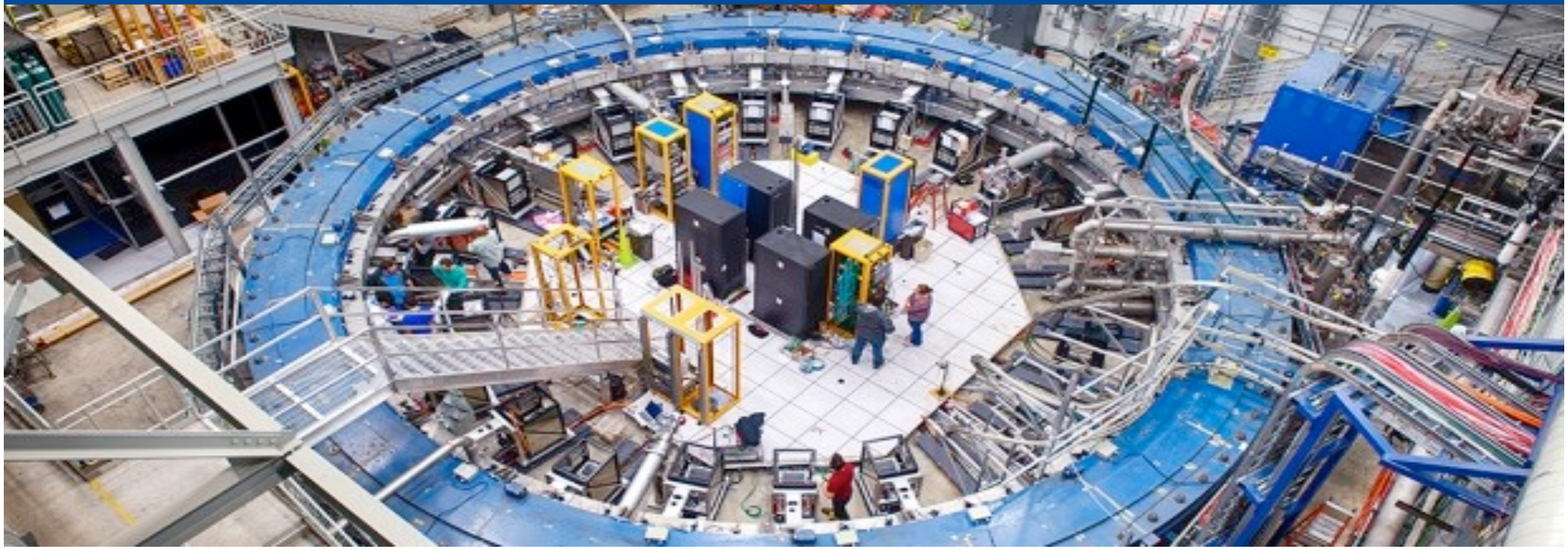


Measuring the muon magnetic anomaly a_μ with the Muon g-2 experiment at Fermilab

Vulcano Conference – 30 September 2022

Marco Incagli – INFN Pisa

What is *"g-2"* ?

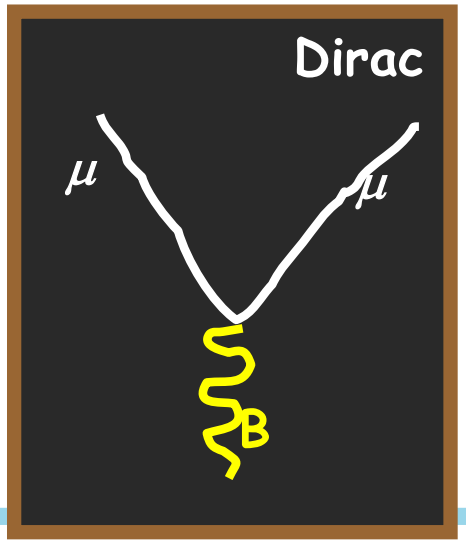
$$\vec{\mu}_p = -g_p \frac{e}{2m_p}\vec{S}$$

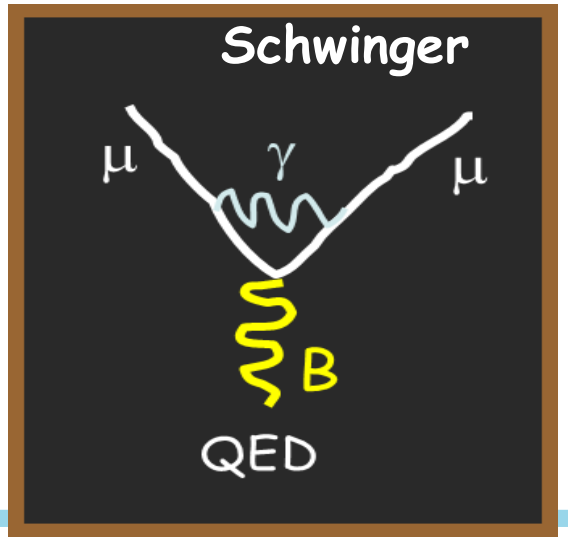
$$a_p = \frac{g_p - 2}{2}$$

- ***g_P*** : proportionality constant between spin and magnetic moment for particle P
- ***a_P*** : magnetic anomaly
- ***a_P*** = 0 at tree level (*purely Dirac particle*)

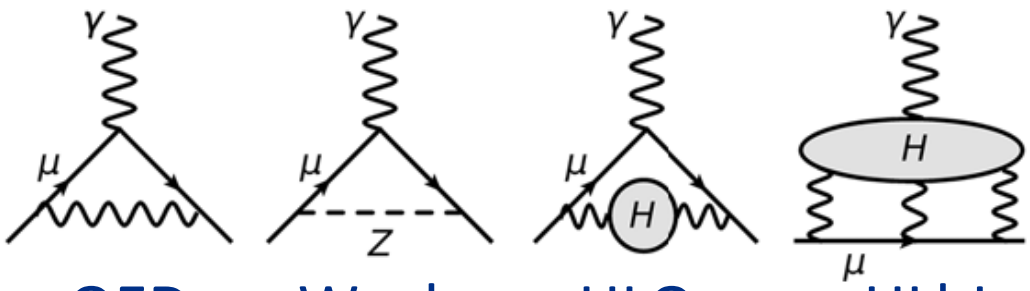
- Using modern language, the term (g-2)/2 reflects the magnitude of the Feynmann diagrams beyond leading order

a = 0 + α/2π + ...





Standard Model determination of a_μ



+ ?

QED	Weak	HLO	HLbL	New Physics.
Known	Known	Data	Models/Lattice	?

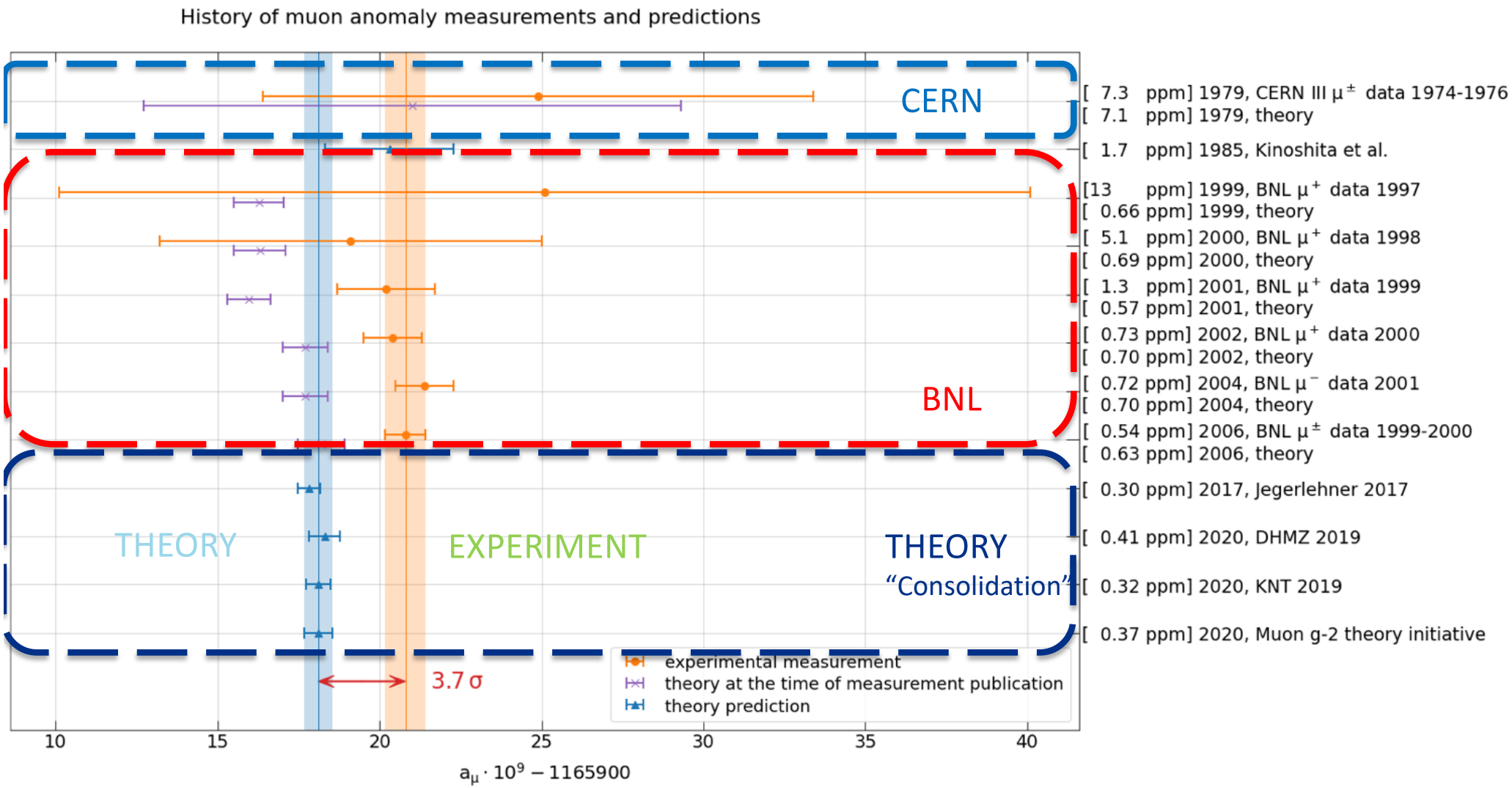
QCD

	Value ($\times 10^{-10}$) units
QED ($\gamma + \ell$)	$11\,658\,471.8951 \pm 0.0009 \pm 0.0019 \pm 0.0007 \pm 0.0077_\alpha$
HVP(lo) Davier17	692.6 ± 3.33
HLbL Glasgow	10.5 ± 2.6
EW	15.4 ± 0.1
Total SM Davier17	$11\,659\,181.7 \pm 4.2$

Theory Initiative White Paper (arXiv 2006:08443)

$a_\mu = (116\,591\,810 \pm 43)\times 10^{-11} \rightarrow 370$ ppb

A rich history of g-2 Theory and Experiment



Tension between theory and experiment

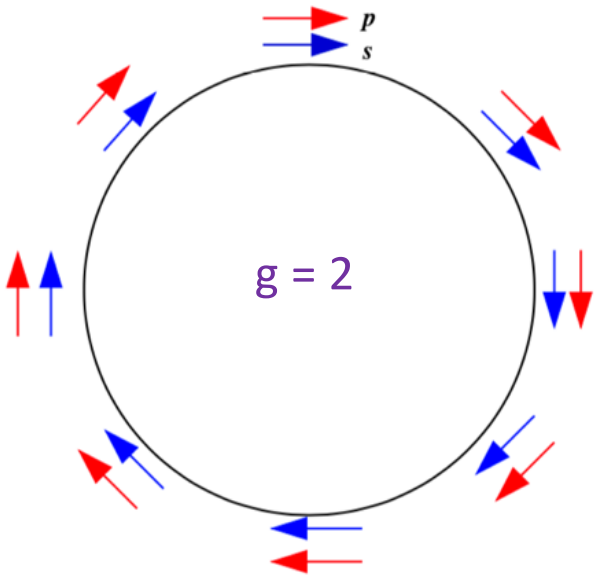
The Fundamental Experimental Principle

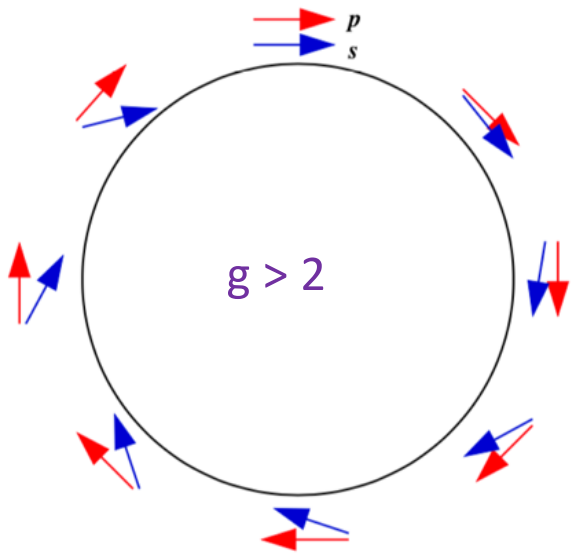
- Difference between spin precession and cyclotron revolution for a muon (charged particle with spin) in a magnetic field*:

$$\omega_a = \omega_s - \omega_c = g\frac{e}{2m}B - \frac{e}{m}B = \frac{g-2}{2}\frac{e}{m}B = a_\mu \frac{e}{m}B$$

*s and p are assumed to be in a plane perpendicular to **B**

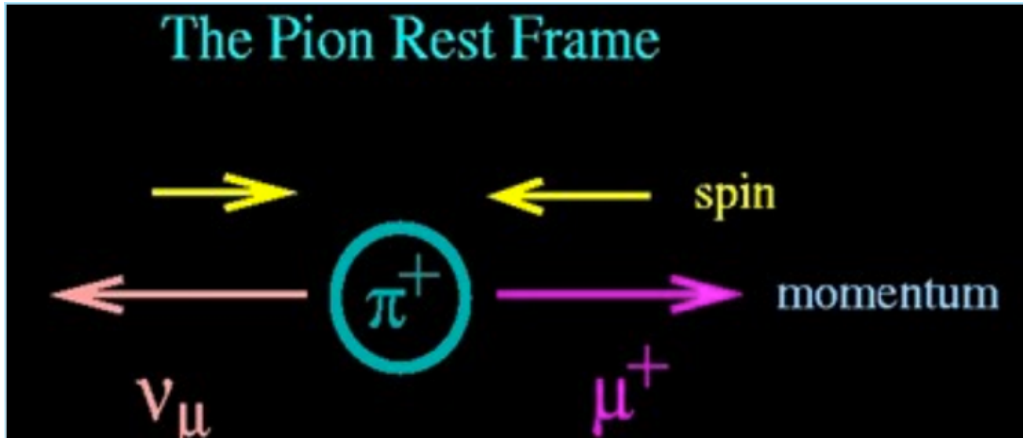
- simple classical calculation
- the relativistic approach provides the same result



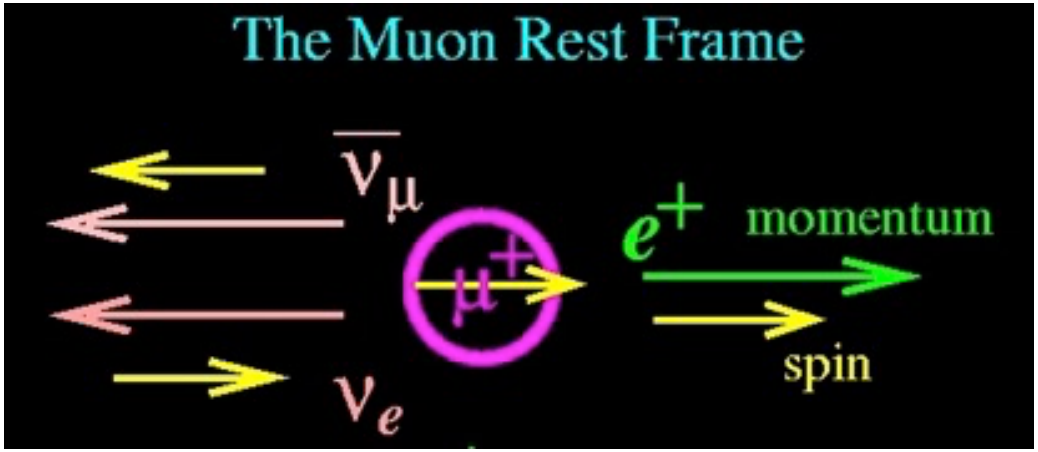


How do we measure the spin direction?

- Use V-A structure of weak decays to build a polarized beam...



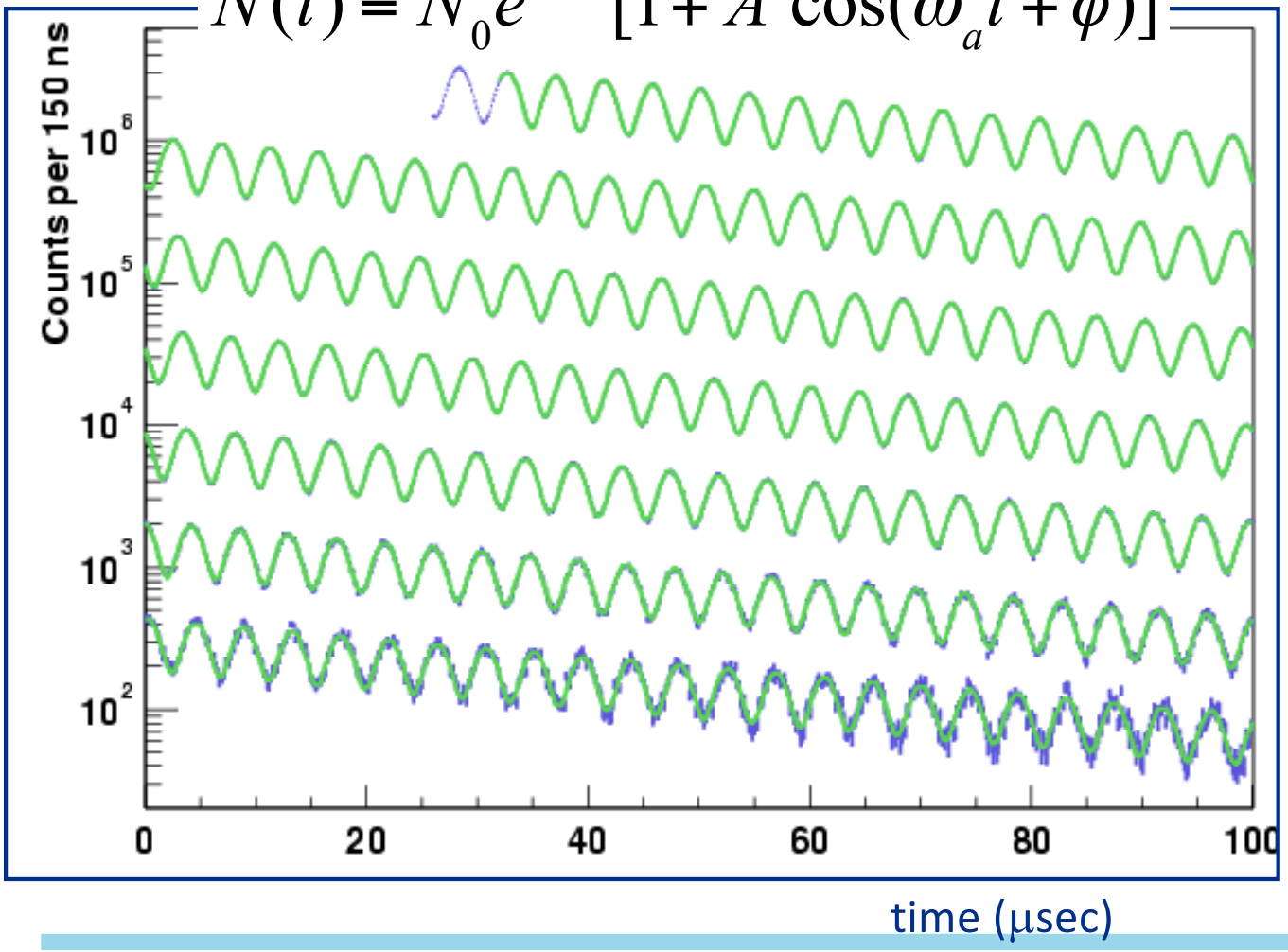
- ... and to measure the muon polarization looking for energetic positrons



Measuring the spin precession

- The number of observed positrons above a threshold energy oscillates with the $\omega_a/2\pi$ frequency due to spin precession

$$N(t) = N_0 e^{-t/\tau}[1 + A\cos(\omega_a t + \phi)]$$



- exponential decay modulated by spin precession
- *note that the x-axis "wraps up" every 100 μsec for a total of ~700 μs → ~10 muon lifetimes*

Extracting a_μ(simplified)

$$\boldsymbol{\omega_a = a_\mu\,(e/m)\,B} \;\rightarrow\; \boxed{\boldsymbol{a_\mu = \omega_a / B\,(m/e)}}$$

by expressing B in terms of the (shielded) proton precession frequency:
$(B = \hbar\omega'_p / 2\mu'_p)$:

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \cdot \frac{\mu'_p}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2} = R'_\mu \cdot \frac{\mu'_p}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}$$

What we measure

External data

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p}$$

ratio of muon to proton precessions in the same magnetic field

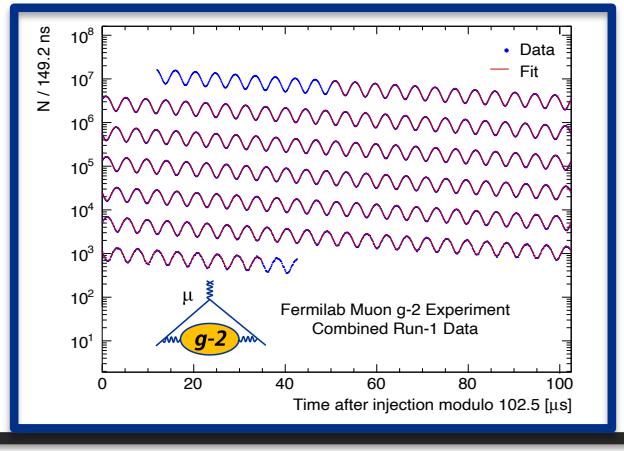
$\tilde{\omega}'_p$ = (shielded) Proton angular velocity **weighted for the muon distribution**

The key ingredients

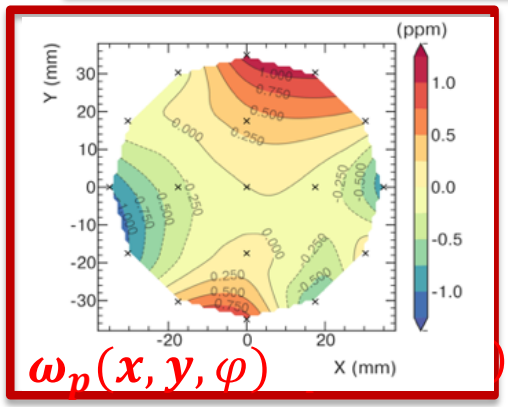
- Neglecting the corrections, the three *key ingredients* to measure the muon magnetic anomaly are:

ω_a=muon spin precession respect to momentum (in B field)

$$\omega_a$$

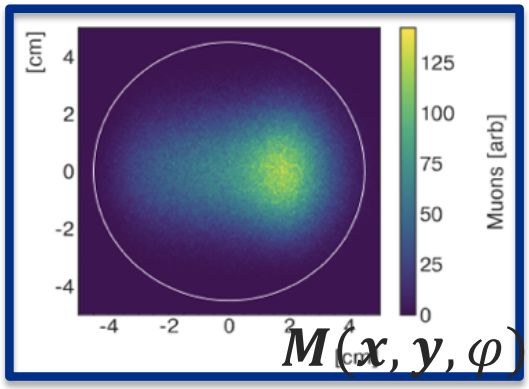


$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \sim$$



$\omega_p(x, y, \varphi)$

$\cdot$

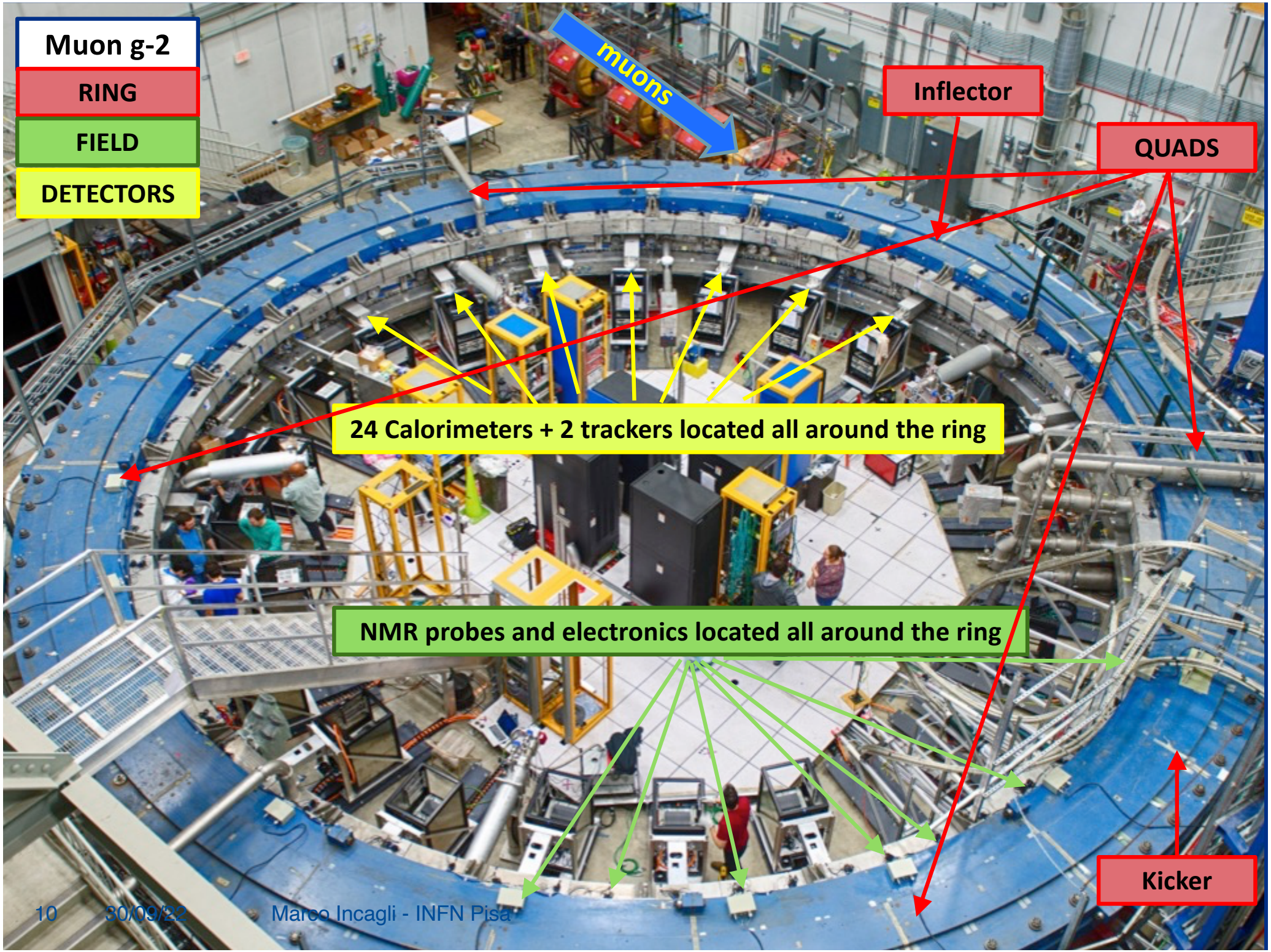


$M(x, y, \varphi)$

$$\tilde{\omega}'_p = \omega'_p(x, y, \varphi) \cdot M(x, y, \varphi)$$

ω'_p=proton precession frequency

M=muon spatial distribution


Muon g-2
RING
FIELD
DETECTORS
muons
Inflector
QUADS
24 Calorimeters + 2 trackers located all around the ring
NMR probes and electronics located all around the ring
Kicker

Additional corrections

- The ratio R'_μ requires additional corrections related to beam dynamics and to magnetic transient fields:

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \cdot \frac{1 + \overbrace{C_e + C_p + C_{ml} + C_{pa}}^{\text{Corrections due to beam dynamics}}}{1 + \underbrace{B_k + B_q}_{\text{Corrections due to transient magnetic fields}}} \cdot f_{clock}$$

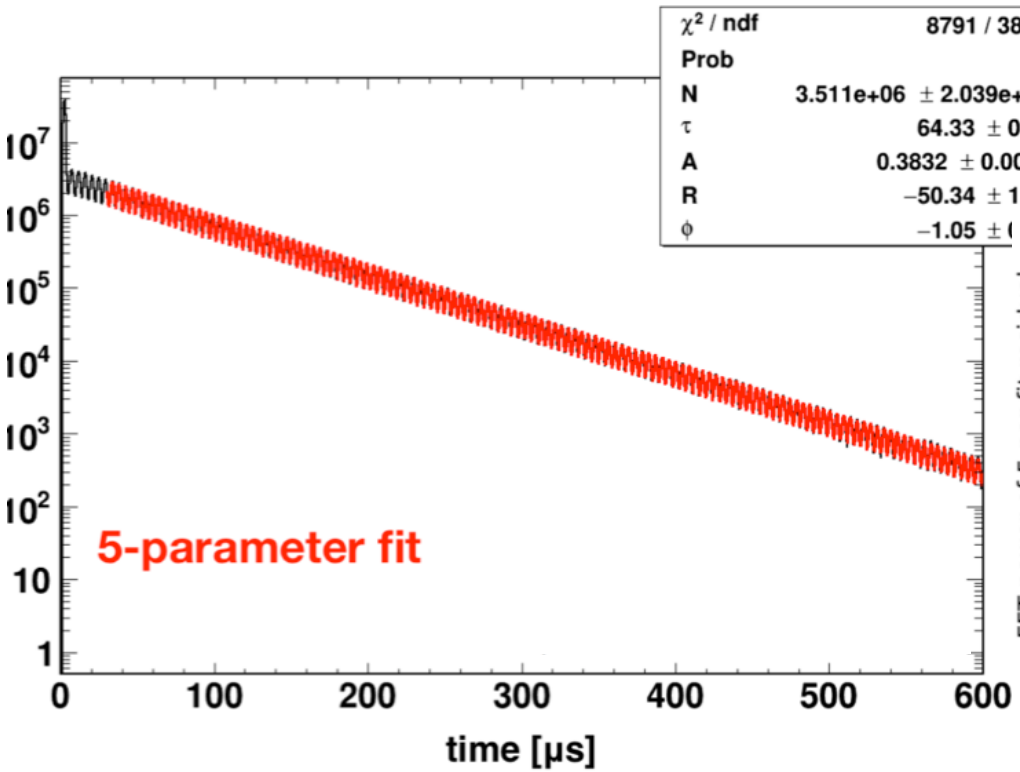
- f_{clock} = *blinding frequency*

Measuring ω_a : 5 parameters fit function

- Fit with simple positron oscillation:

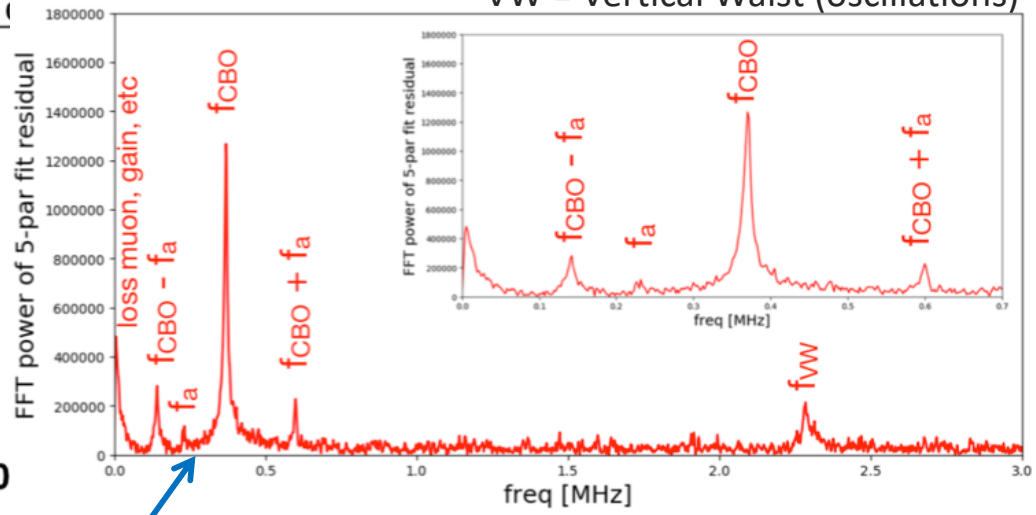
$$N_e(t) = N_0 \exp\left(-t/\tau_{\mu}\right)\left[1 + A\cos(\omega_a t + \varphi)\right]$$

- This simple fit is clearly not sufficient and well defined resonances are observed in the residuals



muon lifetime: $\tau_\mu = \gamma\tau_{0\mu}$ = 64.33 μsec

CBO = Coherent Betatron Ocillations
VW = Vertical Waist (oscillations)



RESIDUALS (in frequency space)

The complete 22 parameters fit function

ω_y, ω_{VW} vertical oscillations

ω_{CBO}, ω_{2CBO} radial oscillations

$$\boxed{N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A \cdot A_{BO}(t)\cos(\omega_a\, t + \phi \cdot \phi_{BO}(t)\,)\,) \cdot N_{\mathrm{CBO}}(t) \cdot N_{\mathrm{VW}}(t) \cdot N_y(t) \cdot N_{2\mathrm{CBO}}(t) \cdot J(t)}$$

$$A_{\mathrm{BO}}(t) = 1 + A_A \cos(\omega_{\mathrm{CBO}}(t) + \phi_A) e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$\phi_{\mathrm{BO}}(t) = 1 + A_\phi \cos(\omega_{\mathrm{CBO}}(t) + \phi_\phi) e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$N_{\mathrm{CBO}}(t) = 1 + A_{\mathrm{CBO}} \cos(\omega_{\mathrm{CBO}}(t) + \phi_{\mathrm{CBO}}) e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$N_{2\mathrm{CBO}}(t) = 1 + A_{2\mathrm{CBO}} \cos(2\omega_{\mathrm{CBO}}(t) + \phi_{2\mathrm{CBO}}) e^{-\frac{t}{2\tau_{\mathrm{CBO}}}}$$

$$N_{\mathrm{VW}}(t) = 1 + A_{\mathrm{VW}} \cos(\omega_{\mathrm{VW}}(t) t + \phi_{\mathrm{VW}}) e^{-\frac{t}{\tau_{\mathrm{VW}}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) t + \phi_y) e^{-\frac{t}{\tau_y}}$$

Red = free parameters

Blue= fixed parameters

$$J(t) = 1 - k_{LM} \int_{t_0}^{t} \Lambda(t) dt$$

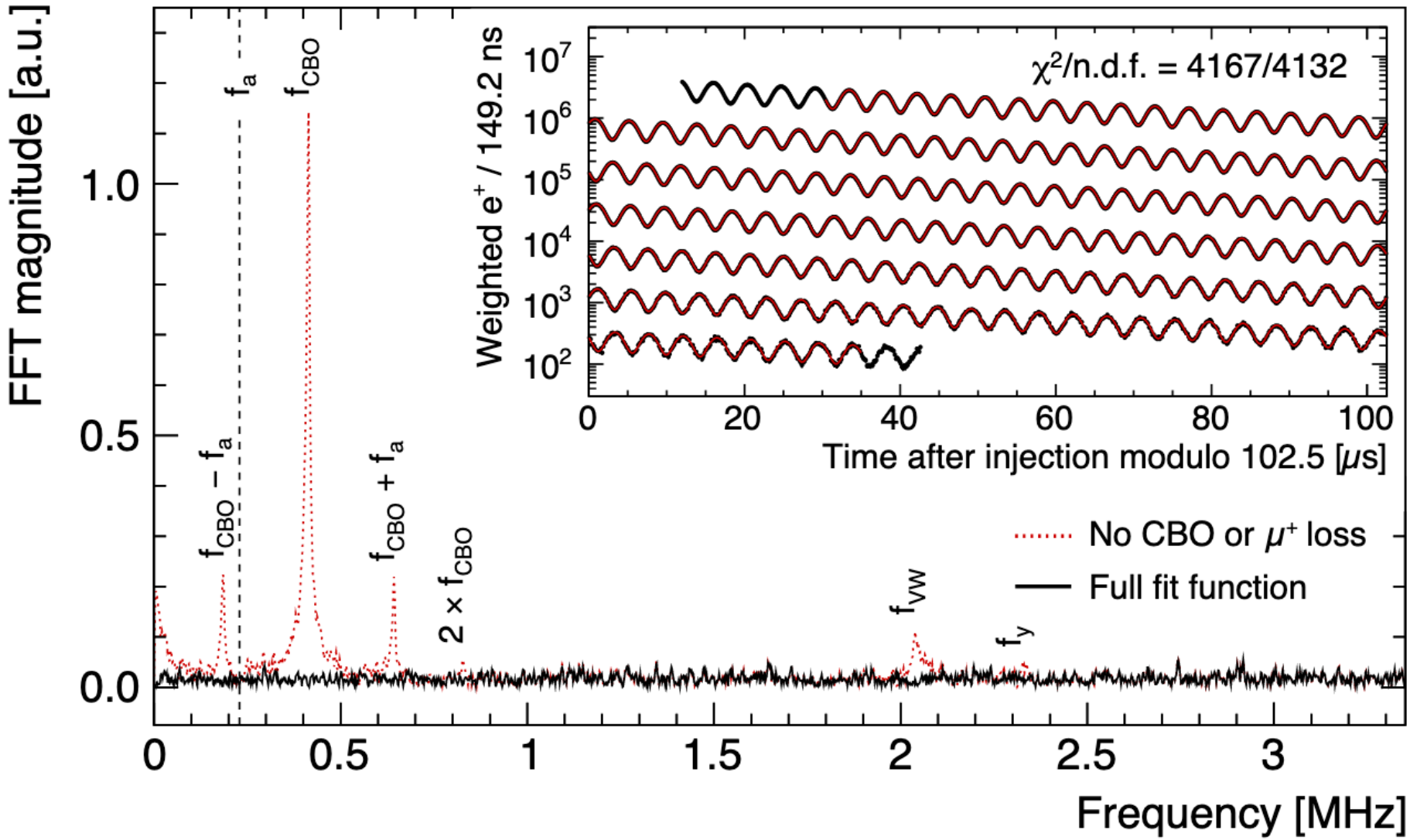
Lost muons (μ hitting collimators)

$$\omega_{\mathrm{CBO}}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{\mathrm{CBO}(t)} \sqrt{2\omega_c / F \omega_{\mathrm{CBO}}(t) - 1}$$

$$\omega_{\mathrm{VW}}(t) = \omega_c - 2\omega_y(t)$$

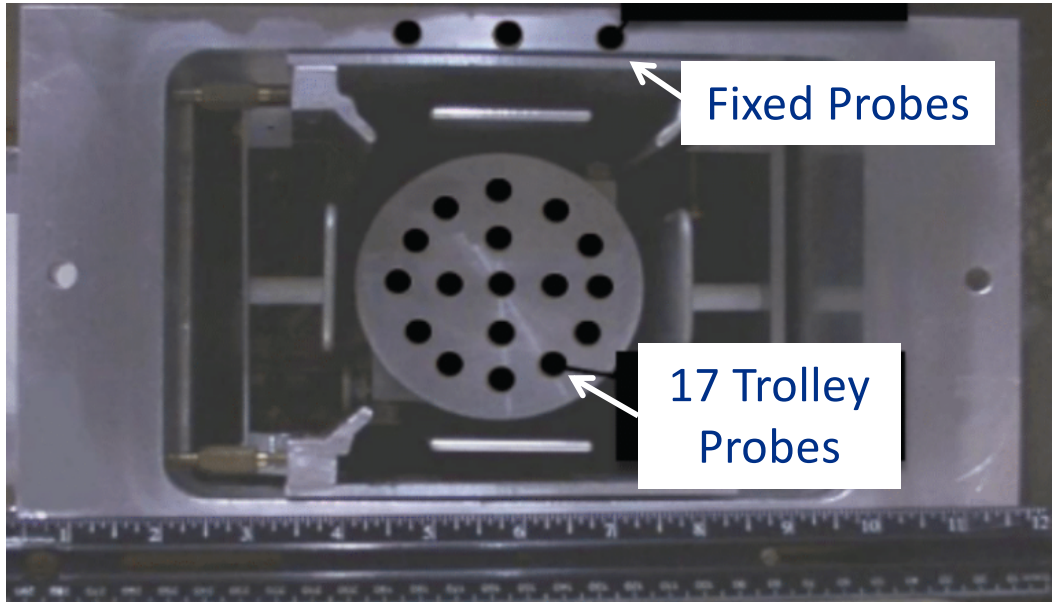
Final fit to get ω_a



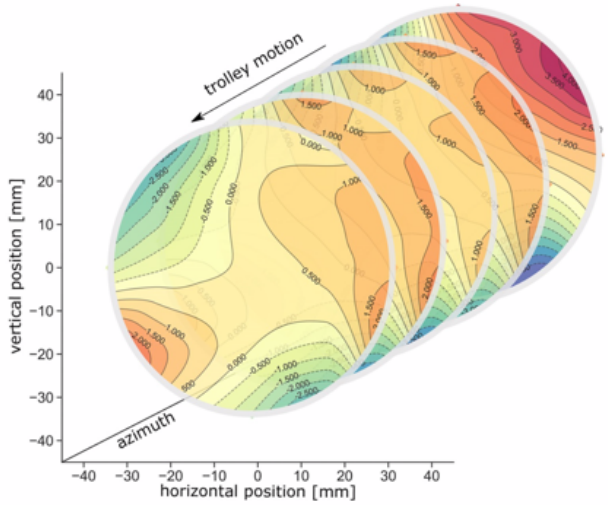
Magnetic field $\vec{B}$ determination

The “trolley”

The "trolley" inside the beam "pipe”



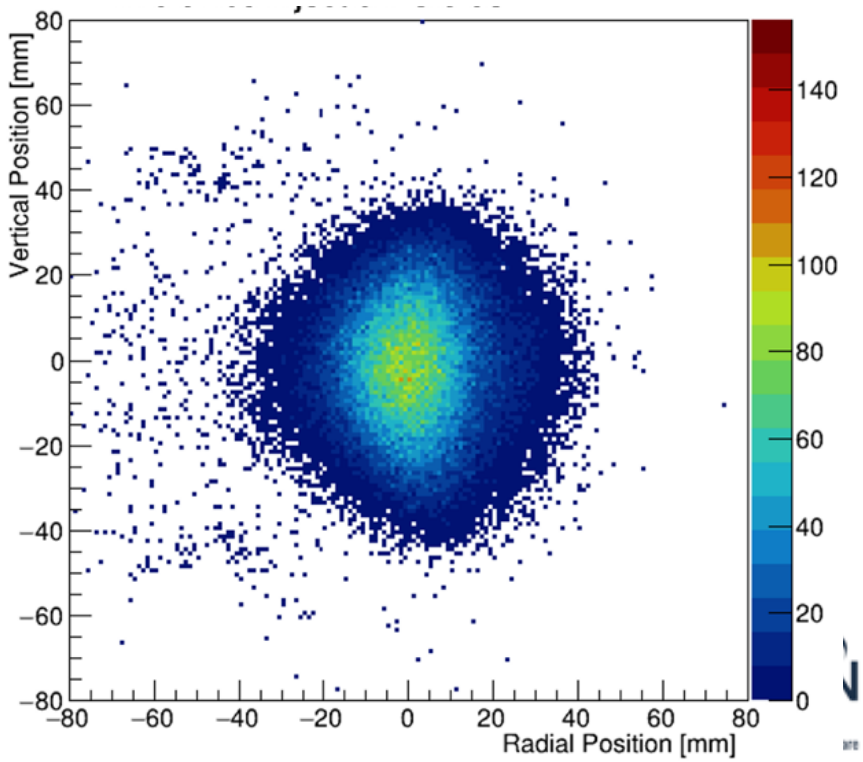
The map



- A Cylinder with 17 **NMR probes** (“trolley”) runs inside the ring every 2-3 days to map the field experienced by muons
- A set of 378 **fixed probes**, located in 72 azimuthal positions, continuosly measures the field
- Absolute probes for calibration tested at Argonne (ANL) magnet

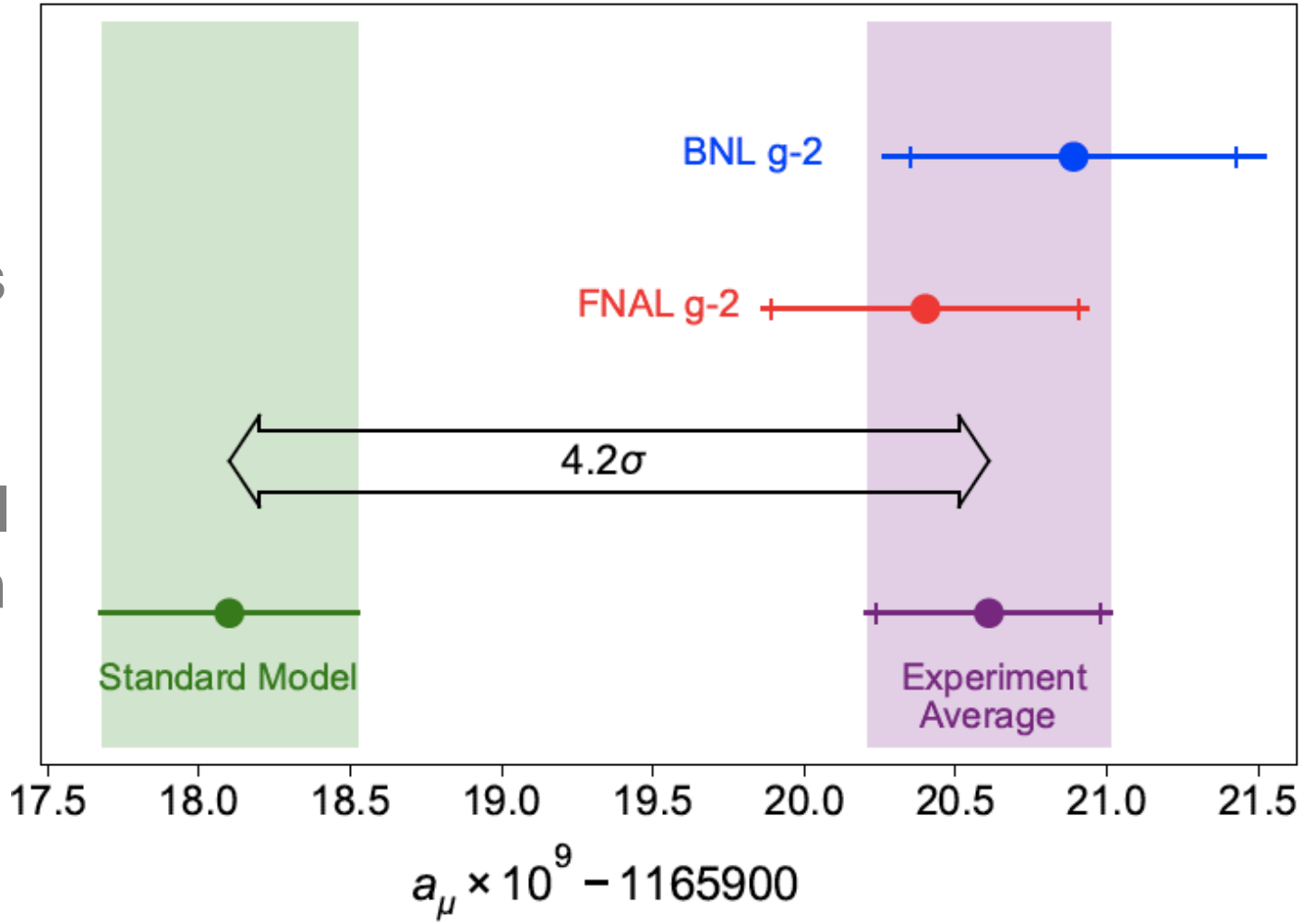
$\omega'_p \rightarrow \tilde{\omega}'_p$: muon distribution inside the Ring

- Two tracker stations, made of straw tube modules, placed at $\phi \sim 180^o$ and $\phi \sim 270^o$, are used to trace back positrons and get the muon distribution
- Use Beam Dynamics models to extrapolate the distribution all around the ring
- Systematic uncertainties mostly due to Beam Dynamics models used for extrapolation and to tracker alignment



a_μ: Unblinding and result

- ***462 ppb total uncertainty***
- The combined a_μ value shows a 4.2σ tension with the standard model 2020 prediction *in the* **Theory Initiative Group White Paper**

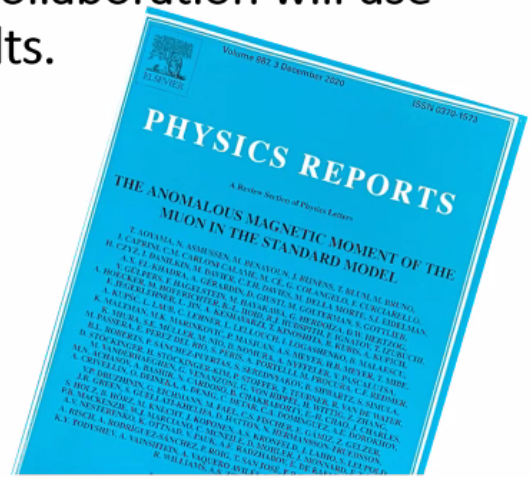


The Theory Initiative

- ~130 physicists collaborated for 3 years [June 2017 – June 2020] in seven workshops to produce a reference number for am to be used by FNAL g-2 experiment as a benchmark

Muon g-2 Theory Initiative defines SM benchmark value that our collaboration will use for comparison. We don't "pick and choose" other individual results.

Group photo from the Seattle workshop in September 2019, https://indico.fnal.gov/event/21626/



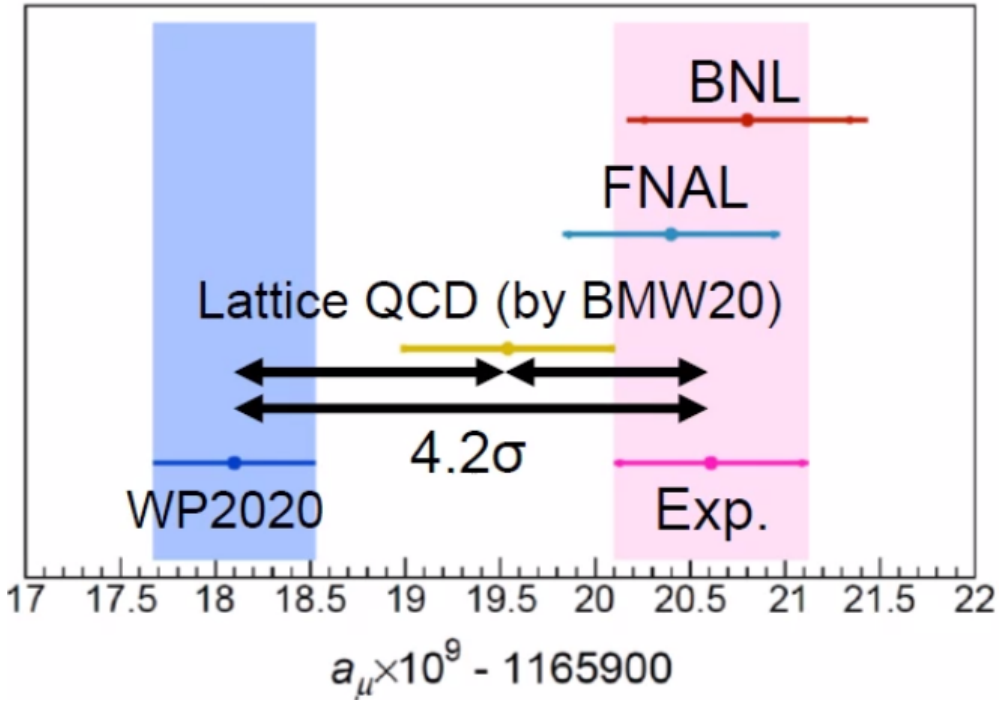
Organizers:
Aida El-Khadra
Martin Hoferichter
DWH

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 8	Eq. (8.14)	279(76)	

WP20

The lattice evaluation of a_μ

- The BMW (Budapest, Marseille, Wuppertal) collaboration published the result of their lattice calculation in april 2021 (pre-print in 2020): *closer to the experimental value than with the Theory Initiative one*



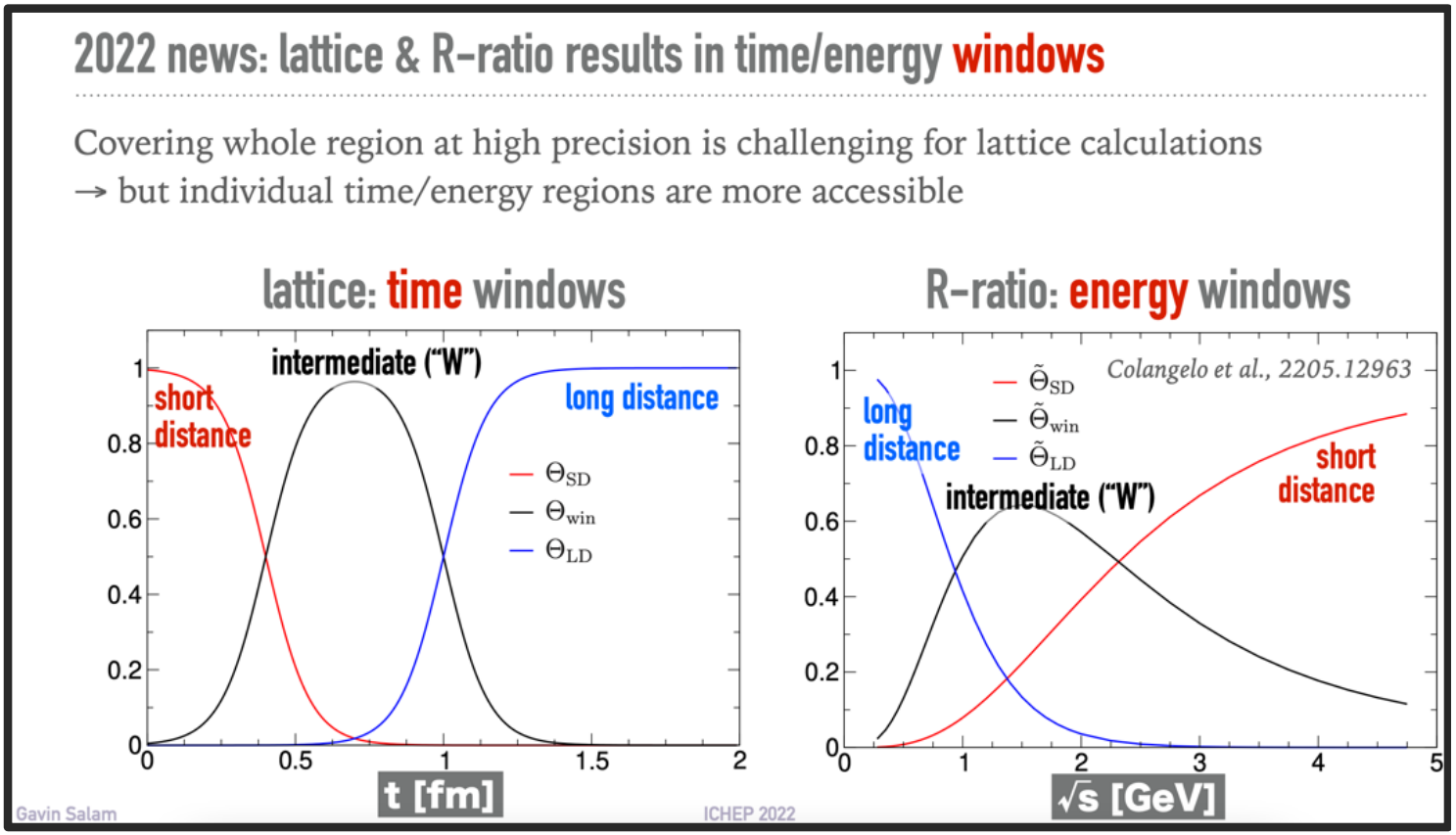
- The Theory Initiative evaluated the (leading order) hadronic contribution to a_μ using the *dispersion integral:*

$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} K(s)\sigma^{had}(s)ds \quad ; \quad with \quad K(s) \propto \frac{1}{s}$$

- which is based on the experimental measurement of the cross section $\sigma^{had}(s) \implies \sigma(e^-e^+ \to hadrons)$

News on lattice calculations (ICHEP 2022)

- Use the *"Intermediate region"*, more under control
- This region corresponds to ~235/700=33% of the full hadronic contribution

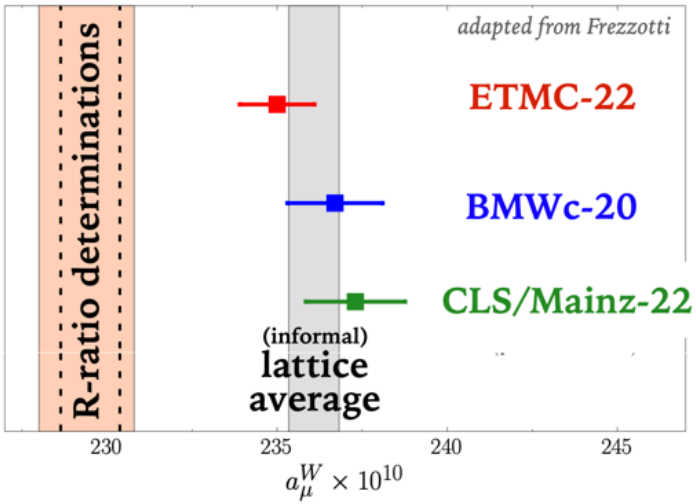


- It (approximately) corresponds to the energy window 1-3 GeV
- Note: the largest contribution to a_μ^{had} still comes from the ***ρ-resonance region***, i.e below 1 GeV

New results from ICHEP22

- Recent lattice results are in agreement with BMW calculation in the intermediate region
- strong tension with the value obtained from e^+e^- data

intermediate window: 2 new lattice results (ETMC-22 & CLS/Mainz-22)



- Highest precision lattice results mutually consistent (more to come soon)
- Difference in this window alone (~7) not enough to explain g_μ-2 (~25), but enhances credibility of full BMWc-20 result
- Tension between lattice & e^+e^- data clearly needs to be understood

- strong tension with a_μ^W (HVP-LO) results driven by experimental e^+e^- data :

at ~ $4.2\sigma_{combined}$ if WP-proc.('22) (2205.12963, Colangelo et al.), see *light-red band*, is used

at ~ $5.8\sigma_{combined}$ if KNT('19-'22) (1911.00367 + private comm.), see *dashed lines*, is used

Gavin Salam ICHEP 2022

Hadronic cross section

- Is it possible a "mistake" in $\sigma^{had}(s)$?
- In principle yes (experimental cuts) but
  - many different experiments should have same bias
  - an upward shift in $\sigma^{had}(s)$ induces an increase of $\Delta\alpha^{had}(M_Z)$

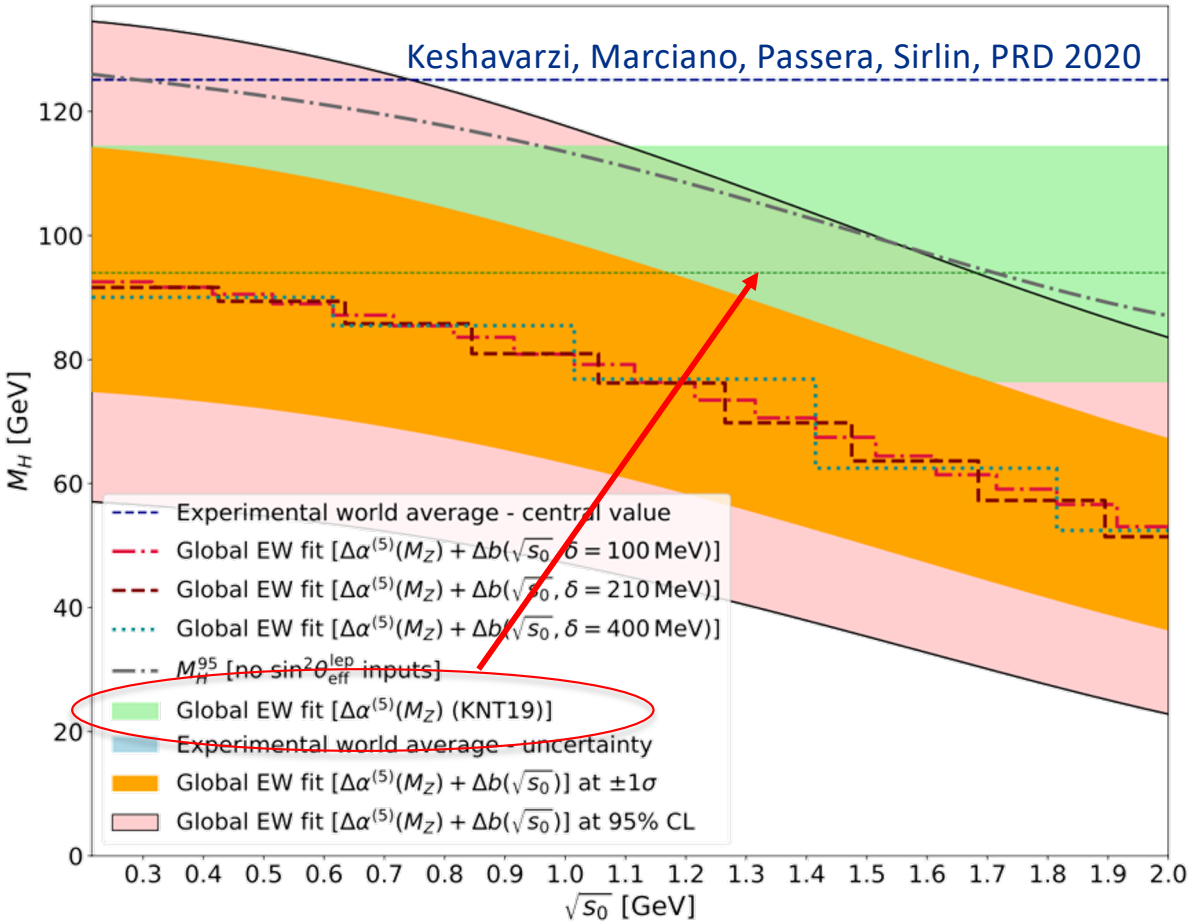
$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} K(s)\sigma^{had}(s)ds \quad ; \quad with \quad K(s) \propto \frac{1}{s}$$

$$\Delta\alpha^{had} = \int_{m_\pi^2}^{\infty} g(s)\sigma^{had}(s)ds \quad ; \quad with \quad g(s) \propto \frac{M_Z^2}{M_Z^2 - s}$$

- Similar dispersion integral with a different kernel function

Hadronic cross section and fine structure constant

- An upward shift in the hadronic cross section to move the theoretical prediction towards the lattice one, or towards the exp. value, also "pushes down" the preferred Higgs Mass value in the global electroweak fit (green band)



Clearly there is something going on there.

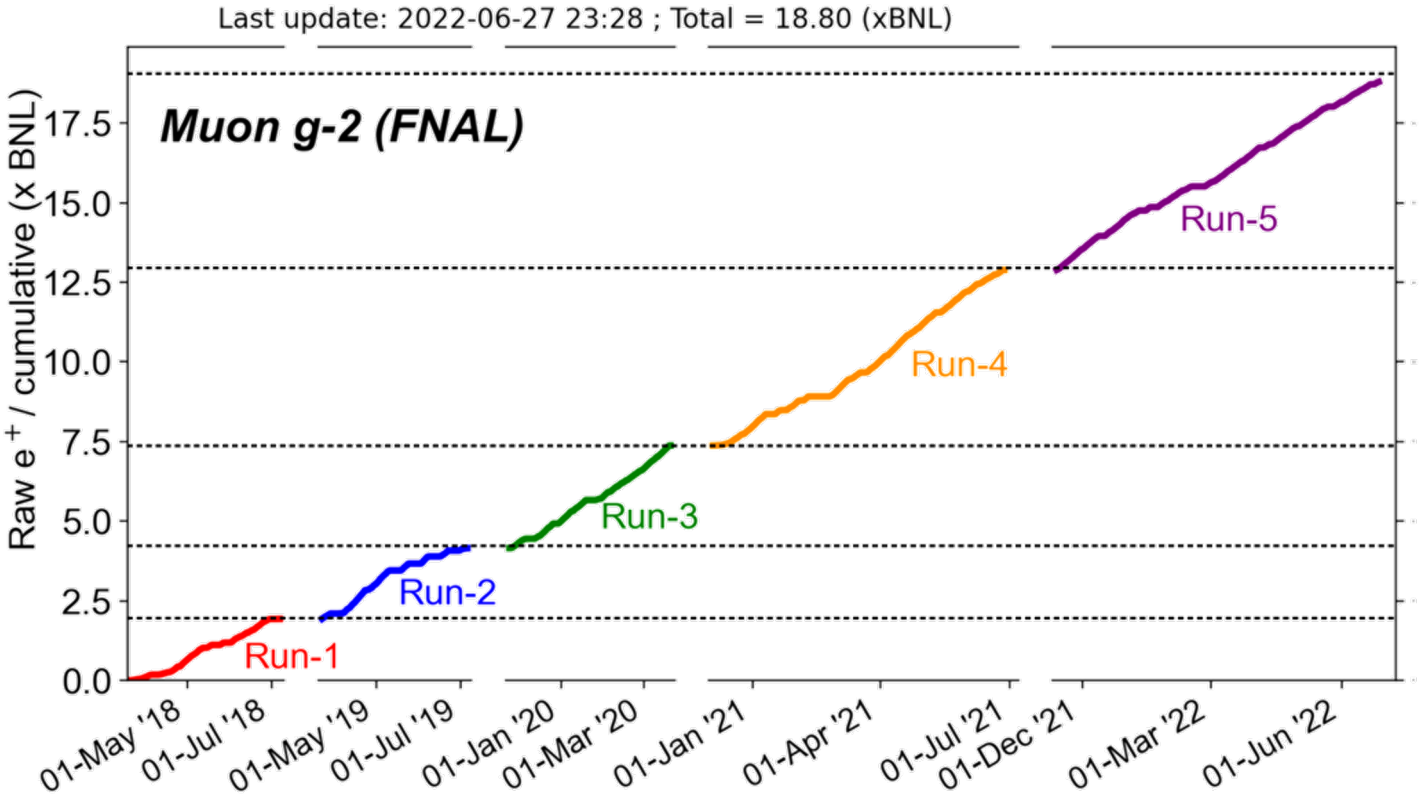
Our task: reduce as much as possible the experimental uncertainty on *g-2* !

FNAL g-2: next publications

Collected data: 18.8 BNL equivalent

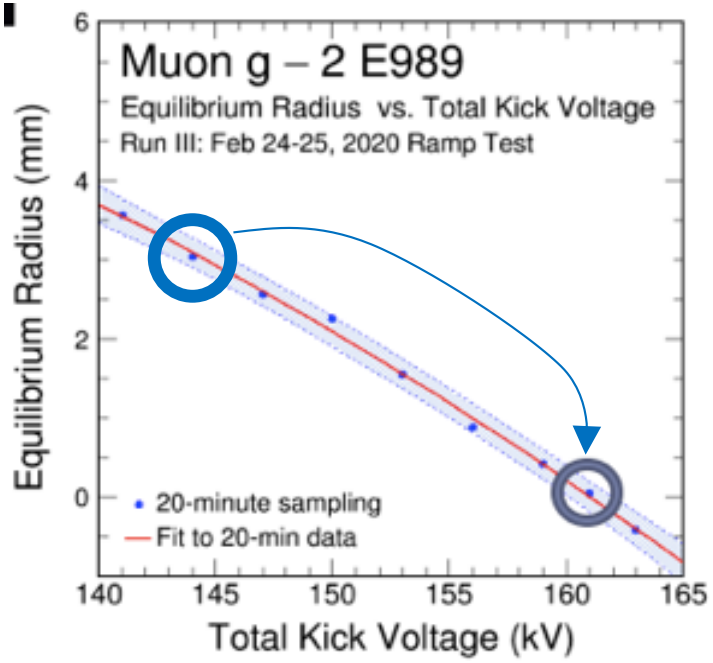
Publications:

- Run1
  7/apr/2021: ±462 ppb
- Run2,3
  spring 2023: ±220 ppb
- Run4,5,6
  beginning 2025: ±140 ppb (maybe ±120 ppb)



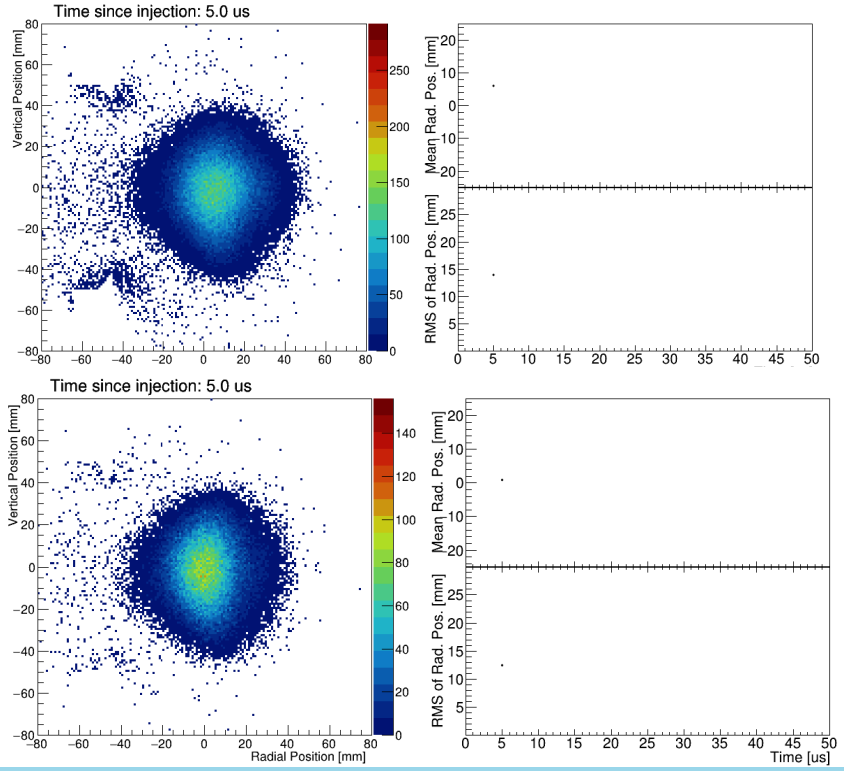
Main improvements wrt Run1

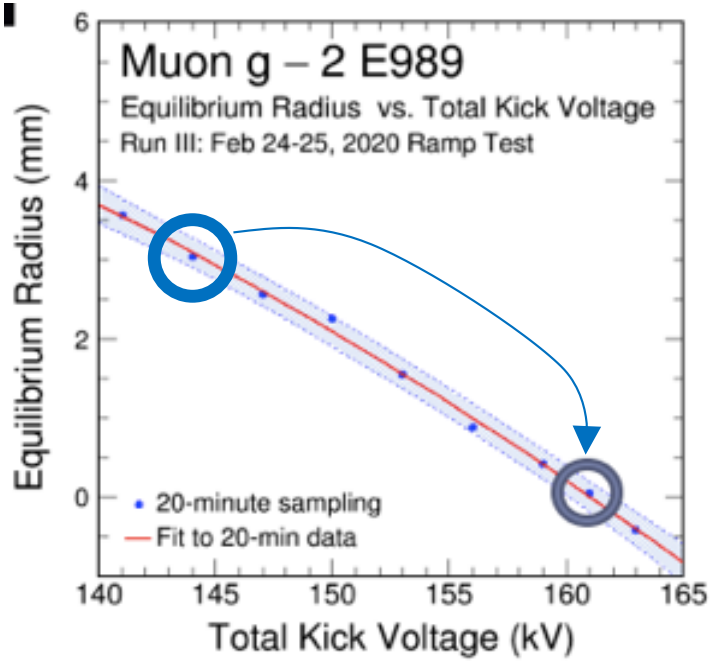
- Thermalization of experimental Hall
- Kicker HV to design value



Main improvements wrt Run1

- Thermalization of experimental Hall
- Kicker strength to design value
- New Radio Frequency System mounted on quadrupoles which reduces Beam Betatron oscillations





Perspectives on final uncertainty Run 1-5

Total Uncertainties

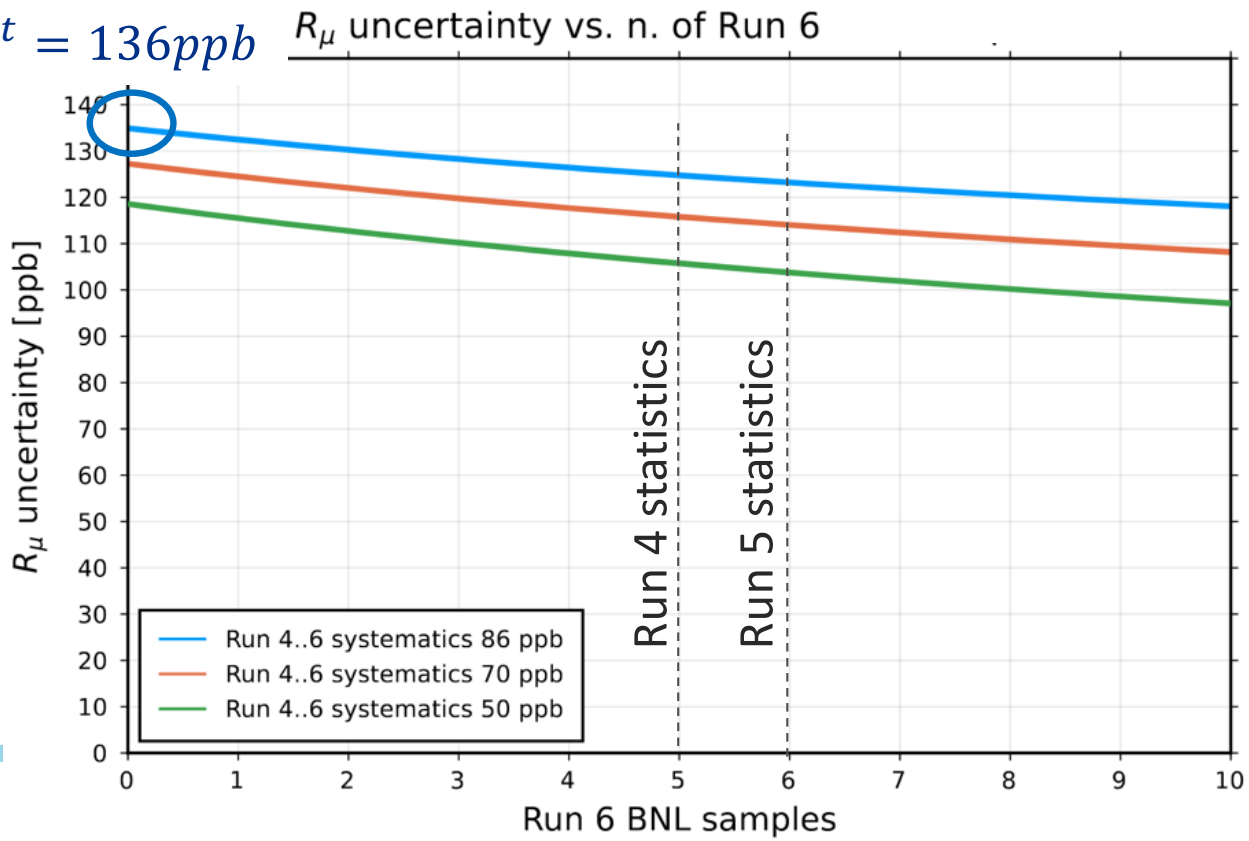
	Run-1	Run-2/3	Run-4/5	Run-1/3	Run-1/5
$\omega_a(stat)$	434	202	119	183	102
$\omega_a(syst)$	109	70	60		
$\omega'_p(tot)$	114	65	62		
R_μ (tot syst)	157	95	86	106	92
Total	**462**	**223**	**147**	**212**	**136**

- The final TDR goal of 140ppb is at reach!
- And we can do more with Run 6 (next slide)..

Run 6

- In August 2022, the new Fermilab Director Lia Merminga confirmed a Run 6 with MU+ in Nov22-Jul23
- The priority, however, will be given to machine development for MU2E experiment
- Aim to reduce systematics, rather than to increase statistics

$\sigma^{tot} = 136ppb$



Conclusions

- There is clearly something going on in the muon anomalous magnetic moment
- Lots of theoretical and experimental activity to pin down the hadronic contribution to a_μ
- The "Muon g-2" experiment has collected data to reduce by a factor of 4 the published uncertainty on a_μ, thus reaching the TDR goal, and it has the potentiality to even gain some 10-20% additional reduction in Run6
- This will be the more precise measurement of the muon anomaly for several decades, thus every possible reduction of the finale uncertainty should (and will) be pursued in the next 2-3 years