Laboratoire Kastler Brossel
Physique quantique et applications

The kaon mass from kaonic atoms: atomic energies and cascade simulations

Paul Indelicato

High precision measurements of kaonic atoms on-line workshop, 09/02/2022

EMMI ENS CNRS SORBONNE UNIVERSITÉ COLLÈGE DE FRANCE 1530

Outline

- Introduction: QED contributions for atoms
- How well do we understand QED in exotic atoms
  - finite nuclear size
  - electron screening
- A few examples
  - Kaonic Ar
  - Kaonic Xe
  - Issues with existing measurements
- Conclusion

Present status of the kaon mass

$K^{\pm}$ MASS

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
493.677±0.016 OUR FIT	Error includes scale factor of 2.8.				
493.677±0.013 OUR AVERAGE	Error includes scale factor of 2.4. See the ideogram below.				
493.696±0.007	[1] DENISOV	91	CNTR	−	Kaonic atoms
493.636±0.011	[2] GALL	88	CNTR	−	Kaonic atoms
493.640±0.054	LUM	81	CNTR	−	Kaonic atoms
493.670±0.029	BARKOV	79	EMUL	±	$e^+ e^- \to K^+ K^-$
493.657±0.020	[2] CHENG	75	CNTR	−	Kaonic atoms
493.691±0.040	BACKENSTO...	73	CNTR	−	Kaonic atoms

Particle Data Group 2020

Present status of the kaon mass

Transition	$E_{\text{calculated}}$	E_{measured}
K^-Pb$(11 \rightarrow 10)$	153.891	153.903 ± 0.008
K^-Pb$(9 \rightarrow 8)$	291.583	291.5800 ± 0.0044
K^-W$(11 \rightarrow 10)$	125.208	125.251 ± 0.024
K^-W$(9 \rightarrow 8)$	237.171	237.206 ± 0.035

K. P. Gall et al., Phys. Rev. Lett. 60, 186 (1988).

ft reflections of the K^--atom line (Fig.
ıergy is 22 105.61 $\pm$ 0.26 eV. The fit of
olated peak, since the instrumental re:

A. S. Denisov et al., JETP Letters 54, 558 (1992).

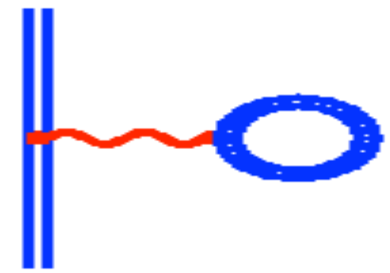
Non-perturbative bound-states QED

Practical calculations and numerical methods

one electron QED at order α

Self Energy

$\frac{\alpha}{\pi}$



Vacuum Polarization

H-like "One Photon" order (α/π)

Many calculations, very accurate down to Z=1

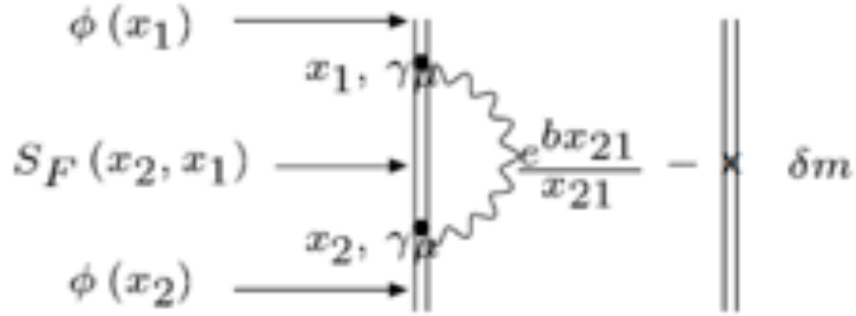
$$\Delta E_{SE} = \frac{\alpha}{\pi}\frac{(Z\alpha)^4}{n^3}F(Z\alpha).$$

$$\frac{\alpha}{\pi} = 2.3 \times 10^{-3}$$

$$E_{NR} = \frac{(Z\alpha)^2}{n^2}$$

Feynman diagrams

- Each contribution to the S-matrix can be decomposed in Feynman diagrams.



Signification of the Feynman diagrams for self-energy.

The Energy shift is given by

$$\Delta E_n = \frac{\alpha}{2\pi i}\int_C dz \int d\vec{x}_2 \int d\vec{x}_1 \phi_n^\dagger(\vec{x}_2)\alpha_\mu G(\vec{x}_2, \vec{x}_1, z)\alpha^\mu \phi_n(\vec{x}_1)\frac{e^{-bx_{21}}}{x_{21}} - \delta m \int d\vec{x}\, \phi_n^\dagger(\vec{x})\beta\phi_n(\vec{x}),$$

where $b = -i\left[(E_n - z)^2\right]^{1/2}$, $\mathrm{Re}(b) > 0$, and $\vec{x}_{21} = \vec{x}_2 - \vec{x}_1$ The photon propagator $\frac{1}{x_{21}} \times e^{-bx_{21}}$ is just the Coulomb potential with a retardation term (speed of light is finite!)

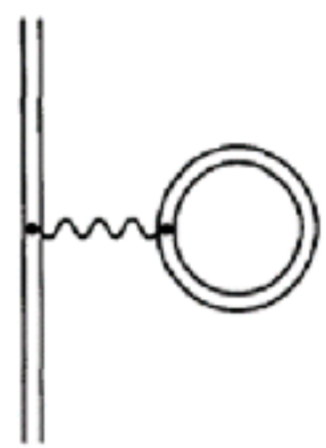
Content of a Feynman diagram: vacuum polarization

- **Vacuum polarization**
  - The electron interacts with a virtual electron-positron (or muon-antimuon) pair in the field of the nucleus

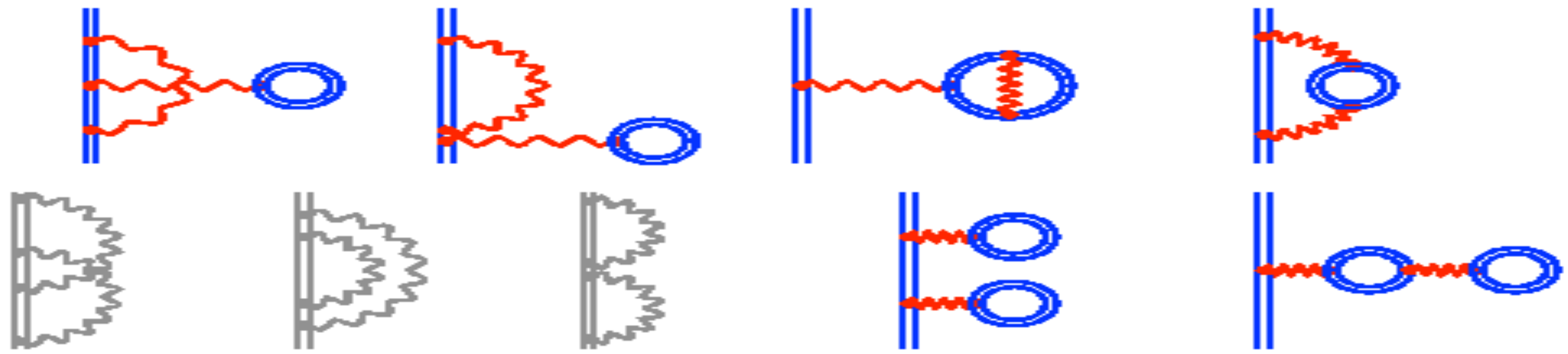
$$E^{(2)}_{\mathrm{VP}} = 4\pi i\alpha \int d(t_2 - t_1) \int d\boldsymbol{x}_2 \int d\boldsymbol{x}_1\, D_{\mathrm{F}}(x_2 - x_1)$$

$$\times\, \mathrm{Tr}[\gamma_\mu S_{\mathrm{F}}(x_2, x_2)]\, \overline{\phi}_n(x_1)\gamma^\mu \phi_n(x_1)$$

→ Potential



one electron QED at order α^2



H-like "Two Photon" order $(\alpha/\pi)^2$

Closed fermion loops 2nd order diagrams

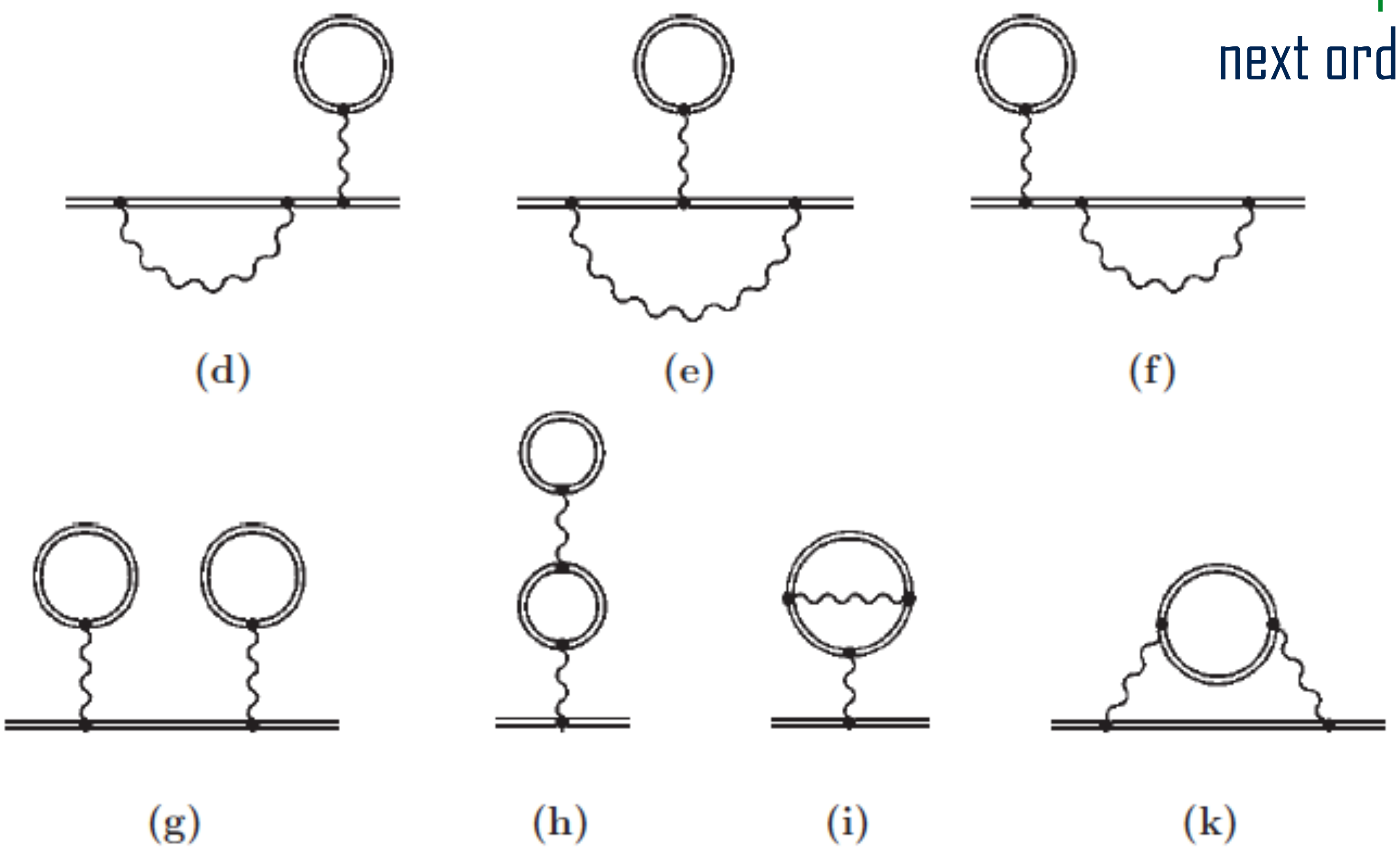
$$\frac{\alpha}{\pi} = 2.3 \times 10^{-3}$$

10 diagrams with ± signs

so compensation

next order?

SE-VP



Uelhing
in Dirac Eq.

VP-VP

Källén & Sabri potential

S(VP)E

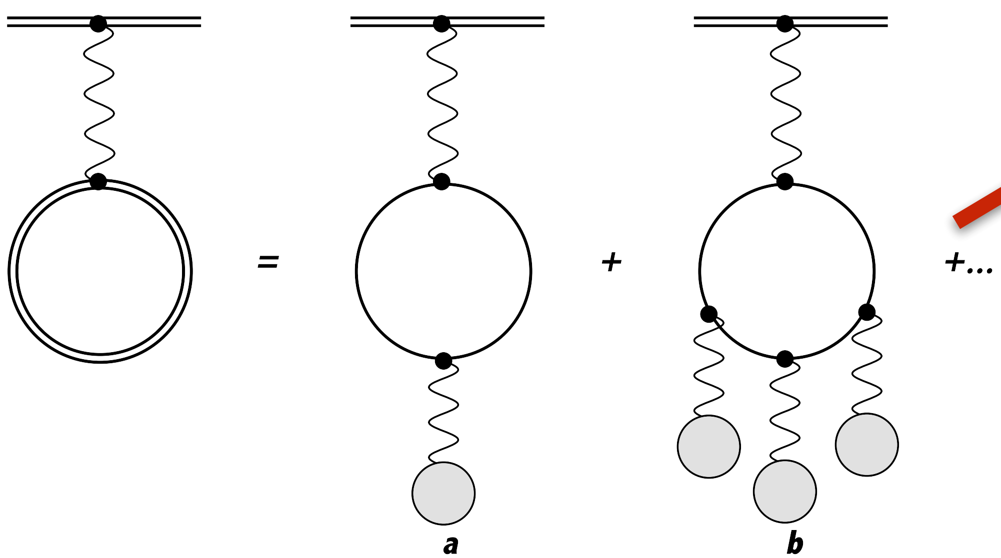
Yerokhin, V. A., P. Indelicato and V. M. Shabaev (2008). " Two-loop QED corrections with closed fermion loops." Physical Review A **77** (6): 062510 (062512).

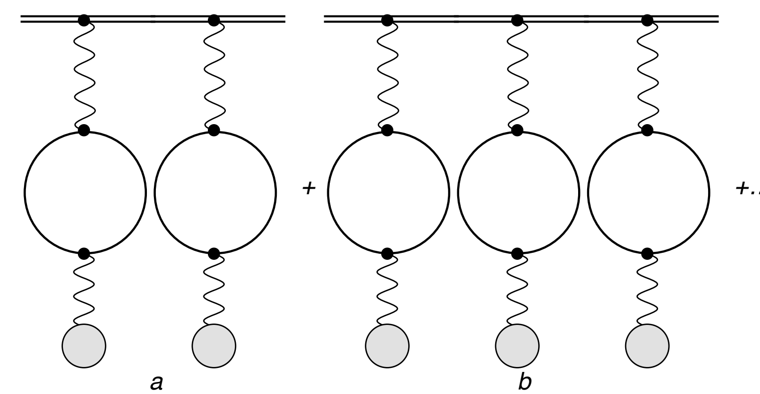
Lindgren, I., H. Persson, S. Salomonson, V. Karasiev, L. Labzowsky, A. Mitrushenkov and M. Tokman (1993). "Second-order QED corrections for few-electron heavy ions: reducible Breit-Coulomb correction and mixed self-energy- vacuum polarization correction." Journal of Physics B: Atomic, Molecular and Optical Physics **26**: L503-L509.

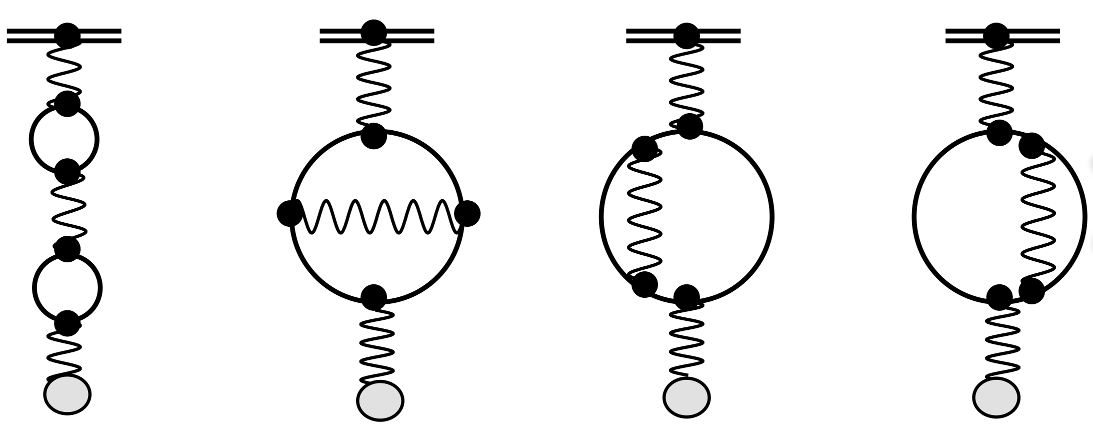
Mitrushenkov, A., L. Labzowsky, I. Lindgren, H. Persson and S. Salomonson (1995). "Second order loop after loop self-energy correction for few-electron multicharged ions." Physics Letters A **200**: 51-55.

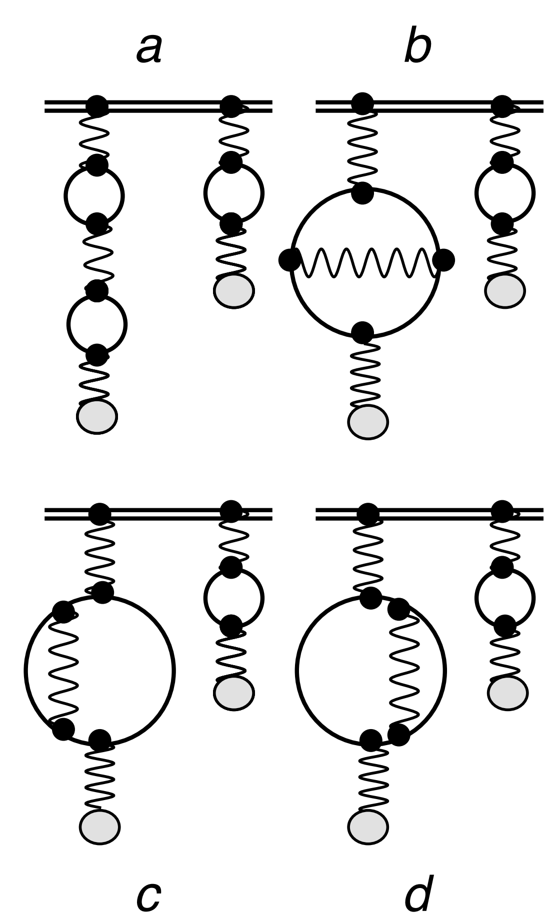
Contributions included to all-order

All-order: the charge distribution is included exactly in the wavefunction and in the operator, when relevant. Higher order Vacuum Polarization contribution included by numerical solution of the Dirac equation





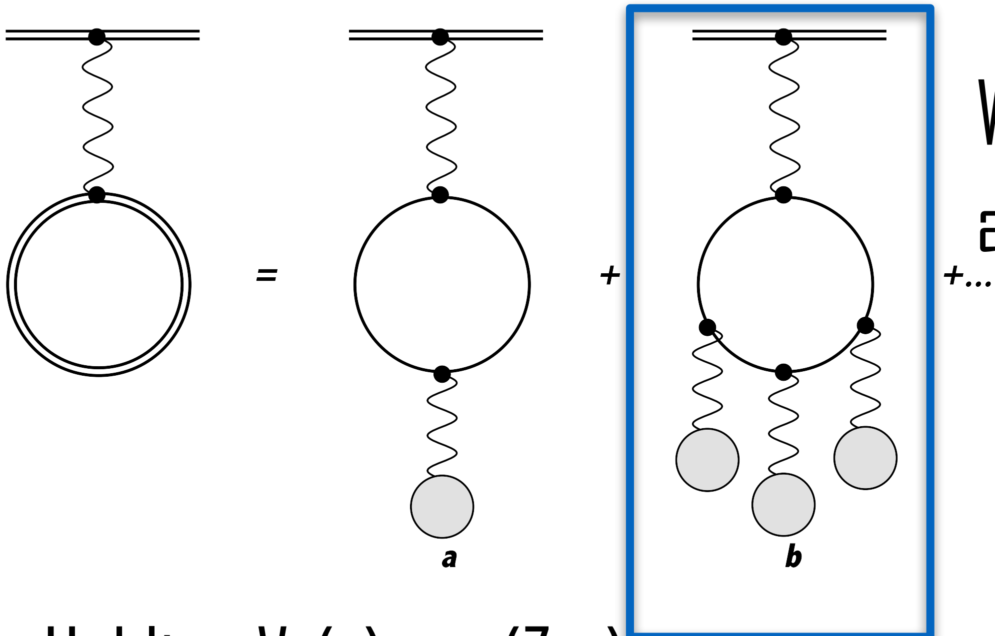




P. Indelicato. Phys. Rev. A **87,** 022501 (2013).

Light by light scattering in muonic atoms

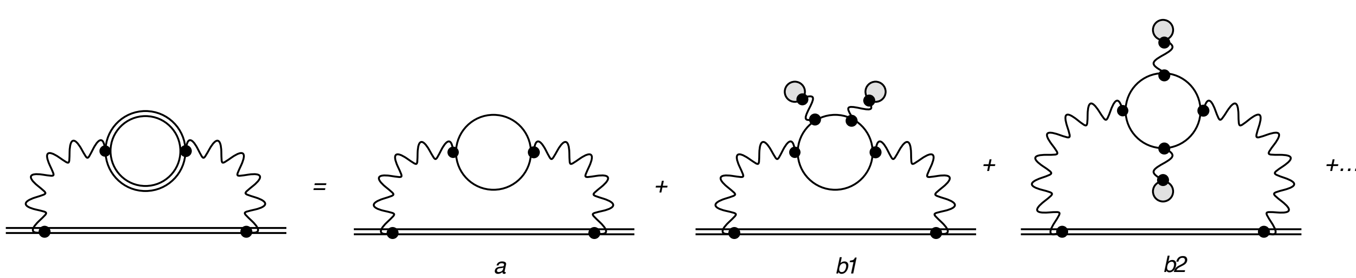
Diagrams with 4 vertex with external lines in an electron-positron loop

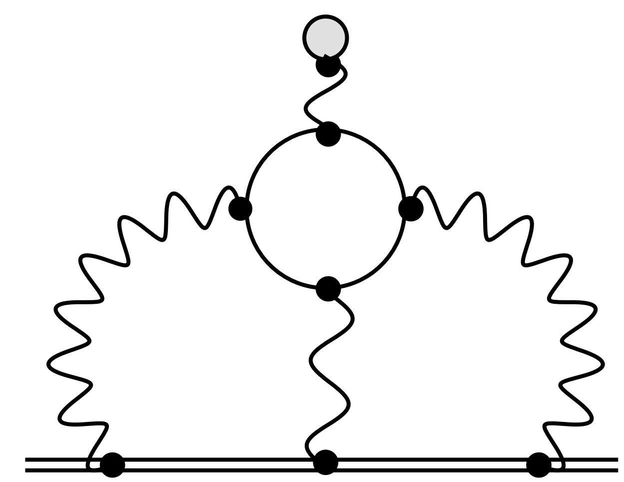


Wichman & Kroll $V_{13}(r) \sim \alpha(Z\alpha)^3$

approx. to $V_{15}(r) \sim \alpha(Z\alpha)^5$ and $V_{17}(r) \sim \alpha(Z\alpha)^7$ also included

Uehling $V_{11}(r) \sim \alpha(Z\alpha)$





a is a renormalization term: no contribution

b1 and b2: virtual Delbrück scattering $V_{22}(r) \sim \alpha^2(Z\alpha)^2$

(part of the expansion of S(VP)E)

3rd order $V_{31}(r) \sim \alpha^3(Z\alpha) \sim V_{13}(r)/Z^2$

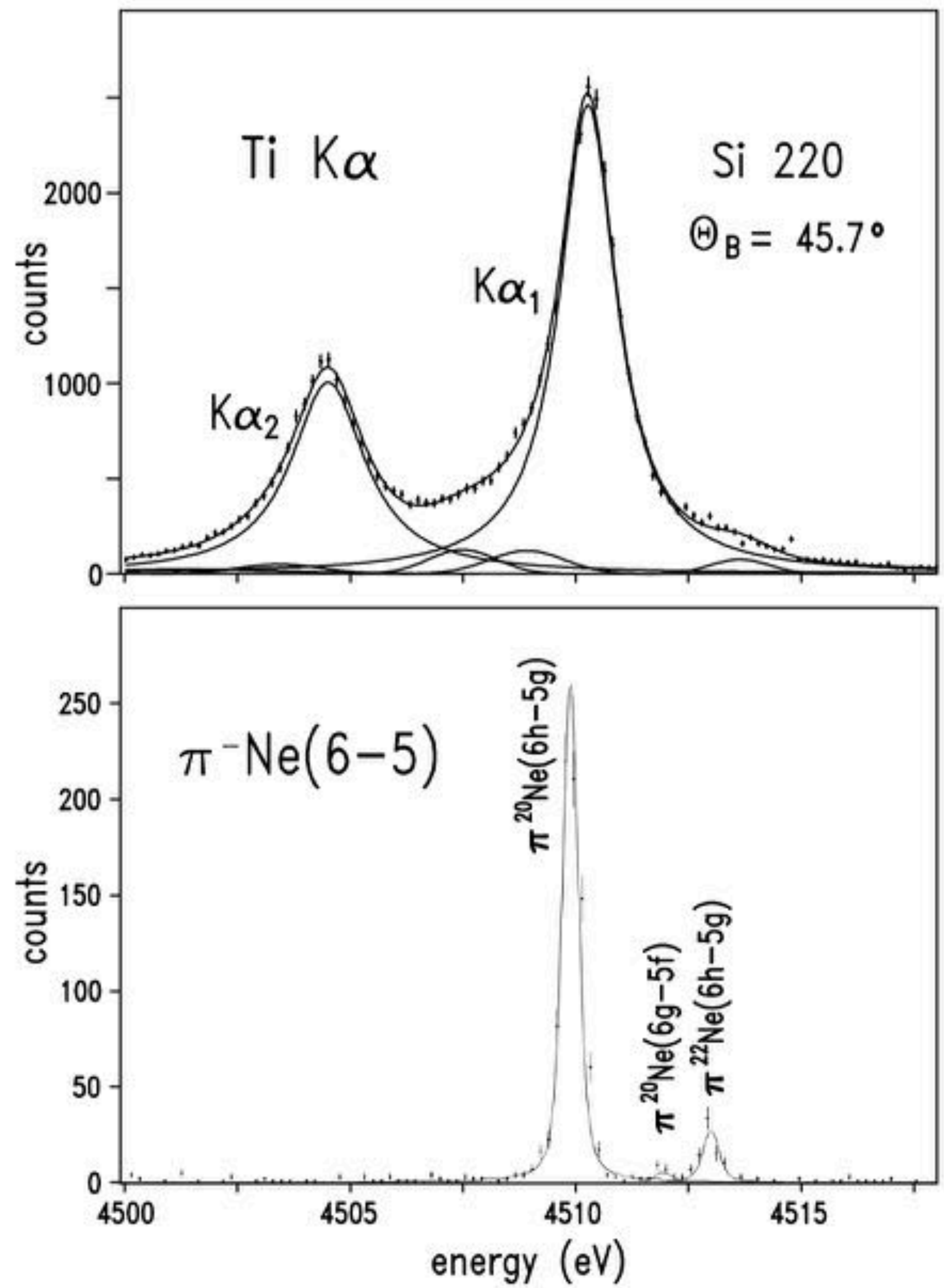
E. Borie and G. A. Rinker, Rev. Mod. Phys. **54**, 67 (1982).
E. Y. Korzinin, V. A. Shelyuto, V. G. Ivanov, R. Szafron, and S. G. Karshenboim, Phys. Rev. A 98, 062519 (2018).

How well do we understand theory of exotic atoms?

A few examples…

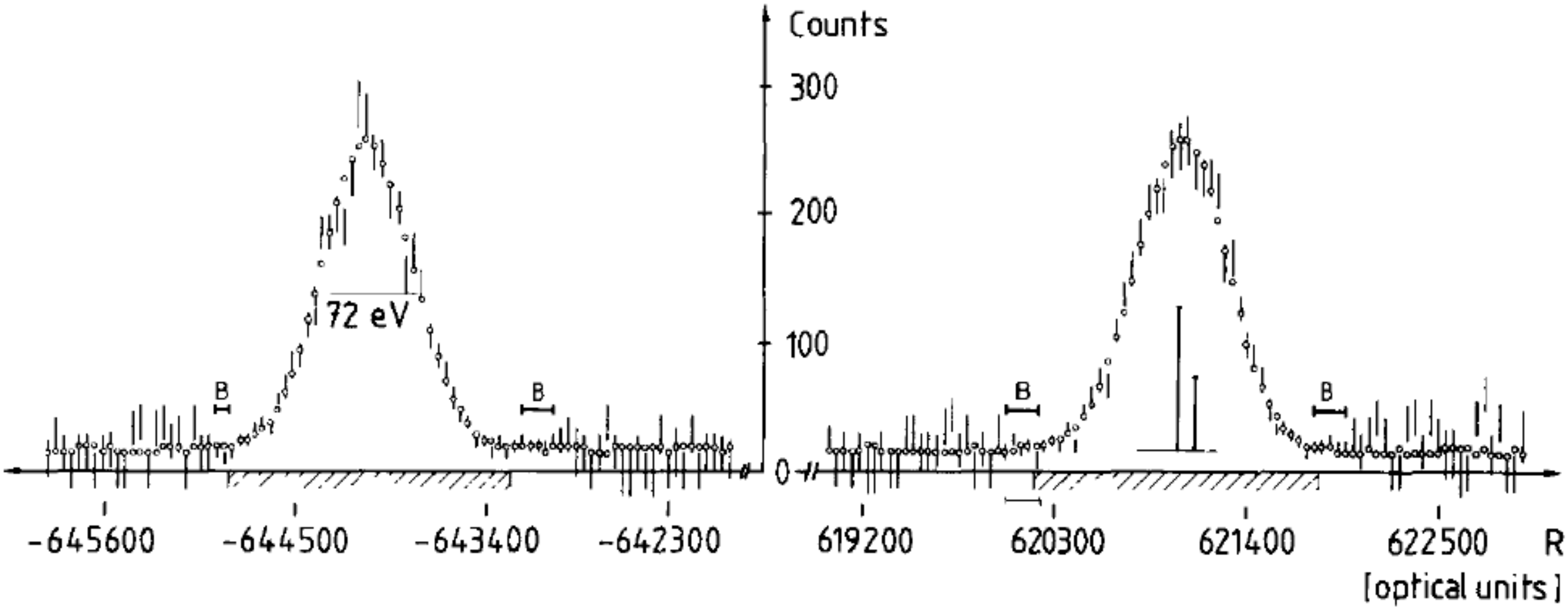
Pionic atoms precision experiment

Possible x-ray reference spectra
Crystal spectrometer used



D. F. Anagnostopoulos, D. Gotta, P. Indelicato, and L. M. Simons, Phys. Rev. Lett. **91**, 240801 (2003).

crystal spectroscopy of muonic Carbon (1984)



(a)

new all order calculations, recalibration of the reference used (^{170}Tm g-ray) for 2010 Si lattice spacing and wavelength to energy conversion

W. Ruckstuhl, B. Aas, W. Beer, et al., Nuclear Physics A **430**, 685 (1984).

Crystal spectrometry of muonic atoms

Z	A	Transition	Exp. Energy (eV)	Err. (eV)	Exp.-Theo.	1st order QED	Higher order QED	FNS	FNS/H.O. QED	1 e- shift	1e- shift / H.O. QED	year
6	12	2p3/2 → 1s1/2	75261.60	0.41	-3.38	368.51	3.73	-402.39	-10786%	-0.008	-0.2%	1982
12	24	3d5/2 → 2p3/2	56215.74	0.39	-0.41	176.83	1.64	-0.81	-50%	-0.358	-22%	1982
12	24	3d3/2 → 2p1/2	56391.93	0.85	0.38	179.03	1.66	-2.73	-164%	-0.358	-22%	1982
14	28	3d5/2 → 2p3/2	76617.7	2.7	0.5	271.95	2.55	-2.25	-88%	-0.438	-17%	1978
14	28	3d5/2 → 2p3/2	76616.6	1.0	-0.6	271.95	2.55	-2.25	-88%	-0.438	-17%	1982
14	28	3d3/2 → 2p1/2	76942.2	1.8	0.4	276.30	2.60	-7.27	-280%	-0.438	-17%	1982
14	28	4f7/2 → 3d5/2	26751.7	1.4	-1.1	44.69	0.40	0.02	3.8%	-1.027	-255%	1984
14	28	4f5/2 → 3d3/2	26779.31	0.81	-1.67	44.93	0.40	0.02	3.7%	-1.027	-254%	1984
15	31	3d5/2 → 2p3/2	88015.6	2.3	-0.1	328.77	3.11	-3.66	-118%	-0.479	-15%	1982
15	31	3d3/2 → 2p1/2	88424.7	7.9	-18.2	334.66	3.18	-11.57	-364%	-0.479	-15%	1982

Experiments performed at the Paul Scherrer Institute with solid targets and transmission crystal spectrometer to measure nuclear radii

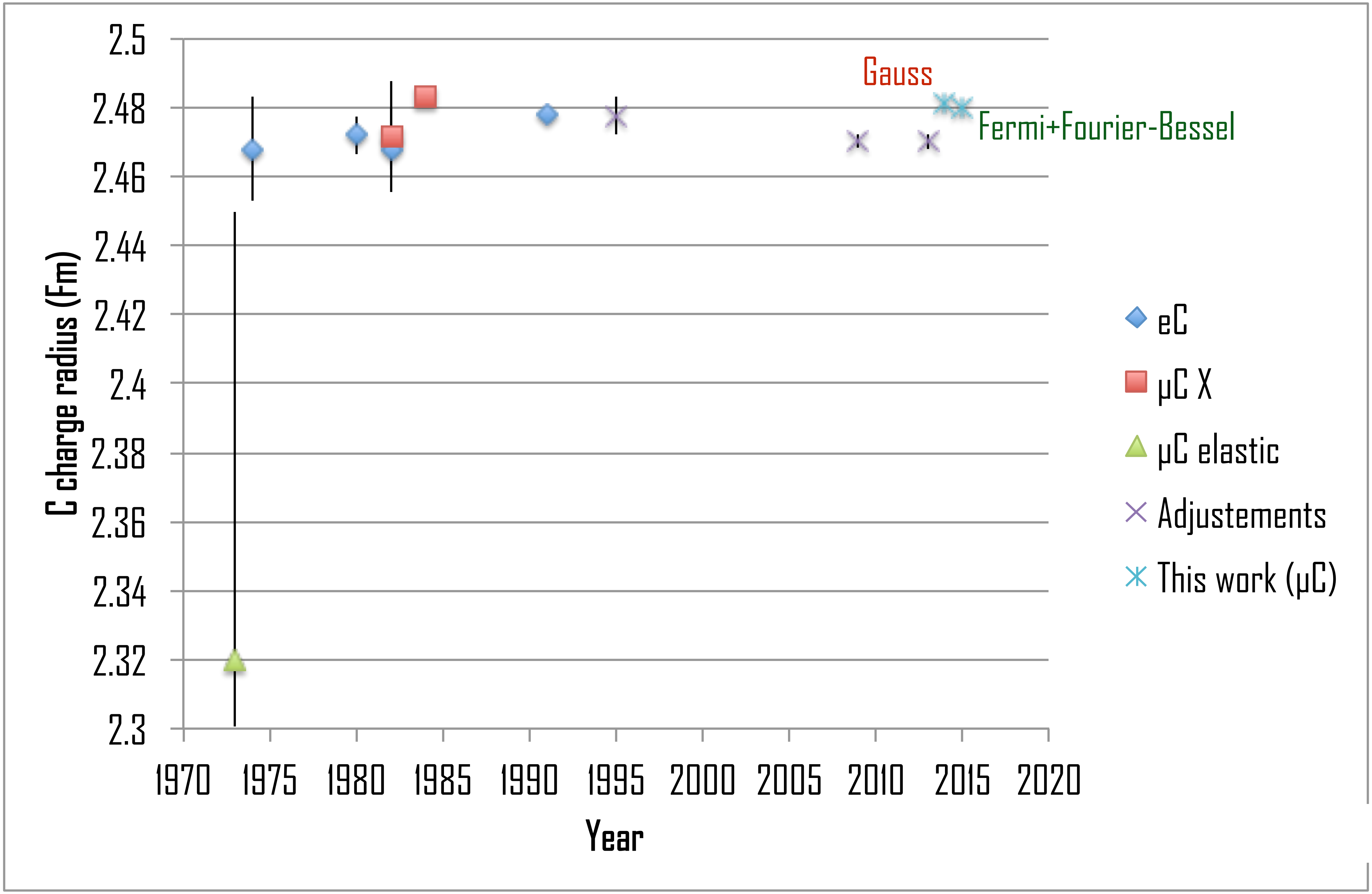
– require intense beams of muons

– strong electron recapture during the cascade (solid), giving shifts

– lower n=1, 2, 3, 4 levels used as the aim was to measure precisely nuclear size

The shift due to uncertainty in the number of electron recaptures in pionic magnesium lead to the error in the pion mass determination

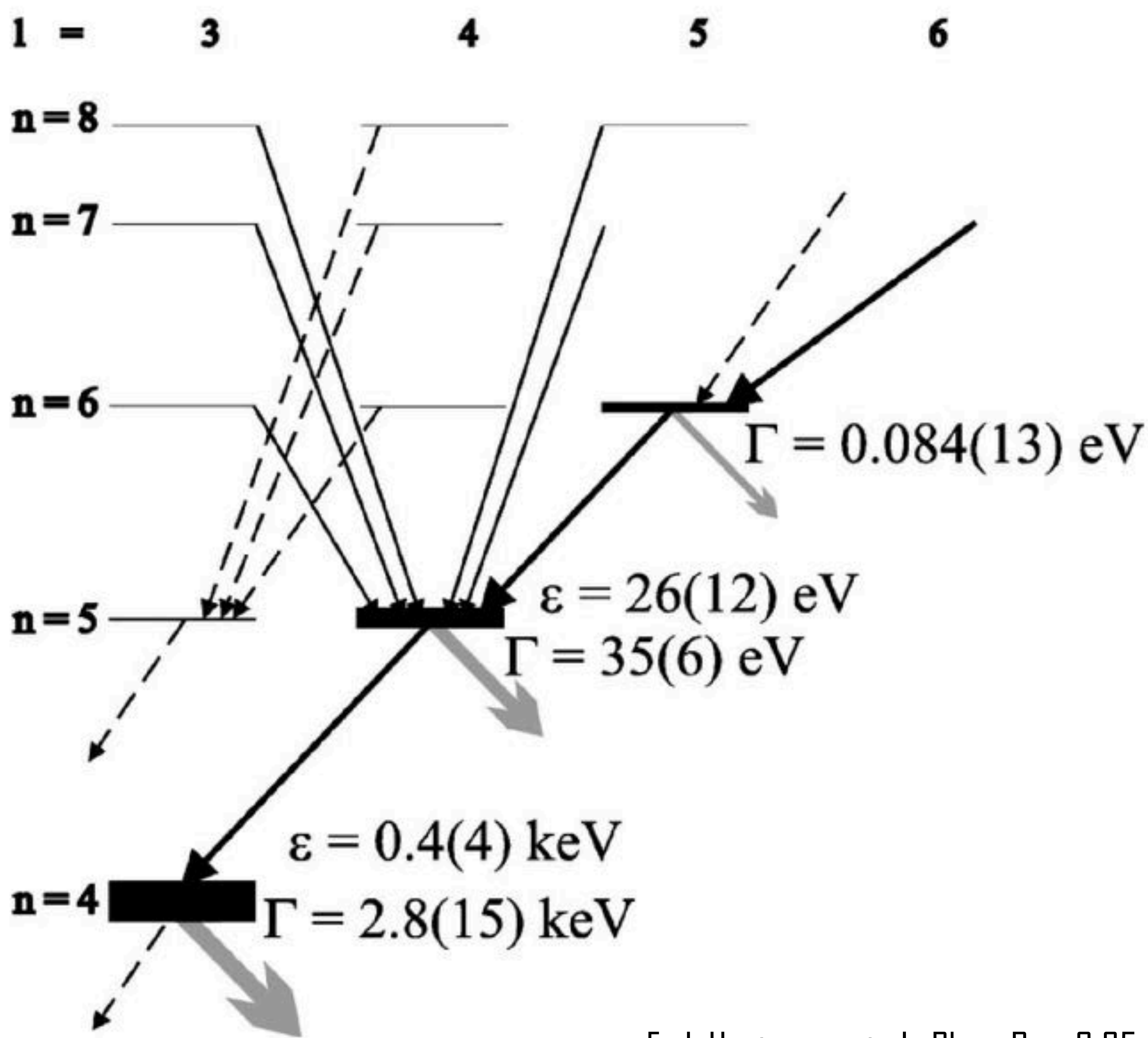
^{12}C charge radius 1974-2015



Strong interaction effects

Measuring strong interaction at low energy

Strong shifts in antiprotonic Ca

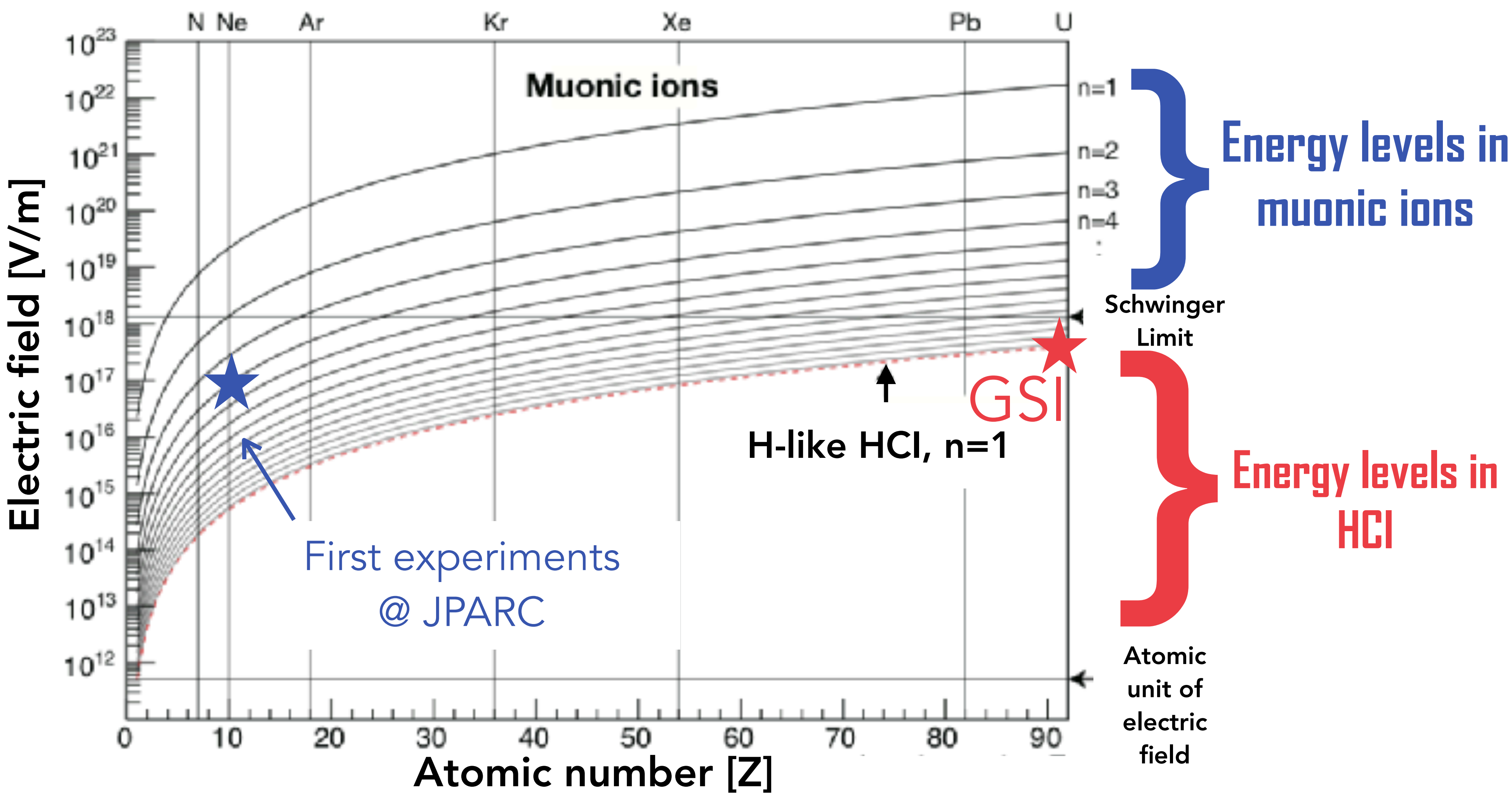


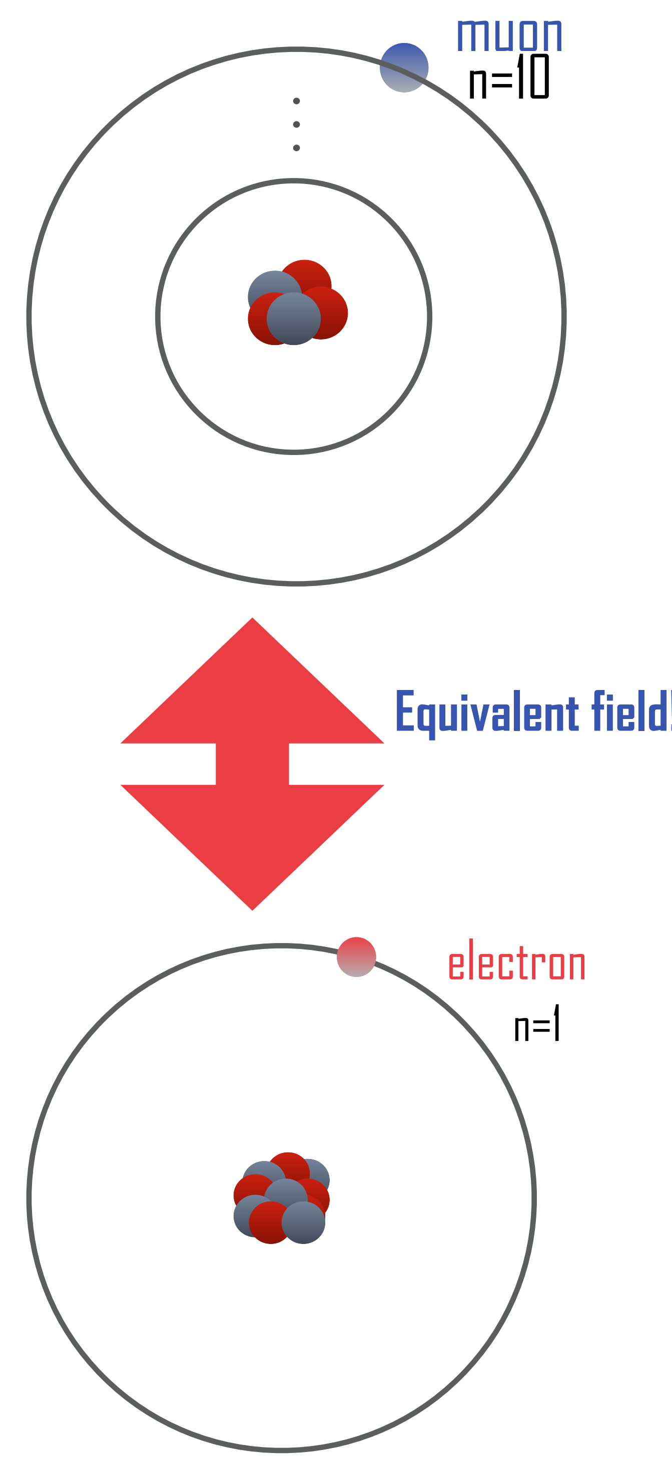
F. J. Hartmann et al., Phys. Rev. C 65, 014306 (2001).

Exotic atoms and QED

Nuclear effects and QED contributions

Strong-field QED with muonic atoms

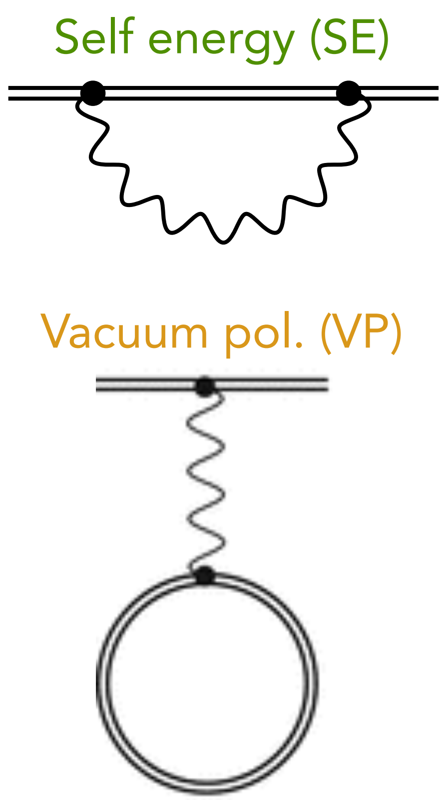


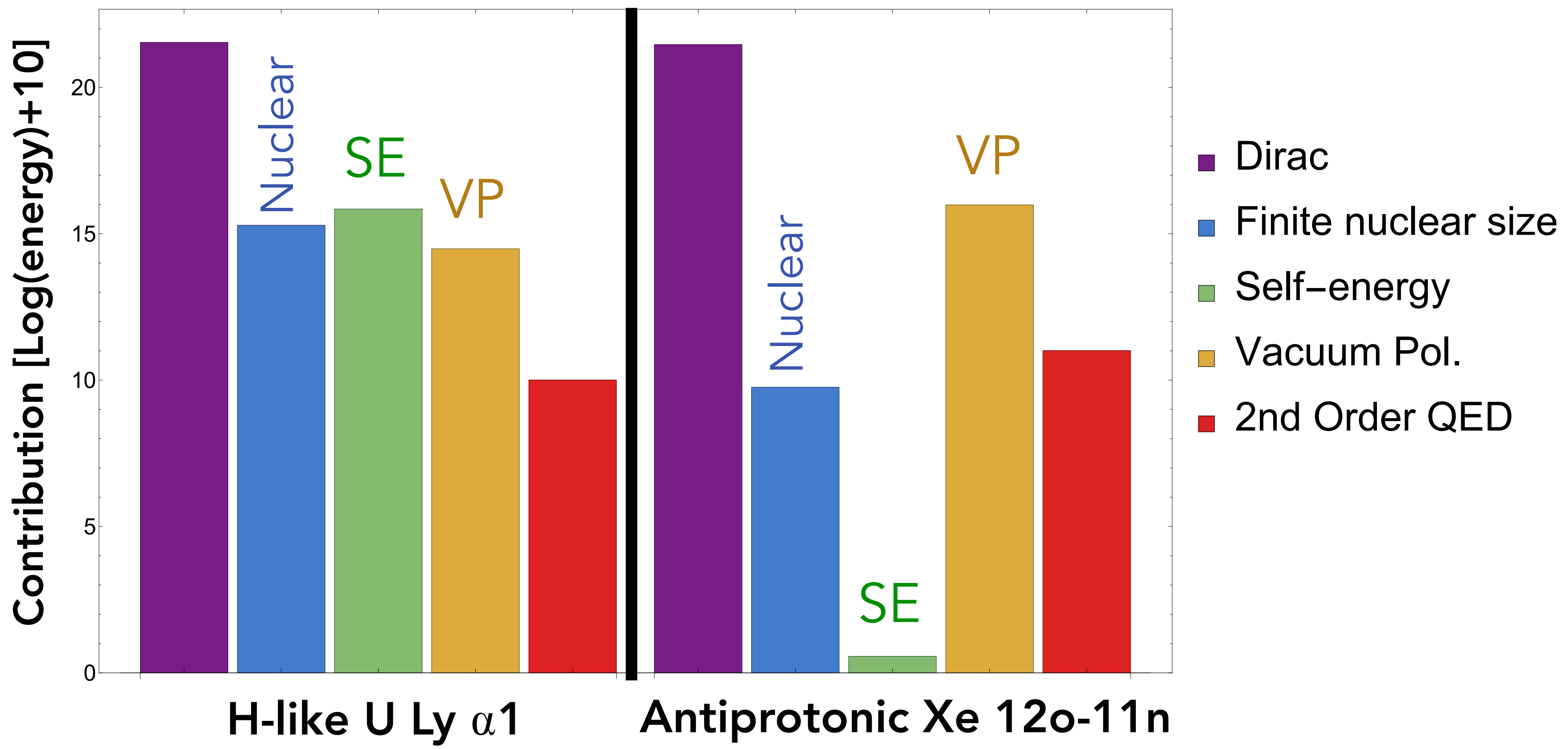


Exotic atoms:
(strong field) x (Rydberg states)
Vanishing nuclear uncertainties!

N. Paul et al, « Testing Quantum Electrodynamics with Exotic Atoms » PRL 126, 173001 (2021)

HCI and Exotic Atoms: a complementary pair





Highly charged ion: SE>VP
Exotic atom: VP>SE

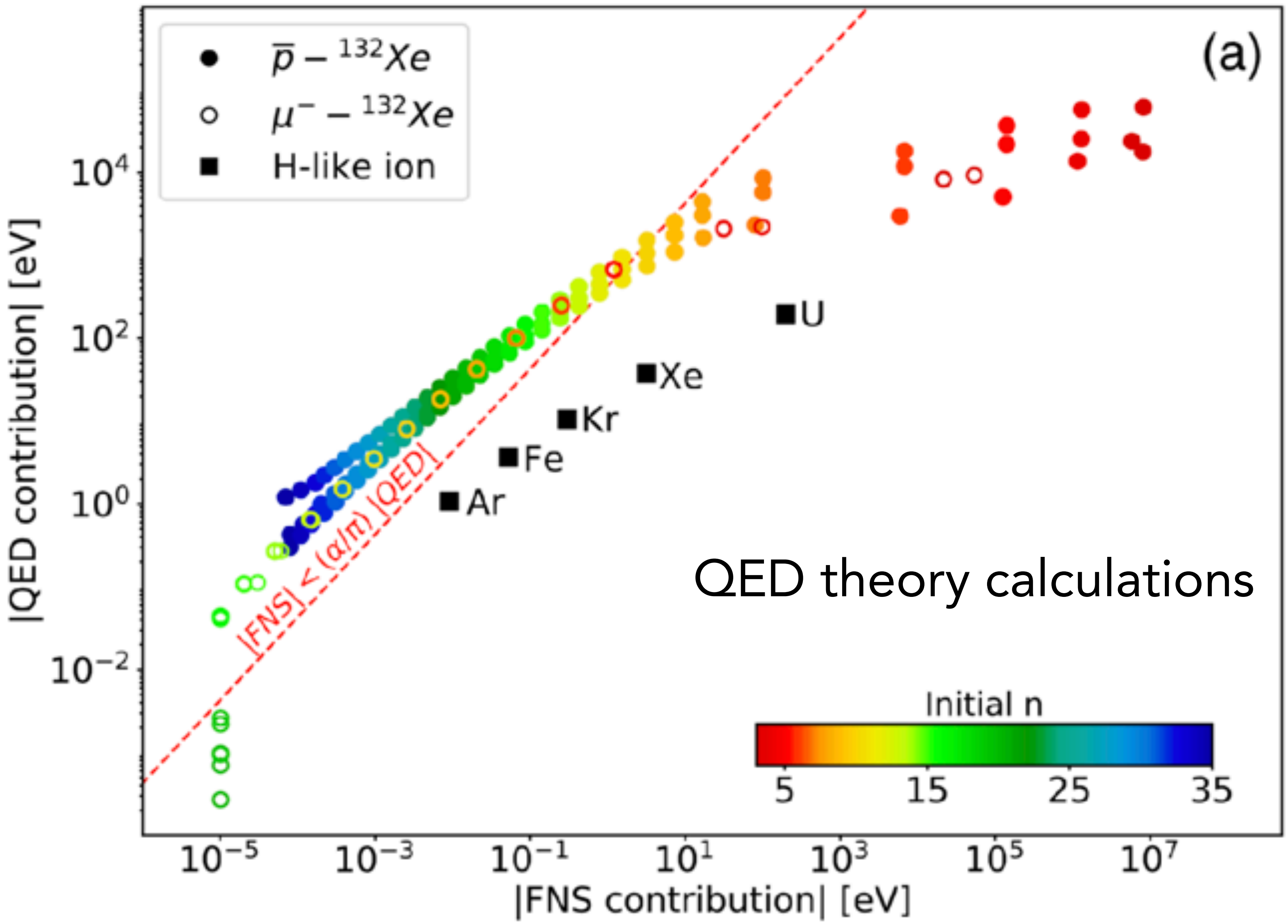
Self-energy is dominant in HCI, vacuum polarization is dominant in exotic atoms

Unique probe of vacuum polarization, *« one of the most interesting phenomena predicted by contemporary quantum electrodynamics »* (Foldy and Eriksen, Physical Review (1954))

Complementary to vacuum studies with high-intensity lasers

For antiprotonic, pionic and kaonic atoms, the finite particle size is also included

QED vs Finite Nuclear Size

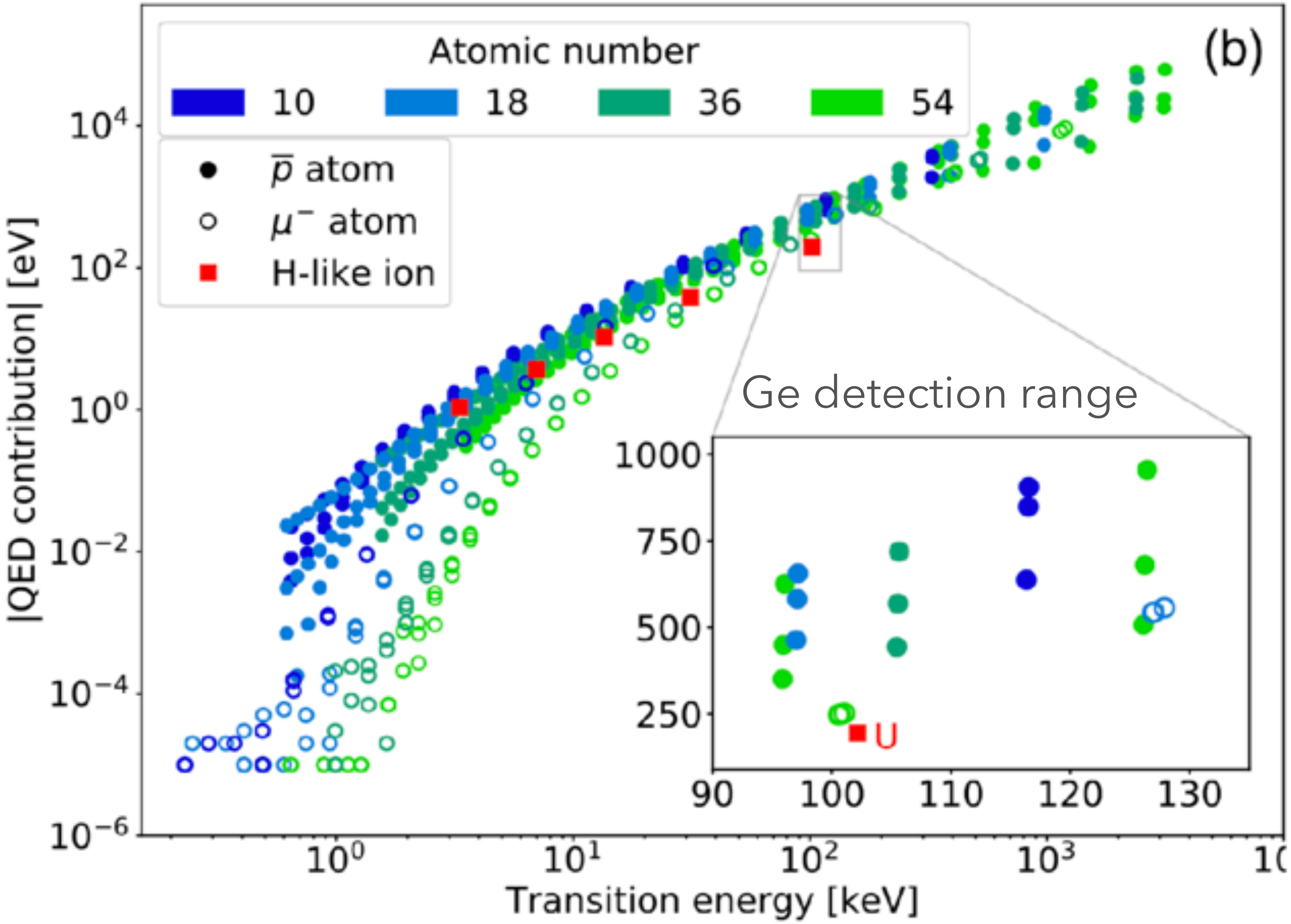


Precision QED calculations performed for radiative cascade in muonic atoms from H → U

Multiconfiguration Dirac Fock code (MCDFGME, P. Indelicato, J.P. Desclaux)

One order of magnitude gain in sensitivity compared to normal H-like ions, by avoiding nuclear physics uncertainties

QED vs Transition Energy



$\bar{p}$

Larger QED contributions to transition energies, accessible for lower-Z ions

Factor of 2-10 gain in QED enhancement for a given detection range

QED effects in exotic atoms are larger for a given Z, thus easier to measure

Comparison with μNe

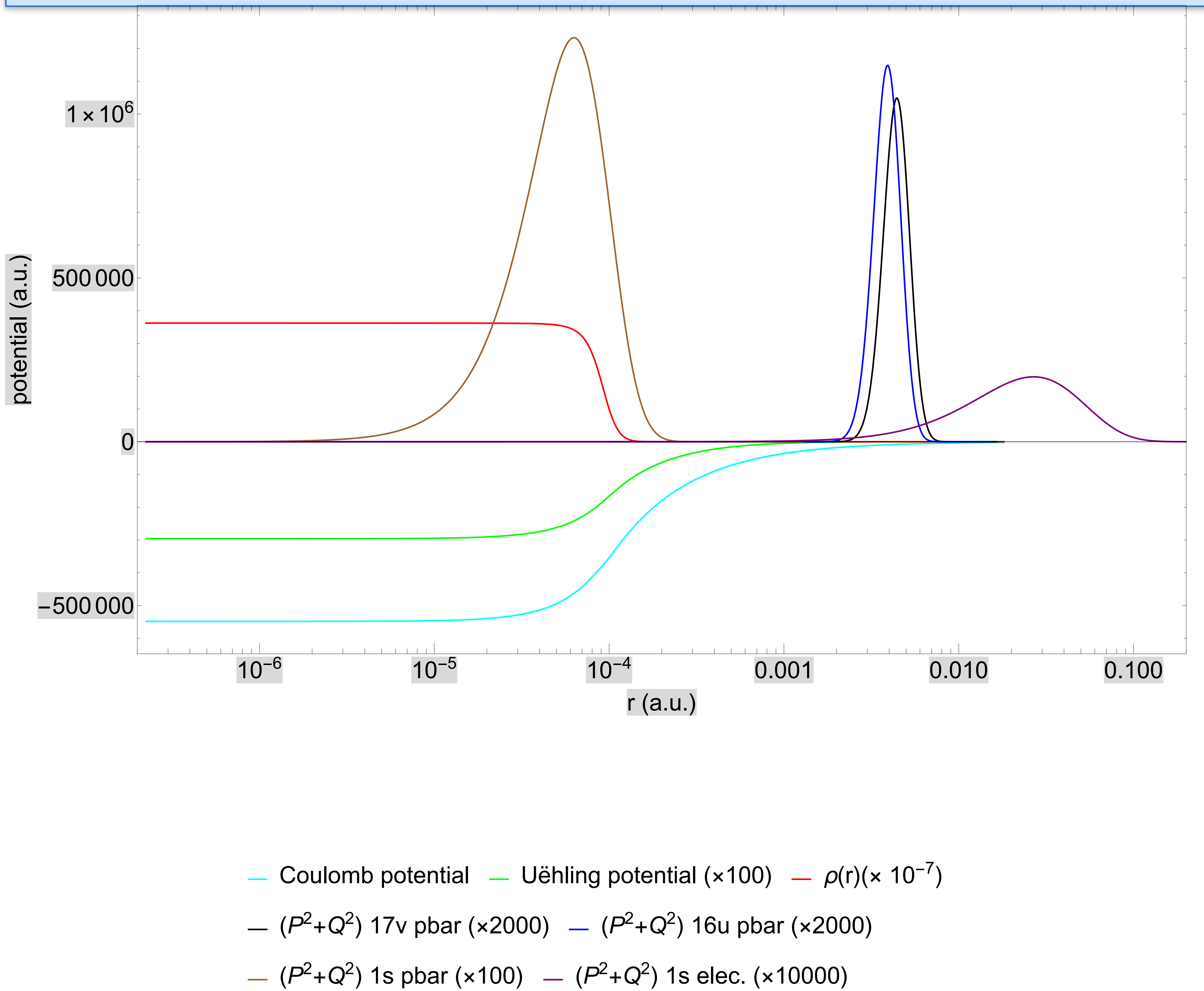
Part.	Z	A	Transition	Theo. Ener. (eV)	1st Order QED	2nd Order QED	p g-2	FNS	FNS / QED	FNS / 2nd order QED	Exp. (eV)	Exp. Err.	Exp. Err. / QED
e-	26	56	2p3/2 → 1s1/2	6973.182	-3.8873	0.0042		-0.0527	1.4%	1255.5%	6972.73	0.24	6.2%
μ-	10	20	5g9/2 → 4f7/2	6297.262	2.3365	0.0229		0.0003	0.01%	1.35%	**6297.275**	**0.079**	**3.4%**
$\bar{p}$	18	40	14r27/2 → 13q25/2	6441.422	5.3566	0.0499	0.2086	0.0010	0.02%	2.06%			
e-	79	197	2p3/2 → 1s1/2	71570.425	-152.0906	0.5816		-49.1287	32.3%	8447%	71573.1	7.9	5%
μ-	74	184	8k15/2 → 7i13/2	73957.183	121.4412	1.2188		0.1005	0.08%	8.2%			
$\bar{p}$	54	132	13q25/2 → 12o23/2	74610.640	274.3166	2.5405	28.1302	0.4259	0.16%	16.8%			
e-	92	238	2p3/2 → 1s1/2	102175.099	-257.2281	1.2278		-198.5110	77.17%	16168%	102178.1	4.3	2%
μ-	54	132	6h11/2 → 5g9/2	100690.898	246.5741	2.4873		0.2473	0.10%	9.9%			
$\bar{p}$	54	132	12o23/2 → 11n21/2	95937.887	398.9032	3.7678	46.7550	0.7860	0.20%	20.9%			
μ-	54	132	5g9/2 → 4f7/2	185826.819	664.5075	7.4047		1.1906	0.18%	16.1%			
μ-	74	184	6h11/2 → 5g9/2	189726.053	612.8893	6.8712		1.1510	0.19%	16.8%			
μ-	92	238	7i13/2 → 6h11/2	176860.622	499.8768	5.6046		0.9234	0.18%	16.5%			

J-PARC

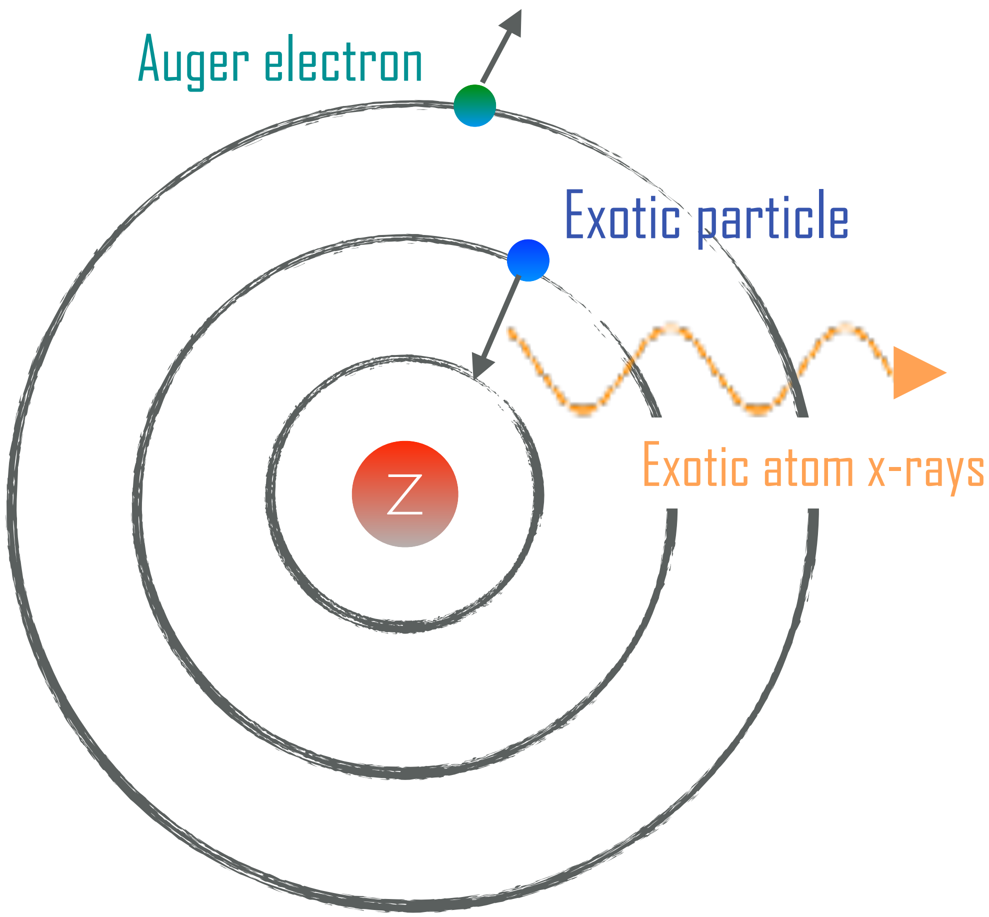
Theory-Exp.: 0.013±0.079 eV

T. Okumura, T. Azuma, D.A. Bennett, P. Caradonna, I. Chiu, W.B. Doriese, M.S. Durkin, J.W. Fowler, J.D. Gard, T. Hashimoto, R. Hayakawa, G.C. Hilton, Y. Ichinohe, P. Indelicato, T. Isobe, S. Kanda, M. Katsuragawa, N. Kawamura, Y. Kino, K. Mine, Y. Miyake, K.M. Morgan, K. Ninomiya, H. Noda, G.C. O'Neil, S. Okada, K. Okutsu, N. Paul, C.D. Reintsema, D.R. Schmidt, K. Shimomura, P. Strasser, H. Suda, D.S. Swetz, T. Takahashi, S. Takeda, S. Takeshita, M. Tampo, H. Tatsuno, Y. Ueno, J.N. Ullom, S. Watanabe, S. Yamada, **Testing Quantum Electrodynamics: High Precision X-ray Spectroscopy of Muonic Neon Atoms**, submitted to PRL, (2021).

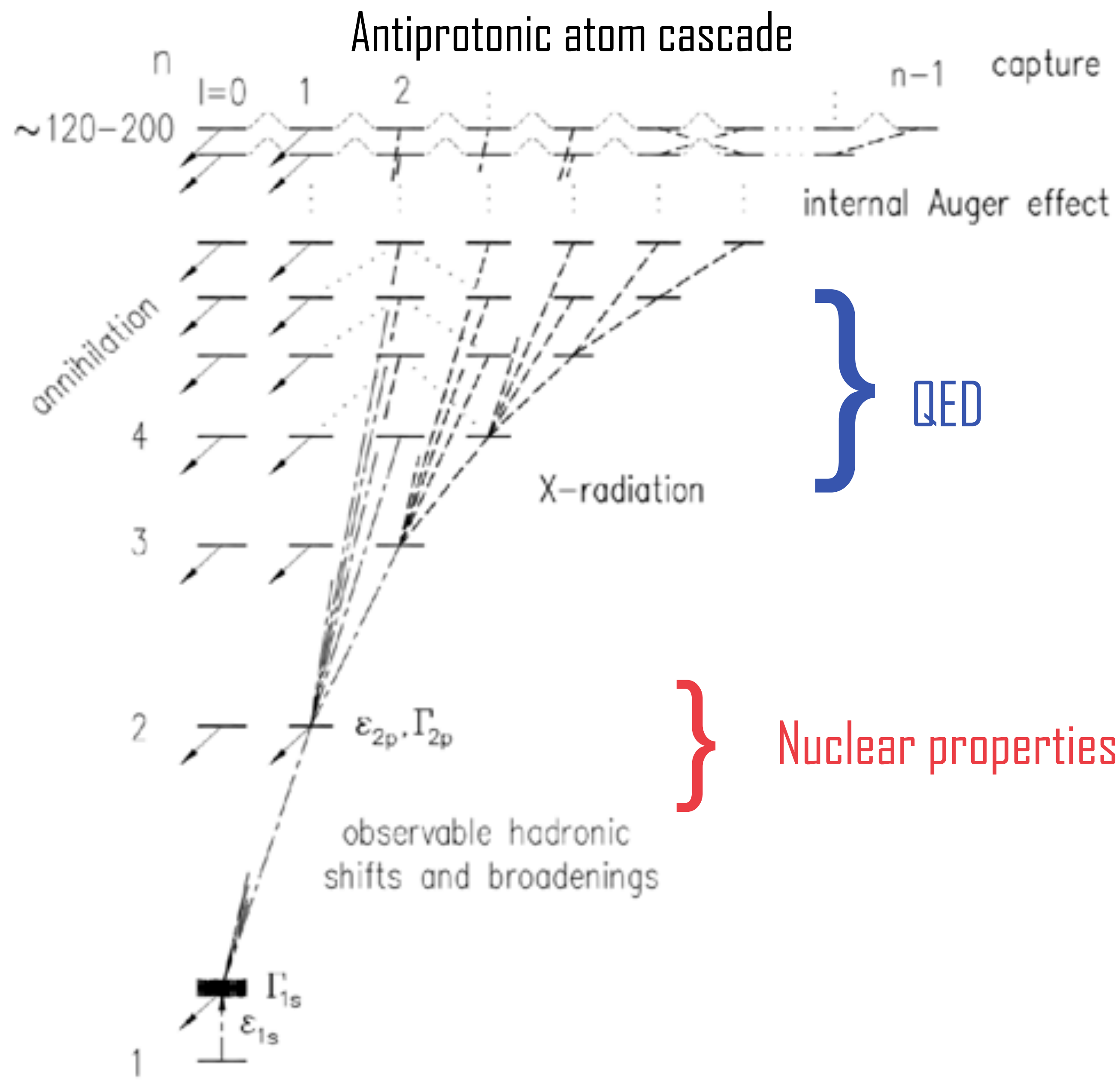
Wavefunctions and potentials



The exotic atom cascade

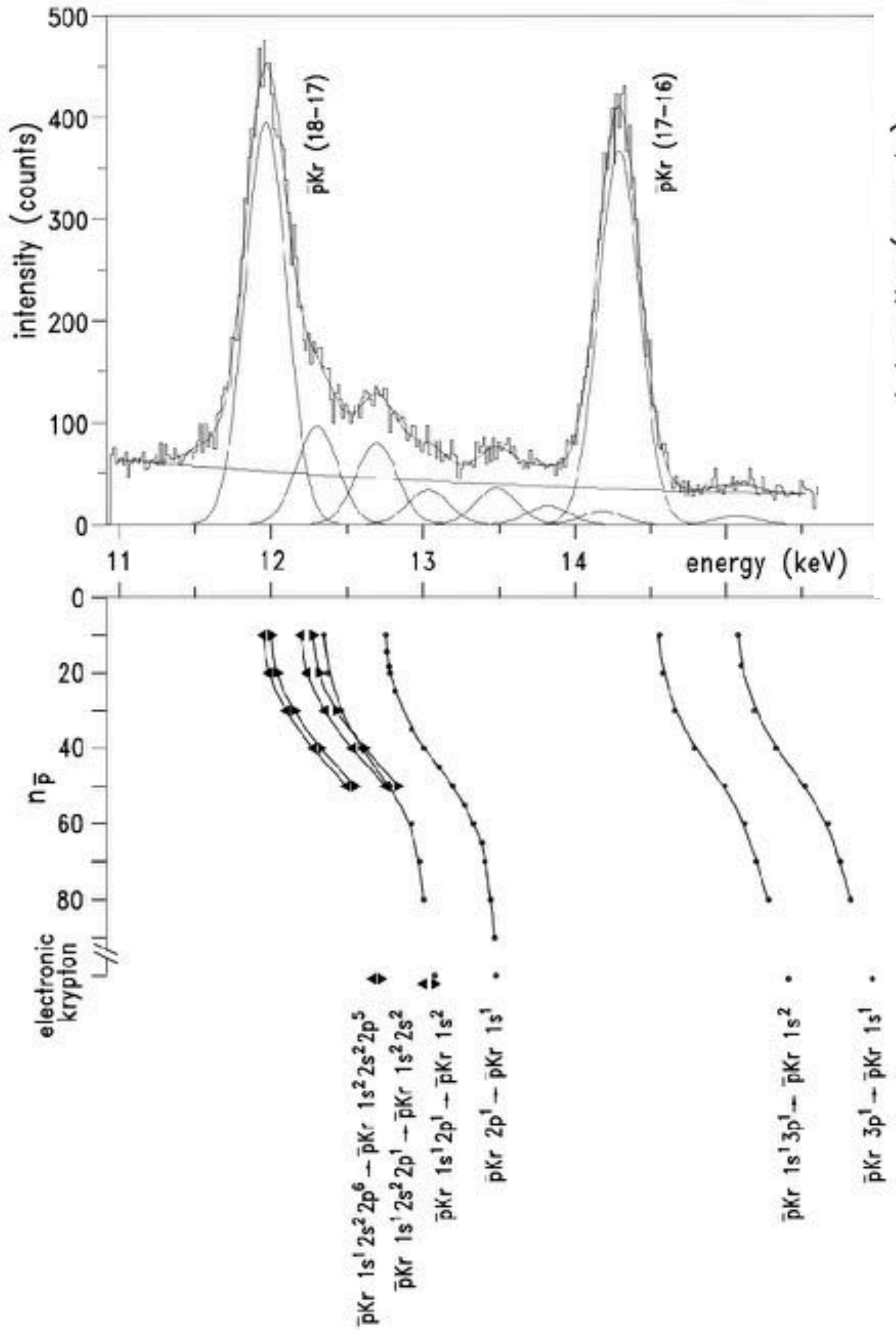


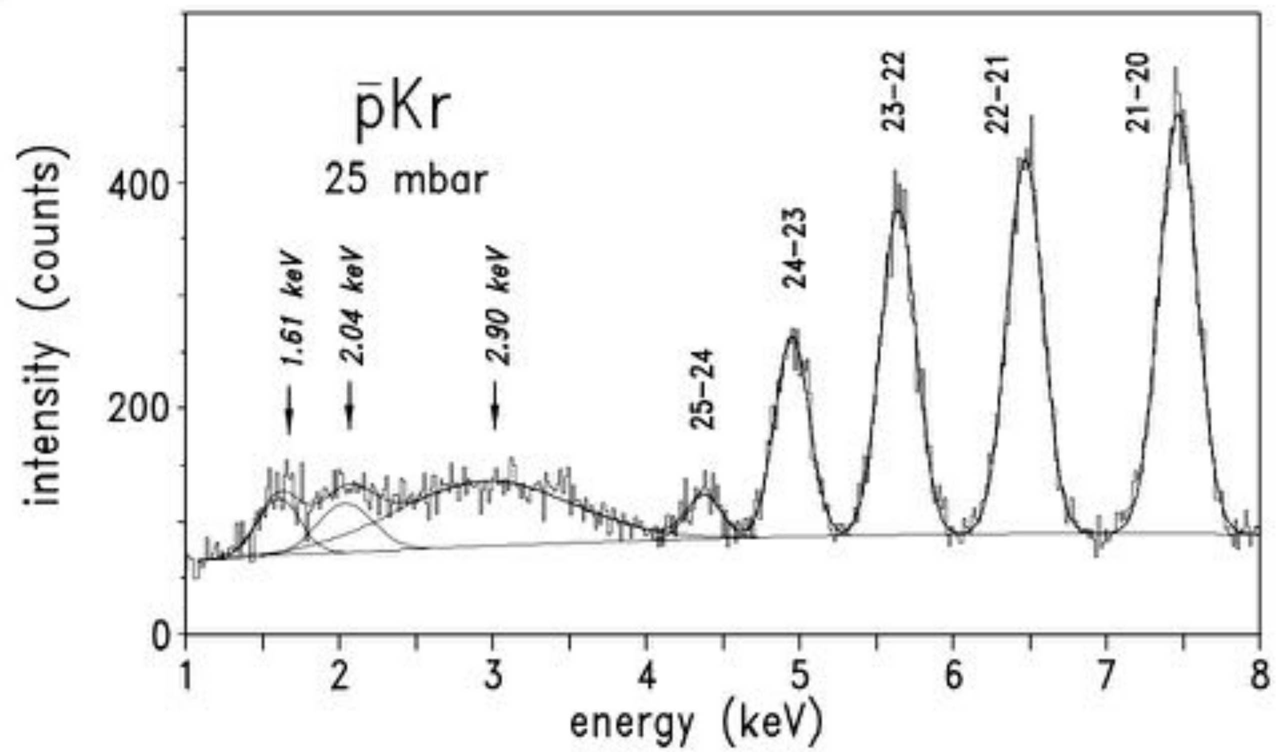
$$n_{exotic} \approx n_{e^-}\sqrt{m_{exotic}/m_{e^-}}$$



- The exotic particle captures onto high-n orbitals
- Decays via Auger electrons and radiatively (X-rays)
- Eventually is either captured by nucleus (muons), or annihilates (antiprotons)

Determining the number of remaining electrons



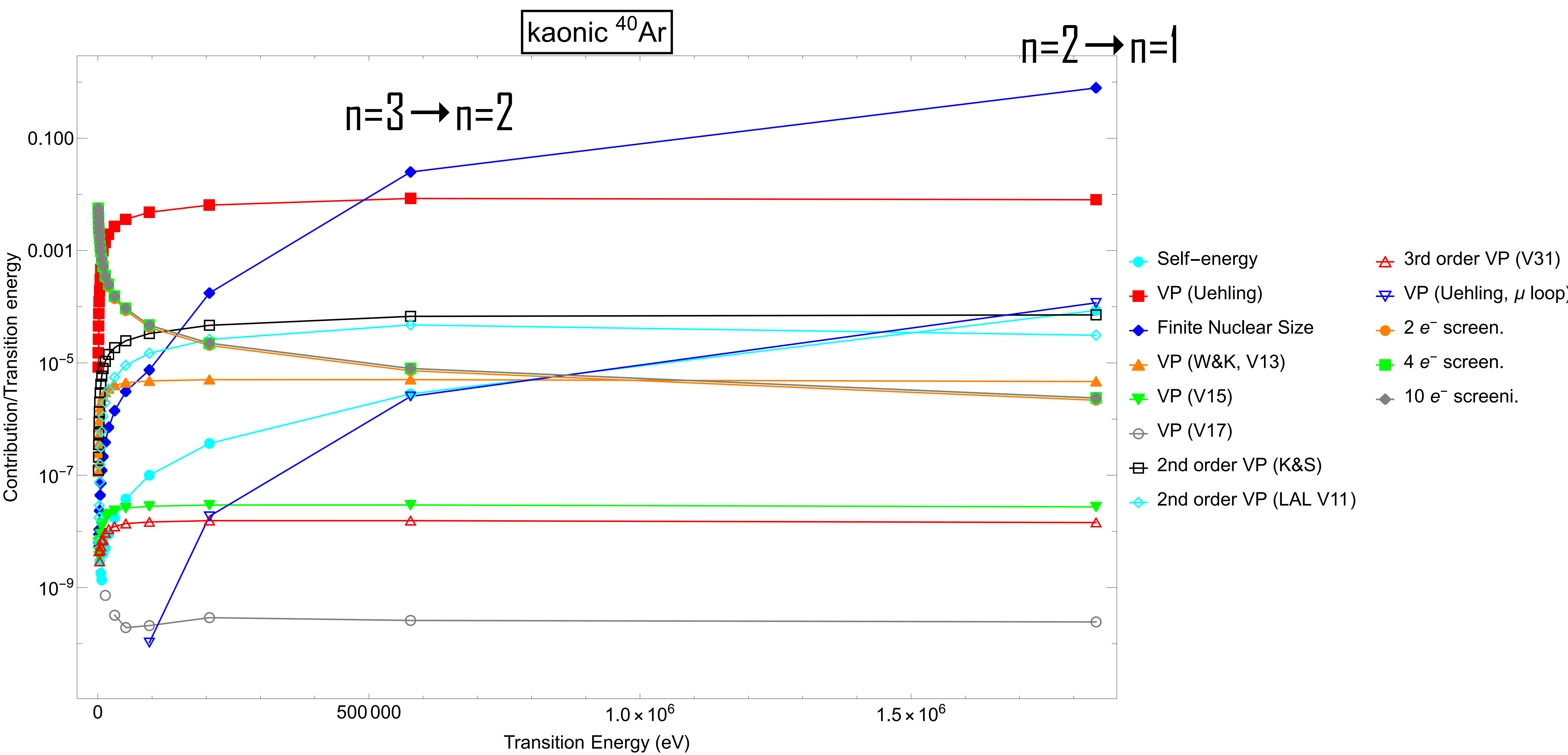


D. Gotta, K. Rashid, B. Fricke, P. Indelicato, and L. M. Simons, Eur. Phys. J D 47, 11 (2008).

Kaonic atoms

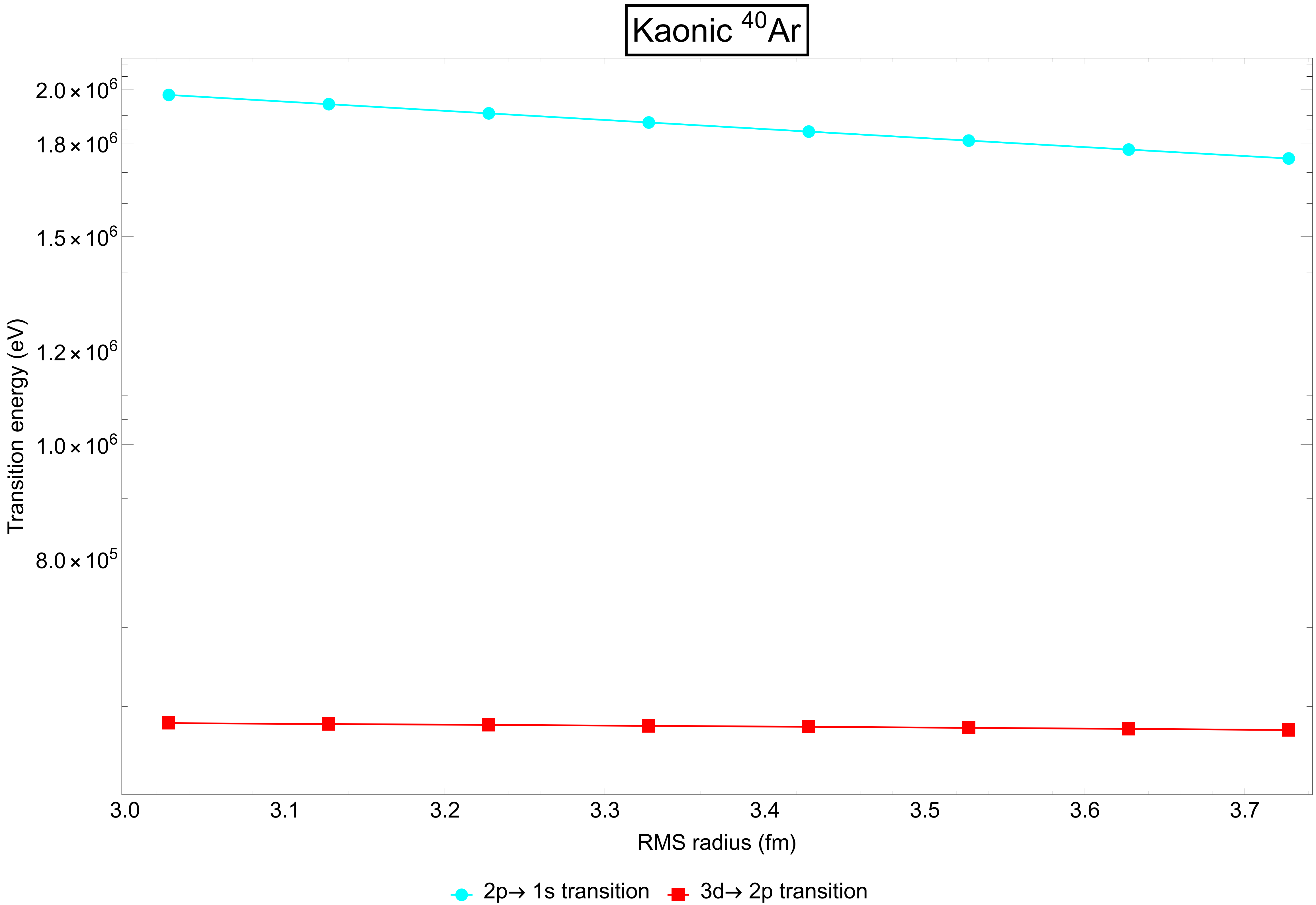
Size of the different effects and properties

Kaonic ^{40}Ar



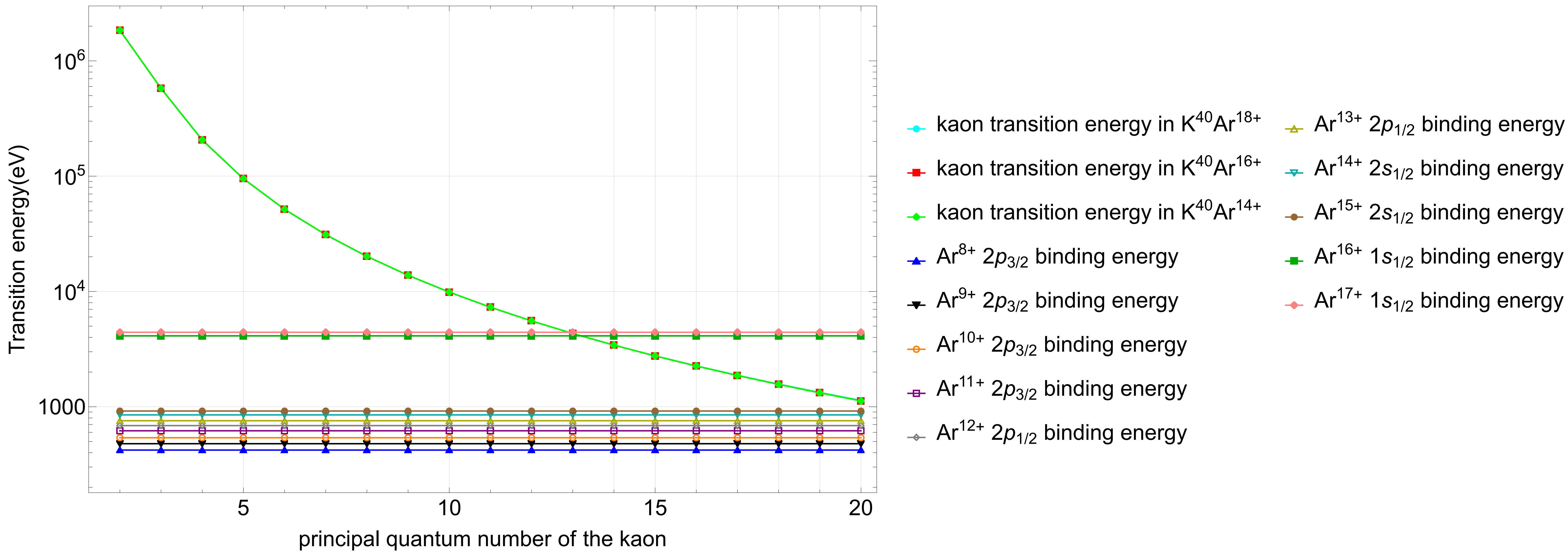
kaonic 40Ar
n=2 → n=1
n=3 → n=2
0.100
0.001
10−5
10−7
10−9
Contribution/Transition energy
0
500 000
1.0 × 10^6
1.5 × 10^6
Transition Energy (eV)
Self−energy
VP (Uehling)
Finite Nuclear Size
VP (W&K, V13)
VP (V15)
VP (V17)
2nd order VP (K&S)
2nd order VP (LAL V11)
3rd order VP (V31)
VP (Uehling, μ loop)
2 e− screen.
4 e− screen.
10 e− screeni.

Kaonic ^{40}Ar



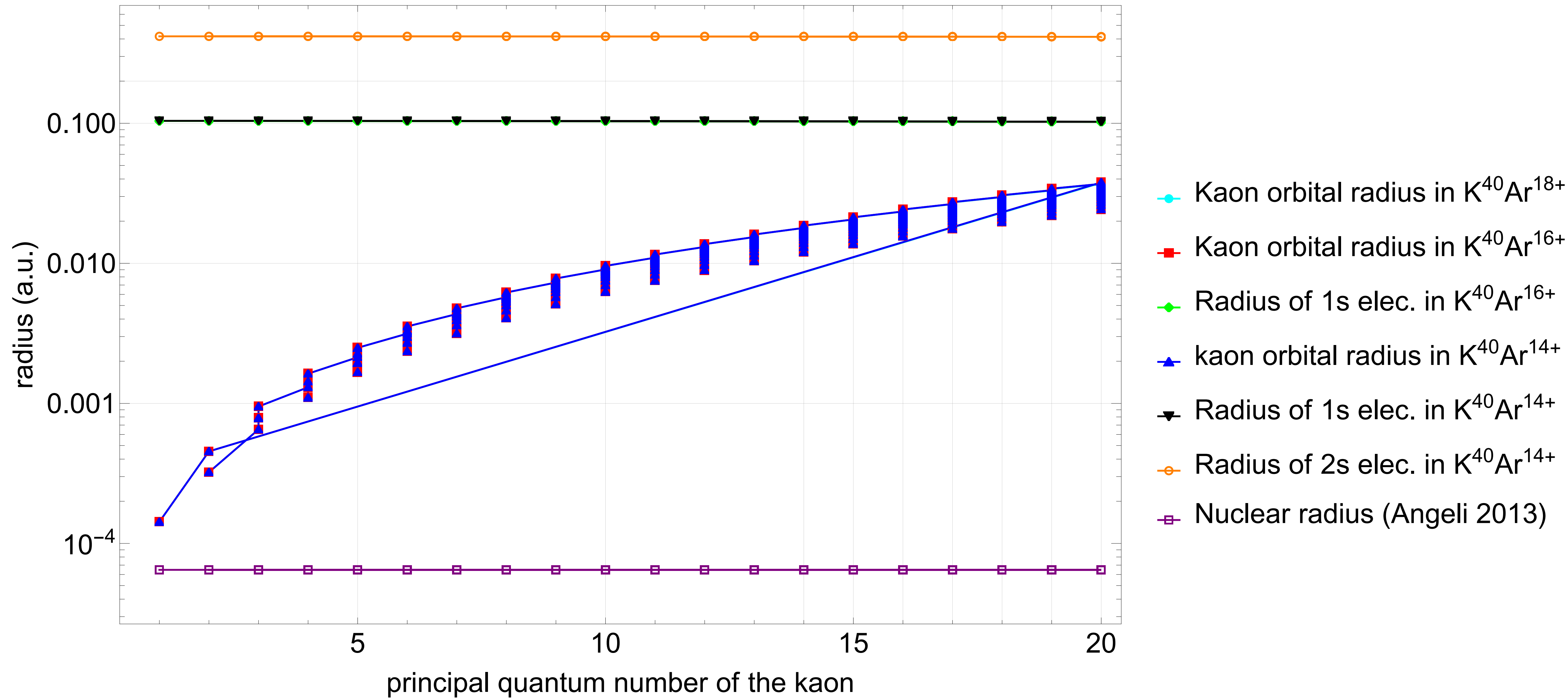
Kaonic ^{40}Ar
Transition energy (eV)
2.0 × 10^6
1.8 × 10^6
1.5 × 10^6
1.2 × 10^6
1.0 × 10^6
8.0 × 10^5
3.0
3.1
3.2
3.3
3.4
3.5
3.6
3.7
RMS radius (fm)
2p→ 1s transition
3d→ 2p transition

Kaonic ^{40}Ar



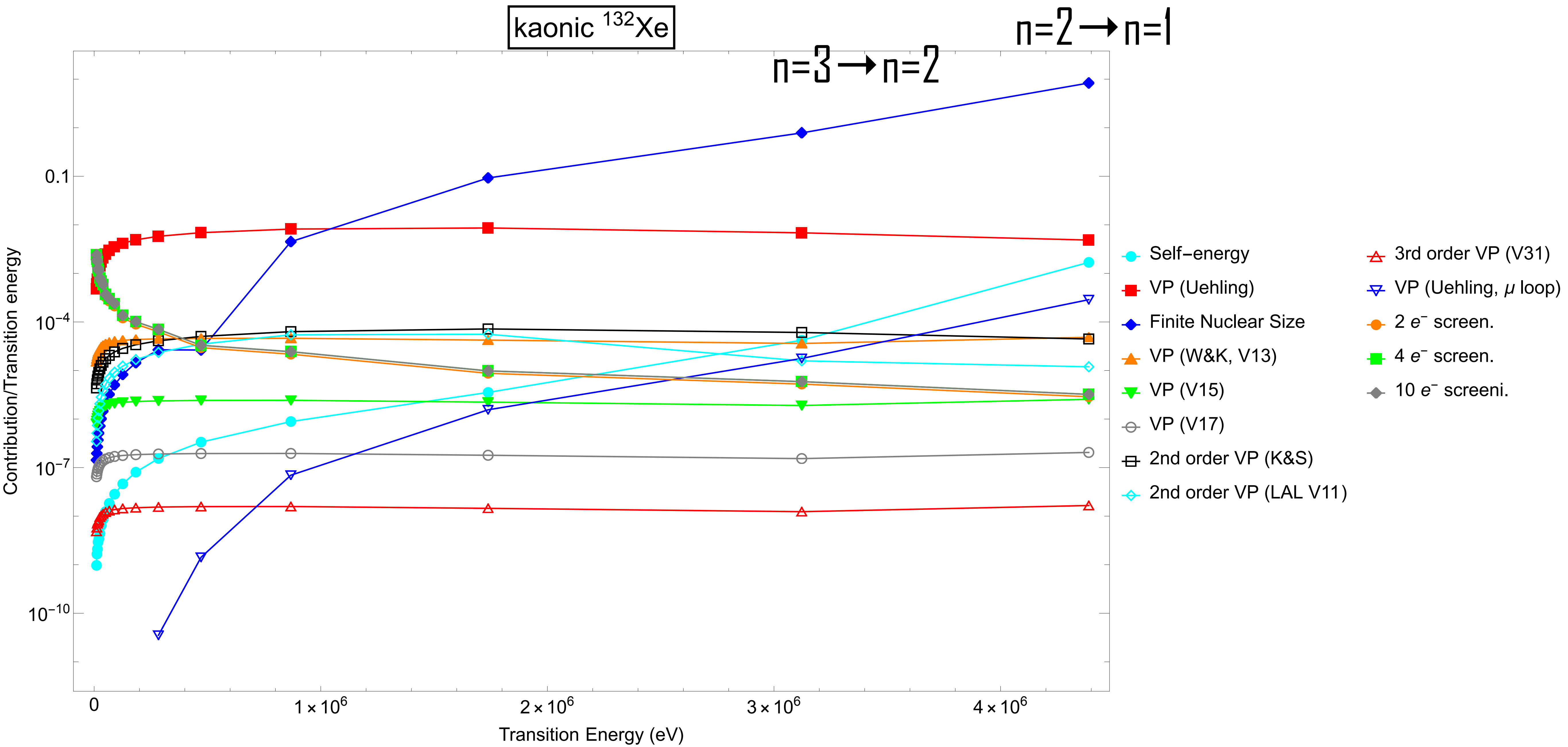
Transition energy(eV)
10^6
10^5
10^4
1000
5
10
15
20
principal quantum number of the kaon
kaon transition energy in K^40Ar^18+
kaon transition energy in K^40Ar^16+
kaon transition energy in K^40Ar^14+
Ar^8+ 2p3/2 binding energy
Ar^9+ 2p3/2 binding energy
Ar^10+ 2p3/2 binding energy
Ar^11+ 2p3/2 binding energy
Ar^12+ 2p1/2 binding energy
Ar^13+ 2p1/2 binding energy
Ar^14+ 2s1/2 binding energy
Ar^15+ 2s1/2 binding energy
Ar^16+ 1s1/2 binding energy
Ar^17+ 1s1/2 binding energy

Kaonic ^{40}Ar

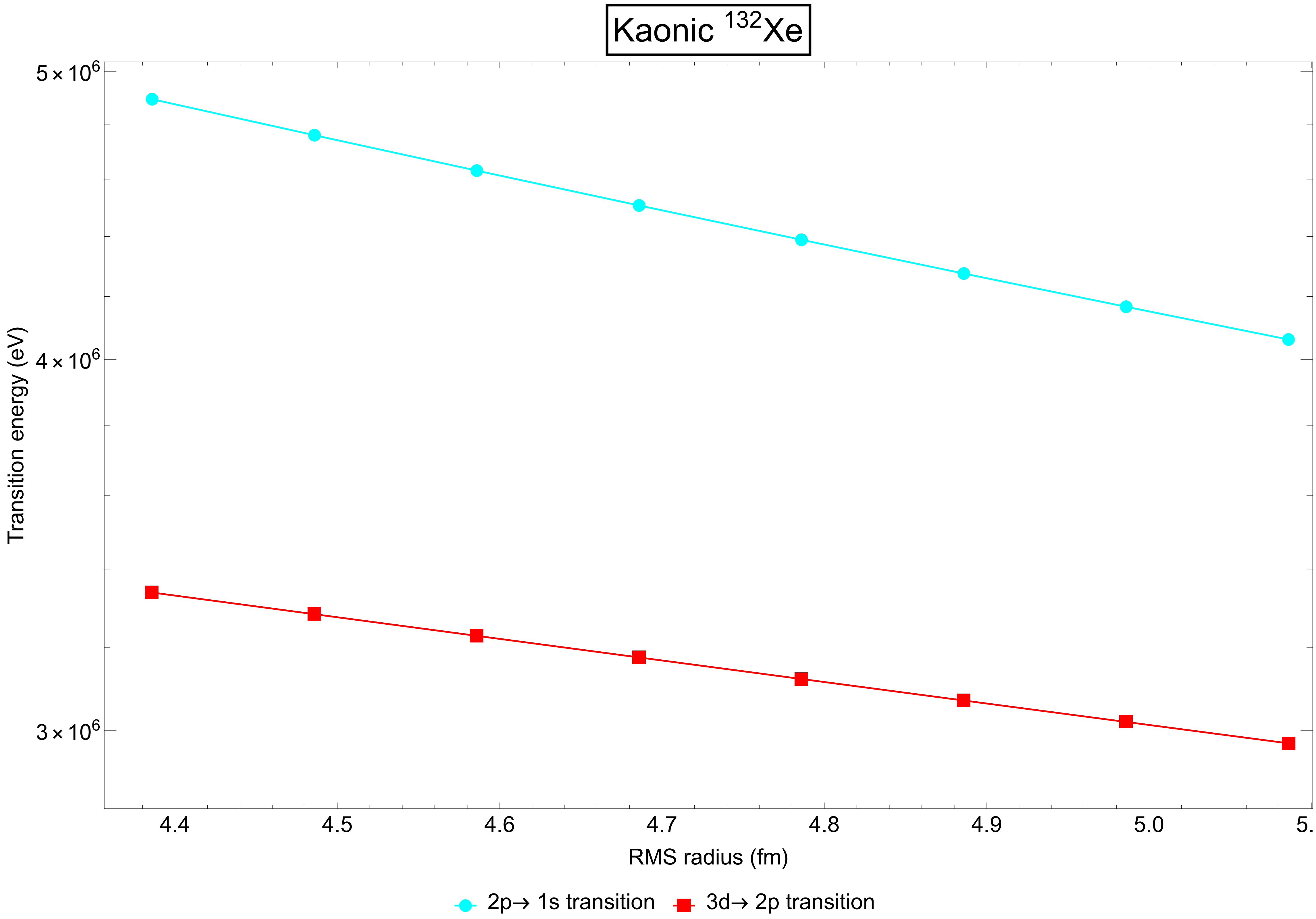


0.100
0.010
0.001
10^-4
radius (a.u.)
5
10
15
20
principal quantum number of the kaon
Kaon orbital radius in K40Ar18+
Kaon orbital radius in K40Ar16+
Radius of 1s elec. in K40Ar16+
kaon orbital radius in K40Ar14+
Radius of 1s elec. in K40Ar14+
Radius of 2s elec. in K40Ar14+
Nuclear radius (Angeli 2013)

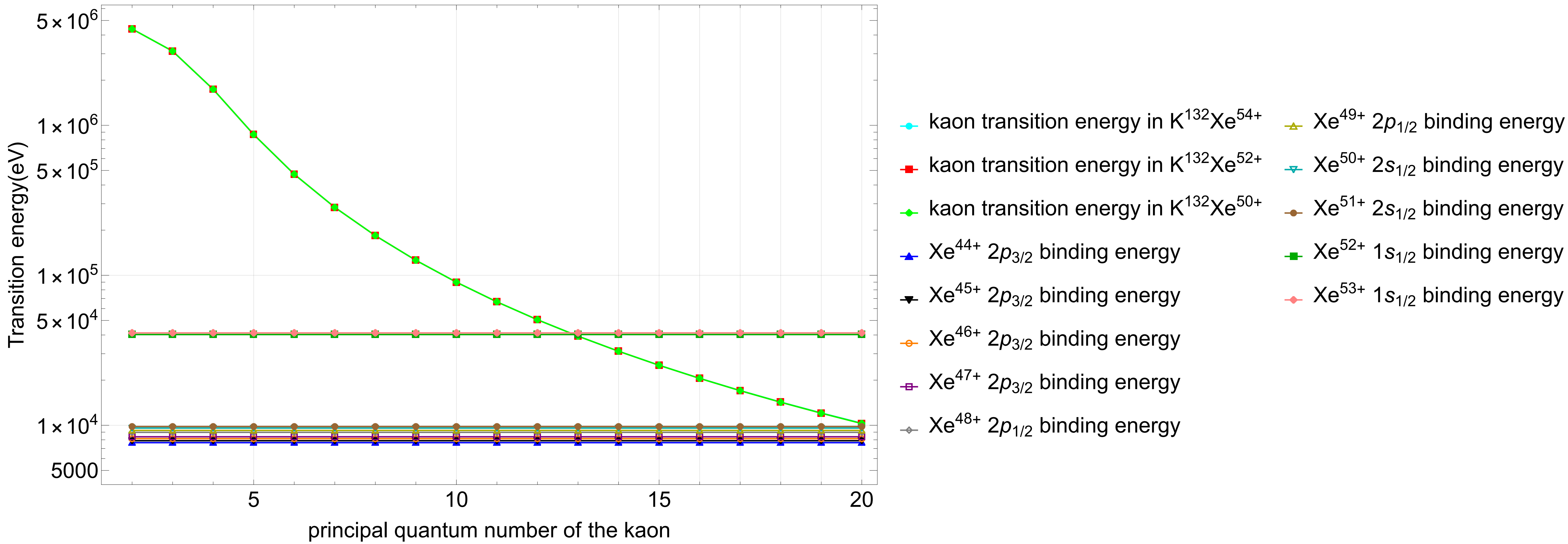
Kaonic ^{132}Xe



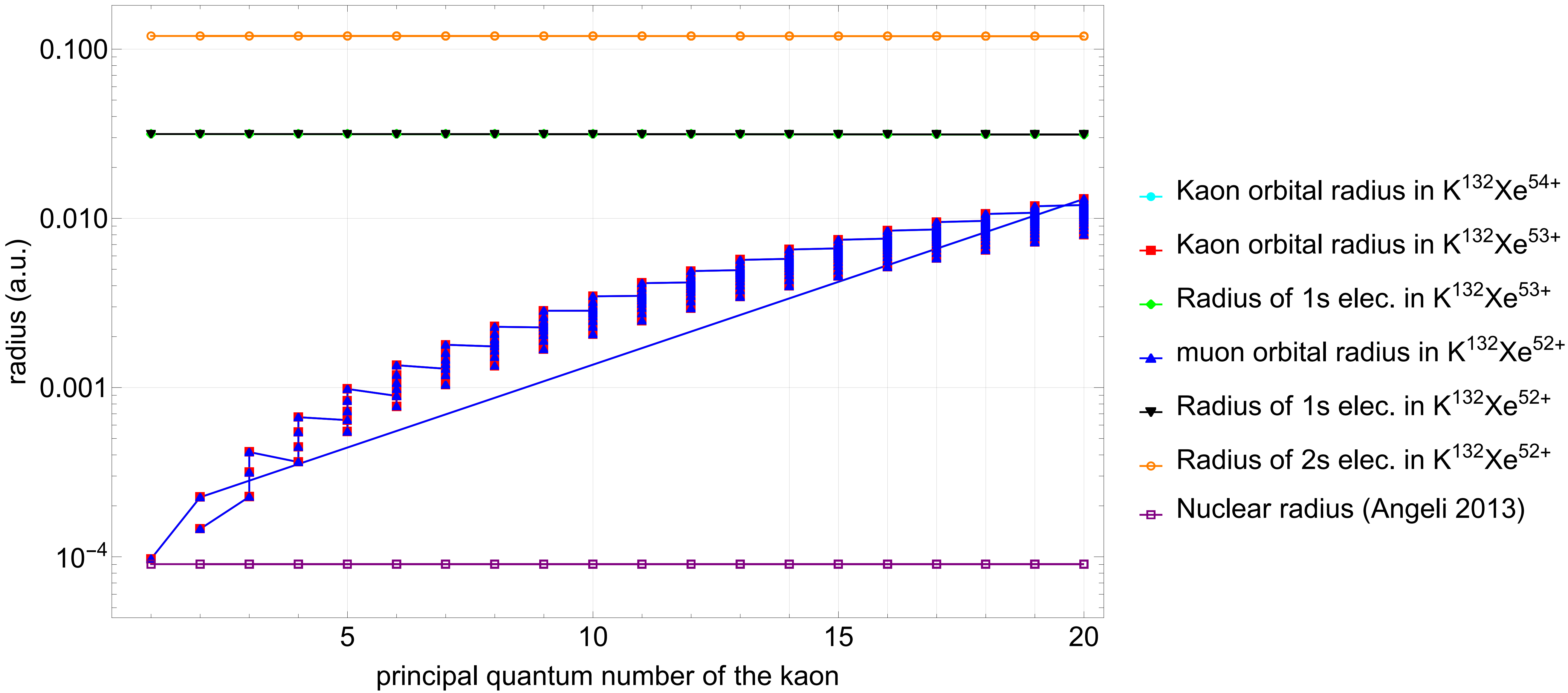
Kaonic ^{132}Xe



Kaonic ^{132}Xe



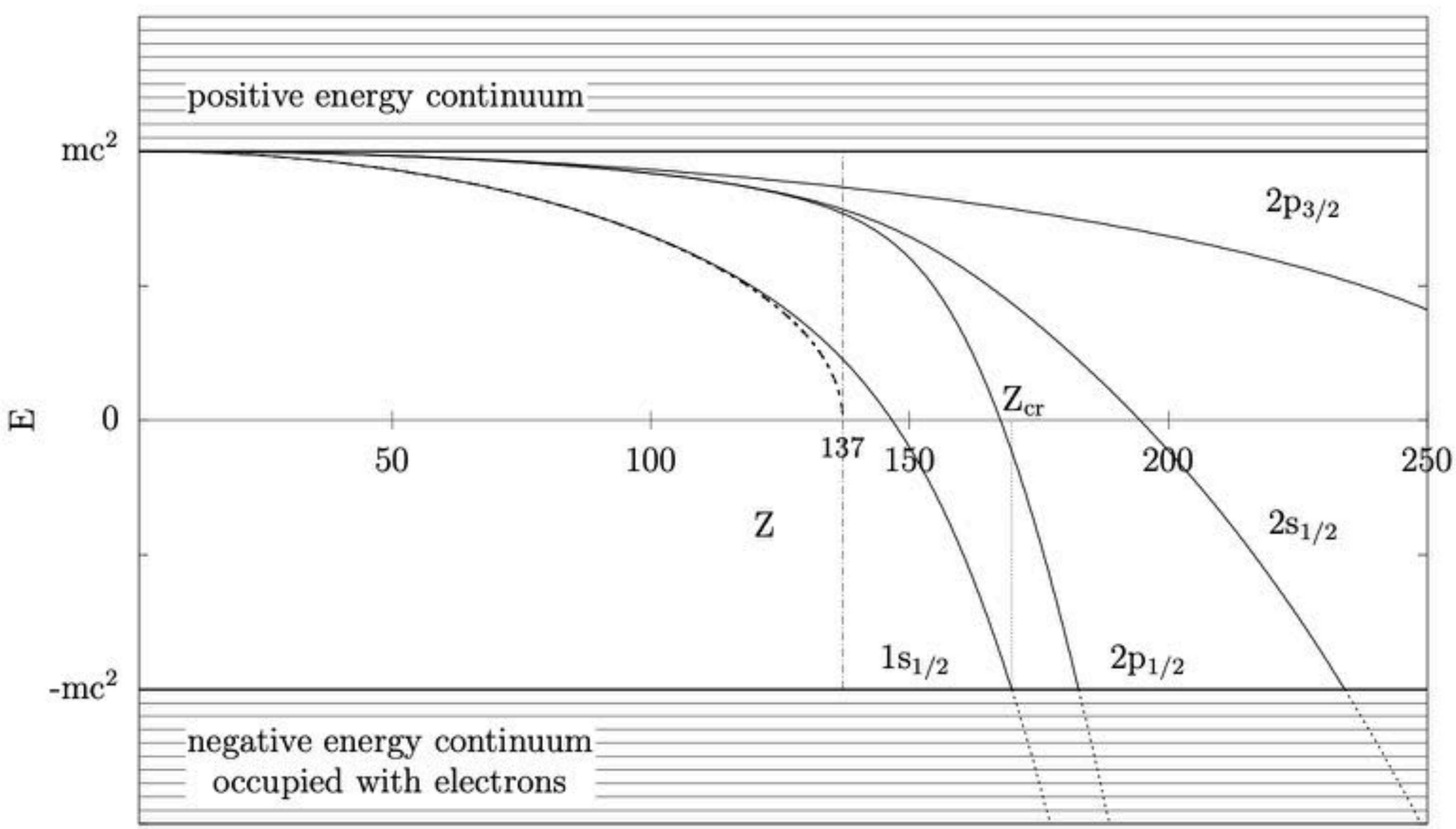
Kaonic ^{132}Xe



Very special behavior at high-Z: Dirac equation

$$E_{n\kappa} = \frac{mc^2}{\sqrt{1+\frac{(Z\alpha)^2}{(n-|\kappa|+\sqrt{\kappa^2-(Z\alpha)^2})^2}}}$$

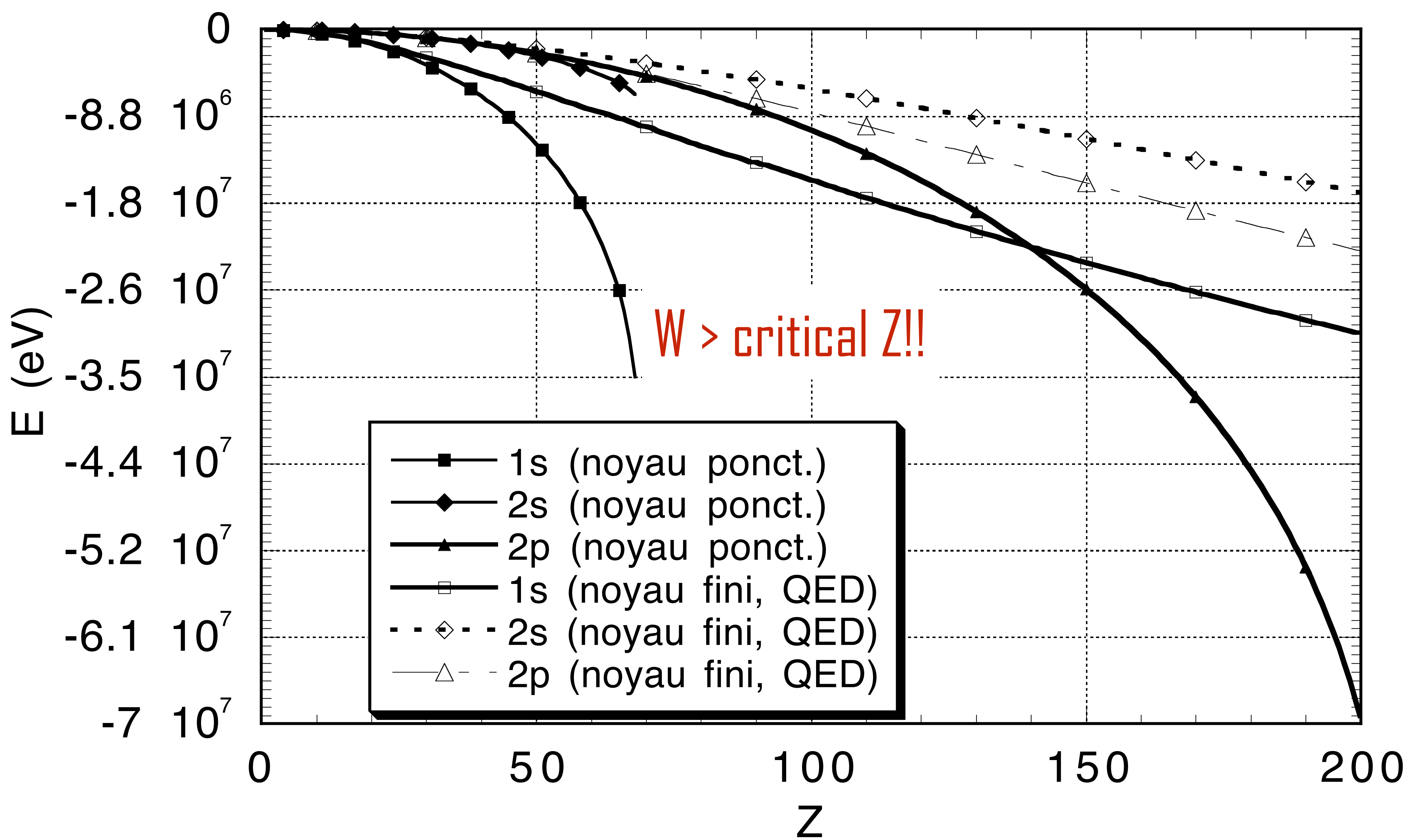
Becomes complex for κ=1 when Z→1/α~137



Very special behavior at high-Z: Klein-Gordon equation

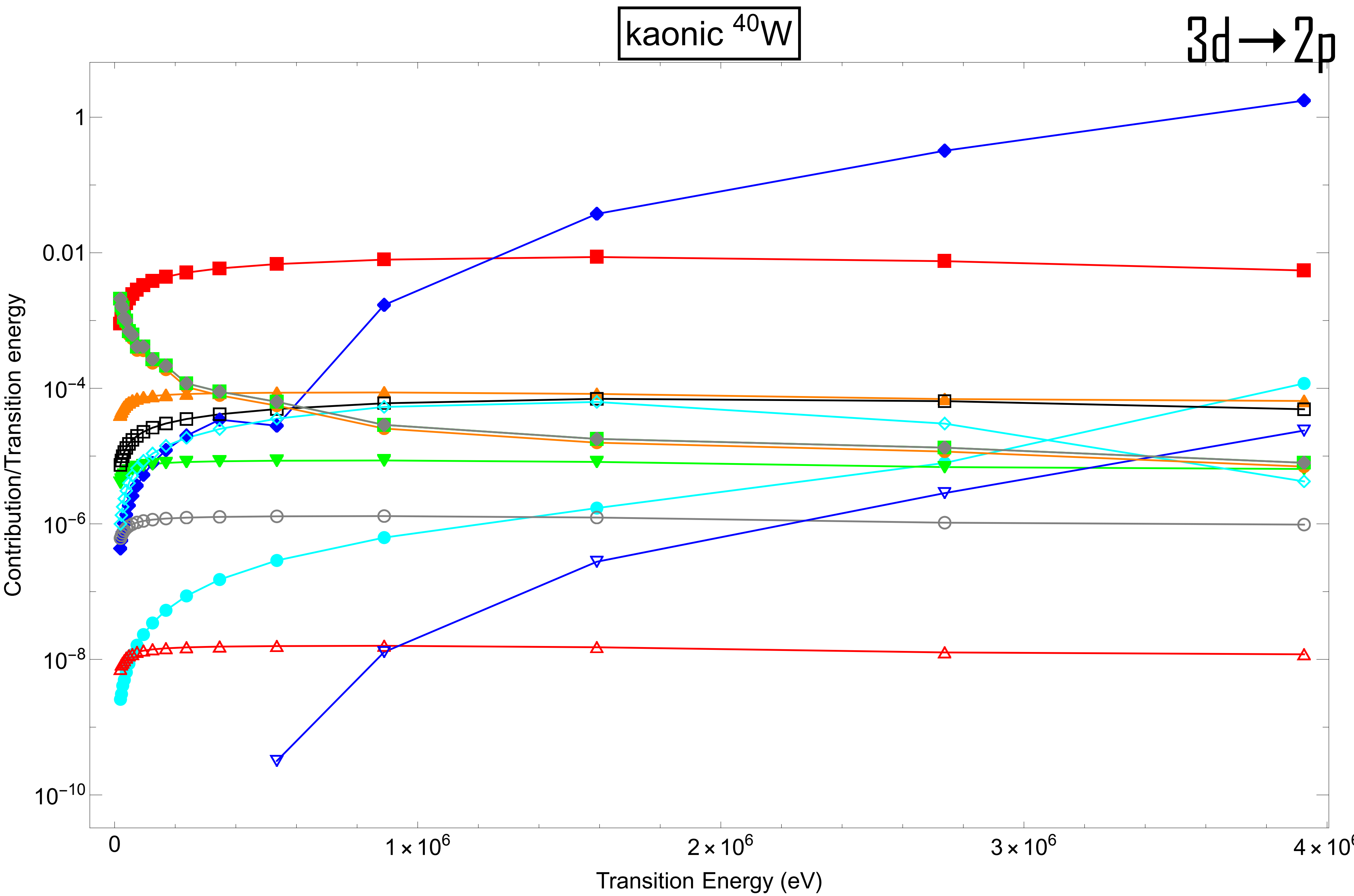
$$\sqrt{(l+1/2)^2-(Z\alpha)^2}$$

Becomes complex for l=0 (1s) when Z→1/2α~68.5



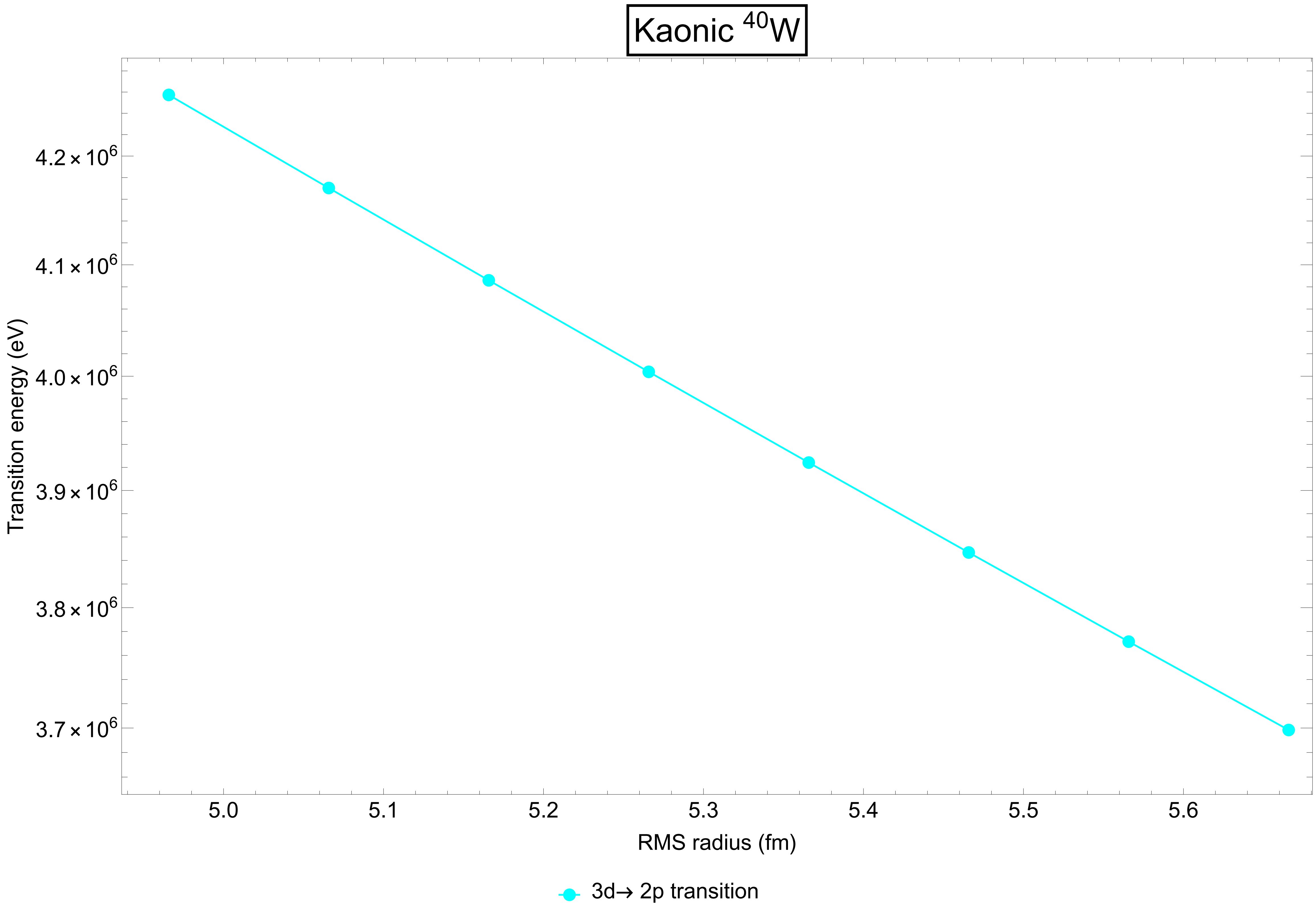
Cannot define finite nuclear 1s size for Z≥69

Kaonic ^{184}W

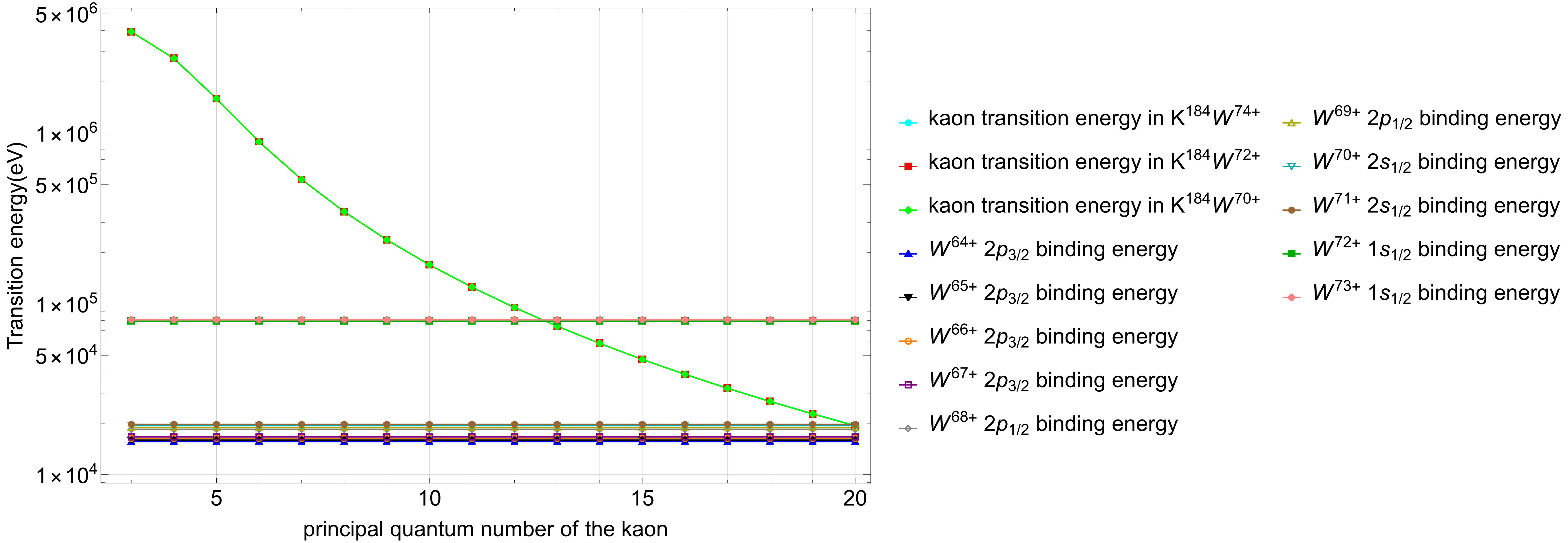


FNS value cannot be calculated for $2p \rightarrow 1s$ ($Z>Z_{crit}$)

Kaonic ^{184}W

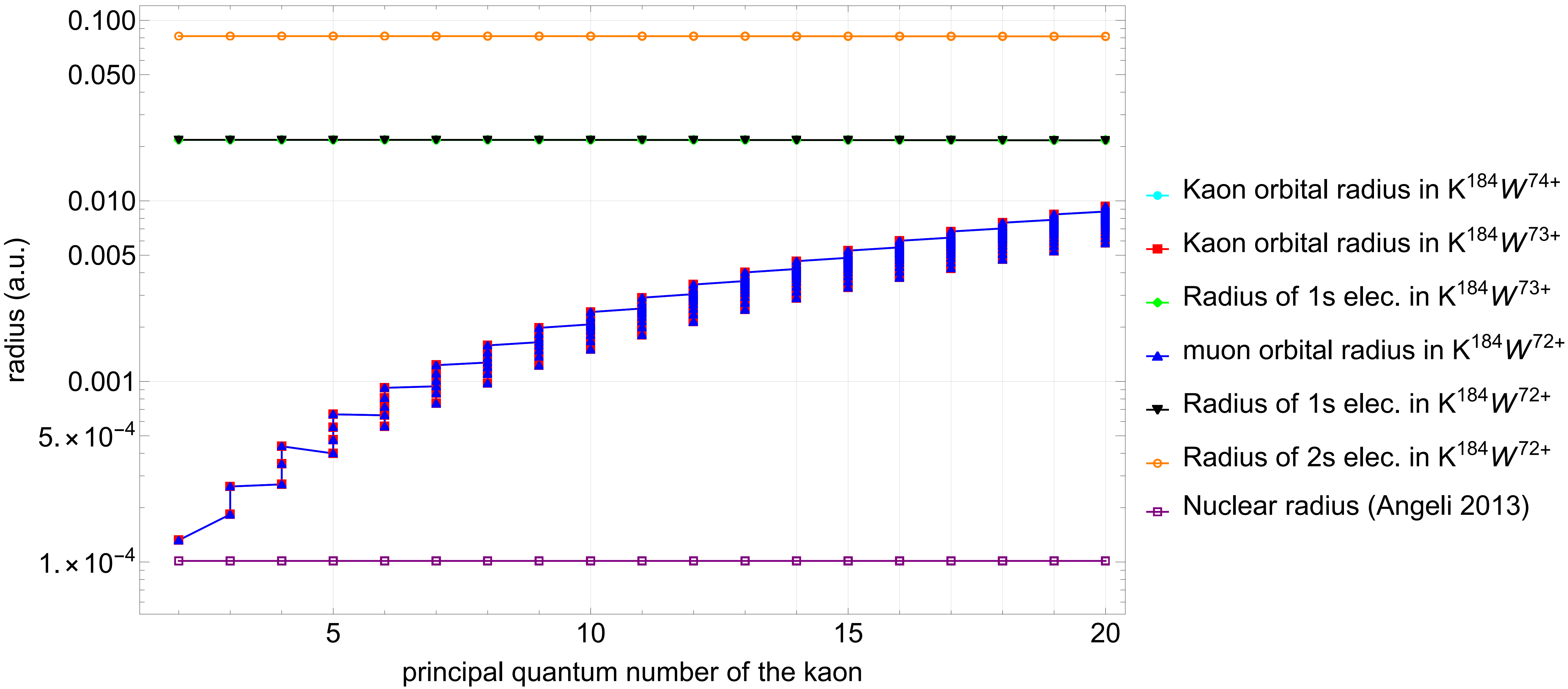

Kaonic ^{40}W
Transition energy (eV)
4.2 × 10^6
4.1 × 10^6
4.0 × 10^6
3.9 × 10^6
3.8 × 10^6
3.7 × 10^6
5.0
5.1
5.2
5.3
5.4
5.5
5.6
RMS radius (fm)
3d→ 2p transition

Kaonic ^{184}W



Transition energy(eV)
5 × 10^6
1 × 10^6
5 × 10^5
1 × 10^5
5 × 10^4
1 × 10^4
5
10
15
20
principal quantum number of the kaon
kaon transition energy in K^184W^74+
kaon transition energy in K^184W^72+
kaon transition energy in K^184W^70+
W^64+ 2p3/2 binding energy
W^65+ 2p3/2 binding energy
W^66+ 2p3/2 binding energy
W^67+ 2p3/2 binding energy
W^68+ 2p1/2 binding energy
W^69+ 2p1/2 binding energy
W^70+ 2s1/2 binding energy
W^71+ 2s1/2 binding energy
W^72+ 1s1/2 binding energy
W^73+ 1s1/2 binding energy

Kaonic ^{184}W



0.100
0.050
0.010
0.005
0.001
5.×10^-4
1.×10^-4
radius (a.u.)
5
10
15
20
principal quantum number of the kaon
Kaon orbital radius in $K^{184}W^{74+}$
Kaon orbital radius in $K^{184}W^{73+}$
Radius of 1s elec. in $K^{184}W^{73+}$
muon orbital radius in $K^{184}W^{72+}$
Radius of 1s elec. in $K^{184}W^{72+}$
Radius of 2s elec. in $K^{184}W^{72+}$
Nuclear radius (Angeli 2013)

Electronic shifts

Element	Electronic core	kaon initial label	kaon final label	Theoretical transition energy	electronic shift	Uncert. due to Kaon mass	Experimental energy	Uncert.	Exp.-Theo.
C	none	4f	3d	22104.792		0.58	22105.61	0.26	0.82
C	1s2 1S0	4f	3d	22104.847	0.056	0.58	22105.61	0.26	0.76
C	1s2 2s2 1S0	4f	3d	22104.932	0.140	0.58	22105.61	0.26	0.68
W	1s2 2s2 1S0	11n	10m	125258.170	-29.257	3.30	125251	24	-7.2
W	1s2 2s2 2p6 1S0	11n	10m	125258.243	-29.183	3.30	125251	24	-7.2
W	1s2 1S0	11n	10m	125261.981	-25.446	3.30	125251	24	-11.0
W	none	11n	10m	125287.426		3.30	125251	24	-36.4
W	1s2 2s2 1S0	9l	8k	237261.029	-18.639	6.25	237171	35	-90.0
W	1s2 2s2 2p6 1S0	9l	8k	237261.087	-18.581	6.25	237171	35	-90.1
W	1s2 1S0	9l	8k	237263.467	-16.201	6.25	237171	35	-92.5
W	none	9l	8k	237279.668		6.25	237171	35	-108.7
Pb	1s2 2s2 2p6 1S0	11n	10m	153901.882	-40.018	4.05	153903.0	8.0	1.1
Pb	1s2 2s2 1S0	11n	10m	153901.937	-39.963	4.05	153903.0	8.0	1.1
Pb	1s2 1S0	11n	10m	153907.433	-34.467	4.05	153903.0	8.0	-4.4
Pb	none	11n	10m	153941.900		4.05	153903.0	8.0	-38.9
Pb	1s2 2s2 2p6 1S0	9l	8k	291595.850	-26.171	7.68	291580.0	4.4	-15.8
Pb	1s2 2s2 1S0	9l	8k	291595.871	-26.151	7.68	291580.0	4.4	-15.9
Pb	1s2 1S0	9l	8k	291599.487	-22.534	7.68	291580.0	4.4	-19.5
Pb	none	9l	8k	291622.021		7.68	291580.0	4.4	-42.0

Conclusion

- The accuracy of QED test in HCI is limited in fact by nuclear corrections
- Circular Rydberg states exotic atoms offer an alternative, without uncertainties due to nuclear corrections
- One can find favorable cases to best measure the kaon mass depending on detector energy range, sensitivity required…