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# Modeling Nuclear Physics Data with Novel Artificial Intelligence Approaches

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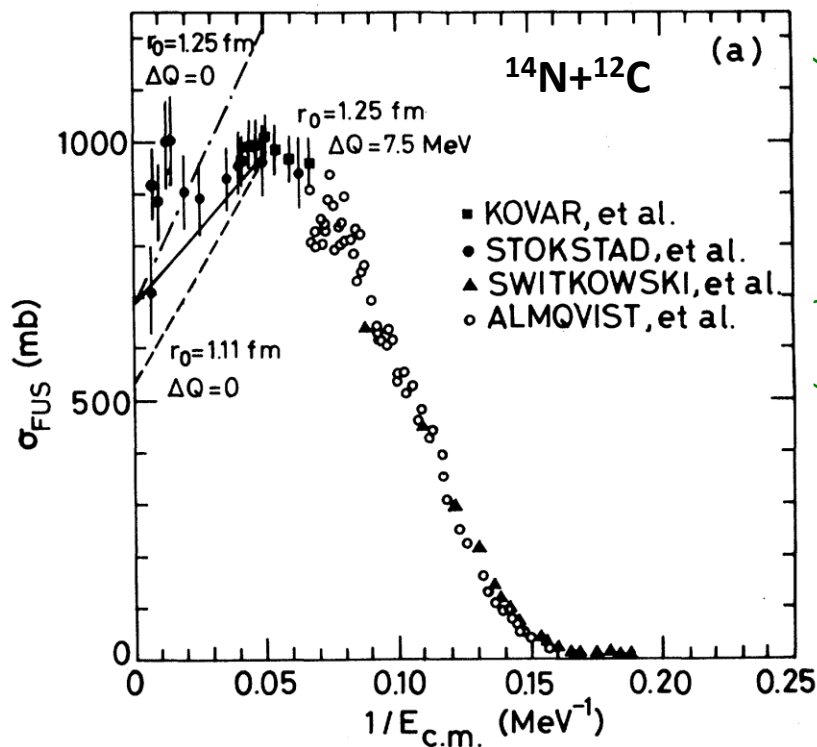
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- **Physics background**
  - state-of-the-art models;
  - problems;
- **Dataset and approach**
  - Nuclear Reaction Video (NRV) Project;
- **Methods: Brain Project**
  - genetic programming: population, crossover, fitness;
  - parallel and distributed implementation;
  - neural networks;
  - data modeling;
- **Results**
  - comparisons with the literature;
  - conclusions and perspectives.

# Heavy-ion fusion cross section at energies above-barrier

Different, complementary, experimental methods can be effectively used to estimate the yield of evaporation residues (gamma-ray analysis, time-of-flight and magnetic spectrometers, charged particle detection with telescope arrays) → heavy-ion fusion cross section from the Coulomb barrier to the onset of multi-fragmentation →  
See e.g. *P. Frobrich, Phys. Rep. 116 (1984) 337.*

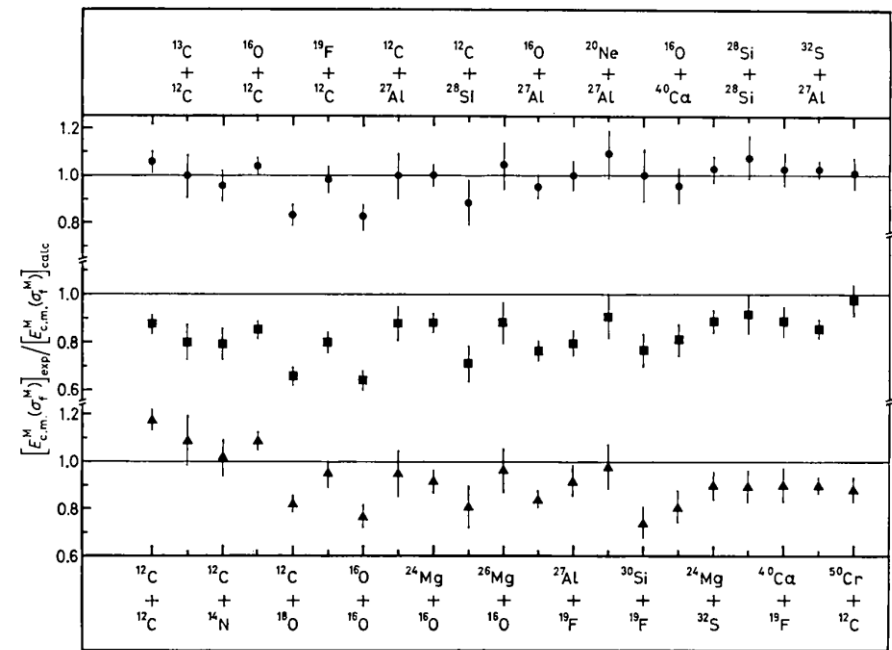
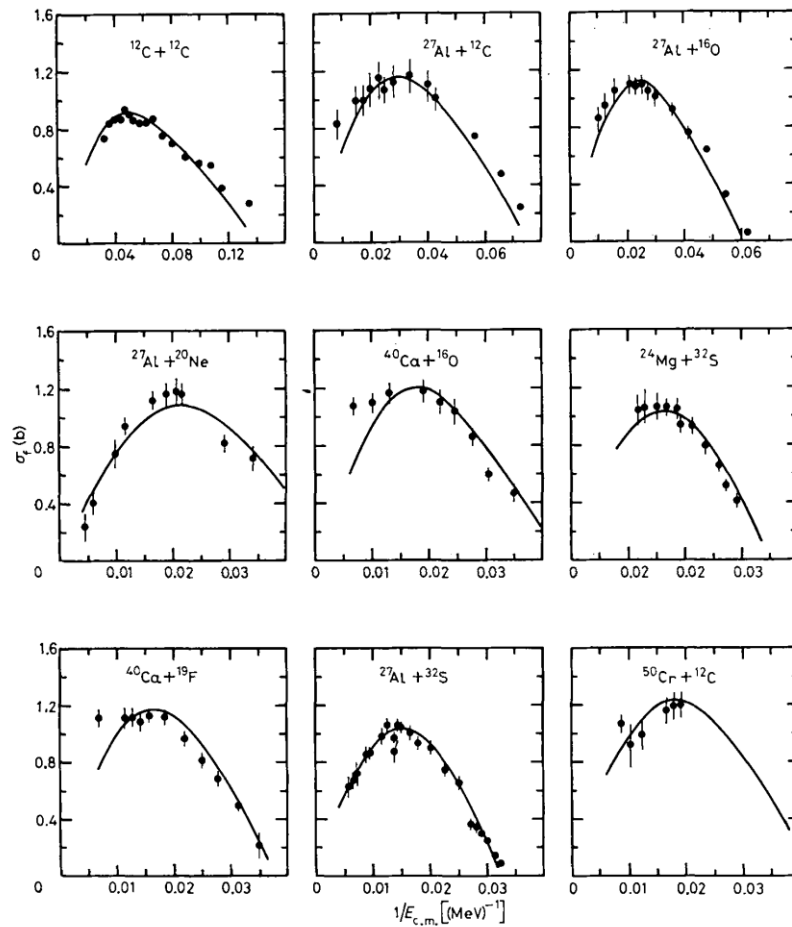


*From S.M. Lee, T. Matsuse and A. Arima  
Phys. Rev. Lett. 45 (1980) 165*

*Models for the description of fusion cross section between heavy-ions.*

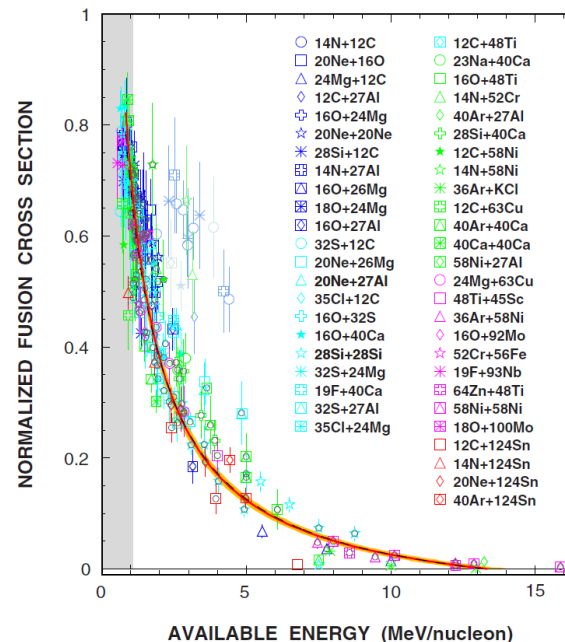
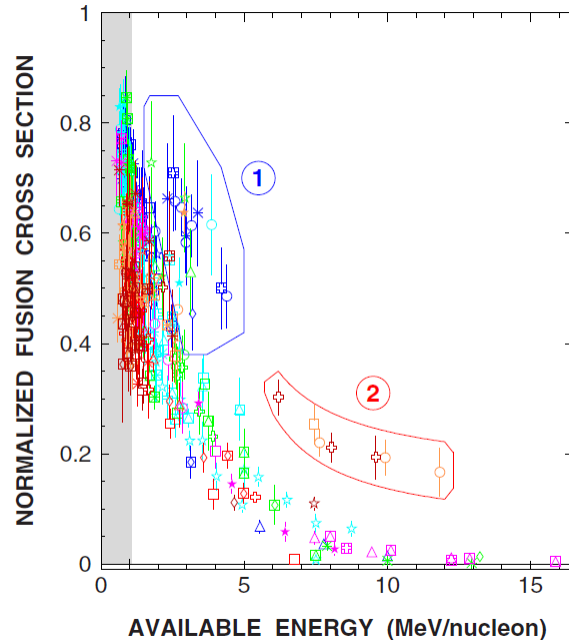
- **Microscopical approaches:** Time-Dependent Hartree Fock (TDHF); Molecular dynamics;
- **Macroscopic models:** critical distance models, limitation to the compound nucleus model (empirical nuclear potentials from semi-classical considerations);
- **Empirical models:** starting from nuclear reaction theory and then optimizing to the experimental data.

Previous data-driven (*phenomenological*) approaches, see e.g. *Porto F and Sambataro S 1984 Nuov. Cim. 83 339* → good description of data around the maxima of the cross section → few datasets in Region III (high energies) and Region I (close to the Coulomb barrier).

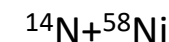
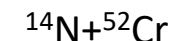
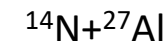
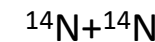
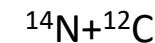
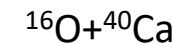
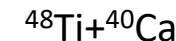
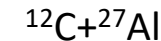


More recently → systematic study of Region III shows discrepancies for some of the systems → further investigation on both experiment and theory is required!

from P. Eudes et al., Phys. Rev. C 90 (2014) 034609.



Fusion cross section in Region III → disagreement with the prediction of state-of-the-art for some collision systems such as:



State-of-the-art artificial intelligence approaches can serve to help describing the cross section between heavy-ions in a broad energy range!

About 50 years of systematics (see e.g. *Nuclear Reactions Video: Karpov A et al., 2017 Nucl. Instrum. Meth. Phys. Res. A 859 112*; *Zagrebaev V et al., 1999 NRV web knowledge base on low-energy nuclear physics* URL <http://nr.v.jinr.ru/> for a complete database) → Possibility to derive new **data-driven** models for the description of the fusion cross section between heavy-ions in Regions I-III.

Experimental data on HI fusion cross sections

Experimental data on  
fusion  
elastic scattering  
evaporation residue

Specify fusion reaction  
(at least one item)

$Z_1$    $A_1$   +  $Z_2$    $A_2$

Search

(Quite recently we started to fill the database. We are very far from finish...)

Show all accumulated data

or choose it from the list  Go

Ordered by P-T combination, compound nucleus, time of publication  
(access to the source may be restricted by owner!)

$^3\text{He} + ^{56}\text{Ni} \rightarrow ^{51}\text{Zn}$ (EvR)	E.F. Aguilera, E. Martinez-Quiroz, R. Chavez-Gonzalez et al.,	Physical Review, C 87 (2013) 14613
$^3\text{He} + ^{167}\text{Er} \rightarrow ^{170}\text{Yb}$ (EvR)	S. Gil, R. Vandenbosch, A. Charlop et al.,	Physical Review, C 43 (1991) 701
$^3\text{He} + ^{181}\text{Ta} \rightarrow ^{184}\text{Re}$ (FF)	F. D. Becchetti, K. H. Hicks, C. A. Fields et al.,	Physical Review, C 28 (1983) 1217
$^3\text{He} + ^{197}\text{Au} \rightarrow ^{200}\text{Ti}$ (FF)	F. D. Becchetti, K. H. Hicks, C. A. Fields et al.,	Physical Review, C 28 (1983) 1217
$^3\text{He} + ^{208}\text{Bi} \rightarrow ^{212}\text{At}$ (FF)	F. D. Becchetti, K. H. Hicks, C. A. Fields et al.,	Physical Review, C 28 (1983) 1217
$^3\text{He} + ^{232}\text{Th} \rightarrow ^{235}\text{U}$ (FF)	F. D. Becchetti, K. H. Hicks, C. A. Fields et al.,	Physical Review, C 28 (1983) 1217
$^4\text{He} + ^{40}\text{Ca} \rightarrow ^{44}\text{Ti}$ (EvR)	K.A. Eberhard, Ch. Appel, R. Bangert et al.,	Physical Review Letters, 43 (1979) 107
$^4\text{He} + ^{44}\text{Ca} \rightarrow ^{48}\text{Ti}$ (EvR)	K.A. Eberhard, Ch. Appel, R. Bangert et al.,	Physical Review Letters, 43 (1979) 107
$^4\text{He} + ^{63}\text{Cu} \rightarrow ^{67}\text{Ga}$ (EvR)	A. Navin, V. Tripathi, Y. Blumenfeld et al.,	Physical Review, C 70 (2004) 44601
$^4\text{He} + ^{65}\text{Cu} \rightarrow ^{69}\text{Ga}$ (EvR)	A. Navin, V. Tripathi, Y. Blumenfeld et al.,	Physical Review, C 70 (2004) 44601
$^4\text{He} + ^{64}\text{Zn} \rightarrow ^{68}\text{Ge}$ (X-rays from EC of EvR)	M. Fischella, V. Scuderi, A. Di Pietro et al.,	Journal of Physics, 282 (2011) 012014
$^4\text{He} + ^{64}\text{Zn} \rightarrow ^{68}\text{Ge}$ (EvR)	V. Scuderi, A. Di Pietro, P. Figuera et al.,	Physical Review, C 84 (2011) 64604
$^4\text{He} + ^{64}\text{Zn} \rightarrow ^{68}\text{Ge}$ (EvR)	A. Di Pietro, P. Figuera, V. Scuderi et al.,	Physics of Atomic Nuclei, 69 (2006) 1366
$^4\text{He} + ^{93}\text{Nb} \rightarrow ^{97}\text{Tc}$ (EvR)	C. S. Palshetkar, S. Santra, A. Chatterjee, K. Ramachandran, Shital Tha.	Physical Review, C 82 (2010) 044608
$^4\text{He} + ^{107}\text{Ag} \rightarrow ^{111}\text{In}$ (FF)	A. Buttkevit, H. H. Duhm, F. Goldenbaum et al.,	Physical Review, C 80 (2009) 37603
$^4\text{He} + ^{139}\text{La} \rightarrow ^{143}\text{Pr}$ (FF)	A. Buttkevit, H. H. Duhm, F. Goldenbaum et al.,	Physical Review, C 80 (2009) 37603
$^4\text{He} + ^{154}\text{Sm} \rightarrow ^{158}\text{Gd}$ (EvR)	S. Gil, R. Vandenbosch, A. J. Lazzarini et al.,	Physical Review, C 31 (1985) 1752
$^4\text{He} + ^{162}\text{Dy} \rightarrow ^{166}\text{Er}$ (EvR)	R. Broda, M. Ishihara, B. Herskind et al.,	Nuclear Physics, A 248 (1975) 356
$^4\text{He} + ^{165}\text{Ho} \rightarrow ^{169}\text{Tm}$ (FF)	A. Buttkevit, H. H. Duhm, F. Goldenbaum et al.,	Physical Review, C 80 (2009) 37603
$^4\text{He} + ^{168}\text{Er} \rightarrow ^{172}\text{Yb}$ (EvR)	S. Gil, R. Vandenbosch, A. Charlop et al.,	Physical Review, C 43 (1991) 701
$^4\text{He} + ^{188}\text{Os} \rightarrow ^{192}\text{Pt}$ (EvR)	A. Navin, V. Tripathi, Y. Blumenfeld et al.,	Physical Review, C 70 (2004) 44601
$^4\text{He} + ^{182}\text{Os} \rightarrow ^{186}\text{Pt}$ (EvR)	A. Navin, V. Tripathi, Y. Blumenfeld et al.,	Physical Review, C 70 (2004) 44601
$^4\text{He} + ^{197}\text{Au} \rightarrow ^{201}\text{Ti}$ (FF)	D. L. Uhl, T. L. McDaniel, J. W. Cobble,	Physical Review, C 4 (1971) 1357
$^4\text{He} + ^{197}\text{Au} \rightarrow ^{201}\text{Ti}$ (FF)	J. Ralarosy, M. Debeauvais, G. Remy et al.,	Physical Review, C 8 (1973) 2372
$^4\text{He} + ^{197}\text{Au} \rightarrow ^{201}\text{Ti}$ (FF)	J. Gindler, H. Munzel, J. Buschmann et al.,	Nuclear Physics, A 145 (1970) 337
$^4\text{He} + ^{197}\text{Au} \rightarrow ^{201}\text{Ti}$ (EvR)	H. E. Kurz, E. W. Jasper, K. Fischer et al.,	Nuclear Physics, A 168 (1971) 129
$^4\text{He} + ^{197}\text{Au} \rightarrow ^{201}\text{Ti}$ (FF)	A. Buttkevit, H. H. Duhm, F. Goldenbaum et al.,	Physical Review, C 80 (2009) 37603
$^4\text{He} + ^{197}\text{Au} \rightarrow ^{201}\text{Ti}$ (FF)	J. R. Huizenga, R. Chaudhry, R. Vandenbosch,	Physical Review, 126 (1962) 210
$^4\text{He} + ^{203}\text{Tl} \rightarrow ^{207}\text{Bi}$ (FF)	J. R. Huizenga, R. Chaudhry, R. Vandenbosch,	Physical Review, 126 (1962) 210
$^4\text{He} + ^{205}\text{Tl} \rightarrow ^{209}\text{Bi}$ (FF)	J. R. Huizenga, R. Chaudhry, R. Vandenbosch,	Physical Review, 126 (1962) 210
$^4\text{He} + ^{206}\text{Pb} \rightarrow ^{210}\text{Po}$ (FF)	J. R. Huizenga, R. Chaudhry, R. Vandenbosch,	Physical Review, 126 (1962) 210
$^4\text{He} + ^{207}\text{Pb} \rightarrow ^{211}\text{Po}$ (FF)	J. Ralarosy, M. Debeauvais, G. Remy et al.,	Physical Review, C 8 (1973) 2372
$^4\text{He} + ^{208}\text{Pb} \rightarrow ^{212}\text{Po}$ (alpha-decay of EvR)	S. M. Lukyanov, Yu. E. Penionzhkevich, R. A. Astabatie et al.,	Physics Letters, B 670 (2009) 321
$^4\text{He} + ^{208}\text{Bi} \rightarrow ^{212}\text{At}$ (FF)	W. G. Meyer, V. E. Viola, Jr., R. G. Clark et al.,	Physical Review, C 20 (1979) 1716
$^4\text{He} + ^{208}\text{Bi} \rightarrow ^{212}\text{At}$ (FF)	D. L. Uhl, T. L. McDaniel, J. W. Cobble,	Physical Review, C 4 (1971) 1357
$^4\text{He} + ^{208}\text{Bi} \rightarrow ^{212}\text{At}$ (FF)	J. Ralarosy, M. Debeauvais, G. Remy et al.,	Physical Review, C 8 (1973) 2372
$^4\text{He} + ^{208}\text{Bi} \rightarrow ^{212}\text{At}$ (FF)	Yu. E. Penionzhkevich, Yu. A. Muzychka, S. M. Lukyanov et al.,	European Physical Journal, A 13 (2002) 123
$^4\text{He} + ^{208}\text{Bi} \rightarrow ^{212}\text{At}$ (FF)	A. S. Fomichev, I. David, Z. Dlouhy et al.,	Zeitschrift für Physik, 351 (1995) 129
$^4\text{He} + ^{208}\text{Bi} \rightarrow ^{212}\text{At}$ (FF)	J. Gindler, H. Munzel, J. Buschmann et al.,	Nuclear Physics, A 145 (1970) 337
$^4\text{He} + ^{208}\text{Bi} \rightarrow ^{212}\text{At}$ (FF)	J. R. Huizenga, R. Chaudhry, R. Vandenbosch,	Physical Review, 126 (1962) 210
$^4\text{He} + ^{232}\text{Th} \rightarrow ^{236}\text{U}$ (FF)	J. Ralarosy, M. Debeauvais, G. Remy et al.,	Physical Review, C 8 (1973) 2372
$^4\text{He} + ^{233}\text{U} \rightarrow ^{237}\text{Pu}$ (FF)	W. G. Meyer, V. E. Viola, Jr., R. G. Clark et al.,	Physical Review, C 20 (1979) 1716
$^4\text{He} + ^{233}\text{U} \rightarrow ^{237}\text{Pu}$ (FF)	H. Freiesleben, J. R. Huizenga,	Nuclear Physics, A 224 (1974) 503
$^4\text{He} + ^{233}\text{U} \rightarrow ^{237}\text{Pu}$ (FF)	J. Gindler, H. Munzel, J. Buschmann et al.	Nuclear Physics, A 145 (1970) 337

Approach: *supervised learning* using ***symbolic regression*** algorithms.

### **Novelties:**

- Deriving mathematical expressions to describe the data → support to theories and models attempting to predict the fusion cross section between heavy-ions;
- Comprehensive analysis of large amount of nuclear data → universal model for the description of the entire dataset;
- Advanced feature selection → allows to inspect the dependence on several variables (including nuclear structure variables).

### **Major challenges:**

- The amplitude of the cross section varies even by several orders of magnitude with the energy;
- Experimental errors associated to each individual data point differ by several orders of magnitude for each data point;
- Resulting models must have physical boundaries and extrapolation capabilities.

Dataset used for model derivation → about 4500 experimental data points.

- Learning dataset:  $Z_p Z_t < 250$  → light-to-medium mass nuclei.
- Testing dataset 1:  $Z_p Z_t \geq 250$  → heavy systems (test the extrapolation towards heavy systems).
- Testing dataset 2:  $Z_p Z_t < 250$  → some of the lighter systems.

Symbol	Description
$\frac{1}{E_{cm}}$	inverse of the collision center-of-mass energy (MeV)
$Z_1$	charge of the projectile
$Z_2$	charge of the target
$A_1$	mass of the projectile
$A_2$	mass of the target
$J_1$	spin of the projectile
$J_2$	spin of the target
$\pi_1$	parity of the projectile (1 for positive parity, -1 for negative parity)
$\pi_2$	parity of the target (1 for positive parity, -1 for negative parity)
$\mu_1$	magnetic dipole momentum of the projectile ( $\mu_N$ )
$\mu_2$	magnetic dipole momentum of the target ( $\mu_N$ )
$\langle r^2 \rangle_1$	rms charge radius of the projectile (fm)
$\langle r^2 \rangle_2$	rms charge radius of the target (fm)
$Q$ -value	fusion $Q$ -value (MeV)
$S_\alpha$	$\alpha$ separation energy of the compound nucleus (MeV)
$S_{\alpha 1}$	$\alpha$ separation energy of the projectile
$S_{\alpha 2}$	$\alpha$ separation energy of the target
$S_{n1}$	one-neutron separation energy of the projectile
$S_{n2}$	one-neutron separation energy of the target
$S_{p1}$	one-proton separation energy of the projectile
$S_{p2}$	one-neutron separation energy of the target
$S_{2n1}$	two-neutron separation energy of the projectile
$S_{2n2}$	two-neutron separation energy of the target
$S_{2p2}$	two-proton separation energy of the projectile
$S_{2p2}$	two-neutron separation energy of the target

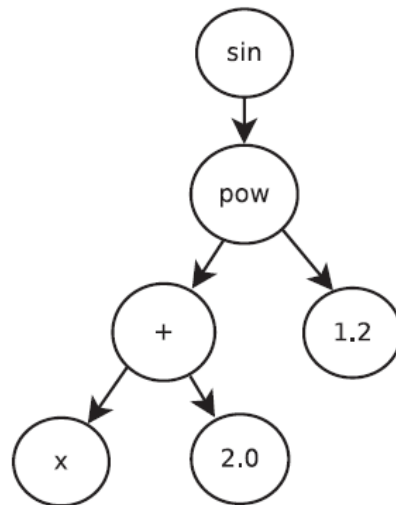


## Brain Project – a neural-genetic tool for the formal modeling of data

Exploits a novel hybridization of genetic programming and artificial neural network → the task is that of symbolic regression. Genetic part → foresees the evolution of tree-like structures representing mathematical expressions → deals with the global search for the maximum of a suitable fitness function; Neural part → deals with the local search for the minimum of the error when the genetic part has identified a good maximum of the fitness function.

Russo M 2016 *Swarm Evo. Comput.* **27** 145

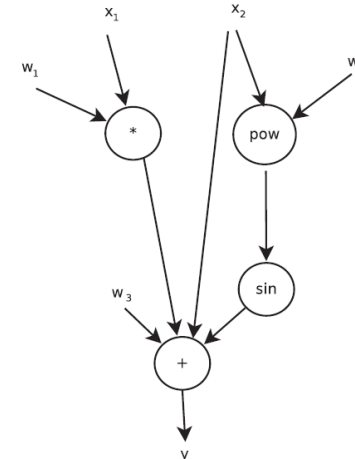
Russo M 2020 *Soft Comput.* **24** 16885–16894



Genetic evolution of tree-like structures representing mathematical expressions



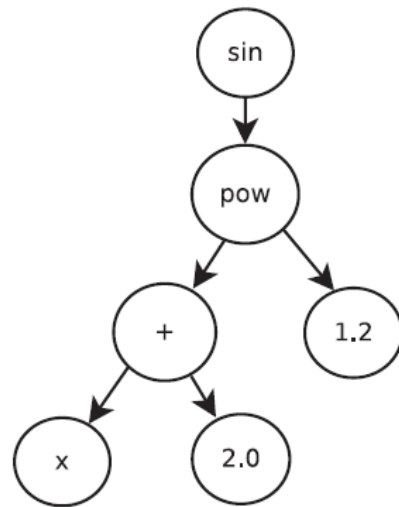
$$y = 2.5 * x_1 + \sin(x_2^{3.8}) + x_2 + 1.5$$



Artificial neural networks to optimize the constants (gradient descent technique)

**Brain Project:** *genetic part* uses genetic → a programming technique that exploits concepts derived from the natural selection (Darwinian Theory of Evolution) to solve optimization problems.

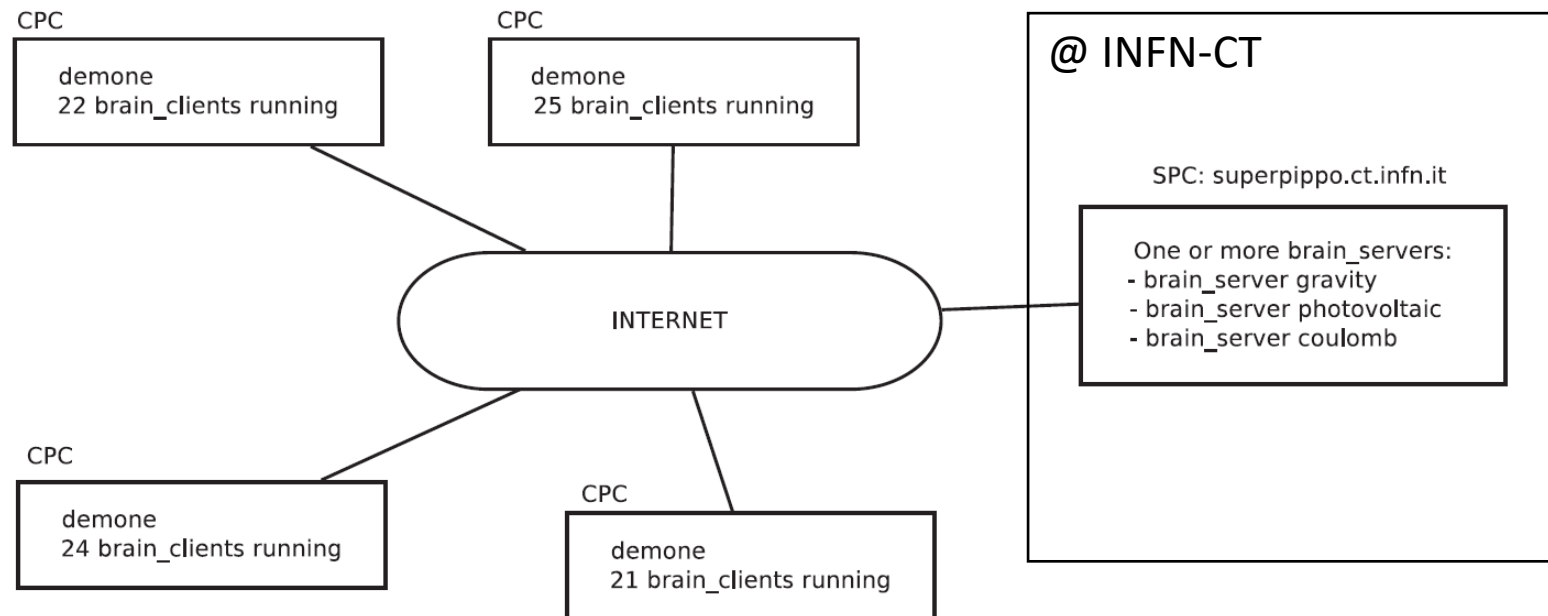
See e.g. John R. Koza, *Genetic Programming – On the Programming of Computers by Means of Natural Selection*, A Bradford Book – The MIT Press (Cambridge, Massachusetts; London, England)



1. Generate an initial population of random trees representing possible solutions to the SR problem being studied;
2. Evaluate each possible solution and assign it a fitness value  $f_{\text{fit}}$ ;
3. Create a new population of trees starting from the previous population using some evolutionary operators, e.g. copy, crossover, mutation, selection and heuristic operators;
4. Return to step two until the termination criterion is met.

Key «ingredients»:

- crossover;
- fitness;
- mutation;
- populations/migration.



*from Russo M 2016 Swarm Evo. Comput. **27** 145*

**Fitness function** → is the function to maximize → it suitably contains the prediction error and a term related to the complexity of the model and/or feature costs.

to tune the desired trade-off  
between accuracy and  
complexity

$$f_{\text{fit}} = 100.0 * \frac{f_e u(f_e) + \alpha f_n u(f_n)}{1 + \alpha} e^{f_e(1 - u(f_e))} e^{f_n(1 - u(f_n))}$$

$$f_e = \frac{e_{\text{max}} - e}{e_{\text{max}}}$$

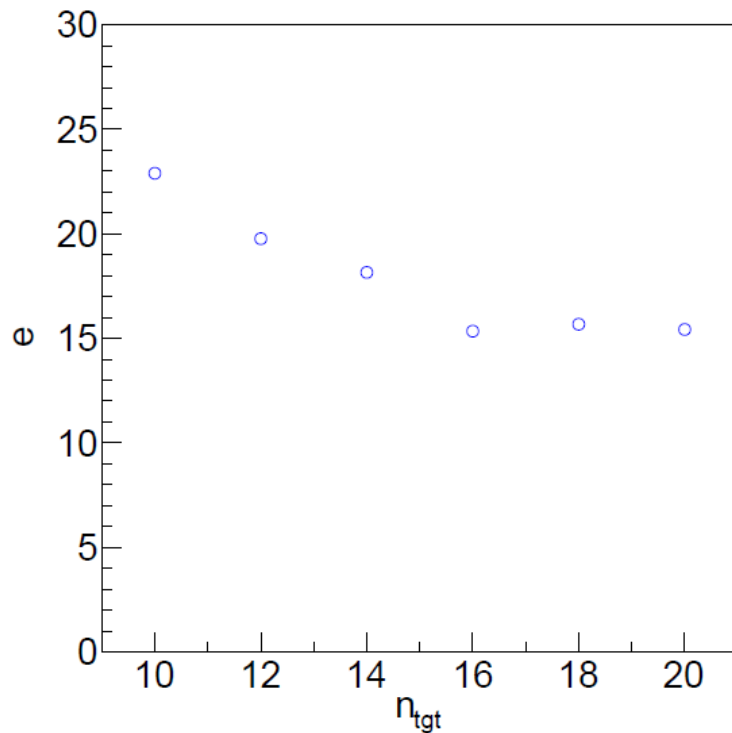
related to the accuracy of the  
model

$$e = 100 \sqrt{\sum_{o=1}^{N_o} \sum_{p=1}^{N_p} \frac{(w_{\text{pat}_p} w_{\text{out}_o} (y_{op}^d - y_{op}^c))^2}{N_o N_{p_{eq}}}}$$

$$f_n = \frac{n_{\text{max}} - n}{n_{\text{max}} - 1}$$

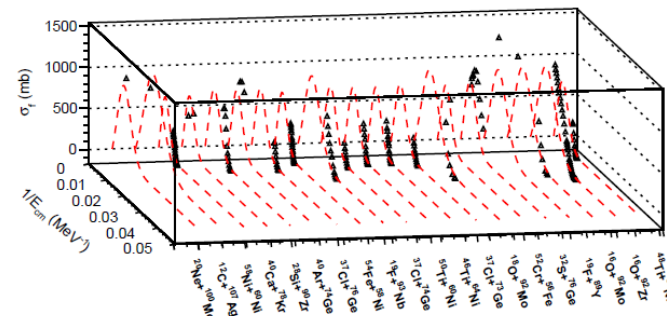
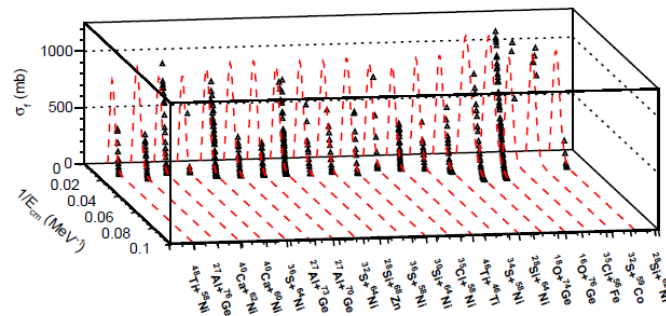
related to the complexity of the  
model

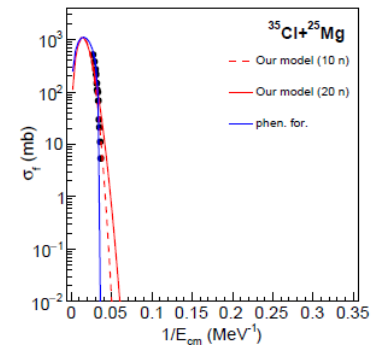
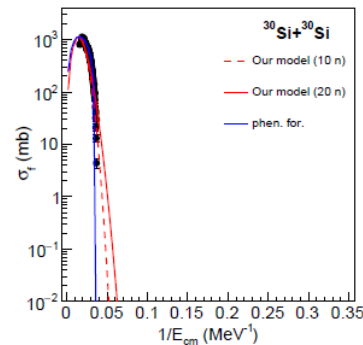
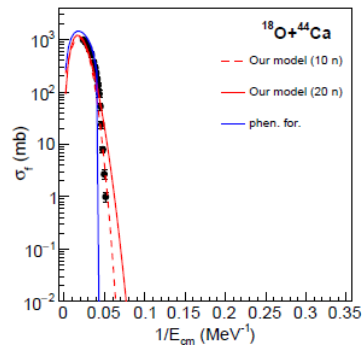
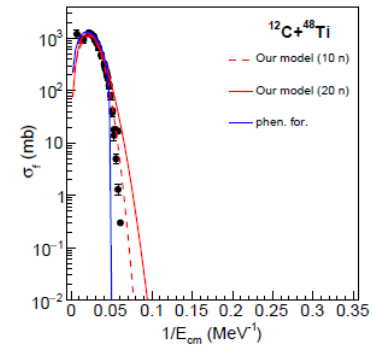
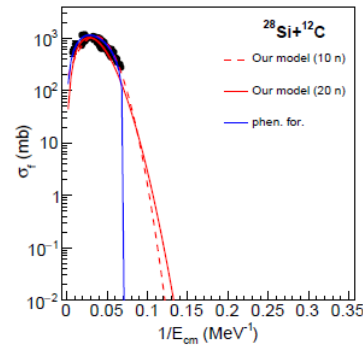
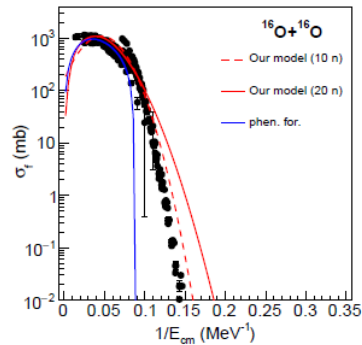
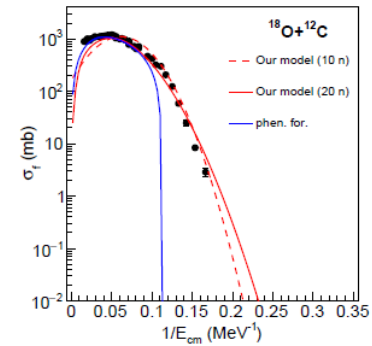
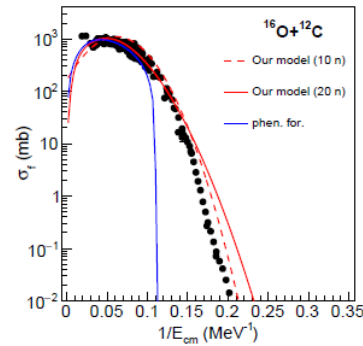
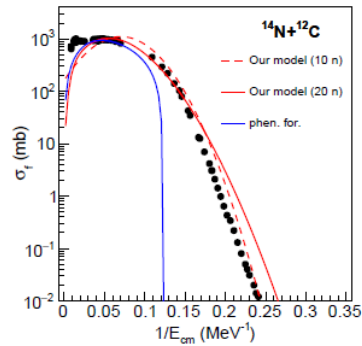
$f_{fit} = f_{fit} \cdot e^{\frac{n_{tgt}-n}{n_{tgt}}} \rightarrow$  required to reach a predefined, target, number of nodes. Brain Project usually tries to optimize the error with a given number of nodes  $\rightarrow$  interesting to more easily tune the complexity of the desired model.

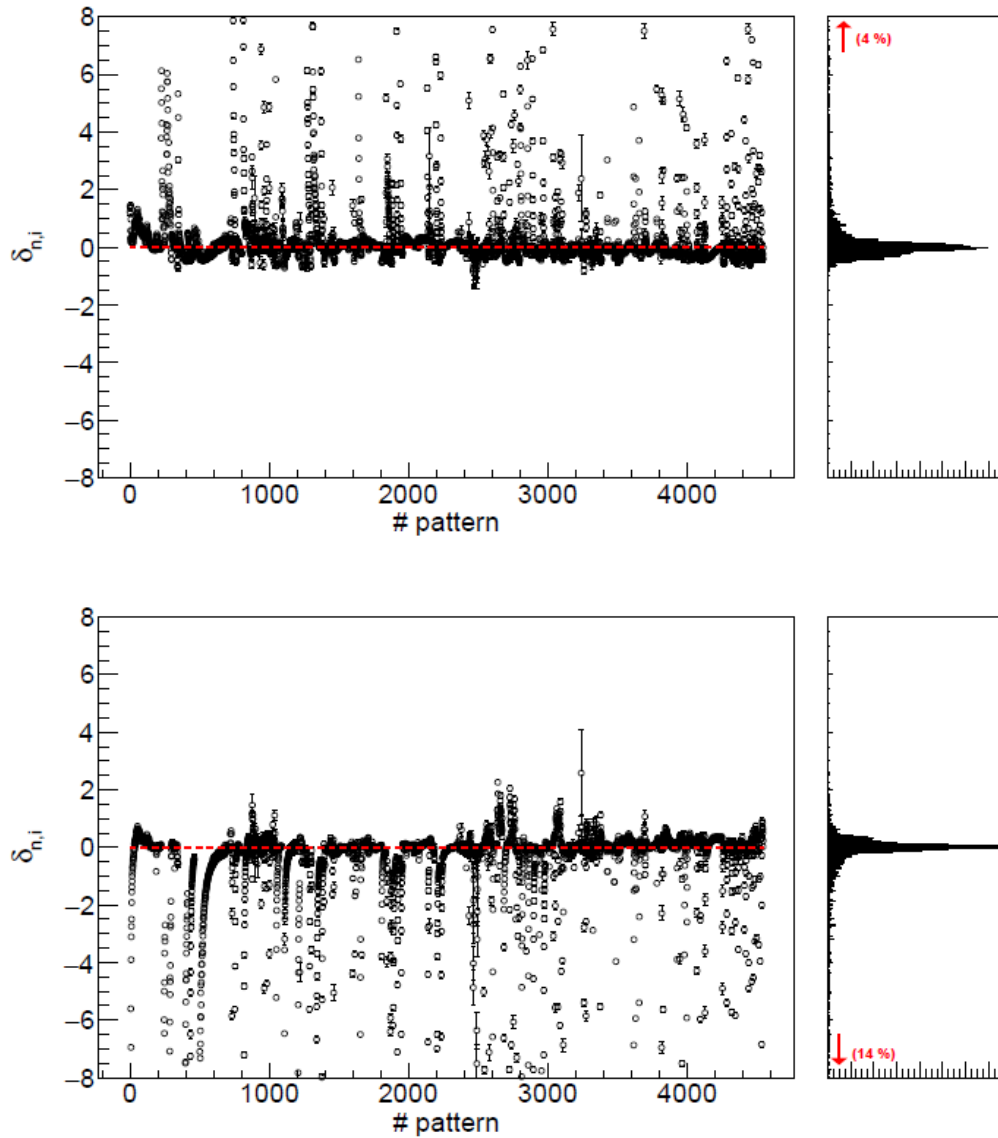


$$\sigma_{fus}^{(n_{tgt}=10)}(E_{cm}) = 1102 \cdot \exp \left[ - \left( 1.39 - 0.469 \cdot Z_2 \cdot Z_1 \cdot \frac{1}{E_{cm}} \right)^2 \right]$$

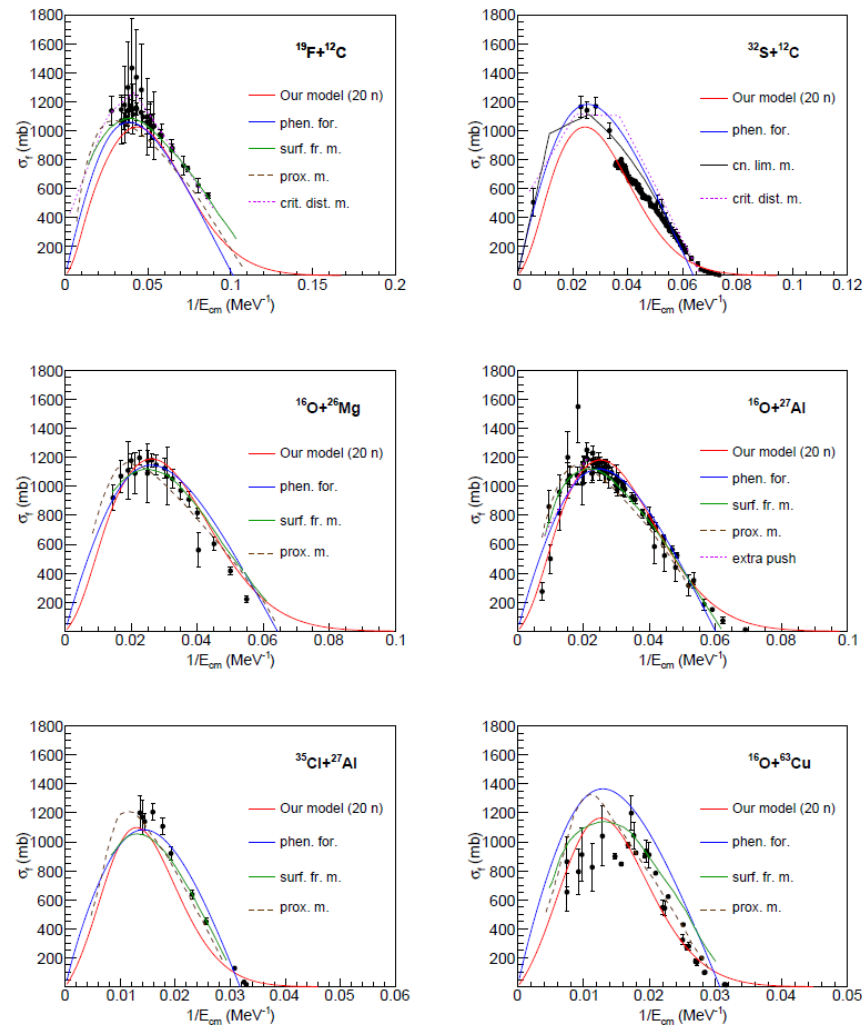
$$\sigma_{fus}^{(n_{tgt}=20)}(E_{cm}) = 144.5 \cdot \operatorname{erf} \left\{ \exp \left[ - \left( 0.129 \cdot \frac{1}{E_{cm}} \cdot Z_1 \cdot (A_2 + \ln(S_{2n_2})) \right)^2 \right] \right\} \cdot e^{\operatorname{erf}\left(102.3 \cdot \frac{1}{E_{cm}}\right) \cdot Z_1 \cdot \frac{1}{E_{cm}} \cdot A_2}$$





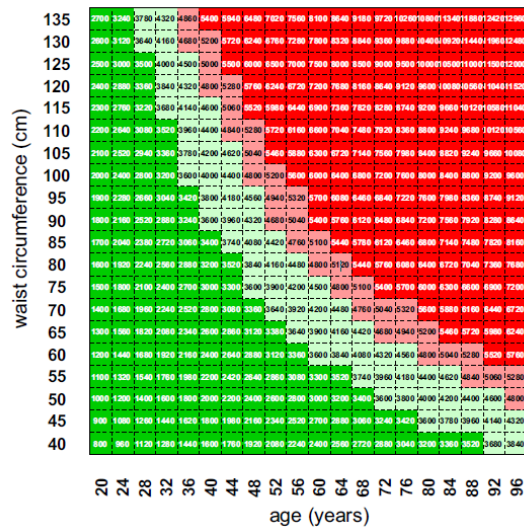




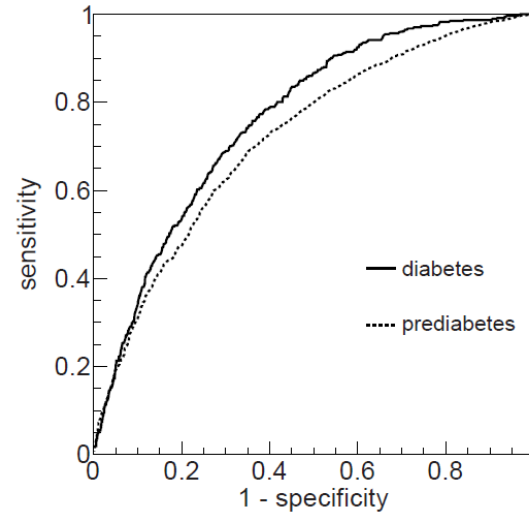


*Daniele Dell'Aquila, Brunilde Gnoffo, Ivano Lombardo, Francesco Porto, Marco Russo, Modeling Heavy-Ion Fusion Cross Section Data via a Novel Artificial Intelligence Approach, arXiv:2203.10367, <https://doi.org/10.48550/arXiv.2203.10367>*

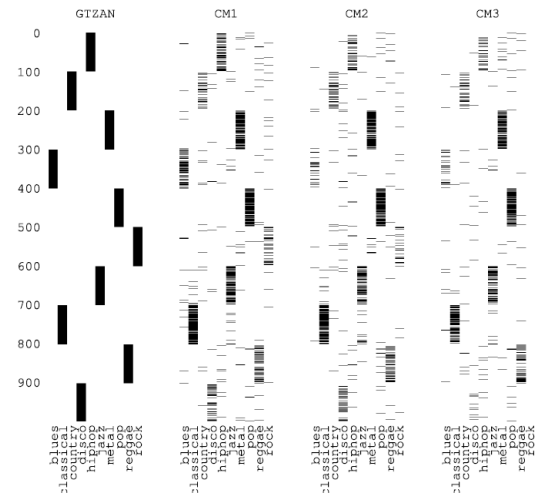
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- Numerous datasets are currently available in nuclear physics, taking advantage of several decades of sophisticated experimental investigations;
- NRV has collected fusion cross section data between heavy-ions → their overall description is still not completely understood by state-of-the-art models;
- Artificial intelligence techniques based on symbolic regression (which exploit genetic programming and artificial neural networks) can serve to effectively model even particularly complex datasets;
- Analytical formulation of the models → useful to spot functional dependencies that can be exploited by theoretical models;
- Advanced feature selection → can serve to guide theoretical models towards the discovery of the dependence on key nuclear physics parameters;
- Many more datasets are yet to be modeled!