Magnetic Turbulence and Cosmic-Ray Small-Scale Anisotropies

Vo Hong Minh Phan

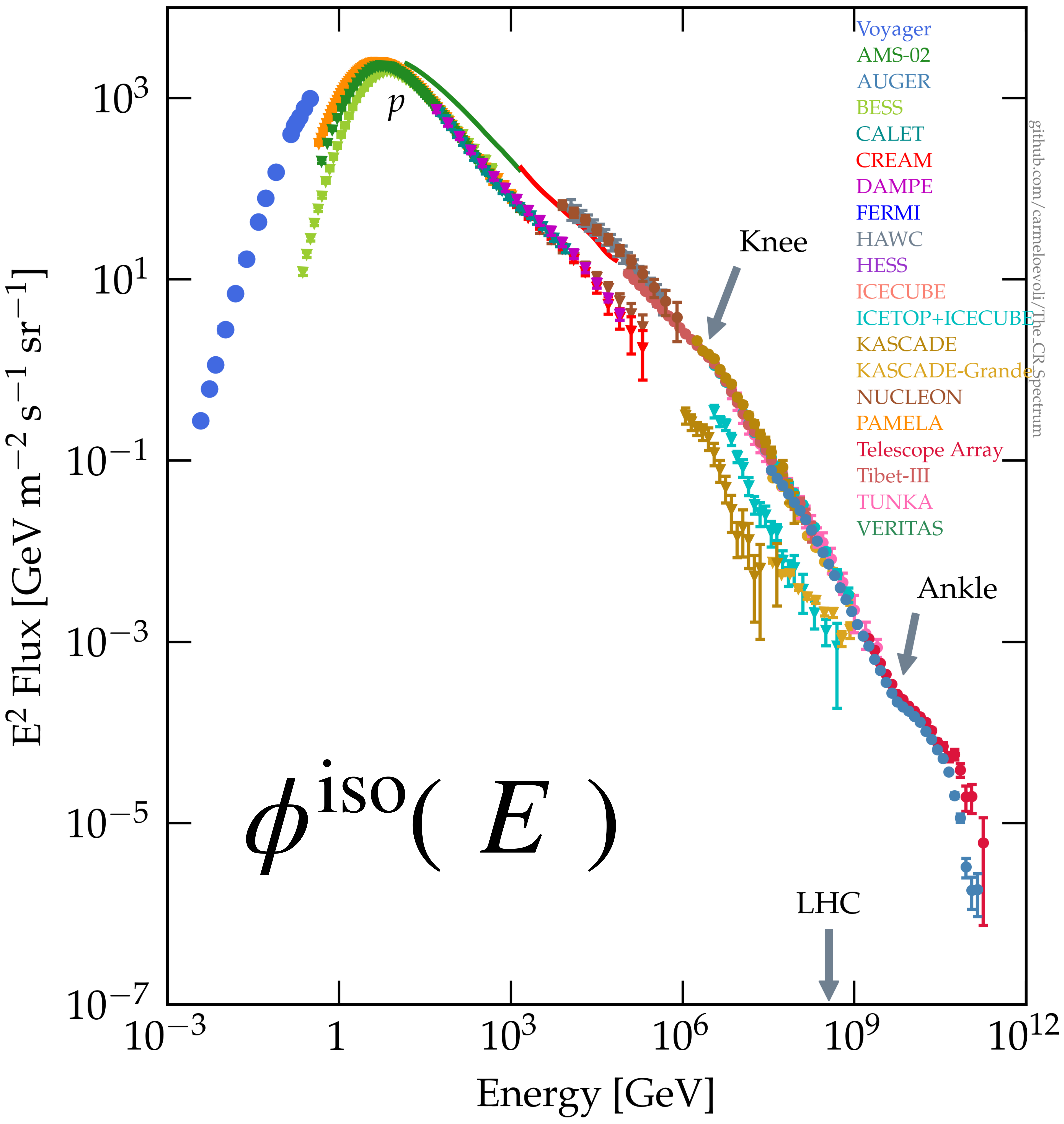
Adopted from Kuhlen, Phan and Mertsch ApJ 2021.

RWTH AACHEN UNIVERSITY

Outline

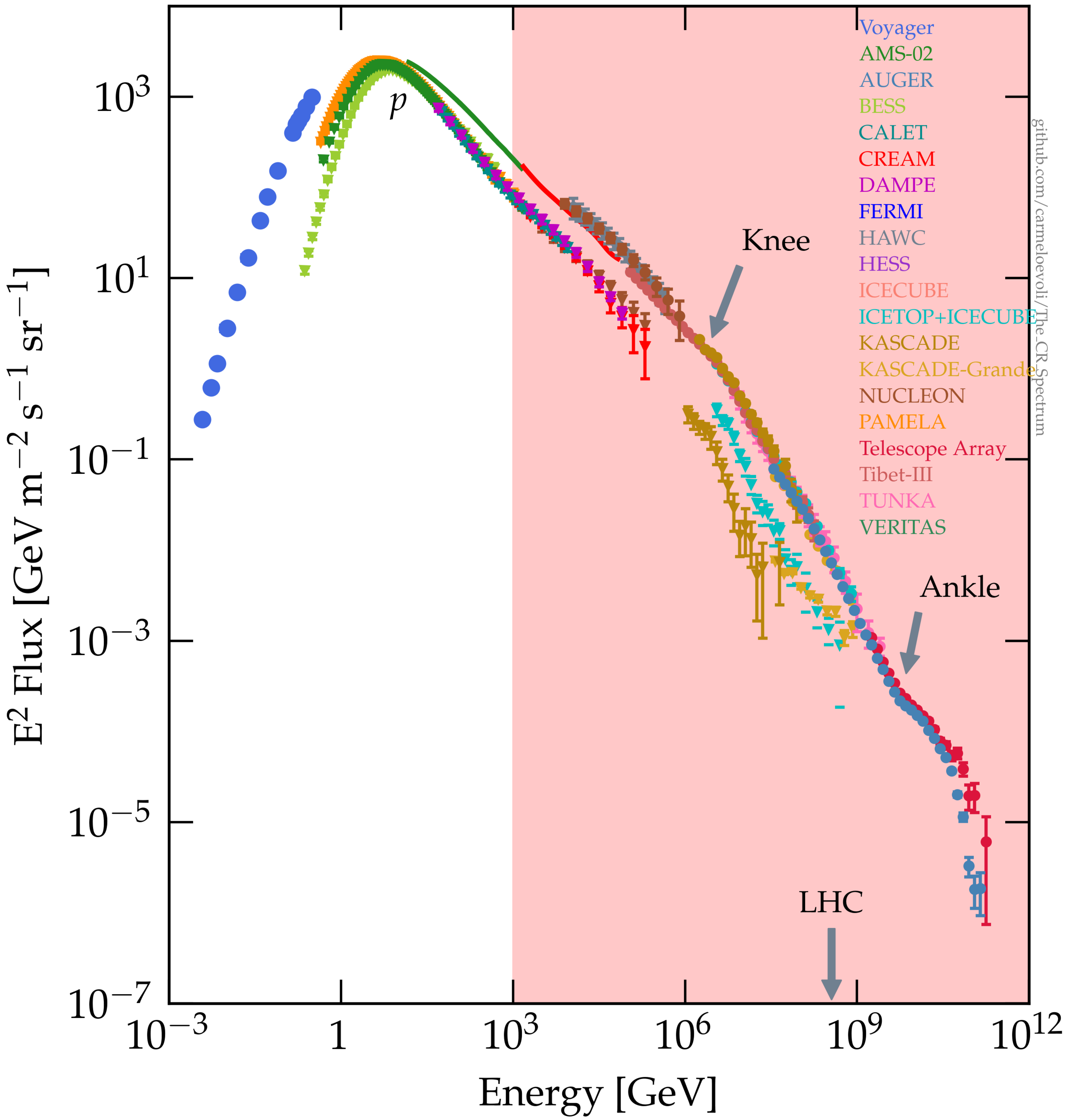
- Introduction to Cosmic-Ray Anisotropies
- Numerical Simulations
- Analytic Theory for Cosmic-Ray Anisotropies
- Summary and Future Perspectives

Cosmic-Ray Flux



Credit: Carmelo Evoli

Cosmic-Ray Flux



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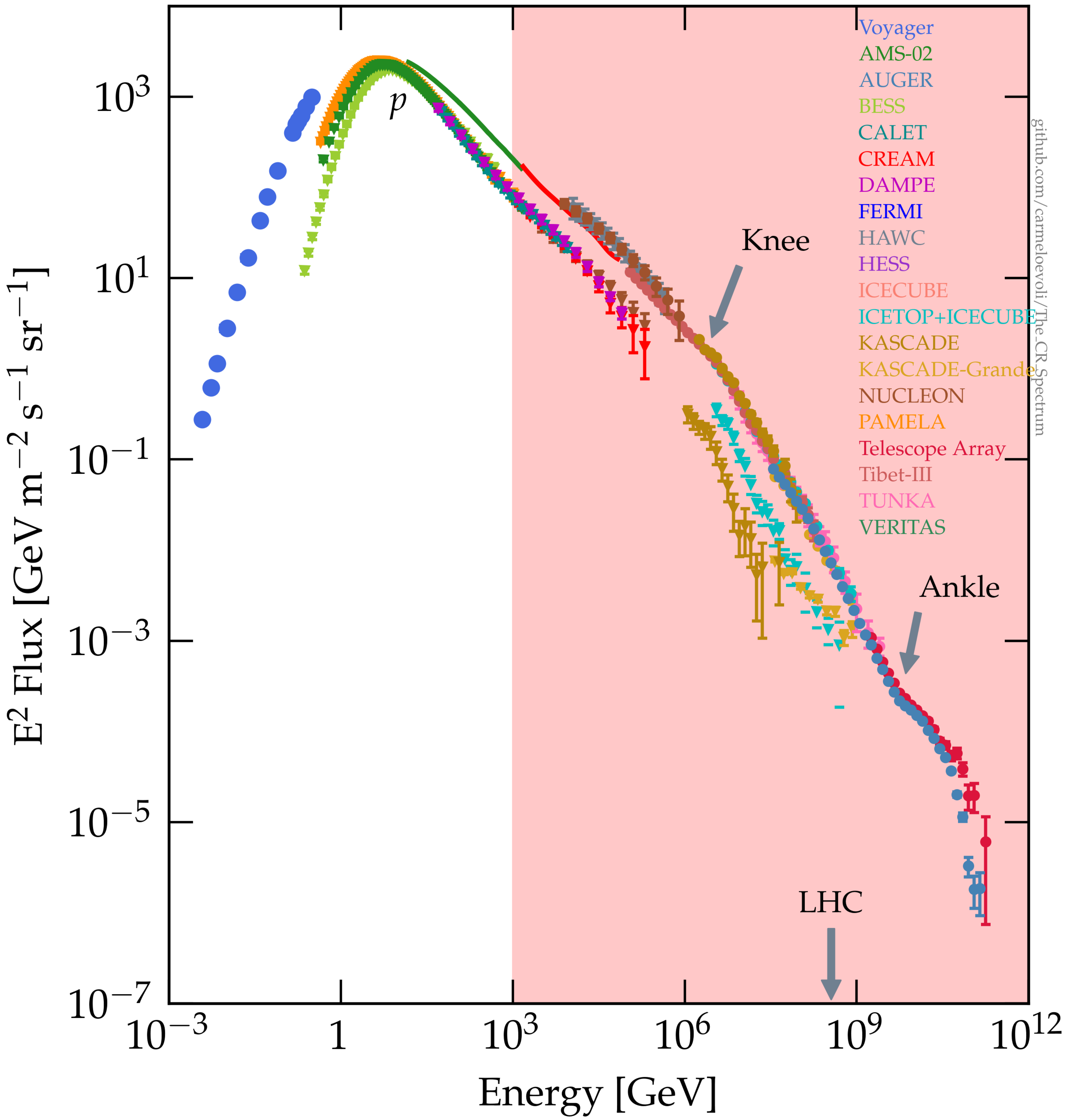
Cosmic-Ray Flux

High statistics and large field of view



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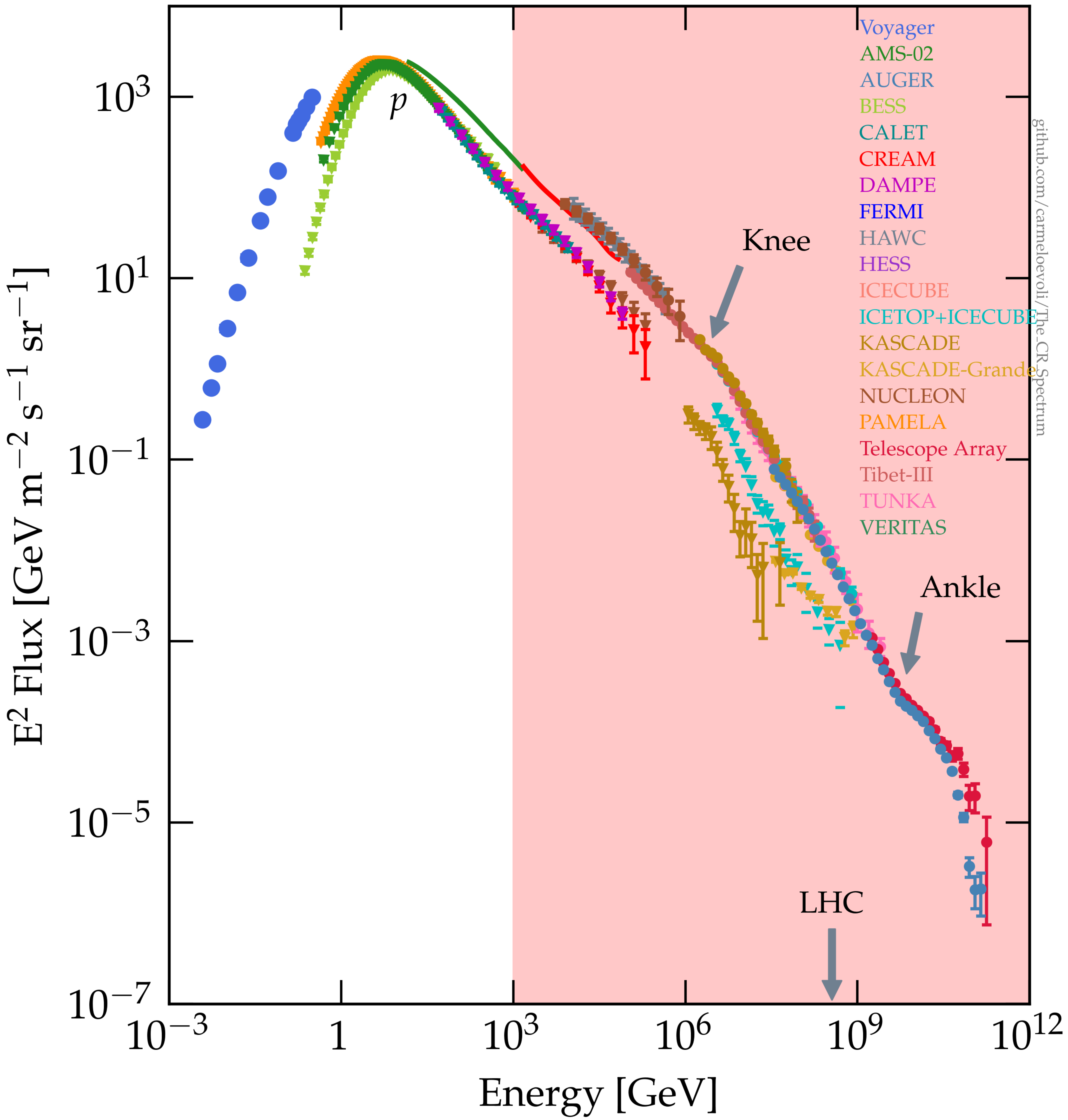
Cosmic-Ray Flux

High statistics and large field of view

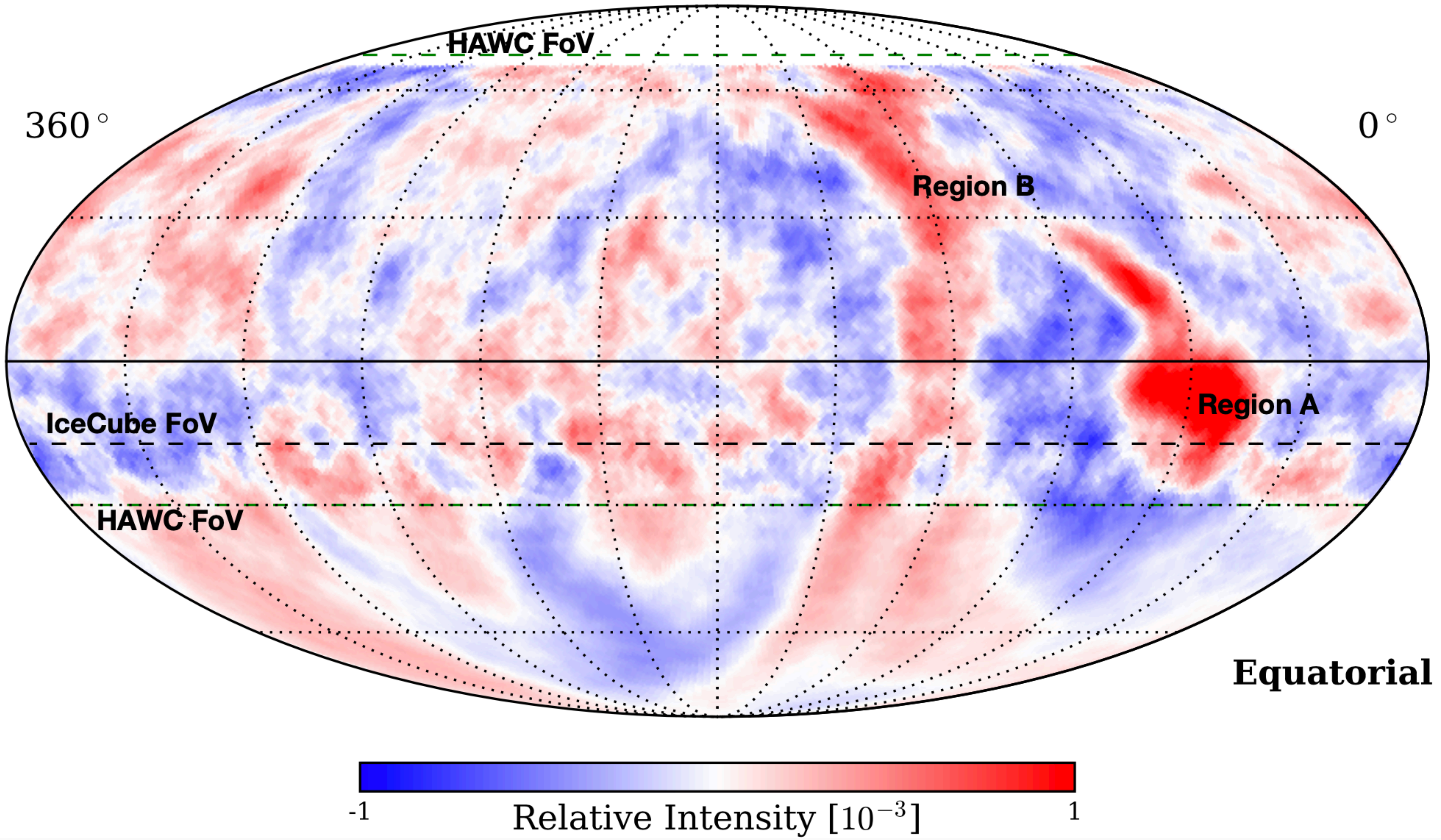
$$\phi(\,\hat{\mathbf{n}}\,,\,E\,)$$



Credit: Carmelo Evoli

Cosmic-Ray Anisotropy

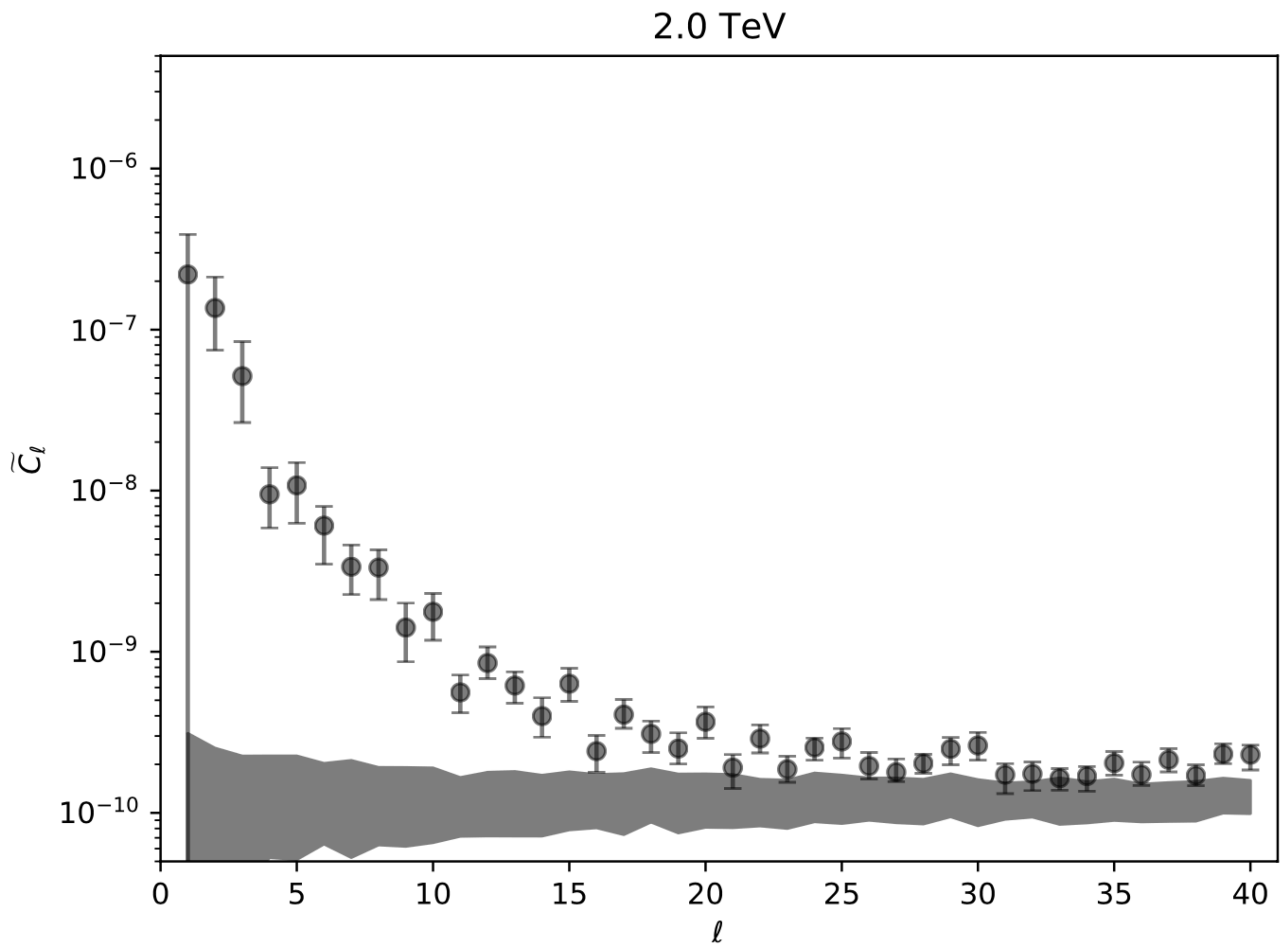
$$\delta I(\hat{\mathbf{n}}, E) = 1 - \frac{\phi(\hat{\mathbf{n}}, E)}{\phi^{\mathrm{iso}}(E)}$$



Credit: IceCube, HAWC, Philipp Mertsch

Angular Power Spectrum

$$C_\ell(E) = \int \mathrm{d}\hat{\mathbf{n}}_1 \, \mathrm{d}\hat{\mathbf{n}}_2 \, P_\ell(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2) \, \phi(\hat{\mathbf{n}}_1, E) \, \phi(\hat{\mathbf{n}}_2, E)$$



Abeysekara et al., ApJ 2018, Giacinti & Sigl, PRL 2012

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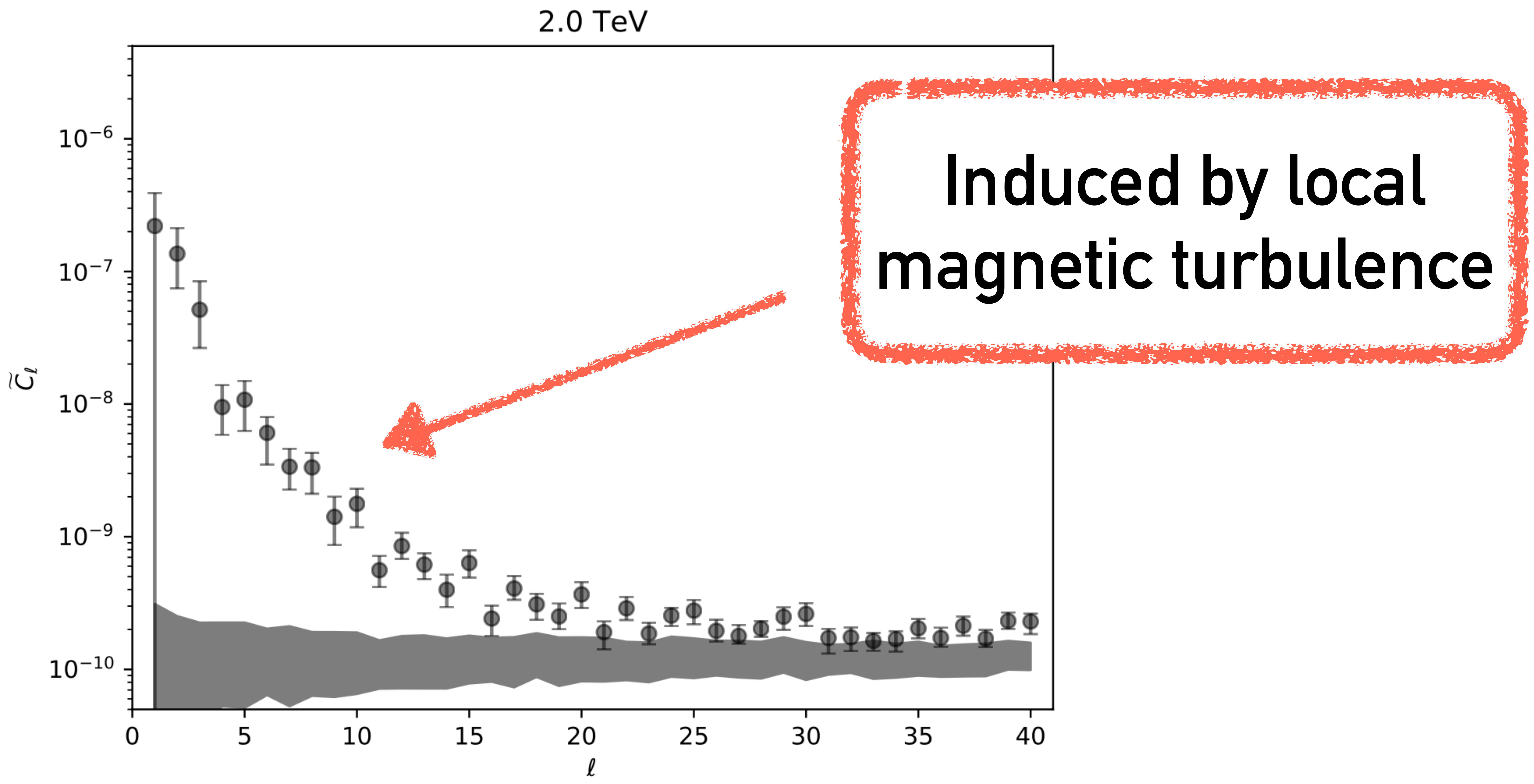
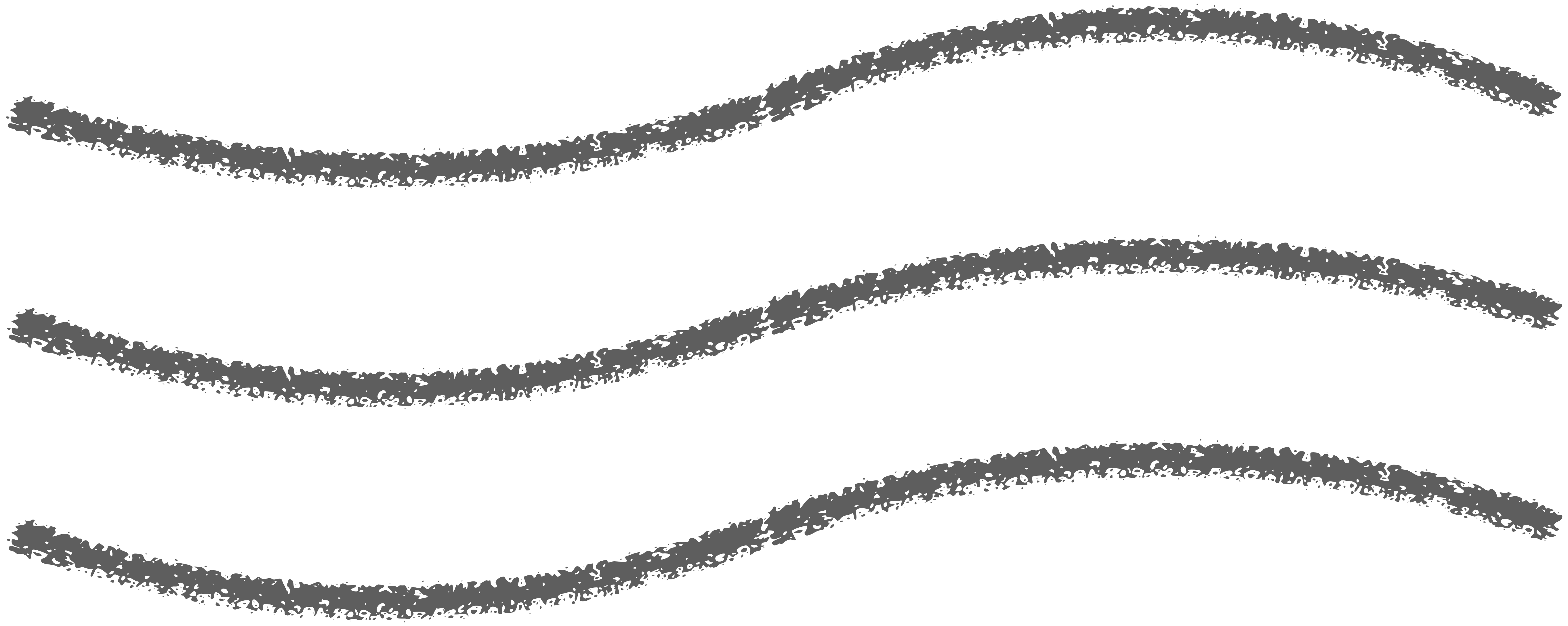


Abeysekara et al., ApJ 2018, Giacinti & Sigl, PRL 2012

Magnetic Turbulence

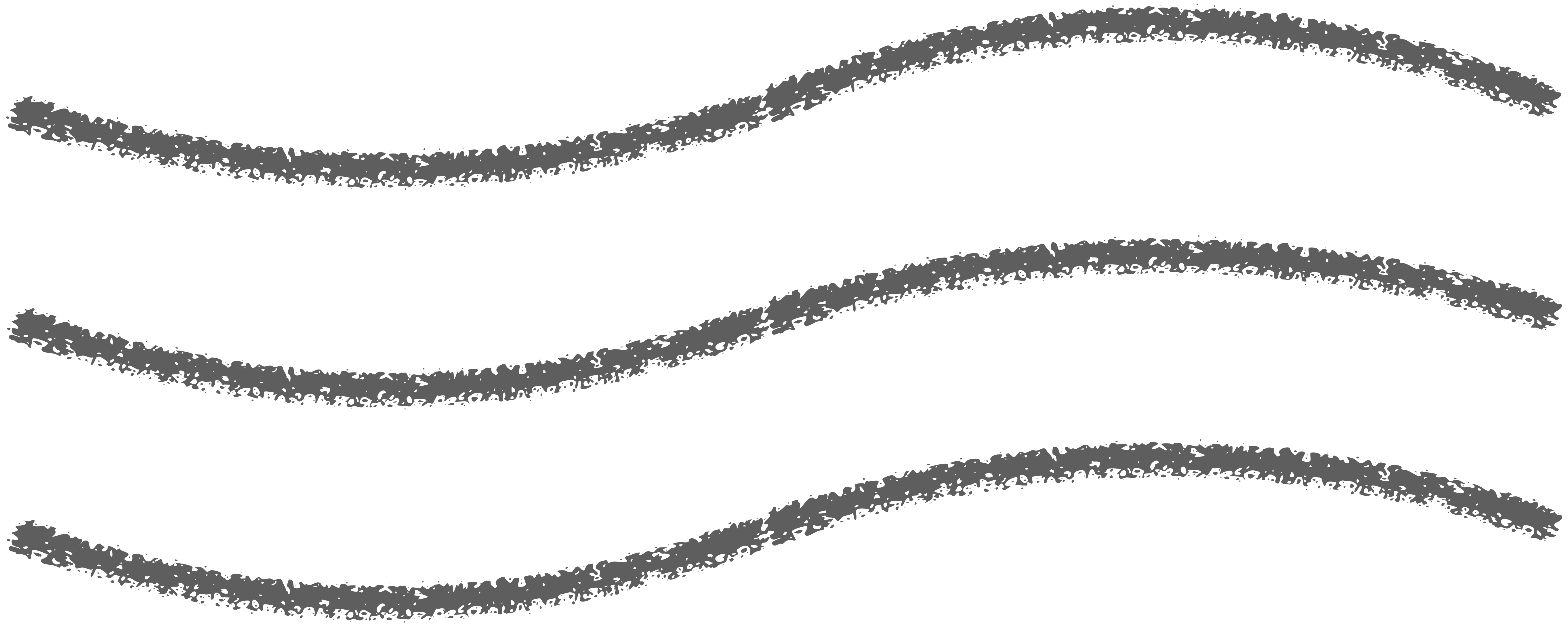
Magnetic Turbulence

$$\mathbf{B}_0 + \delta\mathbf{B}(\mathbf{r})$$



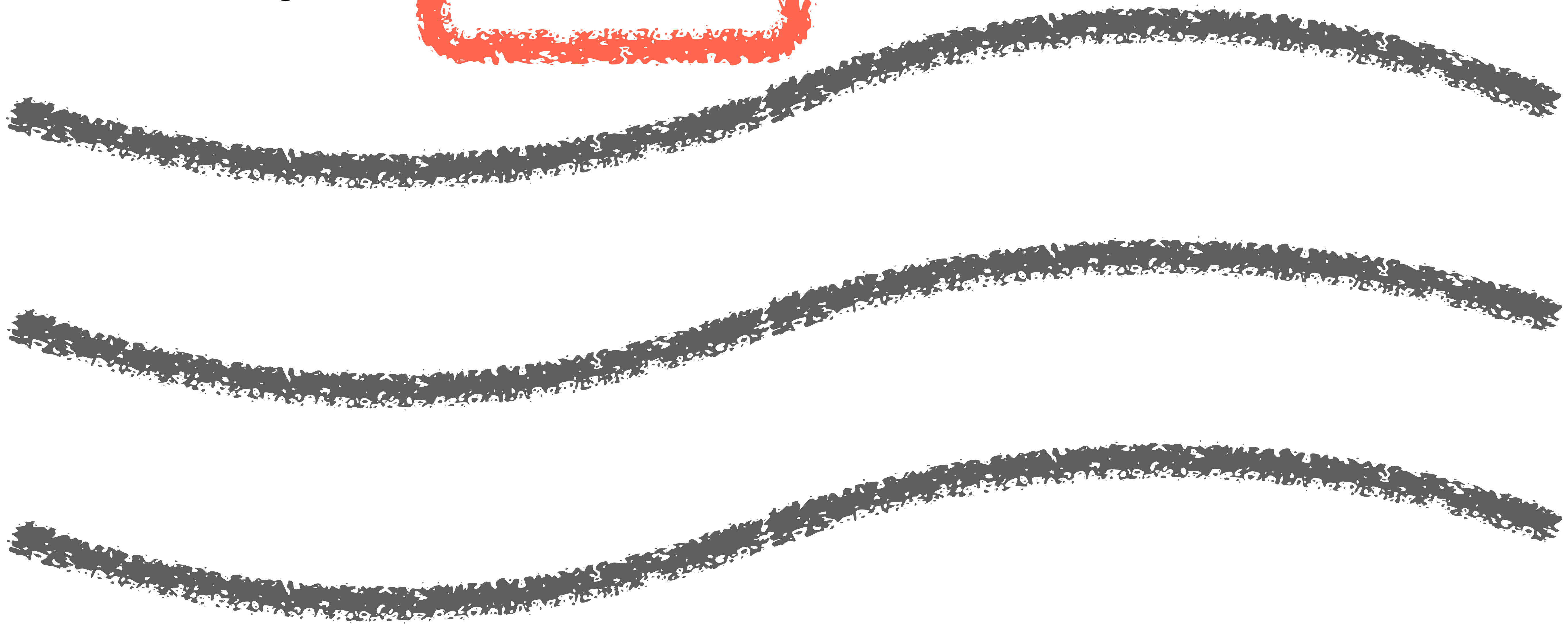
Magnetic Turbulence

$$\mathbf{B}_0 + \delta\mathbf{B}(\mathbf{r}) \longrightarrow \langle \delta\mathbf{B}(\mathbf{r}) \rangle = \mathbf{0}$$



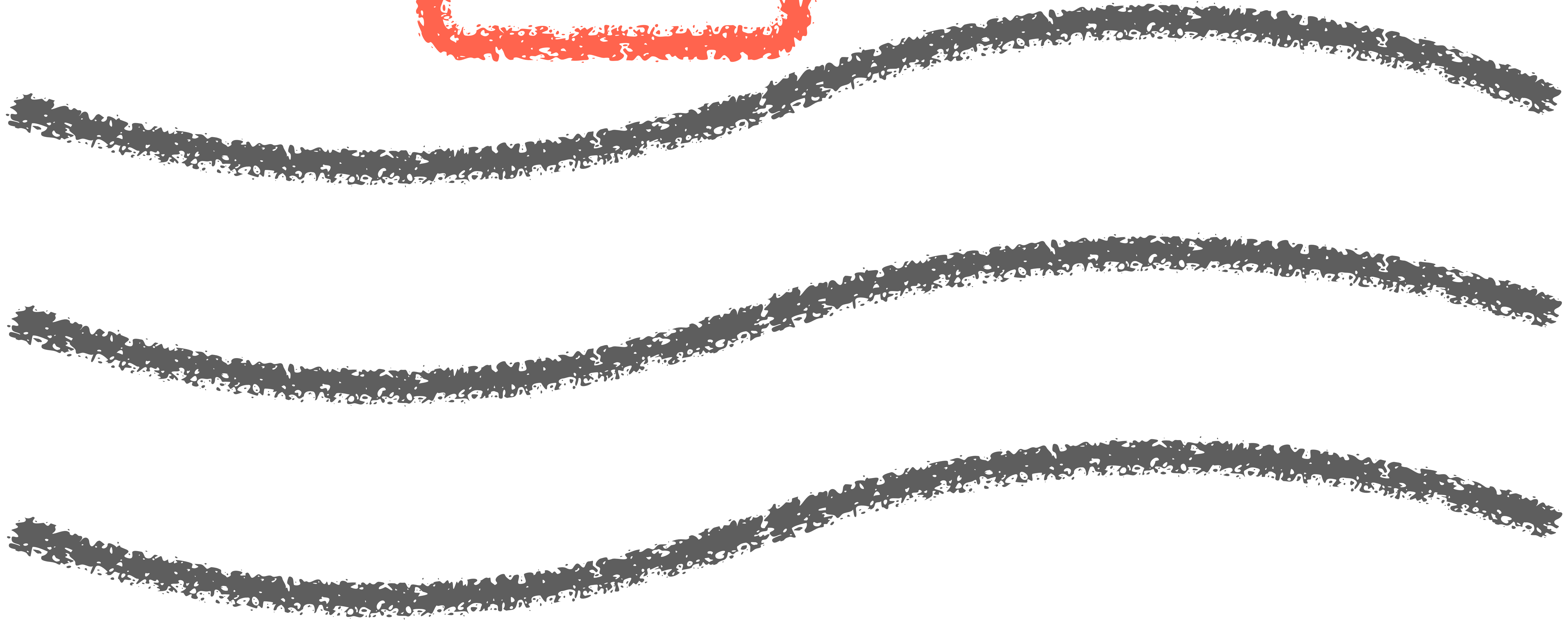
Magnetic Turbulence

$$\mathbf{B}_0 + \delta\mathbf{B}(z)$$



Magnetic Turbulence

$$\mathbf{B}_0 + \delta\mathbf{B}(z)$$ → Slab turbulence



Magnetic Turbulence

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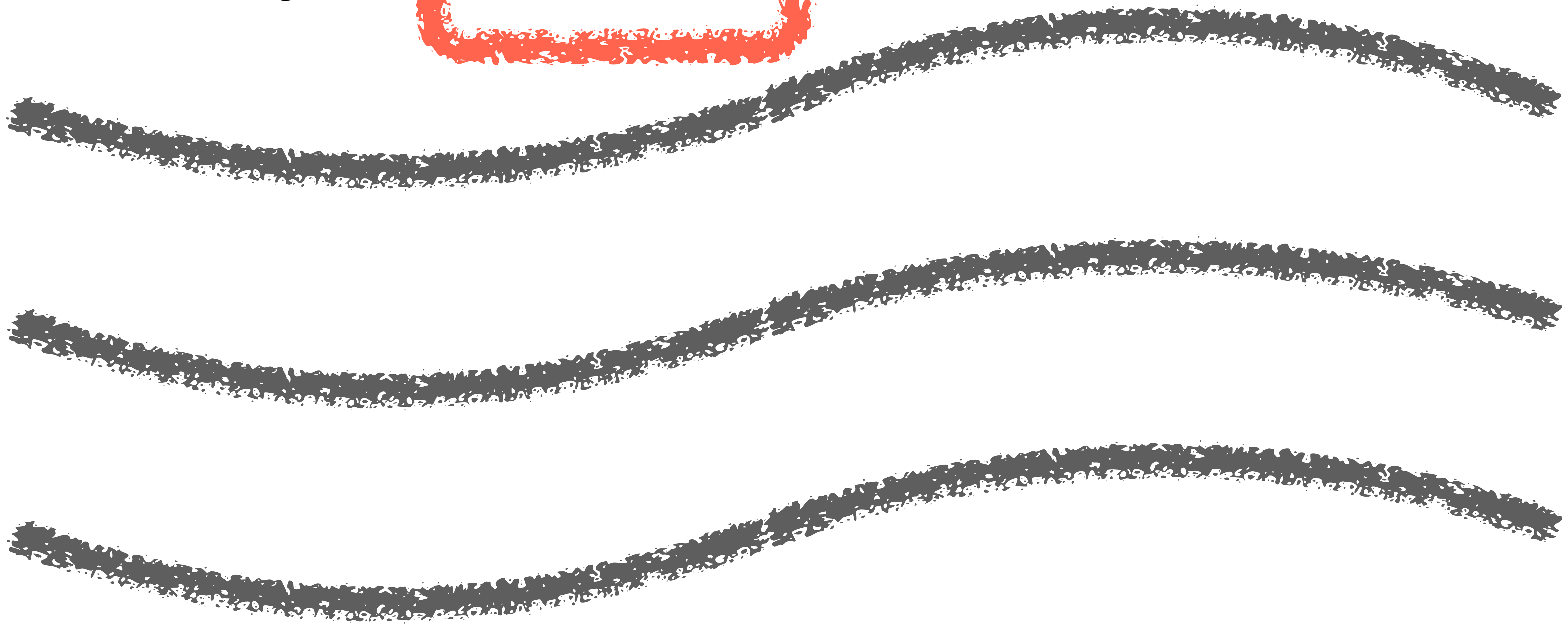

Slab turbulence

$$P(k_{\parallel}) \sim k_{\parallel}^{-5/3}$$

Magnetic Turbulence

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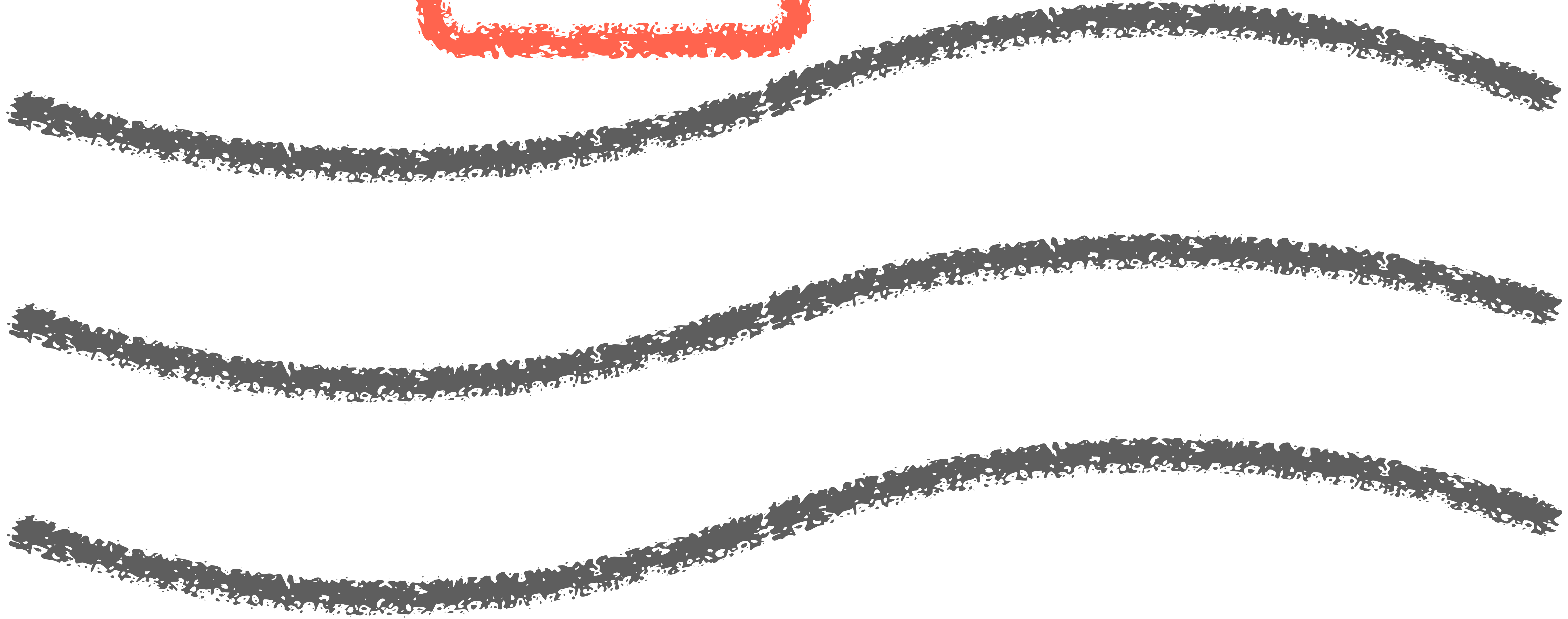

Slab turbulence

$$P(k_{\parallel}) \sim k_{\parallel}^{-5/3}$$

$$K_{zz}(E) \sim E^{1/3}$$

Numerical Simulations: Liouville's Theorem

$$f(\mathbf{r}(t), \mathbf{p}(t), t) = f(\mathbf{r}(t_0), \mathbf{p}(t_0), t_0)$$

Ahlers & Mertsch, ApJL 2015

Numerical Simulations: Liouville's Theorem

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$$f(\mathbf{r}_\odot, \mathbf{p}(t), t) \;=\; f_0 \;+\; \left[\mathbf{r}_\mathbf{i}(t_0) \;-\; 3\hat{\mathbf{p}}_\mathbf{i}(t_0) \cdot \mathbf{K}\right] \;\cdot\; \nabla f_0$$

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Local isotropic cosmic-ray spectrum

Ahlers & Mertsch, ApJL 2015

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Local isotropic cosmic-ray spectrum

Local cosmic-ray gradient

Ahlers & Mertsch, ApJL 2015

Numerical Simulations: Liouville's Theorem

$$f(\mathbf{r}(t), \mathbf{p}(t), t) = f(\mathbf{r}(t_0), \mathbf{p}(t_0), t_0)$$

Trajectories + diffusion coefficient

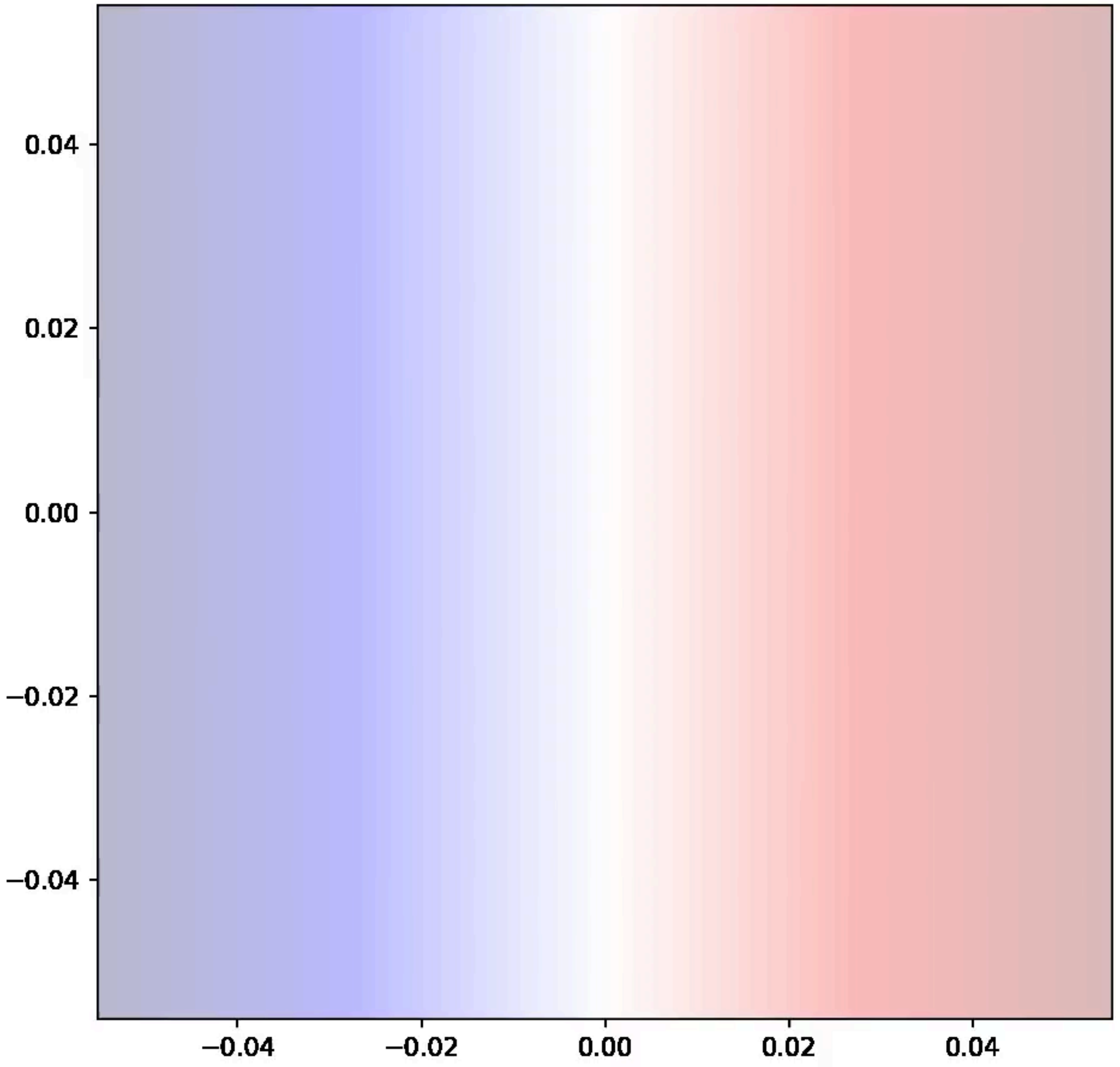
$$f(\mathbf{r}_\odot, \mathbf{p}(t), t) = f_0 + \left[\mathbf{r_i}(t_0) - 3\hat{\mathbf{p}}_\mathbf{i}(t_0) \cdot \mathbf{K}\right] \cdot \nabla f_0$$

Local isotropic cosmic-ray spectrum

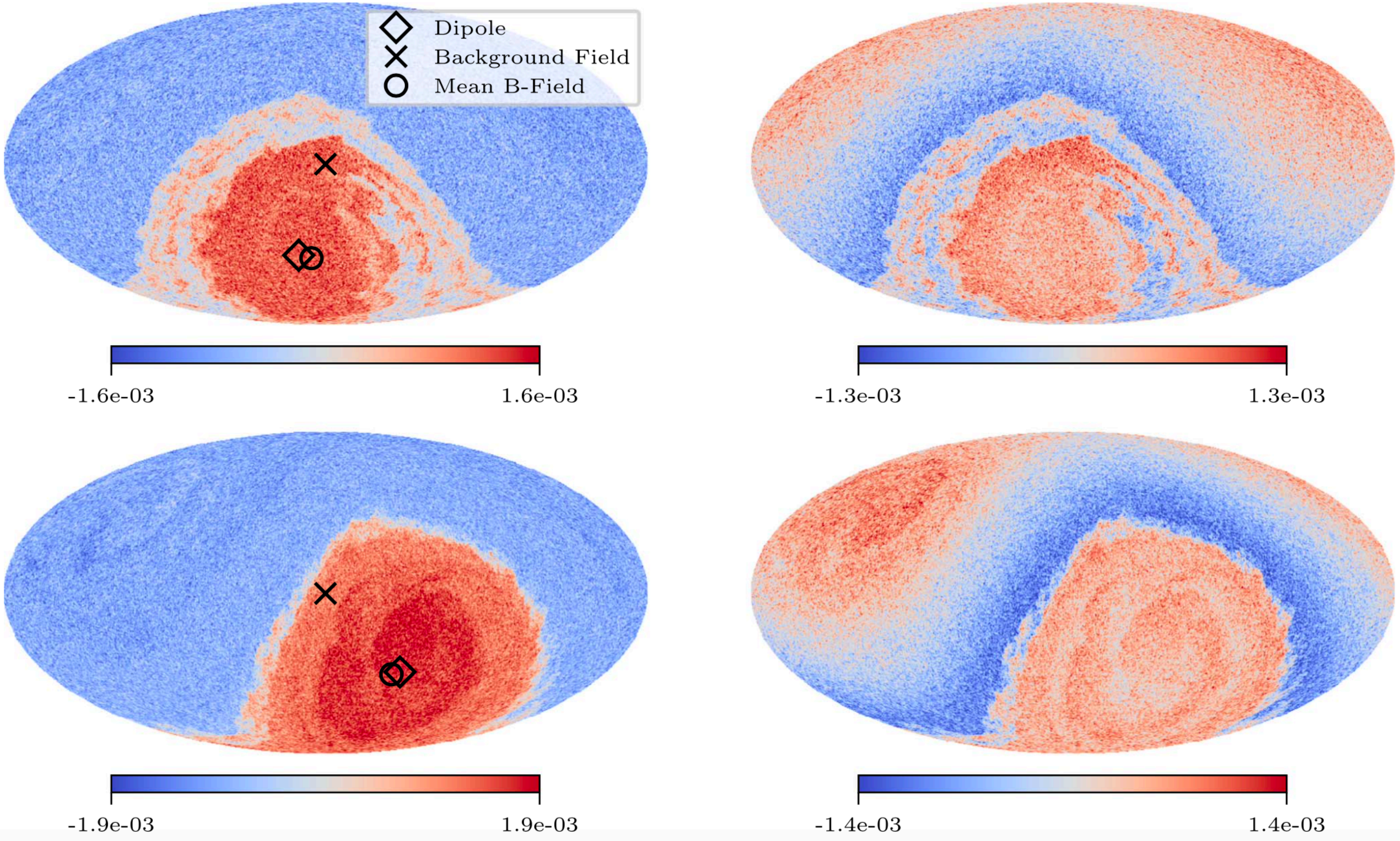
Local cosmic-ray gradient

Test Particle Simulation

Solve Lorentz equation + Liouville's theorem

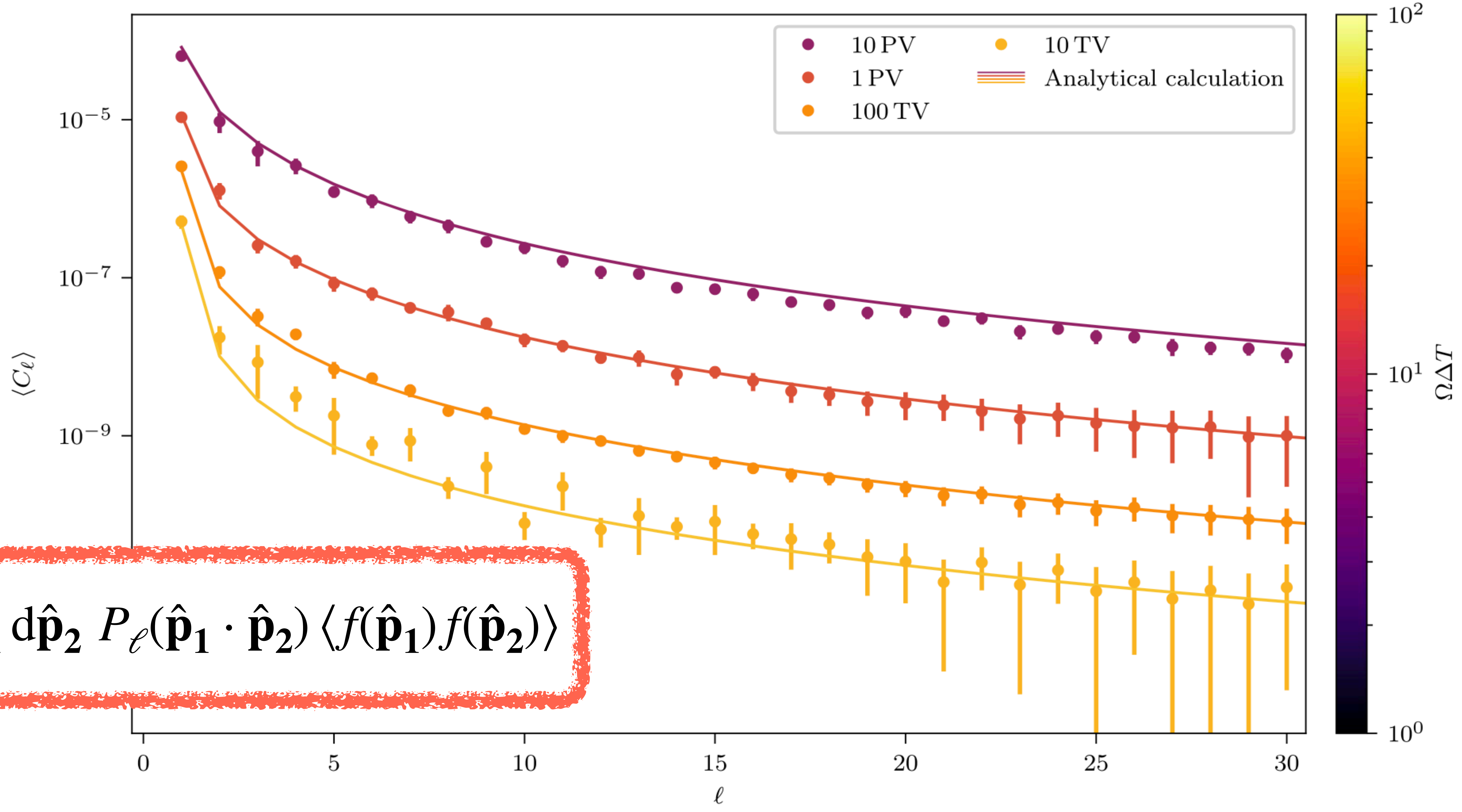


Sky Maps



Kuhlen, Phan, and Mertsch, ApJ 2021

Angular Power Spectrum



$$\langle C_\ell \rangle = \int \mathrm{d}\hat{\mathbf{p}}_1 \, \mathrm{d}\hat{\mathbf{p}}_2 \; P_\ell(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) \, \langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle$$

Kuhlen, Phan, and Mertsch, ApJ 2021, Ahlers & Mertsch, PNPP 2019

Analytic Theory for Cosmic-Ray Anisotropies

$$\langle C_\ell \rangle = \int \mathrm{d}\hat{\mathbf{p}}_1 \, \mathrm{d}\hat{\mathbf{p}}_2 \, P_\ell(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) \, \langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle$$

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Standard approach gives only $\langle f(\hat{\mathbf{p}}_1) \rangle \langle f(\hat{\mathbf{p}}_2) \rangle \leq \langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle$

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Mertsch & Ahlers, JCAP 2019
or Kuhlen, Phan, and Mertsch, ApJ 2021

Mixing Matrix Approach

$$\Lambda_{\ell\ell_0}(\Delta T)\,\langle C_{\ell_0}\rangle = \frac{8\pi}{3}K_{\parallel}\left(\frac{\partial_z f_0}{f_0}\right)\delta_{\ell 1}$$

Kuhlen, Phan, and Mertsch, ApJ 2021

Mixing Matrix Approach

$$\Lambda_{\ell\ell_0}(\Delta T)\langle C_{\ell_0}\rangle = \frac{8\pi}{3}K_{\parallel}\left(\frac{\partial_z f_0}{f_0}\right)\delta_{\ell 1}$$

Mixing matrix

Diagrammatic technique

Kuhlen, Phan, and Mertsch, ApJ 2021

Mixing Matrix Approach

$$\Lambda_{\ell\ell_0}(\Delta T)\langle C_{\ell_0}\rangle = \frac{8\pi}{3}K_{\parallel}\left(\frac{\partial_z f_0}{f_0}\right)\delta_{\ell 1}$$

Mixing matrix

Dipole
source term

Kuhlen, Phan, and Mertsch, ApJ 2021

Mixing Matrix Approach

$$\Lambda_{\ell\ell_0}\langle \Delta T \rangle \langle C_{\ell_0} \rangle = \frac{8\pi}{3} K_{\parallel} \left(\frac{\partial_z f_0}{f_0} \right) \delta_{\ell 1}$$

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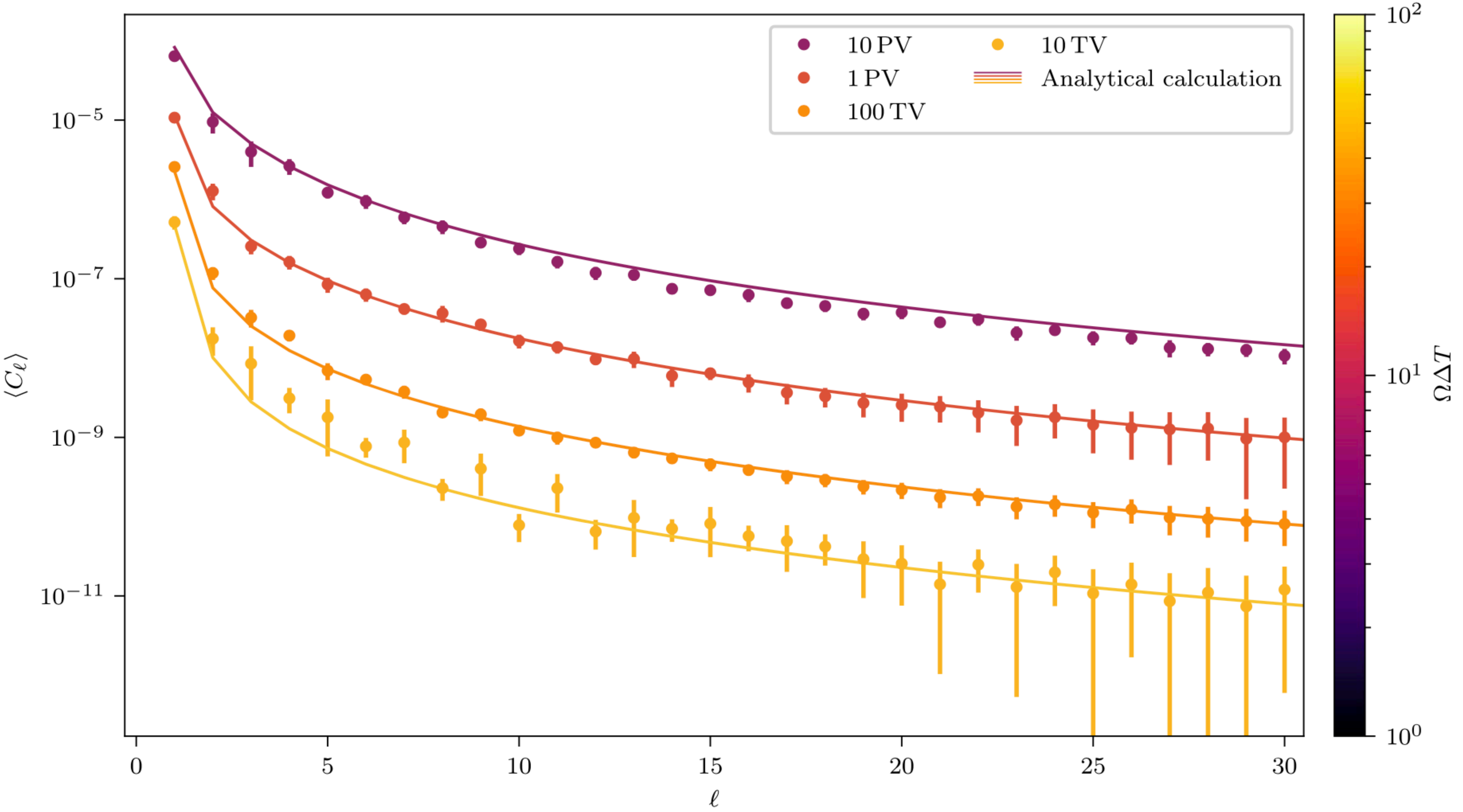
Constrained by numerical simulations

Kuhlen, Phan, and Mertsch, ApJ 2021

Mixing Matrix Approach



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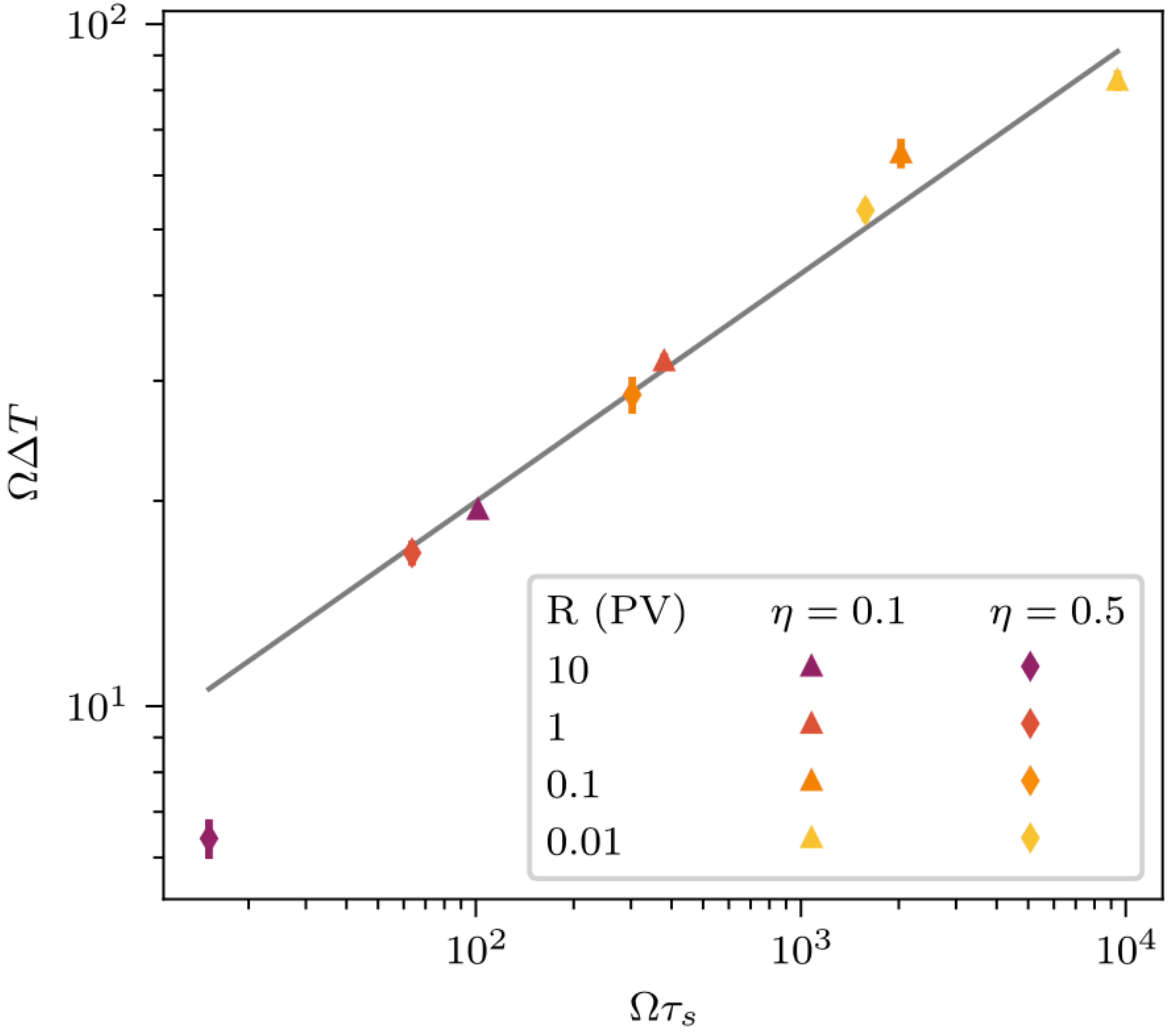
The Parameter $\Omega\Delta T$



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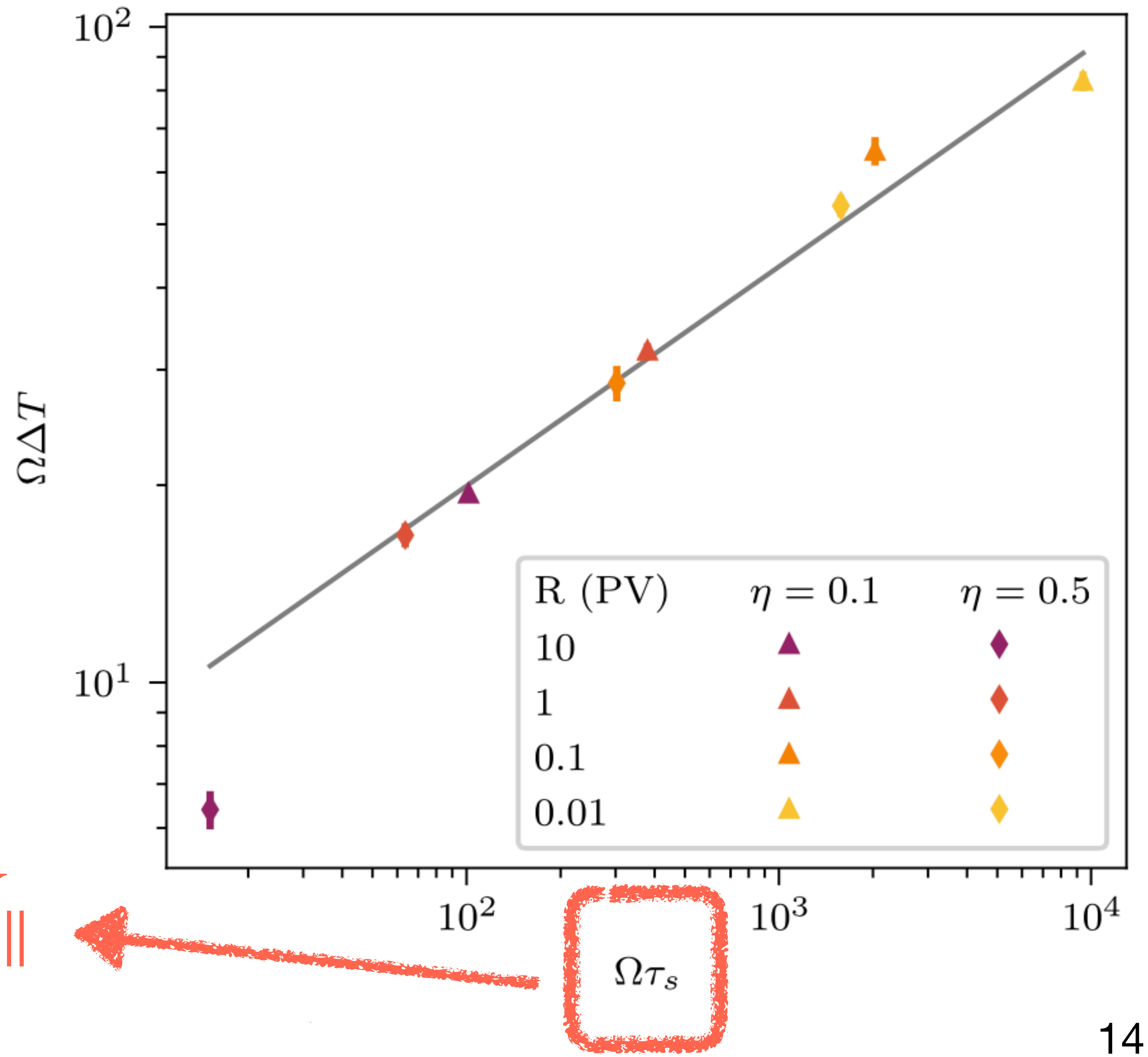

$\tau_s \sim K_\parallel$

The Parameter $\Omega\Delta T$

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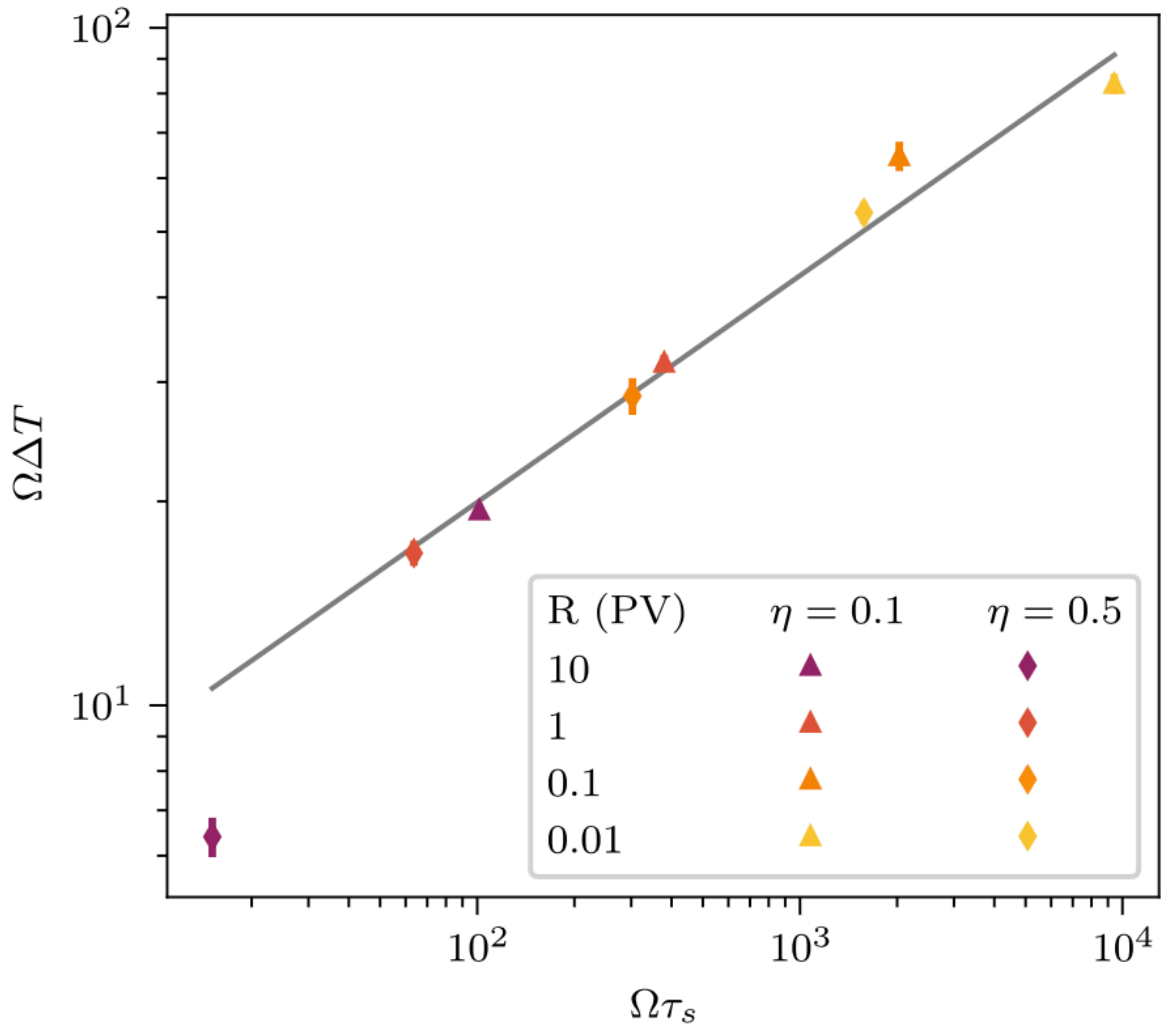

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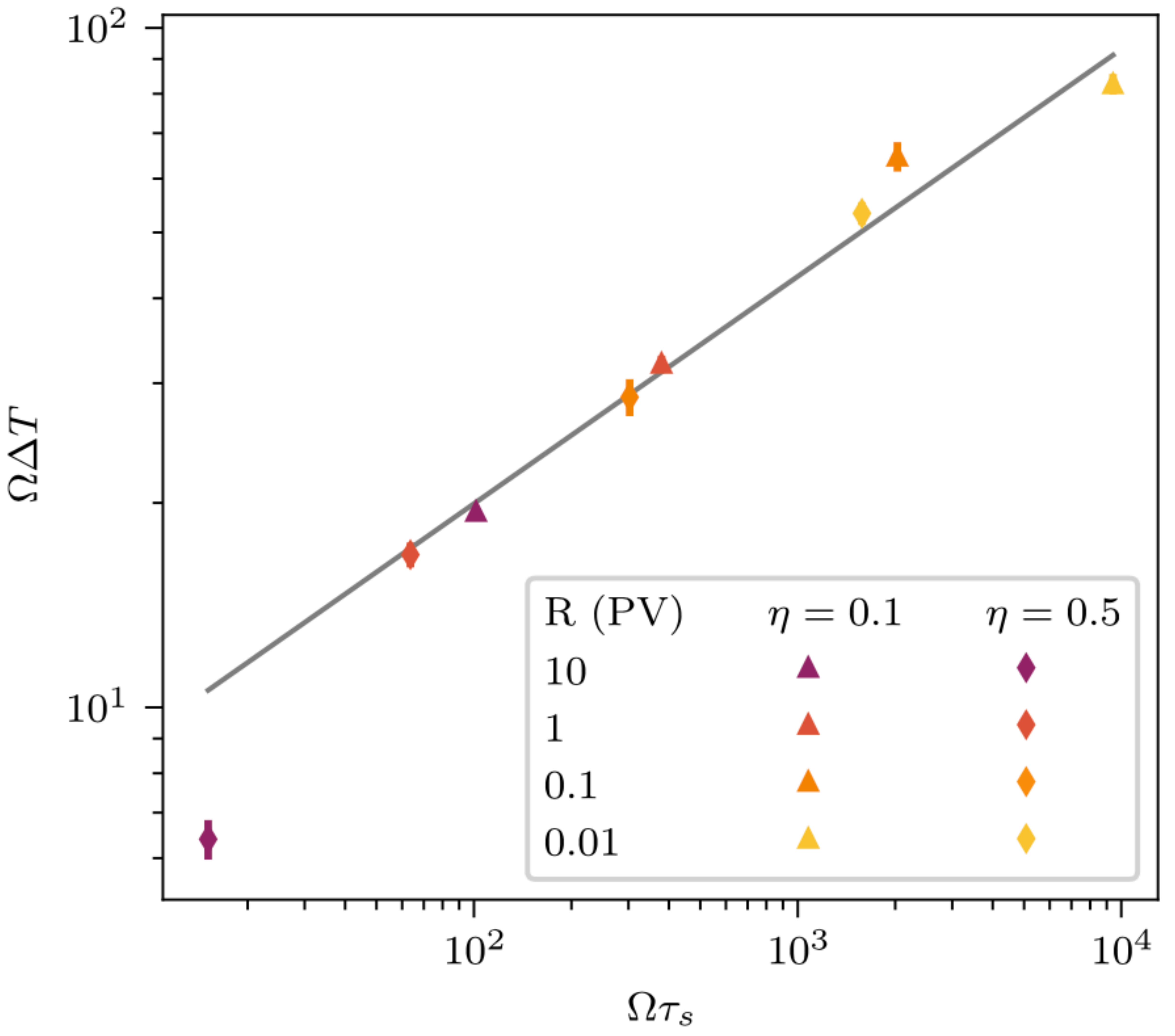
Related to properties of turbulence



The Parameter $\Omega\Delta T$

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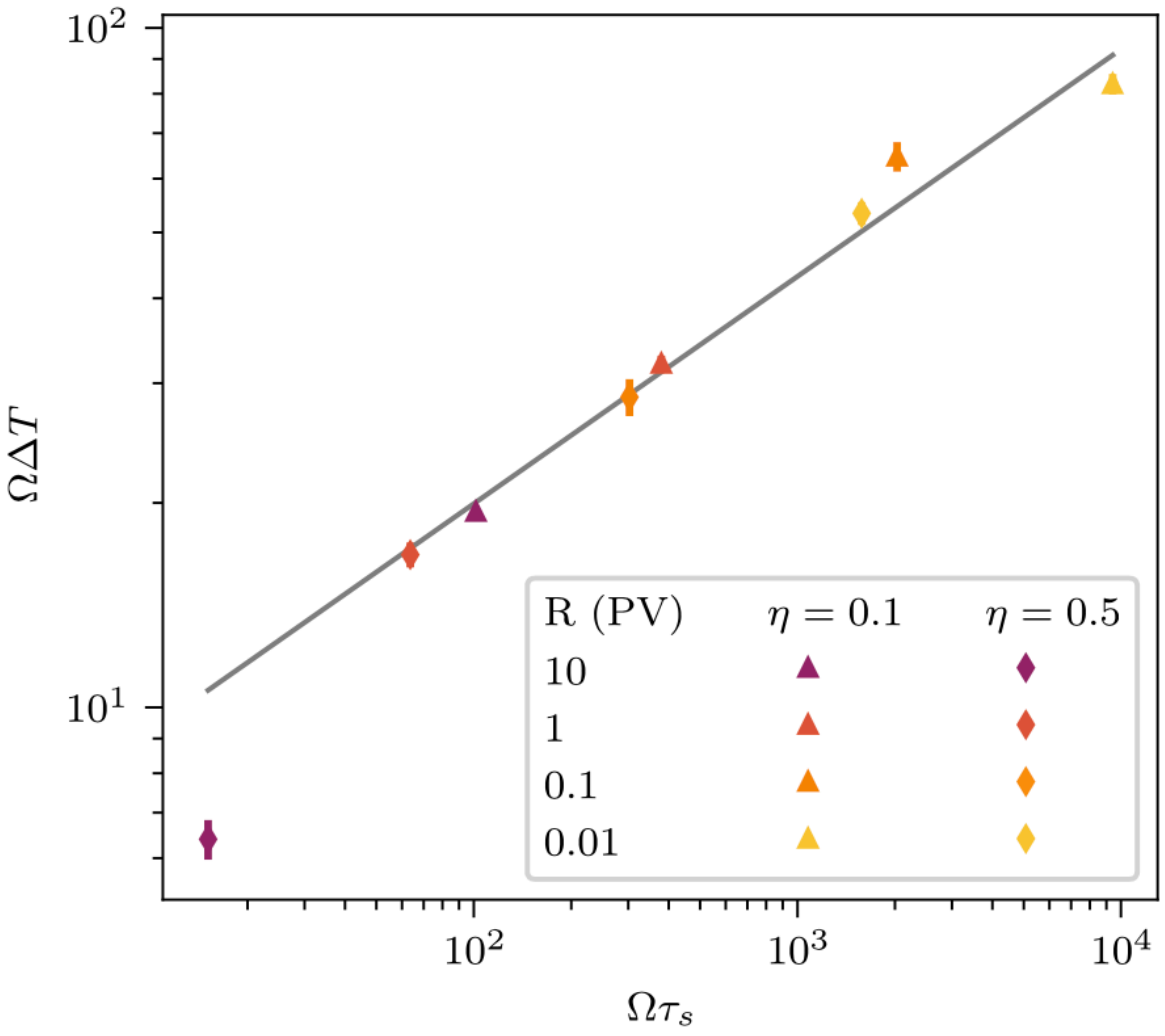
Constrain properties of turbulence



Summary

- Small-scale anisotropies might be induced by magnetic turbulence.
- We perform numerical and analytical analysis to better understand these anisotropies.
- Comparison to data is desired but a few issues have to be taken into account (finite energy resolution, more sophisticated turbulence, sky coverage, and so on).
- Comparison of numerical and analytic approaches might pave the way for constraining turbulence properties in the local interstellar medium.

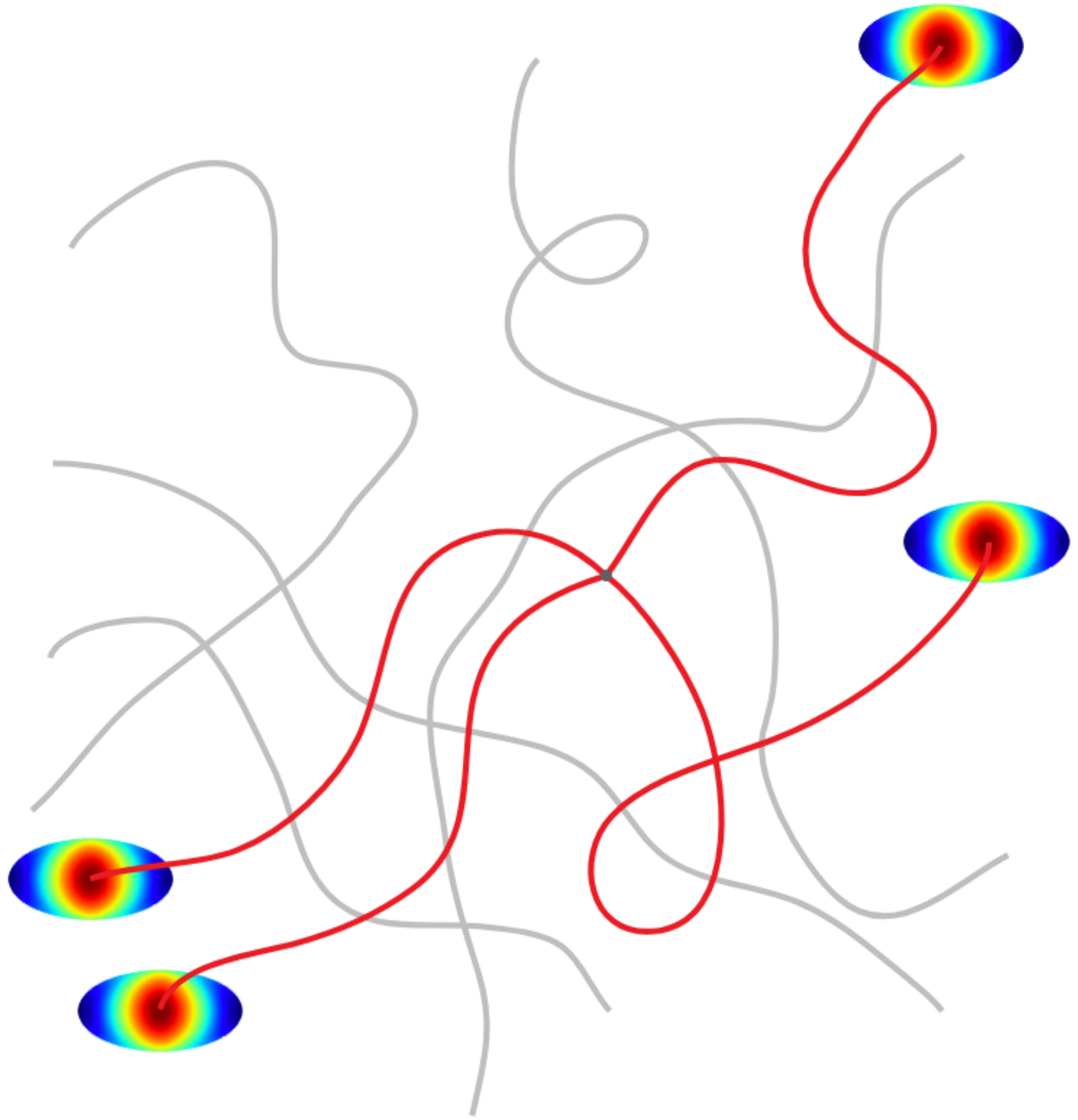

Thank you!
https://www.cloudynights.com/topic/542350-m16-the-pillars-of-creation/page-2

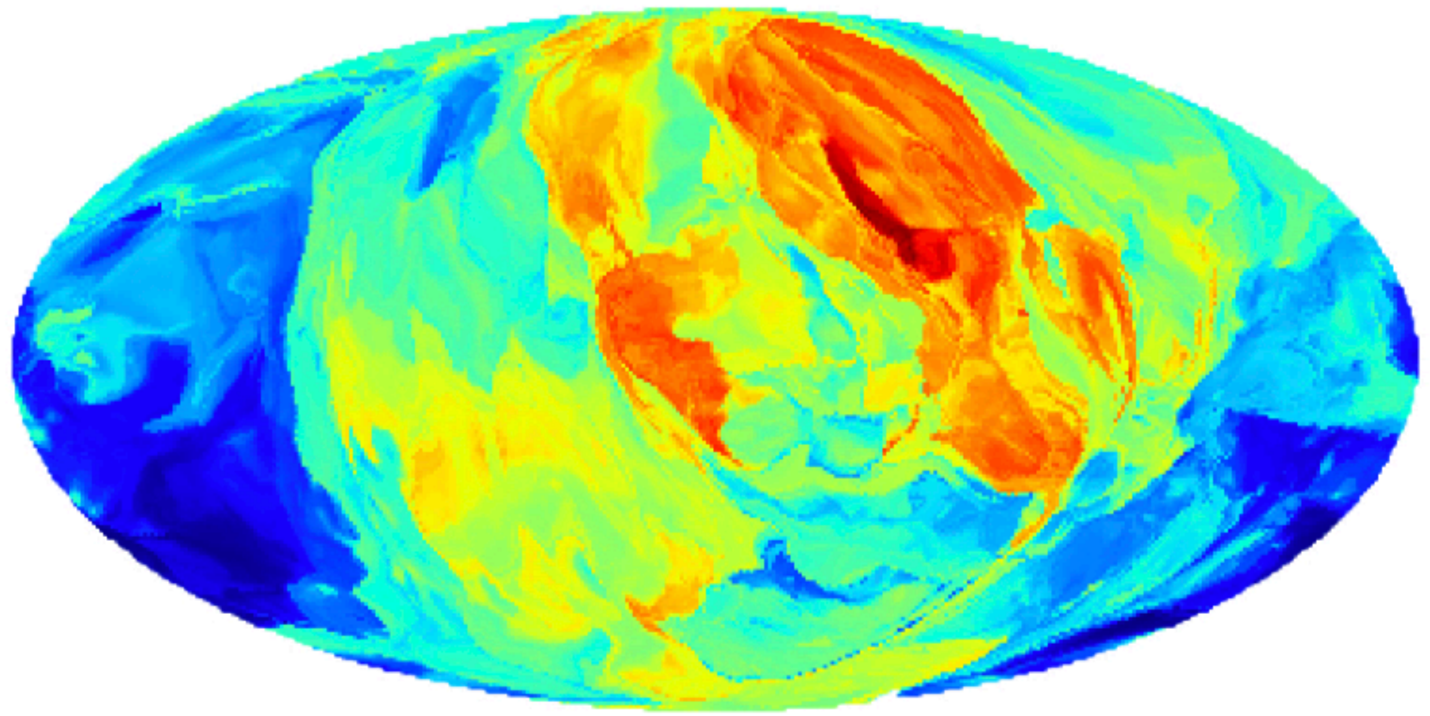
BACK UP

Numerical Simulations: Liouville's Theorem

$$f(\mathbf{r}_\odot, \mathbf{p}(t), t) = f_0 + \left[\mathbf{r_i}(t_0) - 3\hat{\mathbf{p}}_\mathbf{i}(t_0) \cdot \mathbf{K}\right] \cdot \nabla f_0$$

Simulate particle trajectories backward in time!





Credit: Philipp Mertsch

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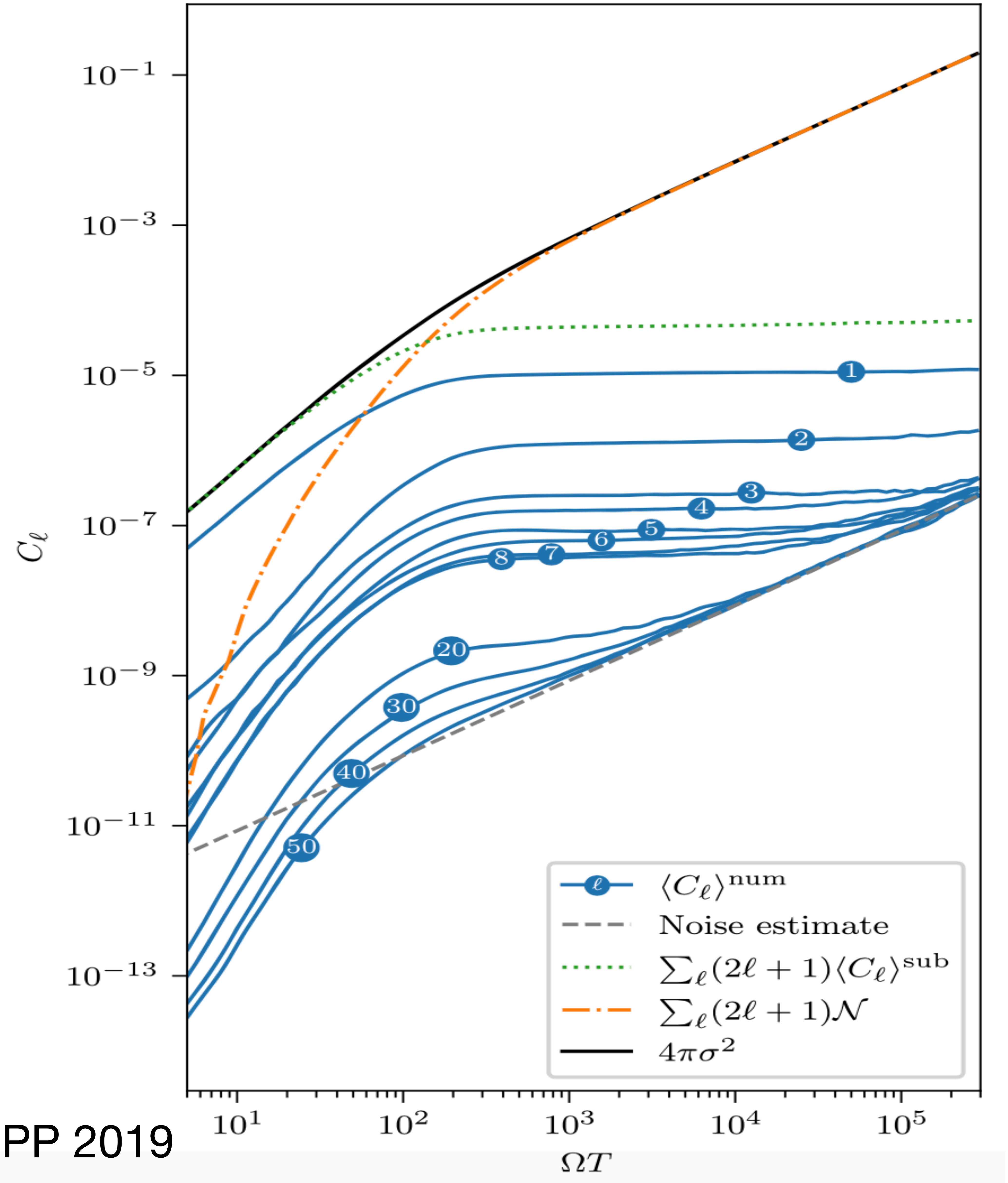
- Finite energy resolution.
- Sky coverage.
- More sophisticated turbulence model.



Kuhlen, Phan, and Mertsch, ApJ 2021, Ahlers & Mertsch, PNPP 2019