation by:
Chrysovalanti Daponta, Stavros
Moustaiz

Terminology 5

$$T_p = T_B = 40 \text{ keV}$$

$$n_B = n_P = 10^{23} \text{ cm}^{-3}$$



Simulation by:
Chrysovalanti Daponta, Stavros
Moustaiz

Introduction - Our non-LTE scenario

We do a parametric scan to try to maximize the number of elastic chain reactions and hopefully achieve even avalanche [although this may not be necessary to achieving energetic gain].

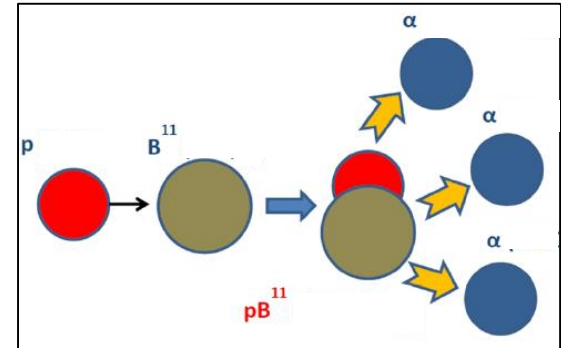
Conditions:

- Thermal plasma background (B+p)
- Super-thermal (external beam) initial protons

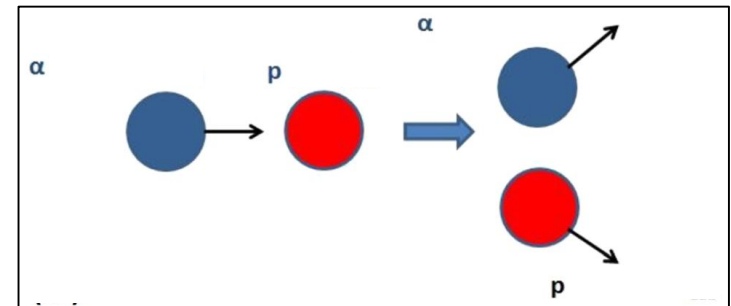
The Main challenges:

- **Stopping power (SP)** – loss of super-thermal velocities – we discuss
- **Radiation losses** – energy loss in a confinement scenario – we do not discuss

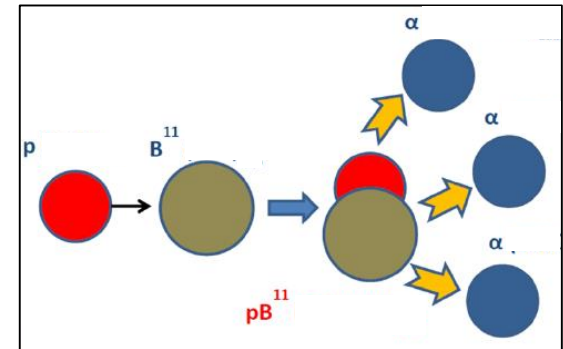
Primary fusion



α elastic collision



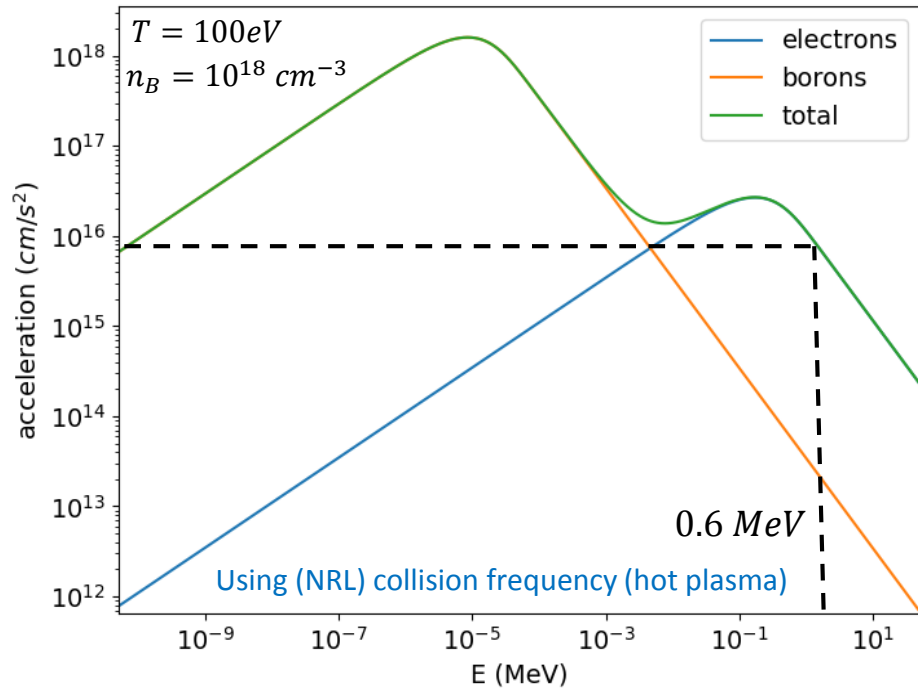
Secondary fusion



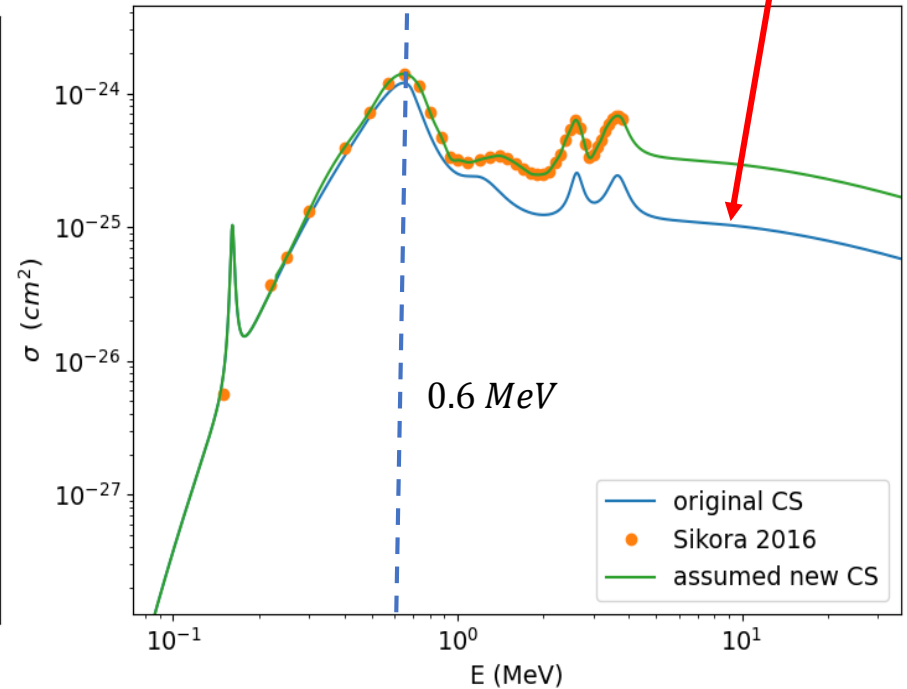
Fusion cross section and SP

The data above ~ 4 MeV is extrapolated

stopping power



Fusion cross section



$$P_f \approx \frac{\text{stopping distance}}{\text{mean free path for fusion}}$$

$$\Delta X = \frac{1}{2} \frac{v_0^2}{SP(n_B)} \quad \text{Stopping distance (constant acceleration)}$$

$$l \approx \frac{1}{n_B \sigma_f(v_{p0})} \quad \text{Mean free path for fusion}$$

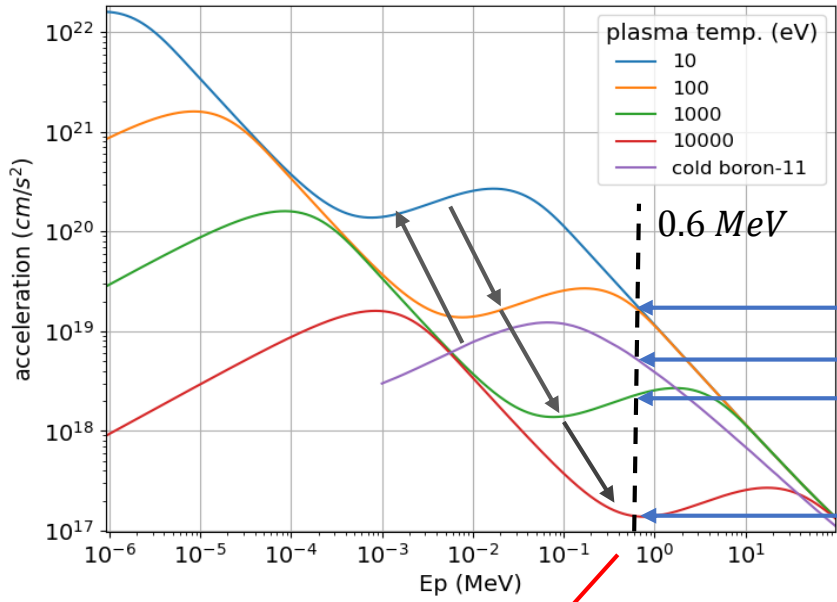
P_f (total fusion probability) is insensitive to the

Total = infinite plasma

The temperature dependence of the stopping power

From (NRL) collision frequency (hot plasma)

$$n_B = n_p = 10^{21} \text{ cm}^{-3}$$



10, 100 eV
Cold, Bethe-bloch
1 keV
10 keV

$T = 10 \text{ keV}$

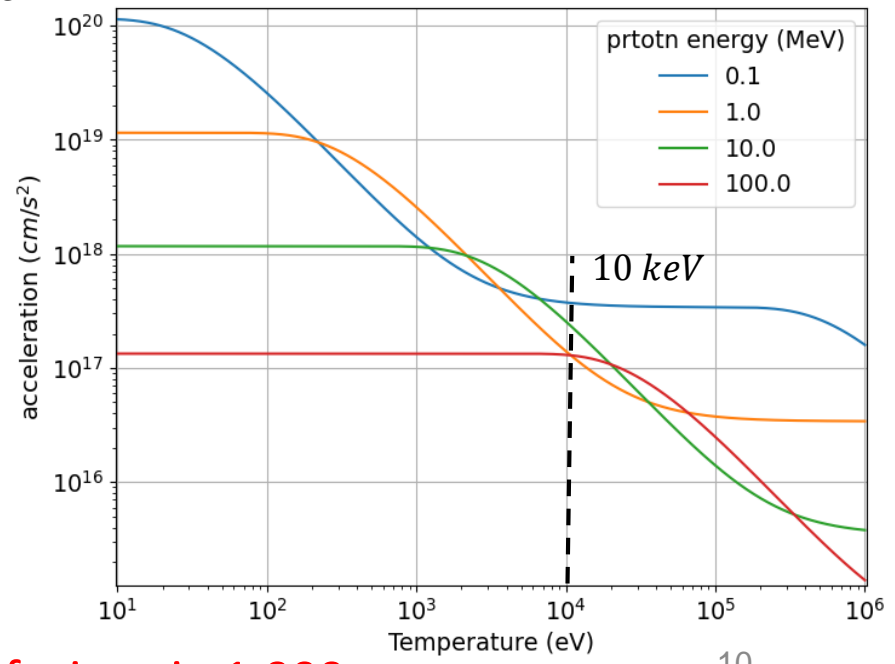
$$S.P(v_0 = 10^9 \text{ cm/s}) \approx 10^{17} \text{ cm/s}^2$$

$$\Delta X = \frac{1}{2} \frac{v_0^2}{S.P} \approx 5 \text{ cm}$$

$$l \approx \frac{1}{n_B \sigma_f(v_{p0})} = 10^3 \text{ cm}$$

$$P_f \approx \frac{\text{stoppind distance}}{\text{mean free path for fusion}} = \frac{5}{10^3} = 5 \times 10^{-3}$$

We need $S.P \approx 10^{15}$ to get $P_f \approx 1$



5 fusions in 1,000 protons

A more accurate calculation of the total fusion probability

Fusion probability in a small interval along the proton track

$$n_B \sigma_f(v_{p0}) v_{p0} dt$$

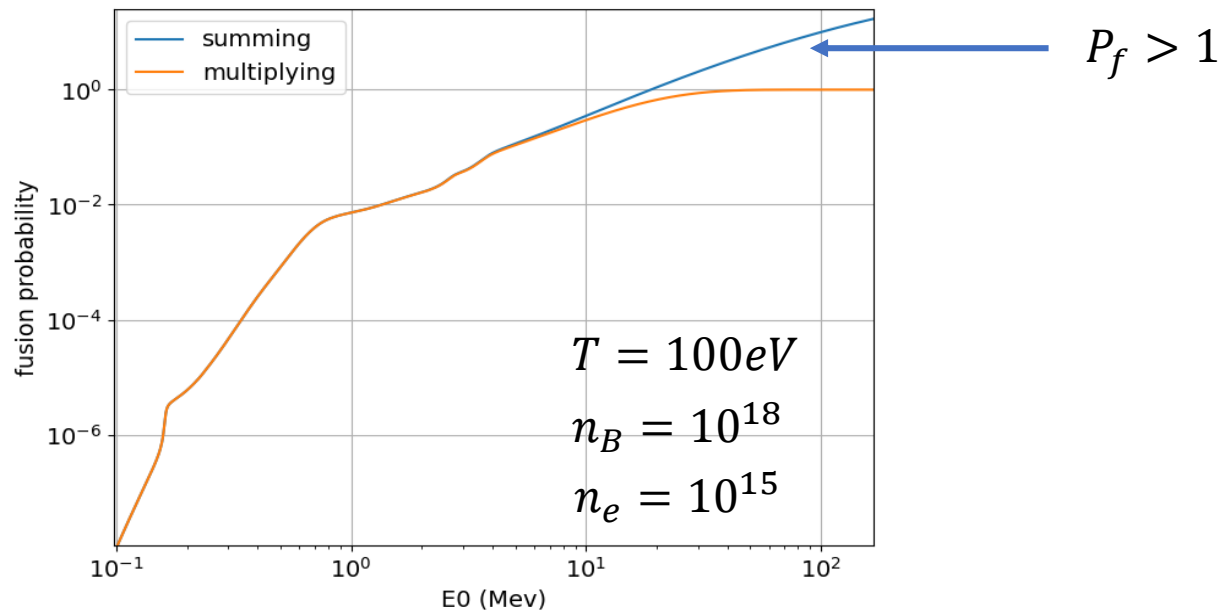
Not - the fusion probability

Not the total probability

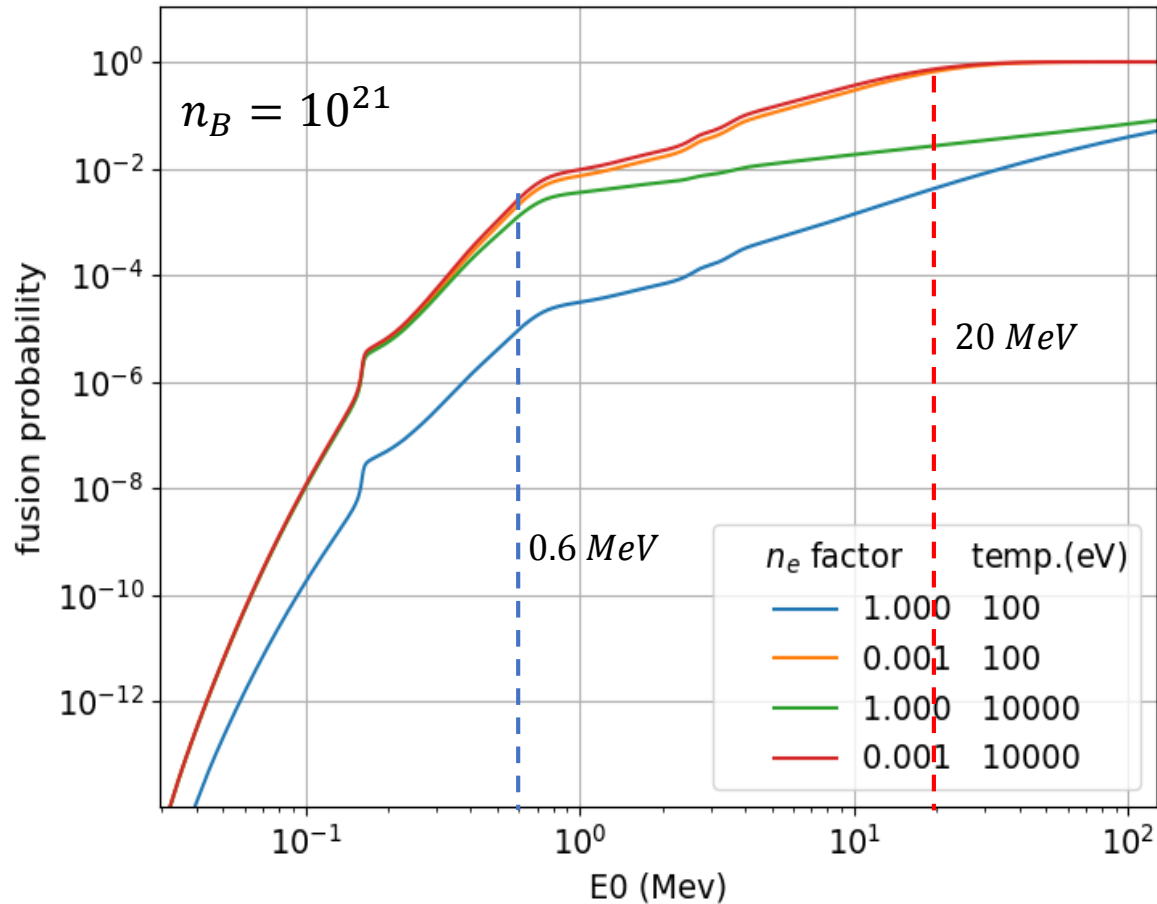
$$P_f(v_0) = \int_0^{t(v_{p0}=0)} n_B \sigma_f(v_{p0}) v_{p0} dt = - \int_{v_{p0}}^0 n_B \sigma_f(v_{p0}) \frac{v_{p0}}{S.P} dv$$

The fusion probability

$$P_f(v_0) = \left(1 - \prod_{v_p} \left(1 - n_B \sigma_f \frac{v_p}{S.P} \Delta v_p \right) \right)$$

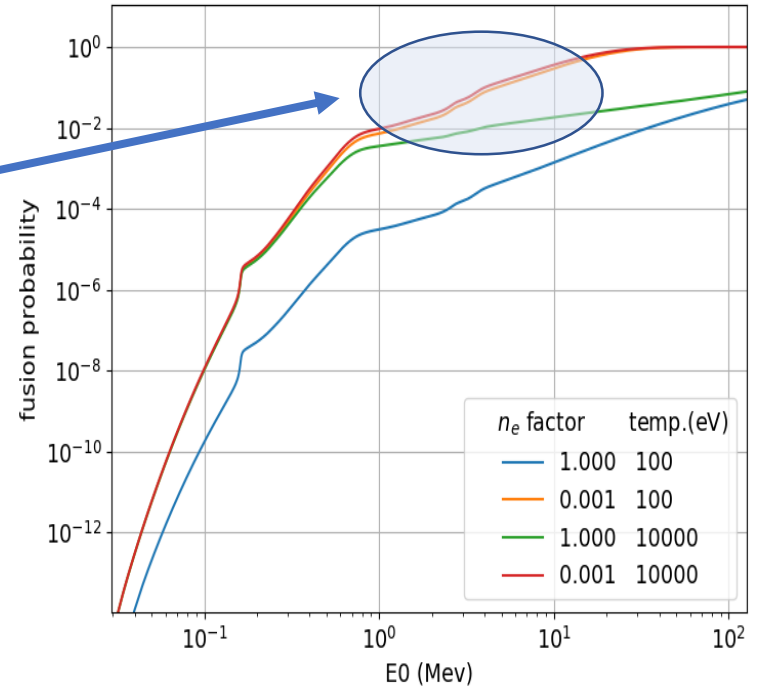
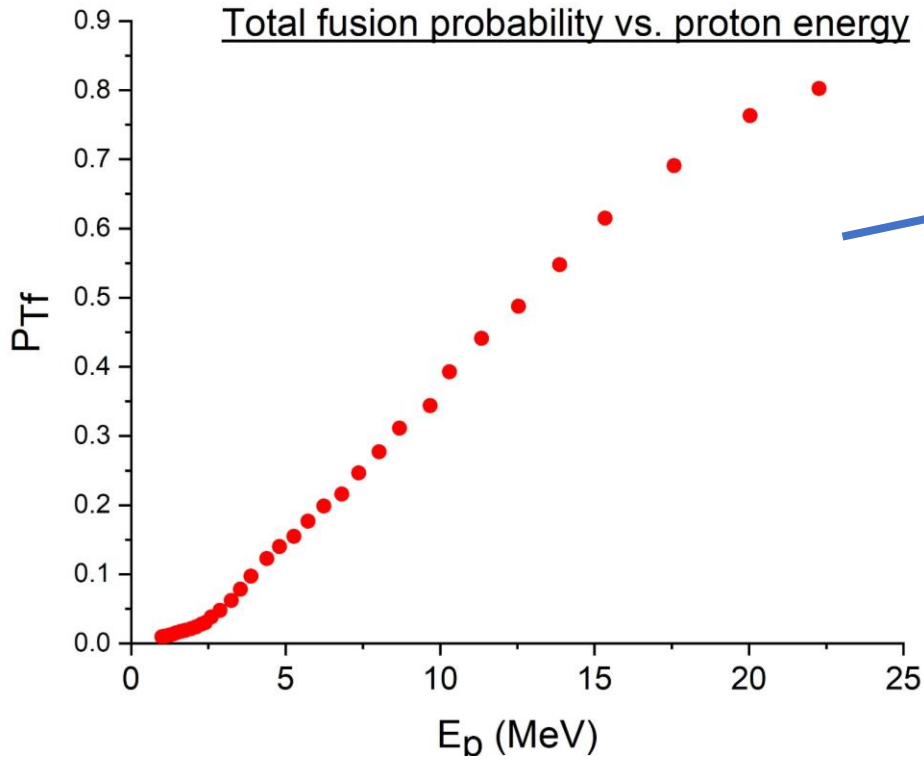


Manipulating the electron density



By reducing the electron density by 3 order of magnitudes we get $P_f \approx 0.8$ for 20 MeV protons already at 100 eV plasma

Total fusion probability vs. proton energy



Starting with

$N_{p0}(v_{max})$ →

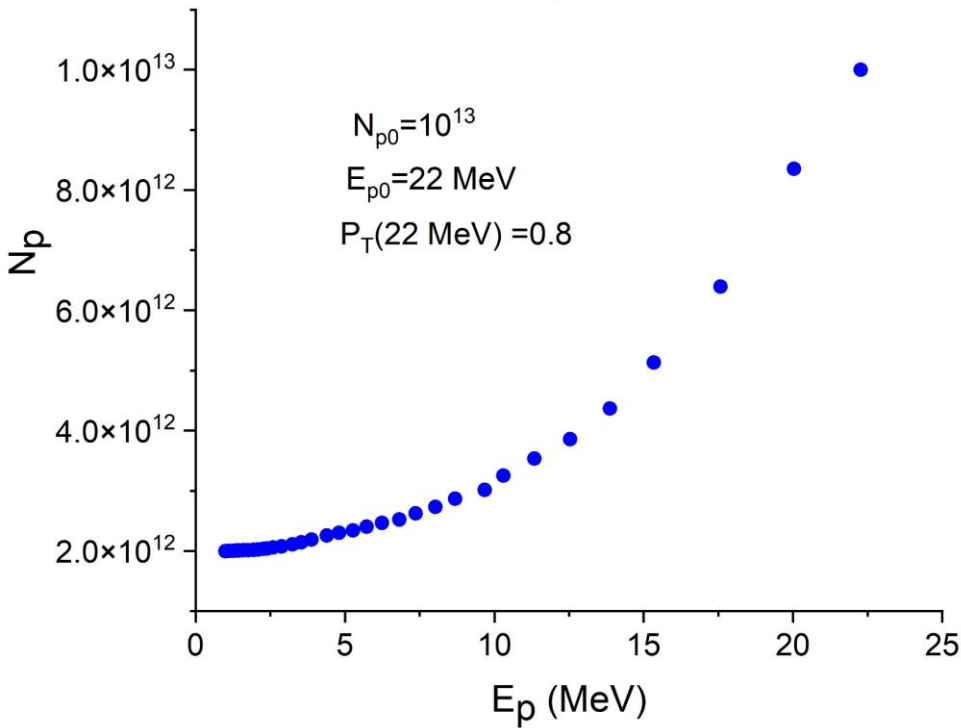
$$\Delta P_f(v, v - dv) = n_B \sigma_f \frac{v}{S.P} dv$$

$$N_f(v, v - dv) = N_p(v) \cdot \Delta P_f(v, v - dv)$$

$$N_p(v - dv) = N_p(v) - N_f(v, v - dv)$$

From v_{max} to 0

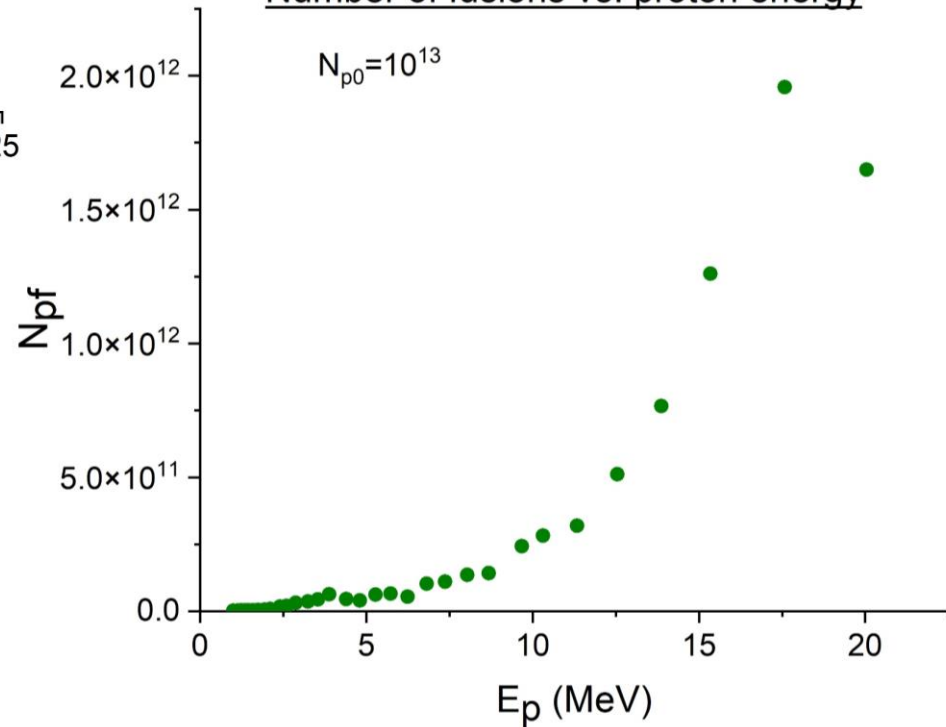
Number of remaining protons vs. proton energy



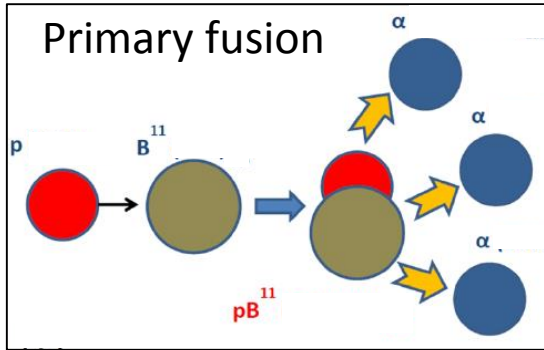
Most of the protons make fusion at high energies

$$\bar{E} = \frac{\sum E \cdot N_{pf}(E)}{\sum N_{pf}(E)} = 17 \text{ MeV}$$

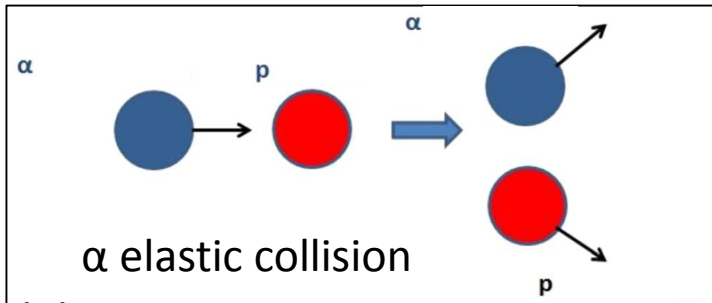
Number of fusions vs. proton energy



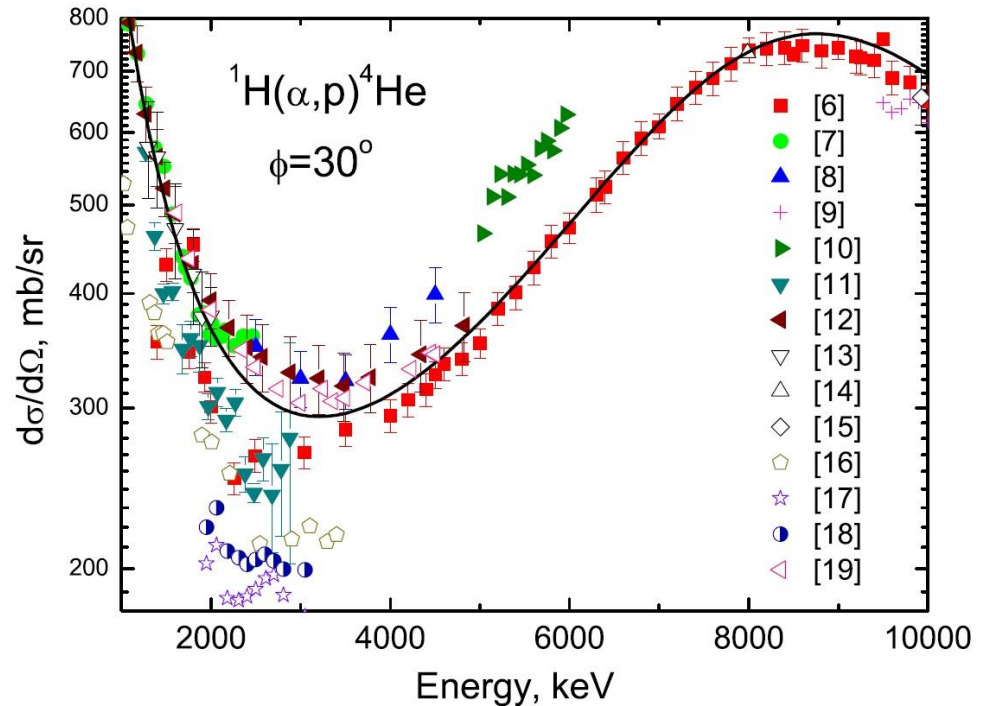
If the protons have high energy in the fusion reaction, then, this energy is transmitted to the alphas which have then higher cross section for elastic collisions.



The high energy of the protons is transmitted to the alpha products



$\rho(\alpha, \alpha)\rho$ Elastic scattering



The elastic cross section is $\sigma \sim 3 - 10 \text{ b}$

A.F. Gurbich, Nuclear Instruments and Methods in Physics Research B, **268**, 1703 (2010).

Multiplication factor 1 (non-neutral plasma)

Simple estimation

Starting with initial number of protons ($N_{p0}(E_{p0}=20\text{MeV})=1 \times 10^{13}$)



Calculate the total number of fusions (fusion probability=0.8)



Calculate the number of alphas from each energy ($3 \cdot N_f$, $2 \cdot 4\text{MeV} + 1 \cdot 1\text{MeV}$)



Split the average proton in fusion (17 MeV) energy evenly between alphas



Calculate the number of elastically scattered protons (we used elastic CS = fusion CS)



Calculate the energy of scattered protons ($16/25 E_\alpha$)



After 2 generations we got 5% (secondary reactions) more than the 0 generation fusions

Multiplication factor 2 (non-neutral plasma)

k= Percentage of secondary reactions



Numerical calculations following the paper by Fabio with:

F. Belloni, Plasma Phys. Control. Fusion, **63**, 055020, (2021)

$$T_e = 10 \text{ keV}, T_i = 1 \text{ keV}$$

$$n_p = 10^{23} \text{ cm}^{-3}$$

$$n_B = 0.2 \times 10^{23} \text{ cm}^{-3}$$

$$n_e = (n_p + 5n_B)\eta_e$$

$$\eta_e = 10^{-3}$$

E_{p0} (MeV)	P_f	$\varepsilon_p = \frac{8.9[\text{MeV}]}{E_{p0}}$	k (multiplication factor)	G_B
10	0.37	0.89	0.0388	0.34
20	0.8	0.44	0.14	0.4
30	1	0.3	0.26	0.37

$$E_i = E_{p0}N_0 \quad \text{Initial beam energy}$$

$$N_i = P_f N_0 \quad \text{Number of initial reactions}$$

$$N_s = kN_i = kP_f N_0 \quad \text{Number of secondary reactions}$$

$$N_T = N_i + N_s = P_f N_0(1 + k) \quad \text{Total number of reactions}$$

$$E_f = 8.9[\text{MeV}] * N_T = 8.9[\text{MeV}] * P_f N_0(1 + k) \quad \text{Total produced fusion energy}$$

$$G_B = \frac{E_f}{E_i} = \frac{8.9[\text{MeV}] * P_f N_0(1+k)}{E_{p0} N_0} = \frac{8.9[\text{MeV}]}{E_{p0}} P_f (1 + k) = \varepsilon_p P_f (1 + k)$$

The cross section for elastic collisions was calculated using Rutherford Eq.

We need to correct for the much higher experimental cross section

Conclusion

- We discussed p-B fusion terminology
- We discussed the temperature and electron density effects on the fusion total probability
- Decreasing the electron density has the advantages of 1) reducing the bremsstrahlung and cyclotron radiation 2) reducing the stopping power (increasing the fusion probability) 3) allows working with low temperature plasmas.
- However, the energy required to produce a non-neutral plasma must be considered.

Thank
You