

# Impact of the charged kaon mass on the charmonium spectrum

(a reminder)

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(In collaboration with Simon Eidelman, 1948-2021 )

To be measured with X-ray transitions  
in  $K^-$  atoms at DAFNE - Siddharta 2

Talk delivered at the Siddharta Workshop: **Kaon mass**, online, 19 April 2021  
For details see attached docs and <http://amsler.web.cern.ch/Highlights/KmassSAB.pdf>

Discrepancy **60 keV** between best measurements

## $K^\pm$ MASS

VALUE (MeV)	DOCUMENT ID	TECN	CHG	COMMENT
<b><math>493.677 \pm 0.016</math> OUR FIT</b>	Error includes scale factor of 2.8.			
<b><math>493.677 \pm 0.013</math> OUR AVERAGE</b>	Error includes scale factor of 2.4. See the ideogram below.			
$493.696 \pm 0.007$	<sup>1</sup> DENISOV	91	CNTR	— Kaonic atoms
$493.636 \pm 0.011$	<sup>2</sup> GALL	88	CNTR	— Kaonic atoms
$493.640 \pm 0.054$	LUM	81	CNTR	— Kaonic atoms
$493.670 \pm 0.029$	BARKOV	79	EMUL	$\pm$ $e^+e^- \rightarrow K^+K^-$
$493.657 \pm 0.020$	<sup>2</sup> CHENG	75	CNTR	— Kaonic atoms
$493.691 \pm 0.040$	BACKENSTO...73	CNTR	—	Kaonic atoms

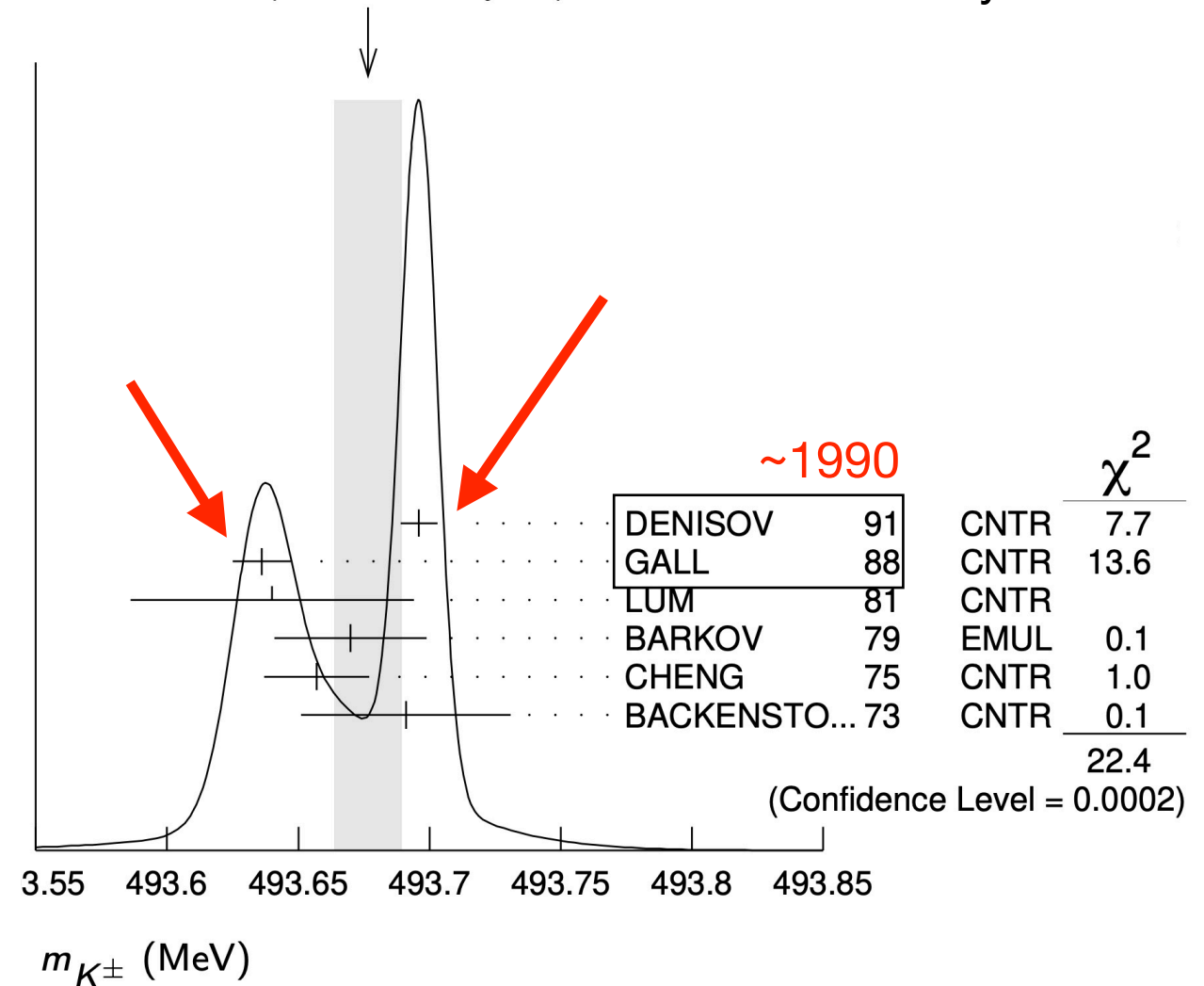
<sup>1</sup> Error increased from 0.0059 based on the error analysis in IVANOV 92.

<sup>2</sup> This value is the authors' combination of all of the separate transitions listed for this

Emulsions:  $K^+$

K atoms:  $K^-$

WEIGHTED AVERAGE  
 $493.677 \pm 0.013$  (Error scaled by 2.4) Errors scaled by x 2.4



**16 keV**  $\rightarrow$  32 ppm, compared to charged pion: 0.18 keV  $\rightarrow$  1.3 ppm

**Goal: solve the discrepancy and try to decrease the uncertainty to below 16 keV**

Many new mesons have been observed since 20 years in the charm spectrum:  $c\bar{c}q\bar{q}$ ,  $c\bar{q}q\bar{c}$  ?

First observation by BELLE in 2002:

decay into  $J/\psi \pi^+ \pi^-$

$$B^+ \rightarrow X(3872) K^+$$

Prominent:  $X(3872) \rightarrow D^0 \bar{D}^{*0}$  dominant

$$\Gamma = 0.96_{-0.18}^{+0.19} \pm 0.21 \text{ MeV} \quad \text{very narrow}$$

Breit-Wigner fit determined by the detector resolution

Not adequate for high resolution experiments  
because of  $D^0 \bar{D}^{*0}$  threshold

Mass spectrum distorted (asymmetric, narrower)  
due to the threshold, unfold resolution, [Flatté analysis](#)

$$\Gamma = 0.22_{-0.08-0.17}^{+0.06+0.25} \text{ MeV}$$

Mass =  $3871.65 \pm 0.06 \text{ MeV}$  (World average 2021)

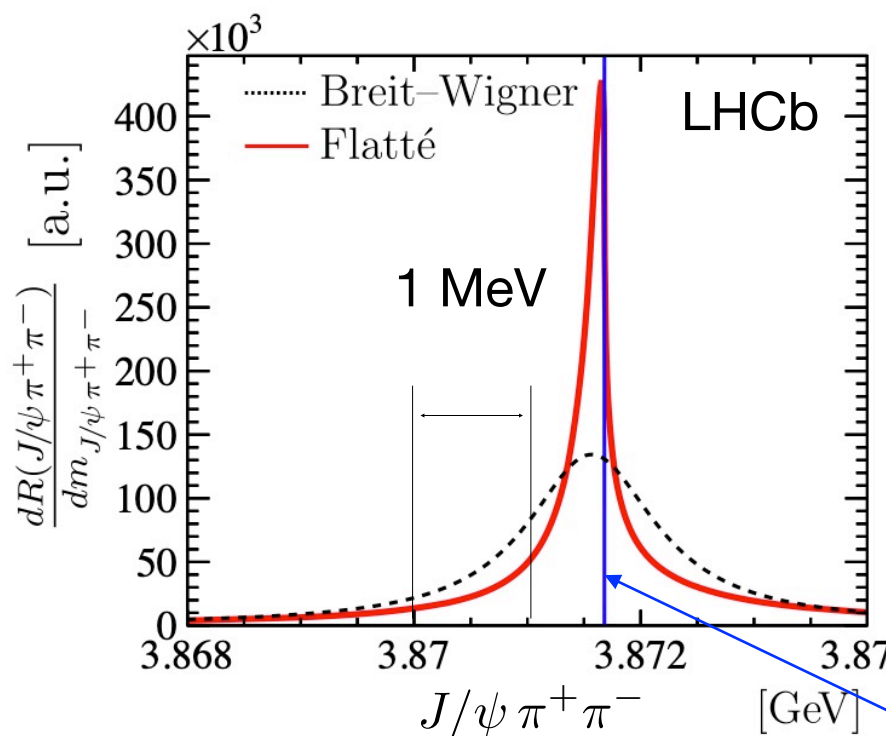
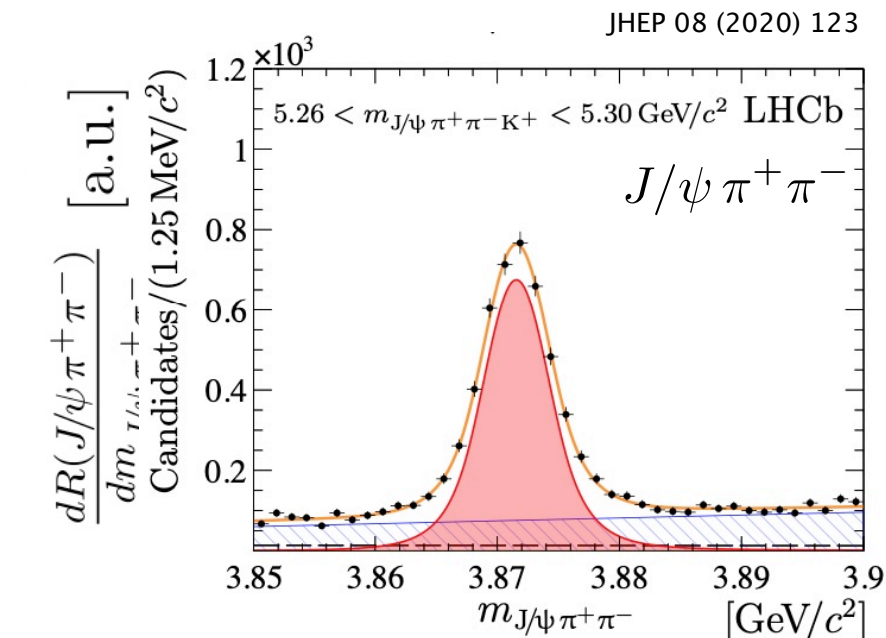
$D^0 \bar{D}^{*0}$  threshold is within the width of X(3872):

$\text{Threshold} - \text{Mass} = 70 \pm 120 \text{ keV (LHCb)}$

Probably [bound “molecule”](#) ( $c\bar{q}q\bar{c}$ ) of two hadrons

Threshold depends on  $D^0$  mass ([known to 50 keV](#))

NB:  $D^{0*} \rightarrow D^0 \pi^0$



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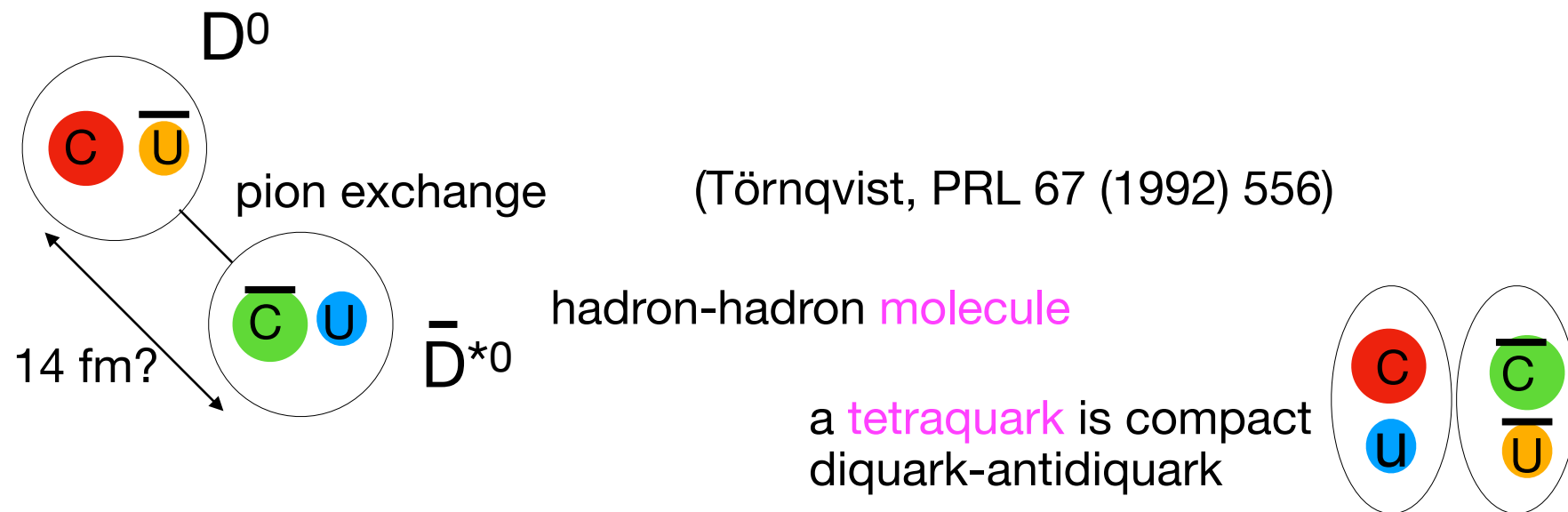
$D^0 \bar{D}^{*0}$

## Size of the X(3872)

Remember the deuteron:  $B = 2.2 \text{ MeV} \longrightarrow R = \frac{1}{\sqrt{2\mu B}} = 4.3 \text{ fm}$

↖ reduced np mass

Assume  $B = 100 \text{ keV}$  for a bound X(3872):  $\longrightarrow$   $R = 14 \text{ fm}$



Precise measurement of mass and threshold location to know the size of the object

Neutral  $D$  ( $D^0, \bar{D}^0$ ) decay dominantly into channels with kaons ( $c \rightarrow s$ )

➡

Precise threshold determination  $\rightarrow$  need more precise  $D^0$  mass (now 50 keV)  
 $\rightarrow$  more precise kaon mass (now 16 keV)

$K$  mass precision  $\rightarrow$   $D$  mass precision: simple simulation of  $D^0$  decay (assume phase space):

For example

keV		$K^-\pi^+$	$K^-\pi^+\pi^0$	$K^-\pi^+\pi^+\pi^-$	$K^+K^-\pi^0$	$K^+K^-\pi^+\pi^-$	$K^+K^-K^-\pi^+$
$\sigma_{m_K}$	b.r. [%]	3.9	14.4	8.2	0.34	0.25	0.023
16		8.0	10.4	11.9 [12.2] <sup>a</sup>	19.0 [25.0] <sup>b</sup>	25.8	43.2
7		3.5	4.6	5.2 [5.2] <sup>a</sup>	9.9	11.3	18.9
3		1.5	2.0	2.2 [2.3] <sup>a</sup>	4.2	4.8 [5.0] <sup>c</sup>	8.1

Table 1: Resolution on  $D^0$ -mass in keV ( $1\sigma$ ) as a function of uncertainty on the  $K^-$ -mass in keV for various decay channels. Square brackets are for the intermediate  $D^0 \rightarrow K^-\rho^0\pi^+$  (b. r. = 6.9%)<sup>a</sup>,  $D^0 \rightarrow \phi\pi^0$  (b. r. = 0.069%)<sup>b</sup>, and  $D^0 \rightarrow \phi\rho^0$  (b. r. = 0.07%)<sup>c</sup>.

Avoid single K channels: dominated by momentum resolution on fast pions

For example: reducing the error on the  $K$ -mass by a factor of 2 would lead to an improvement of 4-5 on the contribution to the  $D^0$  mass (currently 50 keV) in the two-kaon decay mode.

Phase space? The decay channels involve resonances which do not change the results significantly.

$\rho^0$  (a)     $\phi$  (b)     $\phi\rho^0$  (c)

## Conclusions

The last measurements of the charged kaon mass are **very old** (at least 30 years).

There is a **discrepancy of 60 keV** between the two most accurate measurements, each measured with about **10 keV** r.m.s resolution.

This leads to an error of **16 keV** ( $\sigma$ ) on the charged kaon mass and propagates to an error of **50 keV** ( $\sigma$ ) on the  $D^0$  mass.

This uncertainty is propagated to the charmonium spectrum, in particular to precise values of charm-anticharm meson thresholds.

A particular case is that of  $D^0 \bar{D}^{*0}$  which lies within the measured width of the best known candidate for a hadron-hadron molecule, the **X(3872)**.

An improved K-mass measurement would lead to a better interpretation of the X(3872) and a determination of its radius.

Reducing the error on the K-mass by a factor of **2** could lead to an improvement of **4-5** on the  $D^0$  mass in the two-kaon decay mode.

But other systematics (momentum resolutions) may become dominant in re-analyses of old data, or in new measurements.



# Impact of the $K^-$ -mass uncertainty on the $D^0$ -mass

17 April 2021, C. Amsler

The  $D^0$ -mass is currently known to an r.m.s precision of 50 keV (see fig. 1 overleaf). The systematic errors are dominated by the uncertainty on the  $K$ -masses (13 keV for the  $K_S$  and 16 keV for the charged kaon)<sup>1</sup>. The uncertainty on the  $D^0$ -mass has an impact on the masses of all charmed mesons and in particular on the values of the charm-anticharm thresholds, which are relevant to interpret some of the charmed tetraquark candidates as hadronic (charm-anticharm loosely bound) molecules, see e.g. [2].

Since the accuracy on the pion mass is 20 times better than for the kaon mass, one would think that the  $D^0$ -mass should be measured by reconstructing and fitting decay channels that do not involve kaons (such as  $2\pi^+2\pi^-$  with branching ratio (b.r.) of  $7.6 \times 10^{-3}$ ). However, it turns out that these channels are prone to systematic errors from e.g. momentum calibration for the fast pions. In fact, the current best measurement of the  $D^0$ -mass [3] is based on the decay channel  $D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$  (+ charge conjugate) involving only one kaon, but with accuracy limited by the  $K_S^0$ -mass via  $K_S^0 \rightarrow \pi^+ \pi^-$  used for momentum calibration (the  $K_S^0$  mass contributing 52 keV and the charged kaon mass only 12 keV to the  $D^0$ -mass). Adding more kaons to the decay channel increases the contributions to the error on the  $D^0$ -mass, but the smaller energy release ( $Q$ -value) leads to lower systematics in track reconstruction. This is why BaBar [4] and LHCb [5] used the channel  $D^0 \rightarrow K^+ K^- K^- \pi^+$  (both about 50 keV contribution to the  $D^0$ -mass), even if the b. r. is rather low,  $2.3 \times 10^{-4}$ .

I have made a simulation of various  $D^0$ -decay channels (table 1). This was very simply done by assuming phase space distributions and changing the  $K$ -masses and then re-calculating the  $D^0$ -mass, assuming an ideal experiment in which the momenta are measured with infinite precision. Typically  $(6-7) \times 10^5$  decay events were simulated for each channel. As expected, the resolution gets worse with increasing number of kaons, or slower ones. Three cases with intermediate resonances are also given (square brackets). As an illustration, figure 2 shows plots when assuming intermediate resonances.

$\sigma_{m_K}$		$K^- \pi^+$	$K^- \pi^+ \pi^0$	$K^- \pi^+ \pi^+ \pi^-$	$K^+ K^- \pi^0$	$K^+ K^- \pi^+ \pi^-$	$K^+ K^- K^- \pi^+$
	b.r. [%]	3.9	14.4	8.2	0.34	0.25	0.023
16		8.0	10.4	11.9 [12.2] <sup>a</sup>	19.0 [25.0] <sup>b</sup>	25.8	43.2
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$\frac{\sigma_{m_D}}{\sigma_{m_K}}$		0.5	0.66	0.73	1.3	1.6	2.7

Table 1: Resolution on  $D^0$ -mass in keV ( $1\sigma$ ) as a function of uncertainty on the  $K^-$ -mass in keV for various decay channels. Square brackets are for the intermediate  $D^0 \rightarrow K^- \rho^0 \pi^+$  (b. r. = 6.9%)<sup>a</sup>,  $D^0 \rightarrow \phi \pi^0$  (b. r. = 0.069%)<sup>b</sup>, and  $D^0 \rightarrow \phi \rho^0$  (b. r. = 0.07%)<sup>c</sup>.

The results in the first line (for the current  $\sigma_{m_K}=16$  keV) are compatible with the ones from [3] (which quotes 12 keV on  $D^0$  for  $K\pi\pi\pi$ ) and [4, 5] (which quote  $\simeq 50$  keV for  $KKK\pi$ ). For example, assuming that Siddharta measures the  $K^-$ -mass with an r.m.s error of 7 keV as in ref. [6], one would

<sup>1</sup>The uncertainty on the  $D^+$ -mass (fig. 1) can be determined also to within 50 keV from the mass difference  $m_{D^+} - m_{D^0}$  which is known more precisely (15 keV [1]).

contribute only 11.3 keV to the  $D^0$ -mass uncertainty with the channel  $K^+ K^- \pi^+ \pi^-$ , reconstructed in a future experiment. This is much better than the current 50 keV from [4, 5]<sup>2</sup>.

In reality most channels proceed through intermediate resonances, so the numbers are approximate. The best precision achieved in this table is for  $D^0 \rightarrow \phi \rho^0$  which has a very low  $Q$ -value, and by fitting the relatively narrow  $\phi$ .

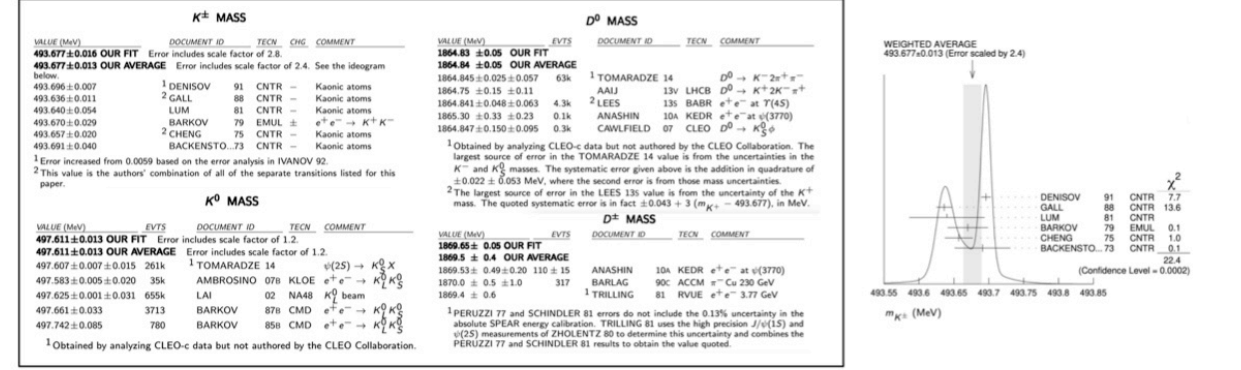


Figure 1: Current values of the  $K^-$ - and  $D$ -masses. The right panel shows the discrepancy between the two most accurate measurements of the  $K^-$ -mass [1], obtained from kaonic X-rays.

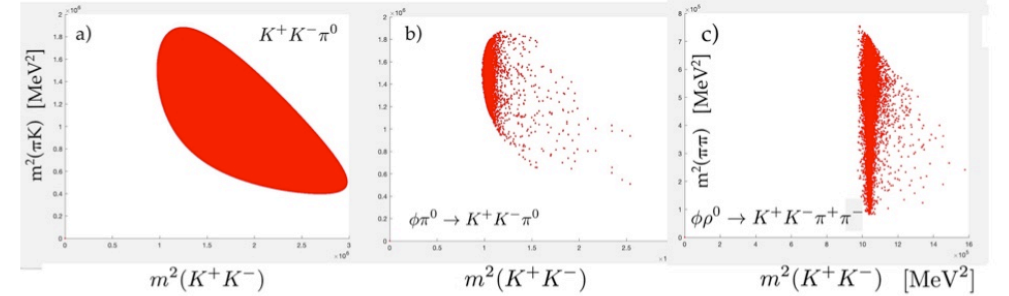


Figure 2: Simulated Dalitz plots for  $D^0 \rightarrow K^+ K^- \pi^0$  phase-space distributed events (a) and through  $\phi \pi^0$  (b). The scatterplot shown in (c) is for  $D^0 \rightarrow \phi(\rightarrow K^+ K^-) \rho(\rightarrow \pi^+ \pi^-)$  ( $Q = 70$  MeV).

## References

- [1] P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020** (2020) 083C01
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- [5] R. Aaij *et al.* (LHCb Collaboration), JHEP **06** (2013) 065
- [6] A.S. Denisov *et al.*, JETP Lett. **54** (1991) 558
- [7] K.P. Gall *et al.*, Phys. Rev. Lett. **60** (1988) 186

<sup>2</sup>Remember that there is a 60 keV discrepancy in the charged kaon mass between refs. [6] and [7] (see fig. 1).



# Charged Kaon Mass

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**Abstract:** We emphasize the importance of remeasuring the charged kaon mass with high precision to resolve the discrepancy between the previous 30-year-old experiments. This uncertainty affects in particular the mass spectrum of states with charmed quarks, and the classification of heavy mesons that cannot be easily reconciled with a standard  $q\bar{q}$  structure.

Pions and kaons are the lightest hadrons. Because of their large lifetime they are practically stable from the point of view of detection and therefore play a special role in experimental high energy physics. While the charged pion mass is known to a relative accuracy of  $1.2 \times 10^{-6}$ , the accuracy of the charged kaon mass is 20 times worse,  $2.6 \times 10^{-5}$  [1]. Note that, because of the longstanding discrepancy between the two most (but very old) precise measurements, the uncertainty of the world average value is inflated by a factor of 2.4 (Figure 1).

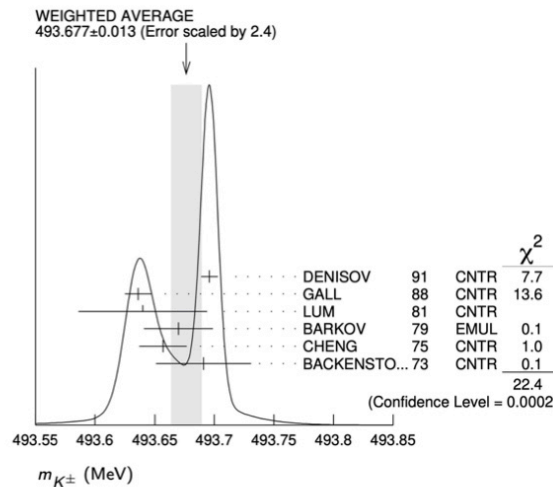


Figure 1: Histogram showing the discrepancy between the various measurements of the charged kaon mass (from [1] where the bibliography can also be found).

The measurements listed in Figure 1 were performed by measuring high-level X-ray transitions following  $K^-$  capture in atoms (such as K, C, Au, Ba, Pb, W) with the exception of ref. [2] who measured the range of stopping  $K^\pm$  in nuclear emulsions. In ref. [3] a crystal spectrometer was used (resolution  $\sim 6$  eV at 22 keV) while the other measurements were performed with Ge detectors with much worse resolutions ( $\sim 1$  keV for 300 keV X-rays). The discrepancy could possibly be due to contaminating nuclear  $\gamma$ -rays in the Ge measurements (for a review see the article by T. G. Trippe in ref. [1]). Thus a remeasurement of the charged kaon mass with modern techniques is highly desirable.

It also turns out that the accuracy of the  $D^0$ -meson mass is limited by the precision of charged kaon mass measurements. In particular, the *systematic* uncertainty of  $m(D^0)$  is dominated by our knowledge of  $m(K^\pm)$  [4, 5]. In turn, the masses of all excited charmed mesons, whose direct measurements are rather uncertain, e.g. those of  $D_1(2420)^0$ ,  $D_2^*(2460)^0$  and  $D_{s1}(2536)^\pm$ , are determined precisely from the fit based on high-precision measurements of mass and mass difference of the  $D^0$ ,  $D^\pm$  and  $D_s^\pm$  states [1]. Thus, the precision on the charged kaon mass appears to be crucial for the precise spectroscopy of charmed mesons.

Furthermore, the precision on  $m(K^\pm)$  directly affects our understanding of the nature of the  $\chi_{c1}(3872)$  meson. This state (formerly called  $X(3872)$ , discovered by the Belle Collaboration in 2003 [6]) was the first enigmatic state whose properties cannot be fully understood in the framework of the quark model. Despite very extensive efforts (the discovery paper with 1880 citations is one of the most cited experimental publications), there is no consensus today about its internal structure. The most popular explanation is that it is a mixture of a regular  $q\bar{q}$  state and a  $D^0\bar{D}^{*0}$  molecule. To test the validity of the molecular hypothesis it is of vital importance to know precisely how far the  $\chi_{c1}(3872)$  state lies from the  $D^0\bar{D}^{*0}$  threshold. Recently LHCb performed a study of  $\chi_{c1}(3872)$  produced in decays of  $B^\pm$  mesons and other  $b$  hadrons [7, 8]. Using the world-largest sample of almost 20k  $\chi_{c1}(3872) \rightarrow J/\psi\pi^+\pi^-$  decays, LHCb performed the most precise measurement of the  $\chi_{c1}(3872)$  mass and of the energy difference  $\delta E = m(D^0) + m(D^{*0}) - m(\chi_{c1}(3872)) = 0.07 \pm 0.12$  MeV. Again, the precision is limited by that of the charged kaon mass.

The precision on the  $D^0$  mass also affects the mixing parameters in the  $D^0-\bar{D}^0$  system [4], and in the long run, a more accurate kaon mass may become interesting for first-principle calculations on the lattice [9].

SIDDHARTA-2 plans to measure kaonic X-rays with a high purity Ge detector and also with an HAPG crystal spectrometer (VOXES), in an attempt to resolve the discrepancy mentioned in Figure 1. In future, one could also consider measuring the range of positive kaons with nuclear emulsions that have benefitted enormously from new technologies and the experience gained with the OPERA experiment. This would provide at the same time a CPT test between positive and negative kaon masses. Finally, a measurement of the muon momentum from stopping  $K^+$  decaying into  $\mu^+\nu$  could be envisaged, similar to the one performed for the  $\pi^+$  [10].

## References

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