Orbifolds in spacetime

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based on work in progress with Roberto Volpato

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Introduction

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- Theories of quantum gravity are believed to have no global symmetries, so that many different aspects may be explained quite generally in terms of the way this conjecture is realized (Harlow, Ooguri,'18, Heidenreich et al., '20).
- The description of GS in terms of topological operators leads naturally to such generalizations as higher-form GS and non-invertible GS. Many aspects of QFTs can be described in terms of the "algebra" of symmetry defects without relying on model-dependent properties.

Higher-form symmetries (HS) (Gaiotto et al., '14)

- A p-form GS is defined by topological operators $U_g(M^{(d-p-1)})$ labeled by the elements g fo the group G, with the fusion algebra

$$U_g(M^{(d-p-1)})U_{g'}(M^{(d-p-1)}) = U_{gg'}(M^{(d-p-1)})\,, \qquad (1)$$

and the Ward identities

$$U_g(S^{d-p-1})\,V(C^{(p)}) = g(V)\,V(C^p)\,, \qquad (2)$$

where $g(V)$ is a representation of g and the sphere S^{d-p-1} links once with the p-dimensional support C^p of the charged operator $V(C^p)$.

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- When $G = U(1)$ and a Noether current J_{p+1} exists, one can write explicitly

$$U_{g=e^{i\alpha}}(M^{(d-p-1)}) = \exp\left(i\alpha\int_{M^{(d-p-1)}} \star J_{p+1}\right) \tag{3}$$

$U(1)$ gauge theory in $4d$

- For pure Maxwell theory $G = U(1)$ in $4d$

$$U^E_{g=e^{i\alpha}}(M^2) = \exp\left(\frac{i\alpha}{g^2}\int_{M^2} \star F_2\right), \; U^M_{g=e^{i\alpha}}(M^2) = \exp\left(\frac{i\alpha}{g^2}\int_{M^2} F_2\right)$$

- Given a Wilson loop labeled by an electric charge $n \in \mathbb{Z}$ if S^2 surrounds $\gamma^{(1)}$,

$$\exp\left(\frac{i\alpha}{g^2}\int_{M^2} \star F_2\right) \exp\left(in \oint_{\gamma^{(1)}} A_1\right) = \exp(in\alpha) \exp\left(in \oint_{\gamma^{(1)}} A_1\right) .$$

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- HS can be coupled to background fields. E.g.

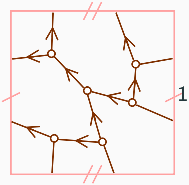
$$\frac{i}{(2\pi)}B_2\wedge F_2 + \frac{in}{(2\pi)}B_2\wedge da, \tag{6}$$

imposing $F_2 + nda = 0$ and making B_2 a $\mathbb{Z}_n$ 2-form gauge field.

The orbifold construction in $2d$ CFT

Given a finite group G formed by symmetry defects g, the correlators of the G-orbifold are described as correlators of the unorbifolded model with a sufficiently fine network of defect lines $Q = \sum_{g \in G} g$ with topological two-to-one junctions $\varphi = \sum_{g,h \in H} \varphi_{g,h}$. For the torus partition function, e.g.

$$Z_{\text{orb}} = \quad \text{[network]}^{1}. \qquad (7)$$



Independence on the choice of defect network implies the associator must give rise to 3-cocycle belonging to the class [1] in $H^3(G, U(1))$ ('t Hooft anomaly). Different choices of $\varphi_{g,h}$ are related by 2-cocycles classified by $H^2(G, U(1))$ (discrete torsion).

[1] Courtesy of Jürg Fröhlich et al.,'09

Orbifold in QFT

Given a d-spacetime-dimensional QFT $\mathcal{C}$ and a finite group G acting on $\mathcal{C}$ by global symmetries, the orbifold construction is a general method to build a new theory $\mathcal{C}/G$ by "gauging" G

- Orbifolding is possible only if the 't Hooft anomaly $\omega \in H^{d+1}(BG, U(1))$ is trivial, i.e. if $\mathcal{C}$ can be coupled to a defect network with consistent junctions.
- Given a foliation $\Sigma \times \mathbb{R}_t$ and the Hilbert space $\mathcal{H}$ of states on space-like Σ, gauging involves projecting on the untwisted sector $\mathcal{H}^G$ of G-fixed points in $\mathcal{H}$ and summing over new twisted sectors $\mathcal{H}_f$, $f \in G$, of states defined up to the action of non-trivial f.
- For a given group G, there might be different and inequivalent ways of integrating over flat connections, classified by $H^d(G, U(1))$.

Orbifold in string theory

- Orbifolds in string theory are realized by carrying out either a QFT orbifold of the worlsheet $2d$ CFT w.r.t. a global symmetry or a geometric orbifold of the string backgrounds w.r.t. some finite group of isometries.

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- Both methods depend on our choice of the fundamental string over other dynamical objects that are present in string theory and therefore it is highly non-trivial to match orbifolds between different duality frames for non-perturbative dualities.
- Spacetime quantum gravity, with asymptotically flat geometry, is characterized by a gauge group H (including higher-group symmetries). In particular, the global worldsheet symmetries are gauged preventing to perform an orbifold as in QFT. The corresponding asymptotic gauge symmetries are however broken by charged states so that they are always present independently of the particular description.

Spacetime orbifold

Spacetime orbifold w.r.t. G should correspond to the worlsheet orbifold w.r.t. the global G worldsheet symmetry.

- We project out all states and operators in a non trivial representation of G, including the ones localized on such extended objects as D-branes, NS5-branes, etc.
- We gauge the global 1-symmetry associated to the Wilson lines in non-trivial representations of G that became non-endable in the previous step and that are projected out . We introduce gauge fields configurations that close only up to gauge transformations.
- For each dynamical object (fundamental string, D-brane, NS-brane) we allow the worldvolume fields to have non-trivial monodromy in G around any non-trivial cycle wrapped and we introduce the codimension 2 operators that create such twist vortices.

Higher-form gauge fields

- The information about a p-form gauge field on a manifold X is encoded in $\check{A} = (\mathrm{N}, \mathrm{A}, \mathrm{F})$, where $\mathrm{F} \in \Omega^{p+1}_{closed}(X)$, $\mathrm{A} : C_p(X) \to \mathbb{R}$ is a p-cochain, mapping a p-chain $M \in C_p(X)$ to $\int_M A$, and with a coboundary operator δ such that, if $\delta\mathrm{A} : C_{p+1}(X) \to \mathbb{R}$, $\int_N \delta\mathrm{A} = \int_{\partial N} \mathrm{A}$ for any $p+1$-chain N, and N is an integral $p+1$-cochain $\mathrm{N} : C_{p+1}(X) \to \mathbb{Z}$, such that $\delta\mathrm{A} = \mathrm{F} - \mathrm{N}$.
- Gauge transformations are given by

$$(\mathrm{N}, \mathrm{A}, \mathrm{F}) \to (\mathrm{N} + \delta\mathrm{n}, \mathrm{A} - \mathrm{n} + \delta\mathrm{a}, \mathrm{F}) \tag{8}$$

  where $\mathrm{a} : C_{p-1} \to \mathbb{R}$ is any $p-1$-cochain and $\mathrm{n} : C_p \to \mathbb{Z}$ is any integral p-cochain.
- Flat gauge fields $\check{A}$ with $\mathrm{F} = 0$ are in one-to-one correspondence with $H^p(X, \mathbb{R}/\mathbb{Z})$. They can be realized as networks of U_g defects stretched in a spatial slice with topological junctions between them.

Type II superstring compactified on S^1

If $z \sim z + 2\pi\ell_z n$, the NS-NS sector includes a 2-form gauge field B_2, the momentum vector A and the winding vector B, whose dynamics is constrained by the Nicolai-Townsend gauge transformations

$$B_2 \mapsto \quad B_2 + \frac{1}{2}\lambda \wedge H_2 + \frac{1}{2}\sigma \wedge F_2 \text{when} \quad \begin{array}{c} A \mapsto A + d\lambda, \\ B \mapsto B + d\sigma \end{array} \tag{9}$$

If

$$\tilde{H}_3 = dB_2 - \frac{1}{2}A \wedge H_2 - \frac{1}{2}B \wedge F_2 + (\text{localized})\,. \tag{10}$$

we can consider the Hodge dual $\check{A}_{\tilde{B}_5} = (\mathsf{N}_{\tilde{B}_5}, \mathsf{A}_{\tilde{B}_5}, \mathsf{F}_{\tilde{B}_5})$, $\mathsf{F}_{\tilde{B}_5} = e^{-2\phi} * \tilde{H}_3$

$$\int_N \mathsf{N}_A \in 2\pi l_z \mathbb{Z}, \quad \int_N \mathsf{N}_{B_1} \in \frac{2\pi}{l_z}\mathbb{Z}, \quad \int_N \mathsf{N}_{\tilde{B}_5} \in 2\pi\mathbb{Z}$$

Higher-group global symmetry

- The higher-group global symmetry is identified by the currents

$$
\begin{aligned}
*J_6 &= i2\pi\ell_z e^{2\phi} * \mathsf{F}_{\tilde{B}_5} & *J_3 &= \tfrac{i}{2\pi}\mathsf{F}_{\tilde{B}_5}\\
*J_{2,e} &= i\left(2\pi\ell_z\right)\ell_z k^2 e^{-2\phi} * F_2 & *J_{7,m} &= \tfrac{i}{2\pi\ell_z}F_2\\
*J_{2,w} &= i\left(2\pi\ell_z\right)\frac{1}{\ell_z}k^{-2}e^{-2\phi} * H_2 & *J_{7,mw} &= \tfrac{i\ell_z}{2\pi}H_2.
\end{aligned}
$$

- In absence of sources

$$
\begin{aligned}
d * J_6 &= -i2\pi\ell_z F_2 \wedge H_2 & d * J_3 &= 0\\
d * J_{2,e} &= i\left(2\pi\ell_z\right)\ell_z H_2 \wedge \mathsf{F}_{\tilde{B}_5} & d * J_{7,m} &= 0\\
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- Chern-Simons terms imply the Maxwell currents $d * J_6$, $d * J_{2,e}$ and $d * J_{2,w}$ are gauge invariant and conserved, but not localized nor quantized.

Definition of topological operators

- The Page currents

$$
\begin{aligned}
d\left(*J_6 + i2\pi\ell_z \mathsf{A}_{B_1\star A_1}\right) &= -i2\pi\ell_z \mathsf{N}_{B_1}\cup\mathsf{N}_{A_1}\,,\\
d\left(*J_{2,e} - i\left(2\pi\ell_z\right)\mathsf{A}_{B_1\star\tilde{B}_5}\right) &= i\left(2\pi\ell_z\right)\mathsf{N}_{B_1}\cup\mathsf{N}_{\tilde{B}_5}\,,\\
d\left(*J_{2,w} - i\left(2\pi\ell_z\right)\frac{1}{\ell_z}\mathsf{A}_{A_1\star\tilde{B}_5}\right) &= i\left(2\pi\ell_z\right)\frac{1}{\ell_z}\mathsf{N}_{A_1}\cup\mathsf{N}_{\tilde{B}_5}\,,
\end{aligned}
$$

  where for two p,q-forms $\check{A}_{A1,A2} = (\mathsf{N}_{A1,A2}, \mathsf{A}_{A1,A2}, \mathsf{F}_{A1,A2})$,

$$
\mathsf{A} = \mathsf{A}_{A_1}\cup\mathsf{N}_{A_2} + (-1)^{p+1}\mathsf{F}_{A_1}\cup\mathsf{A}_{A_2} + Q(\mathsf{F}_{A_1},\mathsf{F}_{A_2})\,, \quad (11)
$$

  are quantized, conserved, well-defined, and invariant under the gauge transformations $\mathsf{A}\to\mathsf{A}+\delta\mathsf{a}$.
- They are not invariant under the gauge transformation $\mathsf{N}\to\mathsf{N}+\delta n$, $\mathsf{A}\to\mathsf{A}+\mathsf{n}$, shifting them by integers so that the corresponding symmetries are completely broken.
- The operator $e^{\frac{2\pi ik}{n}\int_{M(7)}\left(e^{-2\phi}k^2 * F_2 + n\mathsf{A}\right)}$ defines a $\mathbb{Z}_n$ global symmetry.

Introducing sources

- According to the "Completeness Hypothesis" quantum gravity theories should admit states of all possible charges. This is related to the absence of GS, which should therefore either be gauged or broken.
- We introduce p-form currents that are localized on the $9-p$-dimensional worldvolumes of extended objects.
- By consistency Chern-Simons terms require extended objects carry worldvolume degrees of freedom that appear in the gauge invariant combination $\nabla_X Y = dY + X$, where X is a p-form potential and Y is a charged $p-1$ form field on the worldvolume

Equations of motion

$$
\begin{aligned}
d * J_3 &= j_7^F \,, && (12)\\
d * J_{7,m} &= j_3^{KK} \,, && (13)\\
d * J_{7,mw} &= j_3^{NS5} \,, && (14)\\
d\left(*J_6 + i2\pi\ell_z \mathsf{A}_{B_1\star A_1}\right) &= -i2\pi\ell_z \mathsf{N}_{B_1} \cup \mathsf{N}_{A_1} + j_4^{NS5} \\
&\quad +(2\pi)^2\nabla_{B_1}\sigma^{KK} \wedge j_3^{KK} + (2\pi)^2\nabla_A z^{NS5} \wedge j_3^{NS5} \,, && (15)\\
d\left(*J_{2,e} - i\,(2\pi\ell_z)\,\mathsf{A}_{B_1\star\tilde{B}_5}\right) &= i\,(2\pi\ell_z)\,\mathsf{N}_{B_1} \cup \mathsf{N}_{\tilde{B}_5} + j_8^m \\
&\quad -(2\pi)^2\nabla_{B_1} Z^* \wedge j_7^F - (2\pi)^2(\nabla_{\tilde{B}_5} z^{*NS5}) \wedge j_3^{NS5} && (16)\\
d\left(*J_{2,w} - i\,(2\pi\ell_z)\,\frac{1}{\ell_z}\mathsf{A}_{A_1\star\tilde{B}_5}\right) &= i\,(2\pi\ell_z)\,\frac{1}{\ell_z}\mathsf{N}_{A_1} \cup \mathsf{N}_{\tilde{B}_5} + j_8^F \\
&\quad -(2\pi)^2\nabla_A Z \wedge j_7^F - (2\pi)^2\nabla_{\tilde{B}_5}\sigma^{*KK} \wedge j_3^{KK} \,, && (17)
\end{aligned}
$$

where $\star dz = dz'$ on the respective worldvolume.

Orbifold by half-period shift

- We project on configurations of the sources that have even magnetic charges for the gauge field B_1.
- $e^{\pi i \int_{M^{(2)}} H_2}$ implements now a $\mathbb{Z}_2$ 1-form global symmetry that we gauge, so imposing $B_1 = 2\mathcal{B}_1$. Notice this can be seen as the consequence of the gauging of the global quantum symmetry on the worldsheet.
- We can now project on configurations of the sources such that the right-hand side of (22) is even.
- $e^{\pi i \int_{M^{(7)}}(e^{-2\phi} k^2 * F_2 + 2\mathrm{A}_{\mathcal{B}_1 \star \tilde{\mathcal{B}}_5})}$ implements a $\mathbb{Z}_2$ global symmetry, whose gauging implies N_{A_1} is quantized in half-integers.
- We have new Wilson lines carrying odd units of $\mathcal{B}_1$ electric charge and 't Hooft lines corresponding KK monopoles with half-integral unit of magnetic charge for A_1.
- We break the global symmetries by introducing states so that j_8^F and j_3^{KK} are quantized in half-integers.

Orbifolded equations of motion

Renaming $\mathcal{B}_1$ by B_1,

$$d * J_3 = j_7^F \,, \tag{18}$$

$$d * J_{7,m} = \frac{1}{2} j_3^{KK} \,, \tag{19}$$

$$d * J_{7,mw} = 2 j_3^{NS5} \,, \tag{20}$$

$$\begin{aligned} d\left(*J_6 + i2\pi\ell_z \mathsf{A}_{2B_1 \star \frac{1}{2}A_1} \right) &= -i2\pi\ell_z \mathsf{N}_{2B_1} \cup \mathsf{N}_{\frac{1}{2}A_1} + j_4^{NS5} \\ &+ (2\pi)^2 \nabla_{2B_1} \sigma^{KK} \wedge \frac{1}{2} j_3^{KK} + (2\pi)^2 \nabla_{\frac{1}{2}A} z^{NS5} \wedge 2 j_3^{NS5} , \end{aligned} \tag{21}$$

$$\begin{aligned} d\left(*J_{2,e} - i\left(2\pi\ell_z\right) \mathsf{A}_{2B_1 \star \tilde{B}_5} \right) &= i\left(2\pi\ell_z\right) \mathsf{N}_{2B_1} \cup \mathsf{N}_{\tilde{B}_5} + 2 j_8^m \\ &- (2\pi)^2 \nabla_{2B_1} Z^* \wedge j_7^F - (2\pi)^2 (\nabla_{\tilde{B}_5} z^{*NS5}) \wedge 2 j_3^{NS5} \end{aligned} \tag{22}$$

$$\begin{aligned} d\left(*J_{2,w} - i\left(2\pi\ell_z\right) \frac{1}{\ell_z} \mathsf{A}_{\frac{1}{2}A_1 \star \tilde{B}_5} \right) &= i\left(2\pi\ell_z\right) \frac{1}{\ell_z} \mathsf{N}_{\frac{1}{2}A_1} \cup \mathsf{N}_{\tilde{B}_5} + \frac{1}{2} j_8^F \\ &- (2\pi)^2 \nabla_{\frac{1}{2}A} Z \wedge j_7^F - (2\pi)^2 \nabla_{\tilde{B}_5} \sigma^{*KK} \wedge \frac{1}{2} j_3^{KK} , \end{aligned} \tag{23}$$

Conclusions

- The orbifold procedure for quantum gravity we have presented is the direct counterpart of the worldsheet orbifold.
- Whereas for convenience we have used some of the dynamical details of the model, the procedure is expected to be independent of them and rely only on the gauge group structure of quantum gravity. It would be therefore useful to express it solely in terms of topological operators.
- One of the main motivations for the present work is to understand the dependence (or independence) of the orbifold procedure on the duality frame. Even if we have chosen one, we have argued our procedure is largely independent of it.

Conclusions

- This is at odds with known examples where “orbifolds do not commute with dualities”. E.g. the orbifold of $10d$ type IIB string theory for worldsheet parity is type I string theory, whereas the orbifold of IIB by the S-dual $(-1)^{F_L}$ is type IIA.
- Spacetime orbifold could either clarify the obstructions in such cases or suggest different ways to perform the orbifold that are invisible from the worldsheet point of view.

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THANK YOU!