Top Quark Physics at a future LC and the LHC

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LIVERPOOL THEORETICAL PHYSICS

I. Why top?

II. $t\bar{t}$ top-peaks:

- threshold in theory and experimental simulations
- at the ILC and hadron colliders

III. $\tilde{t}\bar{\tilde{t}}$; $t\bar{t}H$; EW+NP couplings

IV. α_s with Giga$Z$

V. Conclusions

What is special about top @ ILC (compared to LHC)?

- Initial $e^+e^-(+\,n\,\gamma)$ state is:
  - colourless; only el.-magn. ISR
  - at fixed, well defined c.m. energy + luminosity (but see below..)
  - tunable w.r.t. its polarization

$\rightsquigarrow$ relatively low multiplicity final states

$\rightsquigarrow$ 'clean' environment, controllable backgrounds and normalizations

- LHC: is there; will mass-produce tops, in a larger kinematic range;
  $\sigma_{t\bar{t}}$: (LHC:Tevatron:ILC) $\sim$ (1000:10:1) $\rightsquigarrow$ millions of top-pairs!
- For precise measurements all three require matching theoretical accuracy
  i.e. higher order calculations, resummations, Effective Field Theories, suitable scheme choices

$e^+e^- \to t\bar{t}$ at the ILC

- Top is the heaviest (SM) particle so far, with Yukawa coupl. y_t of $\mathcal{O}(1)$.
- Top plays a special role in EW precision tests of the SM ($\leftrightarrow$ MSSM);
- very *precise* knowledge of its mass and couplings needed for
  - ⋆ indirect Higgs mass determination / SM consistency:

$$M_W^2(1-\frac{M_W^2}{M_Z^2}) \sim \frac{1}{1-\Delta r}, \quad \Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta_{\rm rem}(m_H), \qquad \Delta\rho \sim m_t^2$$

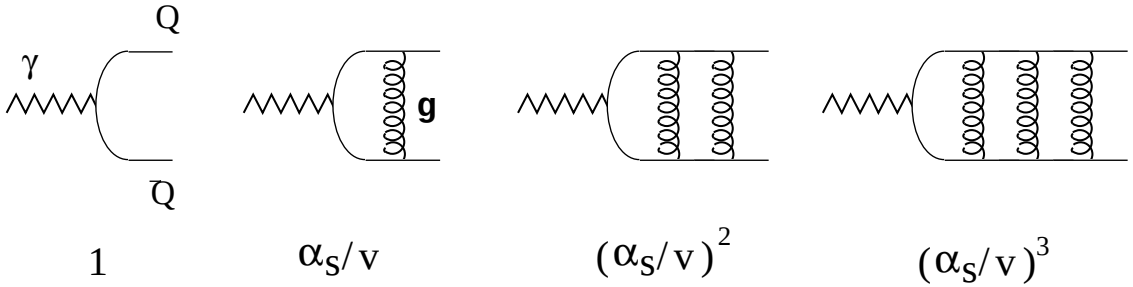
  - ⋆ extrapolation of masses and couplings in GUTs at high energy (via Renormalization Group running) from 'low energy' (SUSY?) parameters

- Threshold scan the only known way to achieve $\Delta m_t \overset{!}{<} 100$ MeV $< \Lambda_{\rm QCD}$:
  $e^+e^- \to t\bar{t}$ means counting $bW^+\bar{b}W^-$ colour singlet states

[At the level $\Delta m_t/m_t < 10^{-3}$ systematic and conceptual problems at hadron colliders: jet energy scale, role of underlying event/soft glue, *which mass?* $m_t^{\rm pole}$?, $m_t^{\rm jet}$, $m_t^{\rm MC}$ ...]

$t\bar{t}$ at threshold: basic Leading Order picture

- Near threshold $\sqrt{s} \sim 2m_t$, and the quarks have a small (non-relativistic) velocity $v = \sqrt{1 - 4m_t^2/s} \sim \alpha_s \ll 1$ $\rightsquigarrow$ quite long time to interact..



Fixed order Perturbation Theory breaks down, gluon exchanges $\sim (\alpha_s/v)^n$ have to be summed $\rightsquigarrow$ Coulombic potential $V_c \sim 1/r$ $\rightsquigarrow$ bound states, can be calculated via

- Coulomb Green function G of the (Leading Order) Schrödinger equation ($E = \sqrt{s} - 2m_t$)

$$\left[-\frac{\boldsymbol{\nabla}^2}{m_t} + V_c(\boldsymbol{r}) - (E + i\Gamma_t) \right] G(\boldsymbol{r}, \boldsymbol{r}', E + i\Gamma_t) \;=\; \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}')$$

- But: including large top decay width $\Gamma_t^{\rm SM} \sim \Gamma^{(0)}_{t\to Wb} = \frac{G_f}{\sqrt{2}} \frac{m_t^3}{8\pi} \sim 1.5\ \text{GeV} \gg \Lambda_{\rm QCD}$

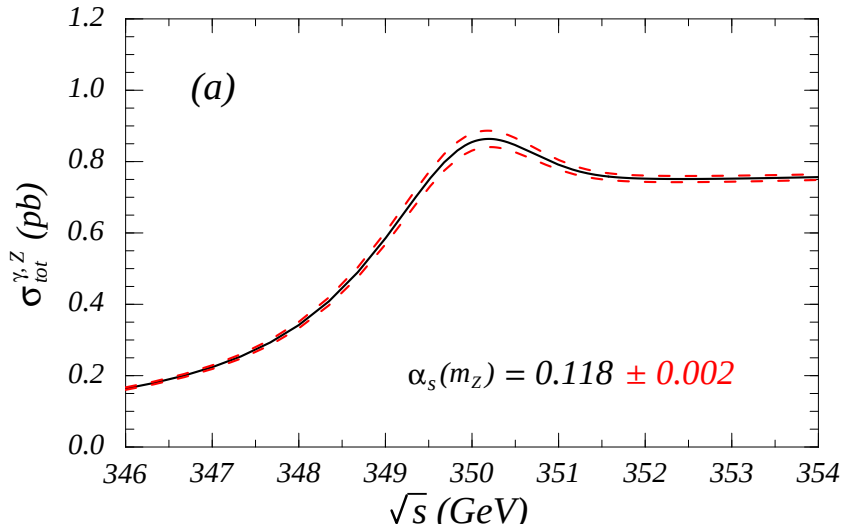
✓ cuts off non-perturbative effects, process calculable in perturbative QCD.

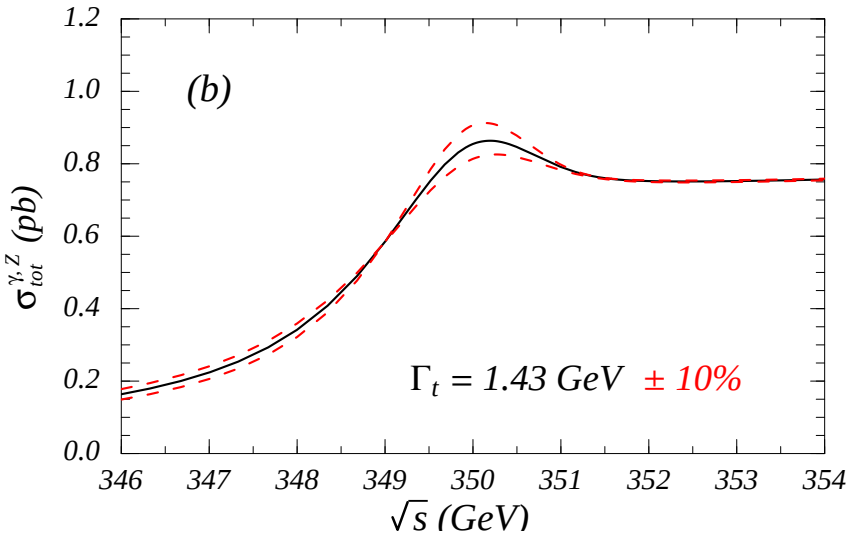
× Resonances smeared out, no 'toponium' spectroscopy, only remainder of $1S$ peak left:

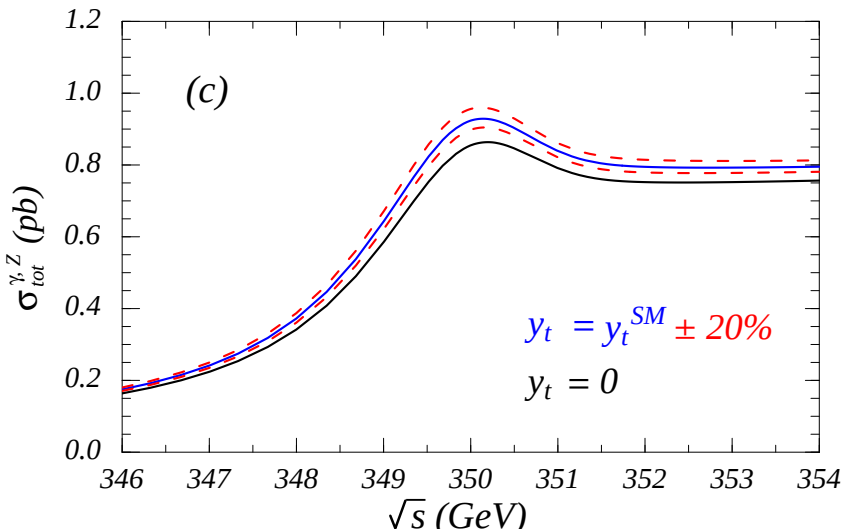
Parameter dependence of total cross section σ_{tot}: α_s, Γ_t, y_t and m_t:

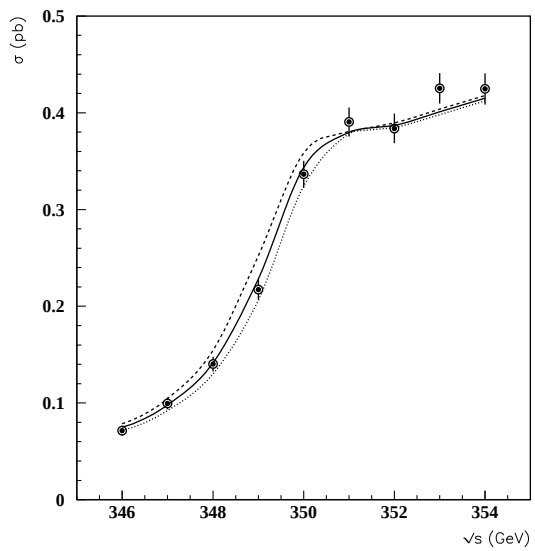
$\sigma_{tot}(e^+e^- \to \gamma^*, Z \to t\bar{t}) \sim \mathrm{Im}G$

(a,b,c) NNLL from Hoang+Manohar+Stewart+TT









Extra Yukawa-potential from H exchange

$\int \mathcal{L} = 100$ fb^{-1}; band: $\Delta m_t = \pm 100$ MeV

Further observables beyond σ_{tot}

⋆ $t\bar{t}$ threshold scan is mainly a *counting experiment* of $bW^+\bar{b}W^-$ colour singlet states. However:

– Cuts needed to select $t\bar{t}$ from background

– Distributions needed to build realistic (higher order) Monte Carlo generators for the signal process

– Use of additional observables [not only $\sigma_{tot}(e^+e^- \to t\bar{t})$] will

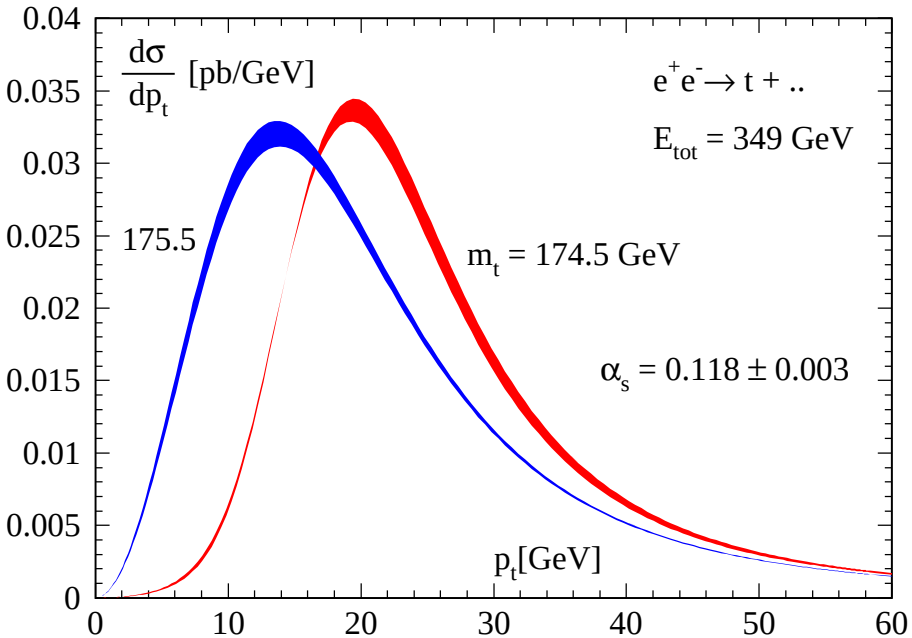
- add information,
- help to disentangle correlations between parameters $\{m_t, \alpha_s, \Gamma_t, y_t\}$,
- increase sensitivity to possible New Physics in production and decay.

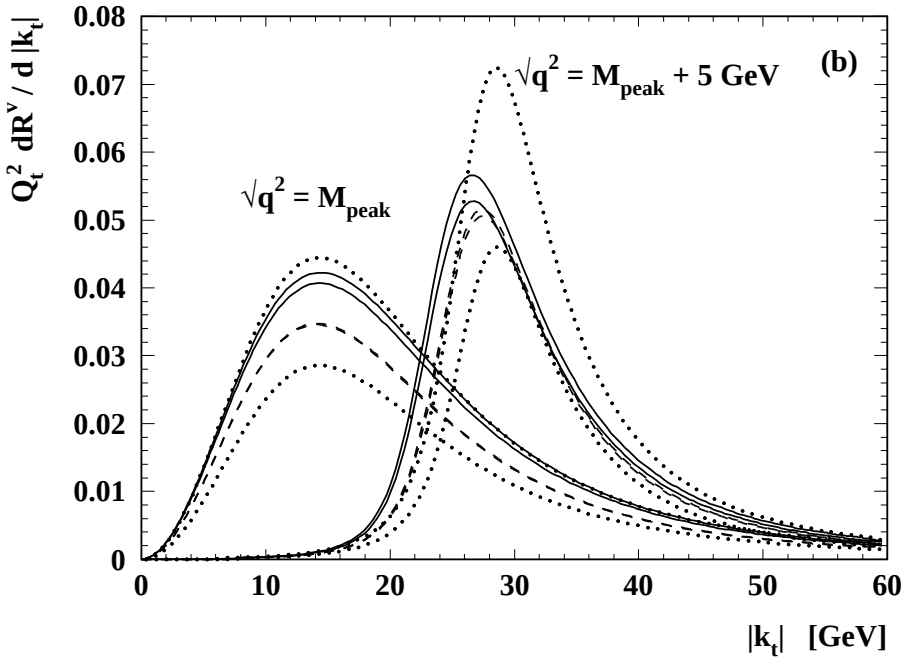
• Observables are e.g. top momentum distribution, Forward-Backward Asymmetry $A_{\rm FB}$, $A_{\rm LR}$, top polarization, W decay lepton spectra ...

- Top momentum distribution $\mathrm{d}\sigma/\mathrm{d}p_t$ ($\sim$ |wave function in momentum space|^2)

$\longrightarrow$ available at NNLO

LO, NLO, NNLO with $\mu = 15...60$ GeV (Hoang+T)

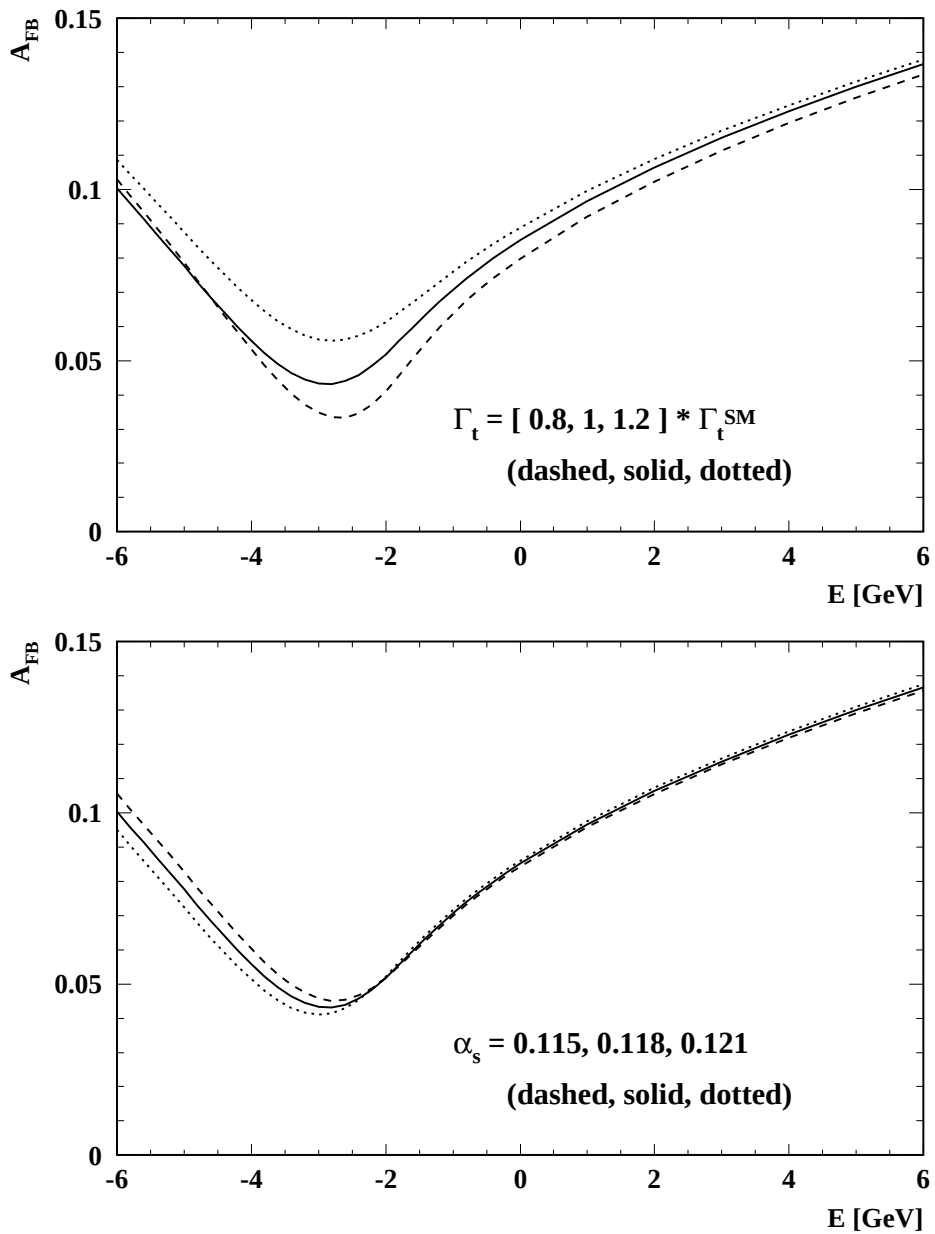




$\longrightarrow$ The peak of the top momentum distribution depends strongly on m_t, but is not very sensitive to α_s ($\rightsquigarrow$ help against correlation of m_t and α_s in σ_{tot})

- Forward-Backward Asymmetry A_{FB} (NNLO)

$t\bar{t}$ production through a virtual Z leads to a (small) P wave contribution. Interference with the leading S wave results in A_{FB}, depending strongly on the width Γ_t, less on α_s.



- Polarization

  – unpolarized beams: -40% (longitudinal) polarized top quarks

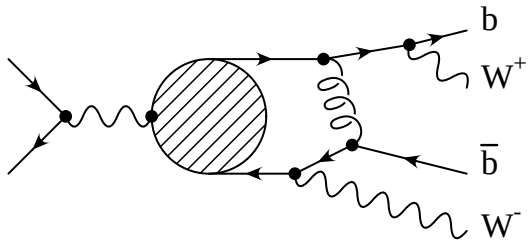
  – polarized beams: highly polarized tops

  $\rightarrow$ all three polarization components calculable (NLO)

  $\rightarrow$ sensitive e.g. to EDM's of top (BSM CP-violation), anomalous coupl. (like $V + A$)

$\longrightarrow$ more/better MCs & experimental analyses needed

- Rescattering corrections



  – cross-talk between $t - \bar{b}$, $\bar{t} - b$ and $b - \bar{b}$

  – strongly suppressed (zero at NLO) for inclusive $\sigma_{\rm tot}$

  – numerical results for rescattering corrections to $\mathrm{d}\sigma/\mathrm{d}p_t$, A_{FB} and top polarization

  NLO, effect typically 10% (needs to be included for realistic Monte Carlo studies)

Effective Field Theories for higher order calculations

- How to calculate *systematically* higher order (relativistic) corrections in α_s (and v) ?
- Threshold Power Counting (fixed order) in α_s and v:

$$R = \sigma_{t\bar{t}}/\sigma_{\mu^+\mu^-} = v \cdot \sum_n \left(\frac{\alpha_s}{v}\right)^n \left[LO\,\{1\}, NLO\,\{v, \alpha_s\}, NNLO\,\{v^2, \alpha_s v, \alpha_s^2\}\right]$$

- Large hierarchy of scales:

$$m_t \sim 175\ \mathrm{GeV} \ \gg\ p_t \sim m_t v \sim 25\ \mathrm{GeV} \ \gg\ E \sim m_t v^2 \sim 4\ \mathrm{GeV} \ \gg\ \Lambda_{\mathrm{QCD}}$$

- Multi-scale problem best treated in the framework of EFT *Non-Relativistic QCD*
  - ⋆ includes a well defined power counting and renormalization
  - ⋆ separates non-dynamical from dynamical d.o.f., making use of the hierarchy of scales (and thus reducing the difficulty of complicated Feynman-graphs)
  - ⋆ sums classes of graphs in an efficient and transparent way
  - ⋆ determines the scales of the couplings involved
  - ⋆ provides a systematic description of $f\bar{f}$ systems in (QED and) QCD (also $b\bar{b}$ or positronium).

- Cross section $R = \textit{matching-coeffs.}(\Lambda) \cdot \mathrm{Im}G(\boldsymbol{r}=0, \boldsymbol{r}'=0, E+i\Gamma_t, \Lambda)$
- Green function G calculable from (NNLO) Schrödinger equation:

$$\left[-\frac{\boldsymbol{\nabla}^2}{m_t}-\frac{\boldsymbol{\nabla}^4}{4m_t^3}+V_c(\boldsymbol{r})+V_{BF}(\boldsymbol{r})+V_{NA}(\boldsymbol{r})-(E+i\Gamma_t)\right] G(\boldsymbol{r},\boldsymbol{r}',E+i\Gamma_t) \;=\; \delta^{(3)}(\boldsymbol{r}-\boldsymbol{r}')$$

  with perturbative Coulomb-, Breit-Fermi- and Non-Abelian potentials, calculated in the EFT.

- *On demand:* 'EFT in a few steps'.

  Otherwise: Results in Next-to-next-to leading order (NNLO): $\longrightarrow$

No details here, but must be mentioned:

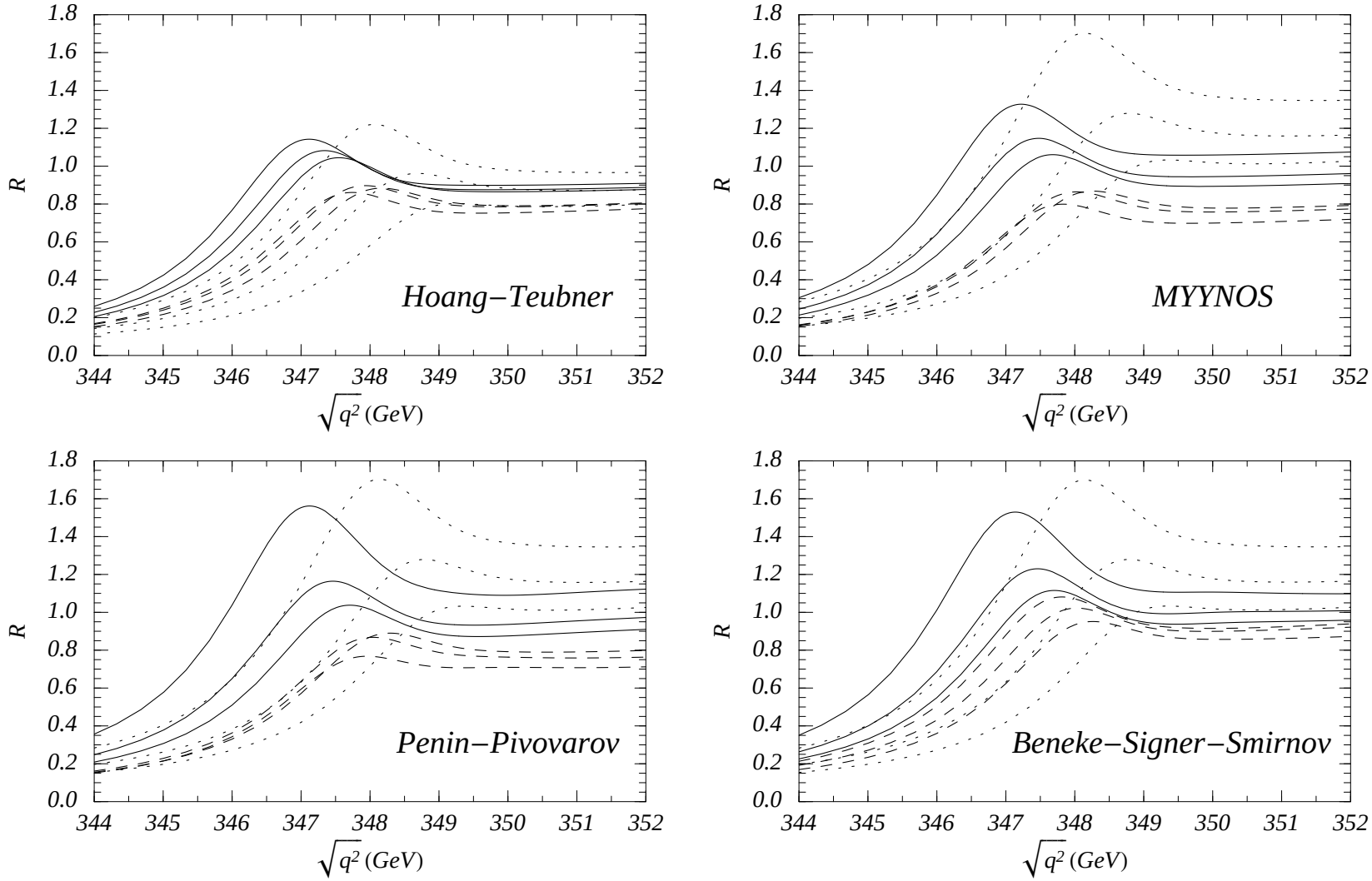
- As of November 2009, the calculation of the perturbative QCD potential to three loops is complete! (more than 20000 FDs)

  [Smirnov, Smirnov, Steinhauser; Anzai, Kiyo, Sumino]

$R \equiv \sigma(e^+e^- \to t\bar{t})/\sigma_{pt}$ in NNLO: Large corrections

1998 – 2000

Four independent groups: H-T, Melnikov-Yelkhovsky-Yakovlev-Nagano-Ota-Sumino, P-P, B-S-S



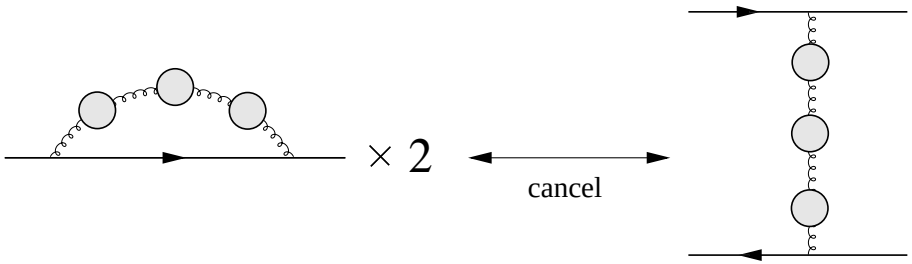
$m_t^{\rm pole} = 175.05$ GeV, $\alpha_s(M_Z) = 0.119$, $\Gamma_t = 1.43$ GeV, $\mu_{\rm soft} = 15,\ 30,\ 60$ GeV; dotted: LO, dashed: NLO, solid: NNLO.

Pole mass scheme. ▶ Are peak-shifts threatening a precise mass determination?

‘Short Distance’ mass schemes

- Origin of the problem of large peak shifts:
  - ⋆ $m^{\rm pole}$ is IR-finite and gauge invariant to all orders in pQCD. But:
  - ⋆ $m^{\rm pole}$ is NO observable
  - ⋆ defined only up to an IR-uncertainty of $\mathcal{O}(\Lambda_{QCD})$ ($\rightarrow$ confinement!).
- Energy of the $1S$ resonance is an IR-safe observable:

$$E_{\rm static}(r) = 2m_t^{\rm pole} + V(r)$$



Cancellation of the (leading) IR ‘Renormalon’ contributions between mass and potential.

⇝ Use a scheme, where also the individual contributions, m_t, V, are IR-safe!

- There are several ‘Short Distance’ mass schemes, e.g.
  - ⋆ ‘kinetic mass’ (Bigi et al., used in B physics),
  - ⋆ ‘Potential Subtracted’ mass (Beneke): $m^{PS}(\mu_f) \;:=\; m^{\rm Pol} + \frac{1}{2}\int_{|q|<\mu_f} \frac{d^3q}{(2\pi)^3}\tilde{V}(q)$,
  - ⋆ ‘$1S$ mass’ (Hoang et al.): $m_t^{1S} \;:=\; \frac{1}{2}M_{t\bar{t}}(1S,$ perturbatively defined for $\Gamma_t \rightarrow 0)$

- pQCD relation to 'high energy' $\overline{MS}$ mass (starting from $m_t^{\overline{MS}}(m_t^{\overline{MS}}) = 165$ GeV):

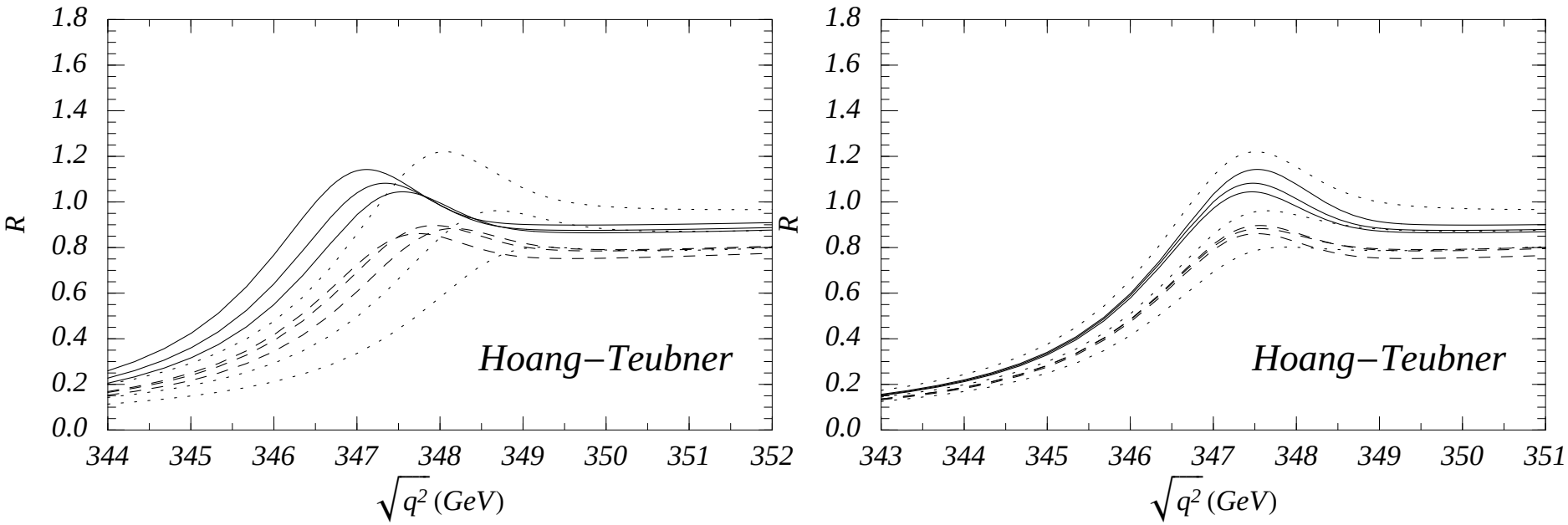
$m_t^{\rm pole} = [165.0 + 7.6 + 1.6 + 0.51]$ GeV Steinhauser+Chetyrkin, Melnikov+Ritbergen

$m_t^{PS}(\mu_f = 20\ {\rm GeV}) = [165.0 + 6.7 + 1.2 + 0.28]$ GeV [LO, NLO, NNLO]

Or for a measured $m_t^{1S,exp.} = 175 \pm 0.1$ GeV (with $\alpha_s(M_Z^2) = 0.118 \pm x \cdot 0.001$):

$m_t^{\overline{MS}} = [175 - 7.58 - 0.96 - 0.14 \pm 0.1 \pm x \cdot 0.007]$ GeV

- Results for $R := \sigma(e^+e^- \to t\bar{t})/\sigma_{pt}$ in the pole and 1S mass schemes:



⇝Position of peak is stabilised, normalization still quite uncertain.

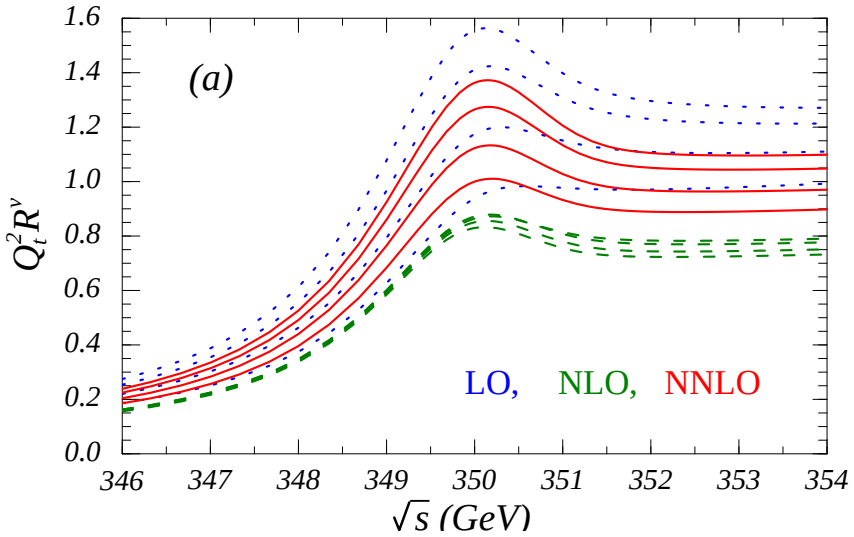
Renormalization Group improved results

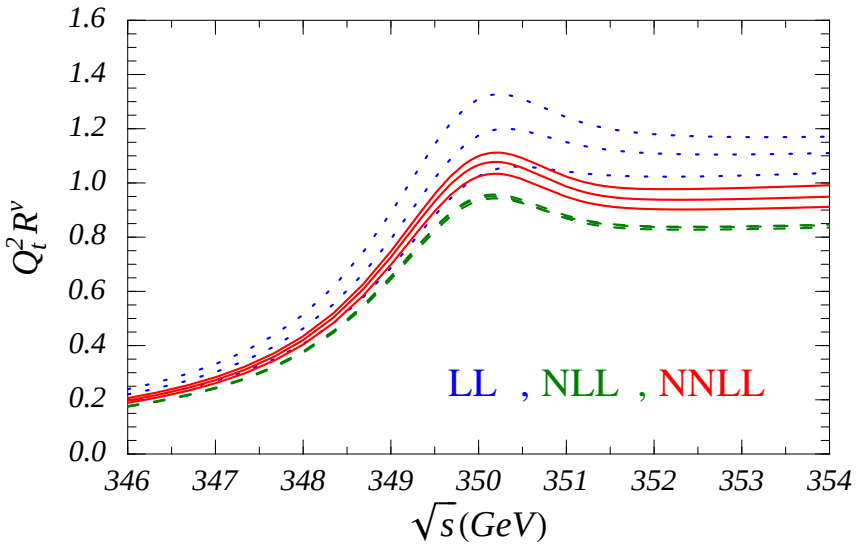
from 2001..

- Within EFT *v/p NRQCD* also summation of large logs possible
- Power Counting, RG improved (large logarithms $\log \frac{m_t^2}{p^2}$, $\log \frac{p^2}{E^2}$, $\log \frac{m_t^2}{E^2}$ summed):

$$R \sim v \cdot \sum_{n,k} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \log v)^k \left[LL\{1\}, NLL\{v, \alpha_s\}, NNLL\{v^2, \alpha_s v, \alpha_s^2\}\right]$$

- Corrections to the normalization and scale dependence stabilized
- Important for determination of Γ_t, y_t, α_s from $t\bar{t}$ threshold scan





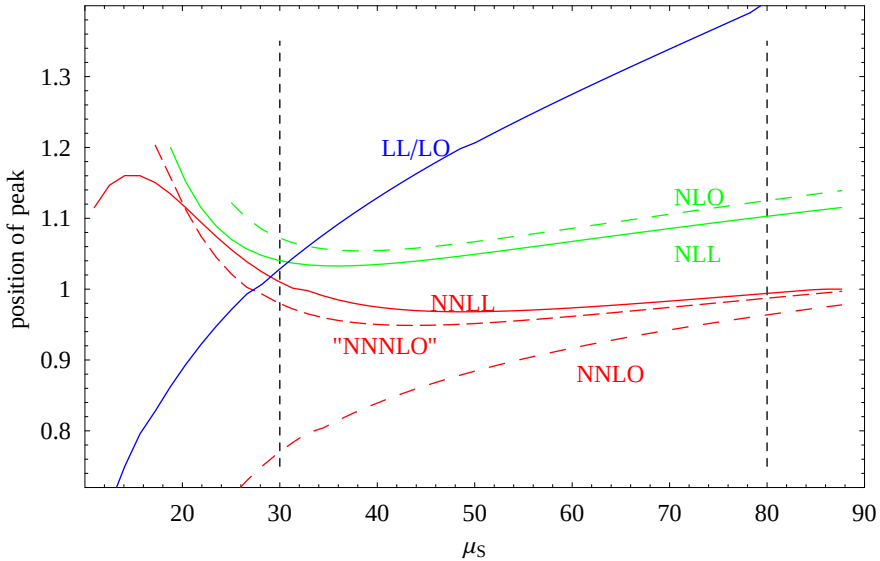
vNRQCD 'NNLL' from Hoang+Manohar+Stewart+T. Later similar results in pNRQCD by Pineda+Signer, following work by Kniehl, Penin, Pineda, Smirnov, Steinhauser.

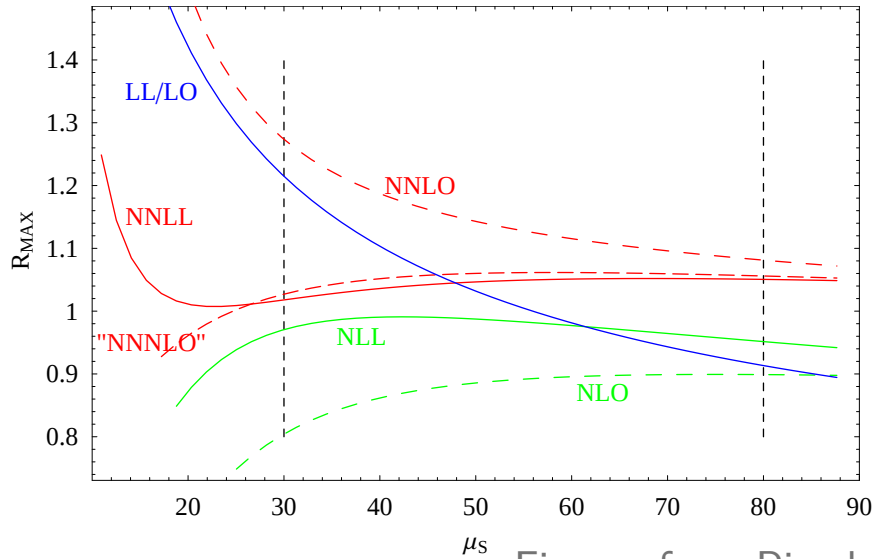
Progress in fixed order: N^{3}LO

Kniehl, Penin, Smirnov, Steinhauser
Beneke, Kiyo, Schuller

- N^{3}LO fixed order calculations for $t\bar{t}$ peak position and height (up to the then missing 3-loop coefficient for Coulomb potential); full cross section shape not yet available
- Peak position and normalization stabilized; convergence of perturbation series; reduced scale dependence

Comparison of RG improved vs. fixed order predictions for peak position and normalization:





Figures from Pineda+Signer

⇝ Summation of leading logs important; fixed order does the job, eventually

EW + non-res. corrs.

Beneke, Jantzen, Ruiz-Femenia; Hoang, Reisser, Ruiz-Femenia

- So far only strong interaction in EFT, but: must include large Γ_t consistently in EFT;
  $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ only approximate; also other QED and weak corrections.
  (EFT power counting: $\alpha_{\rm EW} \sim \Gamma_t/m_t \sim \alpha_s^2 \sim v^2$.)
- In addition to double-res. $t\bar{t} \to W^+W^-b\bar{b}$, also single- and non-res. final states!
- Realistic studies will involve cuts on invariant mass of reconstructed top decay products.
- Very demanding task, as calculations formulated mainly for total cross section;
  no full results yet, great recent progress $\rightsquigarrow$ sizeable correction to the cross section!

relative shifts $\Delta\sigma/\sigma$:

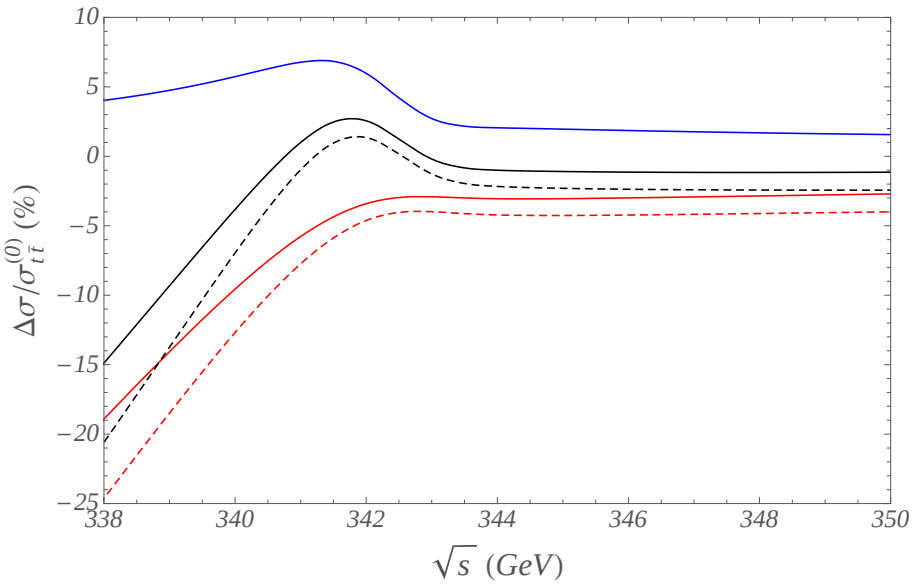
[Beneke et al.]

— blue: QED resonant NLO

— black: combines EW NLO

— red: EW non-resonant NLO

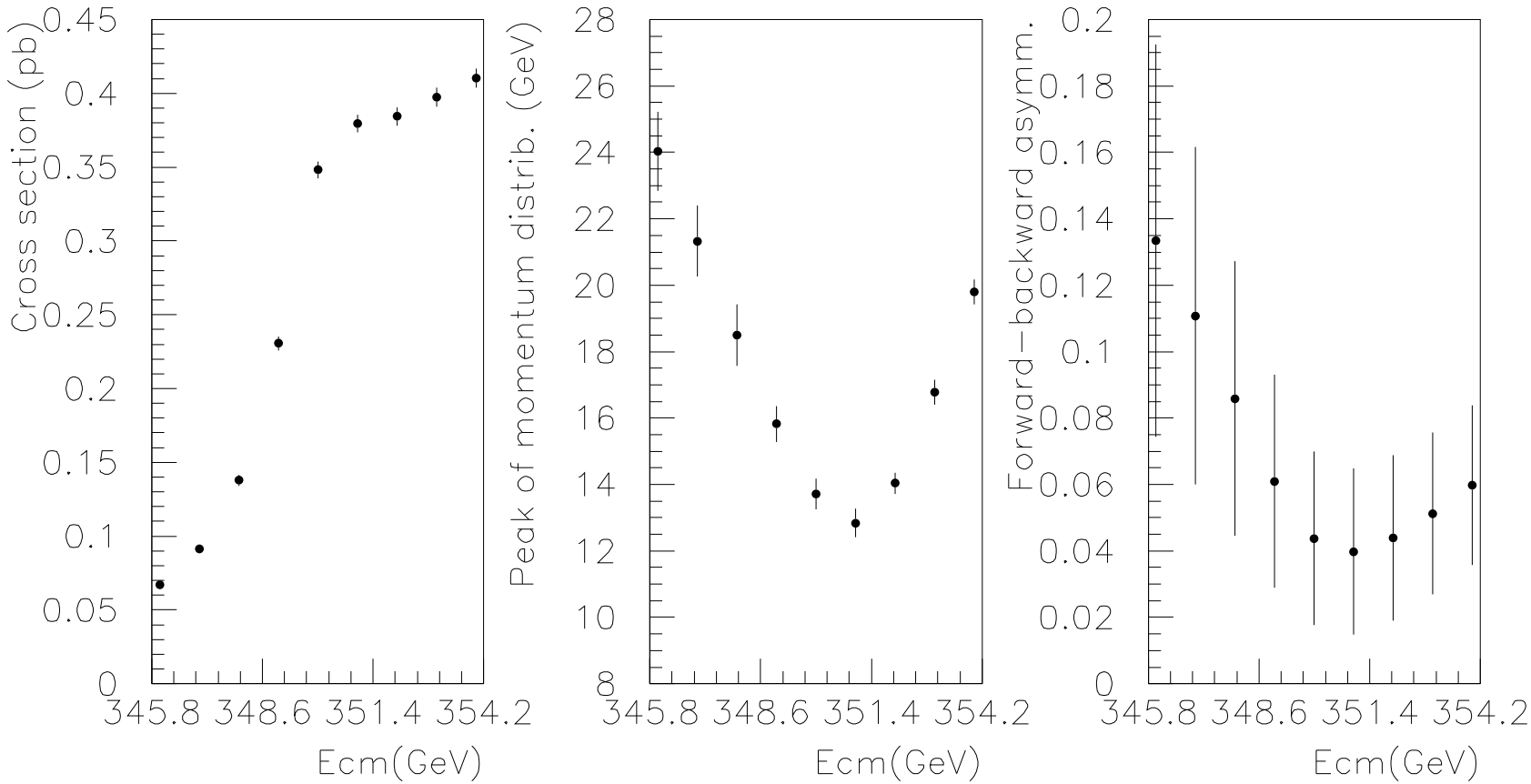
— dashed: $\Delta m_t = 15$ GeV



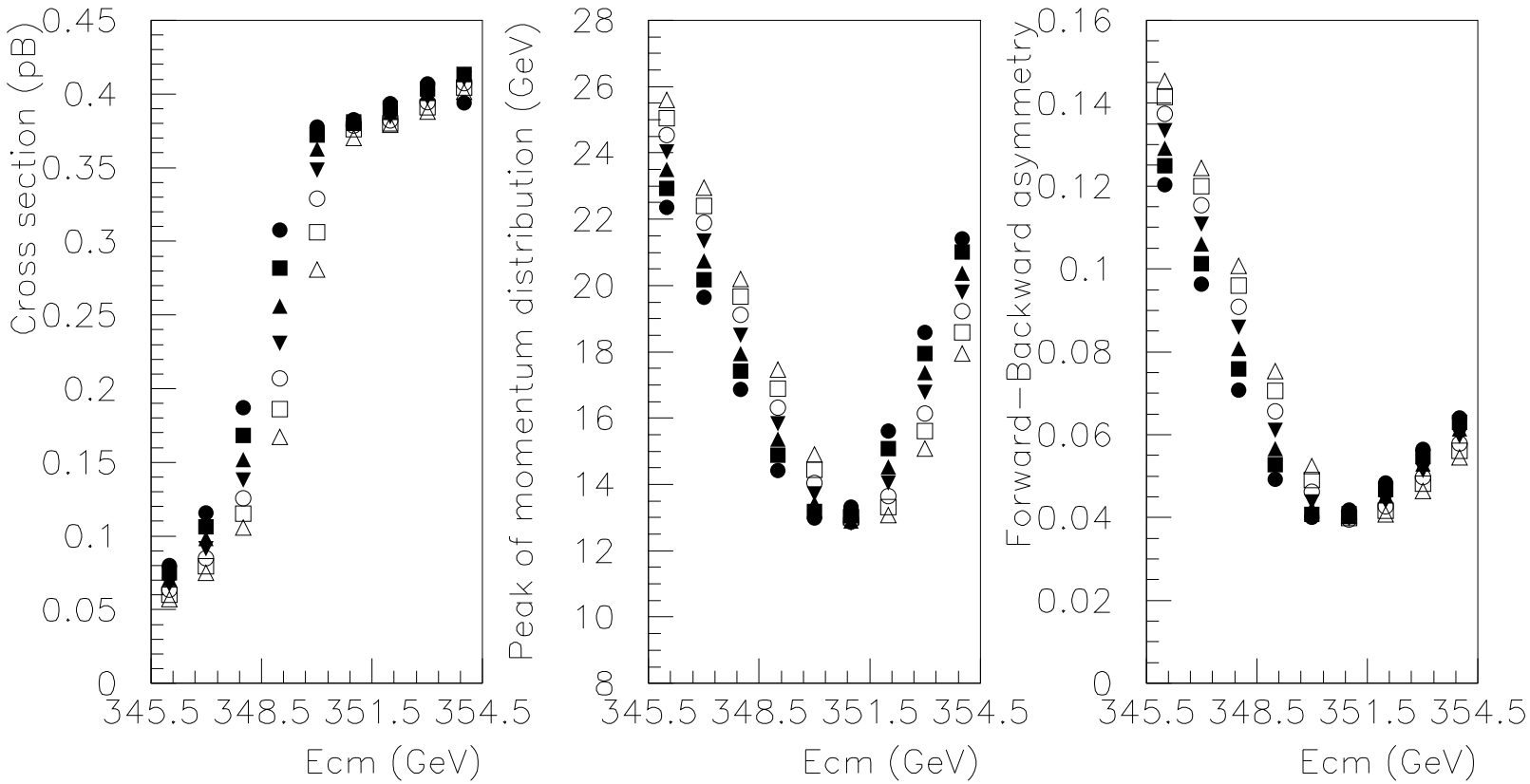
$t\bar{t}$ at ILC: Simulating the threshold scan

Scenario for a 9+1 point threshold scan (total $\mathcal{L} = 300$ fb^{-1}): Martinez+Miquel '02

σ_{tot}, (peak of the) top quark momentum distribution and Forward-Backward Asymmetry as observables, with initial state radiation and beam smearing effects taken into account:



Sensitivity to top mass



- Exp. accuracy possible at ILC (multi-parameter fit, 3% TH-error on σ_{tot} assumed):

$$\Delta m_t \sim 20 \text{ MeV}, \ \Delta\Gamma_t \sim 30 \text{ MeV}, \ \Delta\alpha_s \sim 0.0012, \ \Delta y_t/y_t \sim 35\%$$

— Assumption of known luminosity spectrum.

▶ Recent work on how to extract lumi spectrum, its influence on the threshold study and how to implement it in a more realistic $t\bar{t}$ simulation: Boogert+Gournaris ⟶

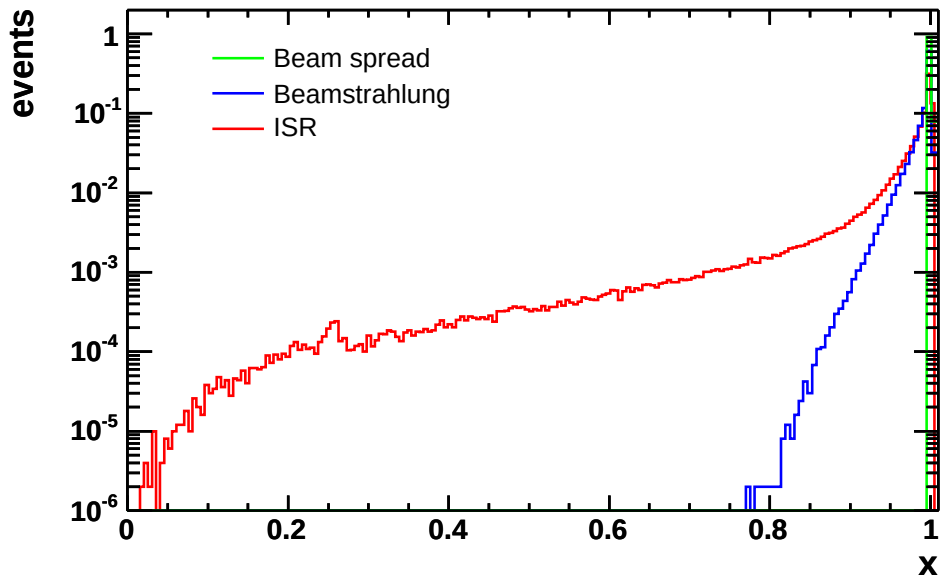
Accurate simulations incl. beam effects for the threshold scan

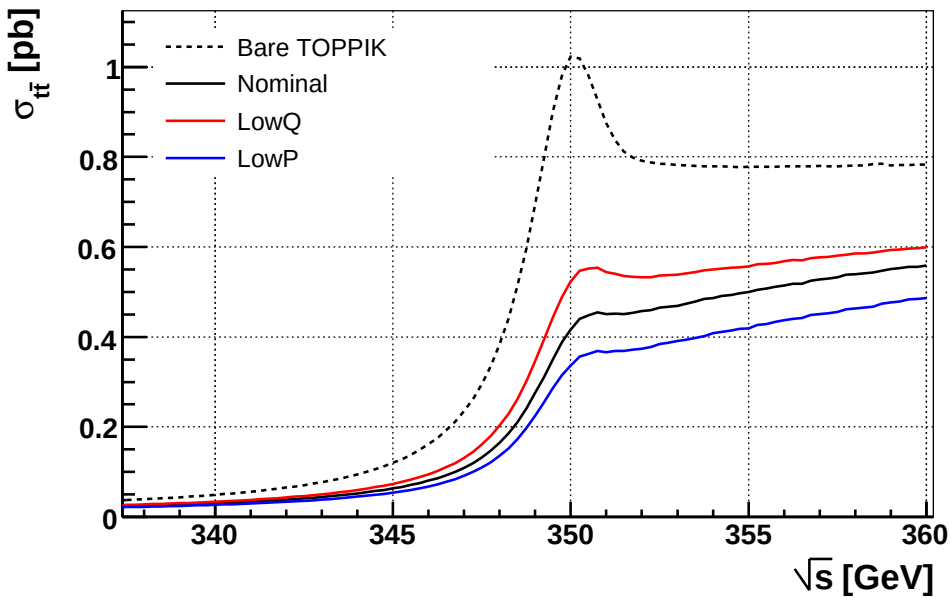
- Precision threshold measurements require:
  - Average c.m. energy $\langle\sqrt{s}\rangle$
    - Use of energy spectrometer
    - Calibrate e.g. with radiative return ($Z\gamma$)
  - Luminosity spectrum $\frac{\mathrm{d}L}{\mathrm{d}\sqrt{s}}$
    - Measure Bhabha acollinearity
      $\rightarrow$ Th.: Higher orders in MC's?
      $\rightarrow$ Detector precision for Bhabha?
  - Calculation of Initial State Radiation
    $\rightarrow$ Theoretical precision of ISR MC's?
- Effect on top cross section:

$$\sigma^{\mathrm{obs}}(\sqrt{s}) = \frac{1}{L_0}\int_0^1 L(x)\,\sigma(x\sqrt{s})\,\mathrm{d}x$$

- Beamstr. depends on machine parameters!

Linear Collider is NOT like LEP!

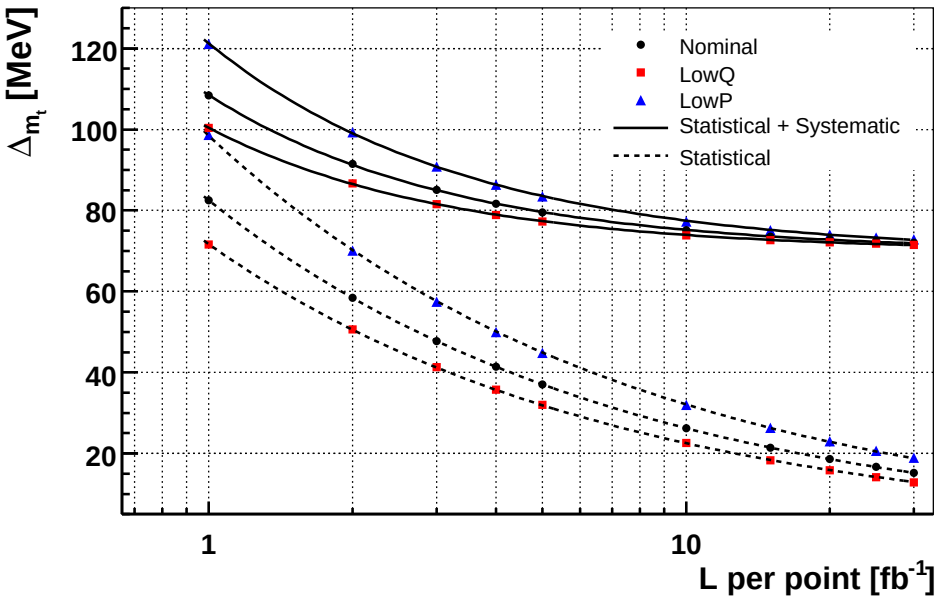




Accurate simulations incl. beam effects for the threshold scan

- Effect on top cross section:
  - Loss in effective luminosity
  - Shift in top mass; systematic error?
- Recent simulations of beam spectra:
  - $\sqrt{s} = 350$ GeV different from higher energies
  - $\mathrm{d}\mathcal{L}/\mathrm{d}s$ parametrizations for 5 designs
  - will allow detailed study of systematic effects
- Development of new $t\bar{t}$ Monte Carlo:
  - for optimizing run params and scan strategy
  - will help to scrutinize detectors concepts
  - no polarization (yet),
    fast for other top studies

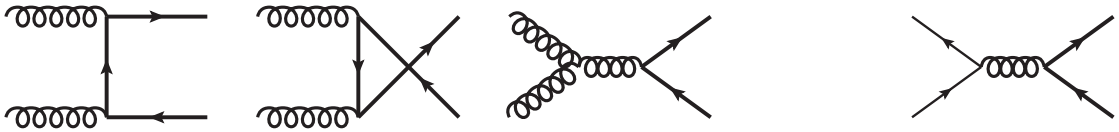
Projected total Δm_t: Boogert, Gournaris



- stat. error from fit: 15 – 100 MeV
- sys. error from $\Delta\sigma_{tot}$: 35 MeV
- absolute beam energy: 35 MeV
- beam spectrum sys.: 2 ... 70 MeV

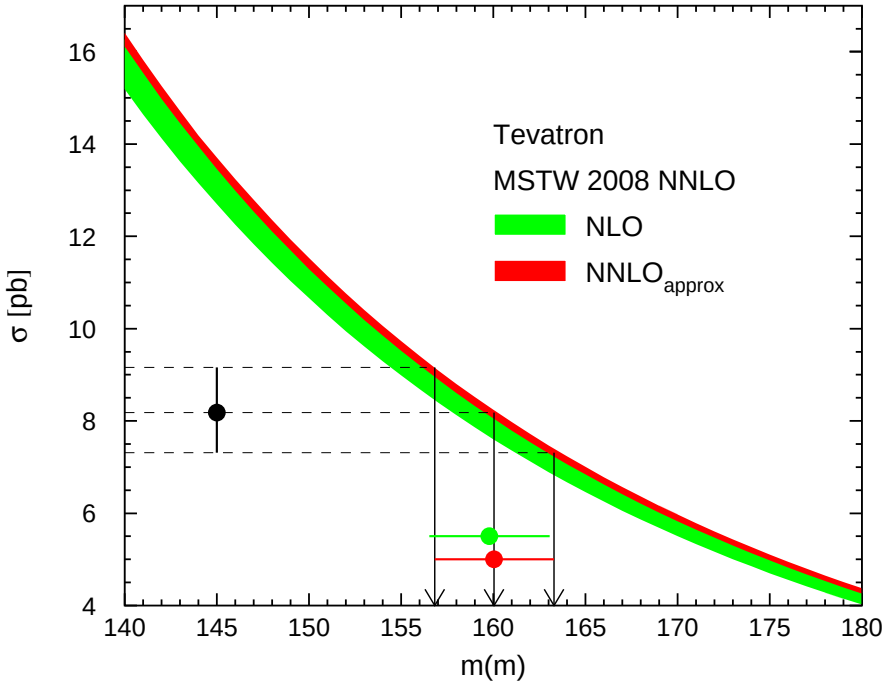
- Top threshold is the benchmark for high precision analyses (W^+W^-, SUSY thresholds).

$t\bar{t}$ at Tevatron and LHC



- Tevatron's $t\bar{t}$ dominated by quark-, LHC's by gluon initial *parton luminosities*

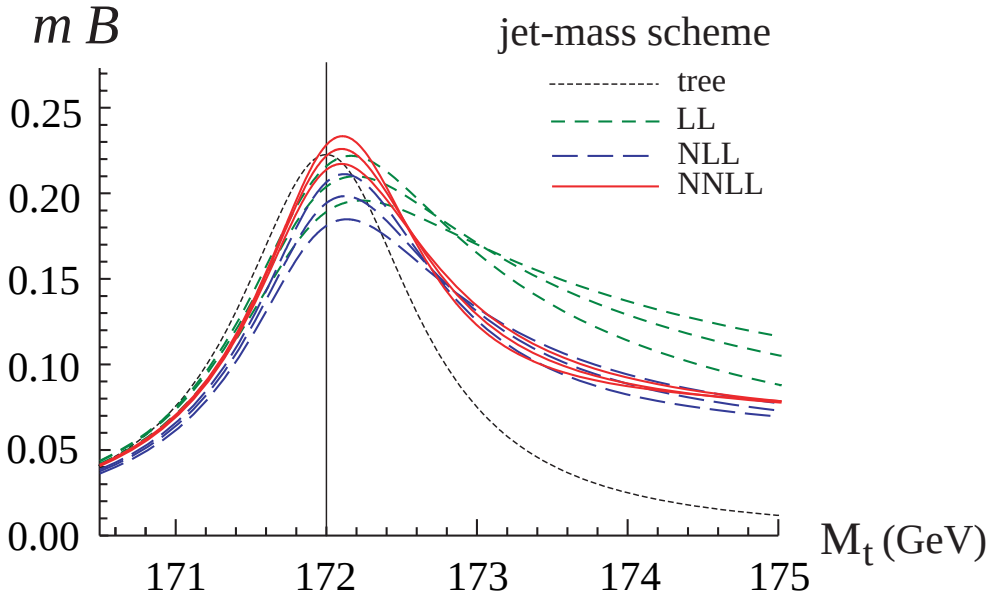
- convolution over x of PDFs
  $\rightsquigarrow$ 'scan' over $\sqrt{\hat{s}}$ implicit
- m_t determination from cross-section possible (though not very precise)
- here: $\overline{\mathrm{MS}}$ (running) mass
  [Langenfeld, Moch, Uwer: $\longrightarrow$]
  $\longrightarrow$ better stability than pole-mass

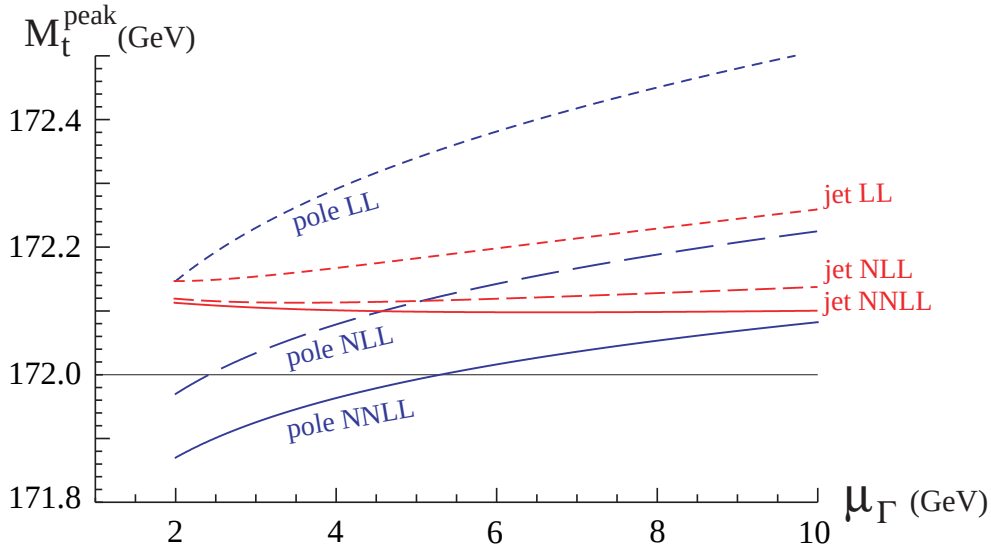


- top 'factories', large statistics $\rightsquigarrow$ m_t reconstructed from top-decays (or 'top-jets') with high precision: $m_t^{\mathrm{Tevatron}} = (173.3 \pm 1.1)$ GeV, a formidable performance!
- BUT: Which mass?

- Jet-masses:

[Fleming, Jain, Hoang, Mantry, Scimemi, Stewart]



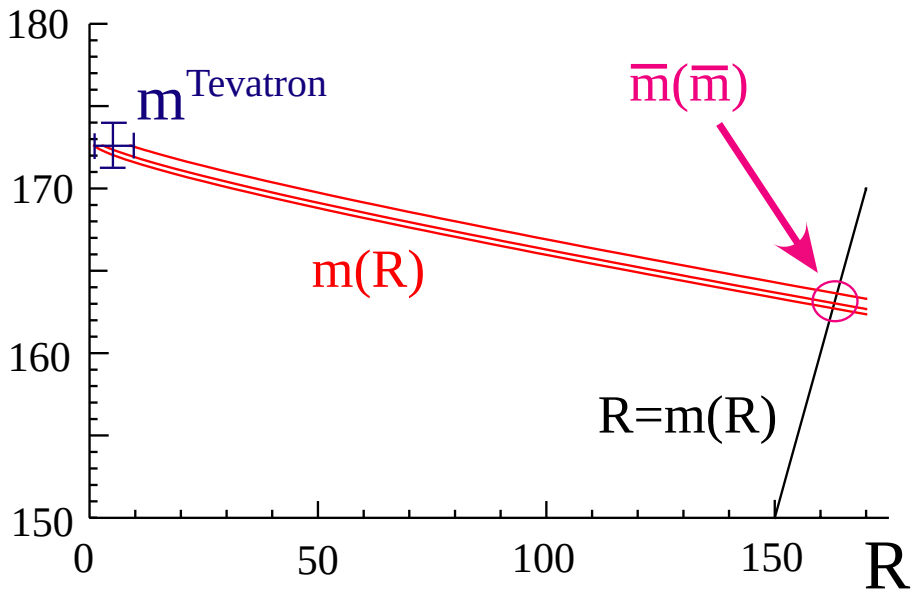


Heavy-quark jet-function and shifts of $M_t^{\rm peak}$

- based on effective theory of SoftCollinearEffectiveTheory
- perturbatively calculable heavy-quark jet-function
- factorisation formula: $\frac{{\rm d}\sigma}{{\rm d}M_t^2 {\rm d}M_{\bar t}^2} = \sigma_0 H_Q H_m \int {\rm d}\ell^+ {\rm d}\ell^- B_+(\ell^+,\Gamma_t) B_-(\ell^-,\Gamma_t) S(\ell^+,\ell^-)$

  with S the non-pert. soft (rad. betw. jets, fragm.) function, B the jet-fctn. (evol.+decay)
- will help to put anticipated m_t determination in continuum on firmer ground

- so far only worked out for e^+e^-, gluon ISR in hadroproduction not yet included in the formalism
- m_t^{Tevatron} relies on MC simulation of $t\bar{t}$ decays $\rightsquigarrow$ $m_t^{\text{PYTHIA,HERWIG}}$
- more similar to jet- than pole-mass?!

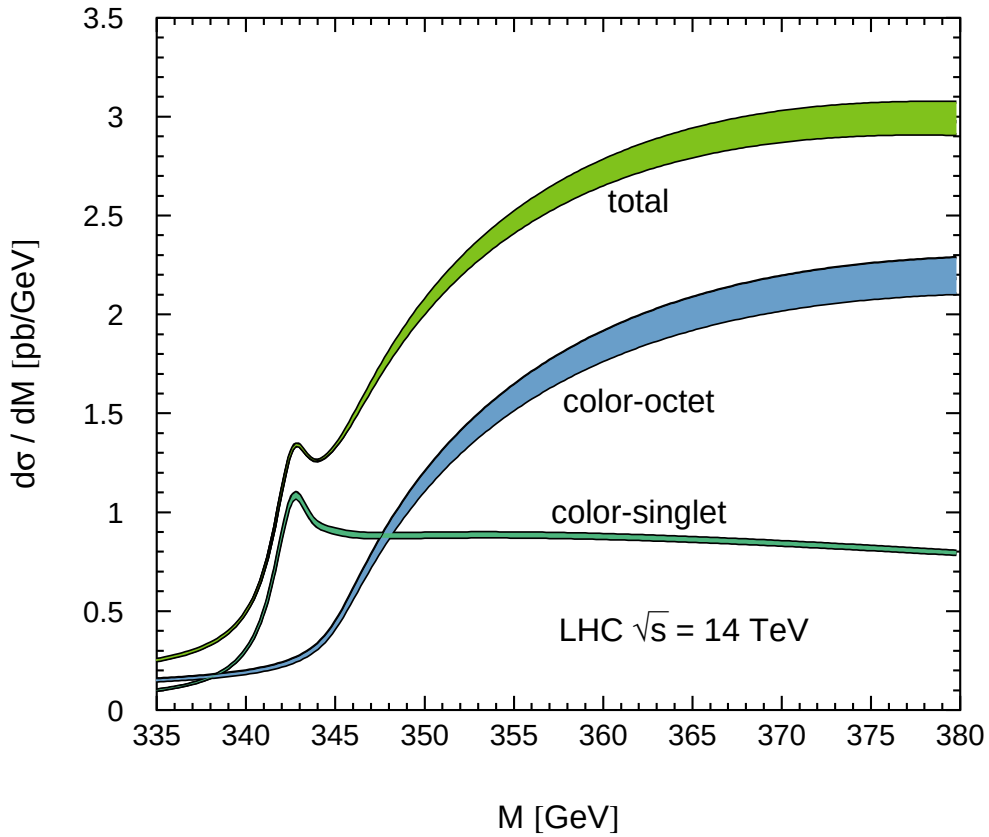


- transition to scale-invariant $\bar{m}(\bar{m}) \sim 163$ GeV then by R-evolution (and agrees with mass determination from xsec.)
- But: How consistent are MCs w.r.t. modelling/tuning of fragmentation and hadronisation? [detailed studies by G. Corcella]
- possibility of bias (or underestimated error); m_t^{Tevatron} should be interpreted with care

• Threshold enhancement in gluon-fusion:

[Hagiwara, Sumino, Yokoya; Kiyo, Kühn, Moch, Steinhauser, Uwer (figures)]

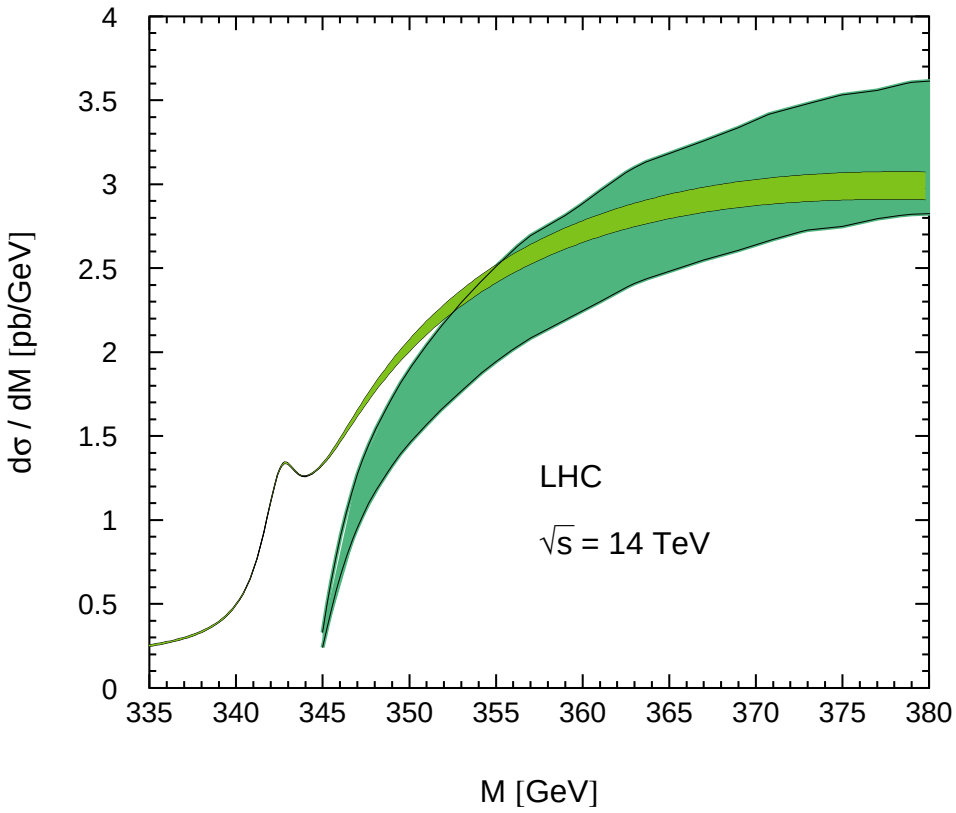
- Colour singlet contribution threshold enhanced (QCD potential attractive for colour singlet, repulsive for octet configuration)
- formalism very similar to e^+e^-; bands indicate uncertainty from scale variation
- contribution below nominal threshold of $\sqrt{\hat{s}} = 2m_t$, shift towards lower $M_{t\bar{t}}$
- possibly relevant effects; may have to be accounted for in precision analyses for m_t

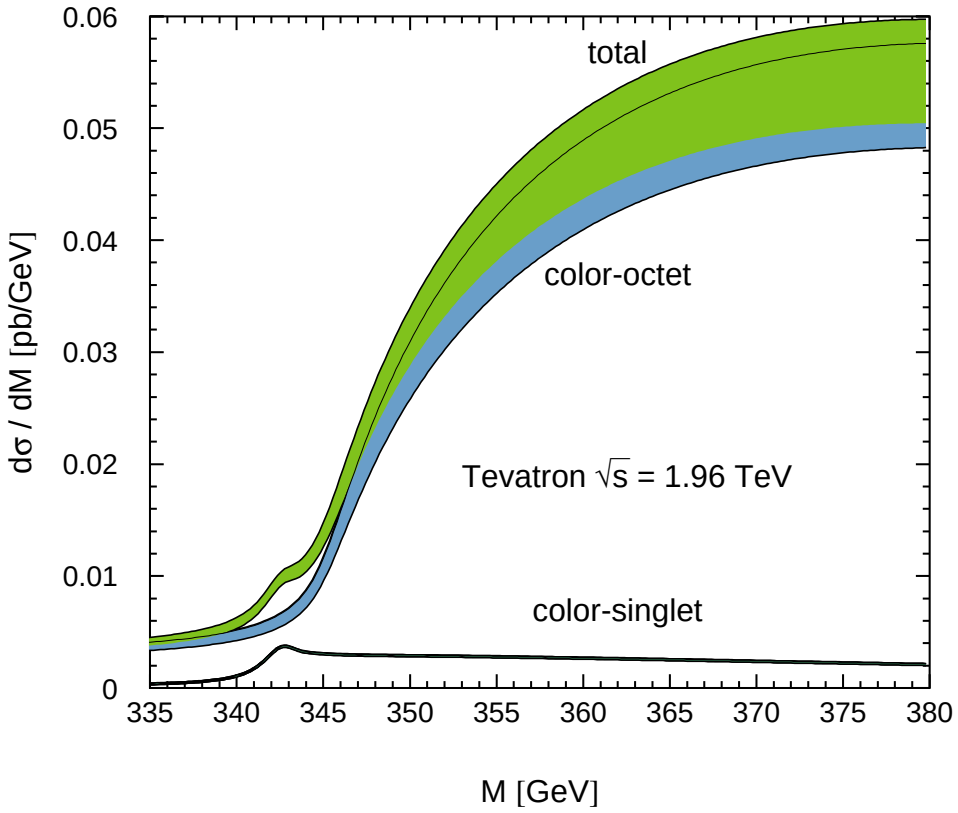


- phase space suppression vs. coulomb enhancement in decays of virtual tops; non-resonant diagrams; ISR ...

• Threshold enhancement in gluon-fusion (contd.)

[Figs. from Kiyo, Kühn, Moch, Steinhauser, Uwer]



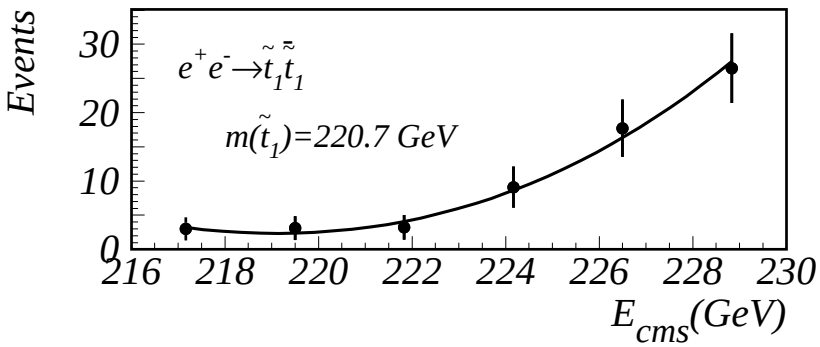


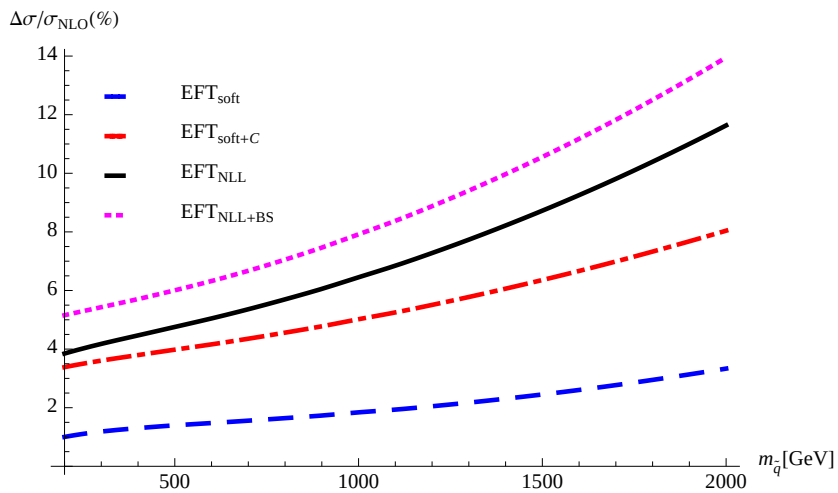
left: transition from threshold to continuum prediction right: Tevatron (scales!)

- less deformation of $M_{t\bar{t}}$ spectrum for Tevatron ($q\bar{q}$ the dominant production channel)
- possible influence on energy calibration, shift in m_t?

$\tilde{t}\bar{\tilde{t}}$ at the ILC

- If $\tilde{t}$ light enough to be pair produced at ILC, one method to determine its mass will be again a $\tilde{t}\bar{\tilde{t}}$ threshold scan, $e^+e^- \to \tilde{t}_1\bar{\tilde{t}}_1 \to c\tilde{\chi}_1^0\bar{c}\tilde{\chi}_1^0$
- In e^+e^-, $\tilde{t}\bar{\tilde{t}}$ produced in a P wave $\rightsquigarrow$ $\sigma \sim v^3$, hence only weak threshold enhancement.
- Exp. analyses by Nowak et al., Sopczak+Carena+Finch+Freitas+Milstene+Nowak:
  - 6 point scan with $\mathcal{L} = 50\,\mathrm{fb}^{-1}$, $P(e^-)/P(e^+) = +80\%/-60\%$ for best S/B ratio
  - Assuming SPS-5 scenario, $m_{\tilde{t}_1} = 220.7$ GeV, $\Gamma_{\tilde{t}_1} \sim 40$ MeV, $m_{\tilde{\chi}_1^0} = 120$ GeV, $\boldsymbol{\Delta m_{\tilde{t}_1} = 1.2}$ GeV:





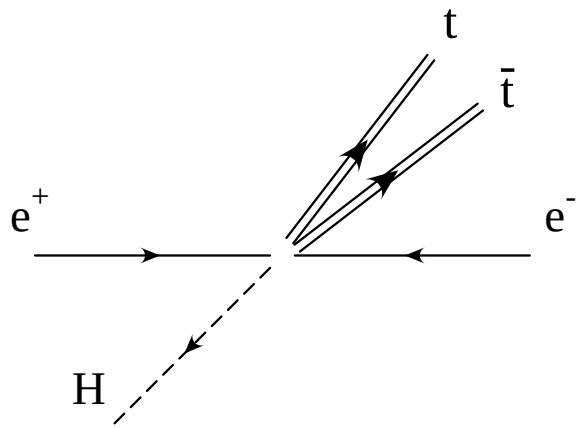
- TH work by Hoang+Ruiz-Femenia, recently Beneke, Falgari, Schwinn:
  Predictions of $\tilde{t}\bar{\tilde{t}}$ to (N)NLL within Effective Field Theory.

Top-Yukawa coupling y_t from $t\bar{t}H$

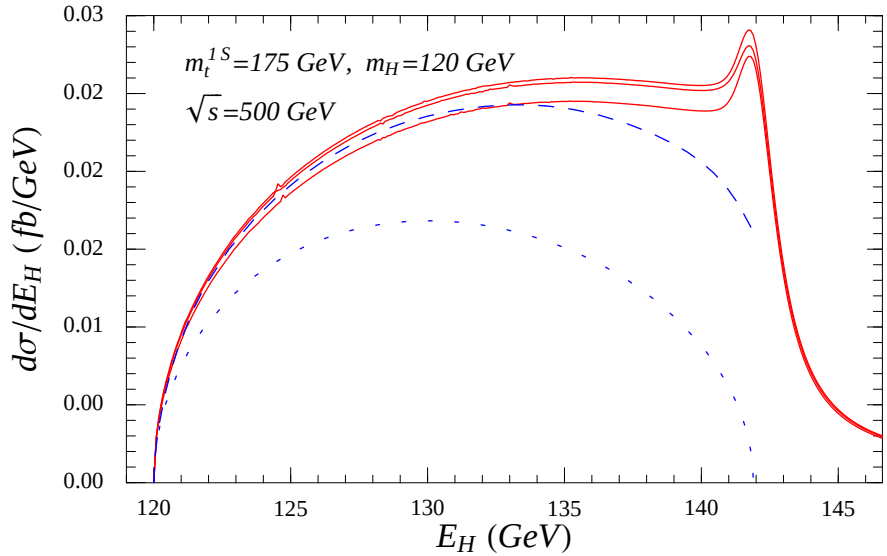
- Aim: measure top Yukawa coupling via $\sigma(e^+e^- \to t\bar{t}H) \sim g^2_{ttH}$ at ILC.
- LHC can get 15% accuracy on g, but only from indirect $gg \to H$ via top triangle.
- Challenging due to complicated final state, low rates, backgrounds,...
- Earlier analysis:
  - ILC (800 GeV, 1000 fb^{-1}): $\Delta g_{ttH}/g_{ttH} \sim 6(10)\%$ for $m_H = 120(190)$ GeV
  - estimate for baseline (500 GeV, 1000 fb^{-1}): $\Delta g_{ttH}/g_{ttH} \sim 24\%$ ($m_H = 120$ GeV)
- But: QCD helps!

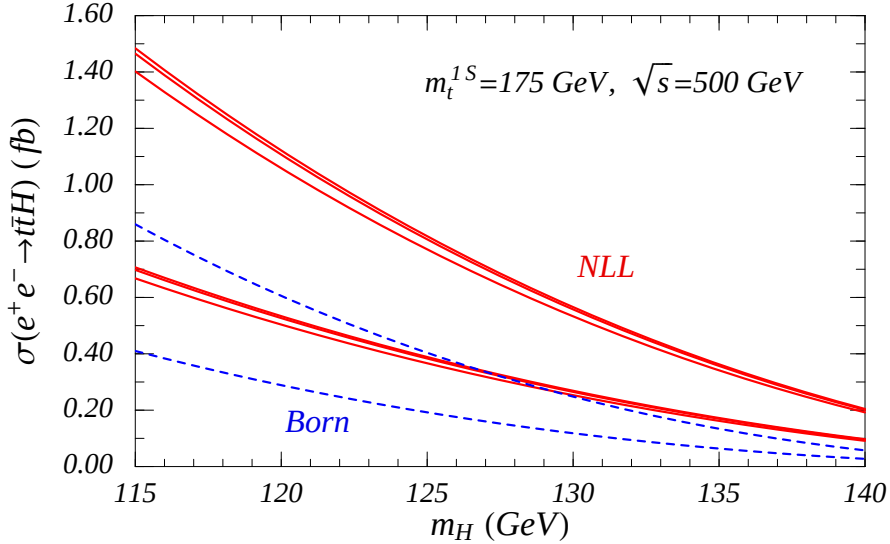
⋆ At $\sqrt{s} \sim 500$ GeV, $t\bar{t}H$ is non-relativistic and dominated by threshold dynamics

⇝ large enhancement, calculable in NRQCD



Calculations by Farrell+Hoang for $t\bar{t}H$ at NLL in vNRQCD:





lower lines: $(P_+, P_-) = (0, 0)$, upper lines: $(P_+, P_-) = (+0.6, -0.8)$

- Choice of (e^+ and e^-) polarization is crucial

▶ Estimates from A. Juste:

– enhancement of σ_{tth} from QCD : $\times 2.4$, from beam polarization: $\times 2.1$

⇝ anticipate $\boldsymbol{\Delta g_{ttH}/g_{ttH} \sim 10\%}$ for baseline ILC, $m_H = 120$ GeV.

▶ New analysis from Yonamine, Ikematsu, Tanabe, Fujii, Kiyo, Sumino, Yokoya:

⇝ $(P_+, P_-) = (+0.3, -0.8)$, 1 ab^{-1}, 500 GeV, fastsim: also 10%.

▶ New other study by Martin+Tabassam ongoing.

EW couplings, NP

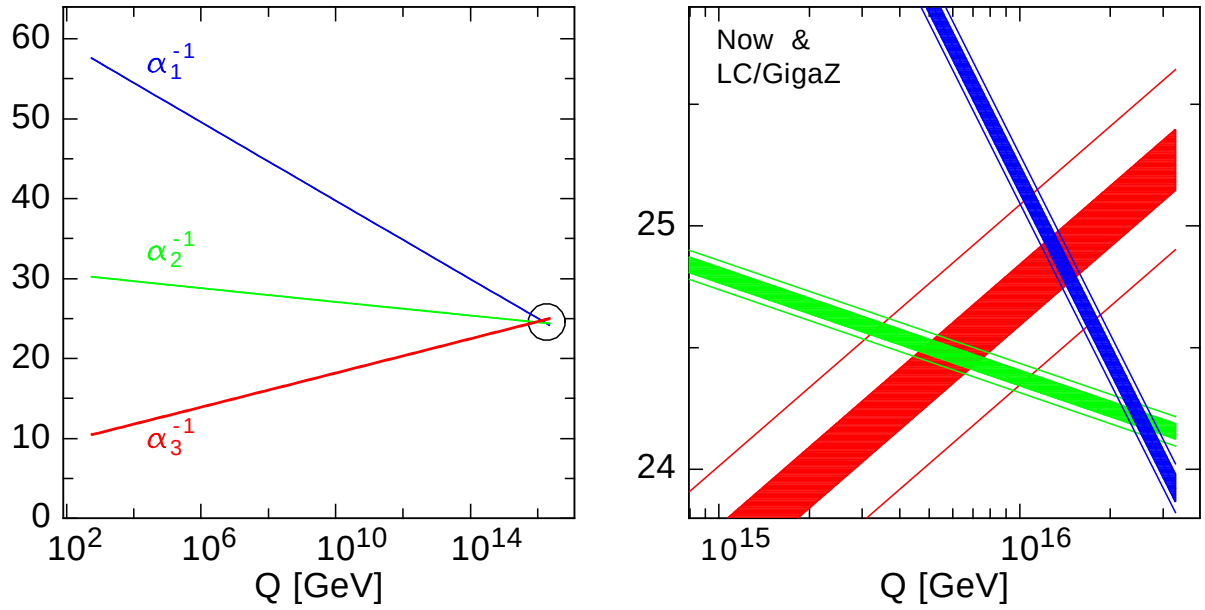
(as reported at IWLC2010 at CERN and LCWS10 in Beijing)

- SM and BSM contributions parametrised by set of general gauge invariant dim-4 operators. Many four-fermion operators can only be tested at the ILC, not at the LHC; for others ILC will improve on LHC's accuracy. [Aguilar-Saavedra]
- Wtb will be measured in single-top production at the LHC (indirect Γ_t determination), Ztt only at the ILC (and has better sensitivity to NP!)
- Study by Doublet, Pöschl, Richard, motivated by various BSM (RS) scenarios which may leave their footprints in $e^+e^- \to t\bar{t}$:
  - expect $\Delta\sigma/\sigma \sim 0.4\%$
  - and $\Delta A_{\rm LR}/A_{\rm LR} \sim 0.7\%$ (stat. only, need polarisation)
- Kühn, Rodrigo: Top charge asymmetry at $\mathcal{O}(\alpha_s^3)$ leads to small $A_{\rm FB}$. Asymmetry measured at Tevatron leaves 2σ room for BSM; axi-gluon, RS KK gauge bosons? ($t\bar{t}$ at LHC and ILC in business for many scenarios.)

  ...

α_s at the percent level

- High precision α_s determination is crucial for accurate predictions of many signal and background processes, e.g. as input in $t\bar{t}$ analyses.
- The current precision of α_s is not sufficient.
- α_s is the least precise input for coupling unification in SUSY, GUT's:



- With 'Giga$Z$' α_s could be improved significantly:

α_s with Giga$Z$

- Giga$Z$ would provide vastly increased statistics and better detector performance than LEP1 $\rightsquigarrow$ Z line-shape observables could be determined MUCH more precisely.
- From $\sigma(e^+e^- \to hadrons)$ and $\sigma(e^+e^- \to l^+l^-)$ one can determine $R = \sigma_{had}/\sigma_{lept}$, the total width Γ_Z, and the Born cross sections σ^0_{had}, $\sigma^0_{lept} = 12\pi\Gamma^2_{lept}/M^2_Z\Gamma^2_Z$ on the Z resonance.
- These observables depend on α_s and can be calculated in perturbative QCD with very high precision and minimal systematic uncertainties:
  - fully inclusive process
  - non-perturbative contributions suppressed by $1/s^2$
  - fixed order perturbative expansion in α_s/π works at its best
  - 2-point correlator known to four-loop accuracy! Chetyrkin+Kühn et al.
- Combining information from all four variables, dominated by R, the estimate of the possible absolute accuracy for α_s at Giga$Z$ is extremely high,

$$\Delta\alpha_s(M_Z) = 0.0005 - 0.0007$$

Conclusions

- Top quark physics at the ILC has moved forward tremendously and has triggered a lot of TH developments.
- TH is typically at next-to-next-to-leading order, but only for inclusive quantities; more/better MC tools will be needed.
- For hadron colliders, NNLO is the next call.
  To fully exploit the top potential of the LHC, a better understanding of soft physics and jets will be required.
- LHC may well deliver more than we now think is possible, but
- ultimately ILC will be *the* precision machine for the determination of SM (and possibly BSM) parameters in the top sector.

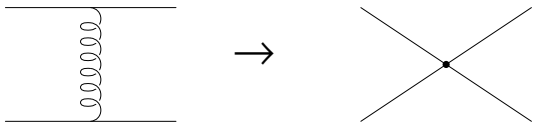
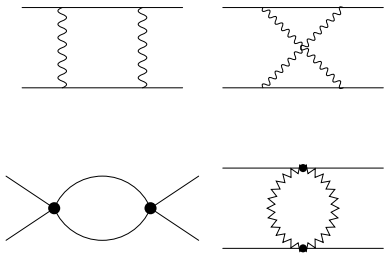
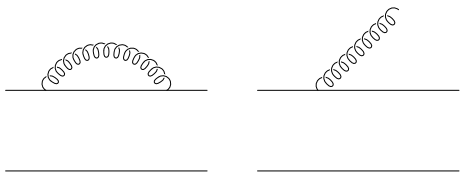
Back-up slides: EFT in a few steps

(velocity) Non-Relativistic QCD in a few steps

Caswell, Bodwin, Braaten, Lepage, Labelle, Grinstein, Rothstein, Luke, Manohar, Savage, Pineda, Soto, Brambilla, Vairo, Stewart, Hoang...

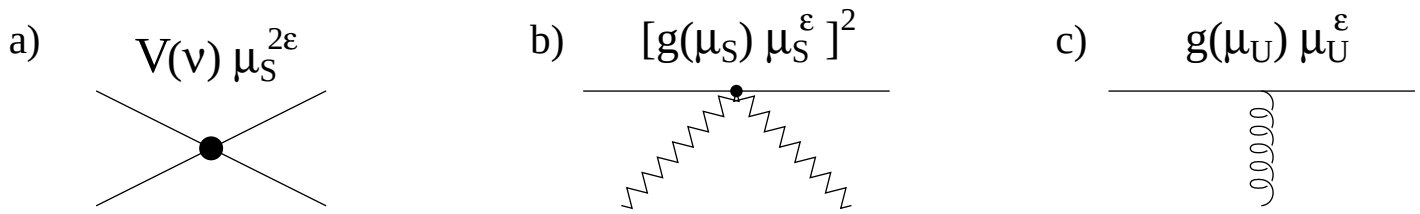
1. Identification of momentum regions ($\longrightarrow$ power counting!)

Beneke, Smirnov

region	$(k^0, \boldsymbol{k})$	on-shell d.o.f.	effects	examples
hard	(m, m)		annihilation, production	
potential	(mv^2, mv)	Quarks ψ_p, χ_p	binding, potential	
soft	(mv, mv)	Gluons A_p^μ	rad. corr. to binding	
ultrasoft	(mv^2, mv^2)	Gluons A^μ	retardation effects	

2. Write down the most general $\mathcal{L}$ for the on-shell d.o.f.

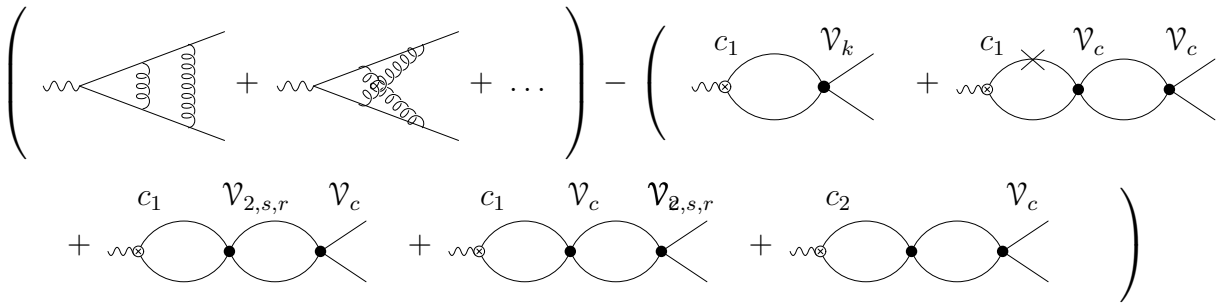
Examples of a) potential, b) soft and c) ultrasoft interactions in vNRQCD:



3. Match the EFT to full QCD at a high scale $\mu = m$

$\longrightarrow$ 'integrate out' hard modes, soft quarks $\rightsquigarrow$ non-dyn. effects in the matching coeffs.

Example for a matching calculation:



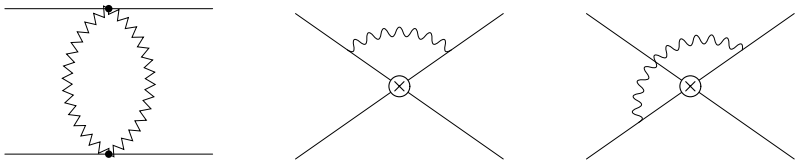
Difference of full QCD and order $\alpha_s^2 v^0$ EFT graphs which gives the two loop matching for $c_1(1)$. The $\times$ denotes an insertion of the $\boldsymbol{p}^4/(8m^3)$ operator.

$\longrightarrow$ Soft gluons are integrated out $\rightsquigarrow$ *'instantaneous'* potentials (cannot be on-shell in a non-rel. $Q\bar{Q}$ system; effective $4f$ vertices!)

$\hookrightarrow$ matrix elements can be calculated by solving a Schrödinger Equation (with higher order potentials and operators for relativistic corrections).

$$cross\ section\ \sim \sum_i matching\ coeffs.\ c_i \cdot matrix\ elements$$

4. Calculate Renormalization Group Equations of all EFT operators (for the process up to a given order in the power-counting in α_s and v).



Example of one-loop running of the potential V_c in QED through soft and ultrasoft photons

5. Evolve from the high matching scale m down to the low (dyn.) scale mv

$\rightsquigarrow$ all large logarithms ($\log \frac{m_t^2}{p^2},\ \log \frac{p^2}{E^2} \sim 4,\ \log \frac{m_t^2}{E^2} \sim 8$) are absorbed ('*summed*') in the potentials and matching coefficients!