Will pulsar timing arrays (PTA) observe the Hellings-Downs curve?

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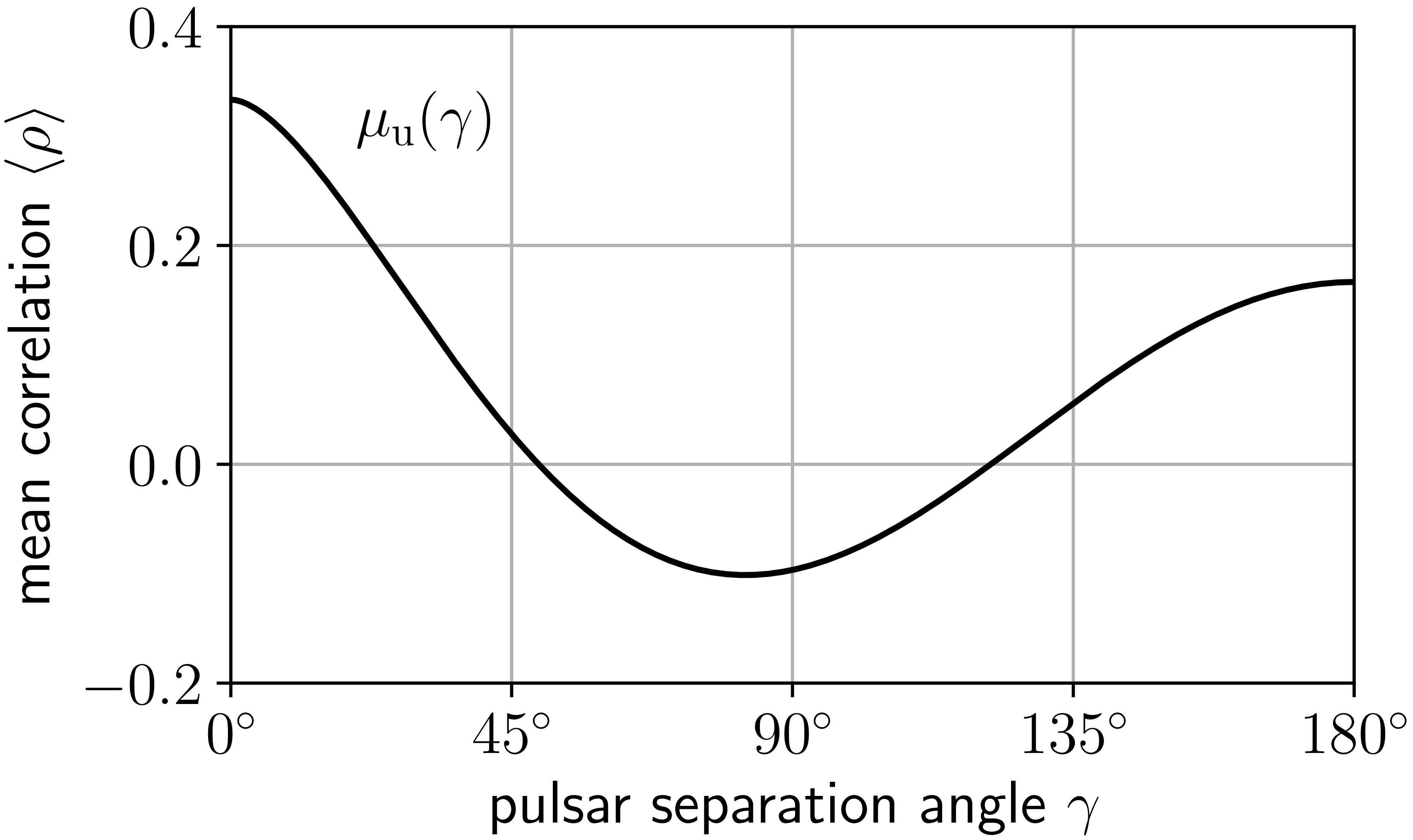
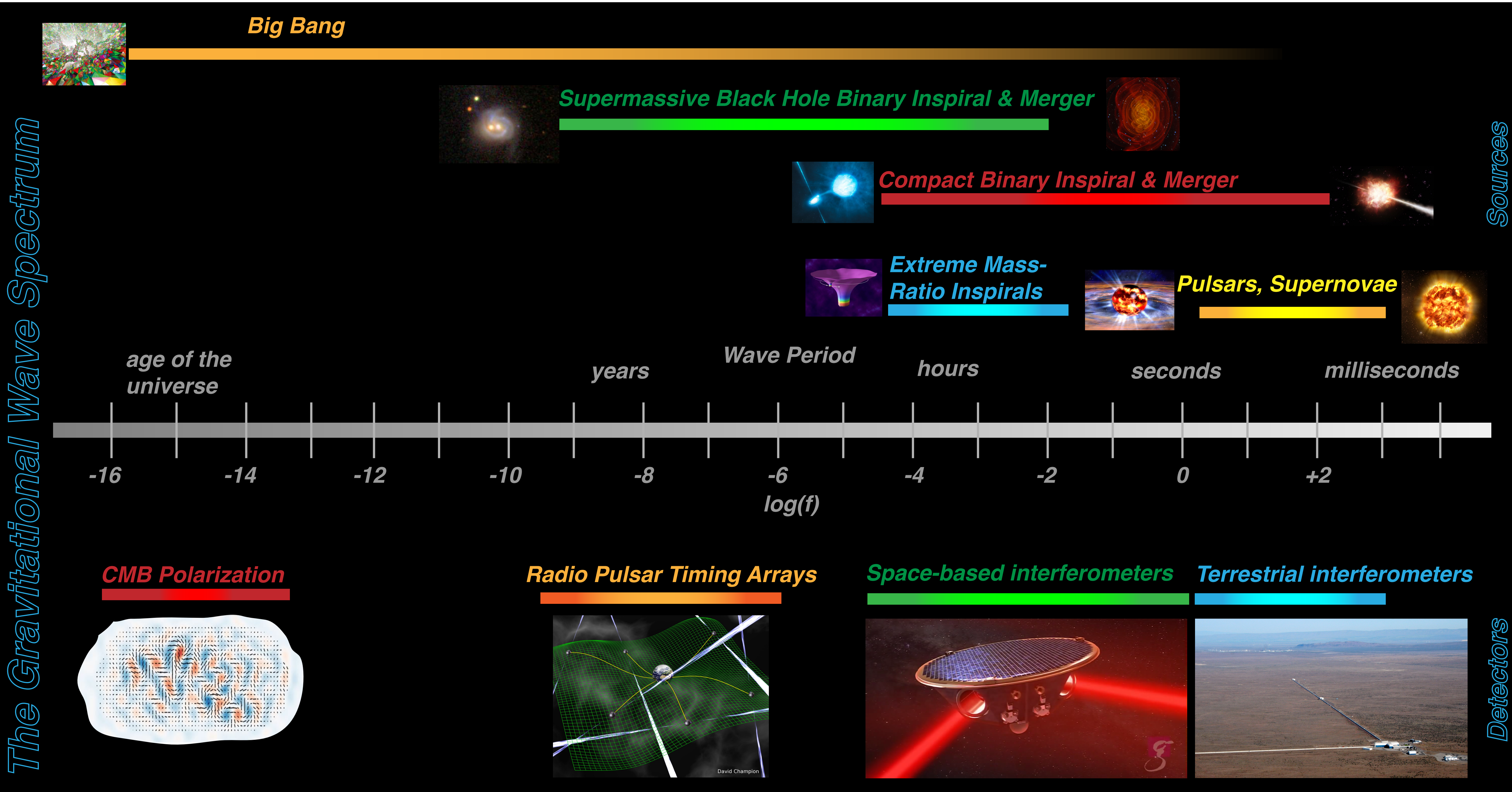
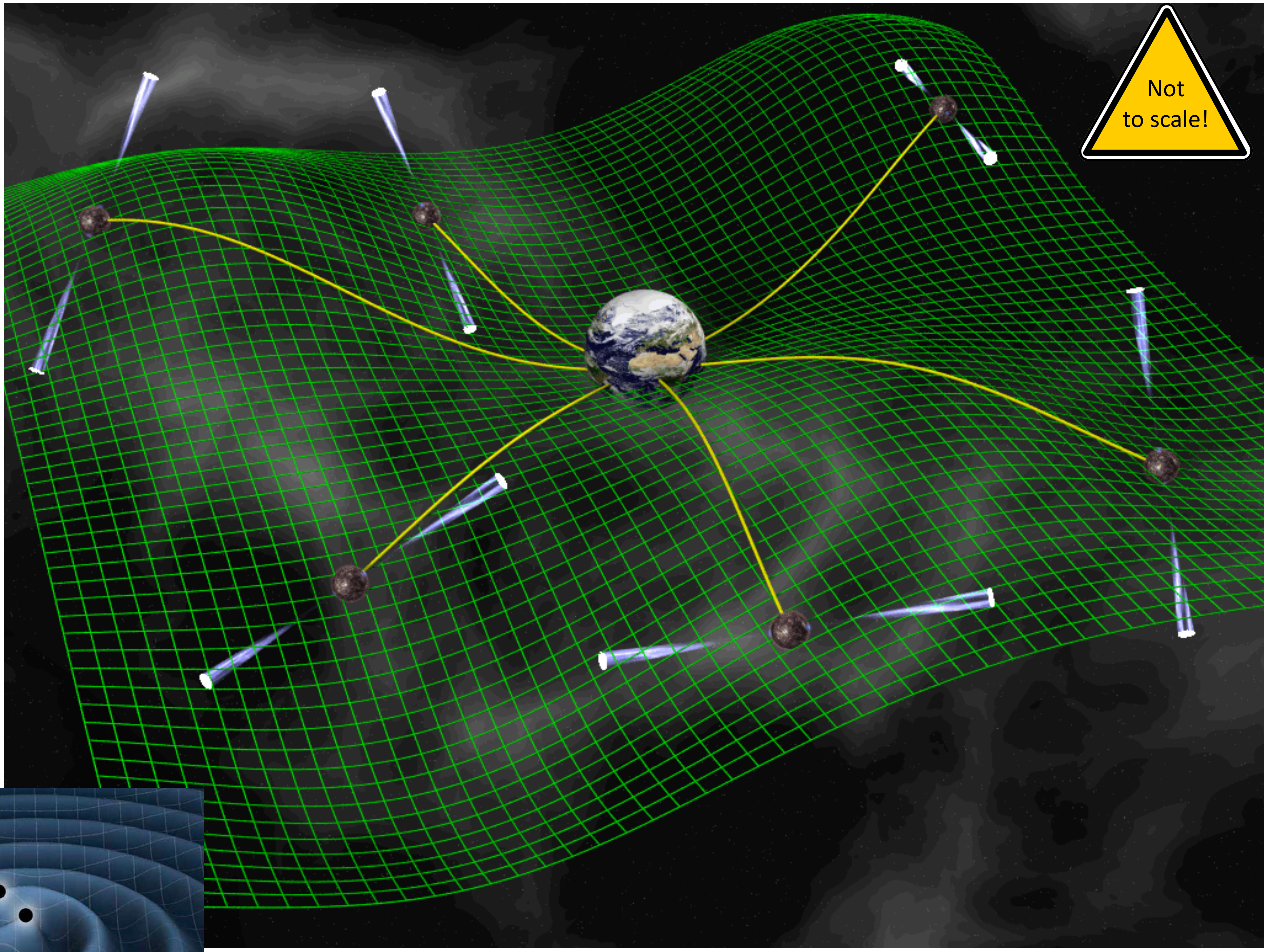


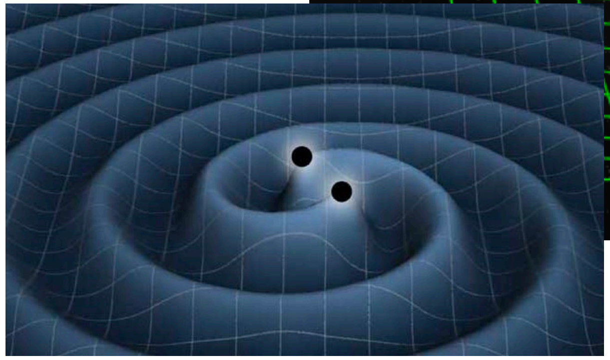
Gravitational Wave (GW) spectrum



Credit: Ira Thorpe

How does a PTA work?





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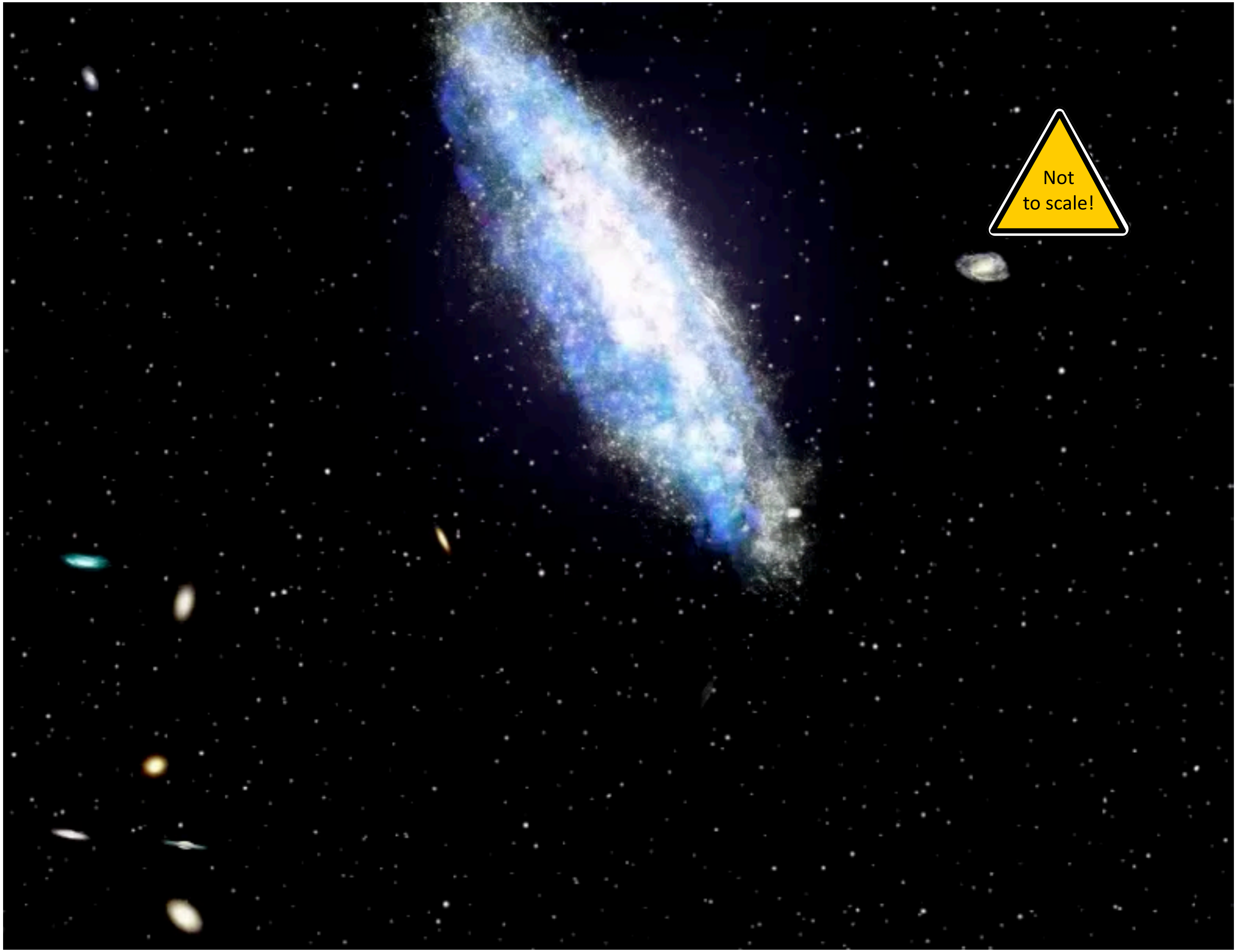
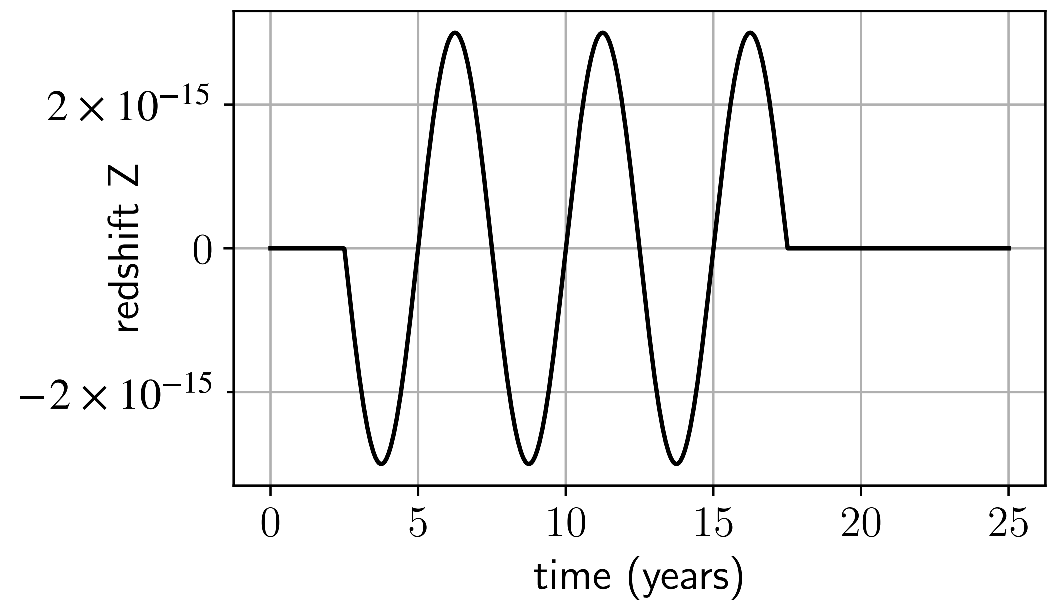
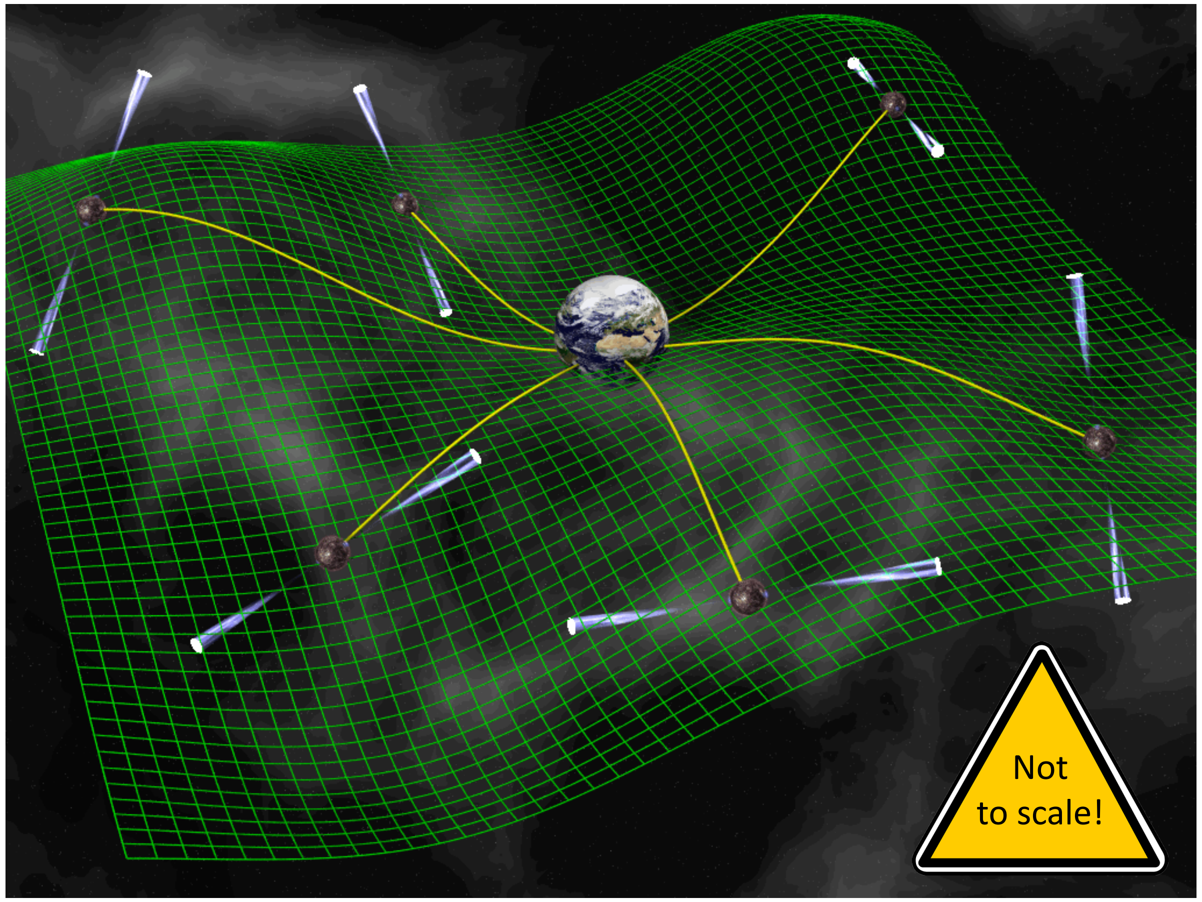


Current PTAs

Data: for each pulsar, construct a decade-long time series of redshifts

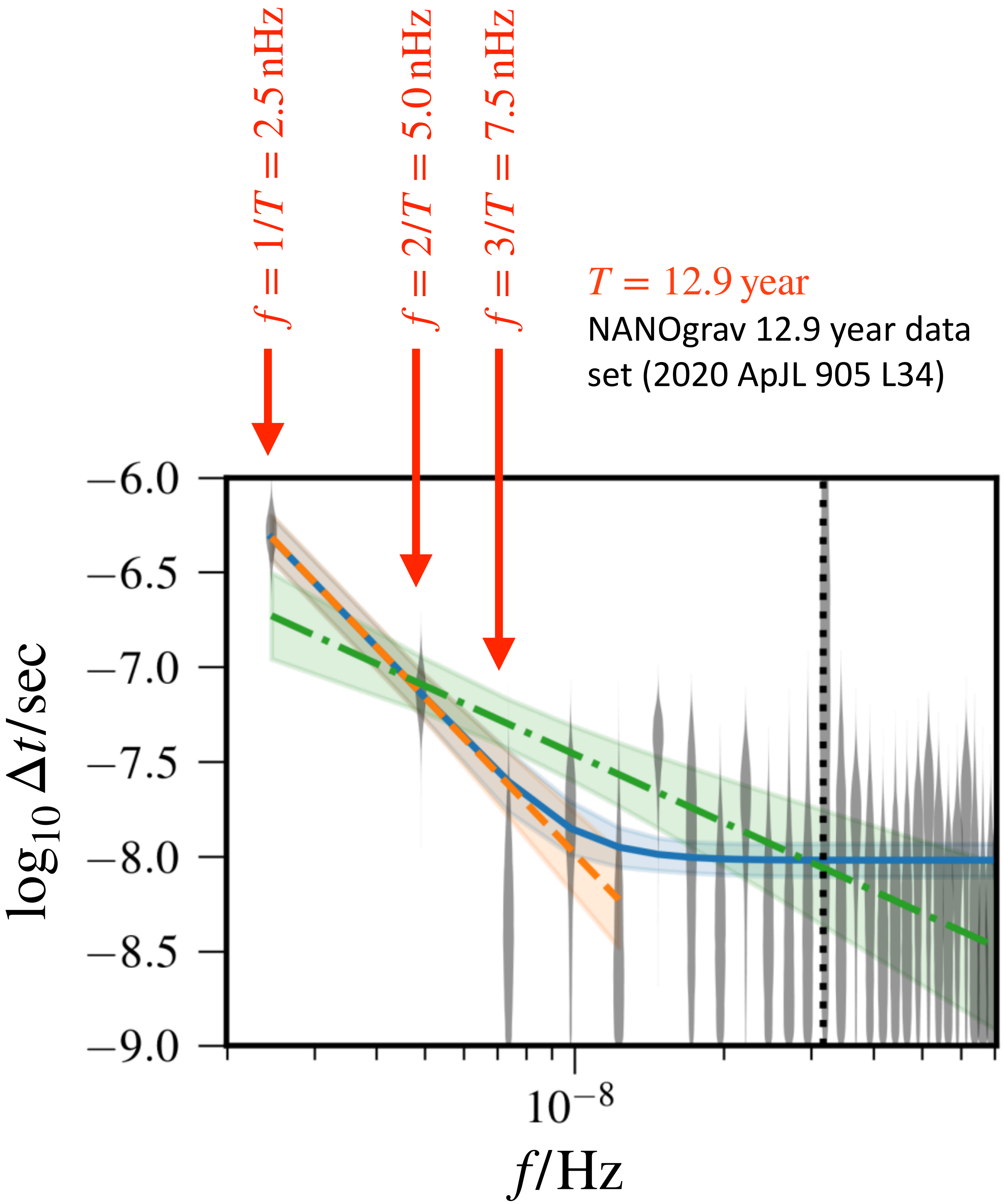


- **EPTA, 42 pulsars**
  European Pulsar Timing Array
- **NANOGrav, 66 pulsars**
  North American Nanoherz Observatory for Gravitational waves
- **PPTA, 26 pulsars**
  Parkes Pulsar Timing Array
- **IPTA, merge to get 88 pulsars**
  International Pulsar Timing Array



Evidence for GWs

- Timing residuals from **all PTA** pulsars show $\Delta t \propto f^{-13/3}$, as expected from massive black hole binary source
- Characteristic strain amplitude $h_c \approx 1.9 \times 10^{-15}$ consistent with expectations for super-massive black hole binary sources



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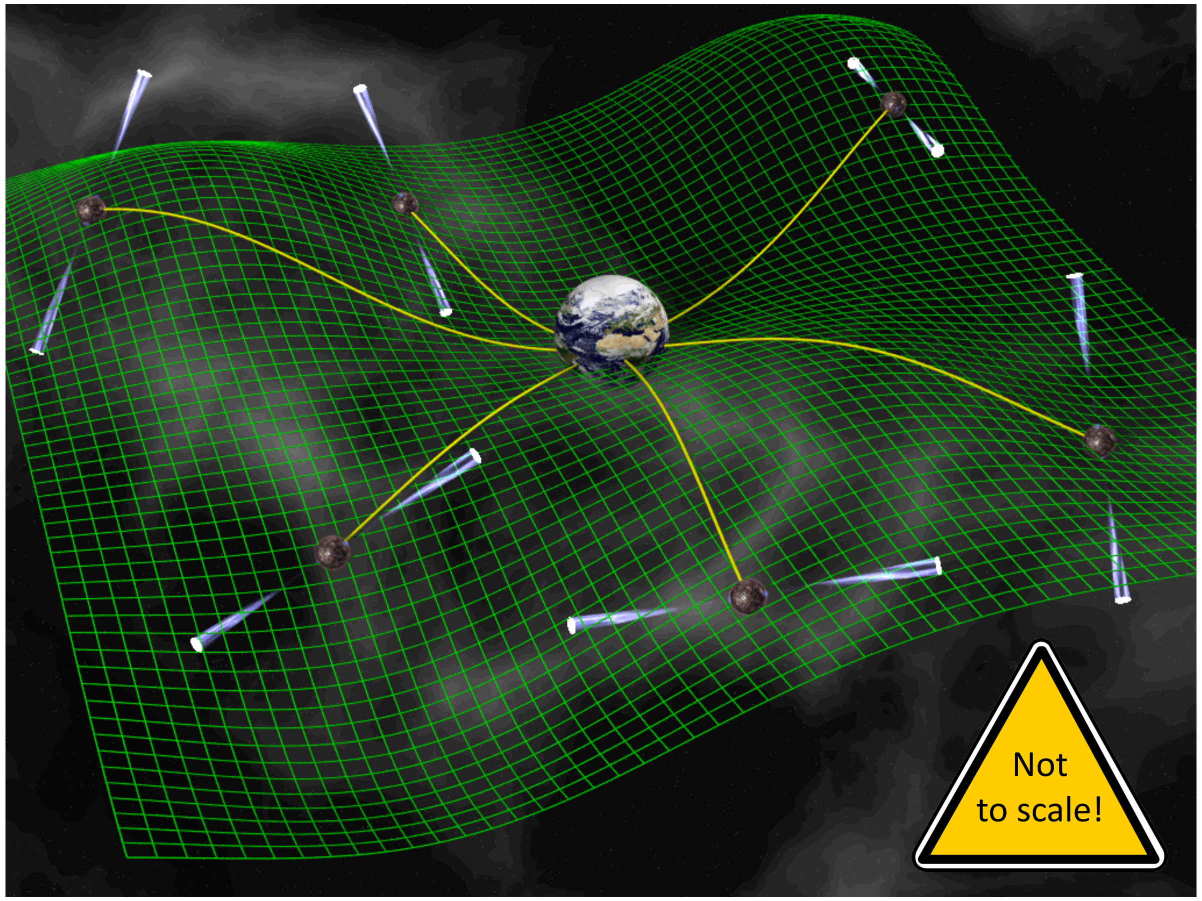
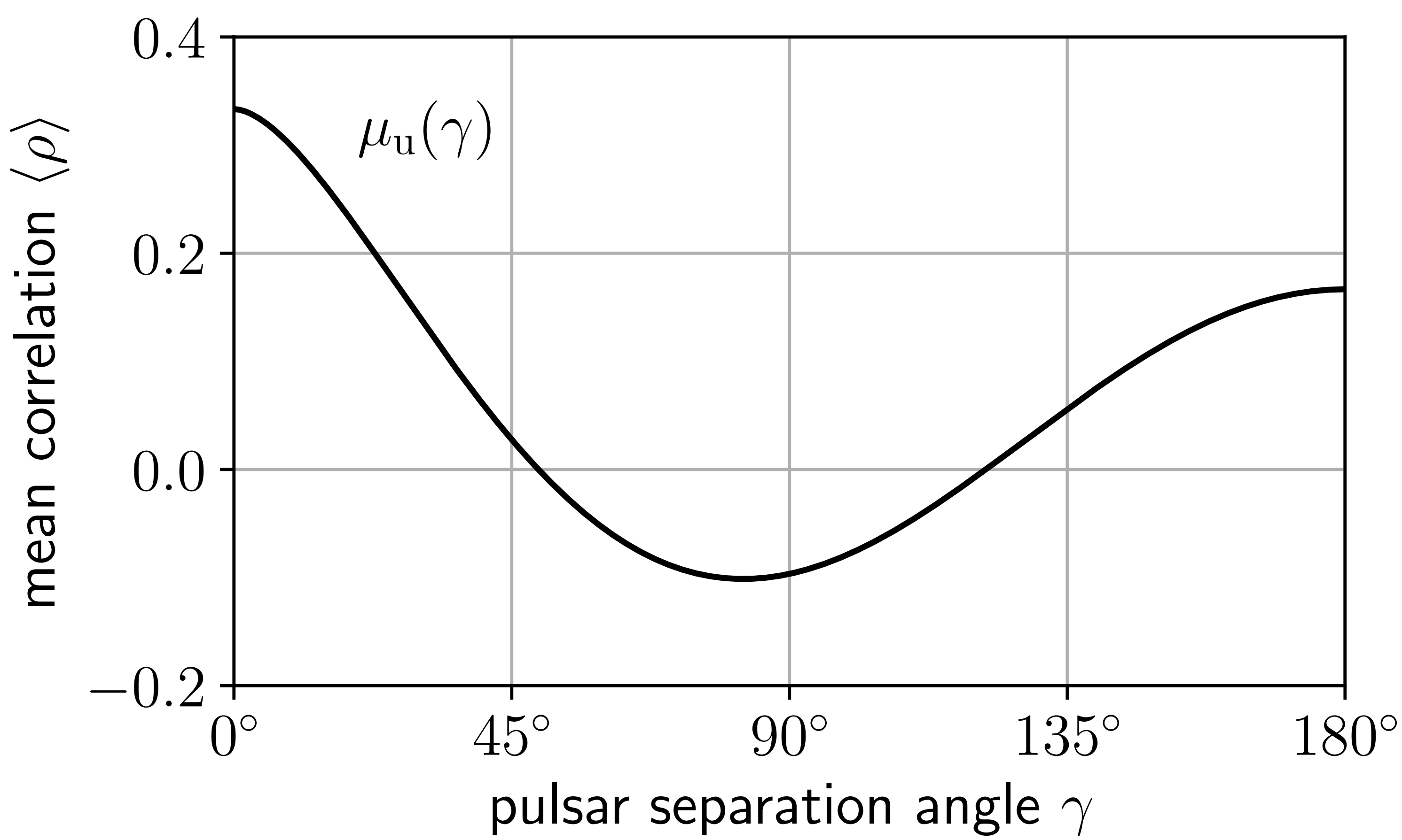


GOAL: detect gravitational wave via the correlations they induce between the different pulsar redshifts (or timing residuals)

Hellings and Downs Curve (1983)



- Pulsar 1 & Pulsar 2 correlation for a single unpolarized distant unit-amplitude GW point source

$$\rho = F_1^+(\Omega)F_2^+(\Omega) + F_1^\times(\Omega)F_2^\times(\Omega).$$

Here Ω is wave direction, and F^+ and $F^\times$ are pulsar response (antenna pattern) functions

- Fix pulsars 1 and 2, separated by angle γ on sky. Hellings and Downs curve (1983) is the **mean correlation** between timing delays of two pulsars, separated in sky direction by angle γ, averaged over source direction

$$\begin{aligned}\mu_u(\gamma) = \langle\rho\rangle &= \frac{1}{4\pi}\int \rho \, d\Omega \\ &= \frac{1}{4} + \frac{1}{12}\cos\gamma + \frac{1}{2}(1-\cos\gamma)\log\left(\frac{1-\cos\gamma}{2}\right)\end{aligned}$$

- Cornish & Sesanna 2013: same result if we fix the source, and average over all pulsar pairs at angle γ
- **Important:** the Hellings-Downs curve is the ***mean*** correlation! Any given pulsar pair will **not** have precisely this correlation.

Questions

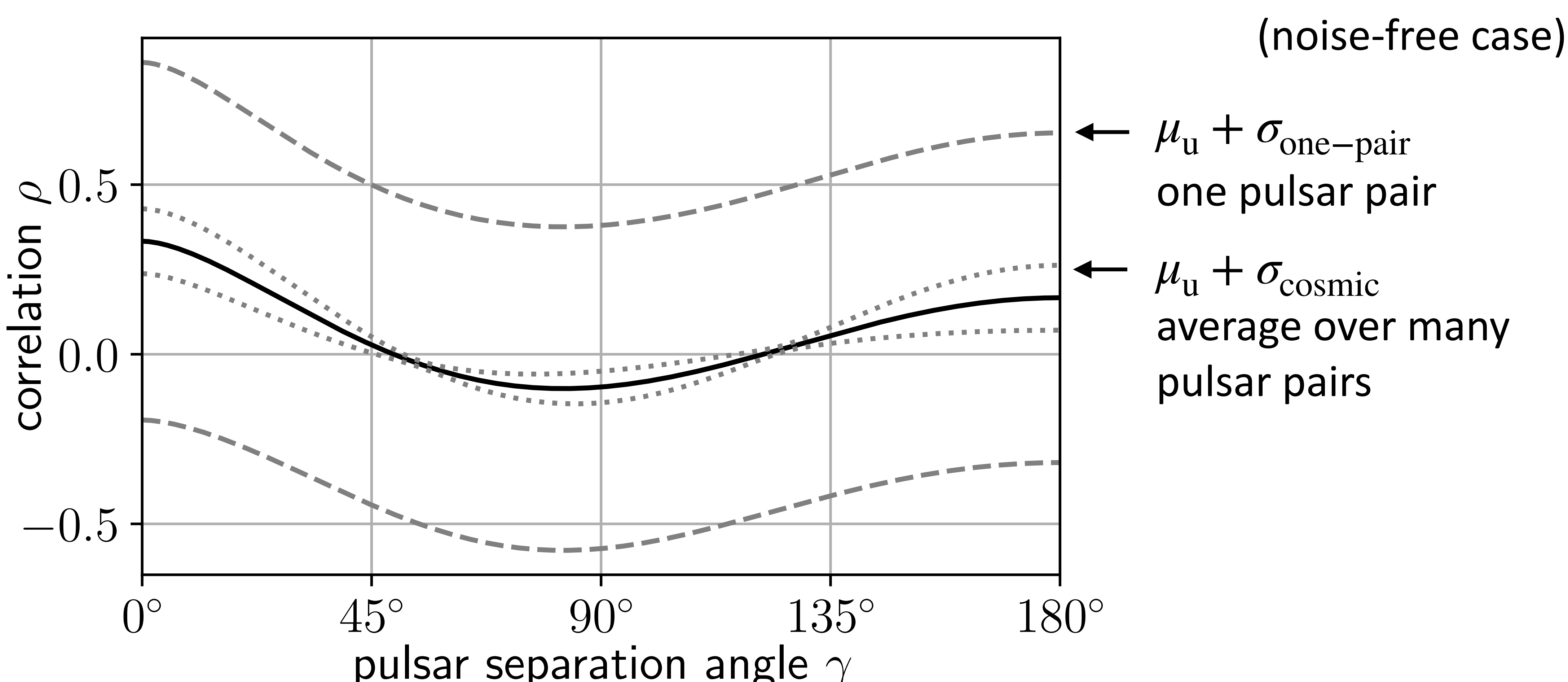
- If we had many noise-free pulsars spread around the sky, and we averaged their correlations in an optimal way, would we find $\mu_{\mathrm{u}}(\gamma)$? Or would it deviate? By how much??
- Given a specific pulsars of the different PTAs (not uniformly distributed on the sky), how do we define the Hellings and Downs correlation? Does it match $\mu_{\mathrm{u}}(\gamma)$?

Variance of Hellings-Downs correlation

- Can compute simple analytic forms. For a single pulsar pair: $\sigma^2_{\text{one-pair}} = \left(\mu_u^2 + 4\mu_u^2(0)\right)/2$
- If we average correlation over all pulsar pairs separated by angle γ **before** computing first and second moments obtain cosmic variance:

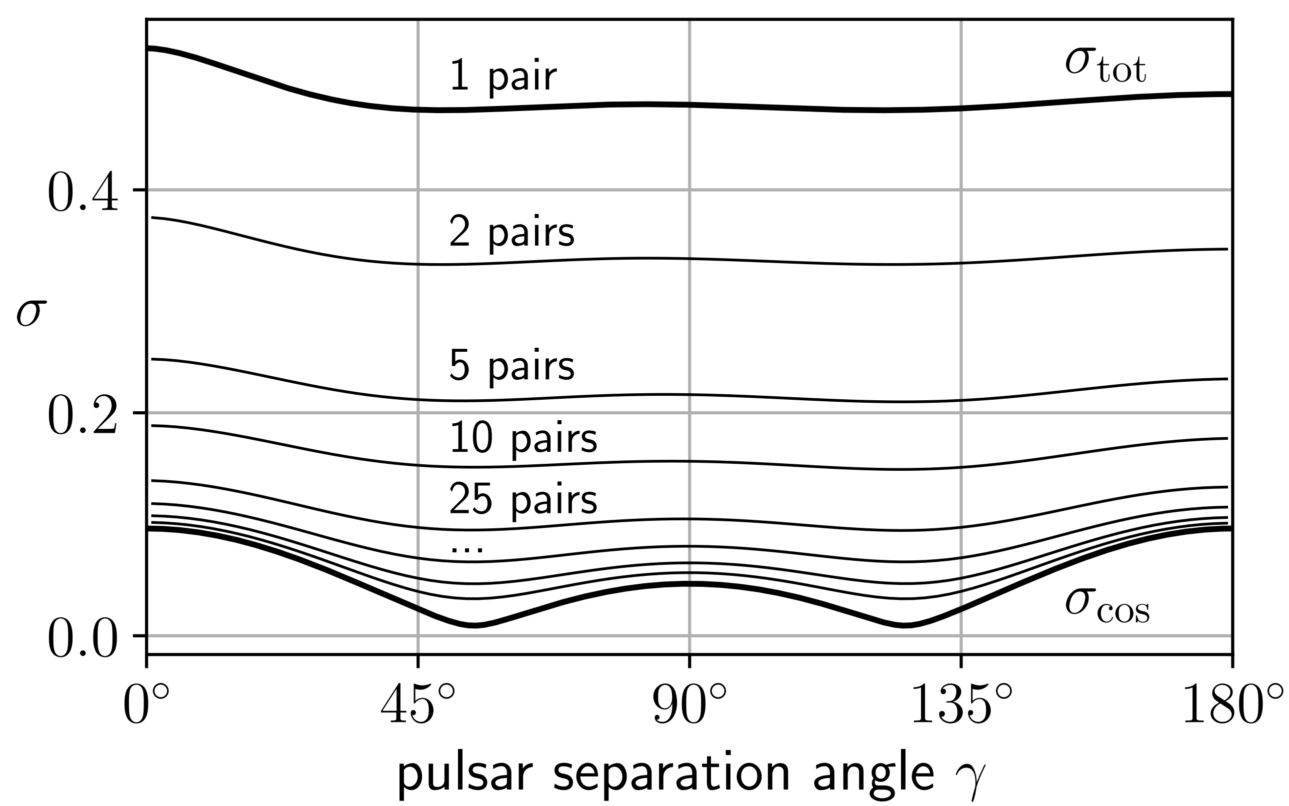
$$\sigma^2_{\text{cosmic}}(\gamma) = -\frac{5}{48} + \frac{49}{432}\cos^2\gamma - \frac{1}{6}(\cos^2\gamma + 3)\log\left(\frac{1-\cos\gamma}{2}\right)\log\left(\frac{1+\cos\gamma}{2}\right) + \frac{1}{12}(\cos\gamma - 1)(\cos\gamma + 3)\log\left(\frac{1-\cos\gamma}{2}\right) + \frac{1}{12}(\cos\gamma + 1)(\cos\gamma - 3)\log\left(\frac{1+\cos\gamma}{2}\right)$$

- In "our" realization of the universe, the pulsar-averaged correlation will not agree exactly with the Hellings-Downs curve. *Fluctuations can not be eliminated by averaging over pulsar pairs.* **The cosmic variance is observable!**



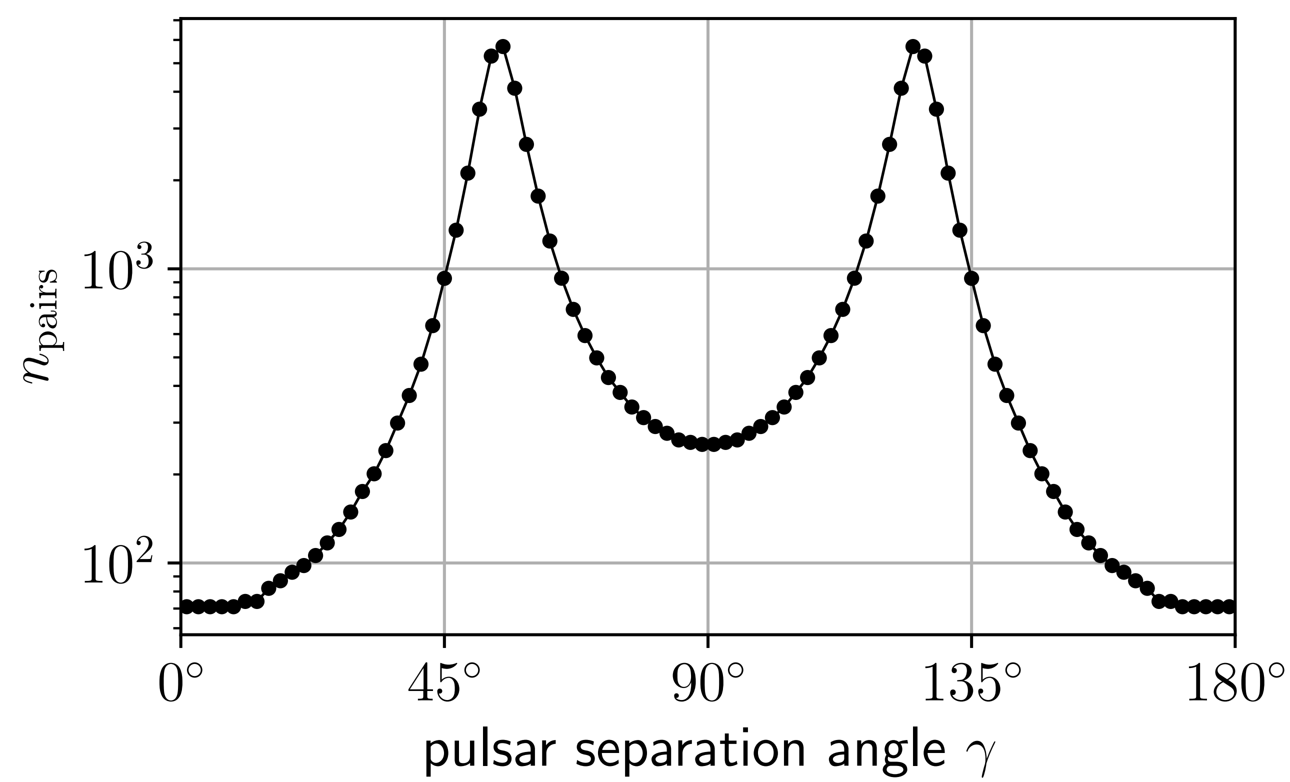
(GW confusion-noise model, $\bar{h}^4/h^4 = 1/2$ and $h^2 = 1$)

Reduction of variance by adding pulsar pairs: approach to cosmic variance

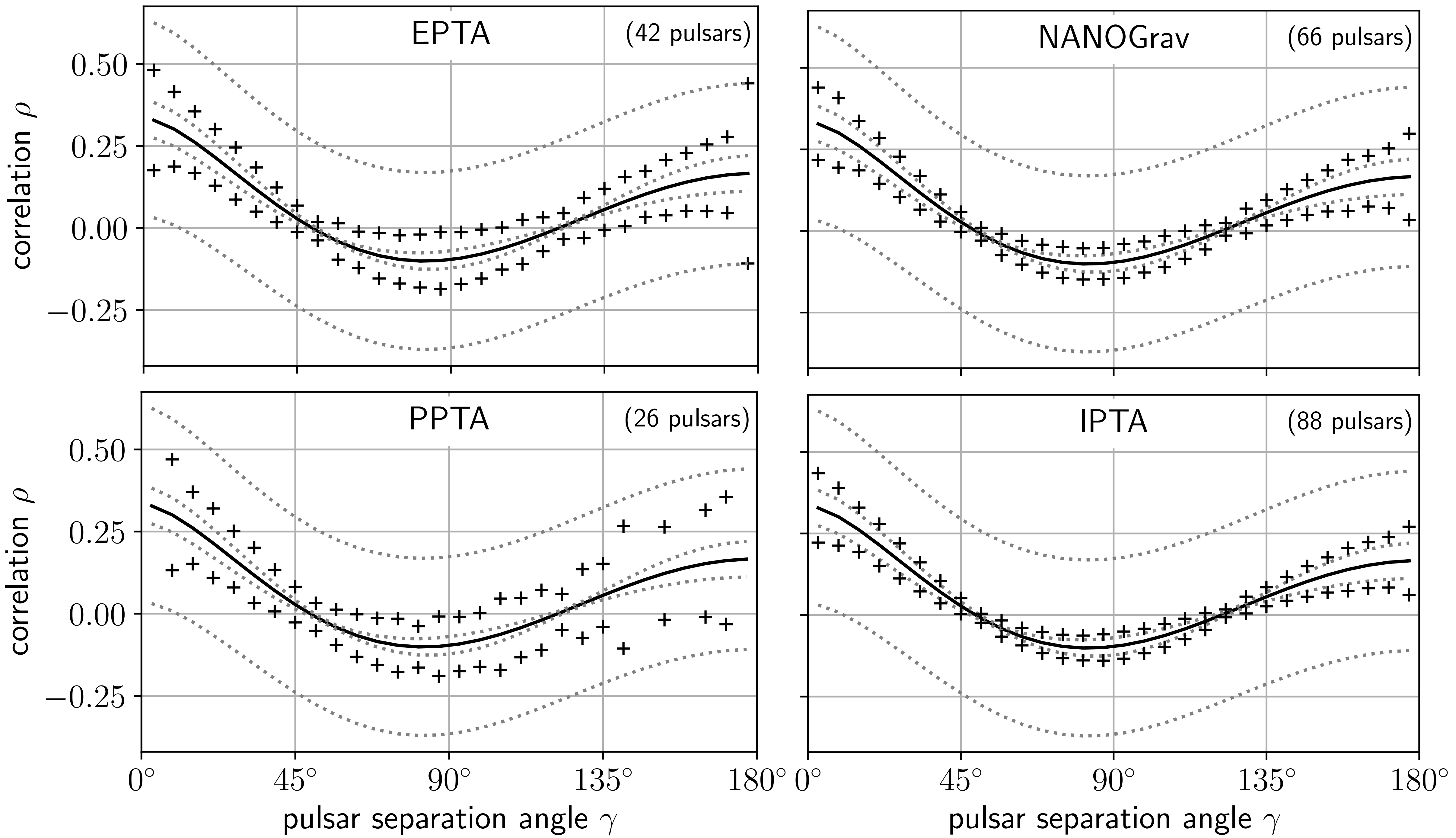


(GW confusion-noise model, $\bar{h}^4 = 1/2$)

number of pairs need for σ^2_{opt} to reach $(1 + 1/e)\,\sigma^2_{\mathrm{cos}}$



Variance of HD correlation for PTA pulsars ($30 \times 6^\circ$ bins, noise-free measurements)



(GW gaussian ensemble, binary inspiral spectrum, timing residual correlations, $\bar{h}^2/h^2 \approx 0.4$ and $h^2 = 1$)

Two papers

- **arXiv:2205.05637,** BA
  Variance in the Hellings-Downs correlation
- **arXiv:2208.07230,** BA & Joseph Romano
  The Hellings and Downs correlation of an arbitrary set of pulsars

Conclusions

- Existing PTAs should detect Hellings and Downs curve once they have enough data
- Even with many pulsars, don't expect PTAs will observe **exactly** the Hellings and Downs curve
- We have analytically predicted the scale of the deviations.
- If the observed deviations are much larger or much smaller than predicted, then our universe does *not* have a GW background described by the Gaussian ensemble (many incoherent SMBH binaries)

THANK YOU