Current status of the first-row CKM unitarity from semileptonic decay processes

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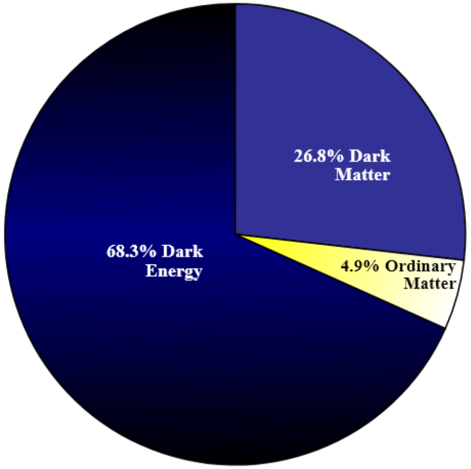
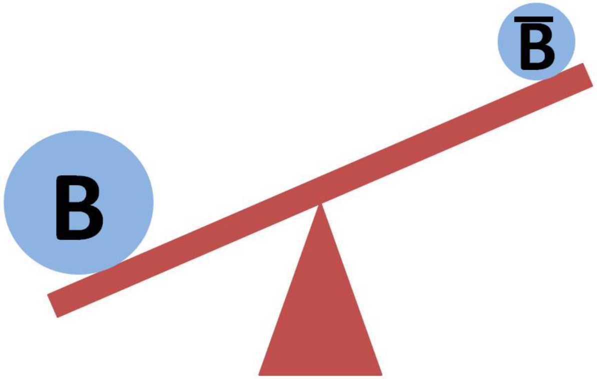
BSM Physics at the Precision Frontier

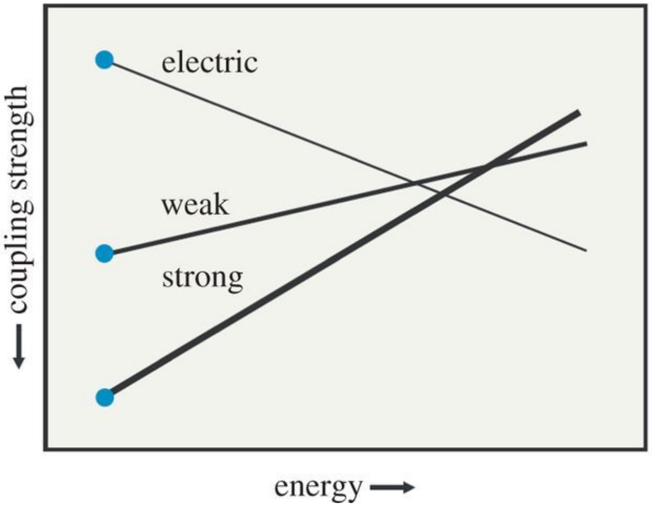
Many unresolved problems call for physics beyond the Standard Model (BSM)



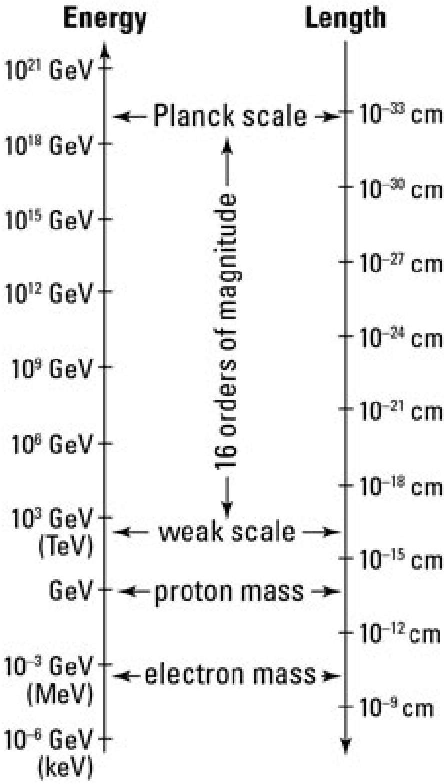
Dark energy, dark matter



Matter-antimatter asymmetry



Unification of forces



Hierarchy problem

BSM Physics at the Precision Frontier

Most of the present anomalies in particle physics arise from **precision experiments**!

- **Muon g-2**: ~4.2σ discrepancy

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 251(41)_{\mathrm{exp}}(43)_{\mathrm{th}} \times 10^{-11}$$

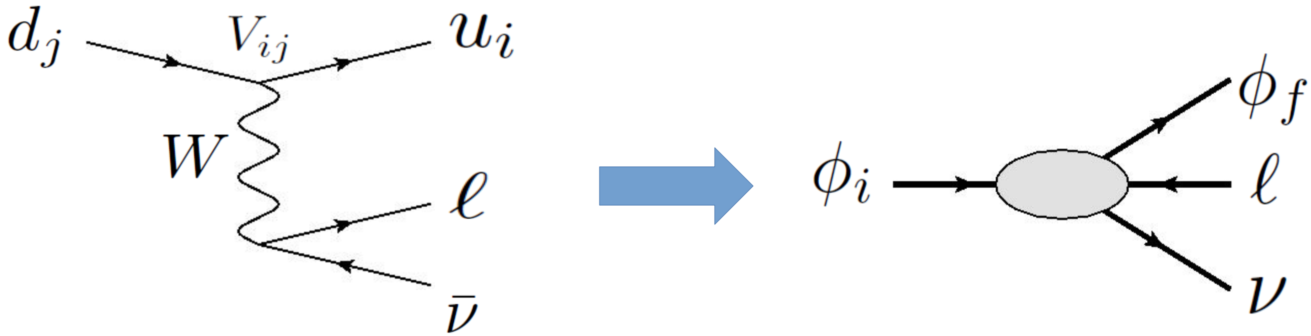
- **B-decay anomalies**: ~3.1σ discrepancy

$$R_K \equiv \frac{\mathcal{B}(\bar{B} \to K\mu^+\mu^-)}{\mathcal{B}(\bar{B} \to Ke^+e^-)} = 0.846^{+0.042}_{-0.039}(\mathrm{stat})^{+0.013}_{-0.012}(\mathrm{syst})$$

Muon g-2 + B-decay anomalies → **“Flavor anomalies”**

…and there is a **THIRD TYPE**!

Anomalies in beta decays



Beta decays had been crucial in the shaping of **Standard Model (SM)**

1930: **Neutrino postulation** by Pauli
1956: Wu’s experiment confirmed **P-violation** in weak interaction (1957 Nobel Prize by Lee and Yang)
1957: Feynman, Gell-Mann, Sudarshan and Marshak: **V-A structure** in the charged weak interaction
1963: **2*2 unitary matrix** by Cabibbo to mix the ΔS=0 and ΔS=1 charged weak current
1973: Kobayashi and Maskawa extended the matrix to 3*3 (**the CKM matrix**), introduced the 3rd generation quarks (Nobel Prize 2008)

$$\psi_{d,f} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_f = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$

The CKM matrix

Anomalies in beta decays

Beta decays place **one of the most stringent tests of SM** through precision measurements of the **first-row CKM matrix elements V_{ud} and V_{us}**

V_{ud}

	$\lvert V_{ud}\rvert$
Superallowed nuclear decays ($0^+ \to 0^+$)	0.97373(31)
Free n decay	0.97377(90)
Mirror nuclei decays	0.9739(10)
Pion semileptonic decay (π_{e3})	0.9740(28)

V_{us}

	$\lvert V_{us}\rvert$
Kaon semileptonic decays ($K_{\ell 3}$)	0.22309(56)
Tau decays	0.2221(13)
Hyperon decays	0.2250(27)

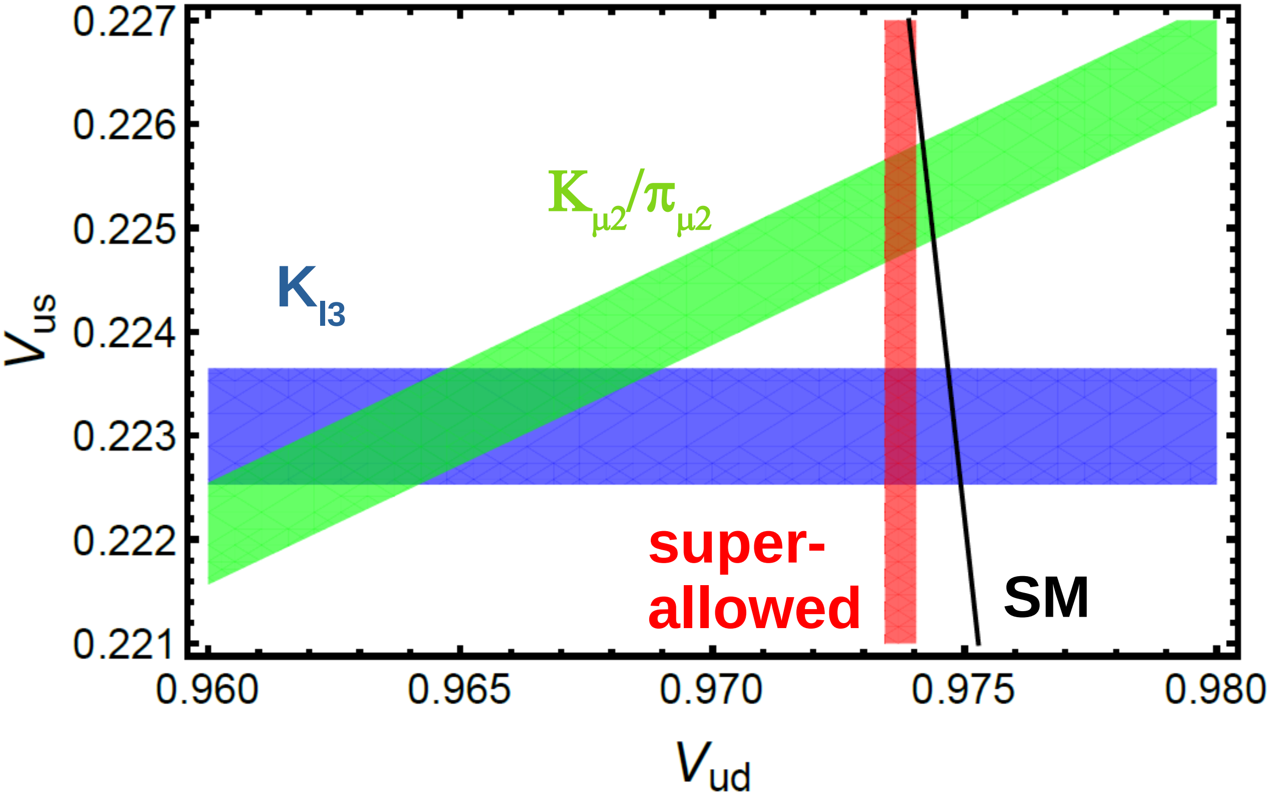
V_{us}/V_{ud}

	$\lvert V_{us}/V_{ud}\rvert$
K/π leptonic decays ($K_{\mu 2}/\pi_{\mu 2}$)	0.23131(51)
K/π semileptonic decays ($K_{\ell 3}/\pi_{e3}$)	0.22908(87)

Anomalies in beta decays

Several **anomalies** are recently observed in the **first-row CKM matrix elements**!

SM prediction: $|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$



"Cabibbo Angle Anomaly (CAA)" ~ 3σ

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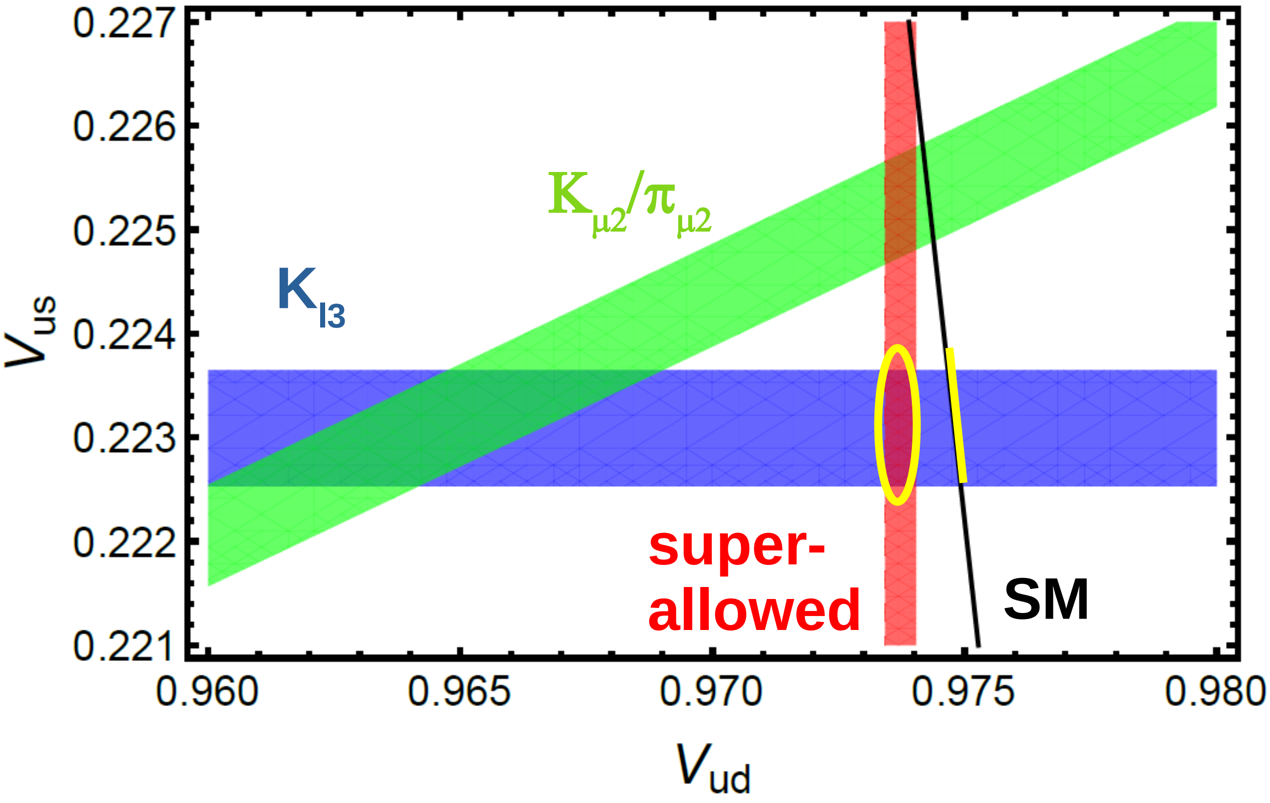

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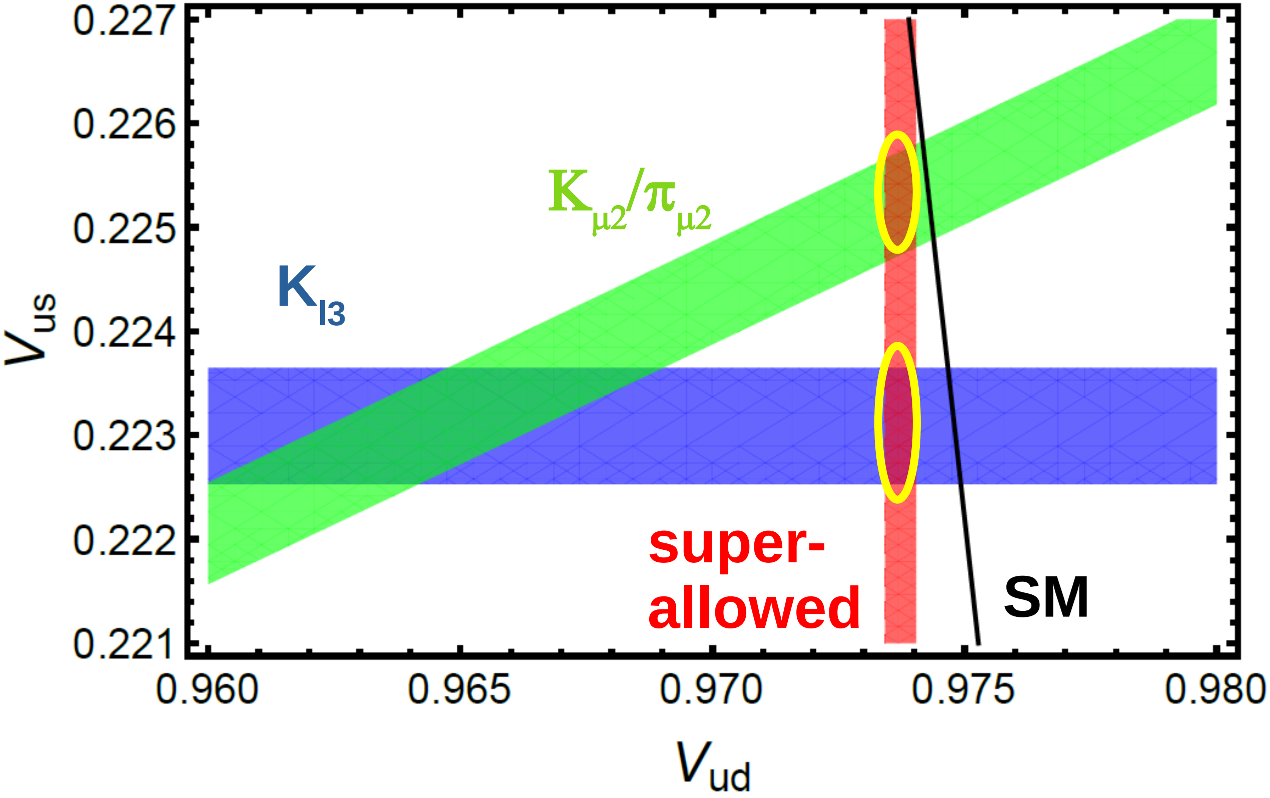


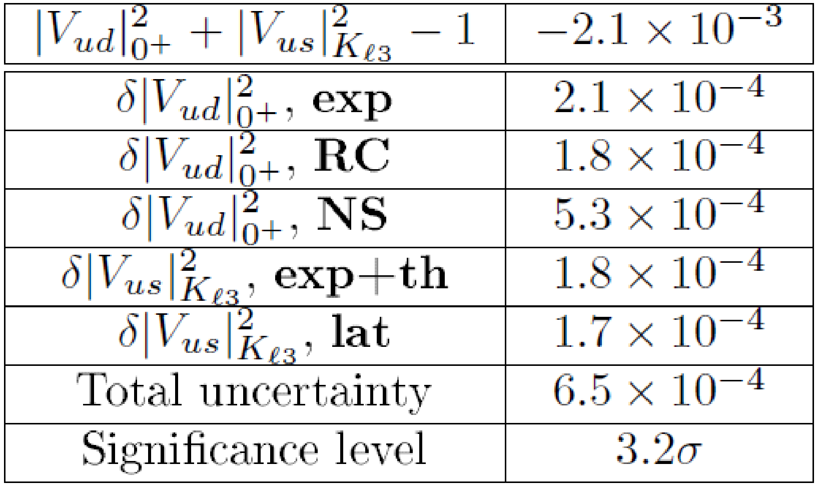
"Cabibbo Angle Anomaly (CAA)" ~ 3σ

Discovery potential of the beta decay anomalies

A concrete example: First-row CKM unitarity with $|V_{ud}|$ from 0^+ beta decay and $|V_{us}|$ from K_{l3} decay

$$\boxed{|V_{ud}|^2_{0^+} + |V_{us}|^2_{K_{\ell 3}} + |\cancel{V_{ub}}|^2 - 1 = -0.0021(7)}$$

SOURCES OF UNCERTAINTY:



$\lvert V_{ud}\rvert^2_{0^+} + \lvert V_{us}\rvert^2_{K_{\ell 3}} - 1$	-2.1×10^{-3}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **exp**	2.1×10^{-4}
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Total uncertainty	6.5×10^{-4}
Significance level	3.2σ

CYS, Galviz, Marciano and Meißner, 2022 PRD

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$\delta|V_{ud}|^2_{0^+}$, **exp:**

Experimental uncertainties in the half-lives of the superallowed beta decays

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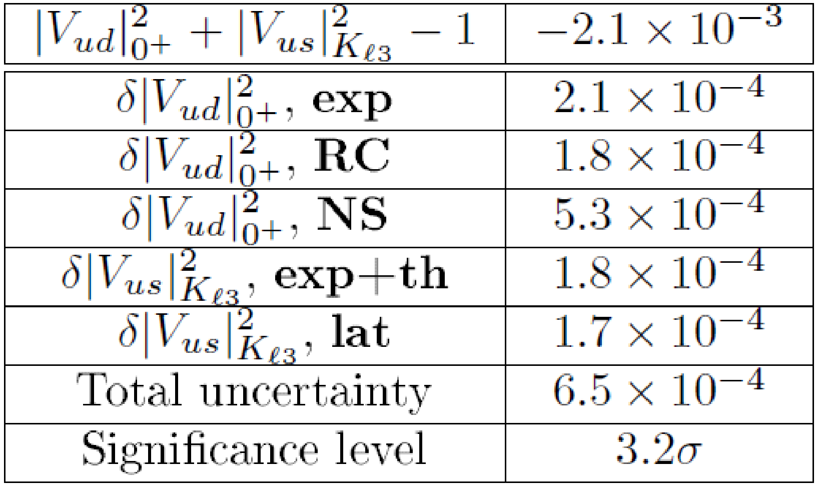
SOURCES OF UNCERTAINTY:

$$\delta |V_{ud}|^2_{0^+}, \ \mathbf{RC:}$$

Theory uncertainties in the single-nucleon radiative corrections (RC)



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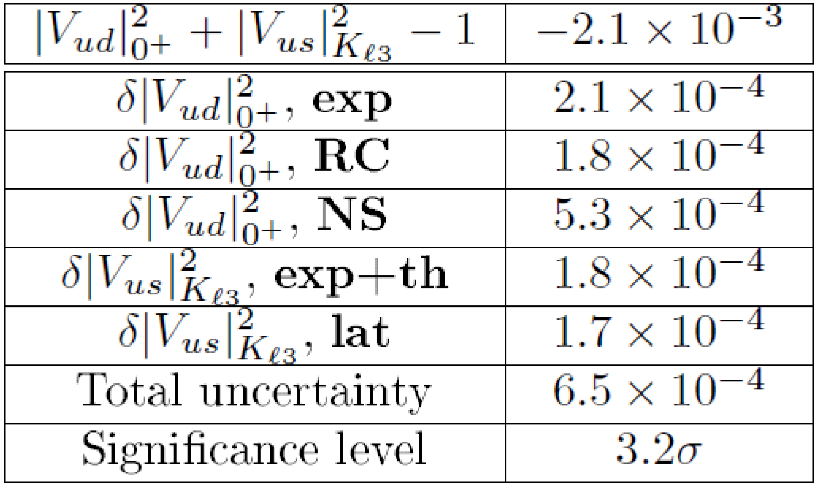
SOURCES OF UNCERTAINTY:

$\delta|V_{ud}|^2_{0+}$, **NS:**

Theory uncertainties in the nuclear-structure (NS) corrections in superallowed beta decays



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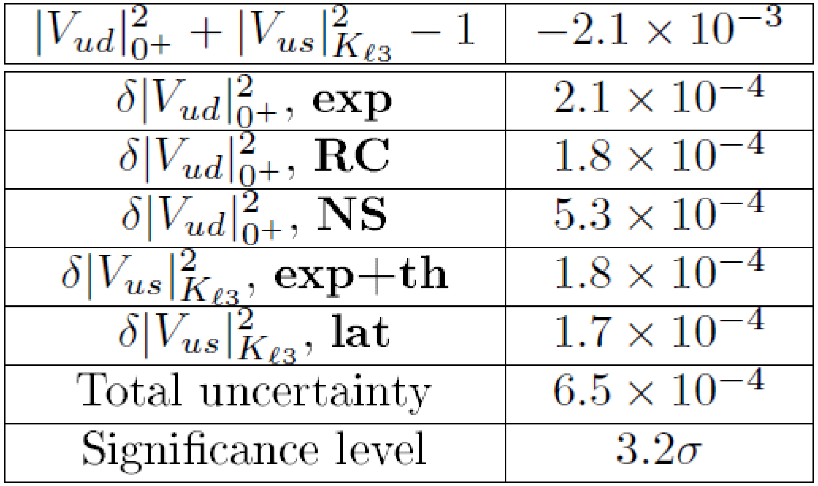
SOURCES OF UNCERTAINTY:

$$\delta|V_{us}|^2_{K_{\ell 3}},\ \mathbf{exp{+}th:}$$

Combined experimental + theory (non-lattice) uncertainties in the K_{l3} decay rate



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SOURCES OF UNCERTAINTY:

$\delta |V_{us}|^2_{K_{\ell 3}}$, **lat:**

Theory uncertainties in the lattice QCD calculation of the $K\pi$ form factor at t=0

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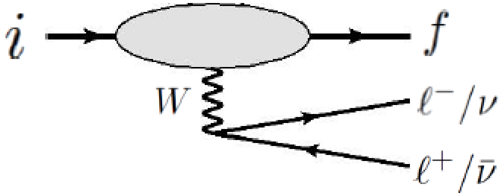
CYS, Galviz, Marciano and Meißner, 2022 PRD

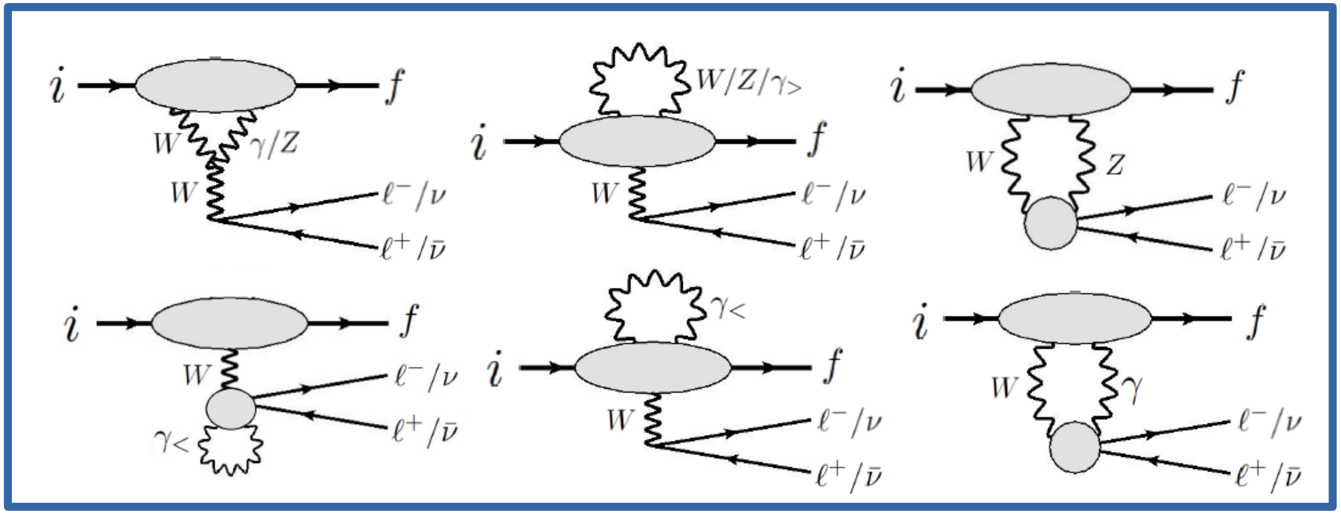
Inputs in nucleon/ nuclear sector (V_{ud})

Single-nucleon radiative corrections (RC)

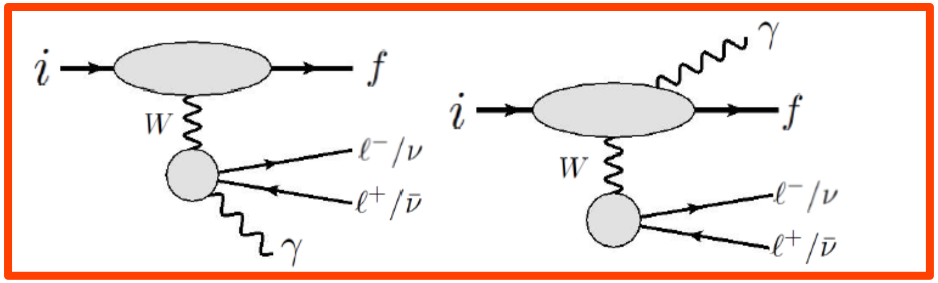


Tree-level diagram

Radiative corrections: Higher-order SM corrections that involve emission + reabsorption of virtual gauge bosons or emission of real photons.



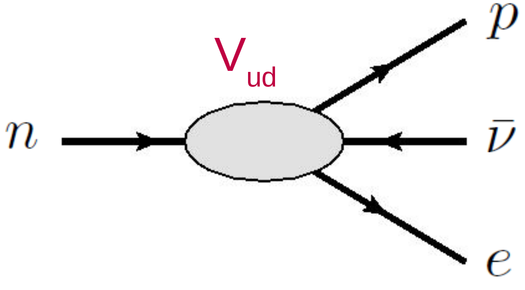
Emission + reabsorption of virtual gauge bosons

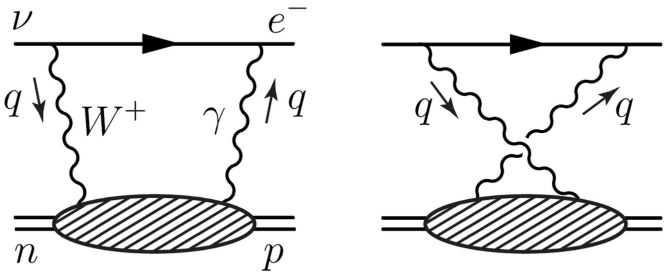


Emission of a real photon (bremsstrahlung)

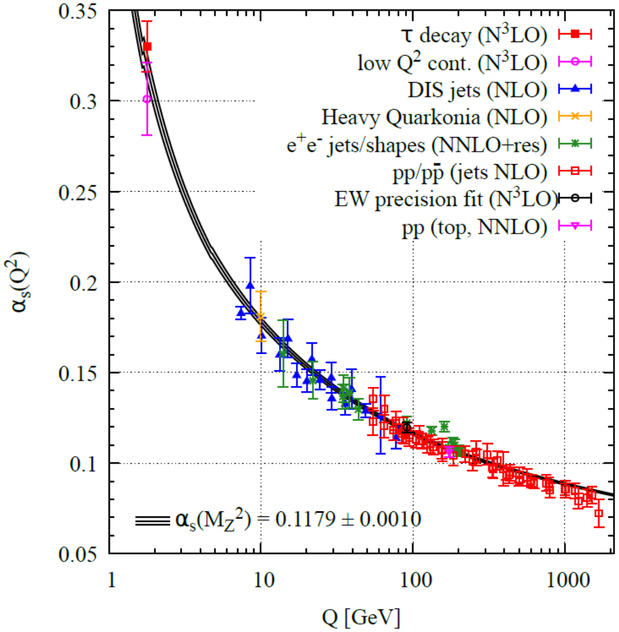
Single-nucleon radiative corrections (RC)

Primary source of uncertainty: the **"single-nucleon axial γW-box diagram"**





$Q^2=-q^2$



Main issue:Strong interactions governed by **Quantum Chromodynamics (QCD)** become non-perturbative at the hadronic scale (Q^2~1 GeV^2)

Major theory challenge in the past 4 decades

Sirlin, 1978 Rev.Mod.Phys

Pre-2018 treatment: Divide the loop integral into different regions of Q^2:

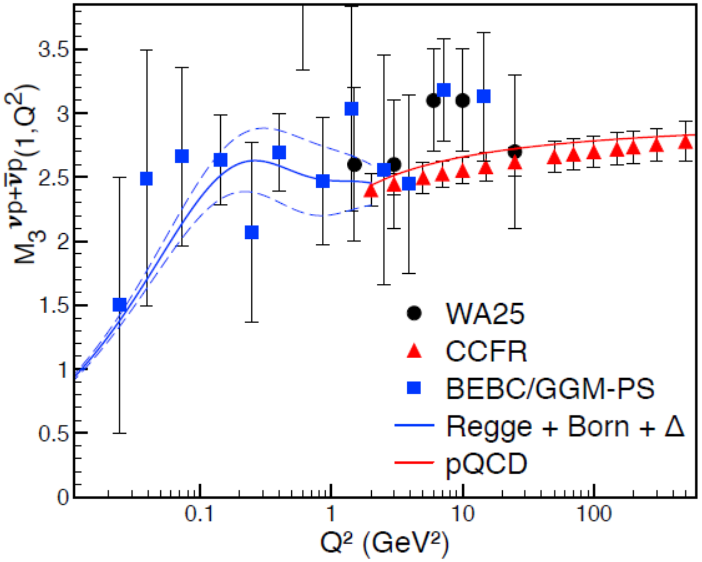
- Large-Q^2: perturbative QCD
- Small-Q^2: elastic form factors
- Intermediate Q^2: Interpolating function

Marciano and Sirlin, 2006 PRL

Single-nucleon radiative corrections (RC)

Year 2018: **Dispersion relation (DR)** treatment --- relate the loop integral to experimentally-measurable structure functions *CYS, Gorchtein, Patel and Ramsey-Musolf, 2018 PRL*

$$\boxed{\Box^V_{\gamma W} = \frac{\alpha_{em}}{\pi \mathring{g}_V}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 dx\,\frac{1+2r}{(1+r)^2}F_3^{(0)}(x,Q^2)}$$



Data input: **Parity-odd structure function F_3** from **neutrino-nucleus scattering**

New treatment led to a **significant change of |Vud|**

|Vud|: $0.97420(21) \rightarrow 0.97370(14)$

Pre-2018 *2018*

unveiling the tension in the top-row CKM unitarity

Confirmation by independent studies:

Czarnecki, Marciano and Sirlin, 2019 PRD
CYS, Feng, Gorchtein and Jin, 2020 PRD
Hayen, 2021 PRD
Shiells, Blunden and Melnitchouk, 2021 PRD

Single-nucleon radiative corrections (RC)

Further application of DR: Radiative corrections to the **Gamow-Teller (GT)** matrix element

Free neutron decay (forward limit):

$$\boxed{\langle p|\, J_W^\mu \,|n\rangle = \bar{u}_p \gamma^\mu \left(\overset{\textbf{Fermi}}{g_V} + \overset{\textbf{GT}}{g_A}\gamma_5\right) u_n}$$

The axial coupling constant g_A can be probed in correlation coefficients of the differential decay rate

$$d\Gamma \propto 1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \hat{e}_s \cdot \left[A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu}\right]$$

The **bare** axial coupling constant was calculated to percent level with **lattice QCD** (sub-percent in near future). Direct comparison with experimental measurement serves as **a strong probe of BSM physics**

To make the comparison rigorous, one needs to understand precisely the full **SM RC to g_A**.

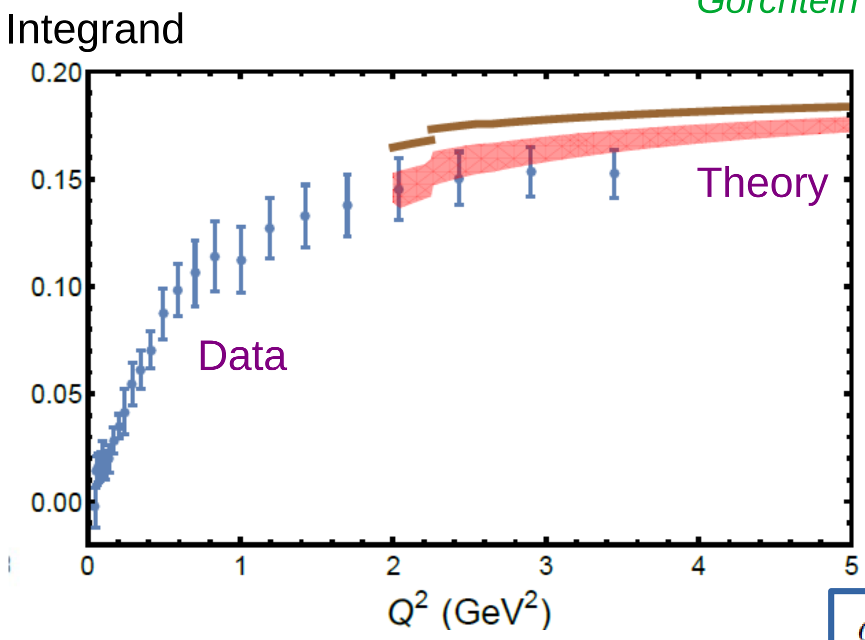
Pioneering work (non-DR): *Hayen, 2021 PRD*

Single-nucleon radiative corrections (RC)

DR formalism:

$$\Box^A_{\gamma W} = -\frac{2\alpha_{em}}{\pi \mathring{g}_A}\int_0^\infty \frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 \frac{dx}{(1+r)^2}\left[\frac{5+4r}{3}\, g_1^{(0)}(x,Q^2) - \frac{4M^2x^2}{Q^2}\, g_2^{(0)}(x,Q^2)\right]$$

Gorchtein and CYS, JHEP 10 (2021) 053



Data input: **Spin-dependent structure functions g_1 and g_2** obtained from **deep inelastic scattering (DIS) experiments**

CLAS Collaboration (Jefferson Lab), EG1b experiment; 2015 PRC and 2017 PRC

Excellent theory precision achieved:

$$\frac{g_A}{g_V} = \left(\frac{g_A}{g_V}\right)_{\text{bare}} \left(1 + 0.13(11)_V(6)_A \times 10^{-3}\right)$$

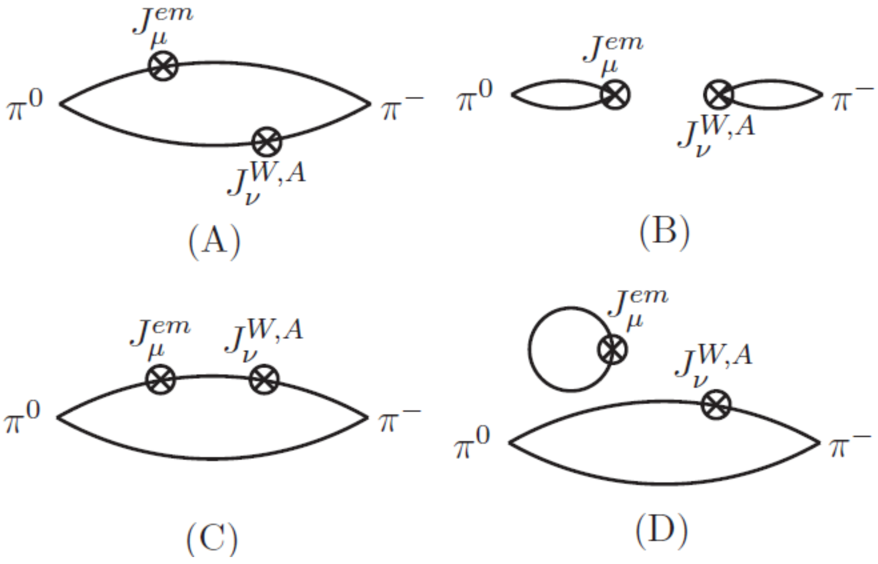
Single-nucleon radiative corrections (RC)

Major limiting factor of the DR treatment: **low quality of the neutrino data** in the most interesting region: **$Q^2 \sim 1\text{GeV}^2$**

Ongoing program: Calculate the box diagram directly with **lattice QCD**

Year 2020: First realistic lattice QCD calculation of the simpler **pion** axial γW-box diagram

Feng, Gorchtein, Jin, Ma and CYS, 2020 PRL



Consequences:

- Significant reduction of the theory uncertainty in **pion semileptonic decay (π_{e3})**
- Indirect implications on the **free-neutron** axial γW-box diagram

CYS, Feng, Gorchtein and Jin, 2020 PRD

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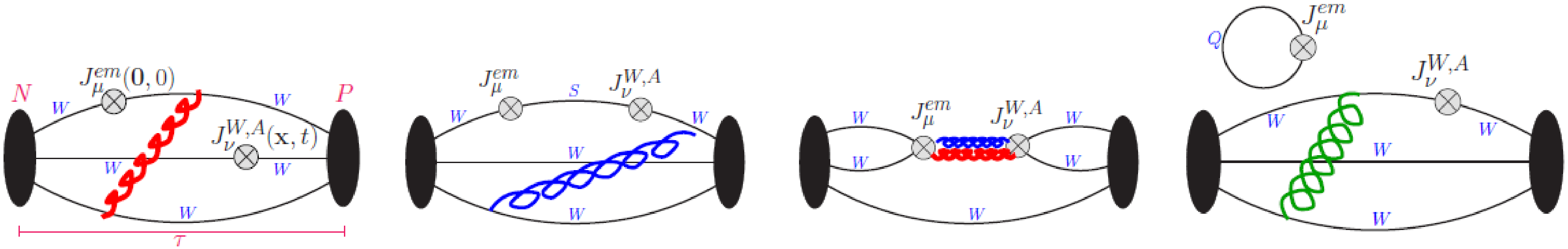
Ongoing program: Calculate the box diagram directly with **lattice QCD**

Neutron axial γW-box diagram is more complicated, but on the way.



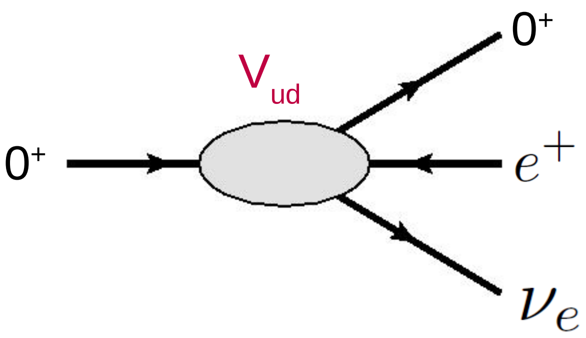
(R. Gupta, Rare Processes and Precision Frontier Townhall Meeting, 2020)

Possible alternative approach using **Feynman-Hellmann theorem (FHT)**

CYS and Meißner, 2019 PRL

Nuclear Structure (NS) corrections

Superallowed $0^+ \to 0^+$ nuclear beta decays provides the best measurement of V_{ud}



Advantages:

1. Conserved vector current (CVC) at tree level
2. Large number of measured transitions, with 15 among them whose lifetime precision is 0.23% or better. Huge gain in statistics.

$T_Z = -1$
$^{10}\mathrm{C} \to {}^{10}\mathrm{B}$
$^{14}\mathrm{O} \to {}^{14}\mathrm{N}$
$^{22}\mathrm{Mg} \to {}^{22}\mathrm{Na}$
$^{26}\mathrm{Si} \to {}^{26}\mathrm{Al}$
$^{34}\mathrm{Ar} \to {}^{34}\mathrm{Cl}$
$^{38}\mathrm{Ca} \to {}^{38}\mathrm{K}$
$T_Z = 0$
$^{26m}\mathrm{Al} \to {}^{26}\mathrm{Mg}$
$^{34}\mathrm{Cl} \to {}^{34}\mathrm{S}$
$^{38m}\mathrm{K} \to {}^{38}\mathrm{Ar}$
$^{42}\mathrm{Sc} \to {}^{42}\mathrm{Ca}$
$^{46}\mathrm{V} \to {}^{46}\mathrm{Ti}$
$^{50}\mathrm{Mn} \to {}^{50}\mathrm{Cr}$
$^{54}\mathrm{Co} \to {}^{54}\mathrm{Fe}$
$^{62}\mathrm{Ga} \to {}^{62}\mathrm{Zn}$
$^{74}\mathrm{Rb} \to {}^{74}\mathrm{Kr}$

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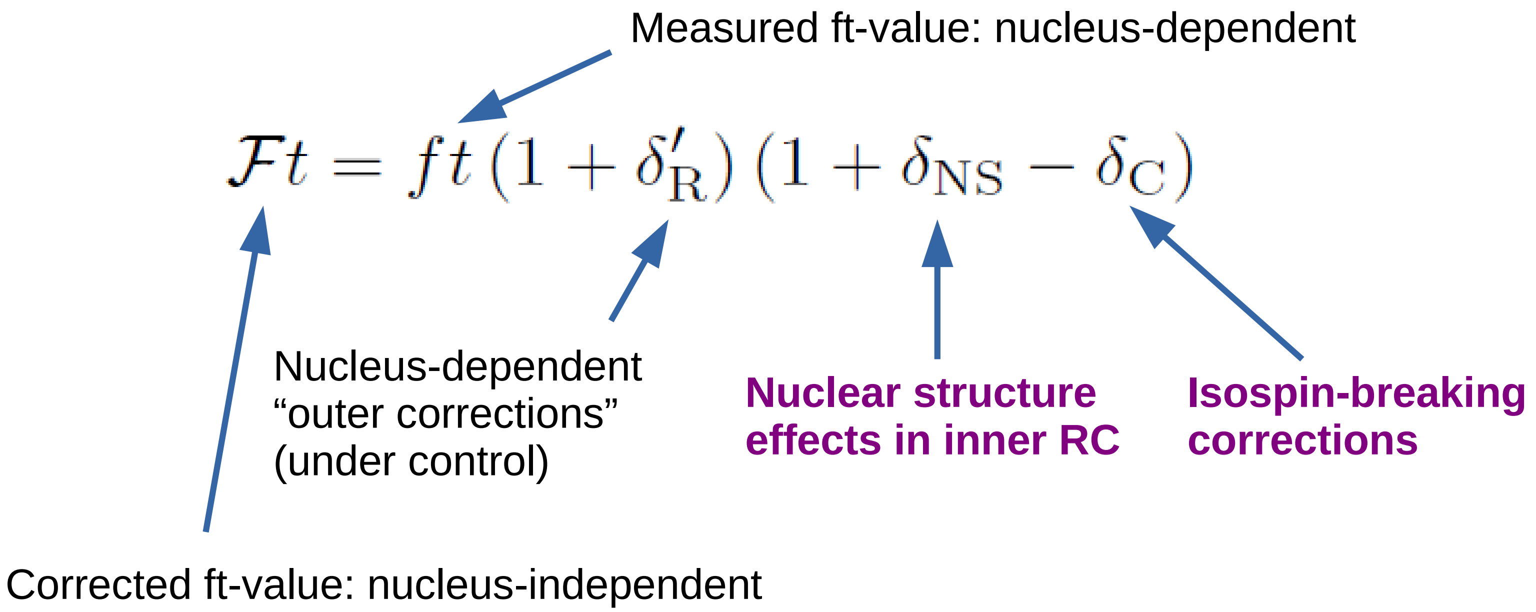
Superallowed $0^+ \rightarrow 0^+$ nuclear beta decays provides the best measurement of V_{ud}

Master formula:

$$\boxed{|V_{ud}|^2 = \frac{2984.43\ s}{\mathcal{F}t\,(1+\Delta_R^V)}.}$$

Single-nucleon RC

Corrected ft (half-life*statistical function)-value:



Measured ft-value: nucleus-dependent

$$\mathcal{F}t = ft\,(1+\delta'_{\rm R})\,(1+\delta_{\rm NS}-\delta_{\rm C})$$

Nucleus-dependent
“outer corrections”
(under control)

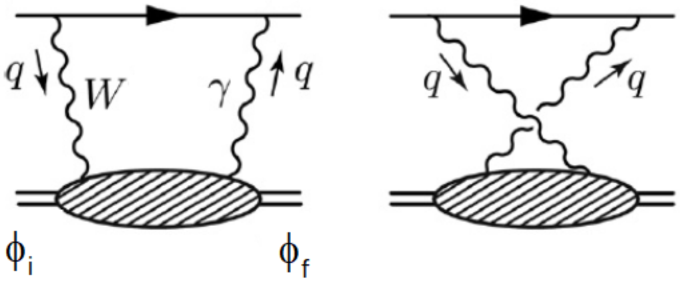
Nuclear structure effects in inner RC

Isospin-breaking corrections

Corrected ft-value: nucleus-independent

Nuclear Structure (NS) corrections

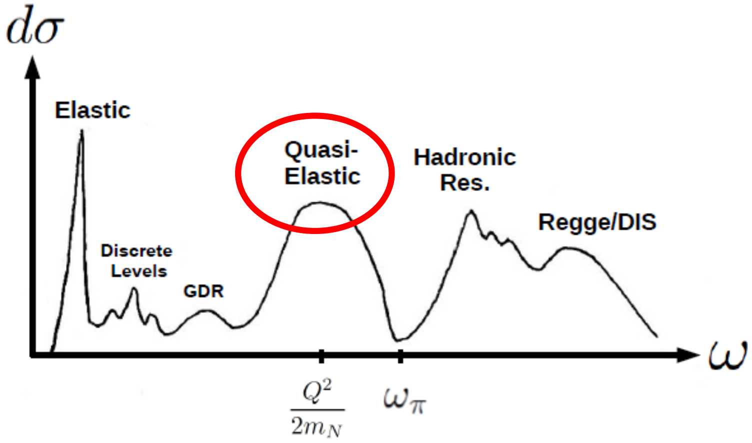
δ_{NS}: **nuclear modifications of the free-nucleon inner RC**

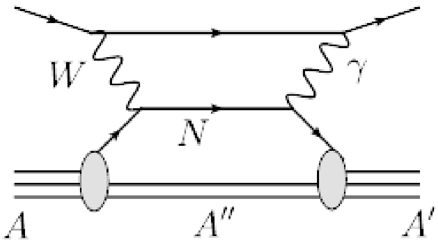


LARGEST source of uncertainty in V_{ud}!

$$\Box^{\text{nucl.}}_{\gamma W} = \Box^{n}_{\gamma W} + \underbrace{\left[\Box^{\text{nucl.}}_{\gamma W} - \Box^{n}_{\gamma W}\right]}_{\delta_{\text{NS}}}$$

- The **low-energy absorption spectrum** is distorted by nuclear corrections
- An important contribution from the **quasi-elastic nucleons** was not properly accounted for in previous nuclear-model calculations, which results in the large uncertainty in δ_{NS}.



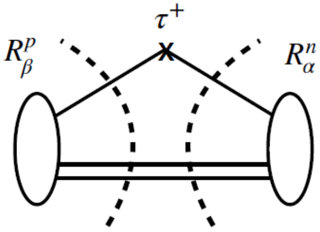


Ab-initio nuclear theory calculations of δ_{NS} urgently needed!

CYS, Gorchtein and Ramsey-Musolf, 2019 PRD; Gorchtein, 2019 PRL

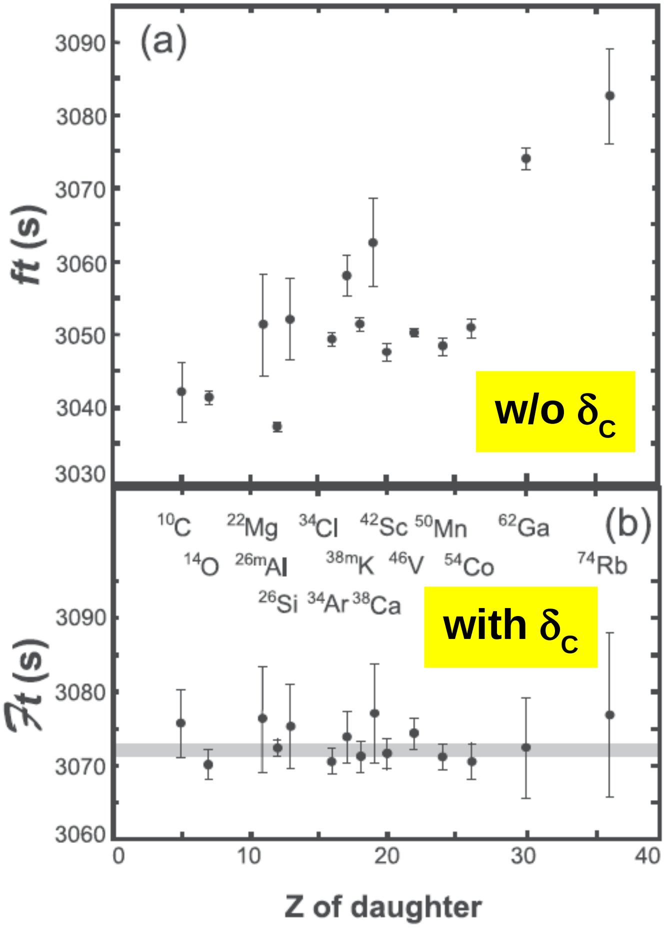
Nuclear Structure (NS) corrections

δ_C: isospin-breaking (ISB) corrections to nuclear wavefunctions

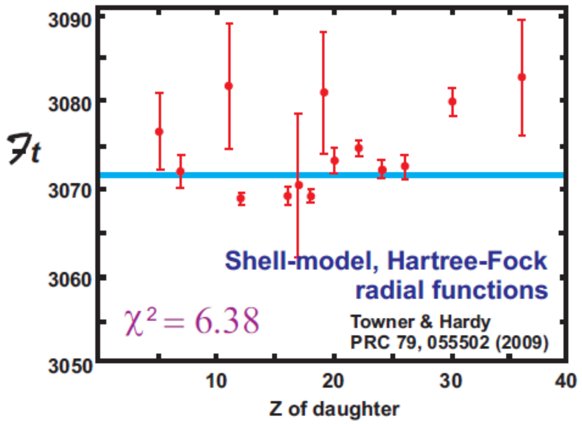


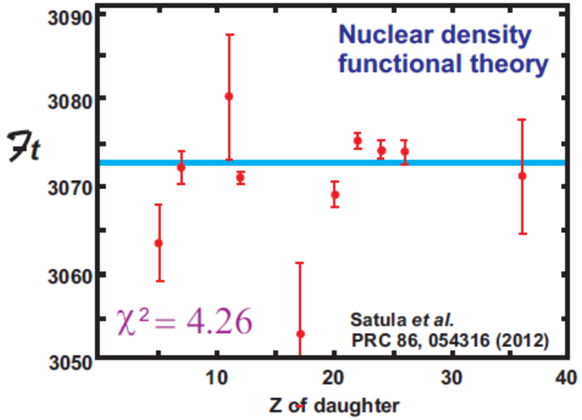
Essential to **align the Ft-values** of different superallowed transitions.

It turns out that such alignment is only achieved within **some specific choices of nuclear models** (e.g. Woods Saxon), but not the others.



Hardy and Towner, 2020 PRC





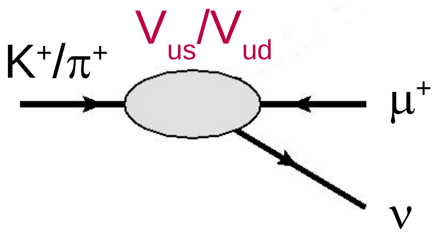
A **model-independent assessment** of δ_C is needed!

Inputs in Kaon/pion sector

(V_{us} and V_{us}/V_{ud})

Kaon/pion leptonic decay ($K_{\mu2}/\pi_{\mu2}$)

$$\boxed{\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = {\underbrace{\left[\frac{\Gamma_{K_{\mu2}}M_{\pi^+}}{\Gamma_{\pi_{\mu2}}M_{K^+}}\right]}_{\text{“axial ratio” } R_A}}^{1/2} \frac{1-m_\mu^2/M_{\pi^+}^2}{1-m_\mu^2/M_{K^+}^2}\left(1-\delta_{\mathrm{EM}}/2\right)}$$



Marciano, 2004 PRL; Cirigliano and Neufeld, 2011 PLB

Lattice QCD inputs: K^+/π^+ decay constants

$$\begin{aligned} N_f &= 2+1+1 &:\quad f_{K^+}/f_{\pi^+} &= 1.1932(21) \\ N_f &= 2+1 &:\quad f_{K^+}/f_{\pi^+} &= 1.1917(37) \\ N_f &= 1 &:\quad f_{K^+}/f_{\pi^+} &= 1.205(18) \end{aligned}$$

FLAG 2021

Electromagnetic RC in ChPT: $\delta_{\mathrm{EM}} = \delta_{\mathrm{EM}}^{K} - \delta_{\mathrm{EM}}^{\pi} = -0.0069(17)$

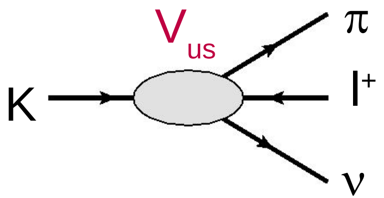
Knecht et al., 2000 EPJC
Cirigliano and Neufeld, 2011 PLB

Advantage: **LECs cancel in the ratio**

Direct lattice QCD calculation of the EMRC+isospin breaking correction (contained in the physical K^+/π^+ decay constants) consistent with ChPT result, with slightly lower uncertainty *Giusti et al, 2018 PRL*

Total: $|V_{us}/V_{ud}| = 0.23131(41)_{\mathrm{lat}}(24)_{\mathrm{exp}}(19)_{\mathrm{RC}}$

Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

Measurements of **branching ratio** exist in all **six channels**:

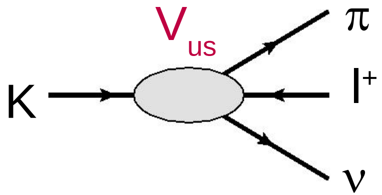
$K_{e3}^L,\ K_{\mu 3}^L$: *PLB632,43(2006), PRD70,092006(2004), ...*

K_{e3}^S : *PLB653,145(2007), PLB636,173(2006), PLB535,37(2002), ...*

$K_{\mu 3}^S$: *PLB804,135378(2020)* ⟵ **New!**

$K_{e3}^+,\ K_{\mu 3}^+$: *JHEP02,098(2008), PRD6,1254(1972), ...*

Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K\ell 3} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU}(2)}^{K\pi}\right)$$

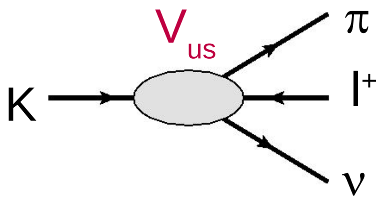
C_K: Known isospin factor

S_{EW}: Short-distance electroweak RCs

$$S_{\mathrm{EW}} = 1.0232(3)$$

Marciano and Sirlin, 1993 PRL

Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K\ell 3} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\rm EW} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1+\delta_{\rm EM}^{K\ell}+\delta_{\rm SU(2)}^{K\pi}\right)$$

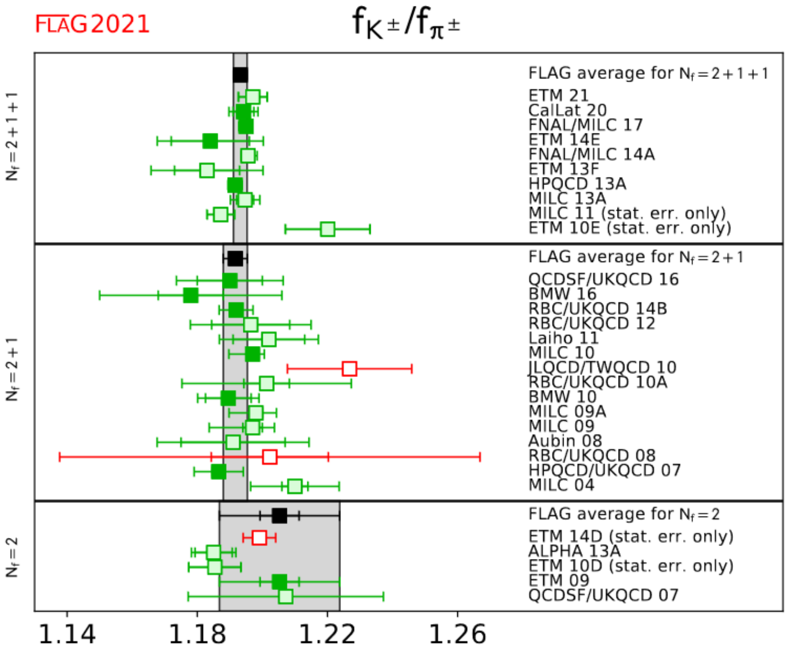
Kπ form factor at t=0: $\langle \pi^-(p')|\, J_W^\mu \,|K^0(p)\rangle = f_+^{K^0\pi^-}(t)(p+p')^\mu + f_-^{K^0\pi^-}(t)(p-p')^\mu$

Lattice QCD inputs:

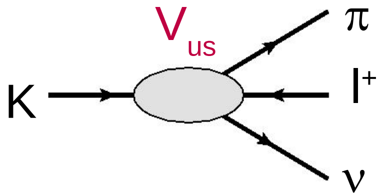
$$N_f = 2+1+1 \quad : \quad f_+(0) = 0.9698(17)$$
$$N_f = 2+1 \quad : \quad f_+(0) = 0.9677(27)$$
$$N_f = 2 \quad : \quad f_+(0) = 0.9560(57)(62)$$

A slight change of **1%** in the central value could lead to **totally different conclusions** on the V_{us} **anomaly** (K_{l3}—$K_{\mu 2}$ discrepancy)

FLAG 2021



Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{Ke3} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

Phase-space factor: $I_{K\ell}^{(0)} = \int_{m_\ell^2}^{(M_K^2 - M_\pi)^2} \frac{dt}{M_K^8} \bar{\lambda}^{3/2} \left(1 + \frac{m_\ell^2}{2t}\right) \left(1 - \frac{m_\ell^2}{t}\right)^2 \left[\bar{f}_+^2(t) + \frac{3 m_\ell^2 \Delta_{K\pi}^2}{(2t + m_\ell^2)\bar{\lambda}} \bar{f}_0^2(t)\right]$

probes the **t-dependence** of the Kπ form factors.

Rescaled
Kπ form factors

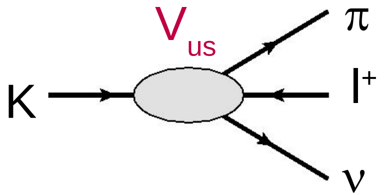
Obtained by fitting to the **K_{l3} Dalitz** plot with **specific parameterizations of f(t)** (Taylor expansion, z-expansion, dispersive parameterization, pole parameterization ...)

The **dispersive parameterization** currently quotes the smallest uncertainty:

Mode	Update
$K^0{}_{e3}$	**0.15470(15)**
$K^+{}_{e3}$	**0.15915(15)**
$K^0{}_{\mu3}$	**0.10247(15)**
$K^+{}_{\mu3}$	**0.10553(16)**

M. Moulson, in the 11th International Workshop on the CKM Unitarity Triangle, 2021

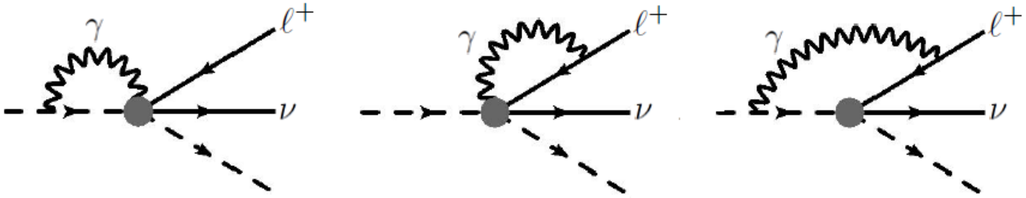
Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K\ell 3} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\rm EW} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \boxed{\delta_{\rm EM}^{K\ell}} + \delta_{\rm SU(2)}^{K\pi}\right)$$

Long-distance electromagnetic RC



	$\delta_{\rm EM}^{K\ell}(\%)$
$K^0_{\mu 3}$	$1.40(19)(11)$
$K^{\pm}_{\mu 3}$	$0.02(19)(16)$
K^0_{e3}	$1.16(2)(1)(1)(2)$
$K^{\pm}_{e3}$	$0.21(2)(1)(1)(4)(1)$

ChPT calculations at $O(e^2p^2)$ + model estimation of the LECs: $\sim 10^{-3}$ error *Cirigliano et al., 2008 JHEP*

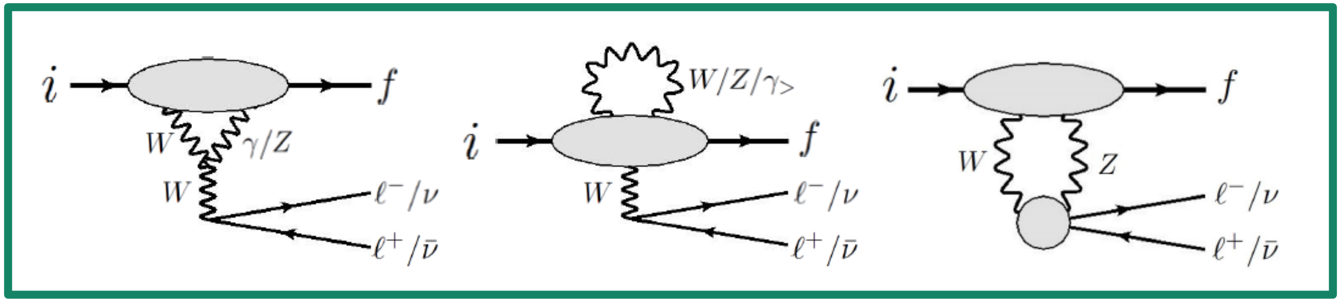
Sirlin's representation + ChPT + lattice QCD: $\sim 10^{-4}$ error *CYS, Galviz, Gorchtein and Meißner, 2021 PLB*
CYS, Galviz, Gorchtein and Meißner, 2021 JHEP

Kaon semileptonic decays (K_{l3})

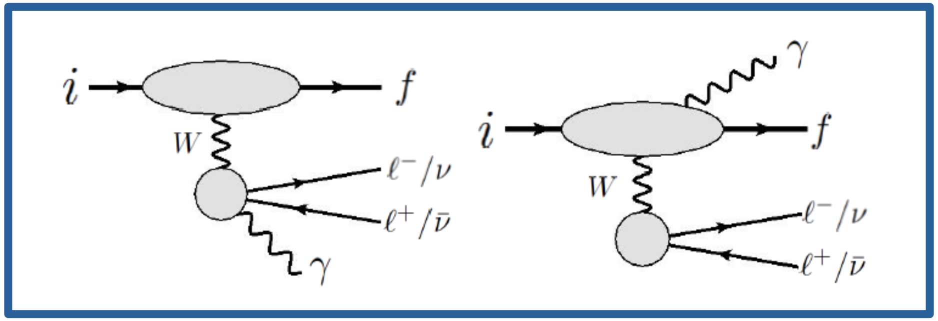
“Sirlin’s representation” of the $O(G_F\alpha)$ electroweak RC:

Sirlin, 1978 Rev.Mod.Phys
CYS, 2021 Particles

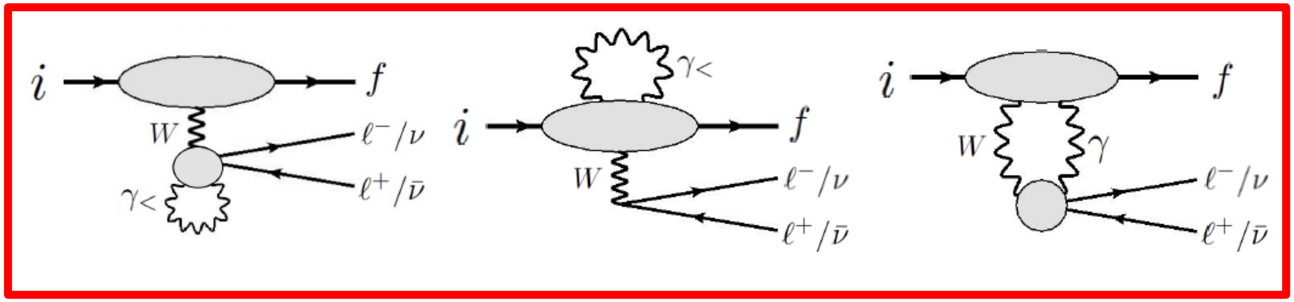
Classifying the full $O(G_F\alpha)$ electroweak RC into **three categories**:



“Weak” RC:
Calculable perturbatively
to satisfactory precision



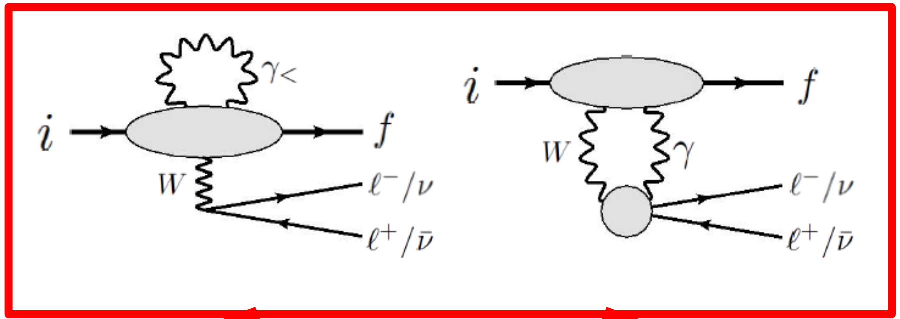
Bremsstrahlung:
Fixed by particles’ charges
to satisfactory precision



(Virtual) electromagnetic RC:
Involve physics at
small Q^2

Kaon semileptonic decays (K_{l3})

Further separation of the non-trivial virtual electromagnetic RC:



$$\left(\delta\mathfrak{M}_2 + \delta\mathfrak{M}^a_{\gamma W}\right)_{\text{int}} - \frac{G_F}{\sqrt{2}}\delta F_3^\lambda L_\lambda$$

Depends on physics at
$Q^2 << 1\text{GeV}^2$.
Fixed by **form factors**
to satisfactory precision

$$\delta\mathfrak{M}^b_{\gamma W}$$

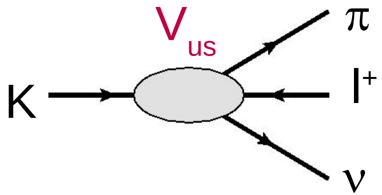
Depends on physics at
$Q^2 \sim 1\text{GeV}^2$.
Fixed by **lattice QCD**
to satisfactory precision

- Significant improvement of the **K_{e3} RC** precision: **10^{-3}** → **10^{-4}**
- Next step: Applying the same framework to **$K_{\mu3}$**

Plans for direct lattice calculations of the full RC: ~10 years to reach 10^{-3} precision

Boyle et al., SnowMass 2021 LoI

Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)$$

ISB correction: presents only in the **K⁺** channel by construction.

$$\delta_{\mathrm{SU(2)}}^{K^+\pi^0} \equiv \left(\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)}\right)^2 - 1 = \frac{3}{2}\frac{1}{Q^2}\left[\frac{\hat{M}_K^2}{\hat{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1 + \frac{m_s}{\hat{m}}\right)\right]$$

(neglecting small EM contributions)

$$Q^2 = (m_s^2 - \hat{m}^2)/(m_d^2 - m_u^2)$$

Most recent lattice QCD inputs: *FLAG 2021*

$$Q = 23.3(5)\ , \quad m_s/\hat{m} = 27.42(12) \qquad N_f = 2+1$$

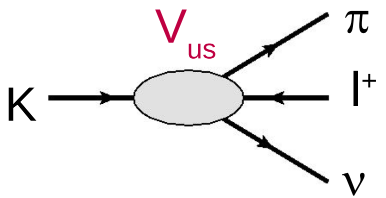
returns: $\delta_{\mathrm{SU(2)}}^{K^+\pi^0} = 0.0457(20)$

Phenomenological inputs from $\eta \to 3\pi$ returns a somewhat larger value:

$$\delta_{\mathrm{SU(2)}}^{K^+\pi^0} = 0.0572(68)$$

Colangelo, Lanz, Leutwyler and Passemar, 2018 EPJC

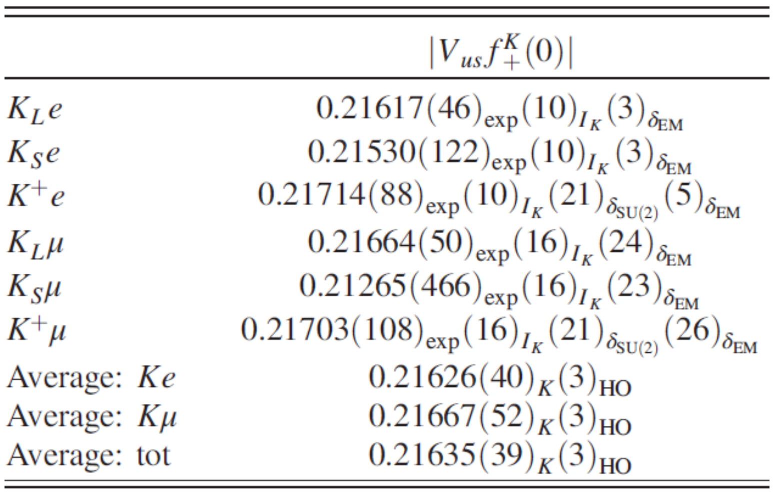
Kaon semileptonic decays (K_{l3})



Master formula:

$$\Gamma_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{192\pi^3} S_{\mathrm{EW}} |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}^{(0)} \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU}(2)}^{K\pi}\right)$$

Averaging over all six channels:



	$\lvert V_{us} f_+^K(0) \rvert$
$K_L e$	$0.21617(46)_{\mathrm{exp}}(10)_{I_K}(3)_{\delta_{\mathrm{EM}}}$
$K_S e$	$0.21530(122)_{\mathrm{exp}}(10)_{I_K}(3)_{\delta_{\mathrm{EM}}}$
$K^+ e$	$0.21714(88)_{\mathrm{exp}}(10)_{I_K}(21)_{\delta_{\mathrm{SU(2)}}}(5)_{\delta_{\mathrm{EM}}}$
$K_L \mu$	$0.21664(50)_{\mathrm{exp}}(16)_{I_K}(24)_{\delta_{\mathrm{EM}}}$
$K_S \mu$	$0.21265(466)_{\mathrm{exp}}(16)_{I_K}(23)_{\delta_{\mathrm{EM}}}$
$K^+ \mu$	$0.21703(108)_{\mathrm{exp}}(16)_{I_K}(21)_{\delta_{\mathrm{SU(2)}}}(26)_{\delta_{\mathrm{EM}}}$
Average: Ke	$0.21626(40)_K(3)_{\mathrm{HO}}$
Average: $K\mu$	$0.21667(52)_K(3)_{\mathrm{HO}}$
Average: tot	$0.21635(39)_K(3)_{\mathrm{HO}}$

CYS, Galviz, Marciano and Meißner, 2022 PRD

With Nf=2+1+1 lattice average of $f_+(0)$:

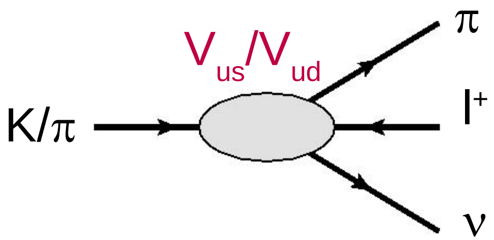
$$|V_{us}|_{K_{\ell 3}} = 0.22309(40)_K(39)_{\mathrm{lat}}(3)_{\mathrm{HO}}$$

Experimental uncertainties apparently dominate in all channels, but one still needs to scrutinize all the **theory inputs** to make sure the **V_{us} anomaly** does not come from some **unexpected, large SM corrections**.

Kaon semileptonic decays (K_{l3})

Vector ratio R_V : A new avenue to determine V_{us}/V_{ud}

$$R_V = \frac{\Gamma(K_{\ell 3})}{\Gamma(\pi_{e3})}$$



Czarnecki, Marciano and Sirlin, 2020 PRD

from R_A $$\left|\frac{V_{us} f_{K^+}}{V_{ud} f_{\pi^+}}\right| = 0.27600(29)_{\rm exp}(23)_{\rm RC} \; ,$$

from R_V $$\left|\frac{V_{us} f_+^K(0)}{V_{ud} f_+^\pi(0)}\right| = 0.22216(64)_{{\rm BR}(\pi_{e3})}(39)_K(2)_{\tau_{\pi^+}}(1)_{{\rm RC}_\pi} \; ,$$ ⟵ **Theoretically cleaner!**

Major limiting factor: **π_{e3} branching ratio** $\mathrm{BR}(\pi_{e3}) = 1.038(6) \times 10^{-8}$

PIBETA, 2004 PRL + recent update

Next-generation experiment (PIONEER) may improve BR (π_{e3}) precision by a factor of 3 or more, making R_V competitive

Aguilar-Arevalo et al., SnowMass 2021 LoI;
Hertzog, in TAU2021

Summary

- Several **anomalies** at the level $\sim 3\sigma$ have been observed in the measurements of the **first-row CKM matrix elements V_{ud} and V_{us}** in **beta decay processes**.
- **SM theory inputs** that require further improvements are:
  - **V_{ud} sector:** RC in single-nucleon and nuclear systems, ISB corrections in nuclear wavefunctions
  - **V_{us} sector:** Lattice inputs of Kaon/pion decay constants and $K\pi$ form factor, RC in leptonic and semileptonic kaon decays, K_{l3} phase-space factor, ISB corrections in $K^{\pm}$ semileptonic decays
- Successful reduction of theory uncertainties above could increase the significance of the anomalies to more than 5σ
- Desirable future **experimental improvements**: K_{l3} and π_{e3} branching ratios, neutron lifetime and g_A, …

Thanks for your attention!