Gravitational Love: The importance of tidal effects for studying binary neutron stars

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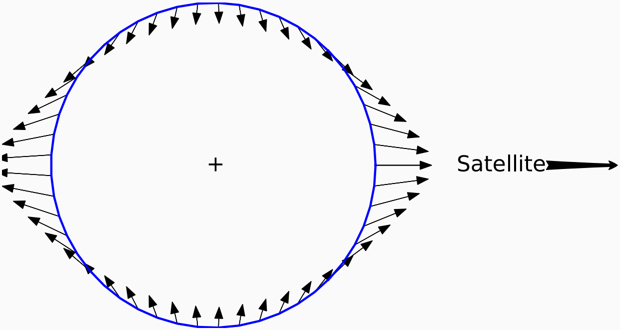
Overview

Love numbers: the Newtonian picture

Love numbers: the relativistic generalization

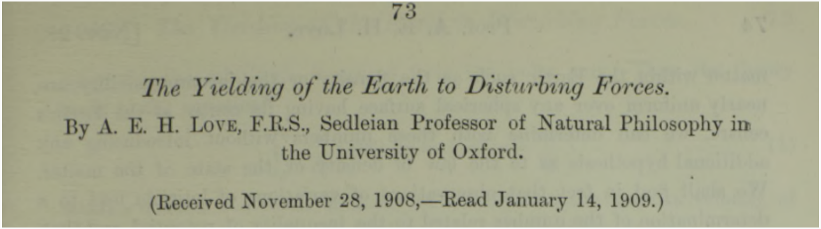
Rotation

Tidal forces and deformations



- How does the body react?
- How does the body's gravitational potential behave now?

Love numbers



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The Yielding of the Earth to Disturbing Forces.

By A. E. H. Love, F.R.S., Sedleian Professor of Natural Philosophy in the University of Oxford.

(Received November 28, 1908,—Read January 14, 1909.)

- k_l: gravitational/tidal Love number
- h_l: surficial Love number

Newtonian Love numbers

Gravitational potential of deformed body:

$$U_{\text{deformed}} = \frac{GM}{r} - \sum_{lm} \frac{4\pi}{2l+1} \frac{GI_{lm}}{r^{l+1}} Y_{lm}$$

External potential:

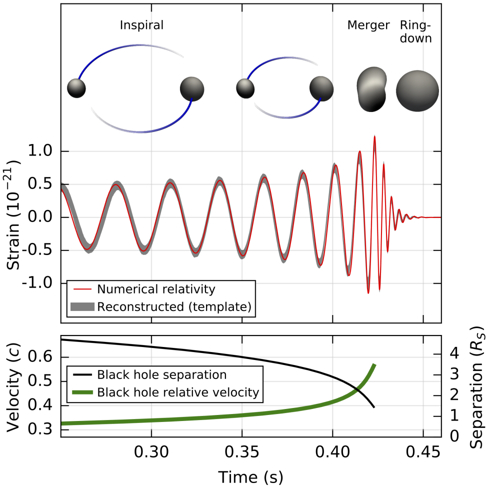
$$U_{\text{tid}} = \sum_{lm} \frac{4\pi}{2l+1} r^l d_{lm} Y_{lm}$$

Love number definition

$$GI_{lm} = 2k_l R^{2l+1} d_{lm}, \quad \delta R = \sum_{lm} \frac{4\pi}{2l+1} \frac{R^{l+2} h_l}{GM} d_{lm}$$

$$k_l = \frac{l+1-\eta_l(R)}{2(l+\eta_l(R))}, \quad h_l = 1 + 2k_l$$

Compact object merger



(B.P. Abbott *et al*, Phys. Rev. Lett. 116, 061102)

Compact object merger

But we need GR!

- Different Love numbers
- Different orbit
- Energy loss through gravitational waves

Relativistic generalization

- Perturbed potential $\longrightarrow$ perturbed metric

$$g_{00} = g_{00}^{(0)} + \sum_l g_{00}^{(l,\text{tidal})} \mathcal{E}^{(l)} + g_{00}^{(l,\text{response})} k_l^{\text{el}} \mathcal{E}^{(l)}$$

$$g_{0\phi} = g_{0\phi}^{(0)} + \sum_l g_{0\phi}^{(l,\text{tidal})} \mathcal{B}^{(l)} + g_{0\phi}^{(l,\text{response})} k_l^{\text{mag}} \mathcal{B}^{(l)}$$

- Poisson equation $\longrightarrow$ Einstein equations
- Euler equation $\longrightarrow$ TOV equation
- Definition of multipole moments

Relativistic generalization

Induced (response) multipole moments (adiabatic relations):

$$M_2 = \lambda_E^2 \mathcal{E}_0^{(2)}$$

$$S_3 = \lambda_M^3 \mathcal{B}_0^{(3)}$$

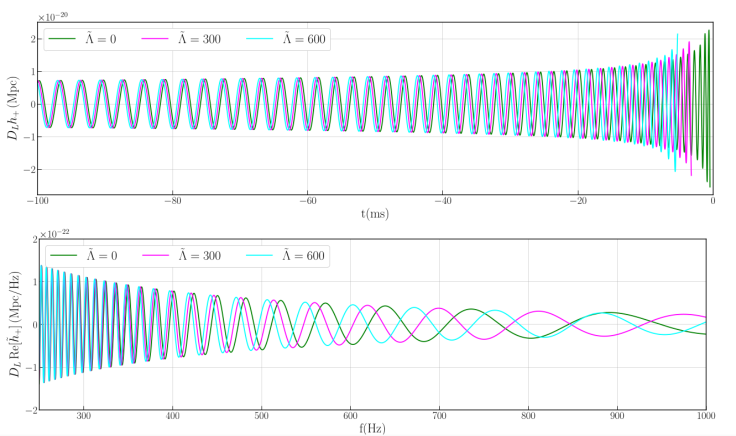
Love number definition

$$\lambda_E^{(l)} = \frac{\partial M_l}{\partial \mathcal{E}_0^{(l)}}, \quad \lambda_M^{(l)} = \frac{\partial S_l}{\partial \mathcal{B}_0^{(l)}}$$

Effect on gravitational wave signal

- Love numbers introduce 5PN correction on the phase of gravitational waves

$$\tilde{h}(f) = \mathcal{A} f^{-\frac{7}{6}} e^{i\psi_{\rm pp} - i\frac{39}{2}\tilde{\Lambda}\left(\frac{\pi M f}{c^3}\right)^{\frac{10}{3}}}$$



Chatziioannou, K., Gen Relativ Gravit 52, 109 (2020)

Effect on gravitational wave signal

- $\Longrightarrow$ Measurement of Love numbers $(\tilde{\Lambda})$ through detection of gravitational waves
- The Love numbers of a compact body give us information about its EoS
    - Stiffer EoS $\longleftrightarrow$ smaller Love numbers
    - Softer EoS $\longleftrightarrow$ larger Love numbers
- Relativistic inverse stellar problem

Now with rotation

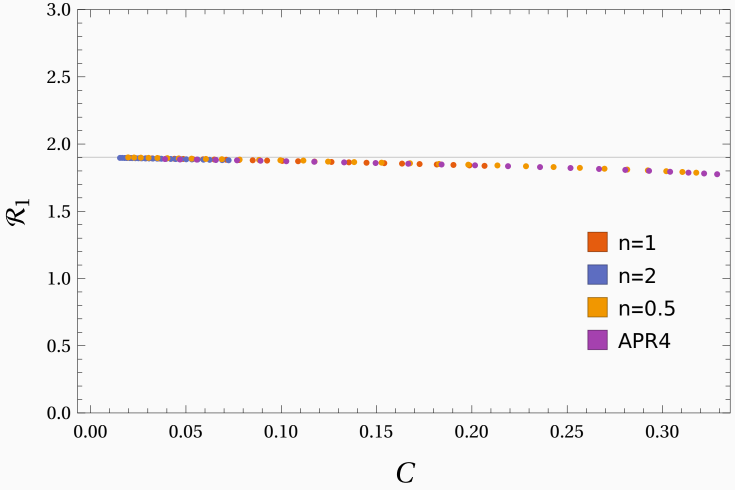
$$g_{00} = g_{00}^{(0,0)} + g_{00}^{(1,0)}\mathcal{E}^{(2)} + \left(g_{00}^{(1,1),\text{tidal}} + \lambda_E^{23} g_{00}^{(1,1),\text{response}}\right)\chi\mathcal{B}^{(3)}$$

$$g_{0\phi} = g_{0\phi}^{(0,0)} + g_{0\phi}^{(1,0)}\mathcal{B}^{(3)} + \left(g_{0\phi}^{(1,1),\text{tidal}} + \lambda_M^{32} g_{0\phi}^{(1,1),\text{response}}\right)\chi\mathcal{E}^{(2)}$$

Rotational tidal Love numbers' definition

$$\lambda_E^{l,l'} = \frac{\partial M_l}{\partial \mathcal{B}_0^{(l')}}, \quad \lambda_M^{l,l'} = \frac{\partial S_l}{\partial \mathcal{E}_0^{(l')}}$$

Hidden symmetry in rotational Love numbers



Hidden symmetry in rotational Love numbers

$$M_2 = \lambda_E^2 \mathcal{E}_0^{(2)} + \chi \lambda_E^{23} \mathcal{B}_0^{(3)}$$
$$S_3 = \lambda_M^3 \mathcal{B}_0^{(3)} + \chi \lambda_M^{32} \mathcal{E}_0^{(2)}$$

$$\frac{\delta \lambda_M^{32}}{\delta \lambda_E^{23}} = \frac{9}{4}\sqrt{\frac{5}{7}}$$

Interaction between multipole moments:

$$\mathcal{L}^{\text{int}} = -\frac{1}{4\lambda_2} Q^{ab} Q^{ab} - \frac{1}{16\sigma_3} S^{abc} S^{abc} + \alpha J^a Q^{bc} S^{abc}$$

Simplification of adiabatic relations

$$\lambda_E^{23} \propto \lambda_M^{32} \propto \lambda_E^2 \lambda_M^3 \alpha$$

Independent of equation of state!

Conclusions

- When a compact body is tidally deformed, its gravitational potential changes
- Different gravitational potential alter the expected gravitational wave signals emitted by compact body mergers
- The phase of these GW signals contain information (Love numbers) about how the bodies were deformed
- The deformations depend on the equation of state of the bodies