Modeling solar modulation in cosmic rays
in light of new data from AMS-02 and PAMELA

Alejandro Reina Conde,
Bruna Bertucci, Emanuele Fiandrini, Nicola Tomassetti, Behrouz Khiali

Istituto Nazionale di Fisica Nucleare, Sezione di Bologna

University of Perugia & INFN; ASI-UniPG 2019-2-HH.0

ASI - SSDC and INFN – Roma Tor Vergata

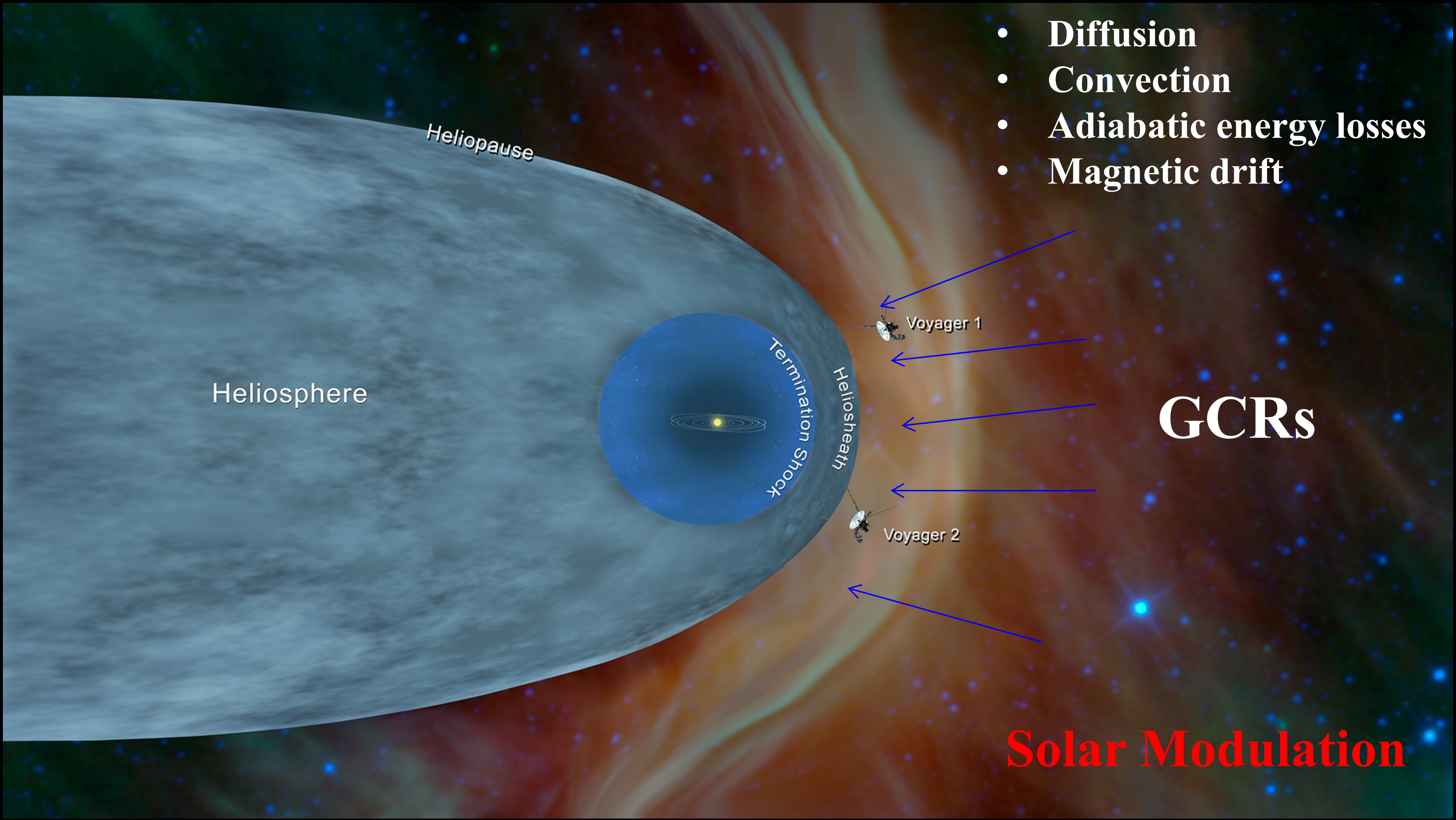
INFN
BOLOGNA
Istituto Nazionale di Fisica Nucleare
Sezione di Bologna

DIPARTIMENTO
DI FISICA E
GEOLOGIA
PERUGIA

CRIS 2022
12th Cosmic Ray International Seminar
Naples, Italy, September 12 -16, 2022

14/09/2022, Naples

GCRs Propagation inside the Heliosphere



- **Diffusion**
- **Convection**
- **Adiabatic energy losses**
- **Magnetic drift**

GCRs

Solar Modulation

GCRs Propagation inside the Heliosphere: Parker’s Equation

$f = f(\vec{r}, p, t)$ is the phase-space density distribution of GCRs averaged over all momentum direction assuming a nearly isotropic distribution of arrival direction of GCRs

rate of change in time of f, in a steady-state solution is equal to zero

diffusion term, account for small scale Heliospheric Magnetic Field (HMF) irregularities, being **K** the diffusion tensor

describes the **adiabatic energy changes** accounting for the energy loss due to the adiabatic expansion of the solar wind

$$\frac{\partial f}{\partial t} = \vec{\nabla} \cdot \left(\boldsymbol{K} \vec{\nabla} f \right) - \vec{V} \cdot \vec{\nabla} f - \langle \vec{v}_d \rangle \cdot \vec{\nabla} f + \frac{1}{3} \left(\vec{\nabla} \cdot \vec{V} \right) \frac{\partial f}{\partial \ln p} + Q(\vec{r}, p, t)$$

convection term, due to the solar wind moving out from the Sun at the velocity V

included to describe any local source (or drain)

particle drift term that account for large scale structures of the HMF, being $\mathbf{v}_d$ the averaged particle drift velocity caused by gradient and curvature drift of charged particle motion in the HMF

GCRs Propagation inside the Heliosphere: Parker’s Equation

$f = f(\vec{r}, p, t)$ is the phase-space density distribution of GCRs averaged over all momentum direction assuming a nearly isotropic distribution of arrival direction of GCRs

rate of change in time of f, in a steady-state solution is equal to zero

diffusion term, account for small scale Heliospheric Magnetic Field (HMF) irregularities, being **K** the diffusion tensor

describes the **adiabatic energy changes** accounting for the energy loss due to the adiabatic expansion of the solar wind

$$\frac{\partial f}{\partial t} = \vec{\nabla} \cdot \left(\boldsymbol{K} \vec{\nabla} f \right) - \vec{V} \cdot \vec{\nabla} f - \langle \vec{v}_d \rangle \cdot \vec{\nabla} f + \frac{1}{3} \left(\vec{\nabla} \cdot \vec{V} \right) \frac{\partial f}{\partial \ln p} + Q(\vec{r}, p, t)$$

convection term, due to the solar wind moving out from the Sun at the velocity V

included to describe any local source (or drain)

particle drift term that account for large scale structures of the HMF, being $\mathbf{v}_d$ the averaged particle drift velocity caused by gradient and curvature drift of charged particle motion in the HMF

Solve it by means of stochastic differential equations (SDEs) using a numerical integration with ∂/∂t=0 with a backward time integration based on SOLARPROP. [Kappl, Comp. Phys. Comm. 207 (2016) 386–399].

Sets of Data

Voyager 1 data:

- **Data outside the Heliosphere, not affected by the solar modulation.**
- **Energies from 140 - 320 MeV.**

AMS-02 data:

- **Data inside the Heliosphere, affected by the solar modulation.**
- **Energies from ~430 MeV to ~60 GeV (≥60 GeV only for the LIS).**
- **May 2011 to May 2017 for a total of 79 Bartels Rotations (BRs, 27 days).**

PAMELA data:

- **Data inside the Heliosphere, affected by the solar modulation.**
- **Energies from ~80 MeV to ~50 GeV.**
- **June 2006 to January 2014 for a total of 47+36 Carrington Rotations (~27 days).**

Sets of Data

Voyager 1 data:

- Data outside the Heliosphere, not affected by the solar modulation.
- Energies from 140 - 320 MeV.

AMS-02 data:

- Data inside the Heliosphere, affected by the solar modulation.
- Energies from ~430 MeV to ~60 GeV (≥60 GeV only for the LIS).
- May 2011 to May 2017 for a total of 79 Bartels Rotations (BRs, 27 days).

PAMELA data:

- Data inside the Heliosphere, affected by the solar modulation.
- Energies from ~80 MeV to ~50 GeV.
- June 2006 to January 2014 for a total of 47+36 Carrington Rotations (~27 days).

Local Interstellar Spectrum (LIS)
Voyager 1
AMS-02 (E ≥ 60 GeV)

Search the best-fit parameters of the model
AMS-02
PAMELA

Parameters Space

A total of **six time-dependent parameters** that are of relevance for the phenomenology of CRs modulation have been identified. Three of them describe the **status of the heliosphere** at a given time, and the other three are related to **diffusion**.

	Min.	Max.	Step
α	**5**	**75**	**10**
B_0 [nT]	**3**	**8**	**1**
A	**-1**	**1**	**2**
K_0 [10^{23} cm^2 s^{-1}]	**0.16**	**1.5**	**0.08**
a	**0.45**	**1.65**	**0.05**
b	**0.45**	**1.65**	**0.05**

Tilt angle
HMF strength
HMF polarity

Normalization factor of $K_{||}$
Spectral index below R_k
Spectral index above R_k
R_k is fixed to 3 GV

Parameters Space

A total of **six time-dependent parameters** that are of relevance for the phenomenology of CRs modulation have been identified. Three of them describe the **status of the heliosphere** at a given time, and the other three are related to **diffusion**.

	Min.	Max.	Step
α	**5**	**75**	**10**
B_0 [nT]	**3**	**8**	**1**
A	**-1**	**1**	**2**
K_0 [10^{23} cm^2 s^{-1}]	**0.16**	**1.5**	**0.08**
a	**0.45**	**1.65**	**0.05**
b	**0.45**	**1.65**	**0.05**

Tilt angle
HMF strength
HMF polarity

Normalization factor of $K_{\parallel}$
Spectral index below R_k
Spectral index above R_k
R_k is fixed to 3 GV

- **120 energy bins ranging from 20 MeV to 200 GeV with log-uniform step.**
- **2 x 10^3 pseudo-particles retro-propagated for each energy bin.**
- **A total of 938,400 simulated fluxes.**

Heliosphere Status Parameters

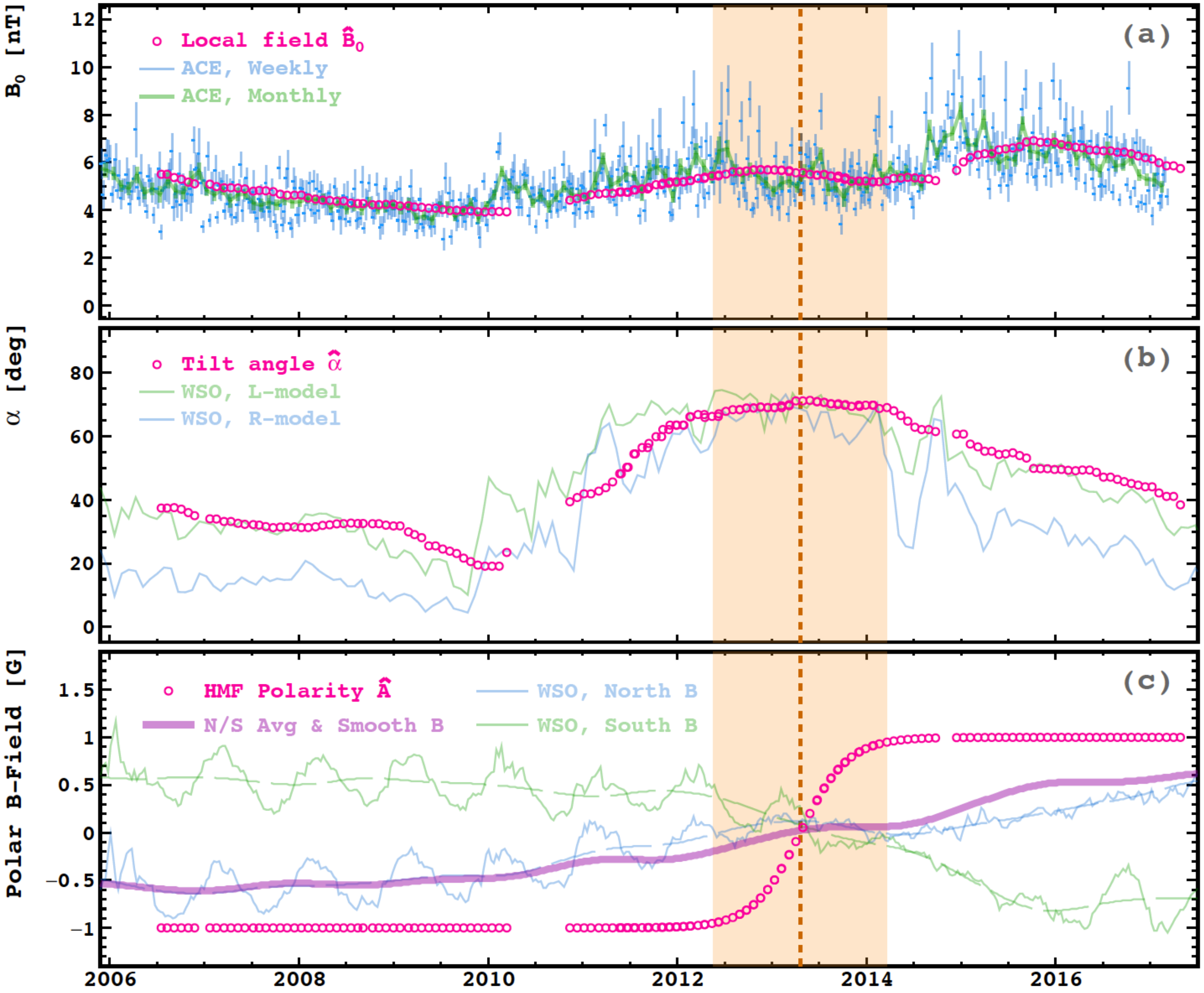
The **perturbations** induced by the **Sun's magnetic activity** take a finite amount of time to establish their **effect in the heliosphere**, which is widely known as **time lag**.

The time needed by the **solar wind (SW) plasma** to transport the magnetic perturbations from the Sun to the Heliopause boundary: between **∼8 months** (fast SW speed) and **∼16 months** (slow SW speed). *[Tomassetti et. al. Astrophys. J. 849, 32 (2017)]*

Heliosphere Status Parameters

The **perturbations** induced by the **Sun's magnetic activity** take a finite amount of time to establish their **effect in the heliosphere**, which is widely known as **time lag**.

The time needed by the **solar wind (SW) plasma** to transport the magnetic perturbations from the Sun to the Heliopause boundary: between **~8 months** (fast SW speed) and **~16 months** (slow SW speed). *[Tomassetti et. al. Astrophys. J. 849, 32 (2017)]*



Backward Moving Average (BMA) for and B0, and A, i.e., a time-average of these quantities calculated over a time window [t - **τ**; t].

$\hat{A}(t) = 1 - P(t)$

T_{rev} mid 2013, $\delta T \cong 3$ months

$$\mathcal{P}(t) = \frac{1}{1 + e^{(t - T_{\mathrm{rev}})/\delta T}}$$

Transport Parameters

e.g. Diffusion Tensor component parallel to HMF
following Potgeiter et al, Sol. Phys. 289, 391 (2014)

$$K_{\parallel} = \boxed{K_0} \frac{\beta}{3} \frac{(R/R_0)^{\boxed{a}}}{(B/B_0)} \left[\frac{(R/R_0)^h + (R_k/R_0)^h}{1 + (R_k/R_0)^h} \right]^{\boxed{\frac{b-a}{h}}}$$

Transport Parameters

e.g. Diffusion Tensor component parallel to HMF
following Potgeiter et al, Sol. Phys. 289, 391 (2014)

$$K_{\parallel} = K_0 \frac{\beta}{3} \frac{(R/R_0)^{a}}{(B/B_0)} \left[\frac{(R/R_0)^h + (R_k/R_0)^h}{1 + (R_k/R_0)^h} \right]^{\frac{b-a}{h}}$$

measures **model**

$$\chi^2(\vec{q}) = \sum_i \frac{[J_d(E_i,t) - J_m(E_i,\vec{q})]^2}{\sigma^2(E_i,t)} \longrightarrow$$

uncertainties

6D vector :
{B_0,α,A,K_0,a,b)

Transport Parameters

e.g. Diffusion Tensor component parallel to HMF
following Potgeiter et al, Sol. Phys. 289, 391 (2014)

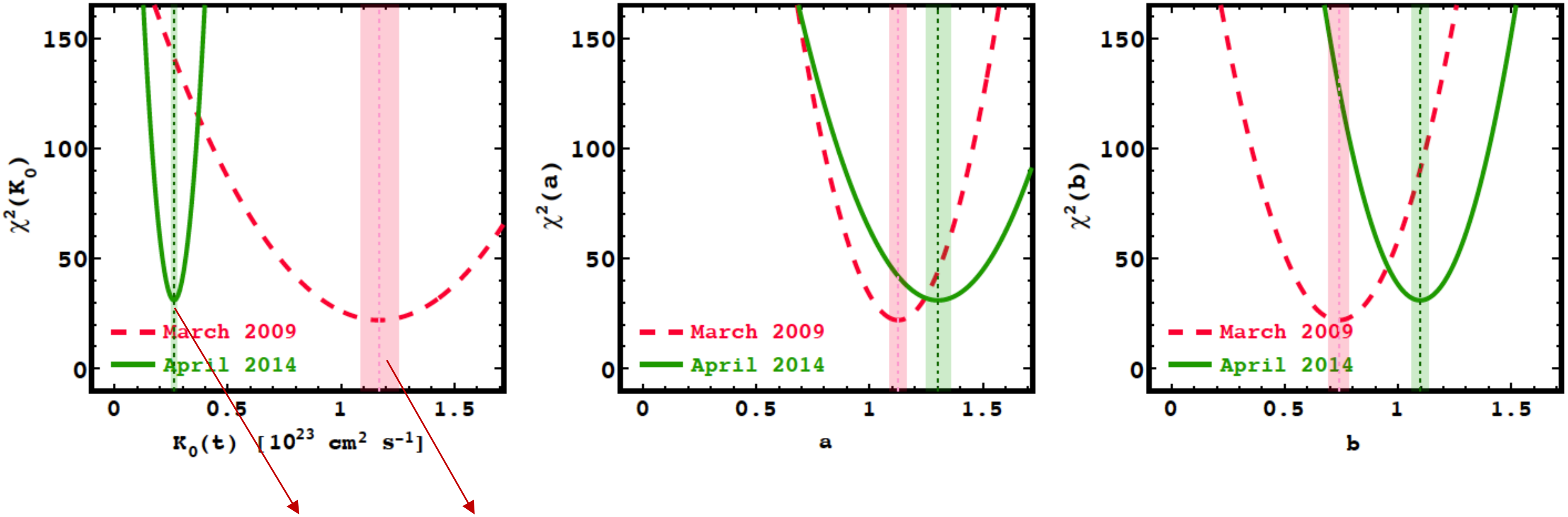
$$K_{\parallel} = K_0 \frac{\beta}{3} \frac{(R/R_0)^a}{(B/B_0)} \left[\frac{(R/R_0)^h + (R_k/R_0)^h}{1+(R_k/R_0)^h} \right]^{\frac{b-a}{h}}$$

measures **model**

$$\chi^2(\vec{q}) = \sum_i \frac{[J_d(E_i,t) - J_m(E_i,\vec{q})]^2}{\sigma^2(E_i,t)}$$

uncertainties

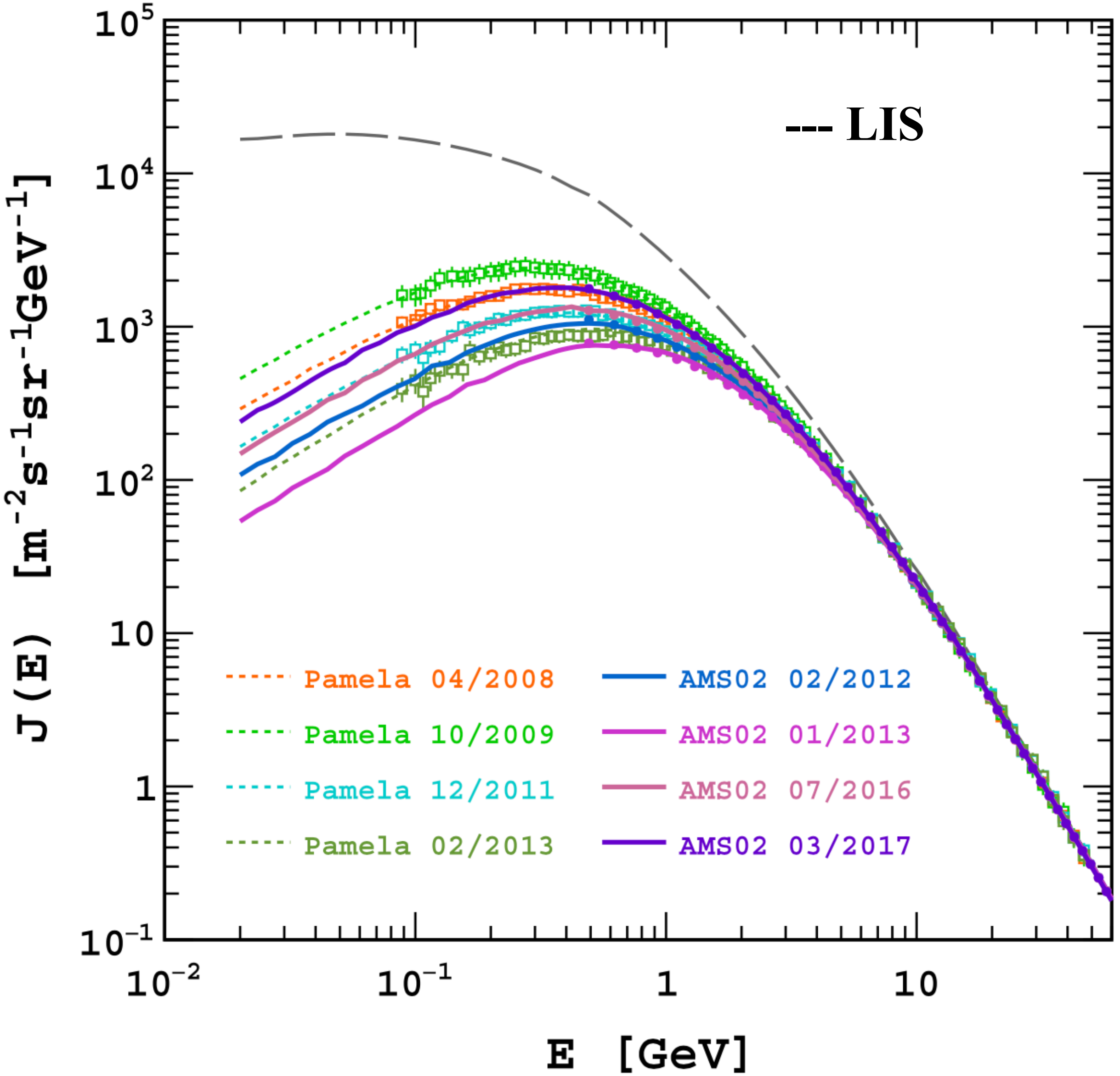
6D vector :
{B$_0$,α,A,K$_0$,a,b)



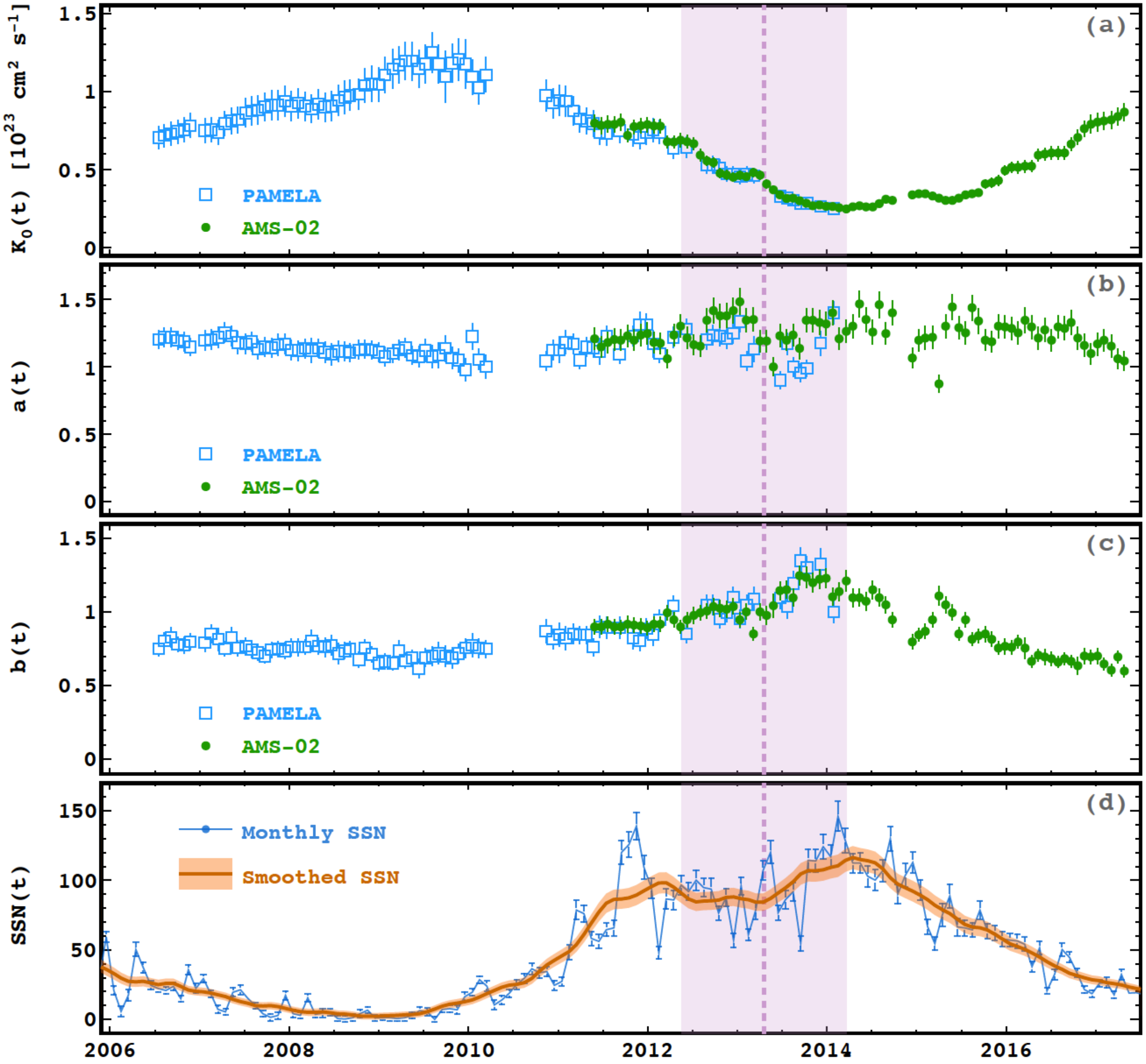
The shaded areas represent the error in the minimum estimation.

Best-Fit Model Fluxes

The flux model is estimated as: $J_{\vec{q}}(E) = J_{\vec{q}^-}(E)\mathcal{P}(t) + J_{\vec{q}^+}(E)[1 - \mathcal{P}(t)]$

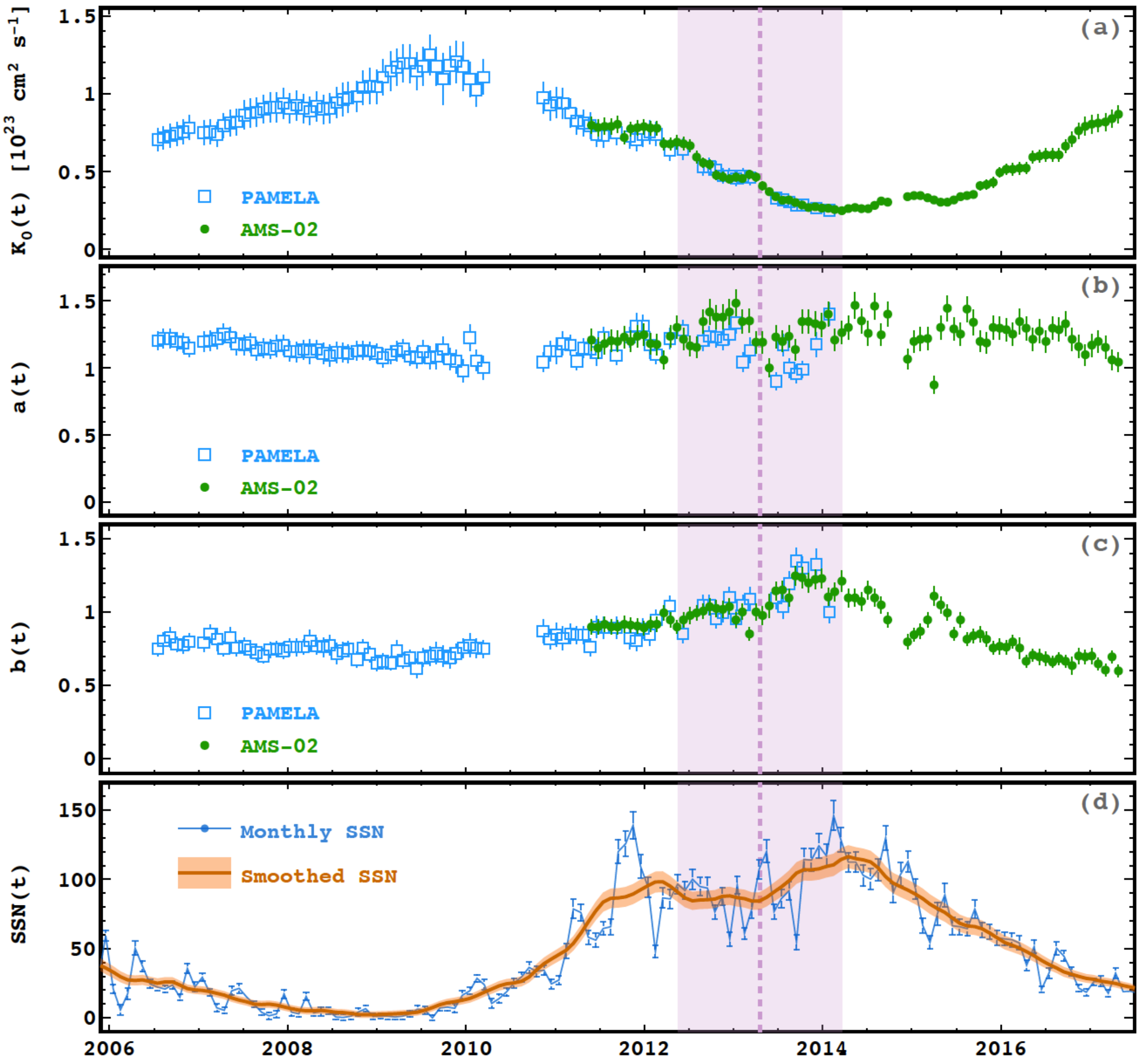


Transport Parameters vs Time



K_0 parameter appears anti-correlated with the monthly SSN.

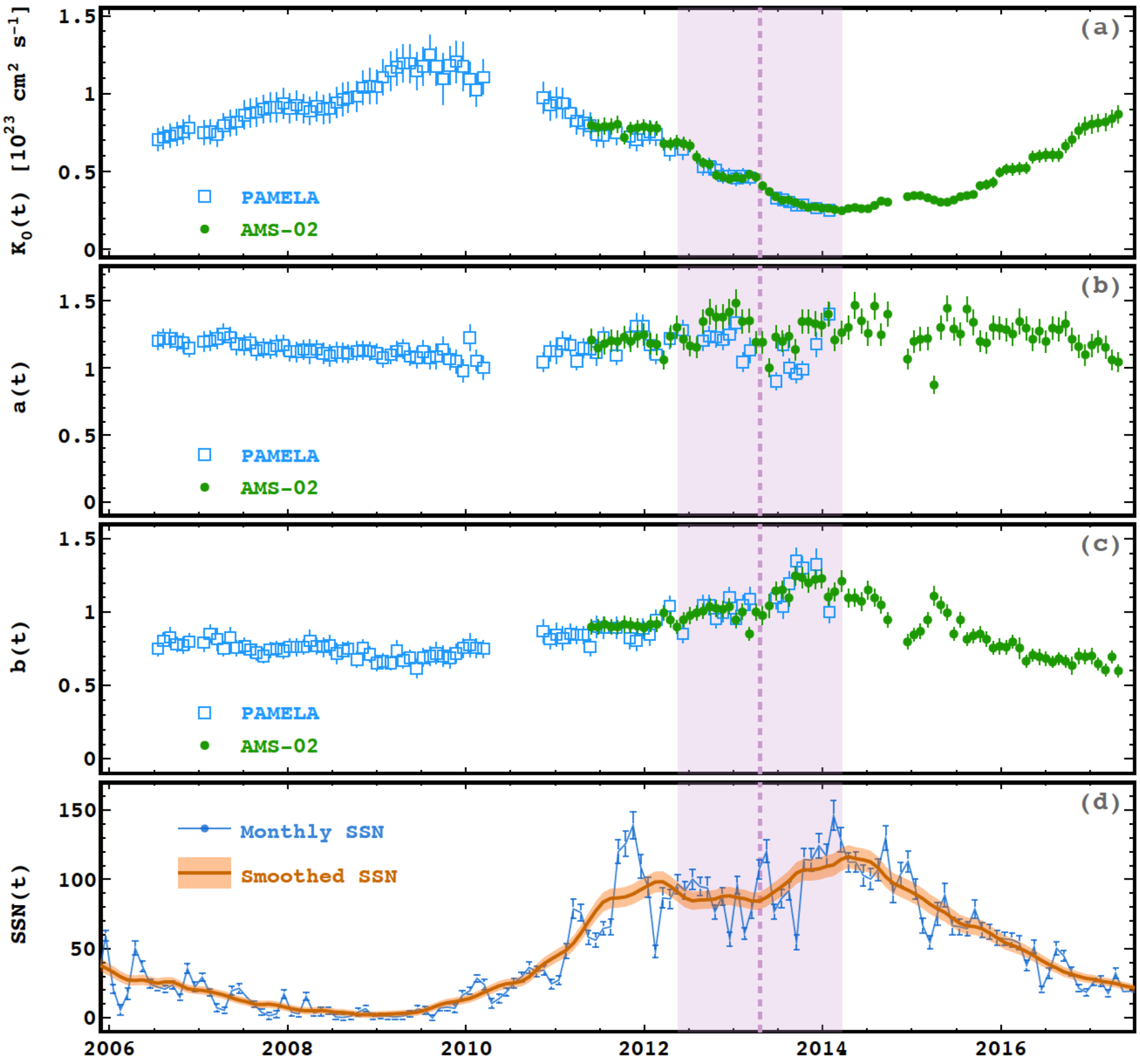
Transport Parameters vs Time



K_0 parameter appears anti-correlated with the monthly SSN.

Larger values of K_0 imply faster CR diffusion inside the heliosphere, causing a milder attenuation of the LIS, i.e., giving a higher flux of cosmic protons in the GeV energy region.

Transport Parameters vs Time

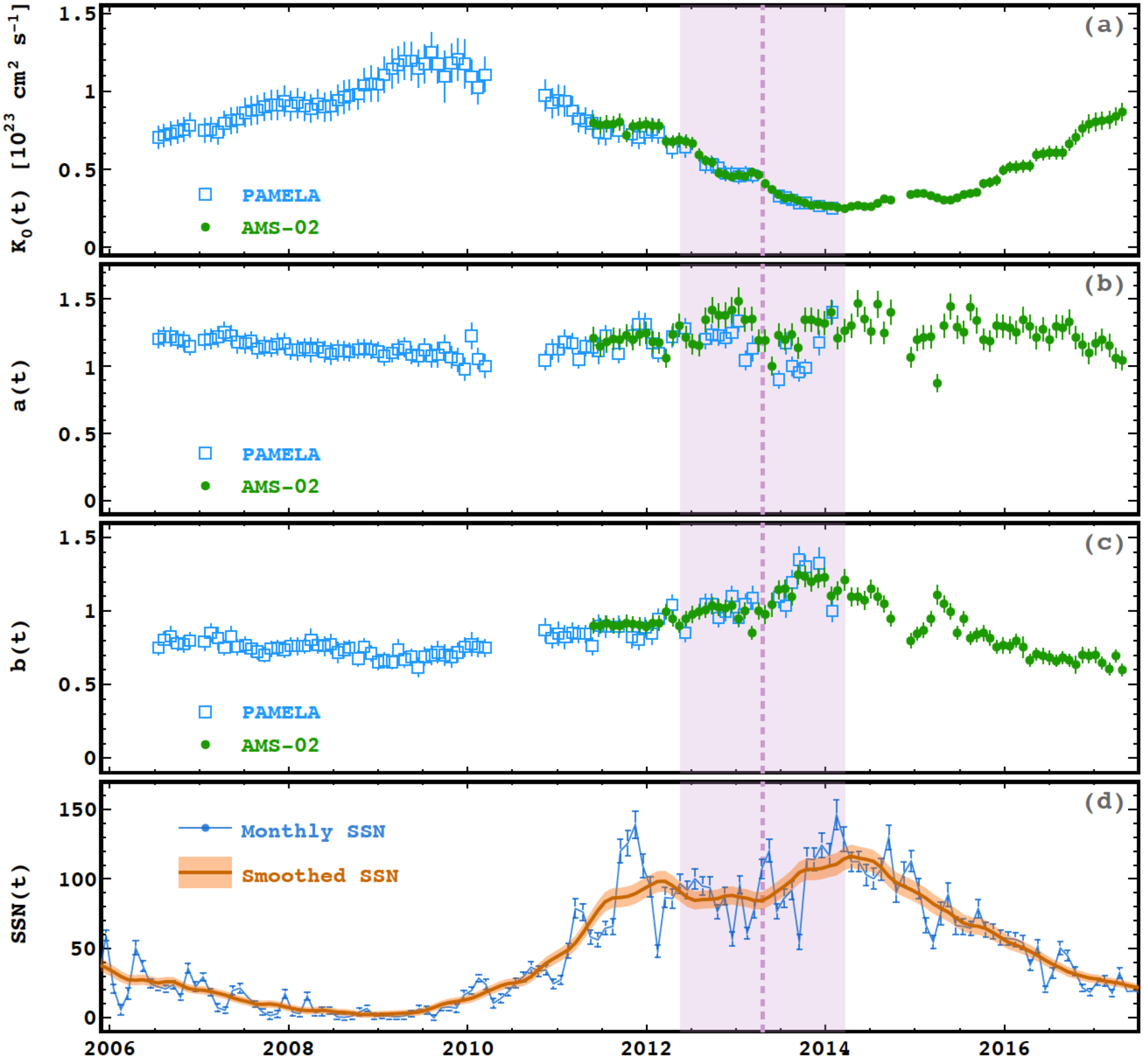


K_0 parameter appears anti-correlated with the monthly SSN.

Larger values of K_0 imply faster CR diffusion inside the heliosphere, causing a milder attenuation of the LIS, i.e., giving a higher flux of cosmic protons in the GeV energy region.

Lower K_0 values imply slower CR diffusion which is typical in epochs of high solar activity where the modulation effect is significant.

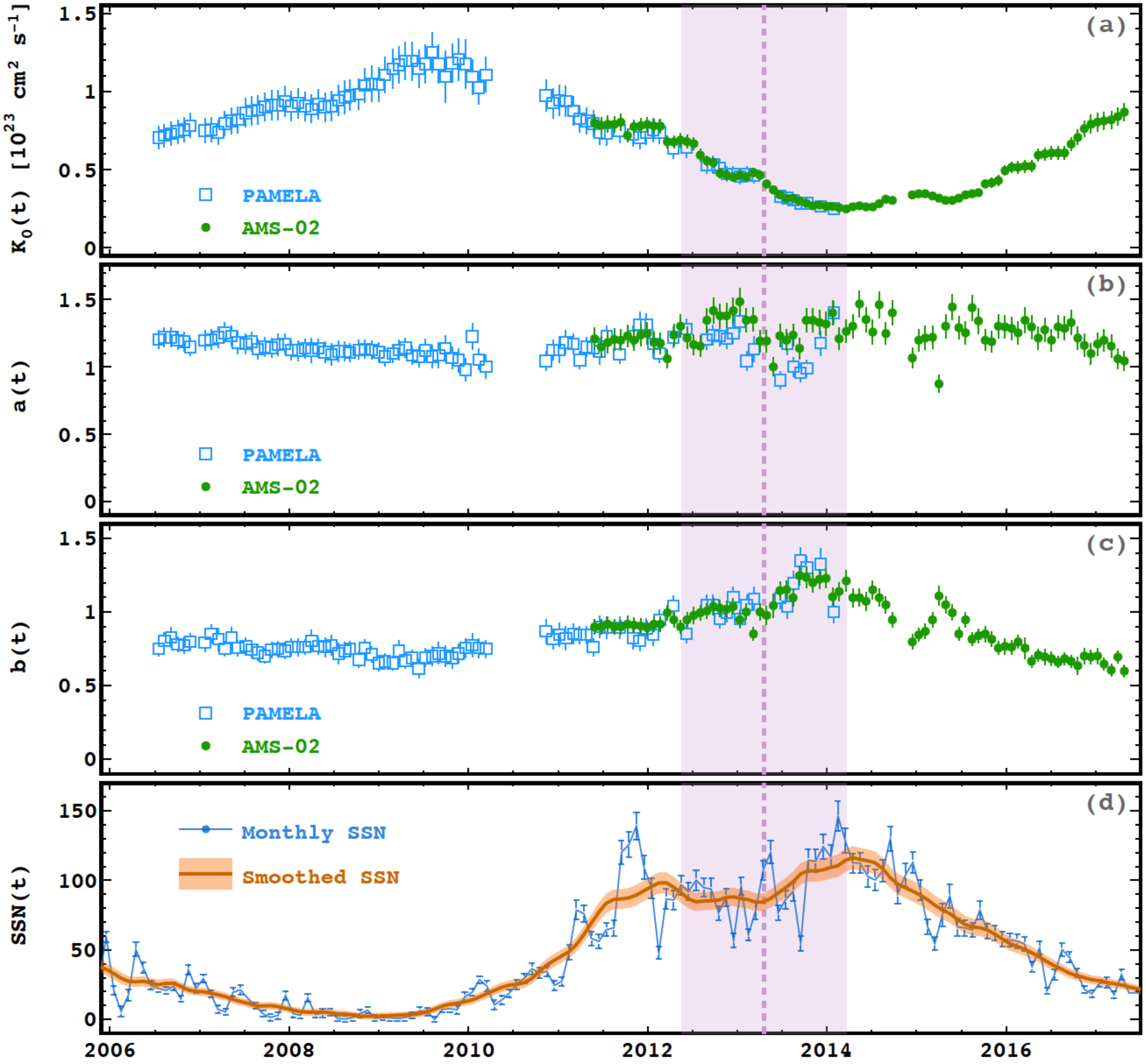
Transport Parameters vs Time



The index a is found to be essentially time independent.

It is poorly constrained in the A > 0 phase, because the AMS-02 data are available only above 1GV of rigidity.

Transport Parameters vs Time



The index a is found to be essentially time independent.

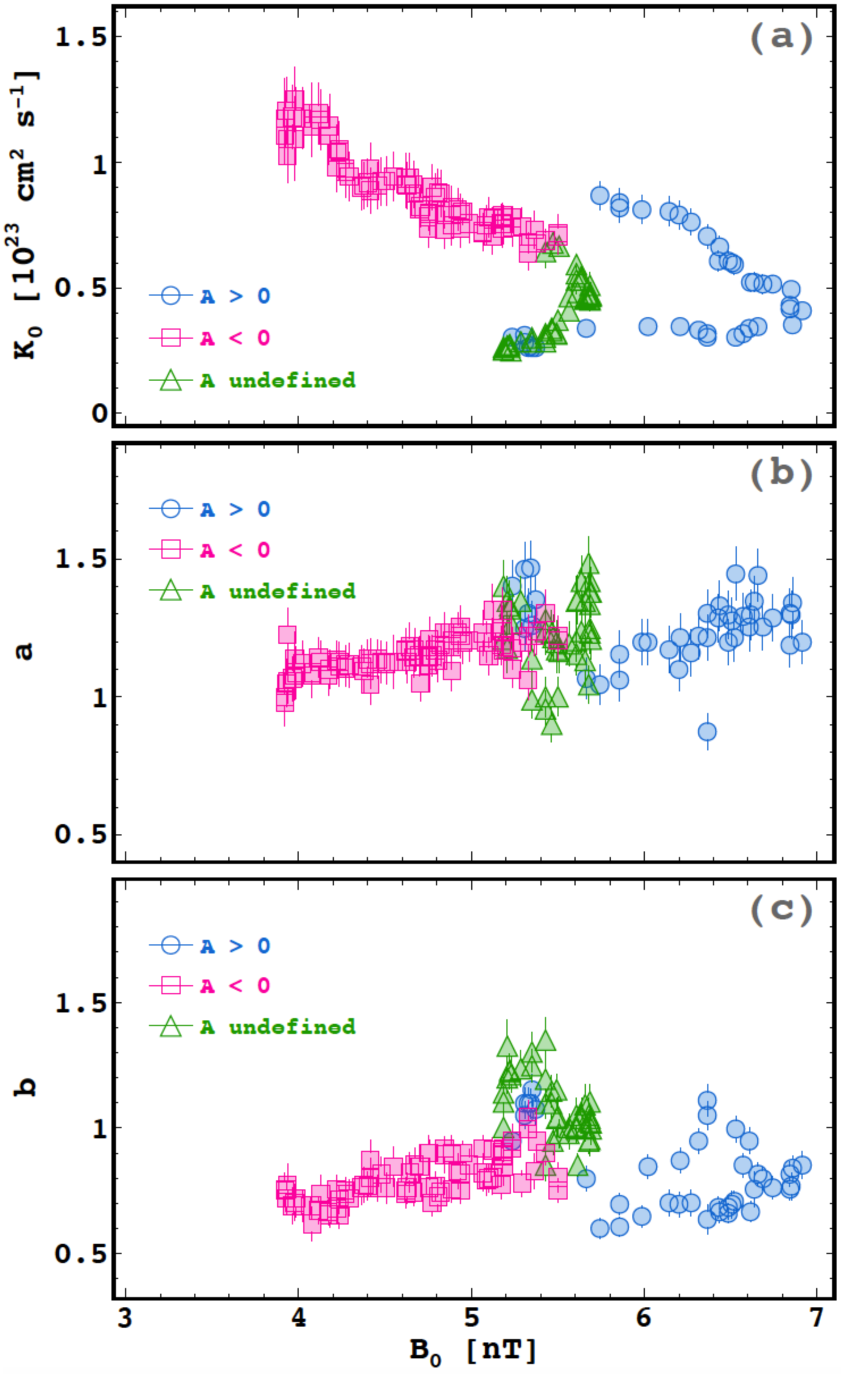
It is poorly constrained in the A > 0 phase, because the AMS-02 data are available only above 1GV of rigidity.

The index b is found to be correlated with the SSN, therefore anti-correlated with K_0.

During solar minimum: K_0 is large and the CR diffusion is fast and its rigidity dependence is shallow ($b \simeq 0.8$).

During solar maximum: K_0 is smaller, CR diffusion is slow and its rigidity dependence is more pronounced ($b \simeq 1.3$).

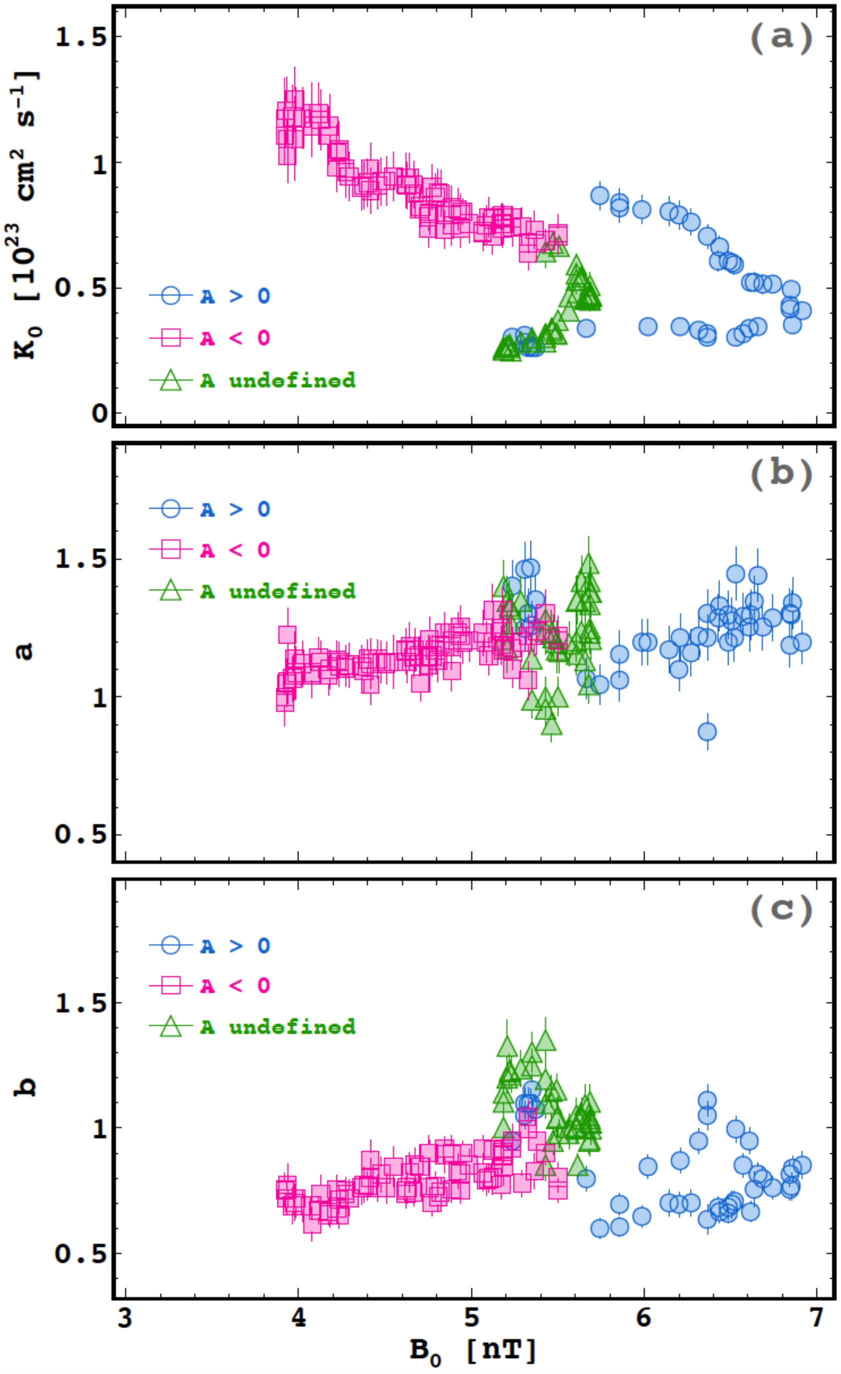
Parameters Cross-correlations



Our results confirm that the relationship between K_0 and B_0 becomes complex when the examination is done over a large fraction of the solar cycle that include polarity changes.

In particular, two distinct relationships can be observed for $A < 0$ and $A > 0$ polarity conditions. *[Wang, B. B. et. al. Phys. Rev. D 100, 063006 (2019)]*

Parameters Cross-correlations



Our results confirm that the relationship between K_0 and B_0 becomes complex when the examination is done over a large fraction of the solar cycle that include polarity changes.

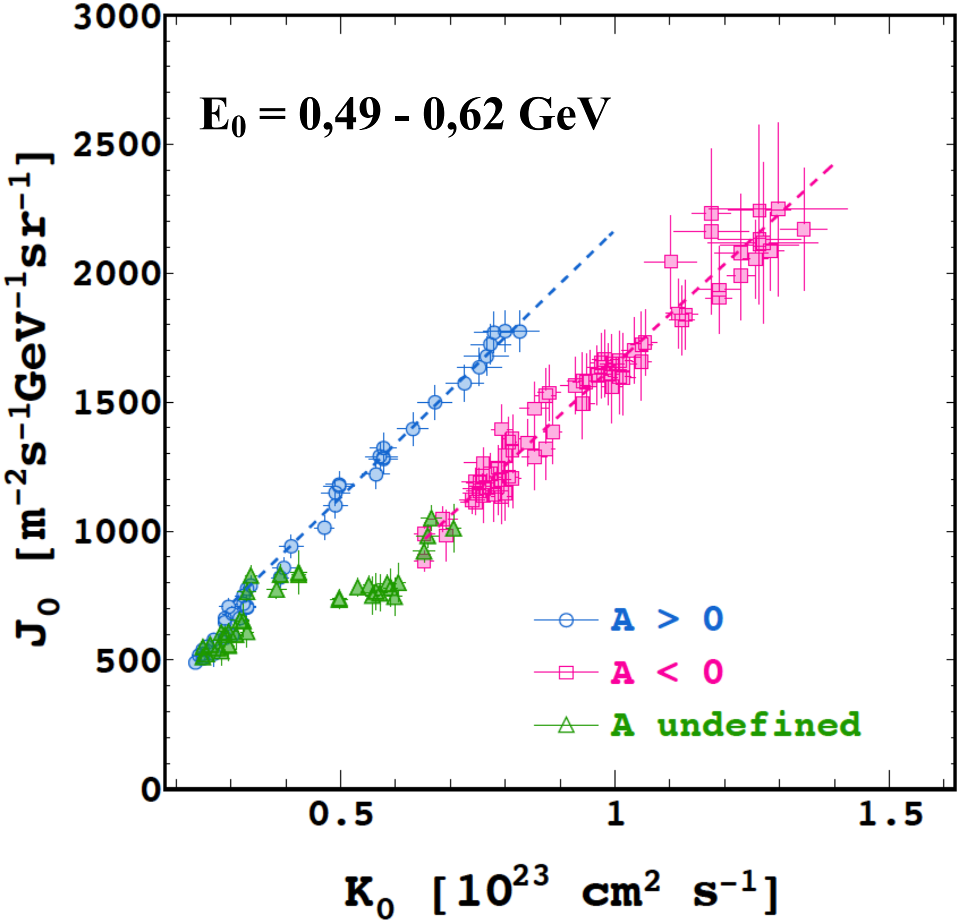
In particular, two distinct relationships can be observed for A < 0 and A > 0 polarity conditions. *[Wang, B. B. et. al. Phys. Rev. D 100, 063006 (2019)]*

Between the spectral index parameters a and b with the HMF magnitude B_0, smoother relationships were found.

Both parameters are seen to depend only weakly on the polarity phase, and no particular cross-correlation is observed between two spectral indices.

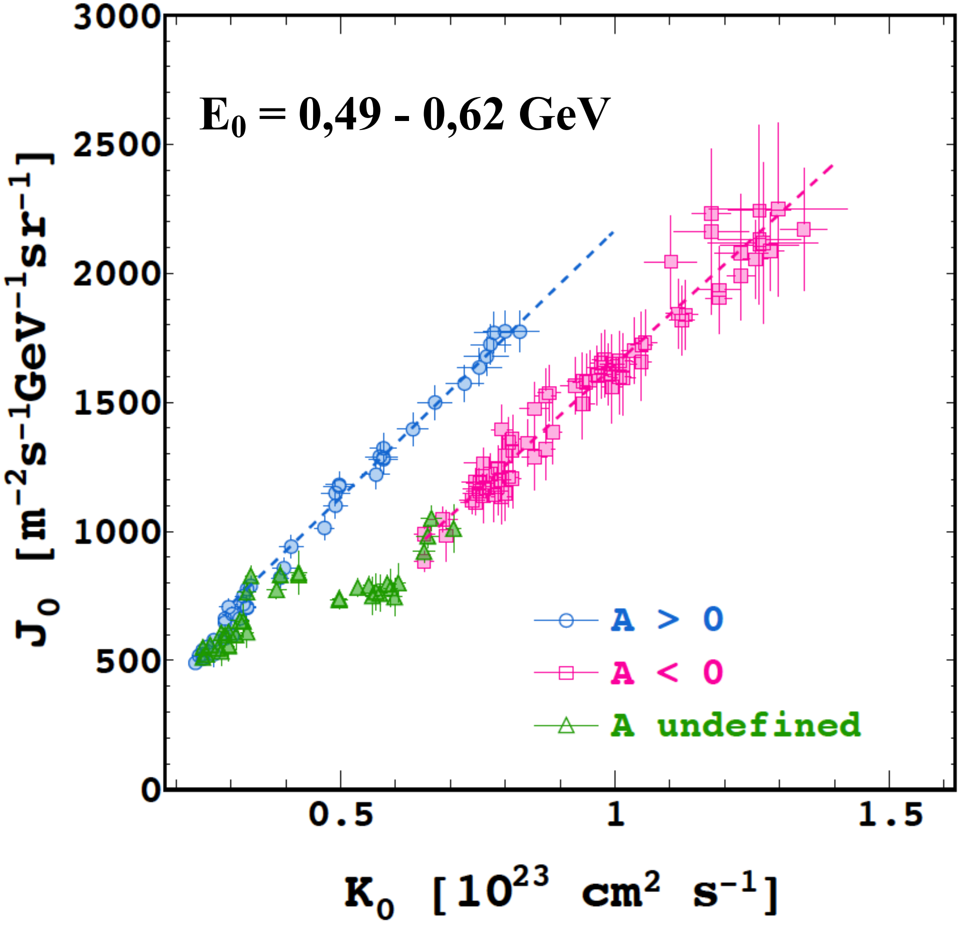
Parameters Cross-correlations

Correlation is remarkably linear during epochs of well defined polarity.



Parameters Cross-correlations

Correlation is remarkably linear during epochs of well defined polarity.



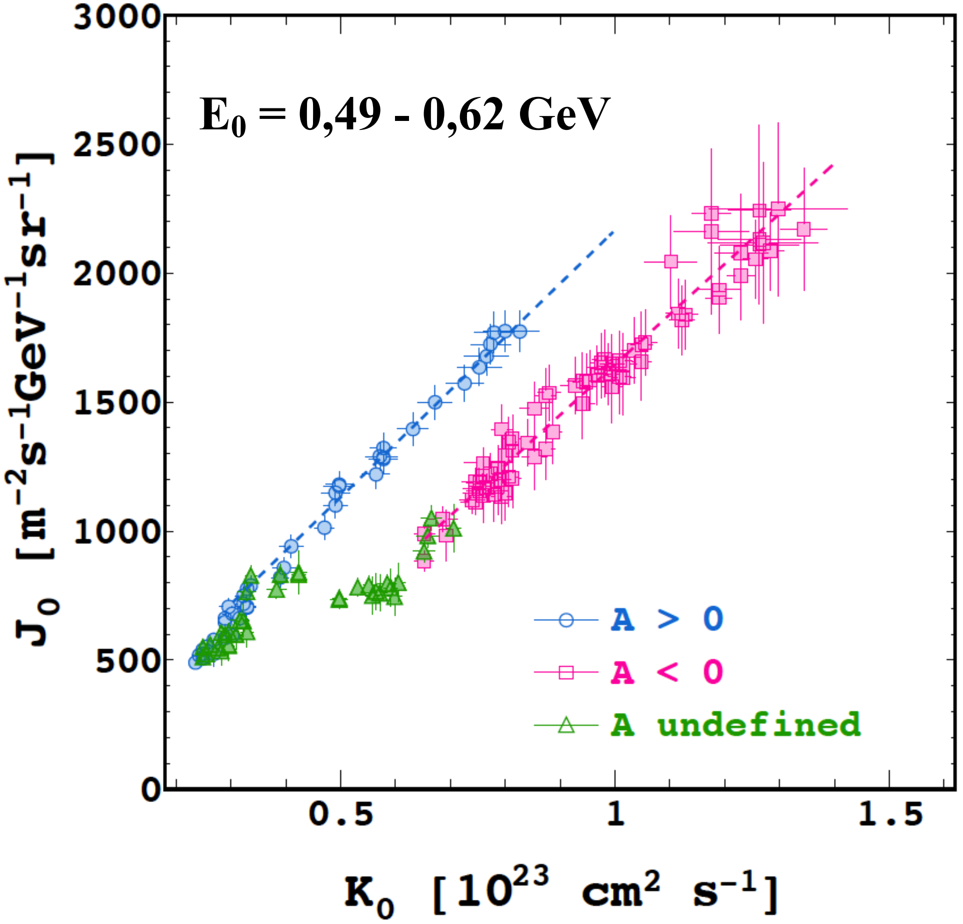
$$J_0(K_0) = \eta K_0 + J_{\mathrm{off}}$$

$$A > 0 \begin{cases} \eta^{+} = (2212 \pm 250)\ \mathrm{x}\ 10^{-23}\ \mathrm{cm}^{-4}\ \mathrm{GeV}^{-1}\ \mathrm{sr}^{-1} \\ J^{+}_{\mathrm{off}} = 46 \pm 21\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}\ \mathrm{GeV}^{-1}\ \mathrm{sr}^{-1} \end{cases}$$

$$A < 0 \begin{cases} \eta^{-} = (1929 \pm 260)\ \mathrm{x}\ 10^{-23}\ \mathrm{cm}^{-4}\ \mathrm{GeV}^{-1}\ \mathrm{sr}^{-1} \\ J^{-}_{\mathrm{off}} = -286 \pm 68\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}\ \mathrm{GeV}^{-1}\ \mathrm{sr}^{-1} \end{cases}$$

Parameters Cross-correlations

Correlation is remarkably linear during epochs of well defined polarity.



$$J_0(K_0) = \eta K_0 + J_{\rm off}$$

A > 0
$\eta^+ = (2212 \pm 250) \times 10^{-23}$ cm^{-4} GeV^{-1} sr^{-1}
$J^+_{off} = 46 \pm 21$ m^{-2} s^{-1} GeV^{-1} sr^{-1}

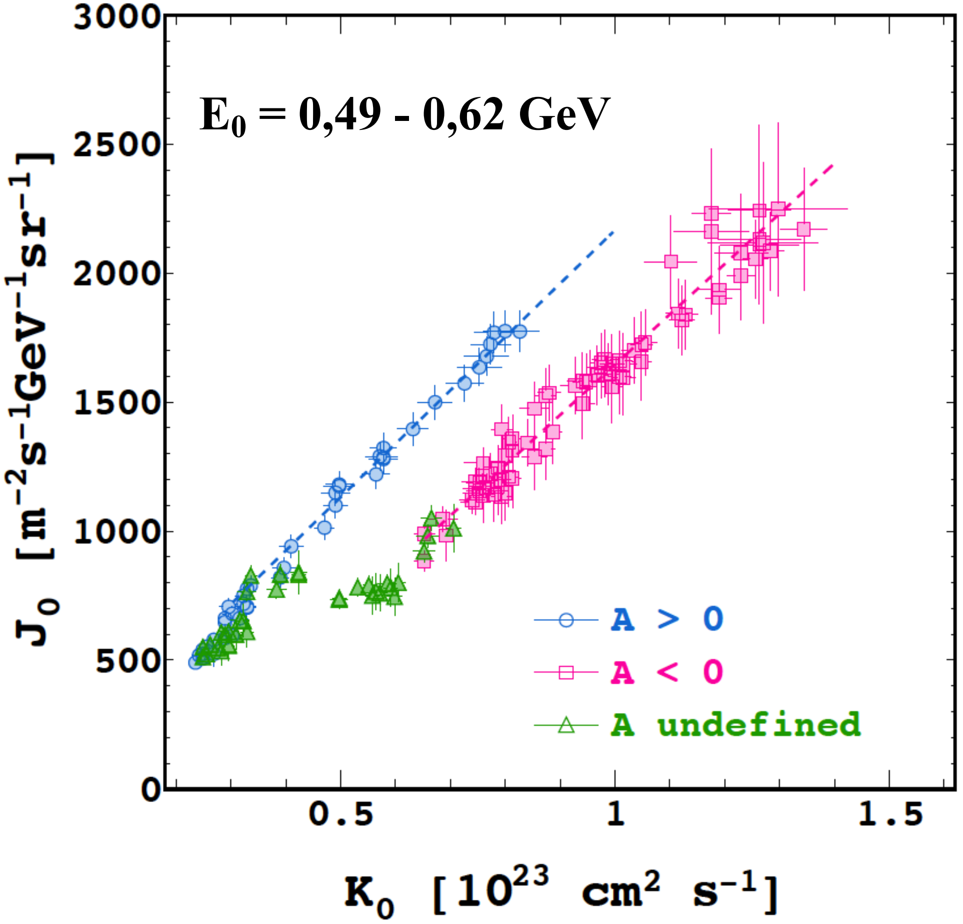
A < 0
$\eta^- = (1929 \pm 260) \times 10^{-23}$ cm^{-4} GeV^{-1} sr^{-1}
$J^-_{off} = -286 \pm 68$ m^{-2} s^{-1} GeV^{-1} sr^{-1}

Within fitting errors the two slopes, η^+ and η^-, are consistent with each other, therefore the slope of $J_0(K_0)$ is polarity and charge-sign independent.

The quantity $\Delta J \equiv J^+_{off} - J^-_{off}$ can be used as a measurement of the net effect of drift on the total CR flux, for a given level of CR diffusion.

Parameters Cross-correlations

Correlation is remarkably linear during epochs of well defined polarity.



$$J_0(K_0) = \eta K_0 + J_{\rm off}$$

A > 0
- $\eta^+ = (2212 \pm 250) \times 10^{-23}\ \mathrm{cm^{-4}\ GeV^{-1}\ sr^{-1}}$
- $J^+_{\rm off} = 46 \pm 21\ \mathrm{m^{-2}\ s^{-1}\ GeV^{-1}\ sr^{-1}}$

A < 0
- $\eta^- = (1929 \pm 260) \times 10^{-23}\ \mathrm{cm^{-4}\ GeV^{-1}\ sr^{-1}}$
- $J^-_{\rm off} = -286 \pm 68\ \mathrm{m^{-2}\ s^{-1}\ GeV^{-1}\ sr^{-1}}$

Within fitting errors the two slopes, η^+ and η^-, are consistent with each other, therefore the slope of $J_0(K_0)$ is polarity and charge-sign independent.

The quantity $\Delta J \equiv J^+_{\rm off} - J^-_{\rm off}$ can be used as a measurement of the net effect of drift on the total CR flux, for a given level of CR diffusion.

Under periods of undefined polarity, the role of drift is not well understood, but the flux J_0 remains correlated with K_0.

Summary & Conclusions

- **A model based in an improved version of SolarProp Framework has been developed.**
- **A total of six time-dependent parameters that are of relevance for the phenomenology of CRs modulation have been identified. Three of them describe the status of the heliosphere (B_0, α, A) at a given time, and the other three are related to diffusion (K_0, a, b).**
- **AMS-02 and PAMELA data has been used to estimate the best-fit parameters for Bartels Rotations between June 2006 and May 2017.**
- **K_0 parameter appears anti-correlated with the monthly SSN. Meanwhile, the index b is found to be correlated with the SSN, therefore anti-correlated with K_0. The index a is found to be essentially time independent.**
- **Correlation between K_0 and J_0 is remarkably linear during epochs of well defined polarity.**
- **The release of more AMS-02 data in Bartels Rotations will help to improve the model and in consequence constraint better these parameters.****

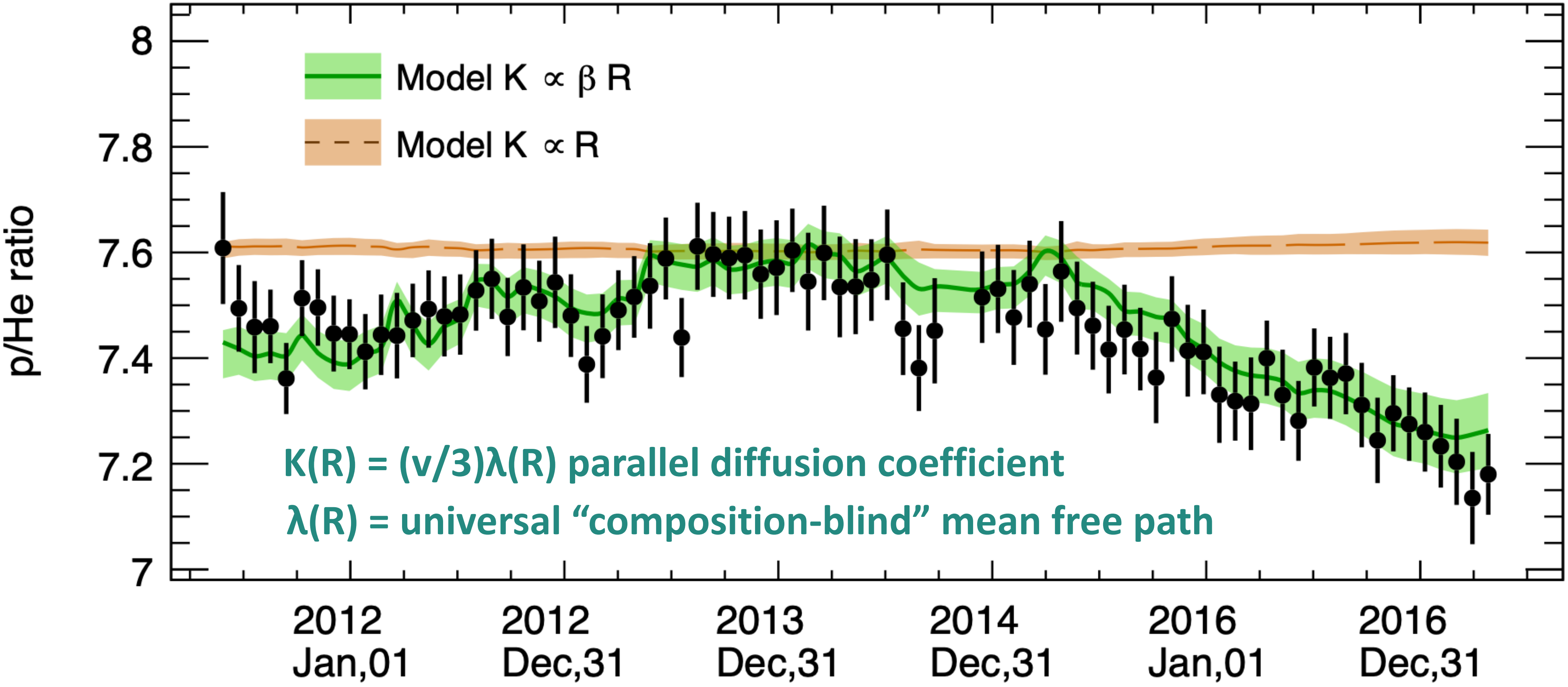
***Federico Donnini talk "Precision Measurement of the Monthly Proton, Helium, Carbon and Oxygen Fluxes in Cosmic Rays with the Alpha Magnetic Spectrometer on the International Space Station"*

Backup

Insights from the p/He ratio: diffusion

The p/He long-term behavior is a signature of *universality* of the CR mean free path λ(R)

NT+ PRL 121 251104 (2018) [arXiv:1811.08909] NT+ ASR 64 2477 (2019)[arXiv:1906.11477]

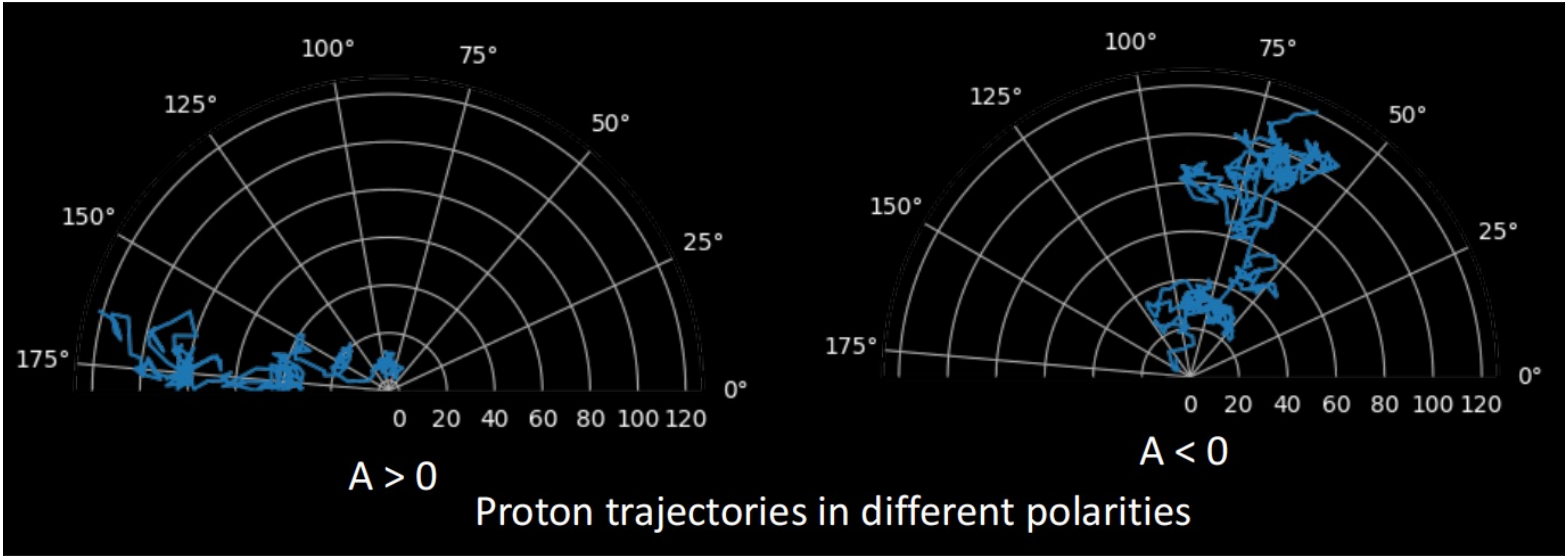


K(R) = (v/3)λ(R) parallel diffusion coefficient

λ(R) = universal “composition-blind” mean free path

Trajectories

Behrouz Khiali presentation at ECRS 2022



Proton trajectories in different polarities

In the A > 0 drift cycle, GCR protons generally drift from the polar regions to reach Earth, whereas, in the A < 0 cycle, they mainly drift along the Heliospheric Current Sheet, taking a much longer time to reach Earth.

Some variables of the model

The HP and TS positions were fixed at $r_{HP} = 122$ AU and $r_{TS} = 85$ AU, deduced from the Voyager-1 observations. The data suggest that the TS may vary over the solar cycle of the order of a few AU, but its impact in the CR fluxes is negligible.

Rigidity break (R_k) represents the scale rigidity value where the CR Larmor radius matches the correlation length of the HMF power spectrum, which is at the GV scale. Regarding the value of R_k, we found that time variations on this quantity do not give appreciable variations in the CR fluxes. ***[Potgieter, M. S. Solar Phys. 289, 391 (2014)]***

Diffusion Tensor in the Model

Particles moving in a magnetic turbulence are pitch-angle scattered by the random HMF irregularities.

This process is described by the three diffusion coefficients $K_{||}$, $K_{r\perp}$ and $K_{\theta\perp}$.

To estimate these coefficients the Quasi Linear Theory (QLT) is used. The QLT has been successful at describing parallel diffusion, specially in its time-dependent and non-linear extensions. *[Jokipii, J. R. 1966, Astrophysical Journal, 146, 480]*

$$K_{||} = K_0 \frac{\beta}{3} \frac{(R/R_0)^a}{(B/B_0)} \left[\frac{(R/R_0)^h + (R_k/R_0)^h}{1+(R_k/R_0)^h} \right]^{\frac{b-a}{h}}$$

QLT

$$K_{\perp\theta} = \xi_{\perp\theta} \times g(\theta) \times K_{||}$$

$$K_{\perp r} = \xi_{\perp r} \times K_{||}$$

Keeping constant values for both $\xi_\perp$ factors implies that $K_{||}$ and $K_\perp$ follow the same rigidity dependence, which may be a simplification in the high rigidity domain. However, simulations based on QLT agree for nearly rigidity independent of ξ, with typical values between 0.02 and 0.04. In the model he have used ξ = 0.02.

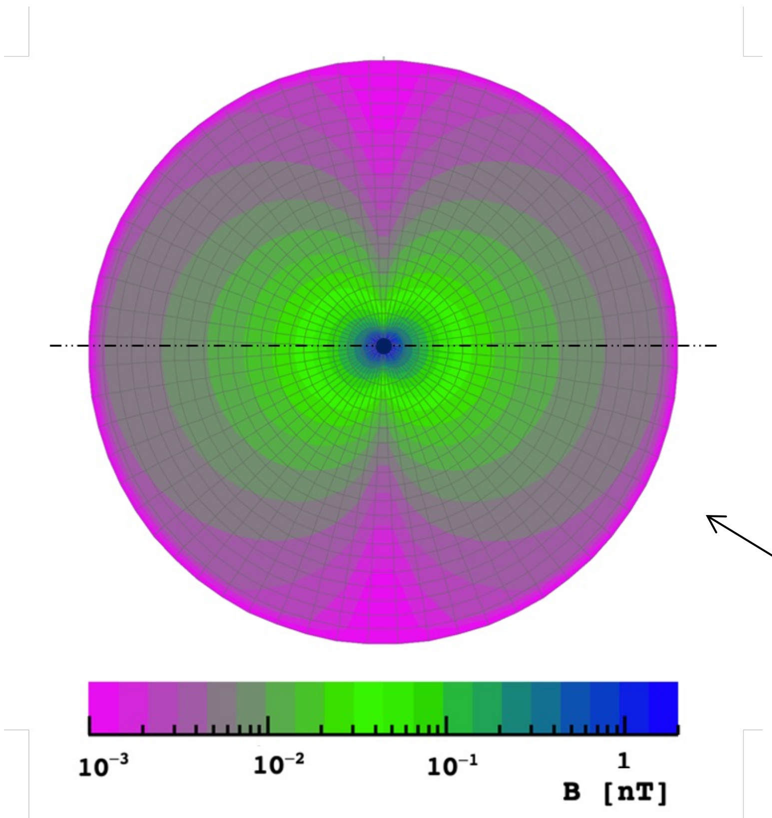
Heliospheric magnetic field

$$\vec{B} = AB_0\left(\frac{r_0}{r}\right)^2(\hat{e}_r - \hat{e}_\phi \tan\psi)[1 - 2H(\theta - \Theta)]$$

determines the position of the wavy HCS

$$\Theta = \pi/2 + \sin^{-1}[\sin(\alpha)\sin(\Omega r/V_{\mathrm{SW}})]$$

tilt angle

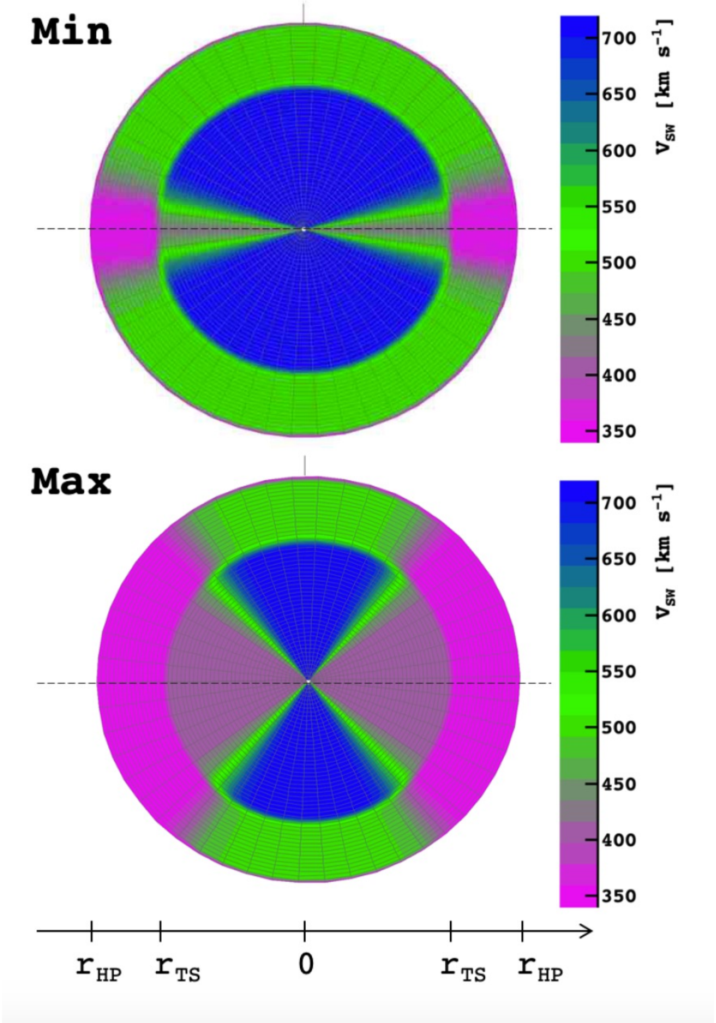


Side view of the HMF model in the (x; z) plane of the heliosphere. The dashed line is the equatorial plane.

$$\tan\psi = \left\{\frac{\Omega(r - r_\odot)}{V} + \left(\frac{r\delta(\theta)}{r_\odot}\right)^2\right\}^{1/2}$$

modifications on **HMF** and **winding angle** are effective only near the polar regions

Solar Wind



Side view of the SW speed profile in the (x; z) of the heliosphere, showing its latitudinal dependence in the typical cases of solar minimum (Min, for α = 10) and solar maximum (Max, for α = 60), where the latitudinal transition from a slow to a fast region depends on the HCS tilt angle.

10º

tilt angle of the HCS

width of the transition

$$\theta_T = \alpha + \delta\alpha$$

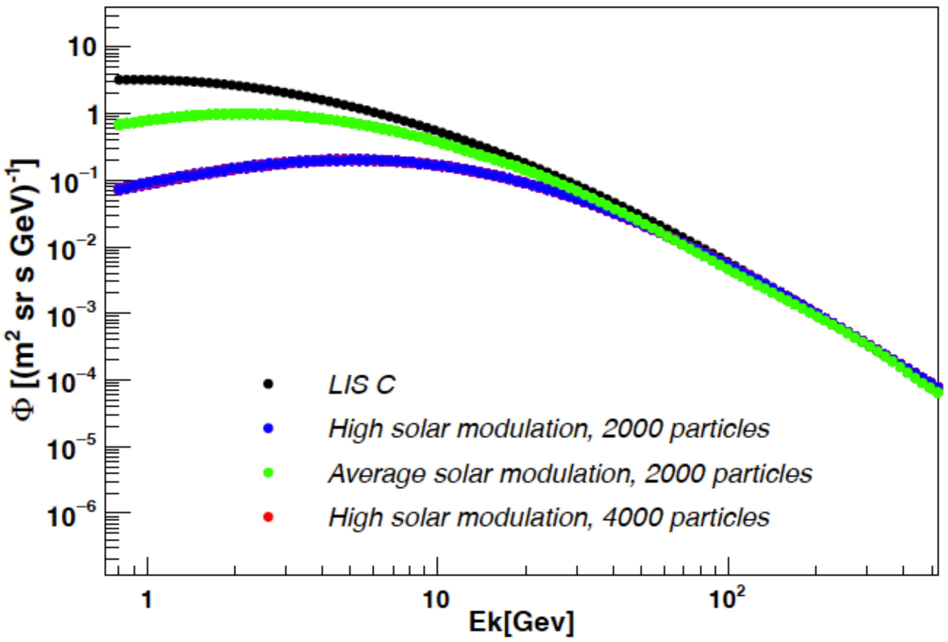
Polar angle at which the Vsw changes from a slow to a fast region

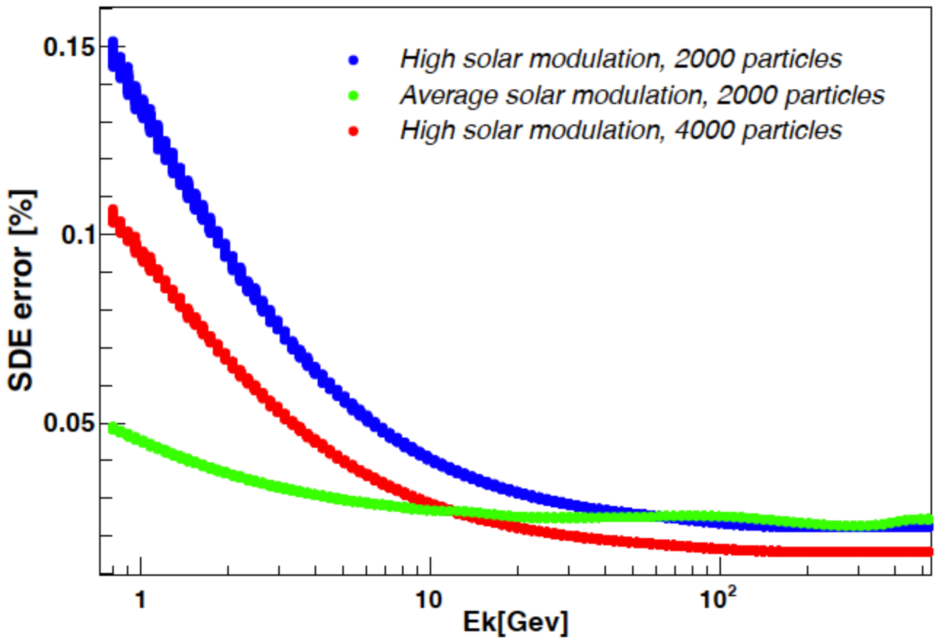
$$V_{SW}(r,\theta) = V_0 \left\{1.475 \mp 0.4 \tanh\left[6.8\left(\theta - \frac{\pi}{2} \pm \theta_T\right)\right]\right\} \times \left[\frac{S+1}{2S} - \frac{S-1}{2S}\tanh\left(\frac{r - r_{TS}}{L}\right)\right],$$

this allow us to use α as a proxy to describe the change in time of θ_T correlated to the solar activity.

SDE error

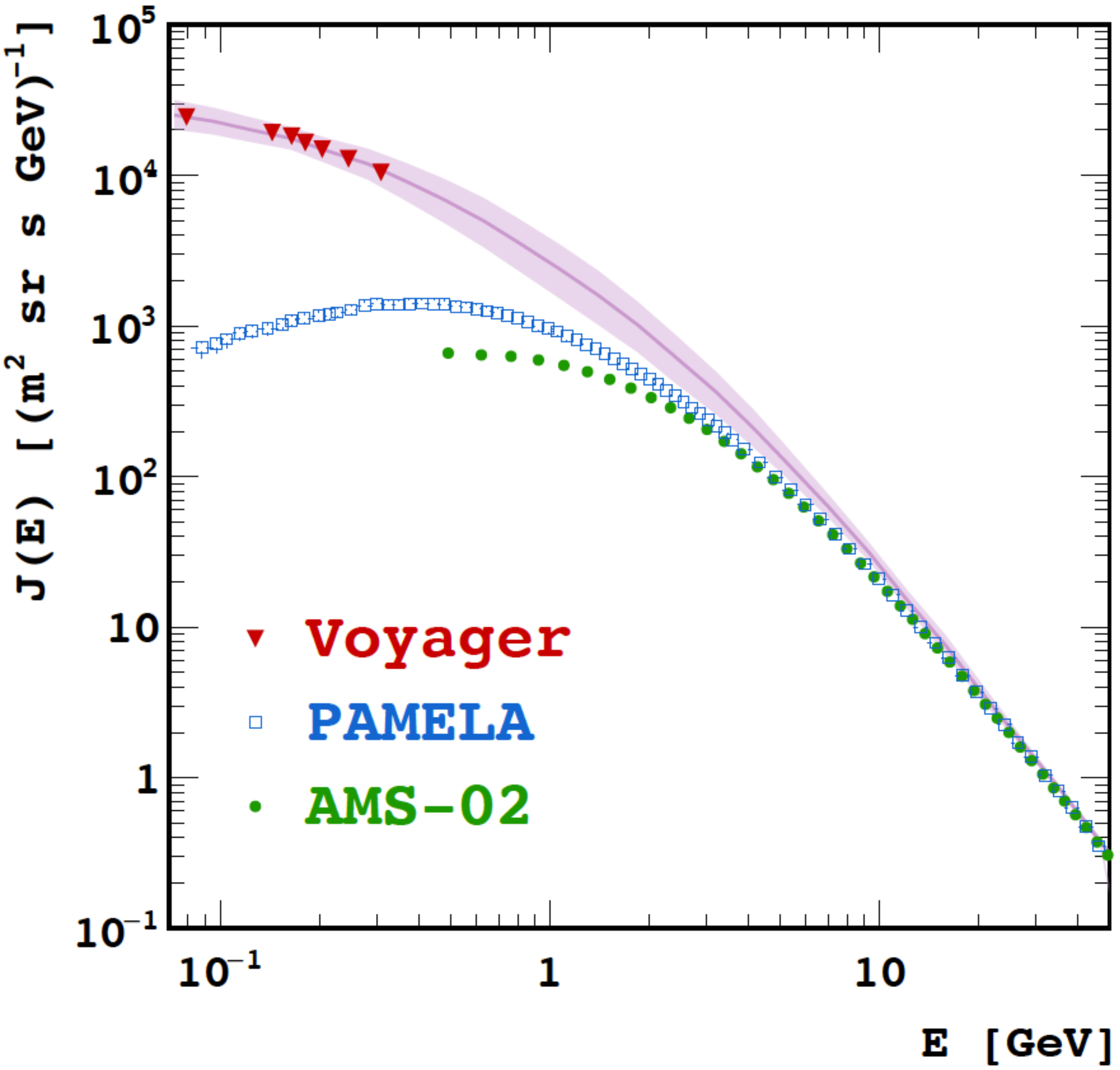
For each selected heliospheric condition q and given energy point E, N_G pseudo-particles are generated from Earth and propagated with random motion backward in time, N of which reach the heliospheric boundary.





The modulated flux can be simply calculated as $J_q = (N = N_G) J_{LIS}$. Therefore, the modulated flux has an associated statistical fluctuation $\propto 1/N^{1/2}$.

Local Interstellar Spectrum



LIS for CR protons that relies on a two-halo model of CR propagation in the Galaxy.

Calculations of the proton LIS were constrained by various sets of measurements: low-energy proton data (at 140 - 320 MeV) collected by Voyager-1 beyond the HP, high-energy proton measurements (E ≥ 60 GeV) made by AMS-02 in low Earth orbit, along with measurements of the B/C ratio from both experiments.

[Feng, J. et. al. Phys. Rev. D 94, 123007 (2016)]

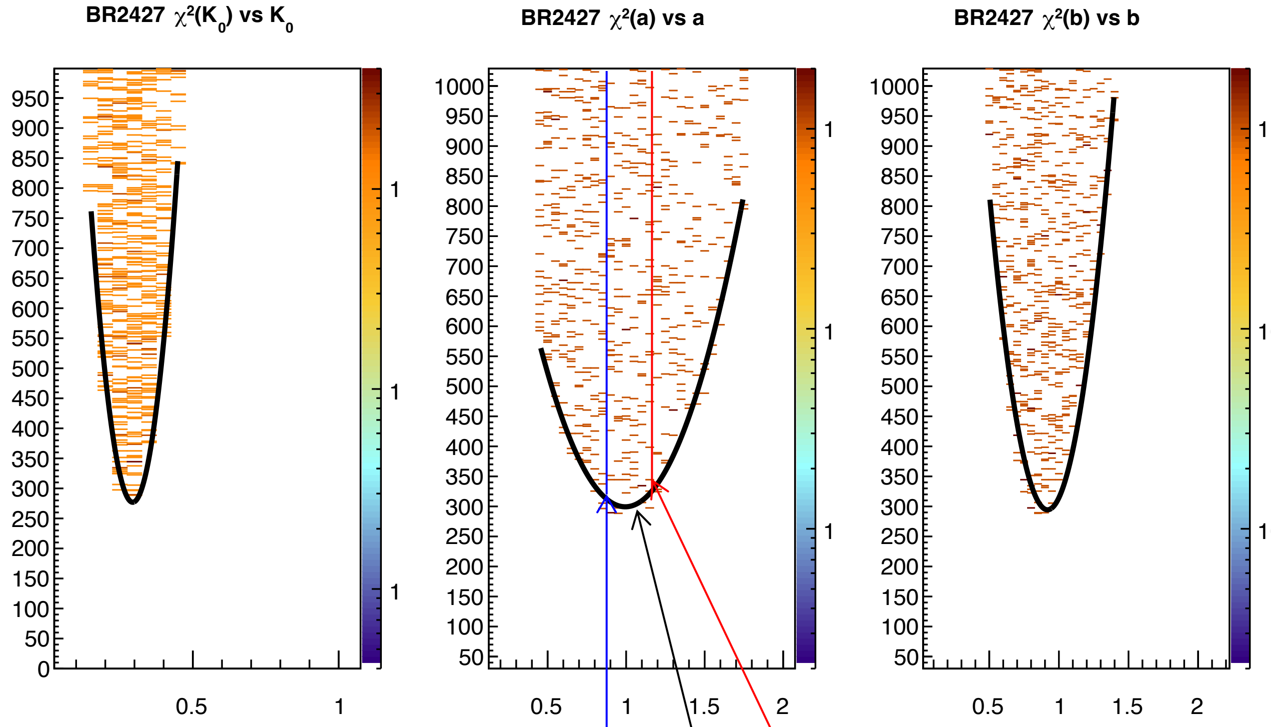
[Tomassetti et. al. Phys. Rev. Lett. 121, 251104 (2018)]

Transport Parameters

IP method

Estimation of the minimum:

- A minimization find the curve that best fit the contour of the distribution, called interpolation method (IP).
- Determination of the minimum, from a parameter scan, and then fitting with the unique parabola that passes through this point and his two neighbors grid values, called discrete method (DC).



Estimation of the error:

$$\sigma^2_{total} = \sigma^2_{param} + \sigma^2_{syst}$$

$$\boldsymbol{\sigma_{param} = \max(\,|x_- - x_{best}|\,,\,|x_+ - x_{best}|\,)}$$

$$\chi^2_{min}(x_\pm) = \chi^2_{min}(x_{best}) + 1,$$

above and below x_{best}

$$\boldsymbol{\sigma_{syst} = |x_{best,IP} - x_{best,DC}|}$$

Transport Parameters: Normalization Factor

The final value of every parameter, x, is evaluated as:

$$x^{\mathrm{Fermi}} = x^{-} \times \mathcal{P}(t) + x^{+} \times [1 - \mathcal{P}(t)]$$