Progress in Perturbative QCD: Tools & Results @ NLO

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- LC10 - INFN LNF - 02.12.2010

LC10 Workshop

New Physics: Complementarities between direct and indirect searches

INFN Istituto Nazionale di Fisica Nucleare Laboratori Nazionali di Frascati

To be held together with the Bruno Touschek Memorial Lectures

INFN - Laboratori Nazionali di Frascati
30th November - 3rd December 2010

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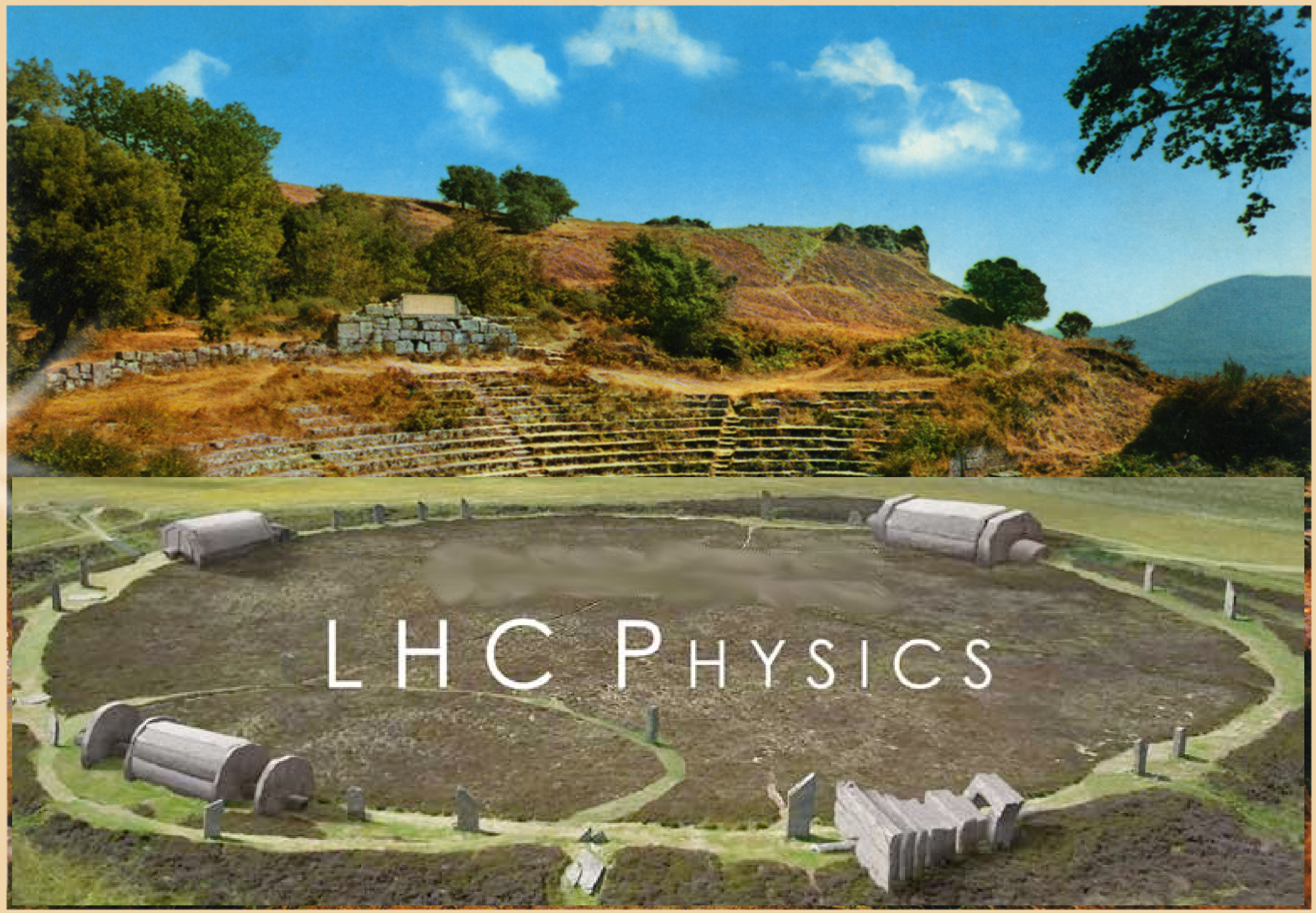

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Outline

- Motivation and State-of-the art
- Unitarity-based Methods vs Theory of Complex Functions
- Analytic Techniques
- Seminumerical Tools
- SAMURAI
  a tool for the seminumerical evaluation of one-loop amplitudes
- Conclusion

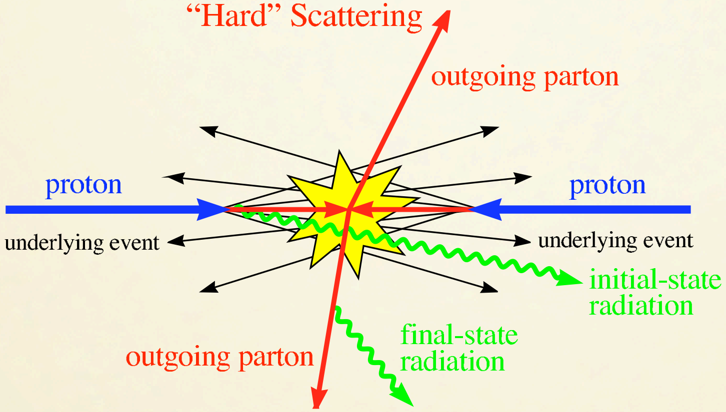
Why NLO ?

- Less Sensitivity to unphysical input scales (renormalization & factorization)
  - first predictive normalization of observables at NLO
  - more accurate estimates of backgrounds to new-physics
  - confidence on cross-sections for precision measurements
- More realistic process modeling
  - initial state radiation
  - jet clustering
  - richer virtuality
- Crossing path with other techniques
  - matching with resummed calculations
  - NLO parton showers

Where NLO ?

- Front-line in Theoretical Particle Physics

@ LHC Phenomenology



Signals:

- Decays: $H \to VV \qquad (V = \gamma, W, Z)$
- $PP \to H + 0,1,2$ jets (Gluon Fusion)
- $PP \to H + 2$ jets (Weak Boson Fusion)
- $PP \to H + t\bar{t}$
- $PP \to H + W, Z$

Backgrounds:

- $PP \to t\bar{t} + 0,1,2$ jets
- $PP \to VV + 0,1,2$ jets
- $PP \to V + 0,1,2,3$ jets
- $PP \to VVV + 0,1,2,3$ jets

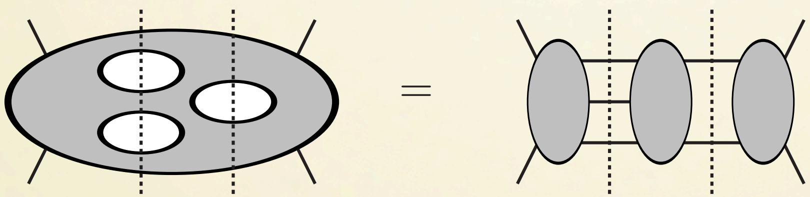
Where NLO ?

- Front-line in Theoretical Particle Physics

@ LHC Phenomenology

@ QFT Stucture

- ElectroWeak Symmetry Breaking: Higgs mechanism
- Beyond the Standard Model (SuSy, Dark Matter, . . .)
- Unveiling the *Iterative Structure* of Scattering Amplitudes in gauge-Theory

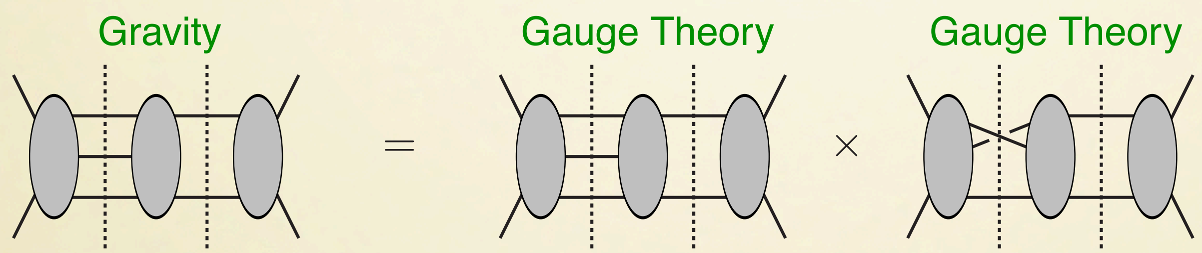


Anastasiou, Bern, Dixon, Kosower
Bern, Dixon, Smirnov;
Bern, Czakon,Dixon, Kosower;
Beisar, Eden, Staudacher;
Drummond, Korchemsky, Sokatchev;
Brandhuber, Heslop, Travaglini;
Alday, Maldacena;
Roiban, Spradlin, Volovich;
....

WHERE NLO ?

- Front-line in Theoretical Particle Physics

@ LHC Phenomenology
@ QFT Stucture
- ElectroWeak Symmetry Breaking: Higgs mechanism
- Beyond the Standard Model (SuSy, Dark Matter, . . .)
- Unveiling the Iterative Structure of Scattering Amplitudes in gauge-Theory
- Exploring the *Finiteness of Supergravity*



Bern, Dixon, Kosower, Perlestein, Rozowski, Roiban;
Bern, Bjerrum-Borh, Dunbar, Ita, Perkins, Risager;
Chalmers; Green, Vanhove, Russo;
Badger, Bjerrum-Borh, Vanhove,
Bern, Carrasco, Johanson;
Arkani-Hamed, Cachazo, Kaplan;
....

Recent Progress

▷ $2 \to 4$ @ NLO

- $pp \to tTbB$ [Bredenstein, Denner, Dittmaier, Pozzorini]
  [Bevilacqua, Czakon, Papadopoulos, Worek]
- $pp \to tT + 2\text{jets}$ [Bevilacqua, Czakon, Papadopoulos, Worek]
- $pp \to bBbB$ (quark-initiated) [Binoth, Greiner, Guffanti, Reuter, Guillet, T. Reiter]
- $pp \to W + 3\text{jets}$ [Berger, Bern Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre]
  [Ellis, Zanderighi, Melnikov]
- $pp \to Z + 3\text{jets}$ [Berger, Bern Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre]
- $pp \to W^+W^+ + 2\text{jets}$ [Melia, Melnikov, Rontsch, Zanderighi]

▷ $1 \to 5$ @ NLO • $e^+e^- \to 5\text{jets}$ [Frederix, Frixione, Melnikov, Zanderighi]

▷ $2 \to 5$ @ NLO • $pp \to W + 4\text{jets}$ [Berger, Bern Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre]

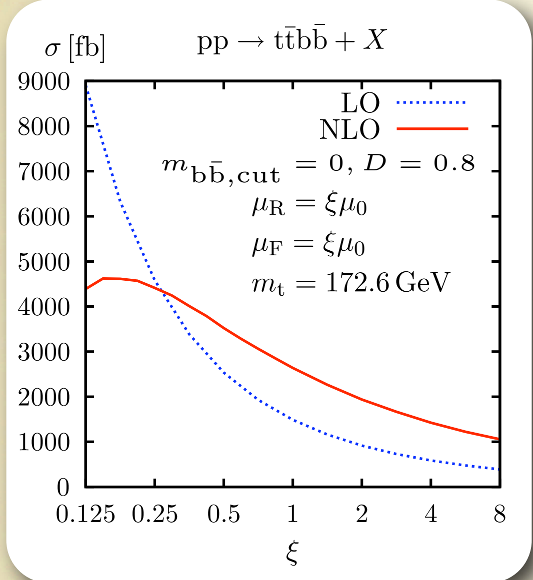
▷... many $2 \to 3$ became available/refined

• $pp \to VV + 1\text{jet}$ • $pp \to V + bB$ • $pp \to tT + 1\text{jet}$ • $pp \to VVV$ • $e^+e^- \to \mu^+\mu^-\gamma$ • $pp \to H + 2\text{jet}$

[Kallweit, Uwer, Campbell, Binoth, Karg, Kauer, Sanguinetti, Ciccolini, Badger, Glover, P.M., Williams, Risager, Sofianatos, Lazopoulos, Petriello, Campanario, Figy, Hankele, Oleari, Zeppenfeld, Ossola, Pittau, Wackeroth, Reina, Weinzierl, Schultze, Actis, Van Hameren, Tramontano, ...]

▷ some analytic results

- $gg \to gggg$ (QCD-virtual) Bern, Dixon, Dunbar, Kosower '96; ... (we are here) ...; Xiao, Yang, Zhu '08
- $\gamma\gamma \to \gamma\gamma\gamma\gamma$ (QED-virtual) Mahlon' 96; Binoth, Gehrmann, Heinrich, P.M. '07
- $pp \to H + 2\text{jets}$ (QCD-Virtual)
  [Badger, Berger, Campbell, Del Duca, Dixon, Ellis, Glover, Risager, Sofianatos, Williams, Zanderighi, P.M.]
- $u\bar{d} \to WbB$ (massive b-pair) [Badger, Campbell, Ellis]



- $pp \to tTbB$

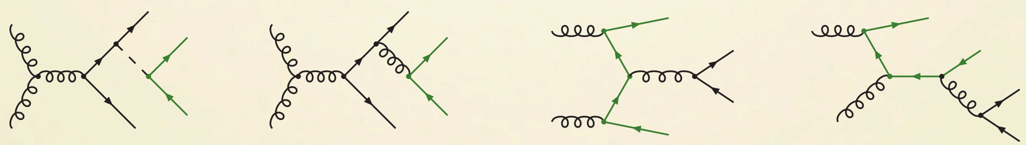
Impact on $t\bar{t}H$ ATLAS studies: $m_{b\bar{b}} > 100\,\mathrm{GeV}$

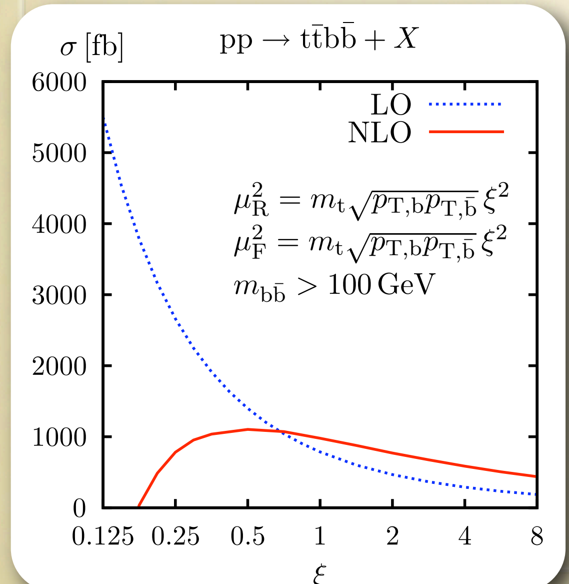
High sensitivity to scale choice

$$\frac{\Delta\sigma_{LO}}{\sigma_{LO}} \simeq \frac{\Delta\alpha_S^4(\mu)}{\alpha_S^4(\mu)} \quad \Rightarrow \quad 78\%\ \text{uncertainty}$$

ATLAS scale choice $\mu_0 = E_{thr}/2 = m_t + m_{b\bar{b}}/2$

QCD dynamics of $t\bar{t}H/t\bar{t}b\bar{b}$ completely different





LO and NLO scale dependence of $\sigma_{t\bar{t}b\bar{b}}$

Variations around new central scale

$$\mu_0^2 = m_t\sqrt{p_{T,b}p_{T,\bar{b}}}$$

Good news for theory: improved convergence

Bad news for experiment: $\sigma_{t\bar{t}b\bar{b}}$ enhanced by factor 2.2[a] wrt LO ATLAS simulations

$\sigma_{t\bar{t}b\bar{b}}$	LO	NLO	NLO/LO
$\mu_{R,F} = E_{thr}/2$	449 fb	751 fb	1.67
$\mu_{R,F}^2 = m_t\sqrt{p_{T,b}p_{T,\bar{b}}}$	786 fb	978 fb	1.24

[a](Partially) taken into account in Fat-Jet analysis!

[Bredenstein, Denner, Dittmaier, Pozzorini]

- $pp \to tT + 2\text{jets}$

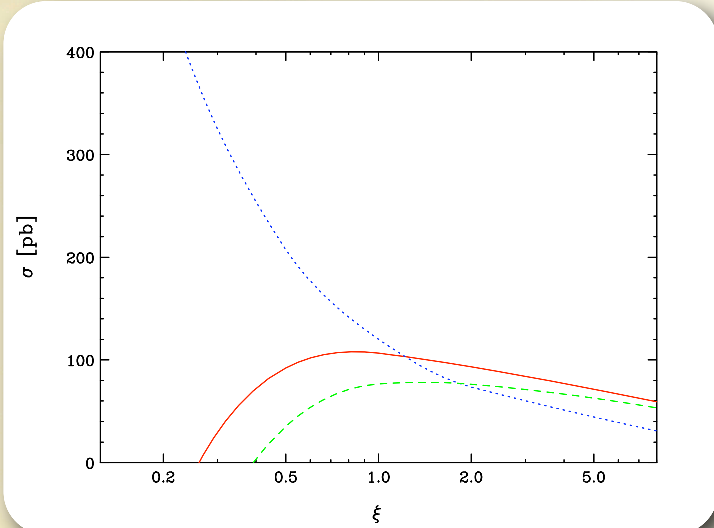


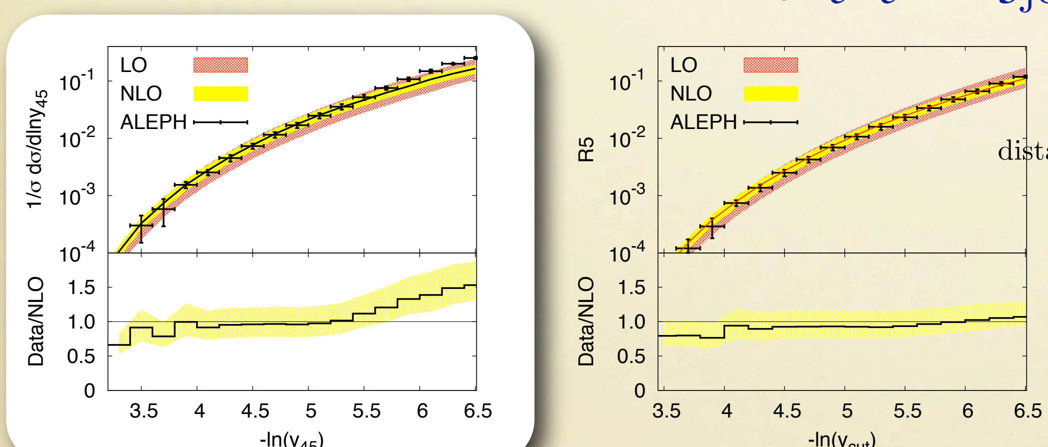
FIG. 2: Scale dependence of the total cross section for $pp \to t\bar{t}jj + X$ at the LHC with $\mu_R = \mu_F = \xi \cdot \mu_0$ where $\mu_0 = m_t$. The blue dotted curve corresponds to the LO, the red solid to the NLO result whereas the green dashed to the NLO result with a jet veto of 50 GeV.

For the evaluation of the NLO corrections, we have used the CTEQ6M parton distribution functions with NLO running of the strong coupling constant. At the central scale $\mu_0 = m_t$, we obtain

$$\sigma^{\rm NLO}_{pp\to t\bar{t}jj+X} = (106.94 \pm 0.17)\ \text{pb}\ ,$$

[Bevilacqua, Czakon, Papadopoulos, Worek]

- $e^+e^- \to 5\text{jets}$



distance between each pair of particles is used in the Durham jet algorithm

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)}{s}(1 - \cos\theta_{ij})\,,$$

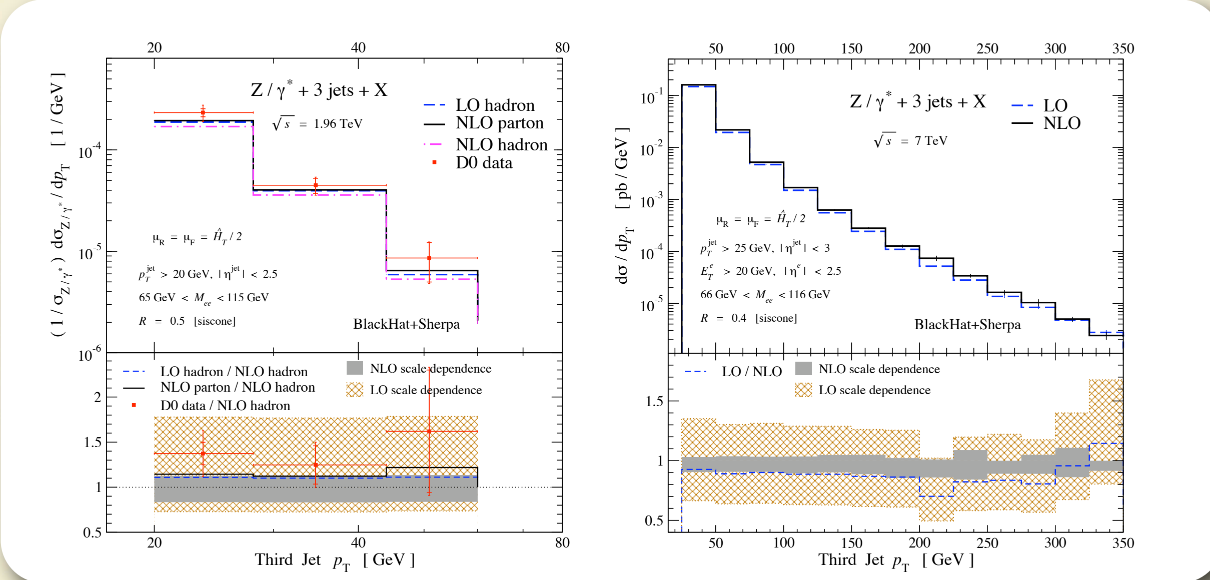
$$\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}\,.$$

Figure 3: ALEPH LEP1 data compared to leading and next-to-leading order predictions in QCD, without hadronization corrections. We use $\alpha_s(M_Z) = 0.130$ at the leading and $\alpha_s(M_Z) = 0.118$ at the next-to-leading order in perturbative QCD. The renormalization scale is chosen to be $0.3M_Z$. The uncertainty bands are obtained by considering the scale variation $0.15\ M_Z < \mu < 0.6\ M_Z$. Solid lines refer to NLO QCD results evaluated with $\mu = 0.3M_Z$.

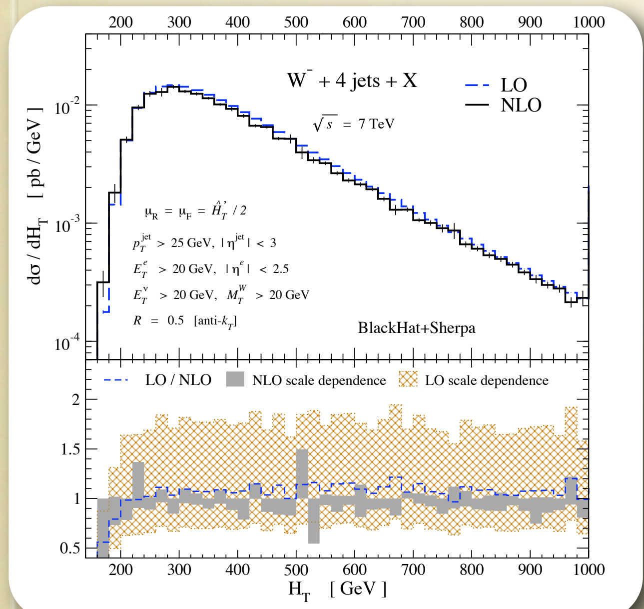
[Frederix, Frixione, Melnikov, Zanderighi]

[Berger, Bern Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre]

- $pp \to Z + 3\text{jets}$



- $pp \to W + 4\text{jets}$



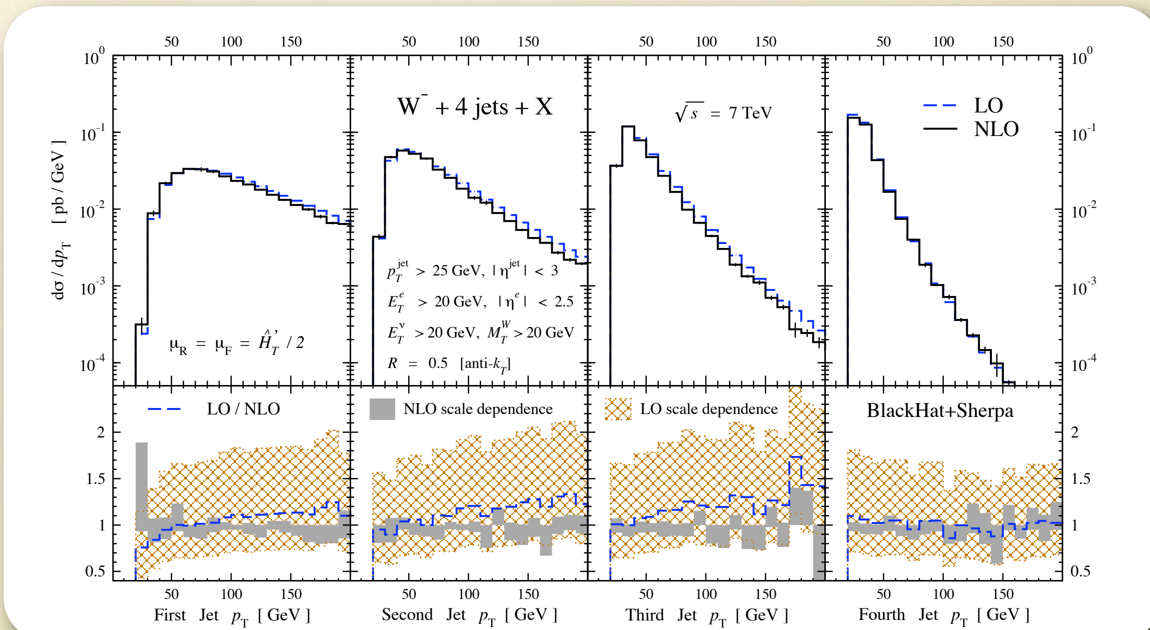


FIG. 3: The H_T distribution for $W^- + 4$ jets.

$\mu = \hat{H}'_T/2$, where $\hat{H}'_T \doteq \sum_j p_T^j + E_T^W \qquad E_T^W = \sqrt{M_W^2 + (p_T^W)^2}$

• $pp \to H + 2\text{jets}$

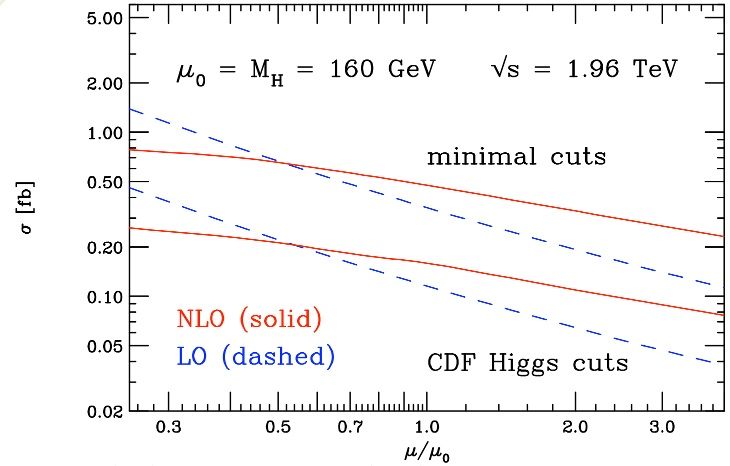
in the limit $m_H < 2m_t$

[Campbell, Ellis, Williams]

Ellis,

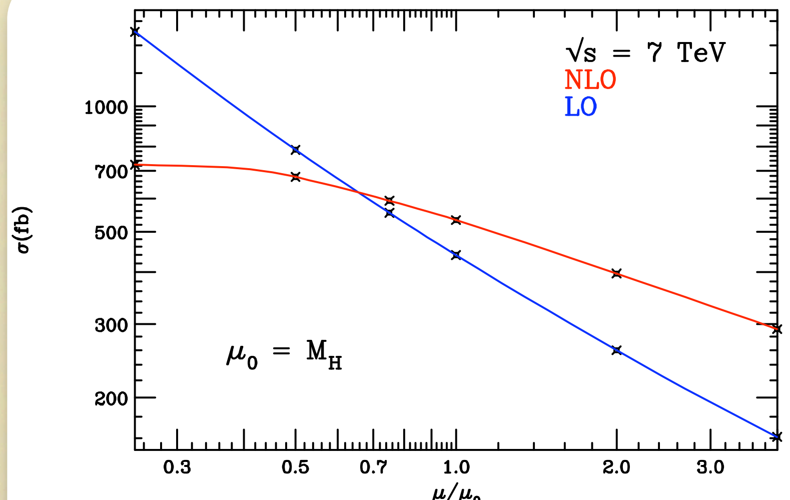
Higgs + 2 jets ☆

- arXiv:0608194v2 was based on a semi-numerical method of calculation of virtual corrections. Code was never released.
- now updated in arXiv:1001.4495 (Campbell,Ellis,Williams), to use compact, analytic expressions for virtual amplitudes.
- Much faster code, obtainable in MCFMv5.7 or greater,
- ~5ms per virtual point, (2.66GHz iMac, gfortran, no opt.)
- Fast enough to include Higgs decays, such as H→WW*→llνν.

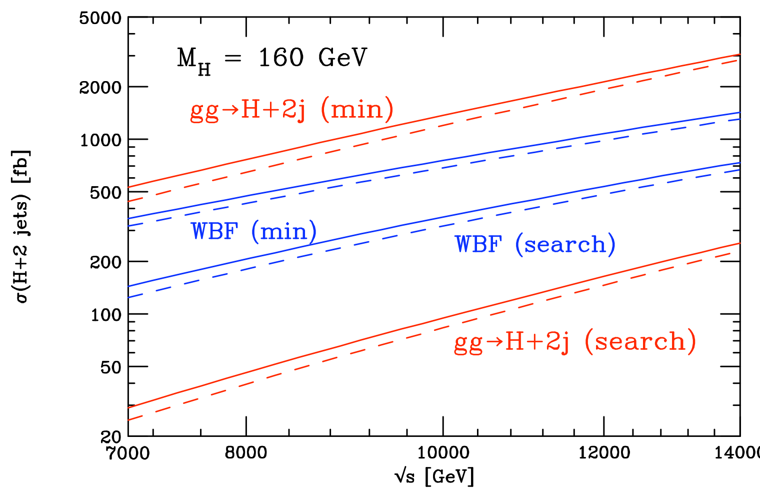


$p_t(\text{jet}) > 15$ GeV, $|\eta_{\text{jet}}| < 2.5$, $R_{\text{jet,jet}} > 0.4$,

Figure 1: Scale dependence for the Higgs + 2 jet cross section, with the Higgs decay into $W^-(\to \mu^- \bar{\nu})W^+(\to \nu e^+)$, at the Tevatron and using the a central scale $\mu_0 = M_H$. Results are shown for the minimal set of cuts in Eq. (2) (upper curves) and for cuts that mimic the latest CDF $H \to WW^\star$ analysis (lower curves).

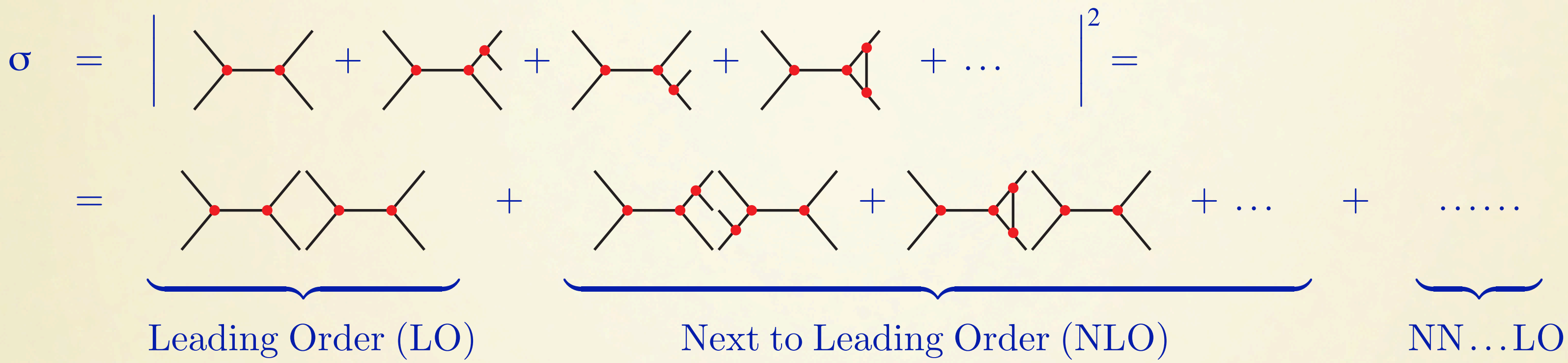


$p_t(\text{jet}) > 40$ GeV, $|\eta_{\text{jet}}| < 4.5$, $R_{\text{jet,jet}} > 0.8$.

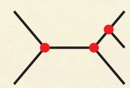


$|\eta_{j_1} - \eta_{j_2}| > 4.2$, $\eta_{j_1} \cdot \eta_{j_2} < 0$,

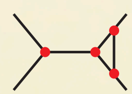
NLO Building Blocks



- ☑ tree-graphs with (n+1)-partons
  - soft/collinear divergences



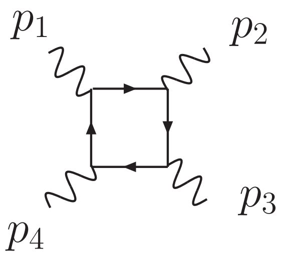
- ☐ virtual-graphs with n-partons
  - divergences from loop-integration



$$\rightarrow \quad I^{\mu\nu\rho\ldots} = \int d^D\ell \, \frac{\ell^\mu \ell^\nu \ell^\rho \ldots}{D_1 D_2 \ldots}$$

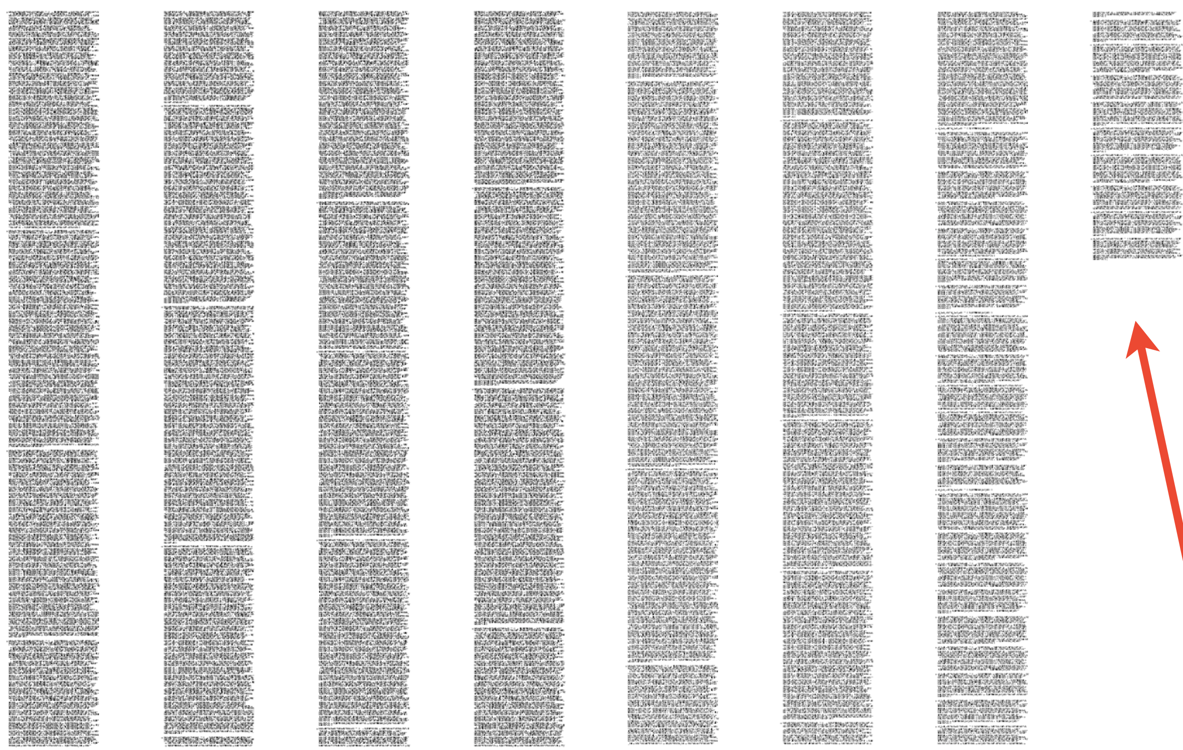
- ☑ extracting IR-singularities from both and combining them
  - phase-space slicing, subtractions, dipoles, antennas

Feynman Integrals Complexity



$$\int \frac{d^{4-2\epsilon}\ell}{(2\pi)^{4-\epsilon}} \frac{\ell^\mu\, \ell^\nu\, \ell^\rho\, \ell^\lambda}{\ell^2\,(\ell-k_1)^2\,(\ell-k_1-k_2)^2\,(\ell+k_4)^2}$$

Passarino-Veltmann reduction



All-plus photon helicity-amplitude = -8 + O(ϵ)

Looking for Simplicity behind Complexity?

Looking for Simplicity behind Complexity?

Use simple tools!

source: Kolb

The Dawn of Simplicity

- momentum of propagating particles

Parametrization in terms of the Isotropic Tetrads [Anderev, Bondarev]

$$\ell_\mu = x_1\, p_\mu + x_2\, q_\mu + x_3\, \varepsilon^+_\mu + x_4\, \varepsilon^-_\mu$$

Pittau, de l'Aguila
Ossola, Papadopoulos, Pittau

$$q^2 = p^2 = \varepsilon^{\pm 2} = 0 = \varepsilon^\pm \cdot p = \varepsilon^\pm \cdot q$$

$$g_{\mu\nu} = \frac{1}{2p\cdot q}\left(p_\mu q_\nu + q_\mu p_\nu - \varepsilon^+_\mu \varepsilon^-_\nu - \varepsilon^-_\mu \varepsilon^+_\nu\right)$$

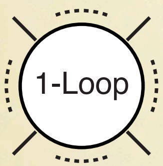
- Spinor-notation

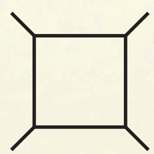
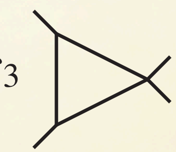
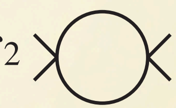
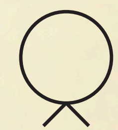
$$p_\mu = \frac{\langle p|\gamma_\mu|p]}{2}\,, \qquad q_\mu = \frac{\langle q|\gamma_\mu|q]}{2}$$

$$\varepsilon^+_\mu = \frac{\langle q|\gamma_\mu|p]}{2}\,, \qquad \varepsilon^-_\mu = \frac{\langle p|\gamma_\mu|q]}{2}$$

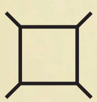
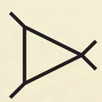
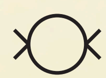
One-Loop Scattering Amplitudes

- n-particle Scattering: $1+2 \to 3+4+\ldots+n$
- Reduction to a Scalar-Integral Basis   Passarino-Veltman



$= \sum_{10^2-10^3} \int d^D\ell \, \frac{\ell^\mu \ell^\nu \ell^\rho \ldots}{D_1 D_2 \ldots D_n} = c_4$  $+\, c_3$  $+\, c_2$  $+\, c_1$ 

- Known: Master Integrals   [QCDLoop - AvH_OLO - GOLEM]

 $= \int d^D\ell \, \frac{1}{D_1 D_2 D_3 D_4}$,    $= \int d^D\ell \, \frac{1}{D_1 D_2 D_3}$,    $= \int d^D\ell \, \frac{1}{D_1 D_2}$,    $= \int d^D\ell \, \frac{1}{D_1}$

- Unknowns: c_i are rational functions of external kinematic invariants

Analytic Unitarity-Methods

- Important for Phenomenology
- Crossing path with Numerical Methods
- Important for understanding the structure of QFT

Process-Independent Strategy

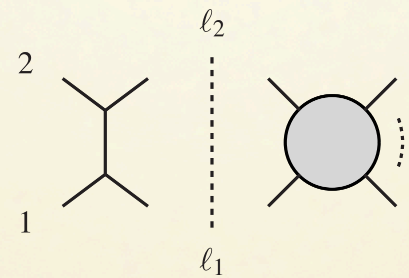
* Properties of the S-Matrix
  - a general mathematical property: **Analyticity** of Scattering-Amplitudes
    ▷ *Scattering Amplitudes are determined by their poles and branch-cuts*
  - a general physical property: **Unitarity** of Scattering-Amplitudes
    ▷ *The residues at poles and branch-points are products of simpler amplitudes, with lower number of particles and/or less loops*

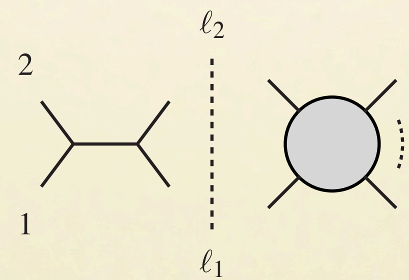
Cutting Rules

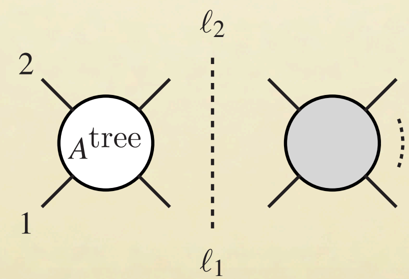
- Discontinuity of Feynman Integrals Landau & Cutkosky

Cut Integral in the P_{12}^2-channel

$$d^4\Phi = d^4\ell_1\, d^4\ell_2\ \delta^{(4)}\left(\ell_1+\ell_2-P_{12}\right)\ \delta^{(+)}\left(\ell_1^2\right)\delta^{(+)}\left(\ell_2^2\right)$$

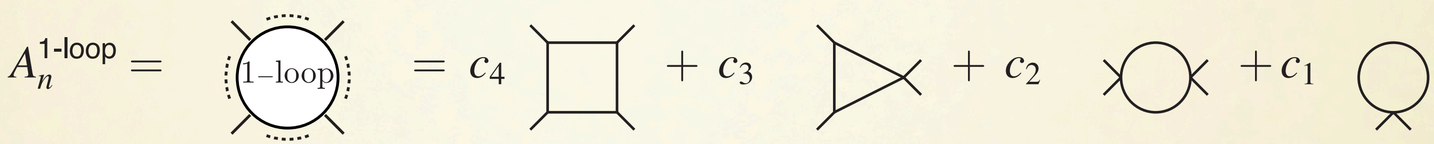
$\Delta(\Gamma_1) =$


$\Delta(\Gamma_2) =$


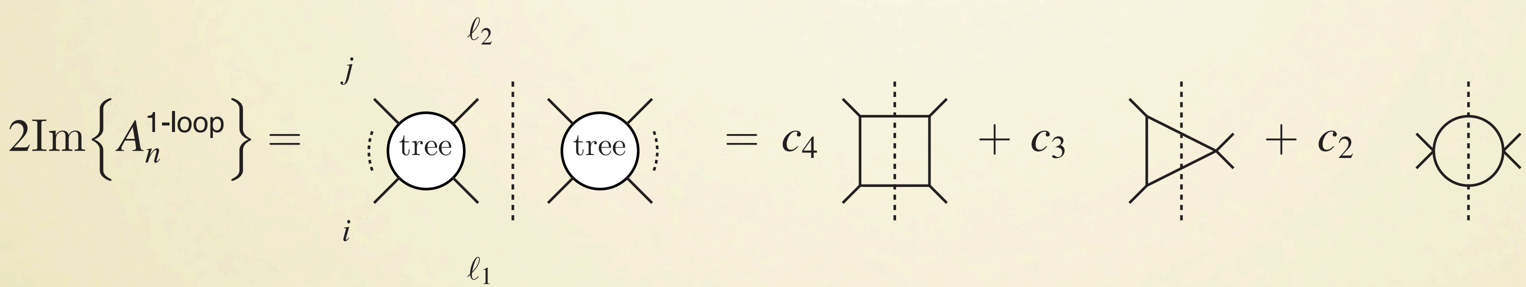
$\Delta(\Gamma_1)+\Delta(\Gamma_2) =$


Unitarity & Cutting Rules

- Optical Theorem from Unitarity $\quad S \equiv 1 + iT \;:\; S^{\dagger}S = 1 \quad \Rightarrow \quad 2\mathrm{Im}T = -i(T - T^{\dagger}) = T^{\dagger}T$
- One-loop Amplitude:



- Discontinuity of Feynman Amplitudes Cutkosky-Veltman; Bern, Dixon, Dunbar & Kosower



$$\text{on} - \text{shell condition :} \quad \frac{1}{(\ell_i^2 - m_i^2 + i0)} \rightarrow \delta\left(\ell_i^2 - m_i^2\right) \qquad (i = 1, 2)$$

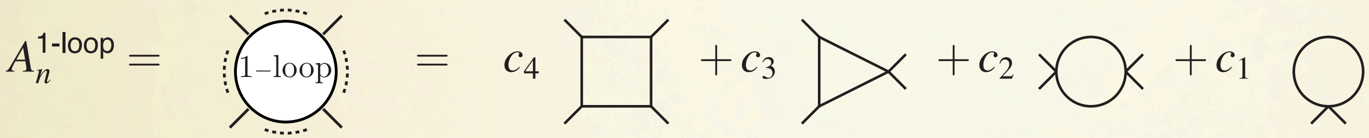
Method ▷ Matching the cuts of any amplitudes onto the cuts of Master Integrals

Advantage 1 ▷ iterative construction: one-loop amplitudes by sewing tree-level amplitudes

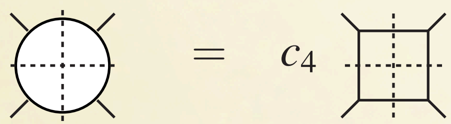
Advantage 2 ▷ simplified input: tree-amplitudes *vs* Feynman graphs
tree-amplitudes are gauge-invariant on-shell quantities,
corresponding to sums of off-shell Feynman diagrams.

The Strategy: Generalised Unitarity

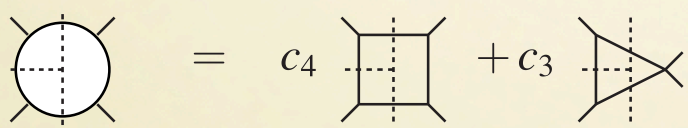
- One-loop Amplitude:



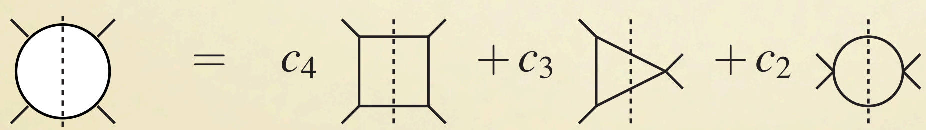
Replacing the original amplitude with simpler integrals fulfilling the same algebraic decomposition



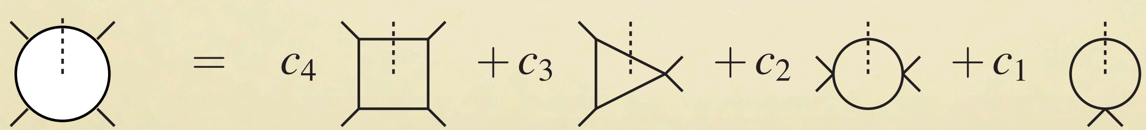
Britto, Cachazo, Feng



Bern, Dixon, Dunbar, Kosower
P.M.
Forde
Bjerrum-Bohr, Dunbar, Perkins



Bern, Dixon, Dunbar, Kosower
Brandhuber, McNamara, Spence, Travaglini
Britto, Buchbinder, Cachazo, Feng, $\oplus$ P.M.
Anastasiou, Britto, Feng, Kunszt, P.M.
Forde; Badger



Glover, Williams
Britto, Feng
Britto, Mirabella

Cut-Conditions

- Loop momentum decomposition

$$q^2 = p^2 = \varepsilon^{\pm 2} = 0 = \varepsilon^{\pm} \cdot p = \varepsilon^{\pm} \cdot q \,, \qquad \boxed{\ell_\mu = x_1\, p_\mu + x_2\, q_\mu + x_3\, \varepsilon^+_\mu + x_4\, \varepsilon^-_\mu}$$

- under Multiple On-shellness Conditions :

- the loop-momentum becomes complex ;
- some of its components (if not all) are frozen;
- the left over free components are *integration*-variable

- On-shell condition

$$\delta\left(\ell_i^2 - m_i^2\right)$$

Cut-Conditions

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- On-shell condition

$$\delta\left(\ell_i^2 - m_i^2\right)$$

- Closer look at the Integrand Structure

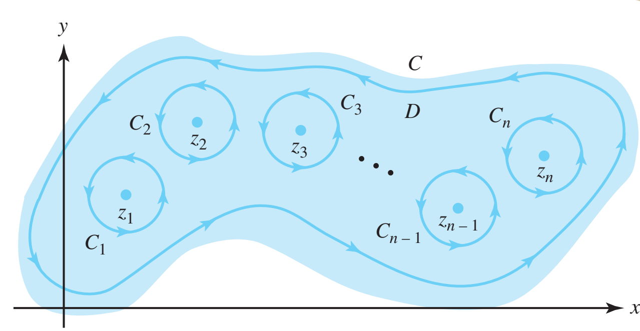
Numerator and denominator of the n-particle cut-integrand are mutivariate-polynomials in $(4-n)$ complex-variables:

$$\mathrm{Cut}_n = \oint dx_1 \ldots dx_{4-n}\ \frac{P(x_1,\ldots,x_{4-n})}{Q(x_1,\ldots,x_{4-n})}$$

▷ Contour Integrals of Rational Functions $\sim$ Integrals by *partial fractioning*

- Residue Theorem

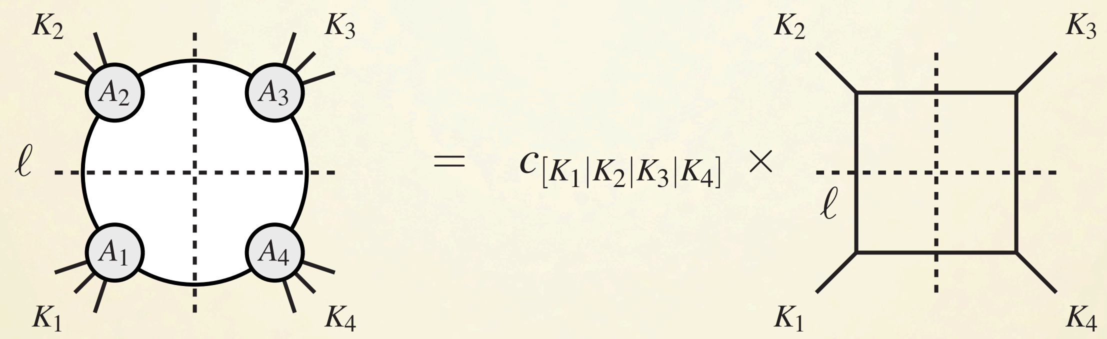
$$\frac{1}{2\pi i}\int_\gamma f(z)\, dz = \sum_{i=1}^{n} \mathrm{Res}(f, z_i)\ .$$



Quadruple-Cut

Britto, Cachazo, Feng (2004)

The discontinuity across the leading singularity, *via* quadruple cuts, is **unique**, and corresponds to the **coefficient** of the master **box**



- 4PLE-cut integrand: $I_4(\ell) = A_1^{\text{tree}} \times A_2^{\text{tree}} \times A_3^{\text{tree}} \times A_4^{\text{tree}}$
- Momentum-decomposition ansatz: $\boxed{\ell_\mu = \alpha_1\, p_\mu + \alpha_2\, q_\mu + \alpha_3 \frac{\langle q|\gamma_\mu|p]}{2} + \alpha_4 \frac{\langle p|\gamma_\mu|q]}{2}}$

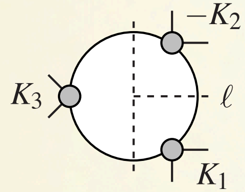
$$p^\mu = \frac{K_1^\mu - (K_1^2/\gamma)K_2^\mu}{1-(K_1^2K_2^2/\gamma)}\,, \qquad q^\mu = \frac{K_2^\mu - (K_2^2/\gamma)K_1^\mu}{1-(K_1^2K_2^2/\gamma)}\,, \qquad q^2 = p^2 = 0\,,$$

- Cut-conditions: $D_1 = D_2 = D_3 = D_4 = 0 \qquad \Leftrightarrow$ coefficient constraints
- Solutions: $\ell_\mu^\pm = \alpha_1\, p_\mu + \alpha_2\, q_\mu + \alpha_3^\pm \frac{\langle q|\gamma_\mu|p]}{2} + \alpha_4^\pm \frac{\langle p|\gamma_\mu|q]}{2}$

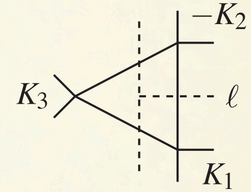
$$c_{[K_1|K_2|K_3|K_4]} = \frac{I_4(\ell_+) + I_4(\ell_-)}{2}$$

Triple-Cut

Forde (2008)



$$= c_{[K_1|K_2|K_3]} \times$$



- 3ple-cut integrand: $I_3(\ell) = A_1(\ell) \times A_2(\ell) \times A_3(\ell)$
- Loop-Momentum decomposition:

$$\boxed{\ell_\mu = \alpha_1\, p_\mu + \alpha_2\, q_\mu + t\, \frac{\langle q|\gamma_\mu|p]}{2} + \frac{\alpha_1\alpha_2}{t}\, \frac{\langle p|\gamma_\mu|q]}{2}}$$

$$p^\mu = \frac{K_1^\mu - (K_1^2/\gamma)K_2^\mu}{1-(K_1^2K_2^2/\gamma)}\,, \qquad q^\mu = \frac{K_2^\mu - (K_2^2/\gamma)K_1^\mu}{1-(K_1^2K_2^2/\gamma)}\,, \qquad q^2 = p^2 = 0\,,$$

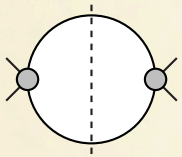
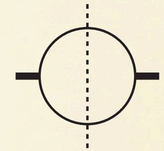
- Cut-conditions: $D_1 = D_2 = D_3 = 0$ $\Leftrightarrow$ coefficient constraints

$$\alpha_1 = \frac{K_1^2(\gamma - K_2^2)}{\gamma^2 - K_1^2K_2^2}\,, \qquad \alpha_2 = \frac{K_2^2(\gamma - K_1^2)}{\gamma^2 - K_1^2K_2^2}\,, \qquad \gamma = (K_1\cdot K_2) \pm \sqrt{\Delta}\,, \qquad \Delta = (K_1\cdot K_2)^2 + K_1^2K_2^2\,.$$

$$c_{[K_1,K_2,K_3]} = \frac{\mathrm{Res}_{t=0}\left\{I_3(\ell^+) + I_3(\ell^-)\right\}}{2} = \frac{\mathrm{Res}_{t=0}\, I_3(\ell^\pm) + \mathrm{Res}_{t=\infty} I_3(\ell^\pm)}{2}$$

Novel Double-Cut

P.M. (2009)

$\Delta = A_L$  $A_R = \displaystyle\int d^4\Phi\ A_L^{\rm tree}(\ell_1)\ A_R^{\rm tree}(\ell_1) \ = \ c_{[K]} \times K$ 

$$\int d^4\Phi = \int d^4\ell_1\ \delta^{(+)}(\ell_1^2 - m_1^2)\ \delta^{(+)}((\ell_1 - K)^2 - m_2^2)$$

- Change of Variables with *special* p and q :

$$p_\mu + q_\mu = K_\mu \ , \qquad \epsilon_+^2 = \epsilon_-^2 \ = \ 0 = \epsilon_\pm \cdot p = \epsilon_\pm \cdot q \ ,$$

$$p^2 = q^2 = 0 \ , \qquad 2\ \epsilon_+ \cdot \epsilon_- \ = \ -K^2 \ .$$

$$2\ p \cdot q = 2\ p \cdot K = 2\ q \cdot K \equiv K^2 \ ;$$

$$\rho = \frac{K^2 + m_1^2 - m_2^2 - \sqrt{\lambda(K^2, m_1^2, m_2^2)}}{2K^2}$$

$$\boxed{\ell_1^\mu = \frac{1-2\rho}{1+z\bar{z}}\left(p^\mu + z\bar{z}\ q^\mu + z\ \epsilon_+^\mu + \bar{z}\ \epsilon_-^\mu\right) + \rho K^\mu}$$

$$\lambda(K^2, m_1^2, m_2^2) \ = \ (K^2)^2 + (m_1^2)^2 + (m_2^2)^2 - 2K^2 m_1^2 - 2K^2 m_2^2 - 2m_1^2 m_2^2$$

massless case: $\rho = 0$

- Simplified parametrization of the Phase-Space

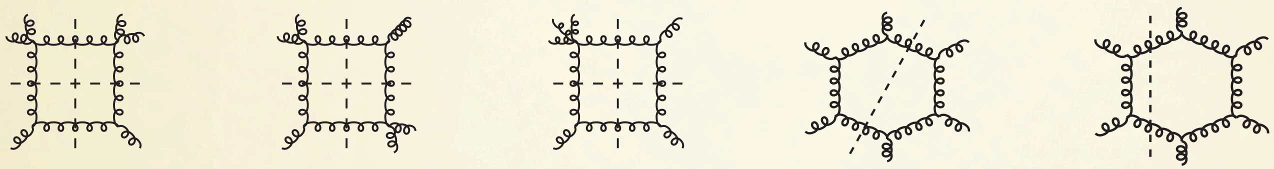
$$\boxed{\int d^4\Phi = (1-2\rho) \iint \frac{dz \wedge d\bar{z}}{(1+z\bar{z})^2}}$$

it is an integral over the complex tangent bundle of the Riemann Sphere

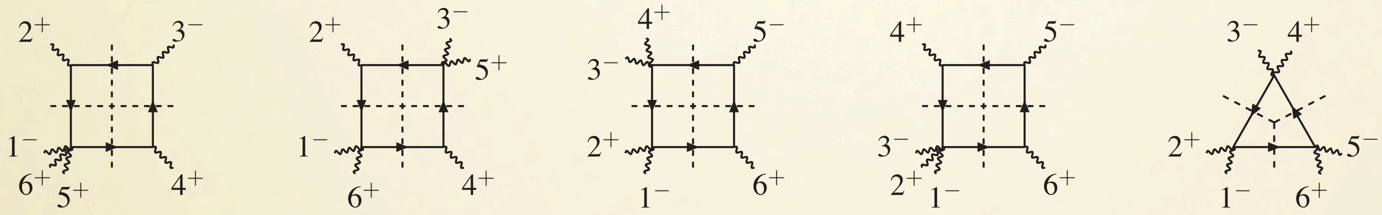
Generalised Cauchy Formula $\quad 2\pi i \mathcal{F}(z_0) = \displaystyle\int_{\partial D} \frac{\mathcal{F}(z)}{z - z_0} dz - \iint_D \frac{\mathcal{F}_{\bar{z}}}{z - z_0} d\bar{z} \wedge dz.$

Early Achievements

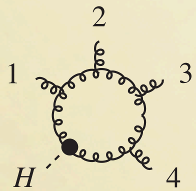
$gg \longrightarrow gggg$ Britto, Feng & P.M. (2006)

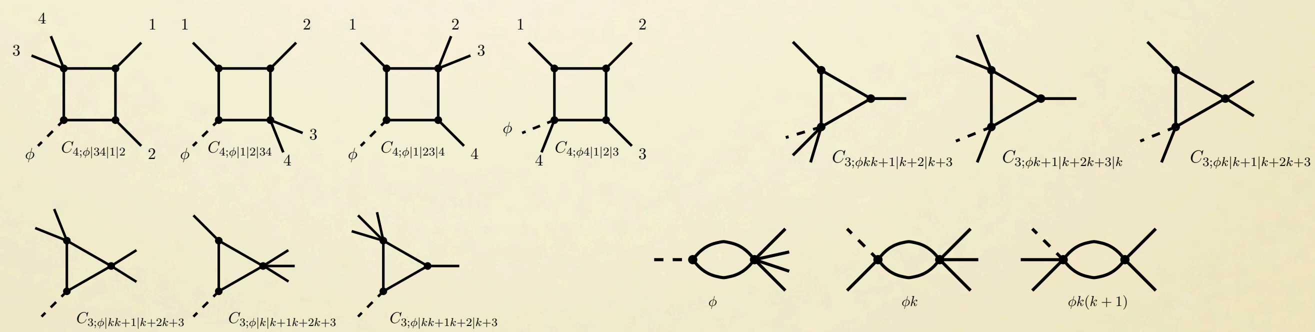


$\gamma\gamma \longrightarrow \gamma\gamma\gamma\gamma$ Binoth, Gehrmann, Heinrich & P.M. (2007)



$gg \longrightarrow Hgg$ Badger, Glover, Williams, P.M. (2008-2009)





ϕ plus four parton amplitudes at one-loop

The helicity amplitudes for $\phi + 4g$ have been calculated,

H amplitude	ϕ amplitude	$\phi^\dagger$ amplitude
$\mathcal{A}(H,+,+,+,+)$	$\mathcal{A}(\phi,+,+,+,+)$ (Berger,Del Duca, Dixon)	$\mathcal{A}(\phi^\dagger,+,+,+,+)$ (Badger,Glover)
$\mathcal{A}(H,-,+,+,+)$	$\mathcal{A}(\phi,-,+,+,+)$ (Berger,Del Duca, Dixon)	$\mathcal{A}(\phi^\dagger,-,+,+,+)$ (Badger,Glover,Mastrolia,CW)
$\mathcal{A}(H,-,-,+,+)$	$\mathcal{A}(\phi,-,-,+,+)$ (Badger,Glover,Risager)	$\mathcal{A}(\phi^\dagger,-,-,+,+)$ (Badger,Glover,Risager)
$\mathcal{A}(H,-,+,-,+)$	$\mathcal{A}(\phi,-,+,-,+)$ (Glover,Mastrolia,CW)	$\mathcal{A}(\phi^\dagger,-,+,-,+)$ (Glover,Mastrolia,CW)

Whilst those with a quark pair and two gluons have also been calculated
$(Q = 1_{\bar{q}}^-, q = 2_q^+)$

H amplitude	ϕ amplitude	$\phi^\dagger$ amplitude
$\mathcal{A}(H,Q,q,+,+)$	$\mathcal{A}(\phi,Q,q,+,+)$ (Berger,Del Duca, Dixon)	$\mathcal{A}(\phi^\dagger,Q,q,+,+)$ (Badger,Campbell,Ellis,CW)
$\mathcal{A}(H,Q,q,-,^-)$	$\mathcal{A}(\phi,Q,q,-,-)$ (Badger,Campbell,Ellis,CW)	$\mathcal{A}(\phi^\dagger,Q,q,-,-)$ (Berger,Del Duca, Dixon)
$\mathcal{A}(H,Q,q,+,-)$	$\mathcal{A}(\phi,Q,q,+,-)$ (Dixon, Sofiantaos)	$\mathcal{A}(\phi^\dagger,Q,q,+,-)$ (Dixon, Sofiantaos)
$\mathcal{A}(H,Q,q,-,+)$	$\mathcal{A}(\phi,Q,q,-,+)$ (Dixon, Sofiantaos)	$\mathcal{A}(\phi^\dagger,Q,q,-,+)$ (Dixon, Sofiantaos)

The $H(\phi)q\bar{q}Q\overline{Q}$ amplitudes have also been calculated, (Ellis, Giele, Zanderighi; Dixon, Sofiantaos)

Those marked in *red* are the most complicated *NMHV* helicity amplitudes and are the main topic of this talk.

http://mcfm.fnal.gov/

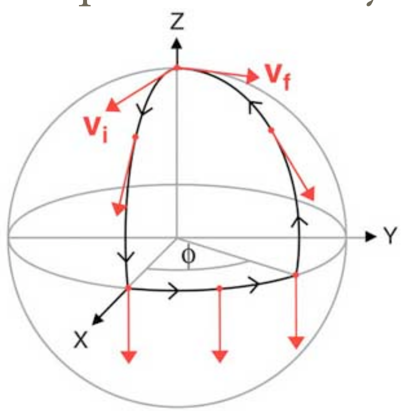
~10 ms/ps-point

Optical Theorem & Berry's Phase

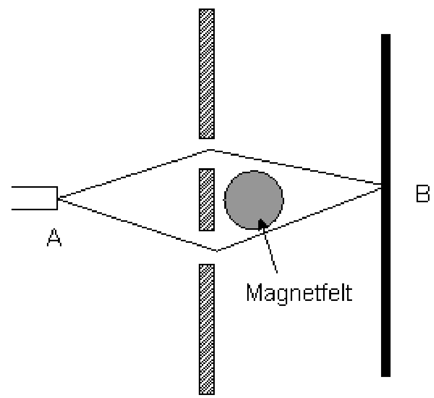
P.M. (2009)

- Geometric Phases

Simple Geometry



Aharonov-Bohm effect



$$\int_\Sigma \nabla \times \mathbf{F} \cdot d\mathbf{\Sigma} = \oint_{\partial\Sigma} \mathbf{F} \cdot d\mathbf{r},$$

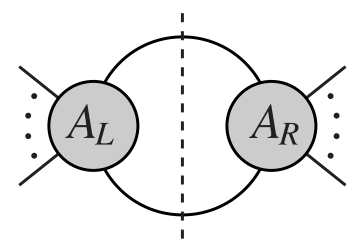
$$\nabla \times \mathbf{A} = \mathbf{B}$$

$$\varphi = \frac{q}{\hbar} \int_P \mathbf{A} \cdot d\mathbf{x},$$

Optical Theorem

$$\begin{aligned} \Delta &= \int d^4\Phi \; A^{*,\mathrm{tree}}_{m\to 2} \; A^{\mathrm{tree}}_{n\to 2} = \\ &= -i\Big[A^{\mathrm{one-loop}}_{n\to m} - A^{*,\mathrm{one-loop}}_{m\to n}\Big] = \\ &= 2\,\mathrm{Im}\Big\{A^{\mathrm{one-loop}}_{n\to m}\Big\} \, , \end{aligned}$$

$$\begin{aligned} \Delta &= (1-2\rho) \iint dz \wedge d\bar{z} \, \frac{A^{*,\mathrm{tree}}_{m\to 2} \; A^{\mathrm{tree}}_{n\to 2}}{(1+z\bar{z})^2} = \\ &= (1-2\rho) \oint dz \int d\bar{z} \, \frac{A^{*,\mathrm{tree}}_{m\to 2} \; A^{\mathrm{tree}}_{n\to 2}}{(1+z\bar{z})^2} \, , \end{aligned}$$



The double-cut is the flux of a 2-form.
The anholonomy phase shift is a consequence of Stokes' Theorem.

Cauchy's Residue Theorem @ Work

Britto, Cachazo, Feng

Boxes

Britto, Cachazo, Feng, Witten

Trees

R.T.

(holomorphic)

Forde

Triangles

Global R.T.

(multivariate holomorphic)

Generalised R.T.

(non-holomorphic)

Higher Loop

Leading Singularities

Arkani-Hamed, Cachazo, Cheung, Kaplan

Tadpoles ???

(Gauss' Theorem)

Bubbles

(Stokes' Theorem)

P.M.

Seminumerical Implementation of Unitarity-based Methods

- CutTools
  [Ossola, Papadopoulos, Pittau]
  - $pp \to tT + 2\text{jets}$ [Bevilacqua, Czakon, Papadopoulos, Worek]

Rat. Term:
Effective-Tree Rules

- BlackHat
  [Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre, Gleisberg]
  - $pp \to W + 3\text{jets}$
  - $pp \to Z + 3\text{jets}$
  - $pp \to W + 4\text{jets}$

Rat. Term:
D-dim Unit. + on-shell

- Rocket
  [Giele, Zanderighi]
  - $pp \to W + 3\text{jets}$ [Ellis, Zanderighi, Melnikov]
  - $e^+e^- \to 5\text{jets}$ [Frederix, Frixione, Melnikov, Zanderighi]

Rat. Term:
D-dim Unitarity

- SAMURAI
  [Ossola, Reiter, Tramontano, P.M.]
  - $pp \to bBbB$ (quark-initiated) [Binoth, Greiner, Guffanti, Reuter, Guillet, T. Reiter]

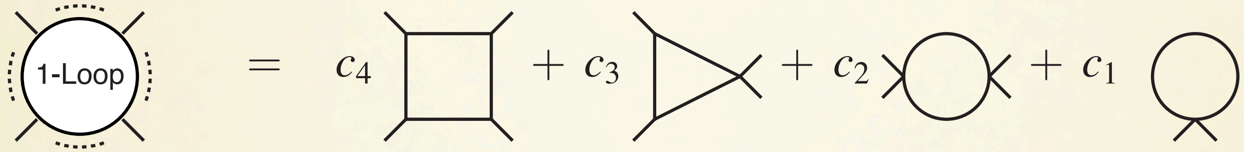
SAMURAI

Scattering AMplitudes from Unitarity-based Reduction Algorithm at the Integrand-level

Ossola, Reiter, Tramontano, & P.M. (2010)

At the Integrand Level

- Reduction to a Scalar-Integral Basis   Passarino-Veltman

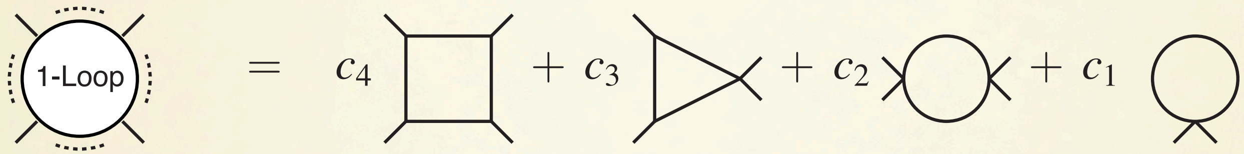


$$\int d^4q\, A(q) \quad = \quad c_4 \int \frac{d^4q}{D_0 D_1 D_2 D_3} + c_3 \int \frac{d^4q}{D_0 D_1 D_2} + c_2 \int \frac{d^4q}{D_0 D_1} + c_1 \int \frac{d^4q}{D_0}$$

- Unknowns: c_i are rational functions of external kinematic invariants

At the Integrand Level

- Reduction to a Scalar-Integral Basis   Passarino-Veltman



$$\int d^4q\, A(q) \quad = \quad c_4 \int \frac{d^4q}{D_0 D_1 D_2 D_3} + c_3 \int \frac{d^4q}{D_0 D_1 D_2} + c_2 \int \frac{d^4q}{D_0 D_1} + c_1 \int \frac{d^4q}{D_0}$$

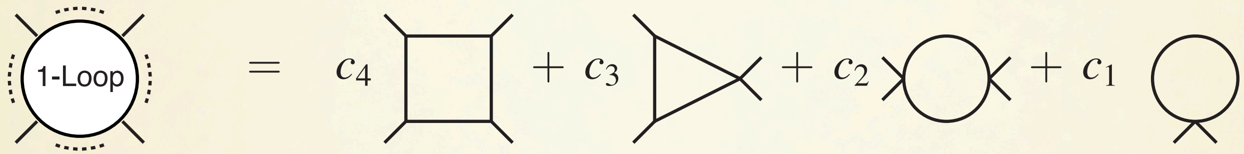
- Unknowns: c_i are rational functions of external kinematic invariants

- At the Integrand-level

$$A(q) \quad \neq \quad \frac{c_4}{D_0 D_1 D_2 D_3} + \frac{c_3}{D_0 D_1 D_2} + \frac{c_2}{D_0 D_1} + \frac{c_1}{D_0}$$

At the Integrand Level

- Reduction to a Scalar-Integral Basis   Passarino-Veltman



$$\int d^4q\, A(q) \;=\; c_4 \int \frac{d^4q}{D_0 D_1 D_2 D_3} + c_3 \int \frac{d^4q}{D_0 D_1 D_2} + c_2 \int \frac{d^4q}{D_0 D_1} + c_1 \int \frac{d^4q}{D_0}$$

- Unknowns: c_i are rational functions of external kinematic invariants

- At the Integrand-level

$$A(q) \;\neq\; \frac{c_4}{D_0 D_1 D_2 D_3} + \frac{c_3}{D_0 D_1 D_2} + \frac{c_2}{D_0 D_1} + \frac{c_1}{D_0}$$

$$= \frac{c_4 + f_4(q)}{D_0 D_1 D_2 D_3} + \frac{c_3 + f_3(q)}{D_0 D_1 D_2} + \frac{c_2 + f_2(q)}{D_0 D_1} + \frac{c_1 + f_1(q)}{D_0}$$

$$\int d^4q\, \frac{f_4(q)}{D_0 D_1 D_2 D_3} = \int d^4q\, \frac{f_3(q)}{D_0 D_1 D_2} = \int d^4q\, \frac{f_2(q)}{D_0 D_1} = \int d^4q\, \frac{f_1(q)}{D_0} = 0$$

$$A(q) \;\equiv\; \frac{\Delta_{0123}(q)}{D_0 D_1 D_2 D_3} + \frac{\Delta_{012}(q)}{D_0 D_1 D_2} + \frac{\Delta_{01}(q)}{D_0 D_1} + \frac{\Delta_0(q)}{D_0}$$

OPP-Integrand Reduction (in a nutshell)

Ossola, Papadopoulos, Pittau

Ellis, Giele, Kunszt

Giele, Kunszt, Melnikov

- OPP-decomposition

$$A_m = \int d^4q \frac{N(q)}{D_0 \ldots D_{m-1}}$$

$$\begin{aligned} N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \Delta_{i_0 i_1 i_2 i_3}(q) \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i \\ &+ \sum_{i_0<i_1<i_2}^{m-1} \Delta i_0 i_1 i_2(q) \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\ &+ \sum_{i_0<i_1}^{m-1} \Delta i_0 i_1(q) \prod_{i\neq i_0,i_1}^{m-1} D_i \\ &+ \sum_{i_0}^{m-1} \Delta i_0(q) \prod_{i\neq i_0}^{m-1} D_i \end{aligned}$$

- $\Delta(q)$ are known polynomials
- c_i are the constant terms of Δ's

▷ Fitting c_i by numerical evaluating $N(q)$ at different values of $q \oplus$ system inversion

- q @ Quadruple-cut: $D_{i_0} = D_{i_1} = D_{i_2} = D_{i_3} = 0$

$$N(q) \quad = \quad \Delta_{i_0 i_1 i_2 i_3}(q) \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i$$

- q @ Triple-cut: $D_{i_0} = D_{i_1} = D_{i_2} = 0$

$$
\begin{aligned}
N(q) \quad & - \quad \sum_{i_0<i_1<i_2<i_3}^{m-1} \Delta_{i_0 i_1 i_2 i_3}(q) \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
& = \quad \Delta_{i_0 i_1 i_2}(q) \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i
\end{aligned}
$$

- q @ Double-cut: $D_{i_0} = D_{i_1} = 0$

$$
\begin{aligned}
N(q) & - \sum_{i_0<i_1<i_2<i_3}^{m-1} \Delta_{i_0 i_1 i_2 i_3}(q) \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
& - \sum_{i_0<i_1<i_2}^{m-1} \Delta_{i_0 i_1 i_2}(q) \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
& = \Delta_{i_0 i_1}(q) \prod_{i \neq i_0, i_1}^{m-1} D_i
\end{aligned}
$$

- q @ Single-cut: $D_{i_0} = 0$

$$
\begin{aligned}
N(q) &- \sum_{i_0<i_1<i_2<i_3}^{m-1} \Delta_{i_0 i_1 i_2 i_3}(q) \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&- \sum_{i_0<i_1<i_2}^{m-1} \Delta_{i_0 i_1 i_2}(q) \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&- \sum_{i_0<i_1}^{m-1} \Delta_{i_0 i_1}(q) \prod_{i \neq i_0, i_1}^{m-1} D_i \\
&= \Delta_{i_0}(q)
\end{aligned}
$$

- OPP-reduction Ossola, Papadopoulos, Pittau (2006)

From the knowledge of the multi-variate polynomial-structure of the Integrand, all n-point coefficients can be determined by fitting a system of polynomial equations.

Advantage ▷ No integration required

Pitfall ▷ Numerical System Inversion $(\Delta \to 0)$

- Improved Reduction with DFT Ossola, Papadopoulos, Pittau, & P.M. (2008)

$$P_m(x) = c_0 + c_1 x + c_2 x^2 + \ldots \; c_m x^m$$

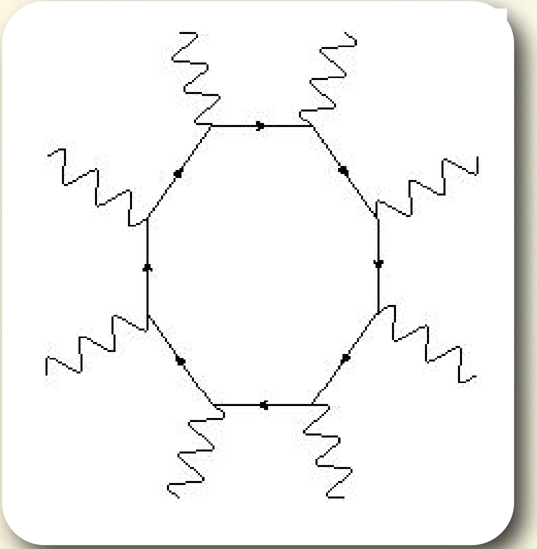
▷ step 1: sample $P_m(x)$ at $(m+1)$ equidistant-points on the unit-circle, $P_{m,k} \equiv P_m(x_k)$,

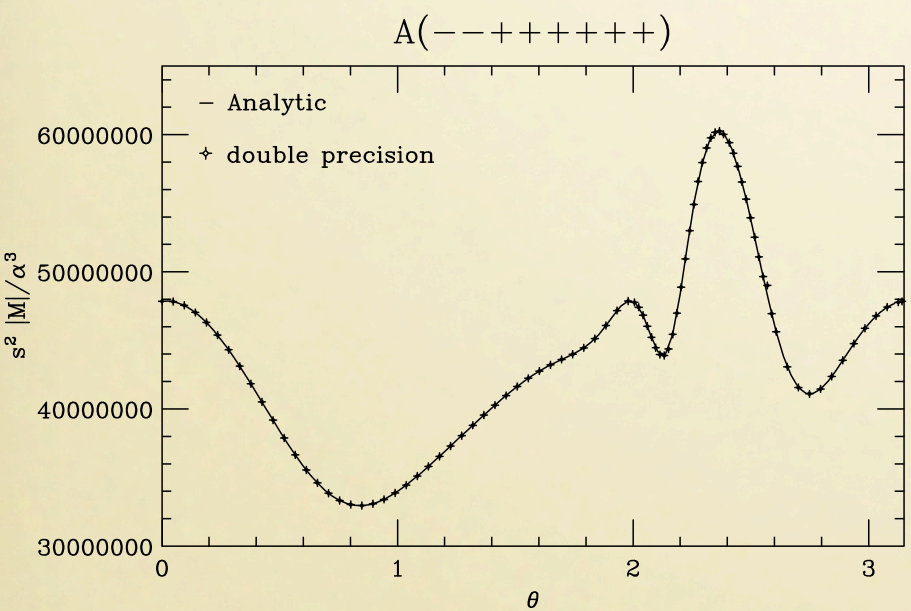
$$x_k = \; e^{-2\pi i \frac{k}{(m+1)}} \qquad (k = 0, \ldots, m) \; .$$

▷ step 2: find c_i from orthogonality (plane-waves):

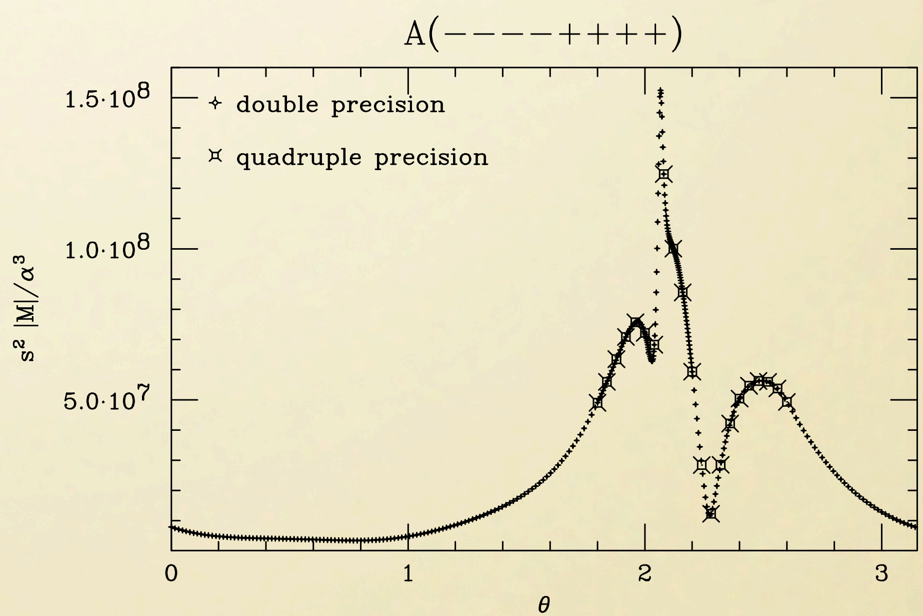
$$c_\ell = \frac{1}{m+1} \sum_{k=0}^{m} P_{m,k} \; e^{2\pi i \frac{k}{(m+1)} \ell}$$

8-Photon in QED



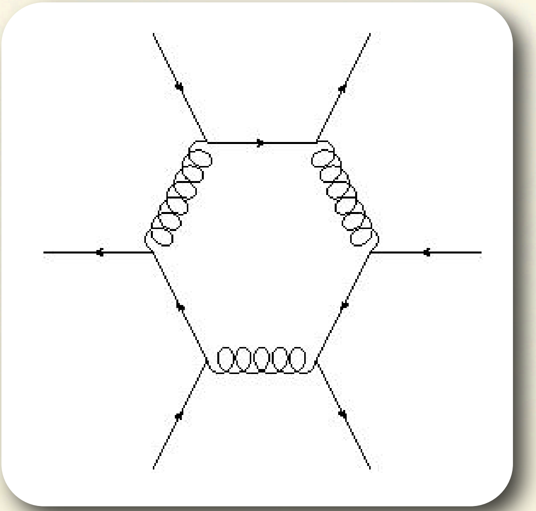


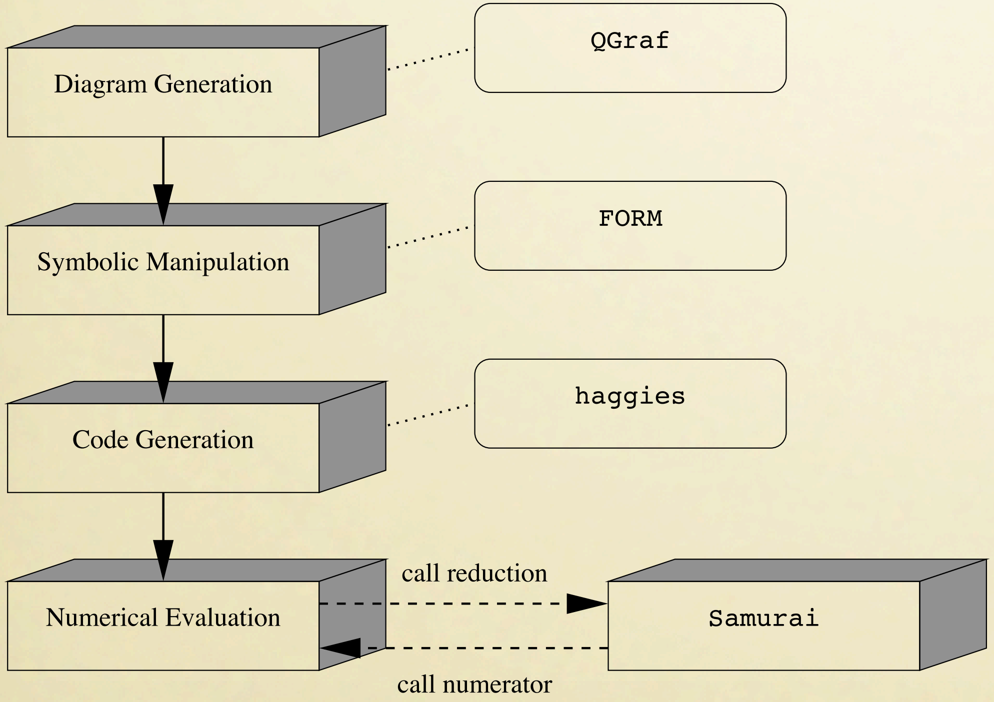
reproducing Mahlon, (1993)

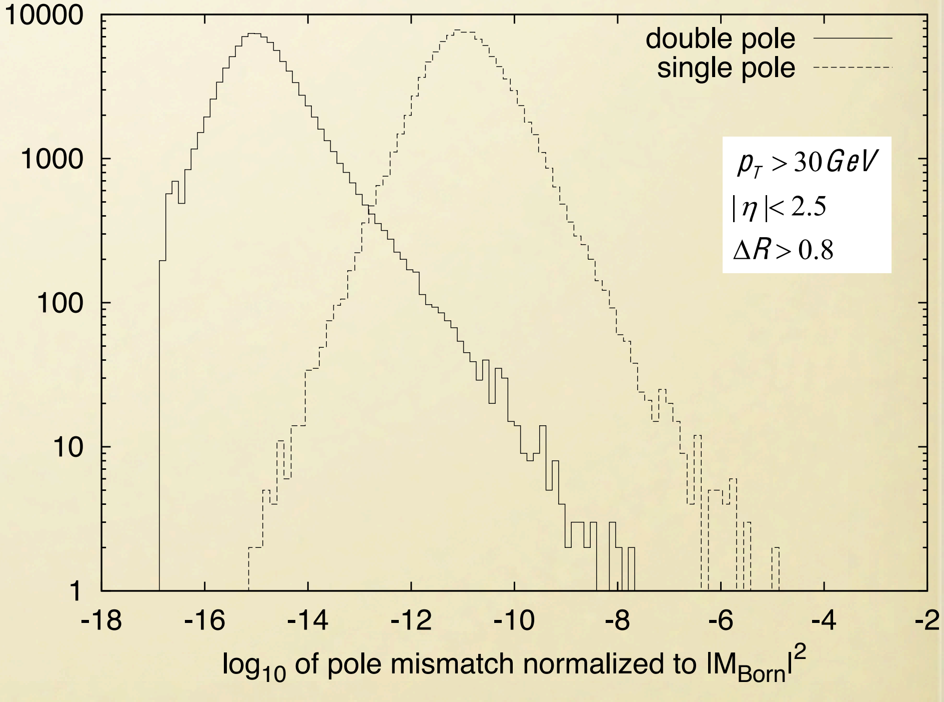


NEW, confirming Badger et al. (2009)

6-Quark in QCD





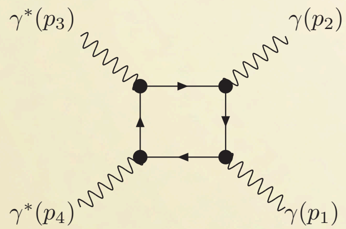


Tensorial Decomposition

Heinrich, Ossola, Reiter, Tramontano

- Dealing with unstable-points
- Tensor Decomposition of N(q): numerical sampling
- Numerical evaluation of Tensor integrals: Golem 95

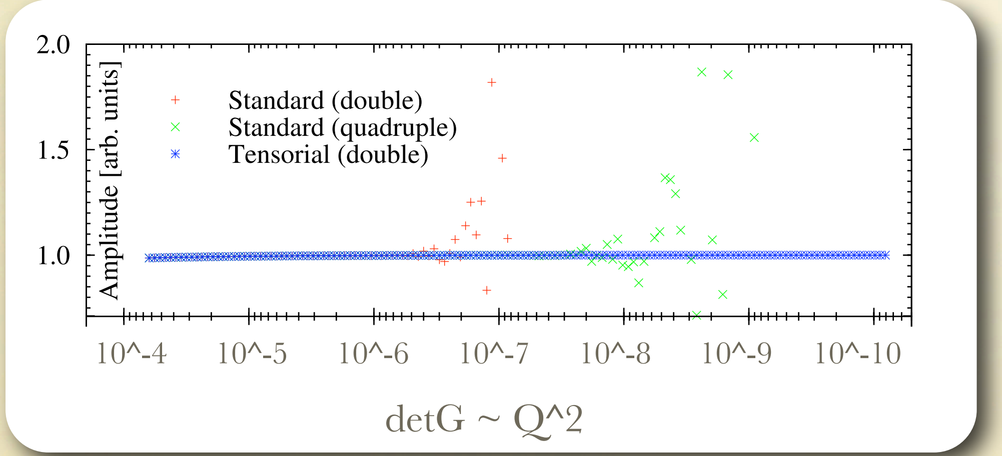
$$\mathcal{N}(q) = \sum_{r=0}^{R} C_{\mu_1 \ldots \mu_r} q_{\mu_1} \ldots q_{\mu_r}$$



$$p_{1,2} = (E, 0, 0, \pm E) \qquad p_{1,2}^2 = 0$$
$$p_{3,4} = (E, 0, \pm Q \sin\theta, \pm Q \cos\theta) \qquad p_{3,4}^2 = m^2$$

vanishing Gram determinant

$$Q \to 0$$



Summary: Unitarity-based Methods

Light-bulbs were not invented by improving candles

T.W. Haensch

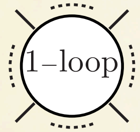
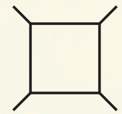
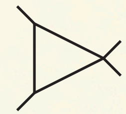
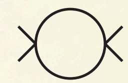
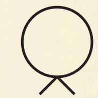
- Jetty backgrounds at LHC demand quantitative control
- NLO QCD results are mandatory
- Novel methods for One-Loop Amplitudes based on Unitarity and Analiticity are giving results

- Analytic Integration and Theory of Complex Functions
- Integration by Series Expansion (Newton's legacy)
- relevant for LHC-Phenomenology: H+2jets in gluon fusion (heavy-top limit)
- relevant for unveiling hidden structures of QFT

- Seminumerical tool for NLO: CutTools, BlackHat, Rocket, ...
- SAMURAI: working within Diagrammatic & Unitarity-based approaches
- applied to: 4-, 6-, 8-photon; Drell-Yan; V+1jet; 5-, 6-gluon; 6-quark; W+2jets prod.
- public Fortran library:

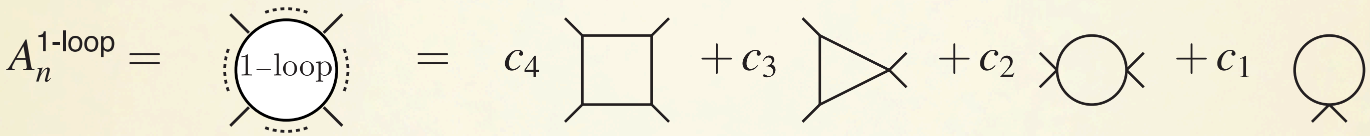
`http://cern.ch/samurai`

Conclusions

$A_n^{\text{1-loop}} =$  $= c_4$  $+ c_3$  $+ c_2$  $+ c_1$ 

- ☑ Improved Tensor Reduction
- ☑ Numerical Unitarity
- ☑ Analytic Unitarity

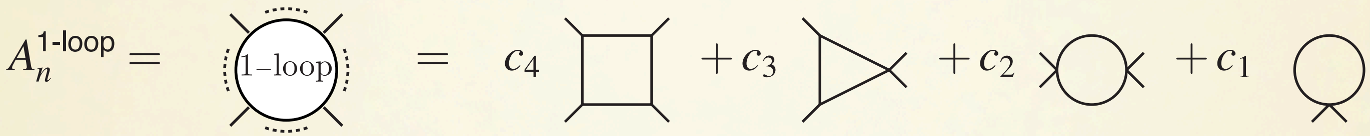
Conclusions



- [x] Feynman Diagrams
- [x] Currents
- [x] Amplitudes

- [x] Improved Tensor Reduction
- [x] Numerical Unitarity
- [x] Analytic Unitarity

Conclusions



- ☑ Feynman Diagrams
- ☑ Currents
- ☑ Amplitudes

- ☑ Improved Tensor Reduction
- ☑ Numerical Unitarity
- ☑ Analytic Unitarity

One-Loop amplitudes...

...out of bottleneck !?!