



UNIVERSITAT DE BARCELONA



Institut de Ciències del Cosmos

One-loop Higgs boson production at the Linear Collider e^+e^- versus $\gamma\gamma$

Joan Solà

sola@ecm.ub.es

HEP Group

Departament d'Estructura i Constituents de la Matèria
Institut de Ciències del Cosmos, Univ. Barcelona

LC10, Lab. Nazionali di Frascati, Nov. 30th-Dec. 3rd 2010

Guidelines of the Talk

- SM, and Physics beyond the SM
- Higgs bosons in the MSSM and the 2HDM
- Higgs boson self interactions
- Quantum effects in $e^+e^- \rightarrow$ Higgs bosons
a window to 2HDM physics?
- Single 2HDM Higgs production in $\gamma\gamma$ -collisions
- Conclusions

Works and collaborators:

Recent works of us on Higgs physics in the Linear Collider:

- D.López-Val, JS, N. Bernal,
Quantum effects on Higgs-strahlung events at
Linear Colliders within the general 2HDM.,
Phys. Rev. D81 (2010) 113005,
arXiv:1003.4312
- D.López-Val, JS, Neutral Higgs-pair production
at Linear Colliders within the general 2HDM: quan-
tum effects and triple Higgs boson self-interactions
Phys. Rev. D81 (2010) 033003,
arXiv:0908.2898
- N. Bernal, D.López-Val, JS
Single Higgs-boson production
through $\gamma\gamma$ -scattering within the general 2HDM,
Phys. Lett. B677 (2009) 39,
arXiv:0901.2257
- R. N. Hodgkinson, D.López-Val, JS
Higgs boson pair production through gauge bo-
son fusion at linear colliders within the general
2HDM.,
Phys. Lett. B673 (2009) 47,
arXiv:0903.4978
- G. Ferrera, J. Guasch, D.López-Val, JS
Triple Higgs boson production in the Linear Col-
lider., Phys.Lett. B659 (2008) 297,
arXiv:0801.3907.

Standard Model

principle of local gauge invariance



symmetry group $SU(2) \times U(1) \times SU(3)_C$



Higgs mechanism and Yukawa interactions
→ masses $M_W, M_Z, m_{\text{fermion}}$

SM { renormalizable quantum field theory
 ↓
 accurate theoretical predictions

detect deviations → “new physics” ?

Theory versus Data

experimental results

New physics?...

$$M_Z \text{ [GeV]} = 91.1875 \pm 0.0021 \quad 0.002\%$$

$$\Gamma_Z \text{ [GeV]} = 2.4952 \pm 0.0023 \quad 0.09\%$$

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23148 \pm 0.00017 \quad 0.07\%$$

$$M_W \text{ [GeV]} = 80.392 \pm 0.029 \quad 0.04\%$$

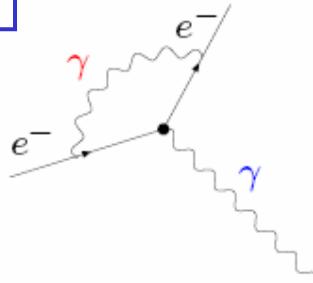
$$m_t \text{ [GeV]} = 170.9 \pm 1.8 \quad 1.05\%$$

$$G_F \text{ [GeV}^{-2}] = 1.16637(1)10^{-5} \quad 0.001\%$$

Precision Physics in the SM

QED:

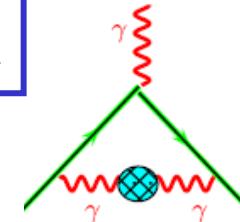
$g - 2:$



$$a = \frac{1}{2}(g - 2)$$

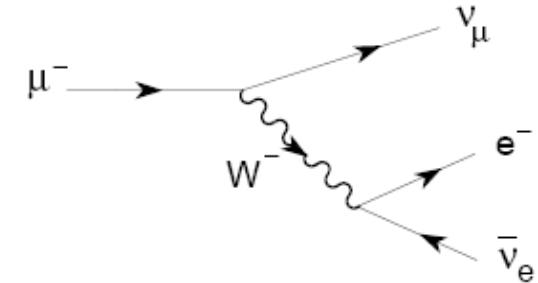
$$a_{\text{exp}} = 1159652188(\pm 4) \times 10^{-12}$$

$$a_{\text{theo}} = 1159652157(\pm 28) \times 10^{-12}$$



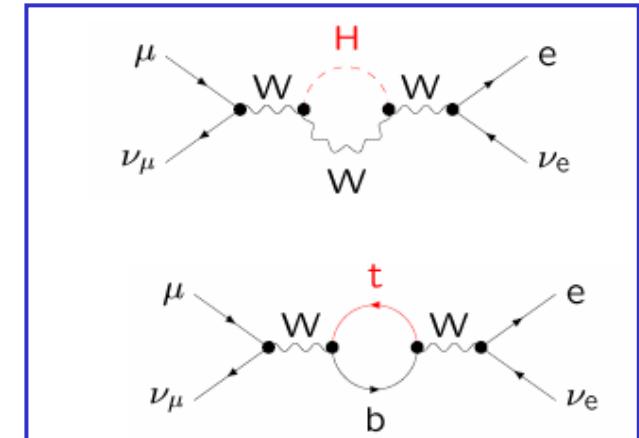
EW:

$$\left\{ \begin{array}{l} G_F \\ M_Z, \Gamma_Z, g_V, g_A, \sin^2 \theta_{\text{eff}}, \dots \\ M_W, m_t \end{array} \right.$$



$$\frac{G_F}{\sqrt{2}} = \frac{\pi \alpha}{M_W^2 (1 - M_W^2/M_Z^2)} (1 + \Delta r)$$

$$\Delta r = \Delta r(m_t, M_H)$$



➤ Higgs bosons in the general 2HDM

- It is the minimal extension of the SM which adds new phenomena – e.g. H^\pm
- It can easily embody new sources of CP violation
- It provides a useful setup for building models of Baryogenesis
- Extended Higgs sectors based on $SU_L(2)$ -Doublet structure easily circumvent the major phenomenological constraints.
-
-
-
- A Two-Higgs Doublet structure arises naturally as a low-energy realization of some more fundamental theories (e.g. in SUSY).

Possible scenarios

Direct search: $M_H > 114$ GeV

Fits to precision data: $M_H \lesssim 180$ GeV

(95% GeV)

- a single light Higgs boson
 - SM Higgs boson?
 - SUSY light Higgs boson?
 H, A, H^\pm heavy (decoupling scenario) $h \sim H_{\text{SM}}$
- a light Higgs boson + more (H, A, H^\pm)
 - SUSY Higgs?
 - non-SUSY 2-Higgs-Doublet model?
- a single heavy Higgs boson ($\gg 200$ GeV)
 - SUSY ruled out in its minimal (MSSM) formulation
 - SM + (?) strong interaction? ($M_h^{\text{MSSM}} < 140$ GeV)
- no Higgs boson
 - strongly interacting weak interaction
new strong force \sim TeV scale

Higgs bosons in the generic 2HDM

2HDM \mathcal{CP} -conserving, gauge invariant, renormalizable potential

$$\begin{aligned}
 V(\Phi_1, \Phi_2) = & \lambda_1 (\Phi_1^\dagger \Phi_1 - v_1^2)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2 - v_2^2)^2 + \lambda_3 \left[(\Phi_1^\dagger \Phi_1 - v_1^2) + (\Phi_2^\dagger \Phi_2 - v_2^2) \right]^2 \\
 & + \lambda_4 \left[(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \right] + \lambda_5 \left[\text{Re}(\Phi_1^\dagger \Phi_2) - v_1 v_2 \right]^2 + \lambda_6 \left[\text{Im}(\Phi_1^\dagger \Phi_2) \right]^2 \\
 \Phi_1 = & \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix} \quad (Y = +1), \quad \Phi_2 = \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix} \quad (Y = +1)
 \end{aligned}$$

MSSM

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \equiv \epsilon \Phi_1^* = \begin{pmatrix} \Phi_1^{0*} \\ -\Phi_1^- \end{pmatrix} \quad (Y = -1) \quad \epsilon = i \sigma_2$$

MSSM with soft-breaking terms. **SUSY** highly restricts the potential:

$$\begin{aligned}
 V_H = & (|\mu|^2 + m_{H_1}^2)|H_1|^2 + (|\mu|^2 + m_{H_2}^2)|H_2|^2 - \mu B \epsilon_{ij} (H_1^i H_2^j + \text{h.c.}) \\
 & + \frac{g_2^2 + g_1^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g_2^2 |H_1^\dagger H_2|^2
 \end{aligned}$$

only gauge self-couplings !!

Physical parameters and fields in the 2HDM

λ_i , ($i = 1 \dots 6$) dimensionless real parameters and $v_{1,2} = \langle \Phi_{1,2}^0 \rangle$

$$\left(v_1^2 + v_2^2 = \frac{G_F^{-1}}{2\sqrt{2}} = 2M_W^2/g^2 \right)$$

$$(M_{h^0}, M_{H^0}, M_{A^0}, M_{H^\pm}, \alpha, \tan \beta, \lambda_5)$$



$$\mathcal{M}^{\text{CP}+} = \begin{pmatrix} 4v_1^2(\lambda_1 + \lambda_3) + v_2^2\lambda_5 & (4\lambda_3 + \lambda_5)v_1v_2 \\ (4\lambda_3 + \lambda_5)v_1v_2 & 4v_1^2(\lambda_2 + \lambda_3) + v_1^2\lambda_5 \end{pmatrix}$$

CP-even sector $\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \text{Re}(\Phi_1^0) - v_1 \\ \text{Re}(\Phi_2^0) - v_2 \end{pmatrix}$

CP-odd $\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \text{Im}(\Phi_1^0) \\ \text{Im}(\Phi_2^0) \end{pmatrix}$

charged $\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \Phi_1^+ \\ \Phi_2^+ \end{pmatrix}$ G^0, G^\pm
Goldstone bosons

➤ Higgs couplings in more general 2HDM's

- Extended Higgs sector \Rightarrow large source of new quantum effects and also of larger Higgs boson production cross-sections
- However, one has to be careful with tree-level **Natural FC+CP** FCNC \Rightarrow **two models or types of 2HDM:** Glashow-Weinberg-Paschos Theorem (1977)
 - In **type I** 2HDM, Φ_2 couples to all the SM fermions; Φ_1 does not couple to them at all $\bar{Q}_L [\tilde{\Phi}_2 U_R, \Phi_2 D_R]$
 - In **type II** 2HDM, Φ_2 couples only to up-like quarks; Φ_1 couples to down-like only.

	type I	type II
$h^0 t\bar{t}$	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$
$h^0 b\bar{b}$	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$
$H^0 t\bar{t}$	$\sin \alpha / \sin \beta$	$\sin \alpha / \sin \beta$
$H^0 b\bar{b}$	$\sin \alpha / \sin \beta$	$\cos \alpha / \cos \beta$
$A^0 t\bar{t}$	$\cot \beta$	$\cot \beta$
$A^0 b\bar{b}$	$-\cot \beta$	$\tan \beta$

Yukawa couplings
2HDM

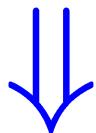
$$\begin{aligned} & \bar{Q}_L \tilde{\Phi}_2 U_R \\ & + \\ & \bar{Q}_L \Phi_1 D_R \\ & \times \left(-\frac{g m_f}{2 M_W} \right) \end{aligned}$$

SM

➤ Higgs bosons self-couplings in the general 2HDM

After **SSB**:

$$(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, v)$$



$$(M_{h^0}, M_{H^0}, M_{A^0}, M_{H^\pm}, \alpha, \tan\beta, \lambda_5)$$

λ_5 is a free parameter
that remains unrelated
to any physical mass or
mixing angle!

$$\lambda_1 = \frac{\lambda_5 (1 - \tan^2 \beta)}{4} + \frac{\alpha_{em} \pi}{2 M_W^2 s_W^2 \cos^2 \beta}$$

$$\times [M_{H^0}^2 \cos^2 \alpha + M_{h^0}^2 \sin^2 \alpha]$$

$$- \frac{1}{2} (M_{H^0}^2 - M_{h^0}^2) \sin 2\alpha \cot \beta \Big] ,$$

$$\lambda_2 = \frac{\lambda_5 (1 - 1/\tan^2 \beta)}{4} + \frac{\alpha_{em} \pi}{2 M_W^2 s_W^2 \sin^2 \beta}$$

$$\times [M_{h^0}^2 \cos^2 \alpha + M_{H^0}^2 \sin^2 \alpha]$$

$$- \frac{1}{2} (M_{H^0}^2 - M_{h^0}^2) \sin 2\alpha \tan \beta \Big] ,$$

$$\lambda_3 = - \frac{\lambda_5}{4} + \frac{\alpha_{em} \pi \sin 2\alpha}{2 M_W^2 s_W^2 \sin 2\beta} (M_{H^0}^2 - M_{h^0}^2)$$

$$\lambda_4 = \frac{2 \alpha_{em} \pi}{M_W^2 s_W^2} M_{H^\pm}^2 ,$$

$$\lambda_6 = \frac{2 \alpha_{em} \pi}{M_W^2 s_W^2} M_{A^0}^2 ,$$

➤ Triple Higgs self-couplings

$h^0 h^0 h^0$	$-\frac{3ie}{2M_W \sin 2\beta s_W} \left[M_{h^0}^2 (2 \cos(\alpha + \beta) + \sin 2\alpha \sin(\beta - \alpha)) - \cos(\alpha + \beta) \cos^2(\beta - \alpha) \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right]$
$h^0 h^0 H^0$	$-\frac{ie \cos(\beta - \alpha)}{2M_W \sin 2\beta s_W} \left[(2M_{h^0}^2 + M_{H^0}^2) \sin 2\alpha - (3 \sin 2\alpha - \sin 2\beta) \frac{2\lambda_5 M_W^2 s_W^2}{e^2} \right]$
$h^0 H^0 H^0$	$\frac{ie \sin(\beta - \alpha)}{2M_W \sin 2\beta s_W} \left[(M_{h^0}^2 + 2M_{H^0}^2) \sin 2\alpha - (3 \sin 2\alpha + \sin 2\beta) s_W^2 \frac{2\lambda_5 M_W^2}{e^2} \right]$
$H^0 H^0 H^0$	$-\frac{3ie}{2M_W \sin 2\beta s_W} \left[M_{H^0}^2 (2 \sin(\alpha + \beta) - \cos(\beta - \alpha) \sin 2\alpha) - \sin(\alpha + \beta) \sin^2(\beta - \alpha) s_W^2 \frac{4\lambda_5 M_W^2}{e^2} \right]$
$h^0 A^0 A^0$	$-\frac{ie}{2M_W s_W} \left[\frac{\cos(\alpha + \beta)}{\sin 2\beta} \left(2M_{h^0}^2 - \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right) - \sin(\beta - \alpha) (M_{h^0}^2 - 2M_{A^0}^2) \right]$

➤ ... more trilinear couplings

$h^0 A^0 G^0$	$\frac{ie}{2M_W s_W} (M_{A^0}^2 - M_{h^0}^2) \cos(\beta - \alpha)$
$h^0 G^0 G^0$	$-\frac{ie}{2M_W s_W} M_{h^0}^2 \sin(\beta - \alpha)$
$H^0 A^0 A^0$	$-\frac{ie}{2M_W s_W} \left[\frac{\sin(\alpha+\beta)}{\sin 2\beta} \left(2M_{H^0}^2 - \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right) - \cos(\beta - \alpha) (M_{H^0}^2 - 2M_{A^0}^2) \right]$
$H^0 A^0 G^0$	$-\frac{ie}{2M_W s_W} (M_{A^0}^2 - M_{H^0}^2) \sin(\beta - \alpha)$
$H^0 G^0 G^0$	$-\frac{ie}{2M_W s_W} M_{H^0}^2 \cos(\beta - \alpha)$
$h^0 H^+ H^-$	$-\frac{ie}{2M_W s_W} \left[\frac{\cos(\alpha+\beta)}{\sin 2\beta} \left(2M_{h^0}^2 - \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right) - (M_{h^0}^2 - 2M_{H^-}^2) \sin(\beta - \alpha) \right]$
$H^0 H^+ H^-$	$-\frac{ie}{2M_W s_W} \left[\frac{\sin(\alpha+\beta)}{\sin 2\beta} \left(2M_{H^0}^2 - \frac{4\lambda_5 M_W^2 s_W^2}{e^2} \right) - \cos(\beta - \alpha) (M_{H^0}^2 - 2M_{H^-}^2) \right]$

➤ Decoupling limit

It corresponds to $\alpha \rightarrow \beta - \frac{\pi}{2}$ and with all masses much larger than M_{h^0}

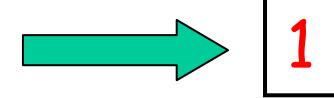
(In the particular case of the MSSM, this limit is correlated with $M_{A^0} \rightarrow \infty$)

In this limit, it is easy to see that

$$\lambda_{h^0 h^0 h^0} \rightarrow \lambda_{HHH}^{\text{SM}} = -\frac{3 e M_H^2}{2 M_W s_W} = \boxed{-\frac{3 g M_H^2}{2 M_W}}$$

and similarly with the Yukawa couplings of the h^0 :

	type I	type II
$h^0 t\bar{t}$	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$
$h^0 b\bar{b}$	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$



➤ Constraints on 2HDM models

➤ rho-parameter:

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1$$

loop corrections

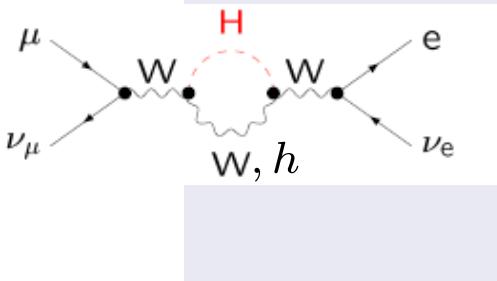
$$\rho = \rho_0 + \delta\rho$$

$$\delta\rho = \left. \frac{\Sigma_Z(k^2)}{M_Z^2} - \frac{\Sigma_W(k^2)}{M_W^2} \right|_{k^2=0}.$$

One-loop corrections induced by Higgs bosons

Barbieri & Maiani, 1983

$$\begin{aligned} \delta\rho_{2HDM} = & \frac{G_F}{8\sqrt{2}\pi^2} \left\{ M_{H^\pm}^2 \left[1 - \frac{M_{A^0}^2}{M_{H^\pm}^2 - M_{A^0}^2} \ln \frac{M_{H^\pm}^2}{M_{A^0}^2} \right] \right. \\ & + \cos^2(\beta - \alpha) M_{h^0}^2 \left[\frac{M_{A^0}^2}{M_{A^0}^2 - M_{h^0}^2} \ln \frac{M_{A^0}^2}{M_{h^0}^2} - \frac{M_{H^\pm}^2}{M_{H^\pm}^2 - M_{h^0}^2} \ln \frac{M_{H^\pm}^2}{M_{h^0}^2} \right] \\ & \left. + \sin^2(\beta - \alpha) M_{H^0}^2 \left[\frac{M_{A^0}^2}{M_{A^0}^2 - M_{H^0}^2} \ln \frac{M_{A^0}^2}{M_{H^0}^2} - \frac{M_{H^\pm}^2}{M_{H^\pm}^2 - M_{H^0}^2} \ln \frac{M_{H^\pm}^2}{M_{H^0}^2} \right] \right\} \end{aligned}$$



Experimental measurements: $|\delta\rho_{2HDM}| \lesssim 10^{-3}$
 $\delta\rho_{2HDM}$ vanish for $M_A \rightarrow M_{H^\pm}$

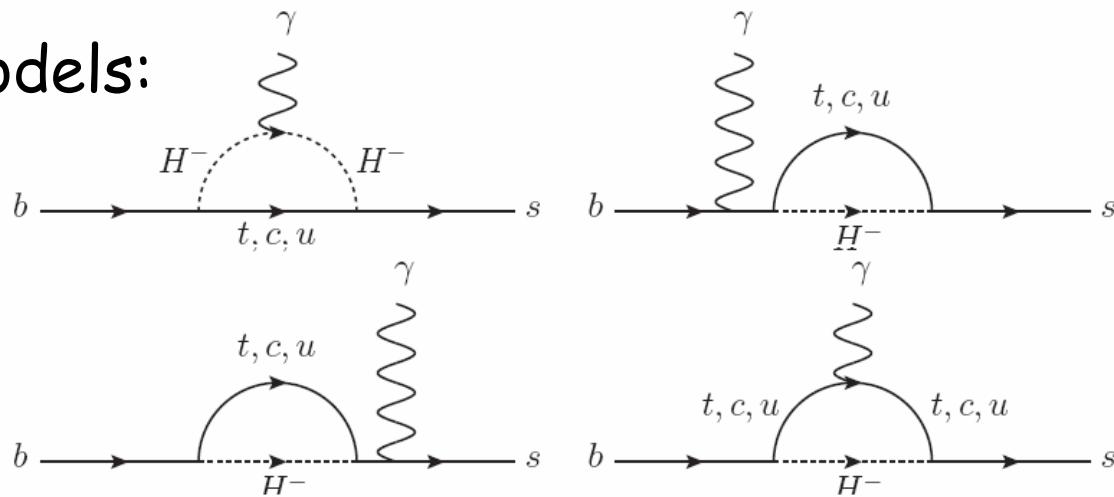
We will demand $\rightarrow M_A \sim M_{H^\pm}$

Restrictions: $\mathcal{B}(b \rightarrow s\gamma)$

- We have strong constraints coming from flavor physics
 - $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) \sim (3.55 \pm 0.25) \cdot 10^{-4}$ from BaBar and Belle
 - $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) \sim (3.15 \pm 0.23) \cdot 10^{-4}$ SM NNLO prediction
- The good agreement between the SM prediction and the experimental result puts severe constraints on the flavor structure of NP models.

New charged-particles contribute to this rare decay.

2HDM models:



Leading-order contributions due to the charged Higgs H^\pm

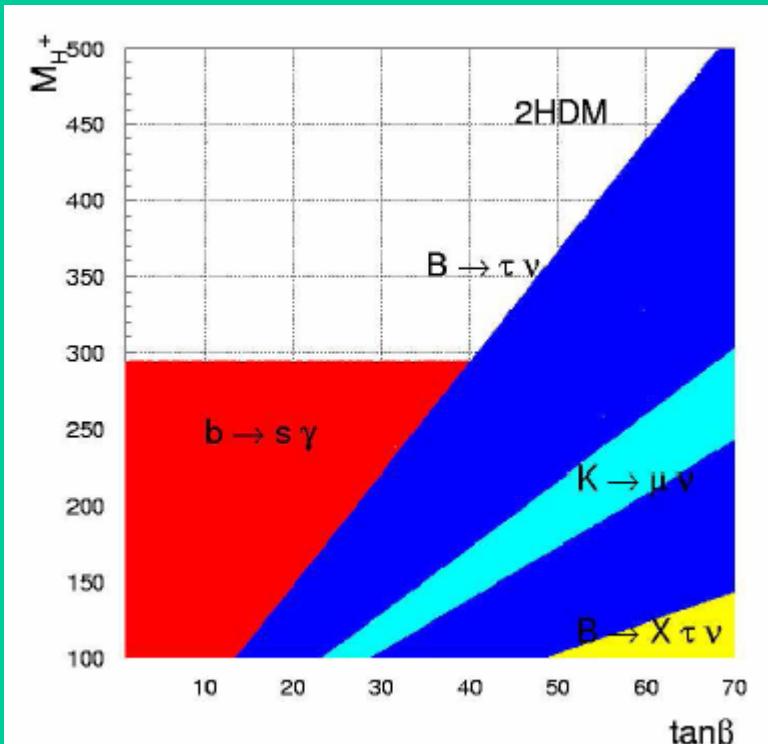
The charged Higgs bosons contribution:

- ✓ positive
- ✓ increases when M_{H^\pm} decreases

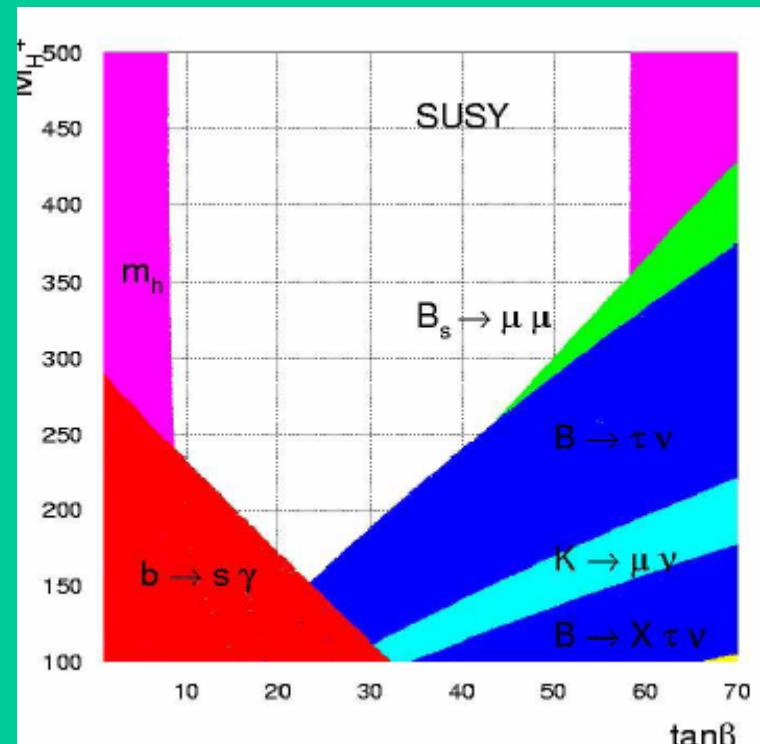
Type-I 2HDM: Couplings $H^\pm qq' \propto 1/\tan\beta$
 Couplings highly suppressed for $\tan\beta > 1$

Type-II 2HDM: Couplings $H^\pm qq' \propto \tan\beta$
 Couplings enhanced for $\tan\beta > 1$
 Restriction → $M_{H^\pm} > 295$ GeV Misiak et al., 2006

Present Constraints from Flavour Physics!



$m_{H^\pm} > 295 \text{ GeV}$



$m_{H^\pm} \gtrsim 130 \text{ GeV}$

F. Mescia (UB), private communication

Deschamps et al, arXiv:0907.5135 [hep-ph] ,

$M_{H^\pm} > 315 \text{ GeV}$

Joan Solà (UB) LC 10

$m_{SUSY} = -2A_{\tilde{t}} = 1 \text{ TeV}$

➤ Perturbativity and unitarity

- Keep the theory within a perturbative regime: $0.1 \lesssim \tan \beta \lesssim 60$

$$h_t = \frac{gm_t}{\sqrt{2}M_W \sin \beta}, \quad h_b = \frac{gm_b}{\sqrt{2}M_W \cos \beta} \rightarrow \frac{gm_b \tan \beta}{\sqrt{2}M_W}$$

- Unitarity bounds: we bound the size of the trilinear Higgs boson couplings by the value of their single SM counterpart at the scale of 1 TeV.

$$|C(HHH)| \leq \left| \lambda_{HHH}^{(SM)}(M_H = 1 \text{ TeV}) \right| = \left. \frac{3 e M_H^2}{2 \sin \theta_W M_W} \right|_{M_H=1 \text{ TeV}}.$$

(This would be at least the simplest unitarity requirement to start with)

➤ Higgs boson and Unitarity

Scattering amplitude for $S_1 S_2 \rightarrow S_3 S_4$. Expansion in partial waves:

$$\mathcal{M} = \sum_{l=1}^{\infty} \mathcal{M}_l \quad \mathcal{M}_l = 16\pi(2l+1)P_l(\cos\theta)a_l$$

$$\sigma(S_1 S_2 \rightarrow S_3 S_4) = \sum_l^{\infty} \sigma_l : \quad \sigma_l = \frac{16\pi}{s}(2l+1)|a_l|^2$$

Unitarity means $SS^\dagger = 1$ with $S = 1 + \mathcal{M}$ $\Rightarrow i(\mathcal{M} - \mathcal{M}^\dagger) = -\mathcal{M}\mathcal{M}^\dagger$

$$\Re(a_l)^2 + \Im(a_l)^2 = |a_l|^2 = \Im(a_l) \quad \Rightarrow \quad \Re(a_l)^2 + \left(\Im(a_l) - \frac{1}{2}\right)^2 = \frac{1}{4}$$

$$|\Re(a_l)| < \frac{1}{2} \quad (\forall l)$$

All scattering lengths bounded by unitarity \Leftrightarrow optical theorem

- ♣ Asymptotic “flatness” of the scattering amplitudes \Leftrightarrow Fermion and gauge-boson couplings to a “Higgs-like object”
- ♣ Conservation of probability for elastic scattering $|\text{Re } a_0| < 1/2$

Lee-Quigg-Thacker bound [’77]

Scattering of longitudinally-polarized W: equivalence theorem

$$a_0(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) \simeq a_0(G^+ G^- \rightarrow G^+ G^-) = \frac{1}{16\pi} \frac{M_H^2}{v^2}$$

$$M_H^2 < 8\pi v^2 = \frac{8\pi}{\sqrt{2}G_F} \simeq (1.2 \text{ TeV})^2$$

- ♣ In the 2HDM; Kanemura, Kubota & Takasugi, 1993; Akeroyd, Arhrib & Naimi, 2000; Horejsi & Kladiva, 2006
- ♣ Higgs and Goldstone boson $2 \rightarrow 2$ S-matrix elements, $S_{ij} \Rightarrow$ Unitarity condition over its eigenvalues $|\alpha_i| < 1/2 \forall i$

➤ More strict unitarity bounds in the 2HDM

Sample of unitarity conditions:

$$\begin{aligned}
 a_{\pm} &= \frac{1}{16\pi} \left\{ 3(\lambda_1 + \lambda_2 + 2\lambda_3) \right. \\
 &\quad \left. \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + \left(4\lambda_3 + \lambda_4 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2} \right)^2} \right\}, \\
 b_{\pm} &= \frac{1}{16\pi} \left\{ \lambda_1 + \lambda_2 + 2\lambda_3 \right. \\
 &\quad \left. \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{(-2\lambda_4 + \lambda_5 + \lambda_6)^2}{4}} \right\}, \\
 c_{\pm} &= d_{\pm} = \frac{1}{16\pi} \left\{ \lambda_1 + \lambda_2 + 2\lambda_3 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{(\lambda_5 - \lambda_6)^2}{4}} \right\},
 \end{aligned}$$



$$|a_{\pm}|, |b_{\pm}|, |c_{\pm}|, |f_{\pm}|, |e_{1,2}|, |f_1|, |p_1| \leq \frac{1}{2}$$

$$a_{\pm} = \frac{1}{16\pi} \left\{ 3(\lambda_1 + \lambda_2 + 2\lambda_3) \right. \\ \left. \pm (\sqrt{9(\lambda_1 - \lambda_2)^2 + (4\lambda_3 + \lambda_4 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2})^2} \right\},$$

$$b_{\pm} = \frac{1}{16\pi} \left\{ \lambda_1 + \lambda_2 + 2\lambda_3 \right. \\ \left. \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{(-2\lambda_4 + \lambda_5 + \lambda_6)^2}{4}} \right\},$$

$$c_{\pm} = d_{\pm} = \frac{1}{16\pi} \left\{ \lambda_1 + \lambda_2 + 2\lambda_3 \right. \\ \left. \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{(\lambda_5 - \lambda_6)^2}{4}} \right\},$$

$$e_1 = \frac{1}{16\pi} \left\{ 2\lambda_3 - \lambda_4 - \frac{\lambda_5}{2} + 5\frac{\lambda_6}{2} \right\},$$

$$e_2 = \frac{1}{16\pi} \left\{ 2\lambda_3 + \lambda_4 - \frac{\lambda_5}{2} + \frac{\lambda_6}{2} \right\},$$

$$f_+ = \frac{1}{16\pi} \left\{ 2\lambda_3 - \lambda_4 + 5\frac{\lambda_5}{2} - \frac{\lambda_6}{2} \right\},$$

$$f_- = \frac{1}{16\pi} \left\{ 2\lambda_3 + \lambda_4 + \frac{\lambda_5}{2} - \frac{\lambda_6}{2} \right\},$$

$$f_1 = f_2 = \frac{1}{16\pi} \left\{ 2\lambda_3 + \frac{\lambda_5}{2} + \frac{\lambda_6}{2} \right\},$$

$$p_1 = \frac{1}{16\pi} \left\{ 2(\lambda_3 + \lambda_4) - \frac{\lambda_5 + \lambda_6}{2} \right\}.$$

➤ Vacuum stability bounds

Minimization of the Higgs potential

We assume that the quartic interaction terms in the potential do not give negative contribution for all directions of scalar fields at each energy scale up to Λ

Require a Higgs potential bounded from below

$$\lambda_1 + \lambda_3 > 0$$

$$\lambda_2 + \lambda_3 > 0$$

$$2\sqrt{(\lambda_1 + \lambda_3)(\lambda_2 + \lambda_3)} + 2\lambda_3 + \lambda_4 + \frac{1}{2} \text{Min}\left(0, \lambda_5 + \lambda_6 - 2\lambda_4 - |\lambda_5 - \lambda_6|\right) > 0$$

Kanemura, Kasai & Okada, 1999

With all these restrictions in mind
we are **ready** for particular calculations
of **2HDM production** at e.g.

- **LHC** {
 - See e.g. S. Kanemura, S. Moretti , Y.Mukai, R. Santos, K.Yagyu, arXiv:0901.0204 [hep-ph];
 - A. Arhrib, R. Benbrik, C.H. Chen, R. Guedes, R. Santos, arXiv:0906.0387 [hep-ph]
 - M. Moretti, S. Moretti, F. Piccinini, R. Pittau, J. Rathsman, arXiv:1008.0820 [hep-ph]
 - etc...

➤ **Linear Colliders (my task here)**

➤ Higgs boson production
at the Linear Colliders
ILC/CLIC
within the general 2HDM

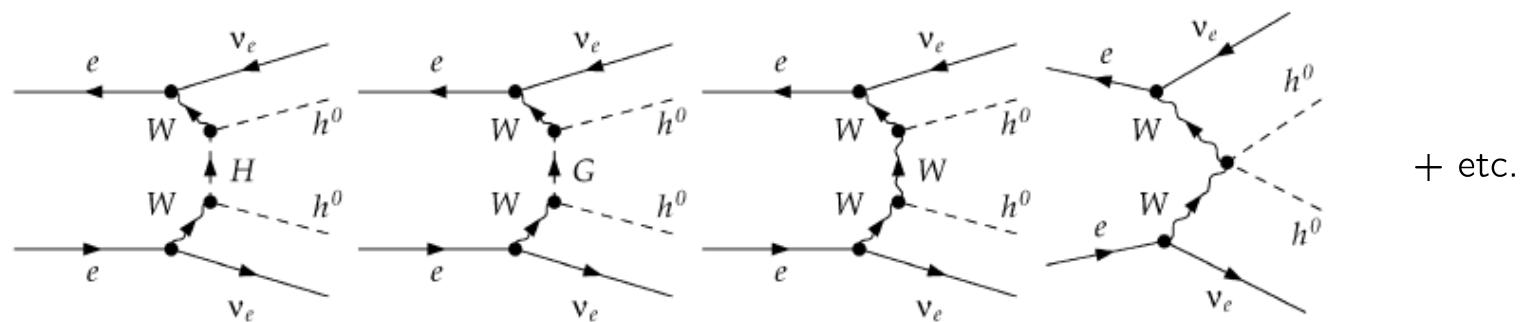
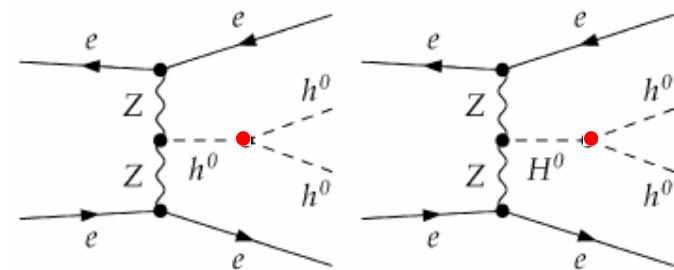
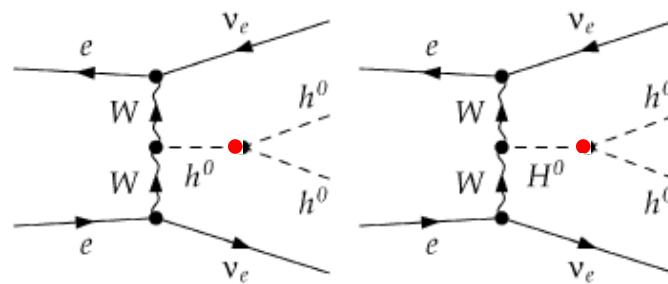
- $e^+e^- \rightarrow ZH$ D. López-Val, JS, N. Bernal, Phys.Rev.D81 (2010) 113005
(at the quantum level)
 - $e^+e^- \rightarrow 2H$ D. López-Val, JS, Phys.Rev.D81 (2010) 033003
 - $e^+e^- \rightarrow 3H$
 - $e^+e^- \rightarrow 2H + X$
 - $e^+e^- \rightarrow \gamma\gamma \rightarrow H$
- } (very promising too!)
N. Bernal, D, López-Val, JS,
Phys. Lett. B677 (2009) 39
N. Hodgkinson, D, López-Val, JS,
Phys. Lett. B673 (2009) 47
G. Ferrera, J. Guasch, D, López-Val, JS,
Phys.Lett.B659: (2008) 297

Extended strategy: $e^+ e^- \rightarrow 2H + X$

➤ Double-Higgs production through gauge boson fusion

$$e^+ e^- \rightarrow V^* V^* \rightarrow h h + X$$

N. Hodgkinson, D. López-Val, JS
Phys. Lett. B677 (2009) 39



CP allowed channels

$$\left. \begin{array}{l} e^+e^- \rightarrow H^\pm H^\mp \\ e^+e^- \rightarrow h^0 A^0 \\ e^+e^- \rightarrow H^0 A^0 \end{array} \right\} \text{LO: tree-level diagrams } (\mathcal{O}(\alpha_{ew}^2))$$

- $C(HH\nu)$ couplings are of purely gauge nature

$$C_{\text{MSSM}}(h^0 A^0 Z) = \frac{e \cos(\beta - \alpha)}{2 \sin \theta_W \cos \theta_W}$$

- ♠ There is no dynamical distinction between the general 2HDM and the MSSM
- ♠ Dedicated studies on radiative corrections in 2H processes are hence mandatory: Chankowski, Pokorski, Rosiek (94, 95); Driesen, Hollik, Rosiek (95, 96); Arhrib, Moulakka (98); Kraft (PhD Thesis, 99), Guasch, Hollik, Kraft (01); Heinemeyer et al (01)

➤ Quantum effects on $e^+e^- \rightarrow 2H$ in the 2HDM

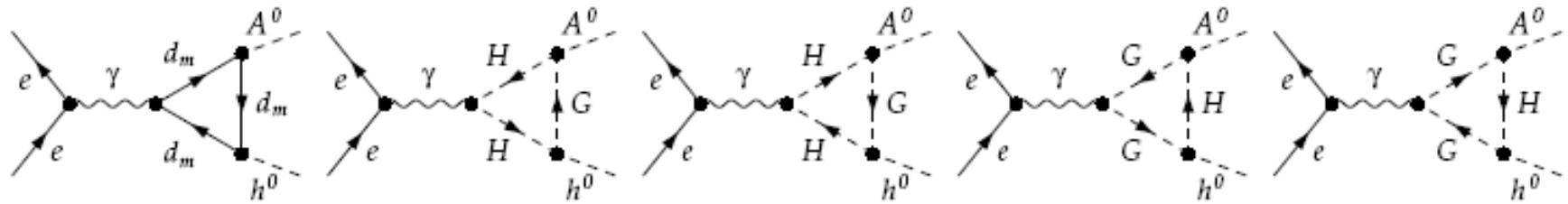
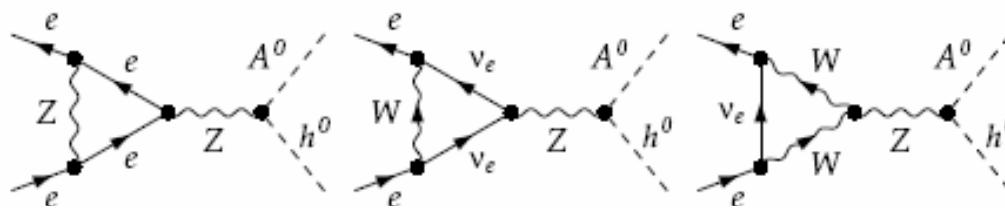
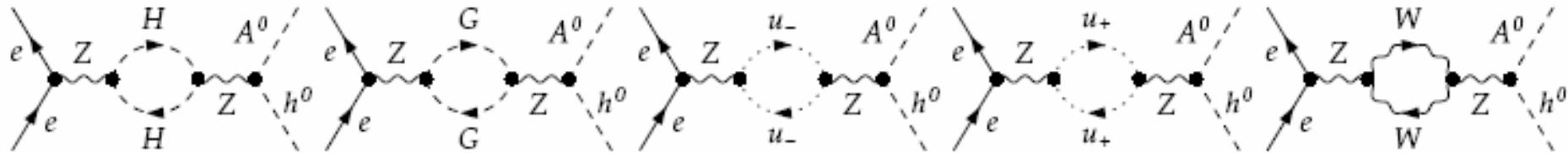
$$e^+ e^- \rightarrow A^0 h^0 \quad (\text{similarly with } e^+ e^- \rightarrow A^0 H^0)$$

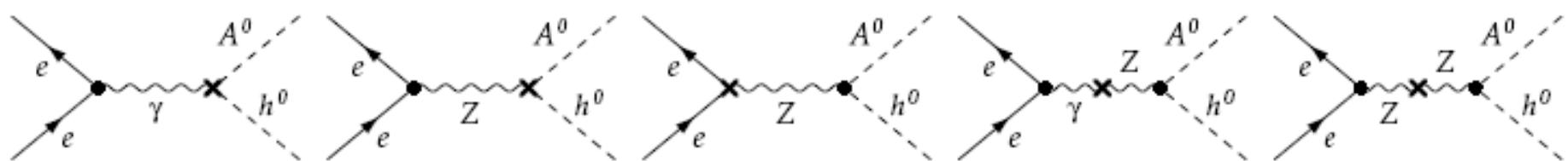
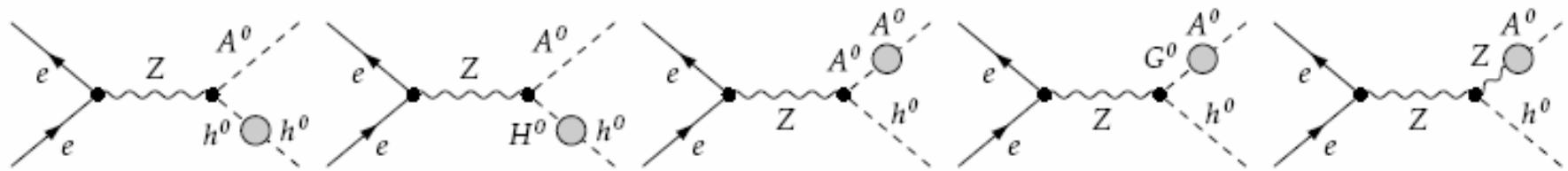
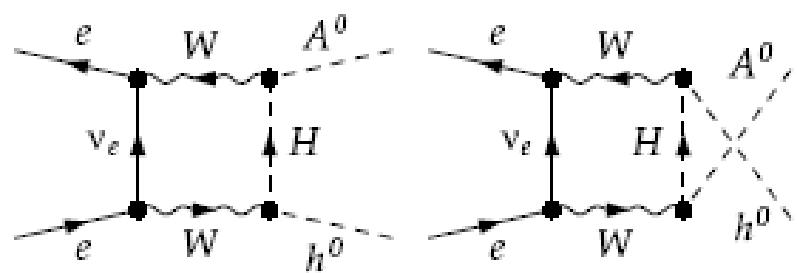
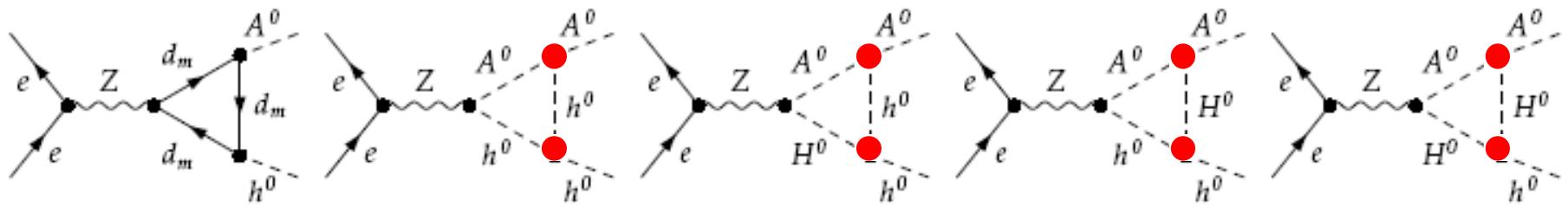
Basic one-loop amplitude:

$$\begin{aligned} M_{e^+ e^- \rightarrow A^0 h^0}^1 &= (M^{1,Z-Z} + M^{1,\gamma-\gamma} \\ &\quad + M^{1,e^+e^-Z} + M^{1,e^+e^-\gamma} \\ &\quad + M^{1,Z^0 A^0 h^0} + M^{1,\text{box}} + M^{1,\text{WF}} + \delta M^1) \end{aligned}$$

The loop correction appears from the interference with the tree-level amplitude: $2\Re e \mathcal{M}^{(0)} \mathcal{M}^{(1)}$

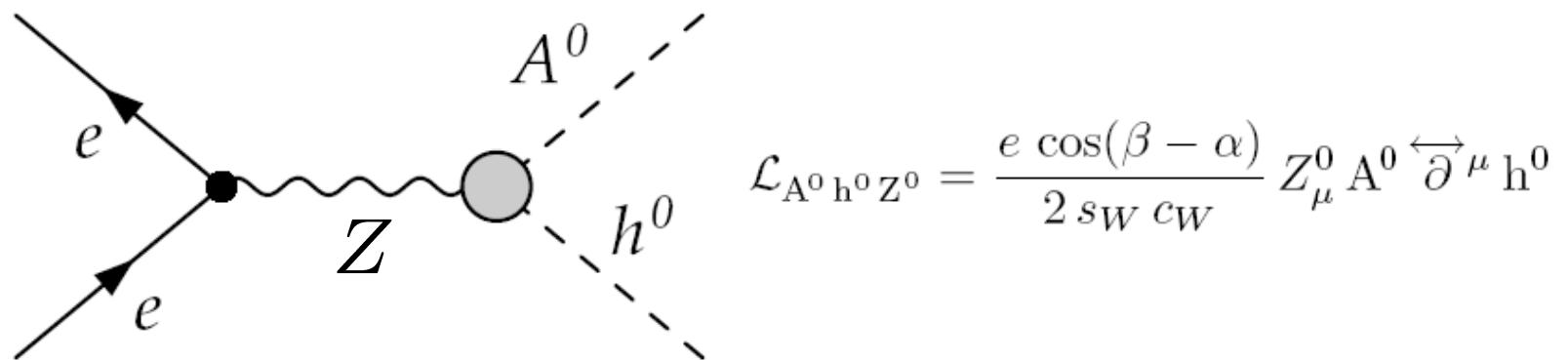
Some of the diagrams involved...





For the complete list see: D, López-Val, JS,
arXiv:0908.2898 [hep-ph]

Apart from the ordinary Zee vertex counterterm, we have e.g.



$$(g_i \rightarrow g_i + \delta g_i , \quad \phi_i \rightarrow (1 + \frac{1}{2} \delta Z_i) \phi_i)$$

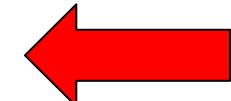
Vertex counterterm:

$$\begin{aligned} \delta \mathcal{L}_{A^0 h^0 Z^0} = & \frac{e \cos(\beta - \alpha)}{2 s_W c_W} Z_\mu^0 A^0 \overleftrightarrow{\partial}^\mu h^0 \left[\frac{\delta e}{e} + \frac{s_W^2 - c_W^2}{c_W^2} \frac{\delta s_W}{s_W} - \right. \\ & - \sin \beta \cos \beta \tan(\beta - \alpha) \frac{\delta \tan \beta}{\tan \beta} + \frac{1}{2} \delta Z_{h^0} + \frac{1}{2} \delta Z_{A^0} + \frac{1}{2} \delta Z_{Z^0} - \\ & \left. - \frac{1}{2} \tan(\beta - \alpha) \delta Z_{H^0 h^0} + + \frac{1}{2} \tan(\beta - \alpha) \delta Z_{A^0 G^0} + \frac{1}{2} \tan(\beta - \alpha) \delta Z_{A^0 Z^0} \right] \end{aligned}$$

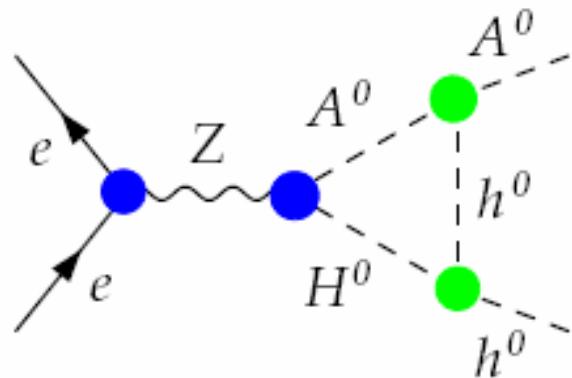
Why we expect important effects in the 2HDM?

$$\begin{aligned}\mathcal{M}_{e^+e^- \rightarrow A^0 h^0}^{(1)} = & \quad \mathcal{M}^{1,Z^0-Z^0} \quad \mathcal{O}(\alpha_{ew}^2) \\ & + \mathcal{M}^{\gamma-Z^0} \quad \mathcal{O}(\alpha_{ew} \alpha_{em}) \\ & + \mathcal{M}^{e^+e^-Z^0} \quad \mathcal{O}(\alpha_{ew}^2) \\ & + \boxed{\mathcal{M}^{A^0 h^0 Z^0}} \quad \mathcal{O}(\alpha_{ew} \lambda_{3H}^2) \\ & + \mathcal{M}^{\gamma h^0 A^0} \quad \mathcal{O}(\alpha_{em} \lambda_{3H}^2) \\ & + \mathcal{M}^{\text{box}} \quad \mathcal{O}(\alpha_{ew}^2) \\ & + \mathcal{M}^{\text{WF}, h^0} \quad \mathcal{O}(\alpha_{ew} \lambda_{3H}^2) \\ & + \mathcal{M}^{h^0 H^0} \quad \mathcal{O}(\alpha_{ew} \lambda_{3H}^2) \\ & + \mathcal{M}^{A^0 Z^0, A^0 G^0} \quad \mathcal{O}(\alpha_{ew} \lambda_{3H}^2)\end{aligned}$$

Leading!



In more detail:



whereas in the MSSM,

$$C_{\text{MSSM}}[h^0 h^0 H^0] = \frac{ie M_W}{2 \sin \theta_W \cos \theta_W} (\cos 2\alpha \cos(\alpha + \beta) - 2 \sin 2\alpha \sin(\alpha + \beta))$$

➤ Renormalization conditions and counterterms

Renormalization of the Higgs sector OS (on-shell) scheme

♠ Higgs fields: 1 WF constant per $SU_L(2)$ doublet

$$\begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix} \rightarrow Z_{\Phi_1}^{1/2} \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix}, \quad \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix} \rightarrow Z_{\Phi_2}^{1/2} \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix},$$

$$(Z_{\Phi_i} = 1 + \delta Z_{\Phi_i})$$

$$\begin{aligned} \spadesuit \quad \tan \beta: \quad & \left. \begin{array}{l} \frac{\delta v_1}{v_1} = \frac{\delta v_2}{v_2} \\ t_{h^0 H^0} + \delta t_{h^0 H^0} = 0 \end{array} \right\} \quad \frac{\delta \tan \beta}{\tan \beta} = \frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1} + \frac{1}{2} (\delta Z_{\Phi_2} - \delta Z_{\Phi_1}) \\ & = \frac{1}{2} (\delta Z_{\Phi_2} - \delta Z_{\Phi_1}) . \end{aligned}$$

Higgs masses (OS scheme)

$$\text{Re } \hat{\Sigma}(M_{h^0}^2) = 0; \text{ Re } \hat{\Sigma}(M_{H^0}^2) = 0; \text{ Re } \hat{\Sigma}(M_{A^0}^2) = 0; \text{ Re } \hat{\Sigma}(M_{H^\pm}^2) = 0$$

$$\text{Re } \hat{\Sigma}'_{A^0 A^0}(k^2) \Big|_{k^2=M_{A^0}^2} = 0 \quad , \quad \text{Re } \hat{\Sigma}_{A^0 Z^0}(k^2) \Big|_{k^2=M_{A^0}^2} = 0$$

$$\delta Z_{\Phi_1} = -\text{Re } \Sigma'_{A^0 A^0}(M_{A^0}^2) - \frac{1}{M_Z \tan \beta} \text{Re } \Sigma_{A^0 Z^0}(M_{A^0}^2)$$

$$\delta Z_{\Phi_2} = -\text{Re } \Sigma'_{A^0 A^0}(M_{A^0}^2) + \frac{\tan \beta}{M_Z} \text{Re } \Sigma_{A^0 Z^0}(M_{A^0}^2)$$



$$\frac{\delta \tan \beta}{\tan \beta} = \frac{1}{2} (\delta Z_{\Phi_2} - \delta Z_{\Phi_1})$$

$$\boxed{\frac{\delta \tan \beta}{\tan \beta} = \frac{1}{M_Z \sin 2\beta} \text{Re } \Sigma_{A^0 Z^0}(M_{A^0}^2)}$$

➤ Mass sets for the numerical analysis

MSSM-like
masses at
one-loop
FeynHiggs
(Heinemeyer et al)

	M_{h^0} [GeV]	M_{H^0} [GeV]	M_{A^0} [GeV]	M_{H^\pm} [GeV]	
Set I	100	150	140	120	type-I
	130	150	200	160	
	150	200	260	300	type-I/II
	95	205	200	215	type-I
	115	220	220	235	type-I
	130	285	285	300	type-I/II

♠ $\sqrt{s} = 500$ GeV

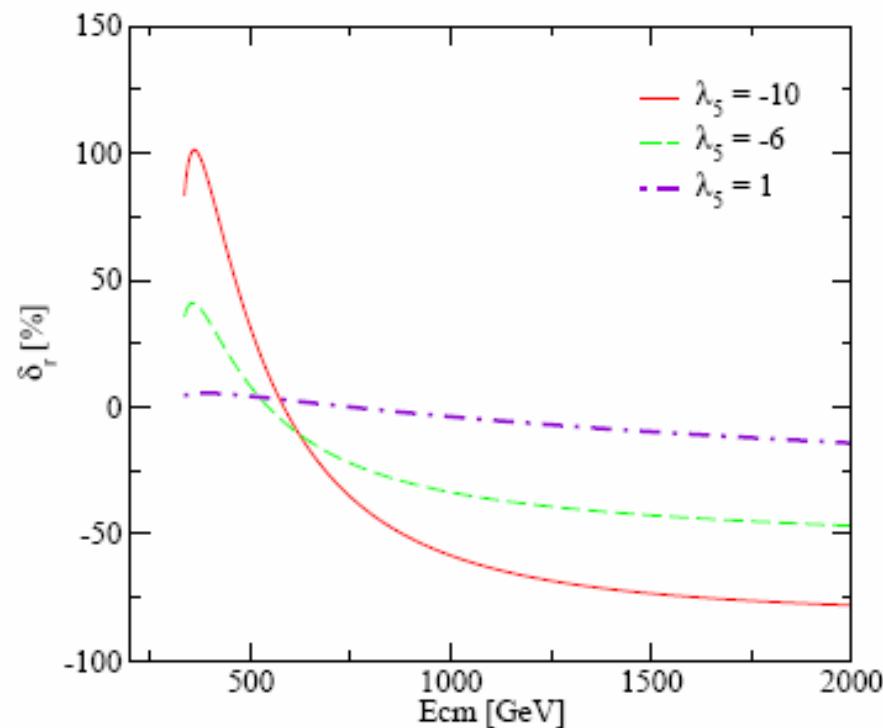
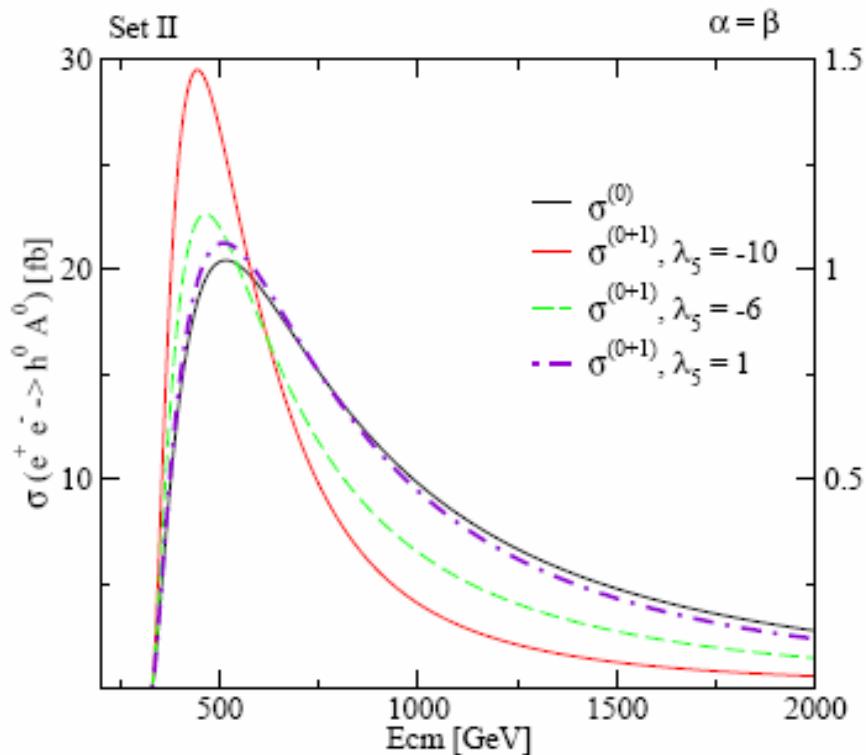
		$\alpha = \beta$	$\alpha = \beta - \pi/3$	$\alpha = \beta - \pi/6$	$\alpha = \pi/2$
Set II	σ_{max} [fb]	26.71	7.34	20.05	13.10
	δ_r [%]	31.32	44.43	31.42	28.81
Set III	σ_{max} [fb]	11.63	3.60	9.08	6.36
	δ_r [%]	35.17	67.59	40.68	47.86
Set IV	σ_{max} [fb]	27.44	12.12	18.37	15.41
	δ_r [%]	12.86	99.42	0.76	26.81

MSSM-like
masses

- Typical maximum cross sections:

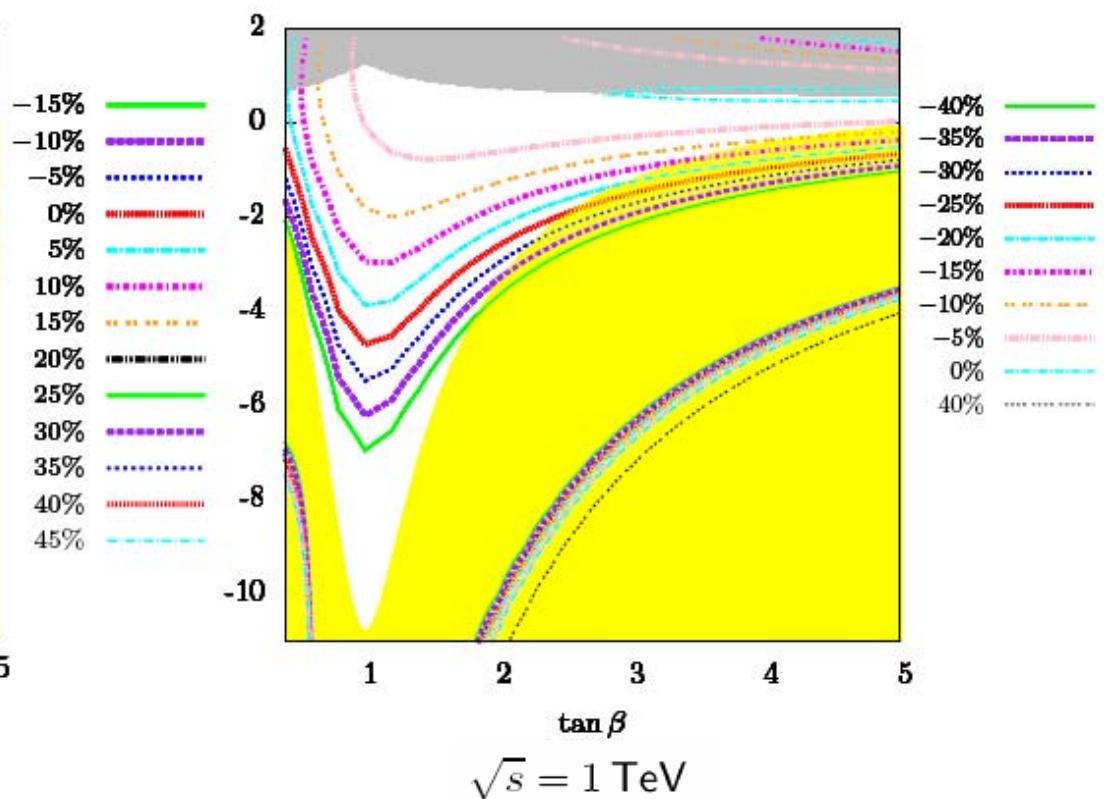
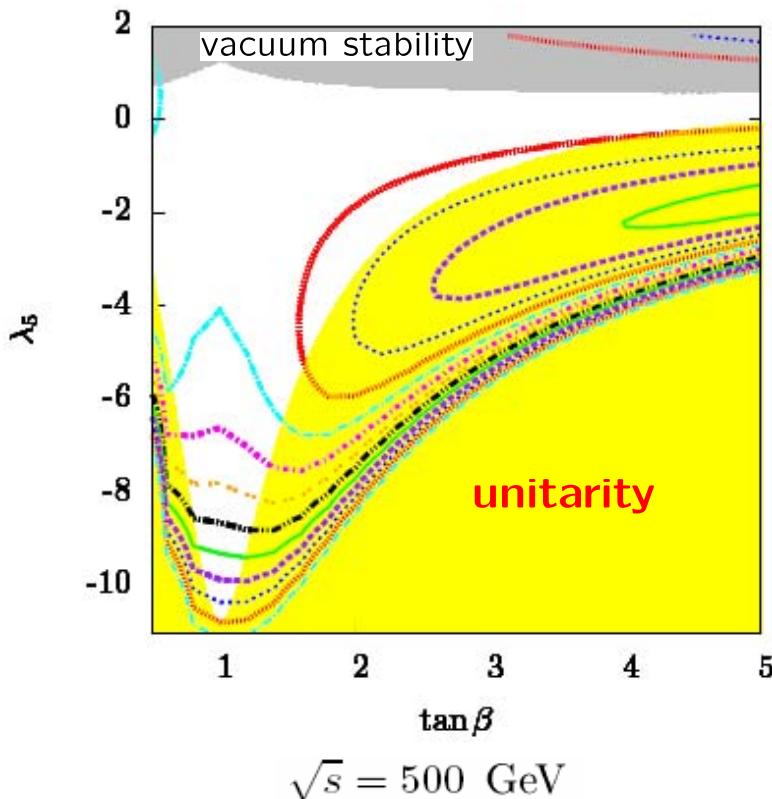
$$\sigma(e^+e^- \rightarrow A^0 h^0) (\sqrt{s} = 0.5 \text{ TeV}) \sim \mathcal{O}(10 \text{ fb}) \quad \sim 10^3 \text{ events per } 100 \text{ fb}^{-1}$$

$\tan \beta$	α	$M_{h^0} [\text{ GeV}]$	$M_{H^0} [\text{ GeV}]$	$M_{A^0} [\text{ GeV}]$	$M_{H^\pm} [\text{ GeV}]$
1	β	130	150	200	160



♠ Radiative corrections over the $\tan \beta - \lambda_5$ plane and its interplay with the unitarity and vacuum stability constraints.

Set II



Dependence of σ with masses, angles etc is
studied in detail in our work: D. López-Val, JS, arXiv:0908.2898 [hep-ph]

Compared predictions for 2H, hZ and 3H events

$\tan \beta$	α	λ_5	M_{h^0} [GeV]	M_{H^0} [GeV]	M_{A^0} [GeV]	M_{H^\pm} [GeV]
1	β	-10	130	150	200	160

Process	$\sigma(\sqrt{s} = 0.5 \text{ TeV})[\text{fb}]$	$\sigma(\sqrt{s} = 1.0 \text{ TeV})[\text{fb}]$	$\sigma(\sqrt{s} = 1.5 \text{ TeV})[\text{fb}]$
$h^0 A^0$	26.71 (31.32%)	4.07	1.27
$H^0 Z^0$	19.09 (-61.56%)	3.73	1.47
$h^0 H^0 A^0$	0.02	5.03	3.55
$H^0 H^+ H^-$	0.17	11.93	8.39
$h^0 h^0 + X$	1.47	17.36	38.01

Great complementarity is observed between the different channels at different energy ranges.

- N. Bernal, D. López-Val, JS, Phys.Lett..B677 (2009) 39
 - + updated analysis presented here
(D. López-Val, JS)

- **Gamma-Gamma fusion: $\gamma\gamma \rightarrow H$ in the 2HDM**

- cf. Maria Krawzyck's talk for the general status of $\gamma\gamma$ physics

➤ Gamma-Gamma fusion: $\gamma\gamma \rightarrow H$ in the 2HDM

- Let us recall that a photon collider is an option of a lepton collider.

It is possible to take into account the conversion $e^+e^- \rightarrow \gamma\gamma$ by the convolution

$$\sigma(e^+e^- \rightarrow \boxed{\gamma\gamma \rightarrow h})(s) = \sum_{\{ij\}} \int_0^1 d\tau \frac{d\mathcal{L}_{ij}^{ee}}{d\tau} \hat{\sigma}_{\eta_i \eta_j}(\gamma\gamma \rightarrow h)(\tau s)$$

- * $\hat{\sigma}_{\eta_i \eta_j}(\gamma\gamma \rightarrow h)$: partonic cross section
- * τ : fraction of the energy carried by the photon
- * \mathcal{L}_{ij}^{ee} stands for the photon luminosity distribution

$$\frac{d\mathcal{L}_{ij}^{ee}}{d\tau} = \int_\tau^1 \frac{dx}{x} \frac{1}{1 + \delta_{ij}} [f_{i/e_1}(x) f_{j/e_2}(\tau/x) + f_{j/e_1}(x) f_{i/e_2}(\tau/x)]$$

- * f_{i/e_1} denotes the photon density functions.

- We use the ones provided by CompAZ

Telnov, 2006 & Żarnecki, 2003

$$\sigma(\gamma\gamma \rightarrow h) = \frac{8\pi^2}{M_h} \Gamma(h \rightarrow \gamma\gamma) \delta(s - M_h^2) (1 + \eta_1 \eta_2) = 8\pi \frac{\Gamma(h \rightarrow \gamma\gamma) \Gamma_h (1 + \eta_1 \eta_2)}{(s - M_h^2)^2 + M_h^2 \Gamma_h^2}$$

One-loop diagrams describing the process $\gamma\gamma \rightarrow h$, within the 2HDM

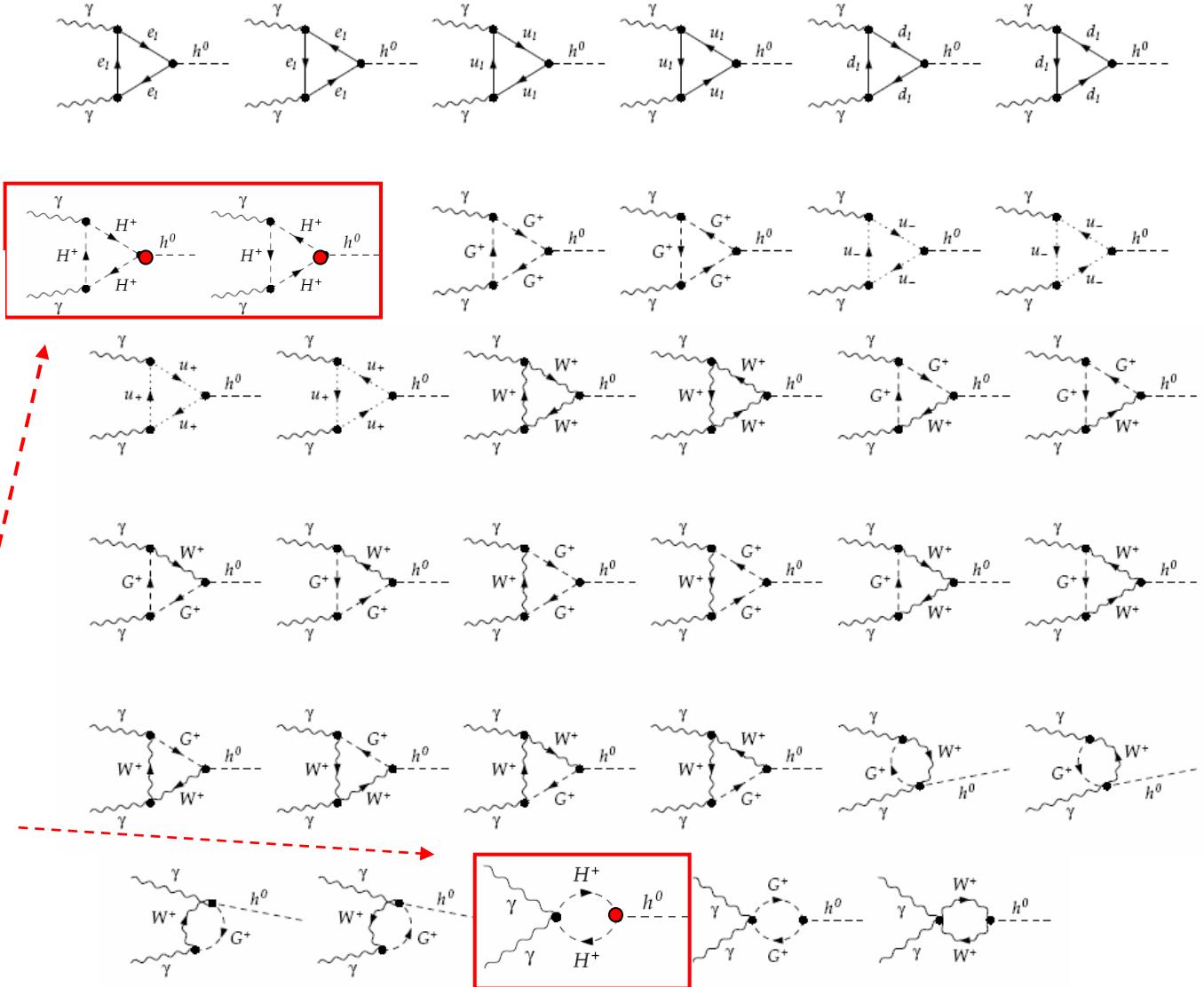
The $\gamma\gamma h^0$ interaction is generated at the quantum level

$$r \equiv \frac{g_{\gamma\gamma h}}{g_{\gamma\gamma H}} = \left[\frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(H \rightarrow \gamma\gamma)} \right]^{1/2}$$

SM contributions:

- * Heavy fermions t, b
- * Vector bosons W^\pm
- * Goldstone bosons G^\pm

+ 2HDM contributions:
* Charged Higgs H^\pm

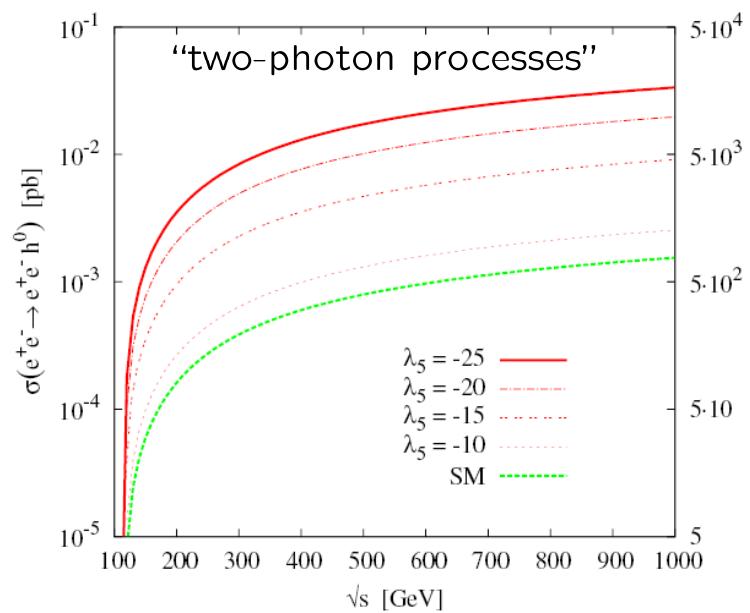
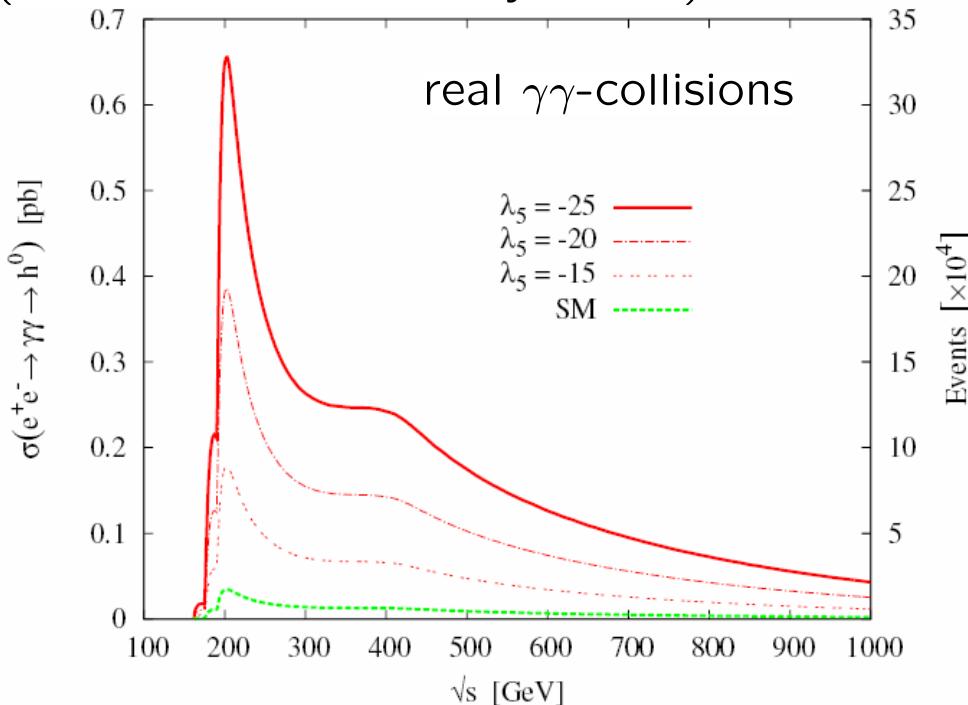


Cross section

2HDM	Set I	Set II	Set III	Set IV
M_{h^0}	115	150	200	200
M_{H^0}	165	200	250	250
M_{A^0}	100	110	290	340
M_{H^\pm}	105	105	300	350

(GeV)

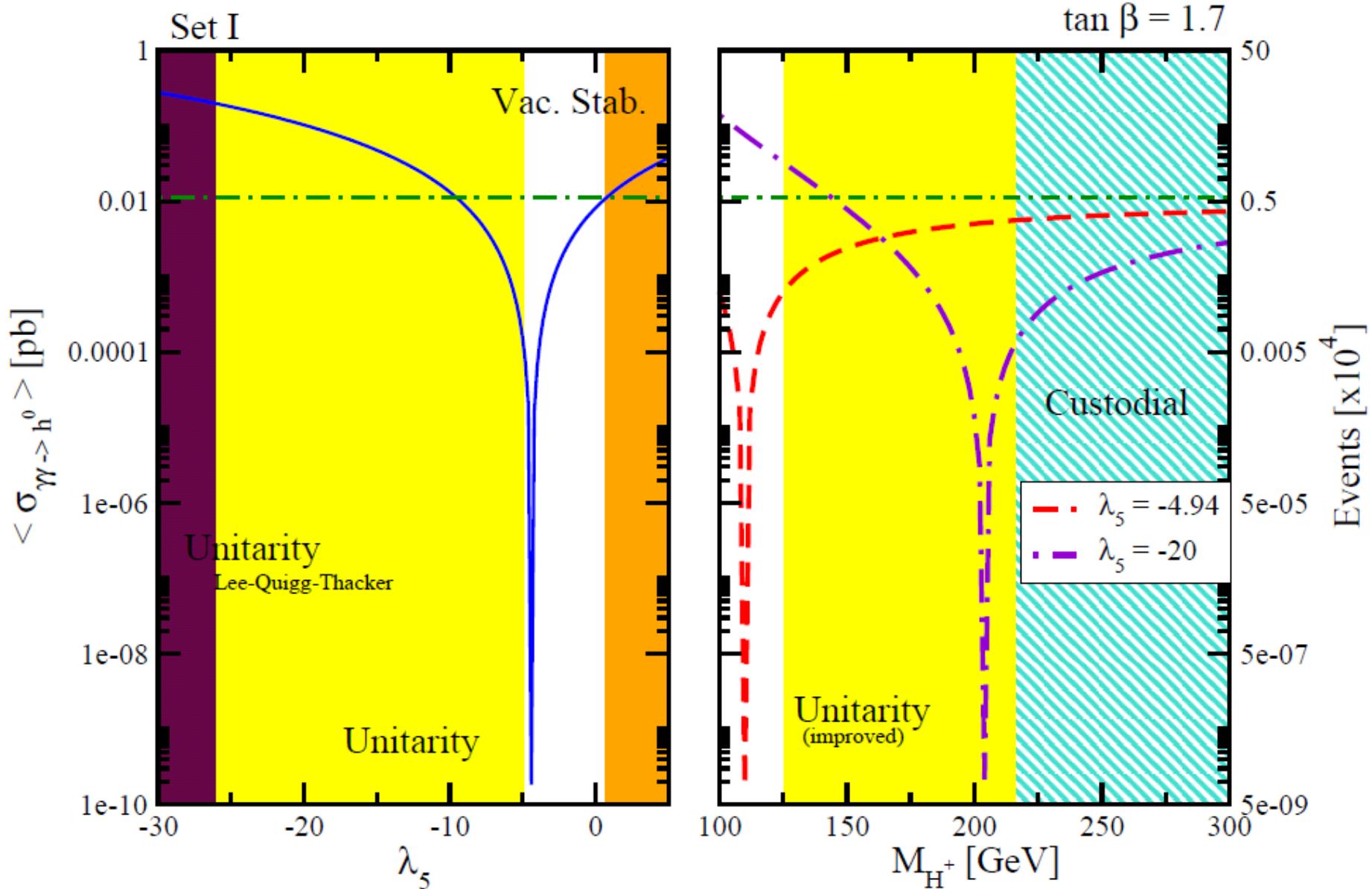
(Set I and LT-like unitarity bounds)



- ▶ The shape of the cross section lead by the parametrization of the photon energy spectrum.
- ▶ Huge number of events for low center-of-mass energy.
- ▶ Due to interference effects, the enhancement capabilities become partially reduced.

* Set I with
 $\sin \alpha = -0.86$,
 $\tan \beta = 1.7$

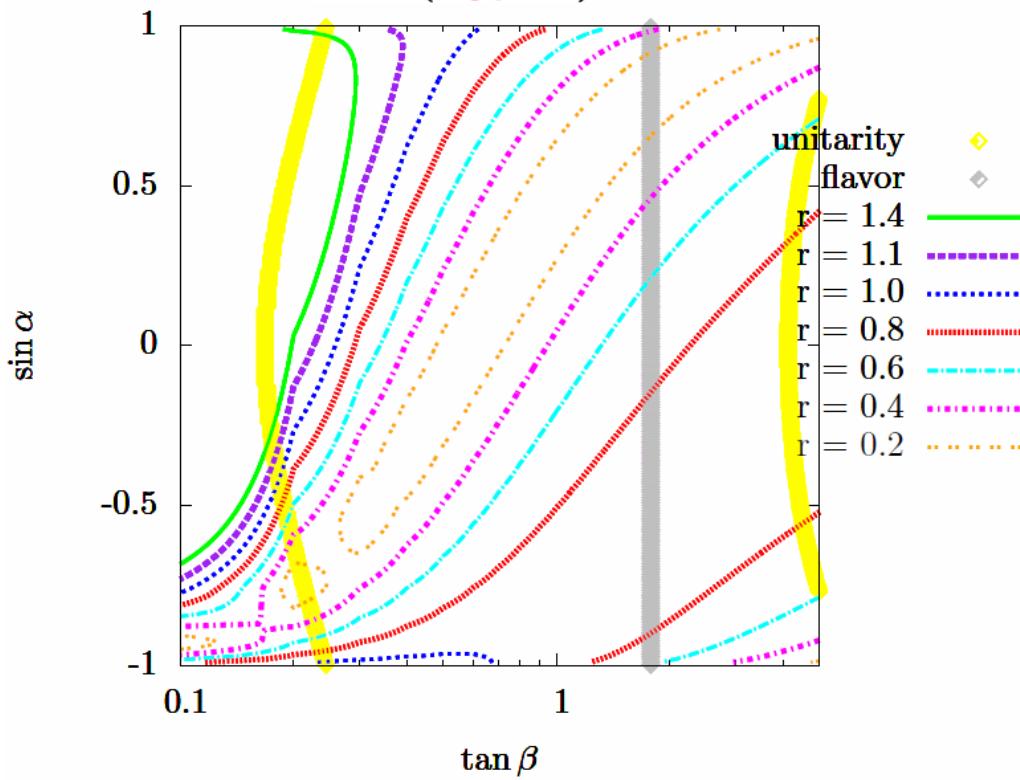
* Luminosity
 $\mathcal{L} = 500 \text{ fb}^{-1}$



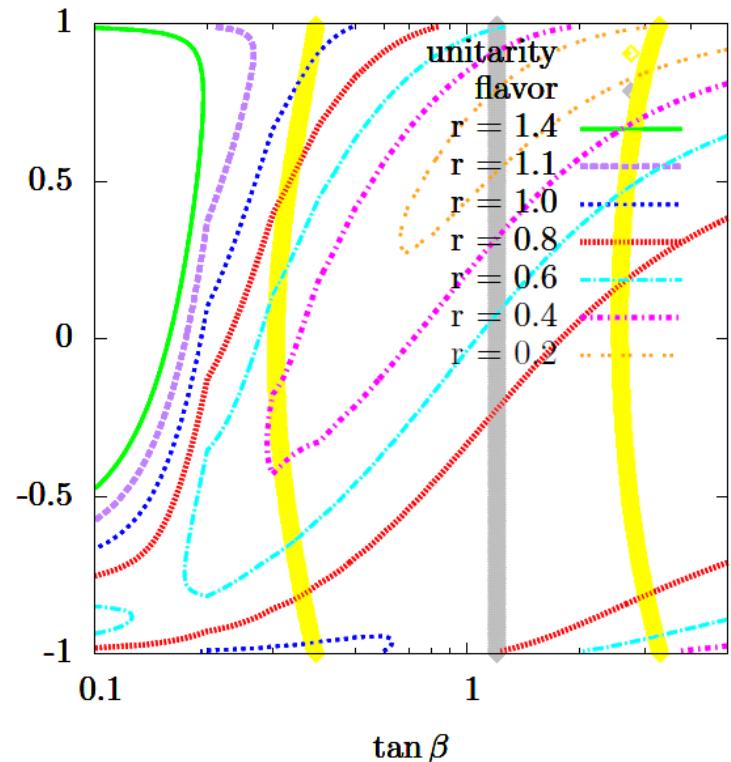
$$\lambda_5 = 0$$

$$r \equiv \frac{g_{\gamma\gamma h}}{g_{\gamma\gamma H}} = \frac{|\mathcal{M}|^{\text{2HDM}}}{|\mathcal{M}|^{\text{SM}}} = \left[\frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(H \rightarrow \gamma\gamma)} \right]^{1/2}$$

Set I (Type I)



Set III (Type II)



yellow bands: Bounds from unitarity

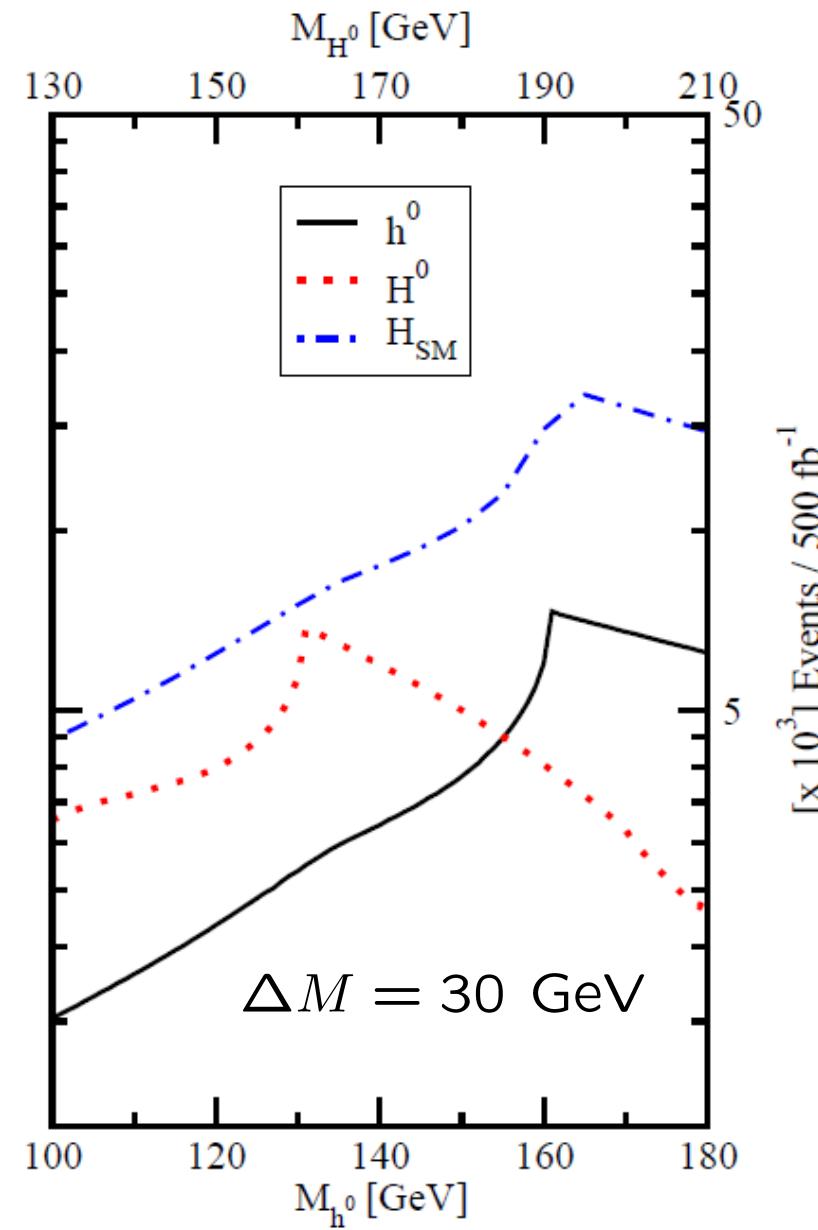
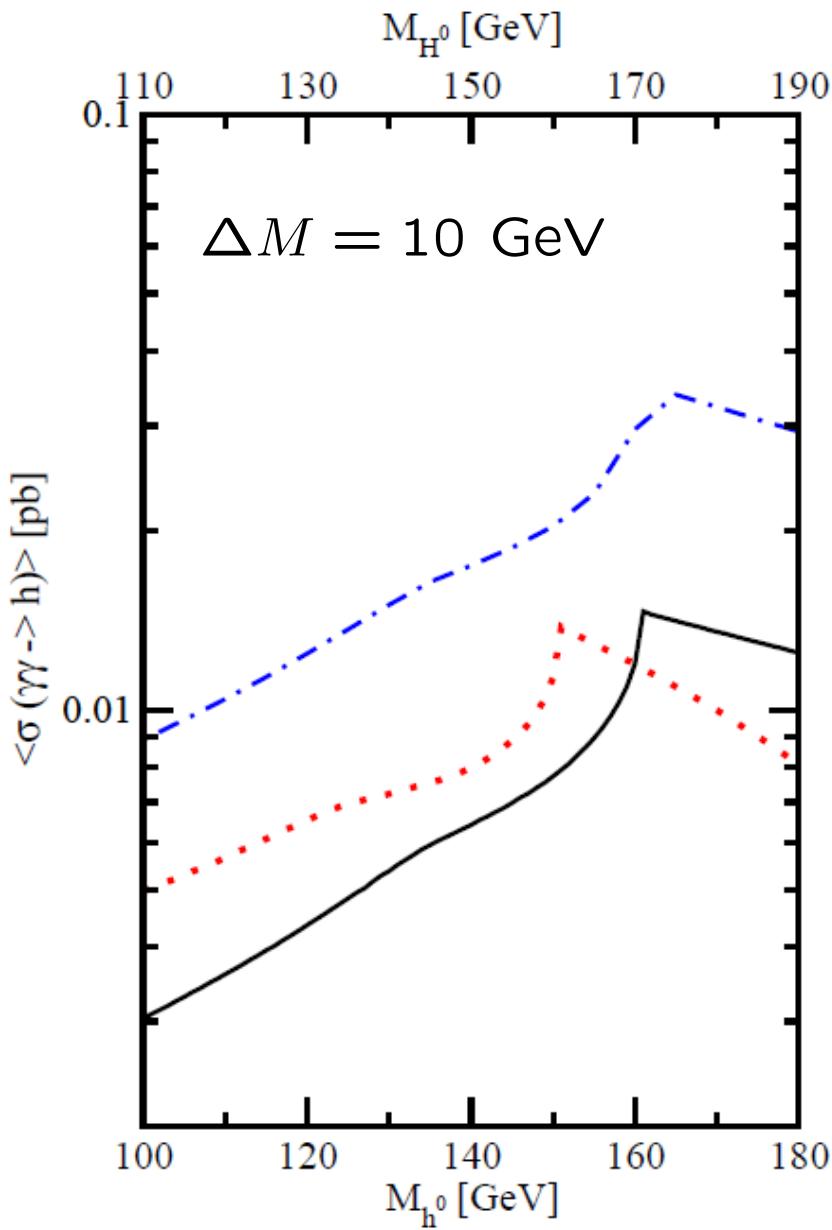
grey band: Lower bound from $B_0 - \bar{B}_0$ ($\sim 1/\tan \beta$)

More strict unitarity bounds
and limits from flavor physics

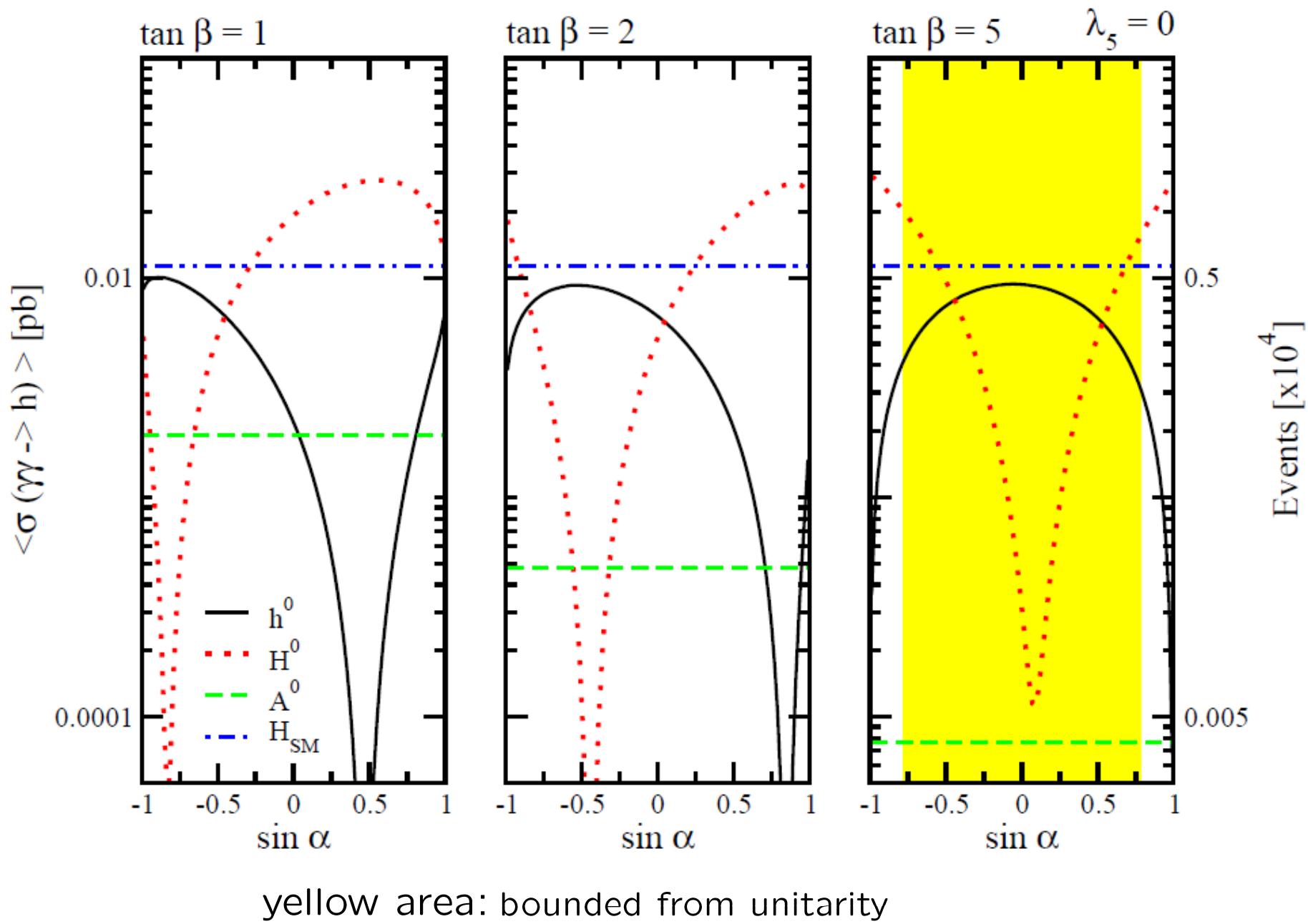


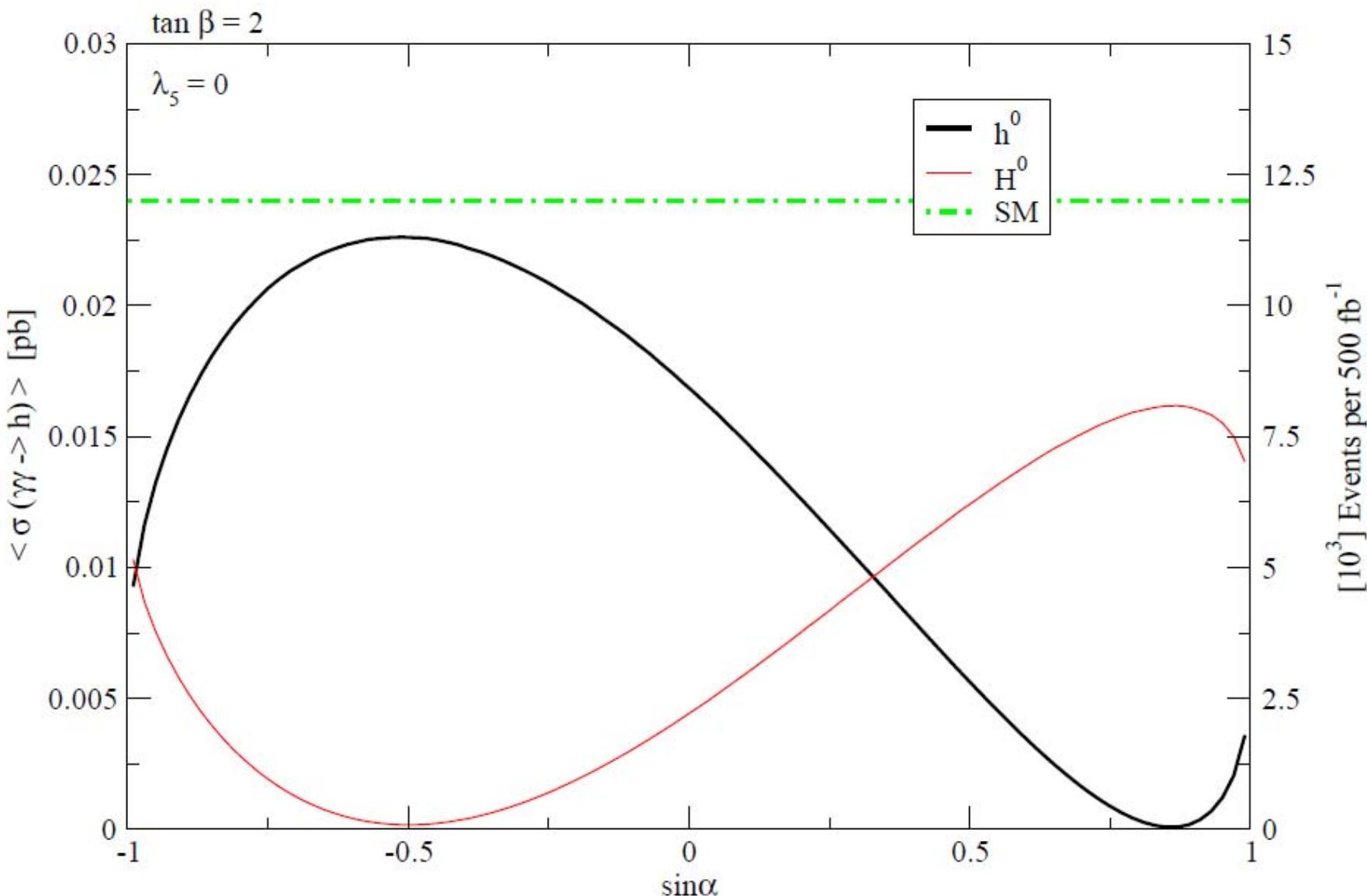
$$r < 1$$

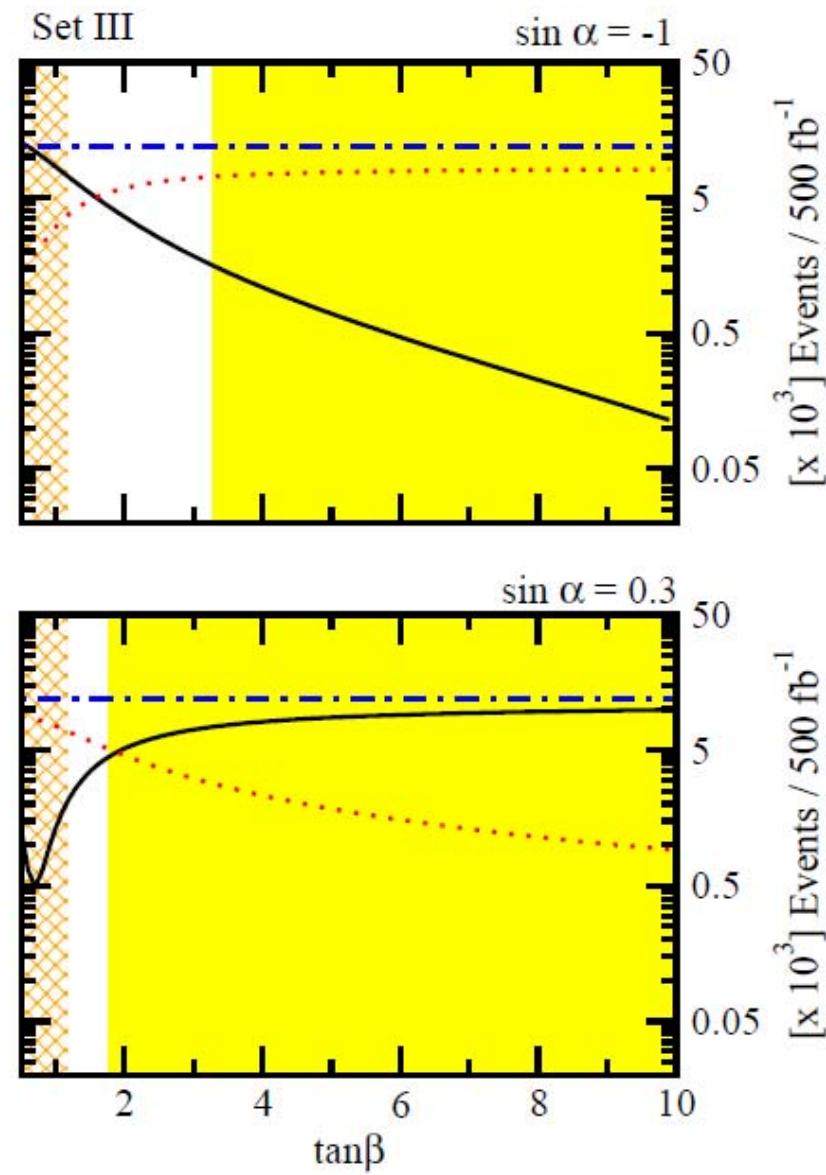
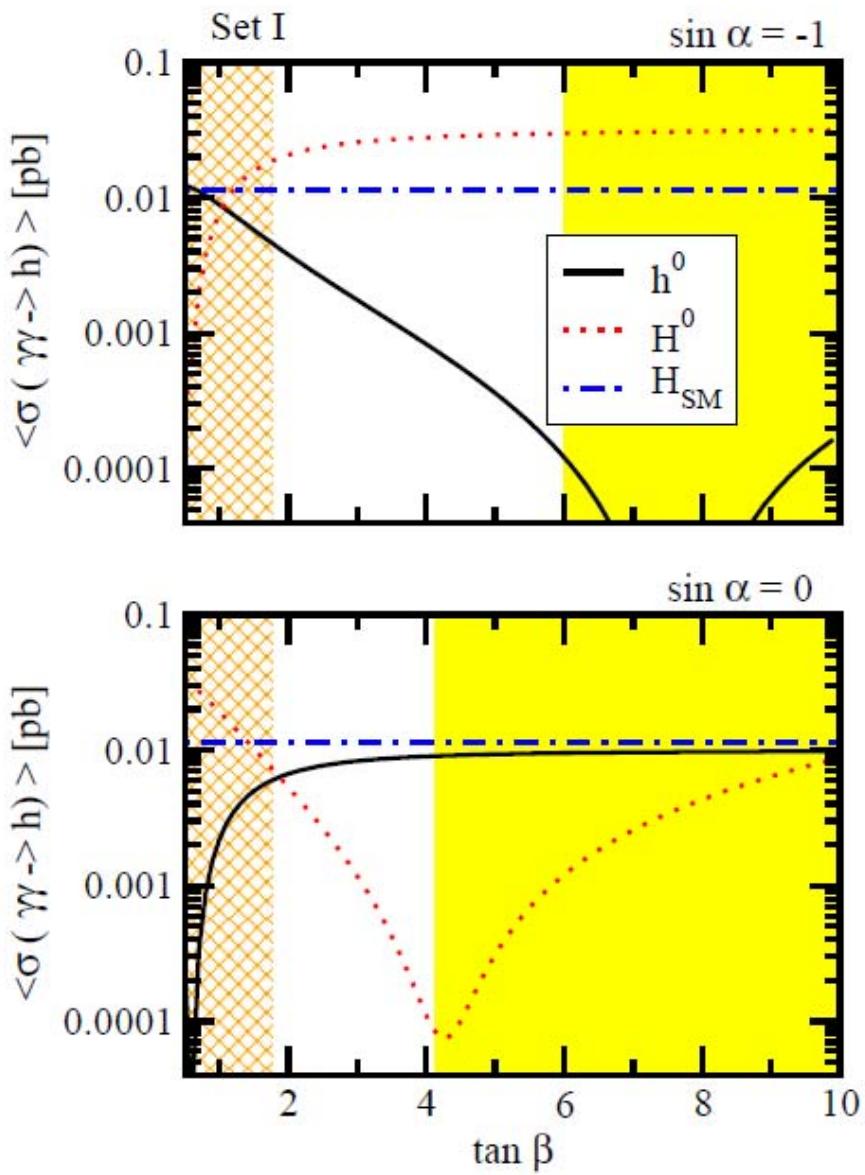
but...



$\sin \alpha = 0.30$, $\tan \beta = 2$ and $\lambda_5 = 0$







yellow bands: Bounds from unitarity

orange bands: Bounds from flavor physics

Conclusions

- General **2HDM models** may offer a **clue** to disentangle hints of physics **beyond** the **SM** at the **LHC** and specially in the linear colliders (**ILC/CLIC**);
- Measurement of **3H** and **2HX** Higgs boson production should allow a **first insight** into the **Higgs potential** through a basic determination of the Higgs boson self-couplings;
- Detailed measurement of **2H** and **HZ** processes can be crucial to **test the 2HDM** models at the **quantum level** ;
- $\gamma\gamma \rightarrow H$ (and $\gamma\gamma \rightarrow 2H$) processes could be the **cleanest** carriers of **new physics**.



The extremely **clean environment** of the linear colliders should allow a comfortable tagging of these processes and might open privileged new vistas into the **structure** of the **Higgs potential**

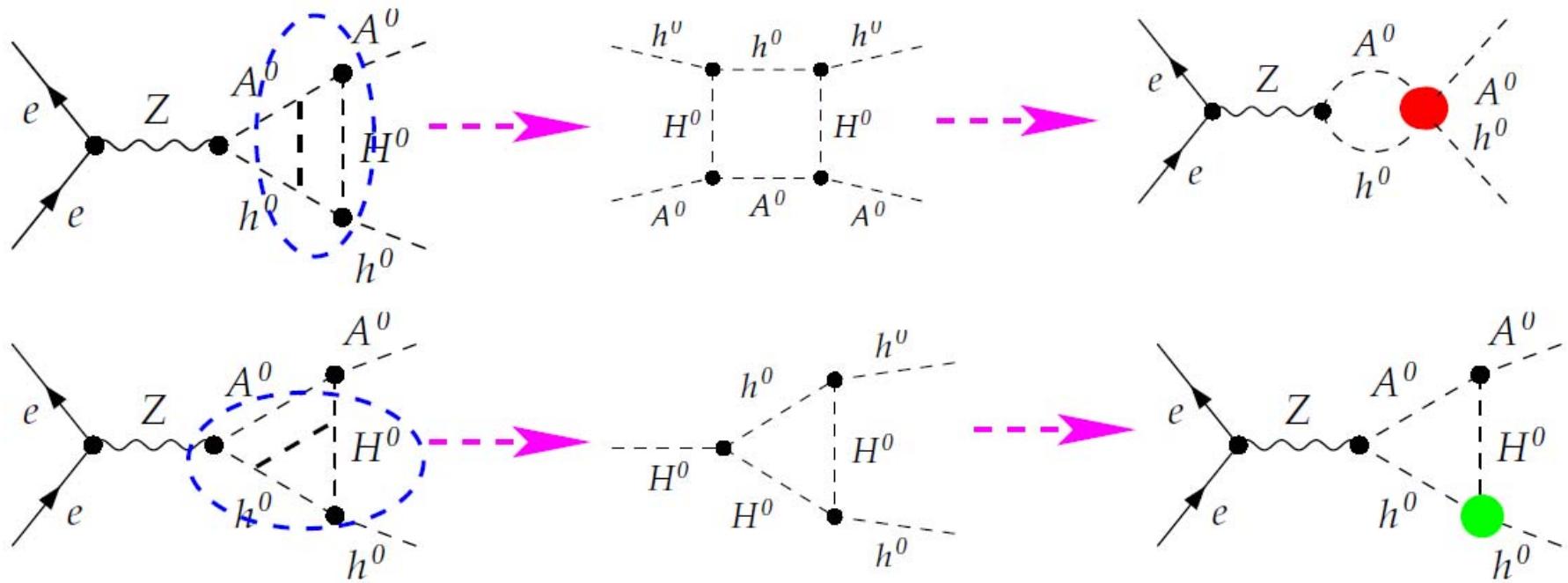


Figure 11.15: Pictorial representation of a sample of the leading two-loop order vertex corrections, which we may rewrite as *improved* one-loop corrections with effective quartic (right corner, above) and triple (right corner, below) Higgs boson self-interactions. These effective Higgs boson self-couplings (center diagrams) track down the leading quantum effects on $e^+e^- \rightarrow h^0A^0$ at two-loops and, indeed, they may be reabsorbed as a finite contribution to the renormalization of the parameter λ_5 , see the text for details.