New dualities from orientifold projections

Salvo Mancani

Theories of the Fundamental Interactions 2022

13 June 2022 - Venice

Based on
M. Bianchi, D. Bufalini, SM, F. Riccioni, hep-th/ 2003.09620
A. Antinucci, SM, F. Riccioni, hep-th/2007.14749
A. Antinucci, M. Bianchi, SM, F. Riccioni, hep-th/2105.06195

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Holographically dual

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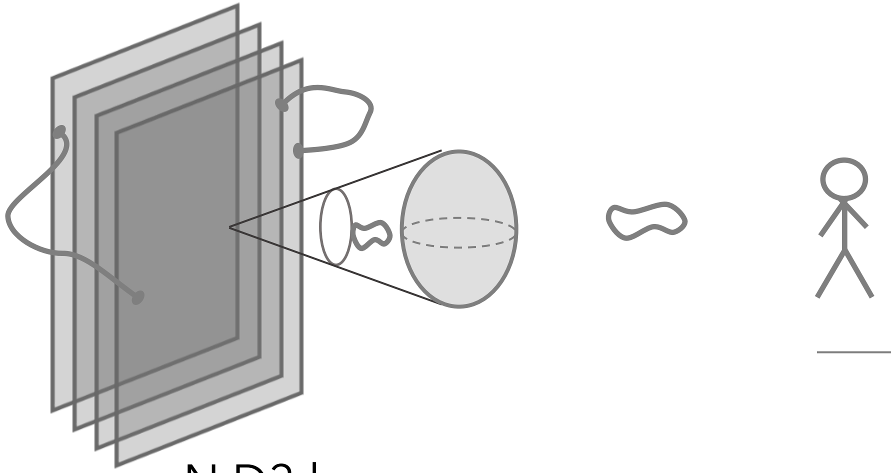
$4\pi g_s = g_{YM}^2$

4d $\mathcal{N} = 4$ SuperConformal gauge theory G=SU(N)



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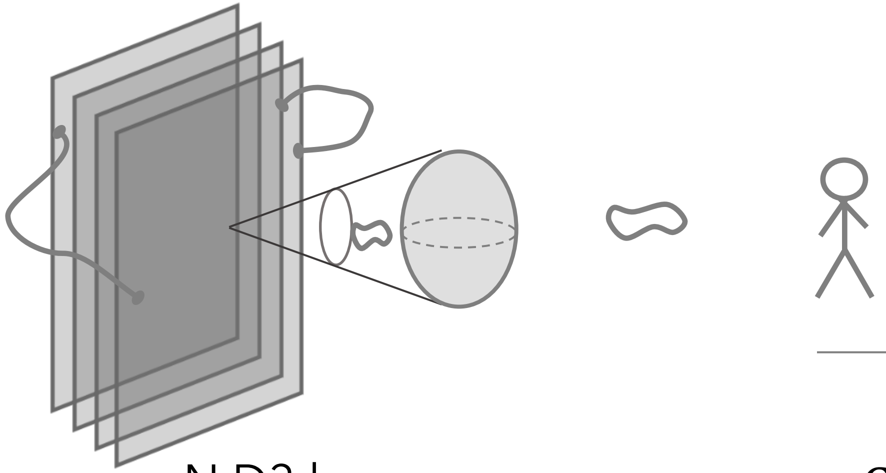


N D3 branes

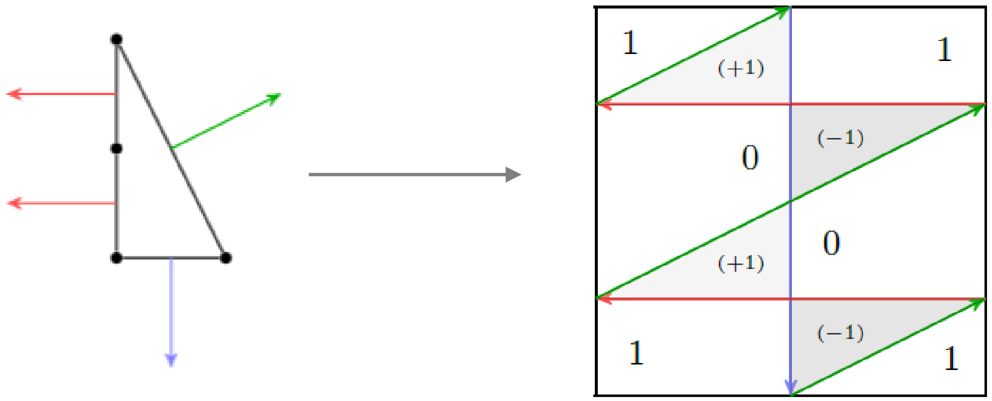
$\mathrm{AdS}_5 \times H^5$

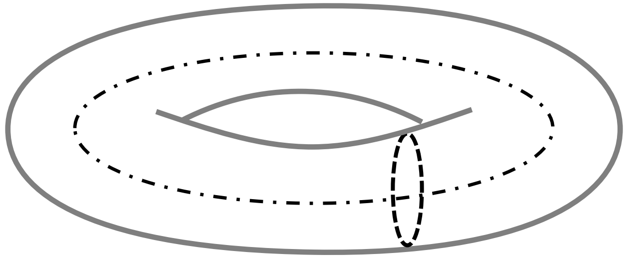
[Morrison, Plesser -1998]

$\mathrm{G} = \prod SU(N)$, $\mathcal{N} = 1,2$

Toric geometry and Brane Tiling

Given a particular geometry, what is the local physics? Focus on Toric geometries:
$U(1)^2 \times U(1)_R$





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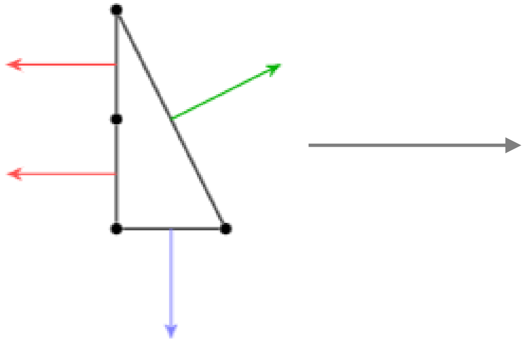
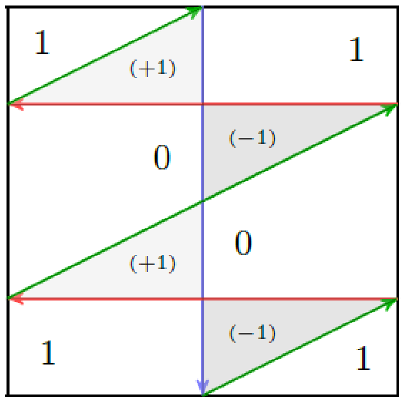
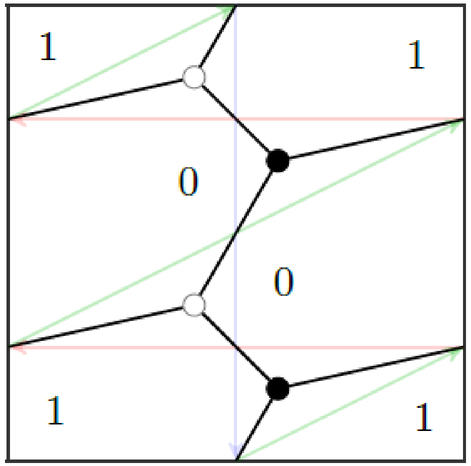


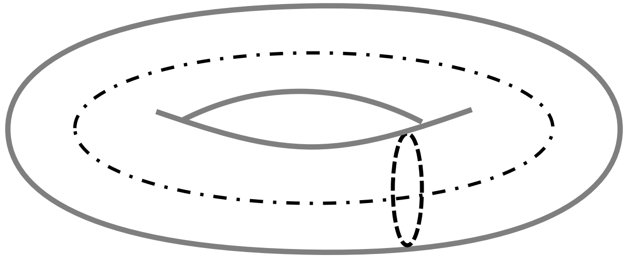


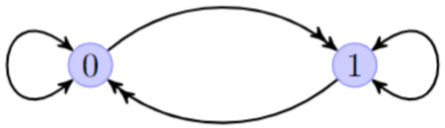
Brane Tiling

[Franco, Hanany, Kennaway, Vegh, Wecht - 2005]



Dictionary:
Face → Gauge group
Edge → Matter field
Node → Interaction term





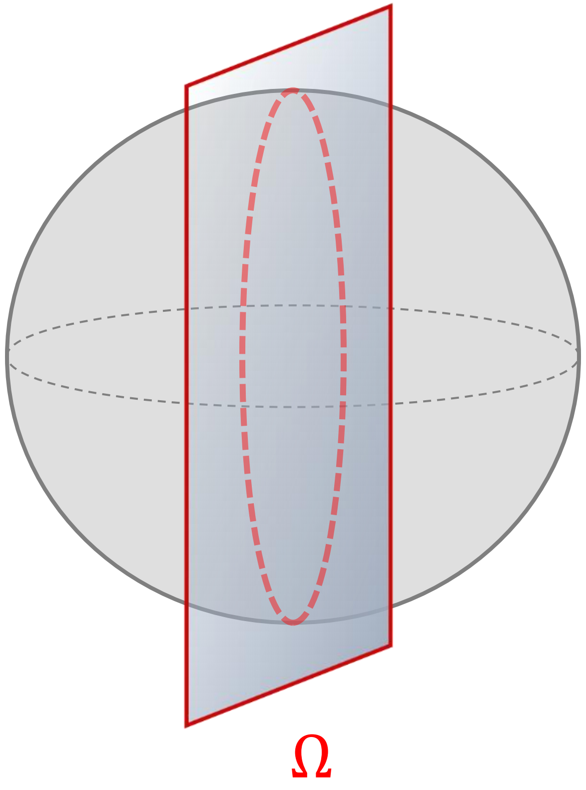
Example: $G = SU(N_0) \times SU(N_1)$

$$W_{\mathbb{C}^3/\mathbb{Z}_2} = \phi_0(X^1_{01}X^2_{10} - X^2_{01}X^1_{10}) + \phi_1(X^1_{10}X^2_{01} - X^2_{10}X^1_{01})$$

Orientifold projection

Involution Ω: $\underbrace{\omega}\ \underbrace{\sigma}\ \underbrace{(-1)^{F_L}}$

- ω — Reverses World-sheet Orientation
- σ — Space-time reflection ⟶ O-planes at the $\mathbb{Z}_2$ fixed points
- $(-1)^{F_L}$ — Projects left-movers



Example:

O3$^{\pm}$ plane, near horizon space $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_2$

and gauge side $\mathcal{N} = 4$ with gauge group $\mathrm{U}Sp(N), SO(N)$ [Witten - 1998]

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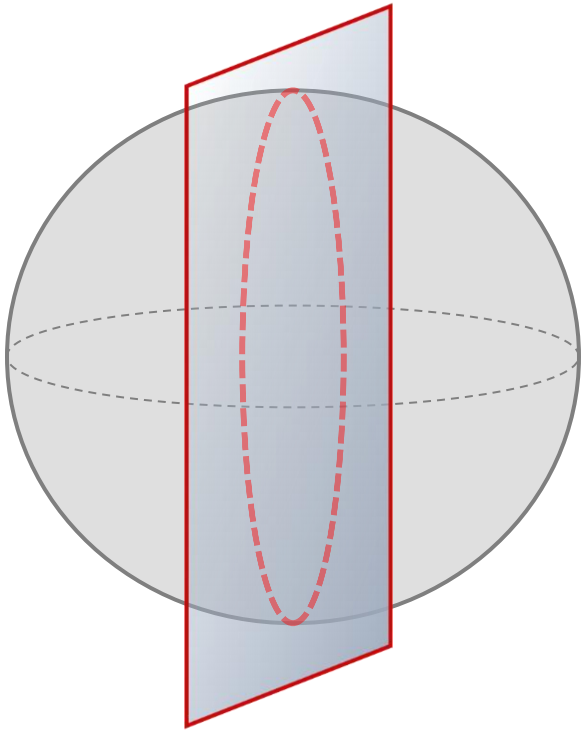


Example:

$O3^{\pm}$ plane, near horizon space $\mathrm{AdS}_5 \times S^5/\mathbb{Z}_2$

and gauge side $\mathcal{N} = 4$ with gauge group $\mathrm{USp}(N)$, $SO(N)$ [Witten - 1998]

Why the orientifold Ω?

- It allows for SO, USp gauge groups and tensor matter fields [Bianchi, Sagnotti – 1990 ; Witten - 1998]
- Present in all attempts to reproduce the MSSM [Wijnholt - 2007]
- It changes the qualitative feature of RG flow and IR dynamics [Argurio, Bertolini - 2017]

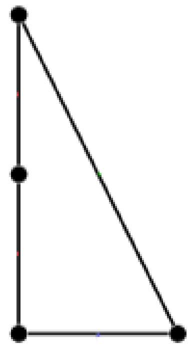
Orientifold projection on Brane Tiling

In [Franco, Hanany, Krefl, Park, Uranga, Vegh – 2007], they study the $\mathbb{Z}_2$ involution of the torus with fixed loci. Such loci correspond to O-planes in the configuration, their charge determines the projection as

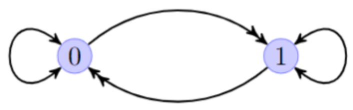
Charge	Face $SU(N)$	Edge Bifundamental ($\square, \bar{\square}$)
+	$SO(N)$	Symmetric $\square\square$
-	$USp(N)$	Antisymmetric $\begin{smallmatrix}\square\\\square\end{smallmatrix}$

Fixed point orientifold on Brane Tiling

Parent:



$\mathbb{C}^3/\mathbb{Z}_2$



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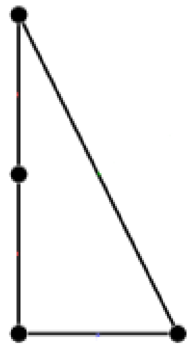

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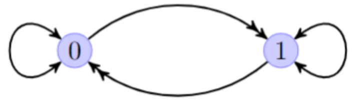

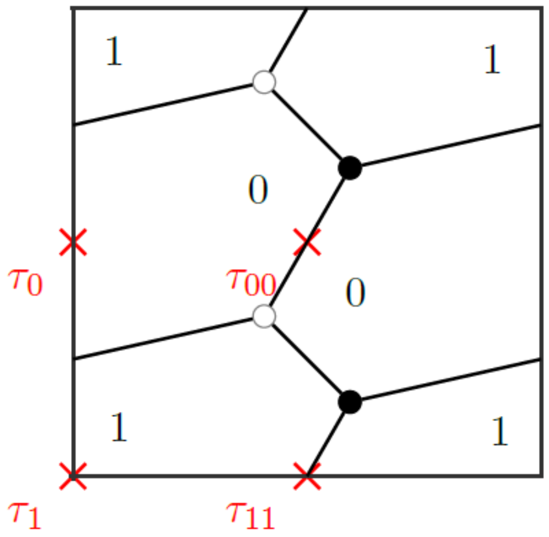

- Black nodes to white nodes
- $\prod \tau = (-1)^{N_W/2}$

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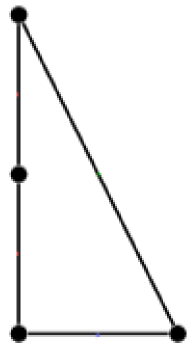

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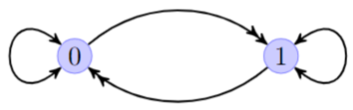

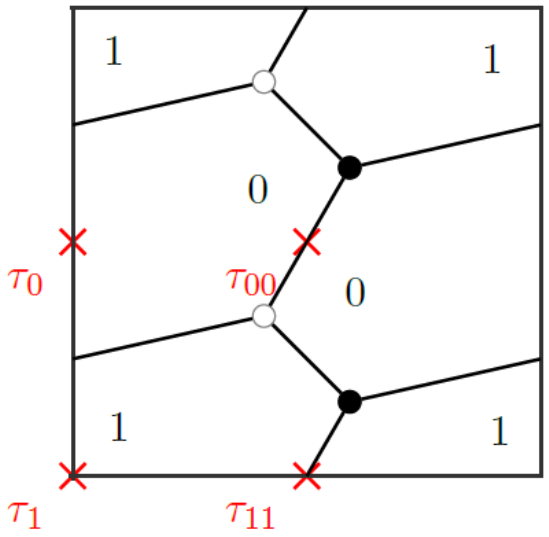

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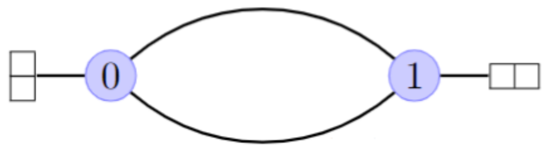
$\Omega = (\tau_0, \tau_{00}, \tau_1, \tau_{11})$



$G = SO(N_0) \times USp(N_1)\,,\ \Omega_A = (+,-,-,+)$

$W^{\Omega_A}_{\mathbb{C}^3/\mathbb{Z}_2} = A_0 X^1_{01} X^2_{10} - S_1 X^2_{10} X^1_{01} \quad , \ \mathcal{N} = 2$

Fixed point orientifold on Brane Tiling

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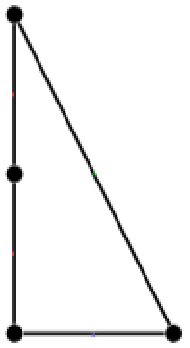

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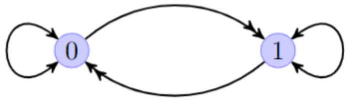

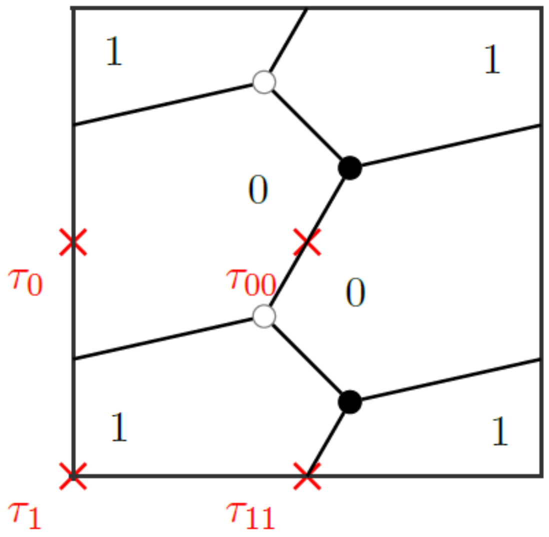

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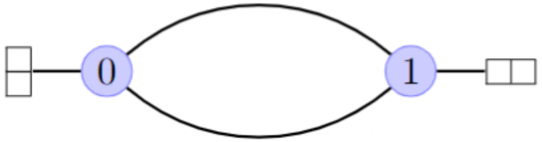

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Fixed lines orientifold on Brane Tiling

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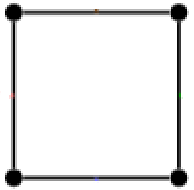
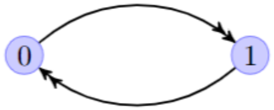


$\mathcal{C}$onifold
$\mathcal{C}$



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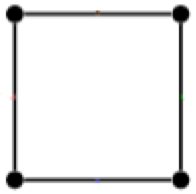

Conifold
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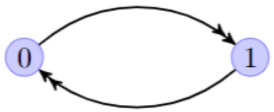

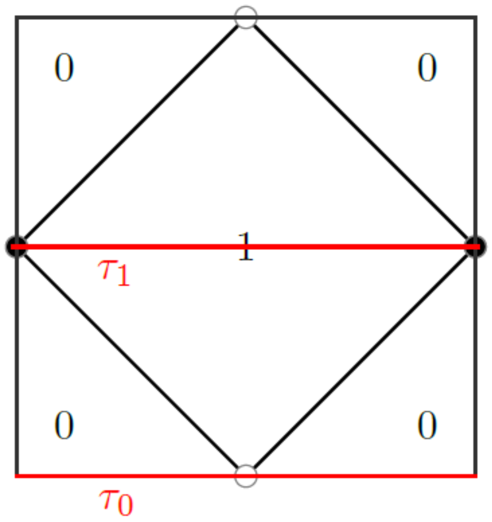

- Black (white) to black (white) nodes

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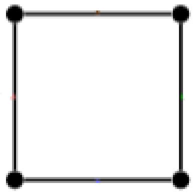

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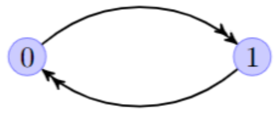

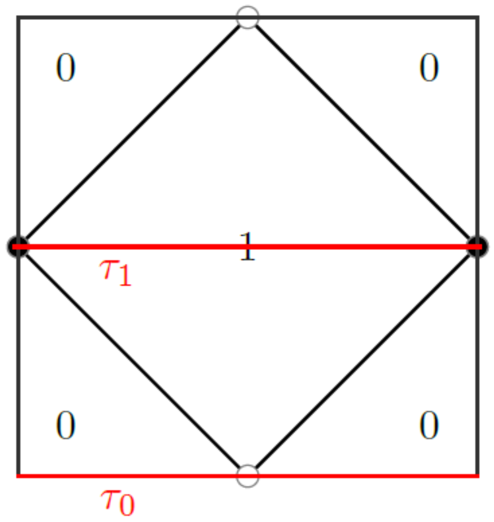

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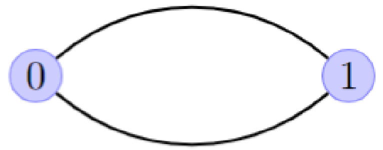

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Supersymmetric Theories

We focus (mainly) on 4d $\mathcal{N} = 1$ SCFT with

- gauge group $G = \prod_a G_a$
- matter fields $X_{ab}, X_{bc}, \ldots$ in some representation ρ of G
- superpotential $W(X)$, e.g. $Tr(X_{ab}X_{bc}X_{ca})$
- global symmetries $\prod_i U(1)_i$, $[SU(n), \ldots]$, R-symmetry $U(1)_R$

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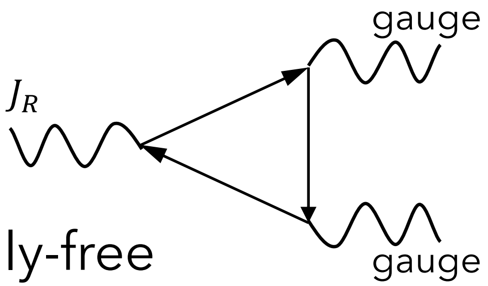
The exact beta-function NSVZ for the gauge coupling g reads

$$\beta_g = -\frac{g^3}{16\,\pi^2}\frac{3\,T_{Adj} - \sum_i T_{\rho_i}(1-\gamma_i)}{1 - T_{Adj}\frac{g^2}{8\,\pi^2}}$$

$$\Delta = \frac{3}{2}R = 1 + \frac{1}{2}\gamma$$

$$\beta_g = 0 \quad \rightarrow \quad T_{Adj} + \sum_i T_{\rho_i}(R_i - 1) = 0 \quad \longrightarrow \quad \text{R-symmetry anomaly-free}$$



SCFT: reach the maximum

Any abelian global factor $U(1)_q$ mixes together with the R-symmetry $U(1)_R$

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For a general SUSY theory, the combination is not uniquely defined: $R = R(x, y, \dots)$.

On the contrary, there is a unique superconformal R-symmetry given by the local maximum [Intriligator, Wecht - 2003] of

$$a = \frac{3}{32}(3\, Tr\, R^3 - Tr\, R)\ ,$$

$$T^\mu_\mu = a\, E_4 + c\, W^2\ ,$$

The central charge a stands as a 'counting' of the d.o.f. and $a_{IR} < a_{UV}$ [Komargodski, Schwimmer - 2003]

Unoriented Conformal Theories (?)

The parent theory is a SCFT.
What is the fate of the conformal invariance after the orientifold involution Ω?

a) Usually broken, e.g. Conifold
b) 'Restored' by the presence of flavour branes [Bianchi, Inverso, Morales, Pacifici - 2014]
c) Same R-charges of the parent (at large N)

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Check the resulting $\mathcal{N} = 1$ theory:

- Find non-anomalous $U(1)$s
- Impose R-symmetry $U(1)_R$ is anomaly-free
- Impose $R(W) = 2$

} $R = R(x, y, \ldots)$ →

- a-maximization on $a = \frac{3}{32}(3\, Tr\, R^3 - Tr\, R)$

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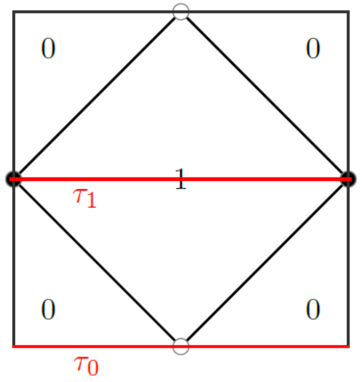
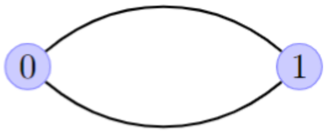
- Usually broken, e.g. Conifold with $N_0 = N_1 = N$





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$$\begin{cases} r_{01}^1 + r_{01}^2 = -1 \\ r_{01}^1 N + r_{01}^2\, N = -(N-2) \\ r_{01}^1 N + r_{01}^2\, N = -(N+2) \end{cases} \quad \rightarrow \quad \text{No solution}$$

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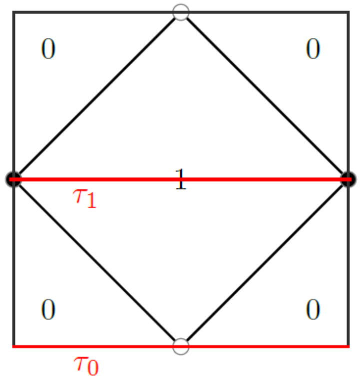

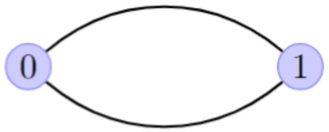

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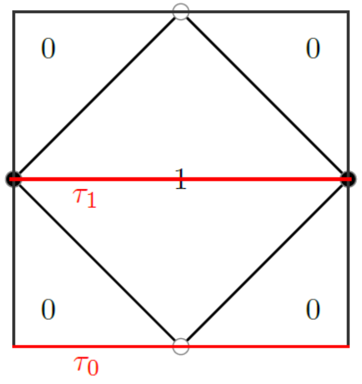
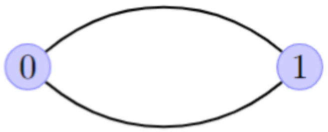
- Same Conformal point of the parent, e.g. Conifold with $N_1 = N_0 - 2$ [Naculich, Schnitzer, Wyllard - 2001]





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$$\begin{cases} r_{01}^{1} + r_{01}^{2} = -1 \\ (r_{01}^{1} + r_{01}^{2})\,(N_0 - 2) = -(N_0 - 2) \\ (r_{01}^{1} + r_{01}^{2})\, N_0 = -(N_0) \end{cases} \quad \rightarrow \quad \begin{cases} r_{01}^{1} = -\dfrac{1}{2} \\ r_{01}^{2} = -\dfrac{1}{2} \end{cases}$$

Unoriented Conformal Theories

Contributions to the central charge are halved at large N, with the same R-charges

$$a^{\Omega} = \frac{9}{32} Tr\, R^3$$

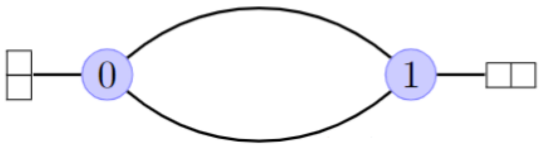
$$= \frac{9}{32}\Big[\ \underbrace{(R_{ab}-1)^3 N_a N_b + \ \dots}_{\text{Bifundamental contribution, half of the parent}} + \underbrace{(R_T - 1)^3 \frac{1}{2} N(N \pm 2)}_{\text{Tensor contribution}} + \underbrace{(N_a^2 - 1) + \ \dots \frac{1}{2} N(N \pm 1)}_{\text{Gauginos contribution}}\ \Big]$$

Bifundamental contribution, half of the parent

Tensor contribution

Gauginos contribution

$$\frac{a^{\Omega}}{a} = \frac{1}{2}$$

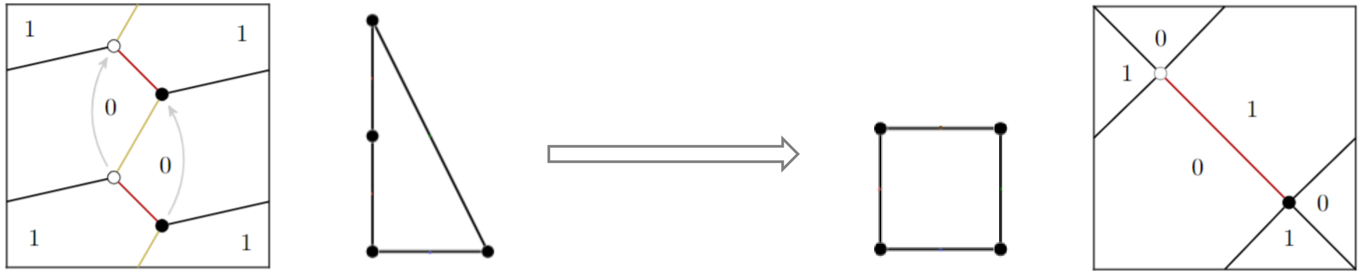


This happens for the choices Ω_A of $\mathbb{C}^3/\mathbb{Z}_2$ and Ω_a of Conifold with $SO(N_0) \times USp(N_0 - 2)$

Mass Deformation and Orientifold

Parent theories: $\mathbb{C}^3/\mathbb{Z}_2$ can be mass deformed via $\Delta W = m^2({\phi_0}^2 - {\phi_1}^2)$.
Below the mass scale, the effective theory is the $\mathcal{C}$onifold [Klebanov, Witten – 1998]

In brane tiling models, the mass term breaks toricity, but can be restored in the effective theory [Bianchi, Cremonesi, Hanany, Morales, Pacifici, Seong – 2014]



$a_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{1}{2}\,N^2$ $\qquad a_{\mathcal{C}} = \frac{27}{64}\,N^2$

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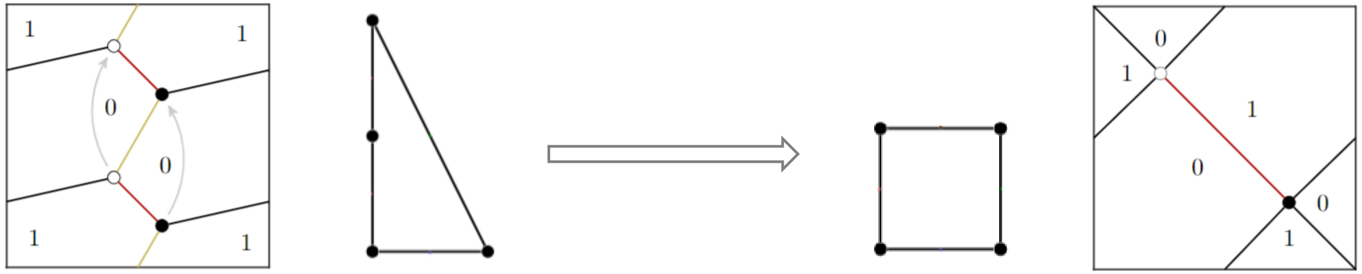

$$a_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{1}{2} N^2 \qquad a_{\mathcal{C}} = \frac{27}{64} N^2$$

$$\frac{a_{\mathcal{C}}}{a_{\mathbb{C}^3/\mathbb{Z}_2}} = \frac{27}{32}$$

[Tachikawa, Wecht - 2009]

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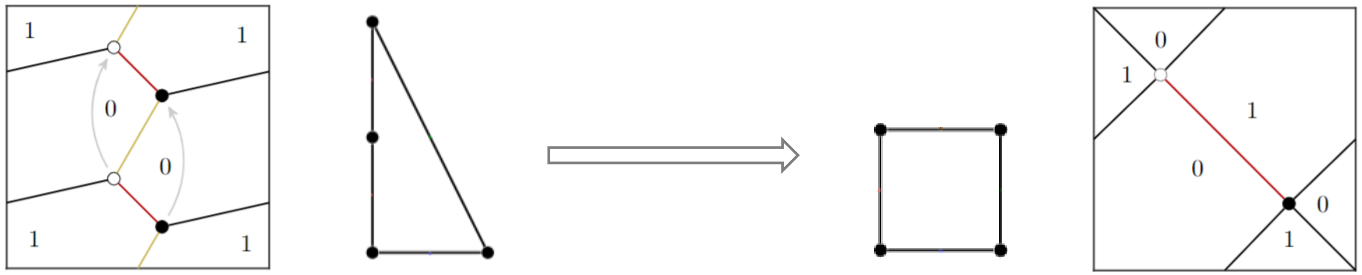

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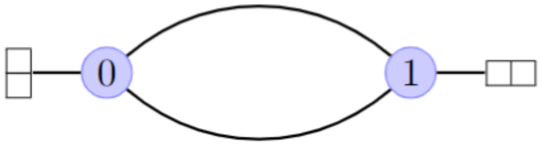
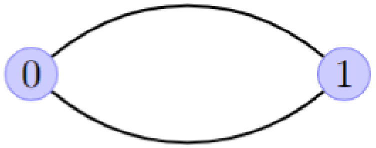
[Tachikawa, Wecht - 2009]

Similarly, the orientifold theories are connected



$\Delta W^{\Omega_A}_{\mathbb{C}^3/\mathbb{Z}_2} = m^2({A_0}^2 - {S_1}^2)$.



$$\frac{a^{\Omega_a}_{\mathcal{C}}}{a^{\Omega_A}_{\mathbb{C}^3/\mathbb{Z}_2}} = \frac{27}{32}$$

$W^{\Omega_A}_{\mathbb{C}^3/\mathbb{Z}_2} = A_0 X^1_{01} X^2_{10} - S_1 X^2_{10} X^1_{01}$

$W^{\Omega_a}_{\mathcal{C}} = X^1_{01}(X^2_{01})^T X^2_{01}(X^1_{01})^T - X_{01}(X^1_{01})^T X^2_{01}(X^2_{01})^T$

A new scenario

What about choice Ω_B for $\mathbb{C}^3/\mathbb{Z}_2$?

$$W^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2} = S_0 X^1_{01} X^2_{10} - A_1 X^2_{10} X^1_{01}$$

$$\begin{cases} a^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{27}{128} N^2 \\ R_S = R_A = 1 \ , \\ R_{01} = R_{10} = \frac{1}{2} , \\ SO(N_0) \times USp(N_0 - 2) \end{cases}$$

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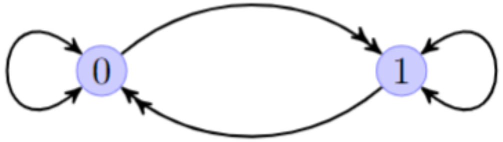
$$a_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{1}{2} N^2$$

$$R_{\phi_0} = R_{\phi_1} = R_{01} = R_{10} = \frac{2}{3} \;,$$

$$SU(N_0) \times SU(N_0 - 2)$$

Compare



$$W_{\mathbb{C}^3/\mathbb{Z}_2} = \epsilon_{ij} (\phi_0 X^i_{01} X^j_{10} + \phi_1 X^i_{10} X^j_{01})$$

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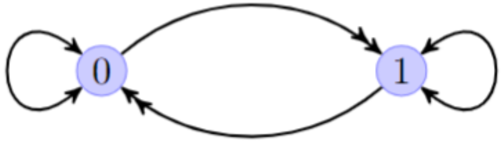


$$W_{\mathbb{C}^3/\mathbb{Z}_2} = \epsilon_{ij}(\phi_0 X^i_{01} X^j_{10} + \phi_1 X^i_{10} X^j_{01})$$

Compare

'Third' scenario : $\dfrac{a^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2}}{a_{\mathbb{C}^3/\mathbb{Z}_2}} < \dfrac{1}{2}$, d.o.f. are more than halved by the orientifold

[Antinucci, SM, Riccioni - 2020 ; Antinucci, Bianchi, SM, Riccioni – 2021]

The third scenario

What is happening ?

- Ω_B is $\mathcal{N} = 1$, set of conditions on R-symmetry fix all R-charges; maximization of a is not needed

- Parent theory (as Ω_A) is $\mathcal{N} = 2$, whose R-symmetry group is $SU(2) \times U(1)_R$. Abelian subgroup of $SU(2)$ mixes with $U(1)_R$: central charge is maximized over 1 variable

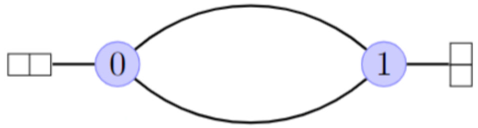
Compare

$$a^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{27}{128} N^2$$

$$R_S = R_A = 1 \ ,$$

$$R_{01} = R_{10} = \frac{1}{2} ,$$

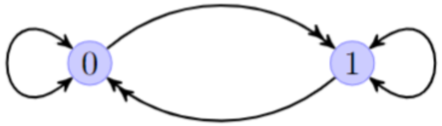
$$SO(N_0) \times USp(N_0 - 2)$$



$$a_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{1}{2} N^2$$

$$R_{\phi_0} = R_{\phi_1} = R_{01} = R_{10} = \frac{2}{3} \ ,$$

$$SU(N_0) \times SU(N_0 - 2)$$



The third scenario

What is happening ?

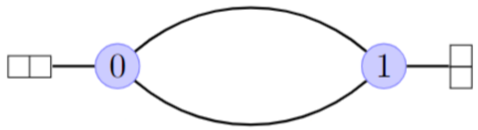
- Ω_B is $\mathcal{N}=1$, set of conditions on R-symmetry fix all R-charges; maximization of a is not needed

- Parent theory (as Ω_A) is $\mathcal{N}=2$, whose R-symmetry group is $SU(2)\times U(1)_R$. Abelian subgroup of $SU(2)$ mixes with $U(1)_R$: central charge is maximized over 1 variable

- Between parent and Ω_B we are losing a $U(1)$!

Compare

$$\left\{\begin{aligned} & a^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{27}{128}N^2 \\ & R_S = R_A = 1\ , \\ & R_{01} = R_{10} = \frac{1}{2}\, , \\ & SO(N_0)\times USp(N_0-2) \end{aligned}\right.$$



$$a_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{1}{2}N^2$$

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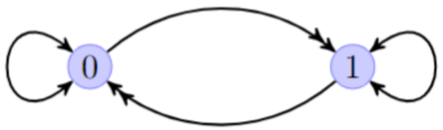


$$\frac{a^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2}}{a_{\mathbb{C}^3/\mathbb{Z}_2}} < \frac{1}{2}$$

Unoriented IR Duality

Compare

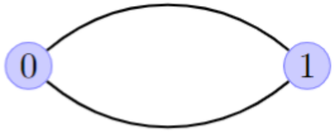
$$\begin{cases} a^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{27}{128} N^2 \\ R_S = R_A = 1\ , \\ R_{01} = R_{10} = \frac{1}{2}, \\ SO(N_0) \times USp(N_0 - 2) \end{cases}$$

$$W^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2} = S_0 X^1_{01} X^2_{10} - A_1 X^2_{10} X^1_{01}$$

$$a^{\Omega_a}_{\mathcal{C}} = \frac{27}{128} N^2$$

$$R_{01} = R_{10} = \frac{1}{2}\ ,$$

$$SO(N_0) \times USp(N_0 - 2)$$



$$W^{\Omega_a}_{\mathcal{C}} = X^1_{01} (X^2_{01})^T X^2_{01} (X^1_{01})^T - X_{01} (X^1_{01})^T X^2_{01} (X^2_{01})^T$$

Not the whole story yet…

- The matter content of orientifold Ω_B of $\mathbb{C}^3/\mathbb{Z}_2$ and Ω_a of $\mathcal{C}$ differ by tensor fields… whose contribution to anomalies is $(R-1) = 0$ (their contribution to the index as well)

- Orientifold Ω_B of $\mathbb{C}^3/\mathbb{Z}_2$and Ω_a of $\mathcal{C}$ share the same 't Hooft anomalies, central charge a and index

Unoriented IR Duality

Compare

$$a^{\Omega_B}_{\mathbb{C}^3/\mathbb{Z}_2} = \frac{27}{128} N^2$$

$R_S = R_A = 1$,

$R_{01} = R_{10} = \frac{1}{2}$,

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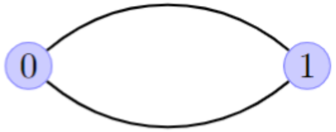

$$W^{\Omega_a}_{\mathcal{C}} = X^1_{01}(X^2_{01})^T X^2_{01}(X^1_{01})^T$$
$$- X_{01}(X^1_{01})^T X^2_{01}(X^2_{01})^T$$

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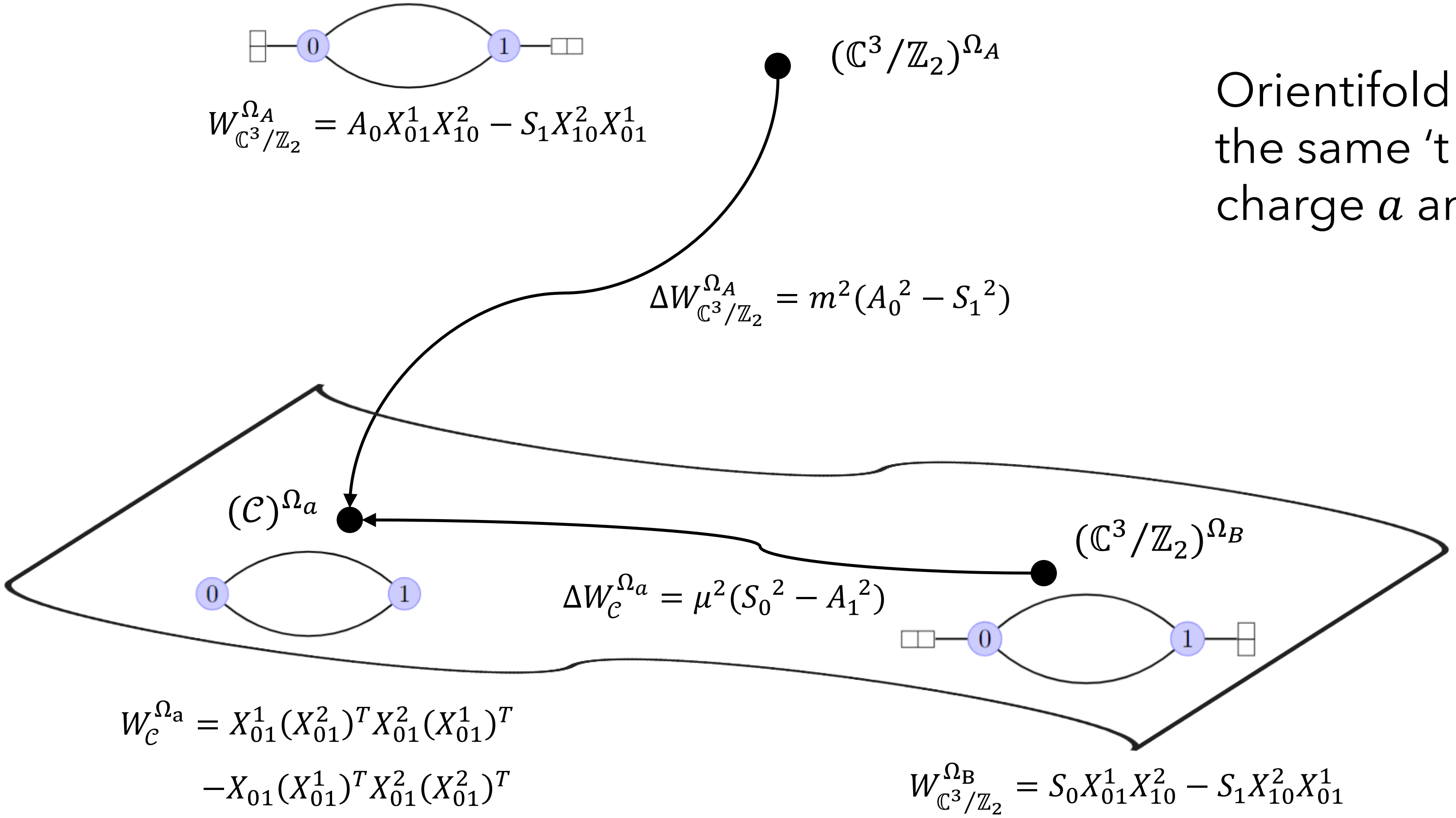
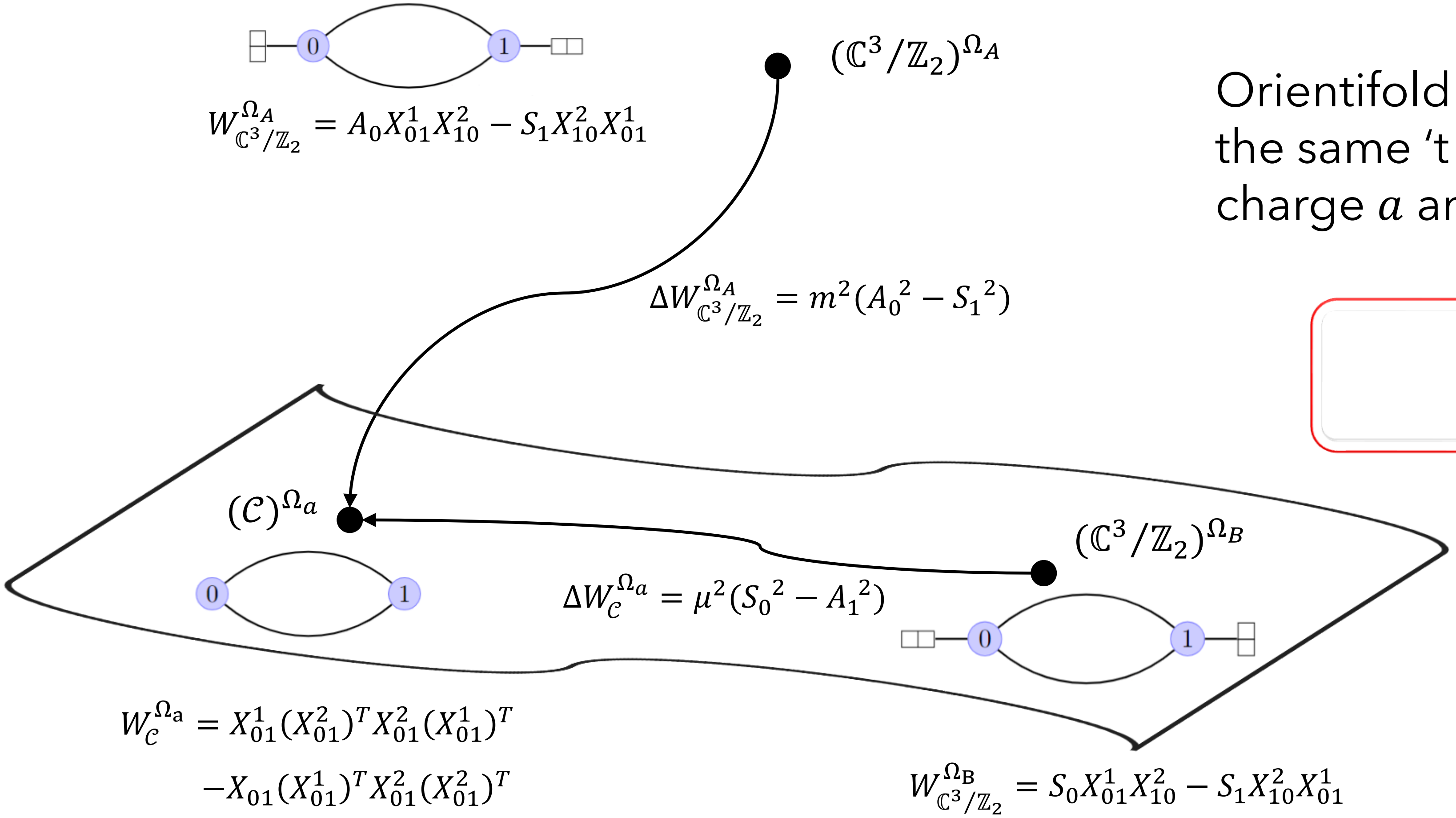
- Orientifold Ω_B of $\mathbb{C}^3/\mathbb{Z}_2$and Ω_a of $\mathcal{C}$ share the same 't Hooft anomalies, central charge a and index
- Smells like a duality!

Unoriented IR Duality



Orientifold Ω_B of $\mathbb{C}^3/\mathbb{Z}_2$ and Ω_a of $\mathcal{C}$ share the same 't Hooft anomalies, central charge a and index

Unoriented IR Duality



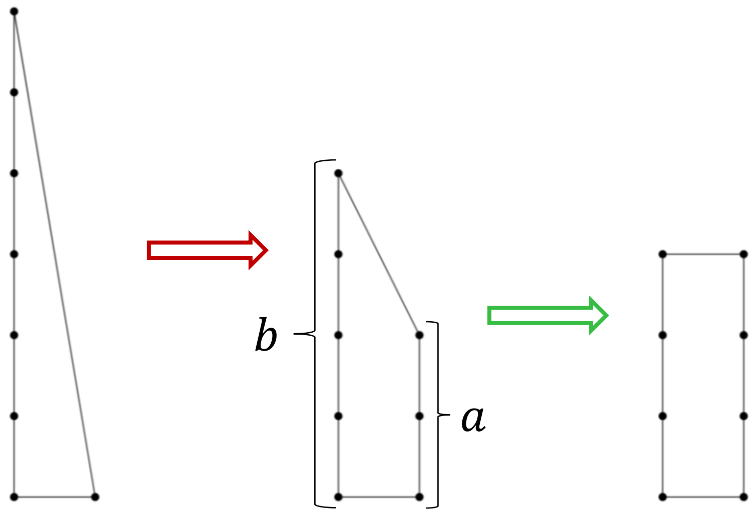
Orientifold Ω_B of $\mathbb{C}^3/\mathbb{Z}_2$ and Ω_a of $\mathcal{C}$ share the same 't Hooft anomalies, central charge a and index

Conformal duality

A class of dual families

It can be generalized to $(L^{a,b,a})^{\Omega}$

[Antinucci, Bianchi, SM, Riccioni – 2021]



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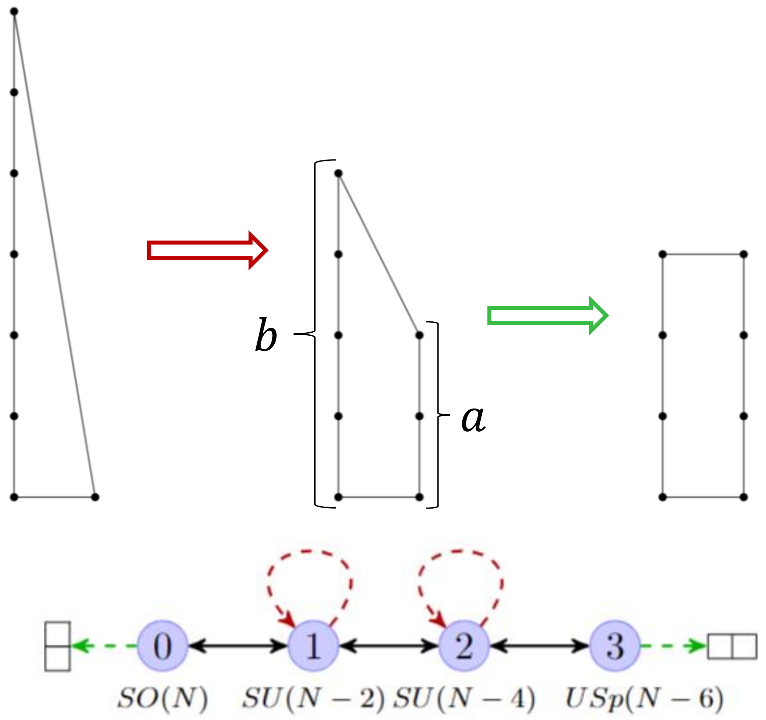

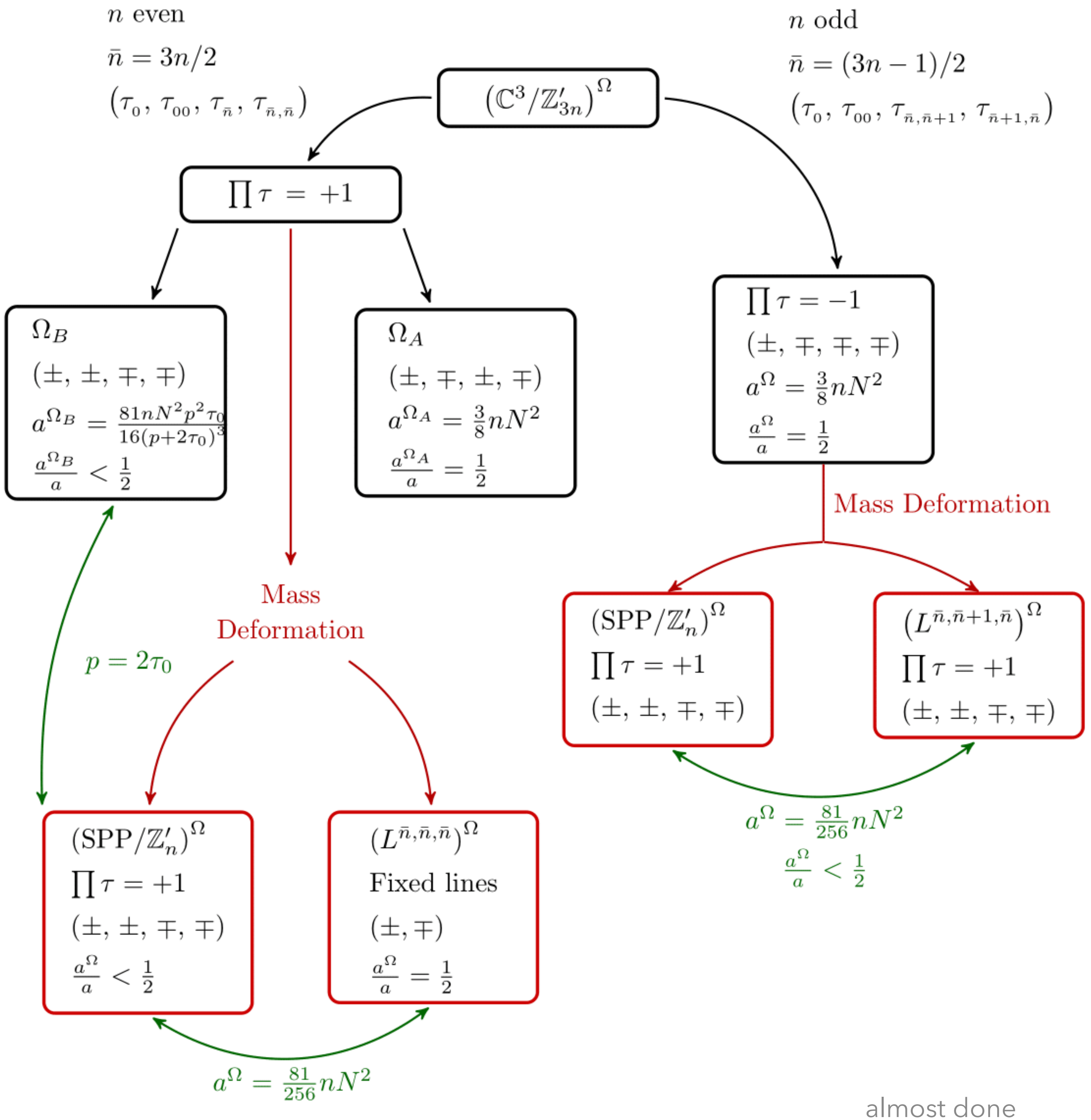

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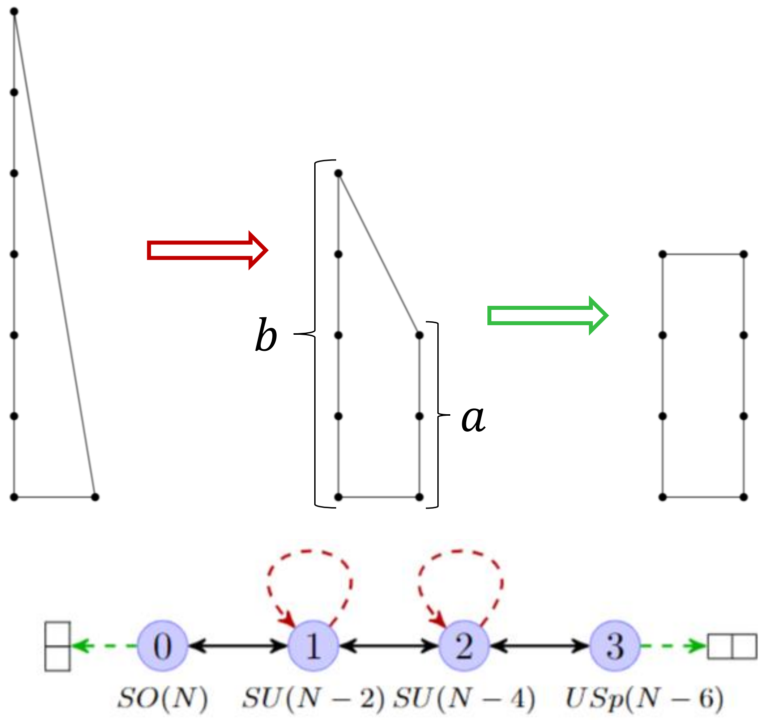
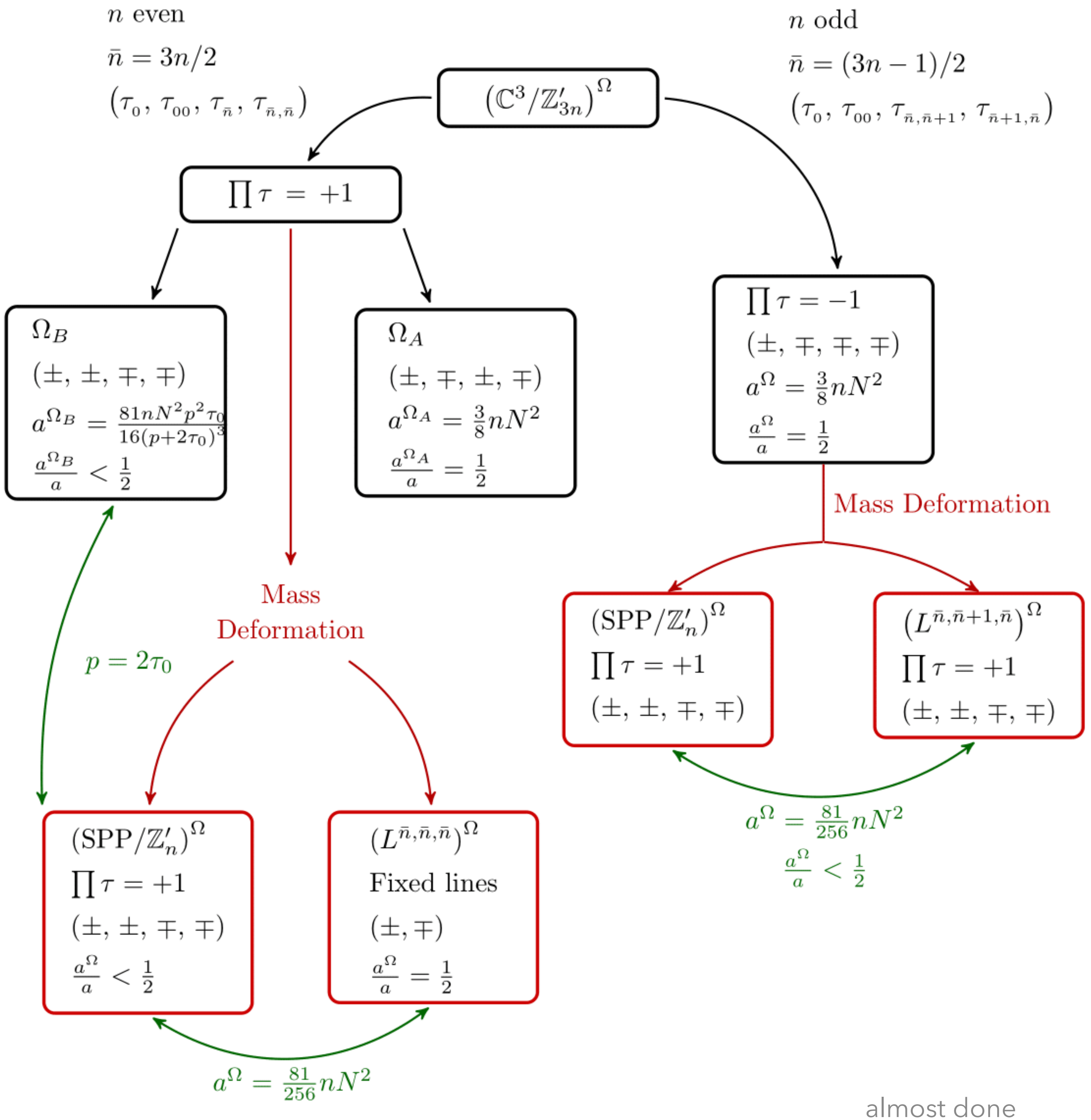
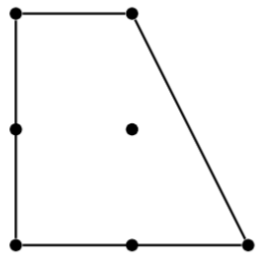


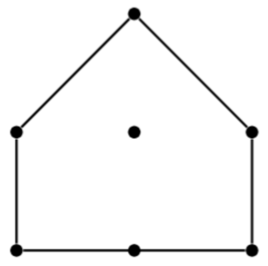
[Amariti, Fazzi, Rota, Segati – 2021]
generalised further this picture and explored the origin of the duality



The only chiral pair: Pseudo del Pezzo



PdP_{3c}



PdP_{3b}

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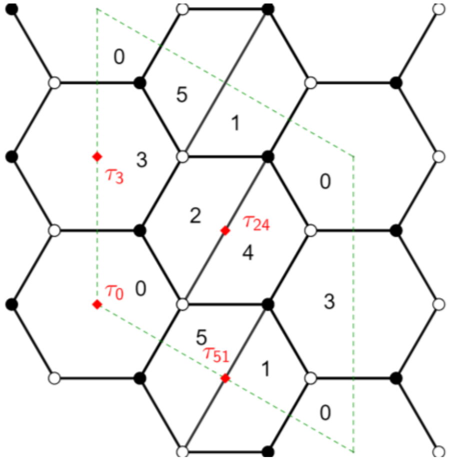

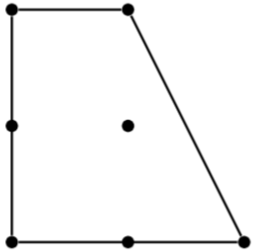

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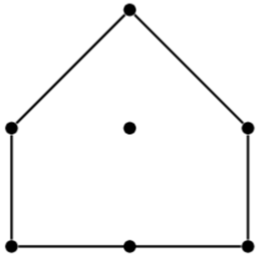

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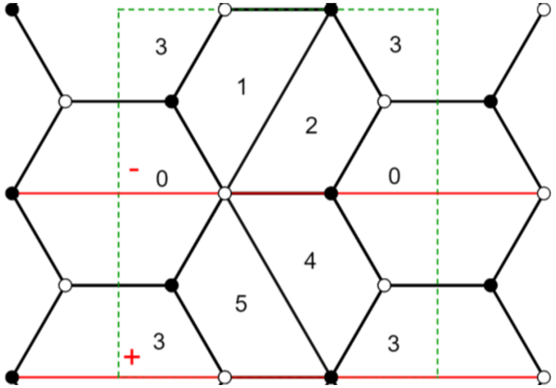

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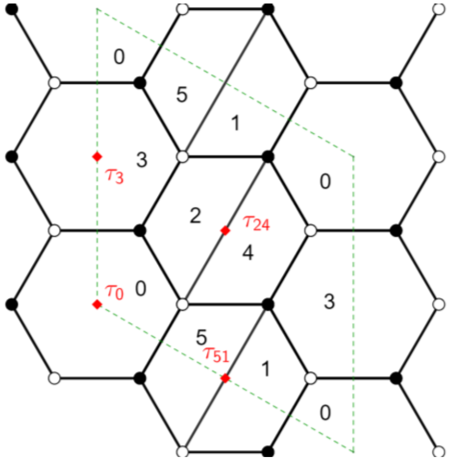

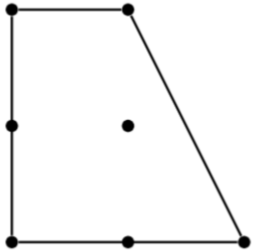

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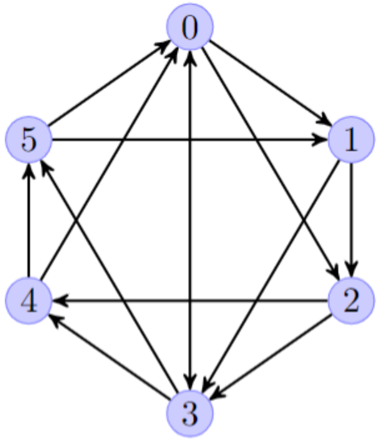

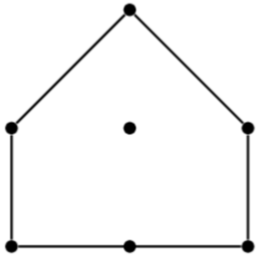

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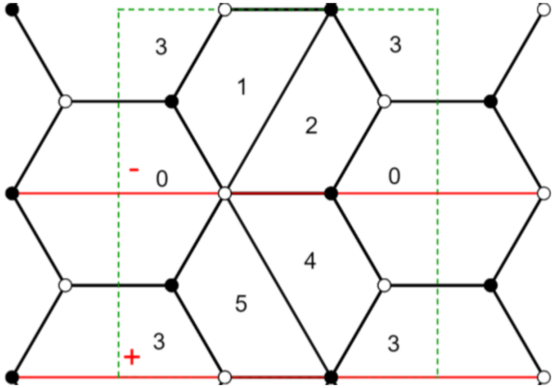

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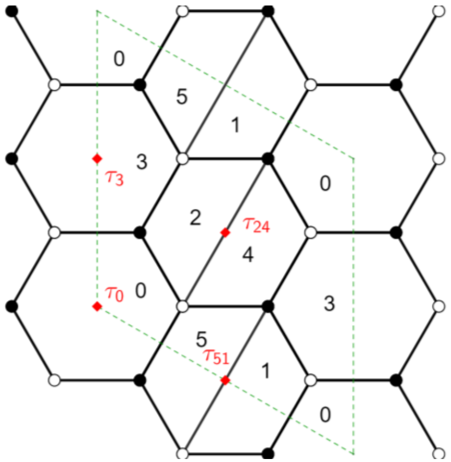

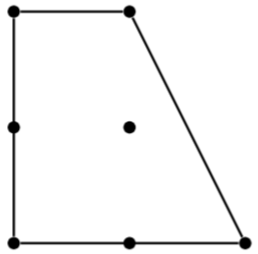

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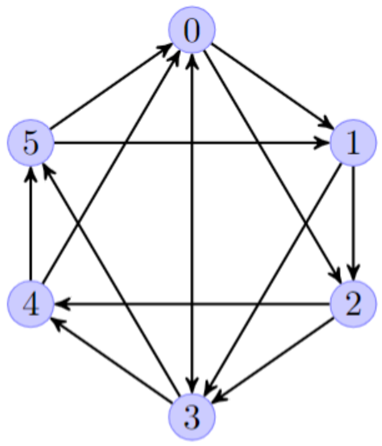
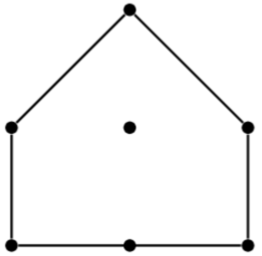
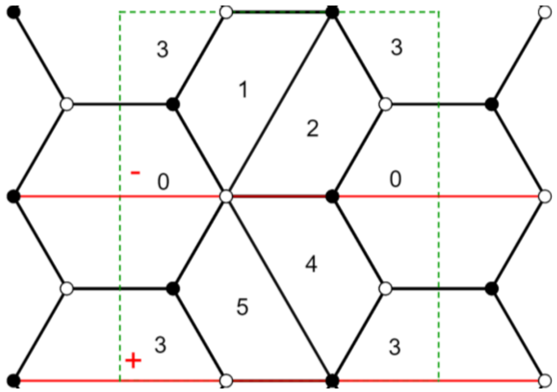




PdP_{3b}



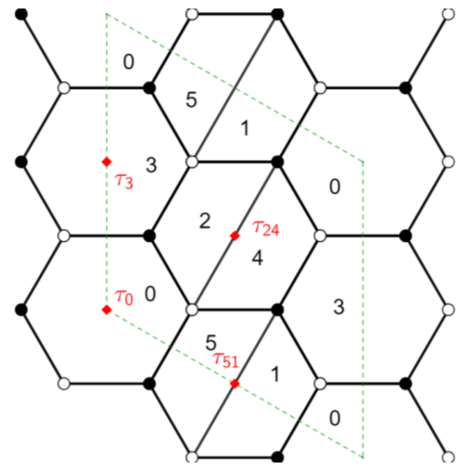
$\Omega = (\tau_0, \tau_3, \tau_{24}, \tau_{51})$

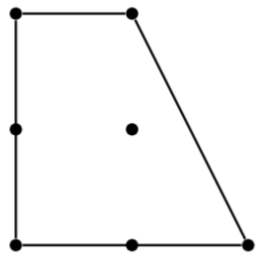
$\Omega = (\tau_0, \tau_3)$

$$W^{\Omega}_{PdP_{3c}} = X_{03}X_{01}X_{13} - X_{02}X_{23}X_{03}$$
$$+ X_{02}A_{22}(X_{12})^T(X_{01})^T - \tilde{S}_{11}X_{13}(X_{23})^T(X_{12})^T$$

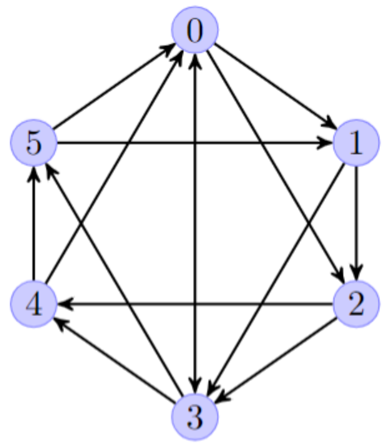
$$W^{\Omega}_{PdP_{3b}} = -X_{03}X_{01}X_{13} + X_{02}X_{23}X_{03}$$
$$+ X_{01}X_{12}A_{22}(X_{12})^T(X_{01})^T - (X_{23})^T(X_{12})^T\tilde{S}_{11}X_{12}X_{23}$$
$$+ (X_{13})^T\tilde{S}_{11}X_{13} - X_{02}A_{22}(X_{02})^T$$

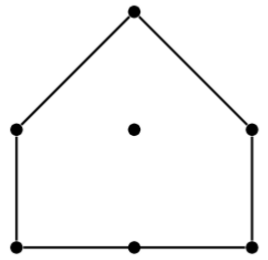
The only chiral pair: Pseudo del Pezzo



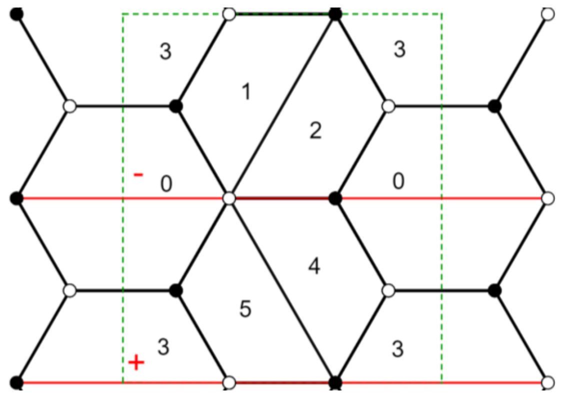


PdP_{3c}





PdP_{3b}



$\Omega = (\tau_0, \tau_3, \tau_{24}, \tau_{51})$

$\Omega = (\tau_0, \tau_3)$

$$W^{\Omega}_{PdP_{3c}} = X_{03}X_{01}X_{13} - X_{02}X_{23}X_{03}$$
$$+ X_{02}A_{22}(X_{12})^T(X_{01})^T - \tilde{S}_{11}X_{13}(X_{23})^T(X_{12})^T$$

$$W^{\Omega}_{PdP_{3b}} = -X_{03}X_{01}X_{13} + X_{02}X_{23}X_{03}$$
$$+ X_{01}X_{12}A_{22}(X_{12})^T(X_{01})^T - (X_{23})^T(X_{12})^T\tilde{S}_{11}X_{12}X_{23}$$
$$+ (X_{13})^T\tilde{S}_{11}X_{13} - X_{02}A_{22}(X_{02})^T$$

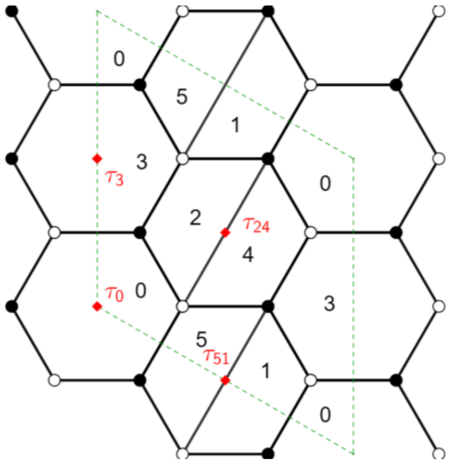
$\Omega_A = (+, -, -, +)$

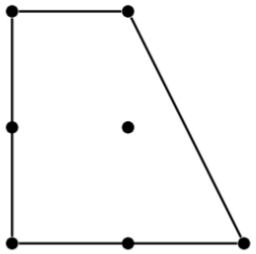
$SO(N) \times SU(N) \times SU(N+2) \times USp(N+2)$

$\Omega_B = (-, +, -, +)$

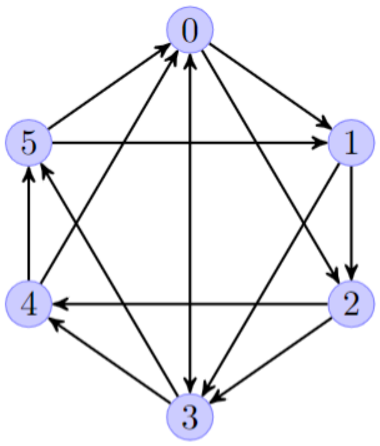
$USp(N-2) \times SU(N) \times SU(N) \times SO(N+2)$

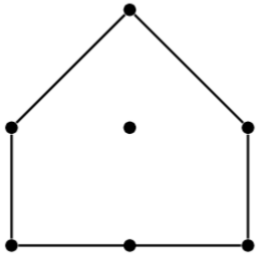
The only chiral pair: Pseudo del Pezzo



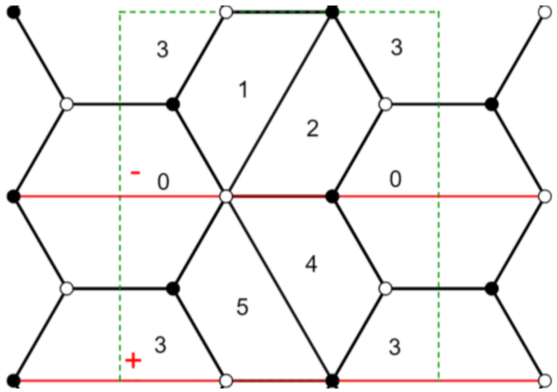


PdP_{3c}





PdP_{3b}



$\Omega = (\tau_0, \tau_3, \tau_{24}, \tau_{51})$

$\Omega = (\tau_0, \tau_3)$

$$W^{\Omega}_{PdP_{3c}} = X_{03}X_{01}X_{13} - X_{02}X_{23}X_{03} + X_{02}A_{22}(X_{12})^T(X_{01})^T - \tilde{S}_{11}X_{13}(X_{23})^T(X_{12})^T$$

$$W^{\Omega}_{PdP_{3b}} = -X_{03}X_{01}X_{13} + X_{02}X_{23}X_{03} + X_{01}X_{12}A_{22}(X_{12})^T(X_{01})^T - (X_{23})^T(X_{12})^T\tilde{S}_{11}X_{12}X_{23} + (X_{13})^T\tilde{S}_{11}X_{13} - X_{02}A_{22}(X_{02})^T$$

$\Omega_A = (+,-,-,+)$

$SO(N) \times SU(N) \times SU(N+2) \times USp(N+2)$

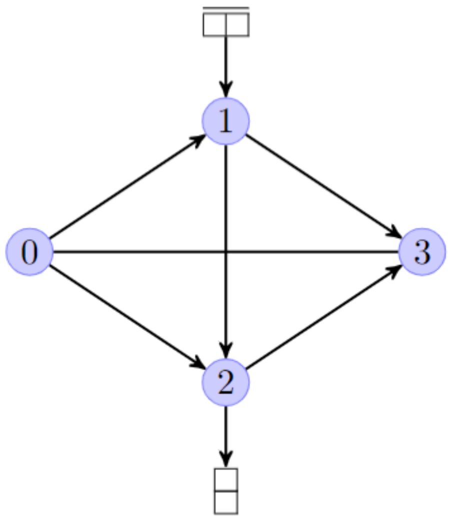
$\Omega_B = (-,+,-,+)$

$USp(N-2) \times SU(N) \times SU(N) \times SO(N+2)$

$\Omega = (-,+)$

$USp(N-2) \times SU(N) \times SU(N) \times SO(N+2)$

The only chiral pair: Pseudo del Pezzo



The only chiral pair: Pseudo del Pezzo

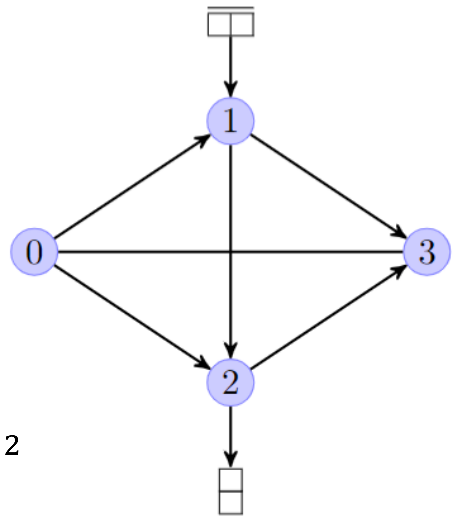
$\Omega_A = (+,-,-,+)$

$SO(N) \times SU(N) \times SU(N+2) \times USp(N+2)$

$R_{03} = 2 - \frac{2\sqrt{3}}{3}$

$R_{12} = R_{22} = R_{11} = 1 - \frac{\sqrt{3}}{3}$

$R_{01} = R_{02} = R_{13} = R_{23} = \frac{\sqrt{3}}{3}$ $\qquad a^{\Omega_A} = \frac{3\sqrt{3}}{8}\mathrm{N}^2$



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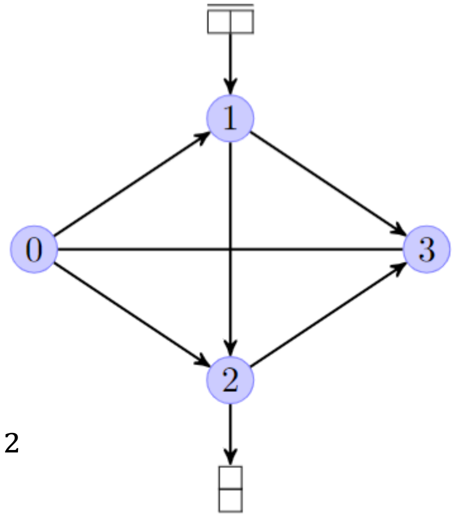


$\Omega_B = (-,+,-,+)$

$USp(N-2) \times SU(N) \times SU(N) \times SO(N+2)$

$R_{12} = 7 - 3\sqrt{5}$

$R_{02} = R_{03} = R_{13} = 3 - 3\sqrt{5}$

$R_{01} = R_{23} = R_{22} = R_{11} = 2\sqrt{5} - 4$

$\frac{a^{\Omega_B}}{a} < \frac{1}{2}$ $\quad \frac{a^{\Omega}}{a} = \frac{1}{2}$

$a^{\Omega_B} = a^{\Omega} = \frac{27}{8}\left(5\sqrt{5} - 11\right)\mathrm{N}^2$

$\Omega = (-,+)$

$USp(N-2) \times SU(N) \times SU(N) \times SO(N+2)$

$R_{12} = 7 - 3\sqrt{5}$

$R_{02} = R_{03} = R_{13} = 3 - 3\sqrt{5}$

$R_{01} = R_{23} = R_{22} = R_{11} = 2\sqrt{5} - 4$

The third scenario

Is the third scenario ‘safe’?

- R-charges are different than the parent and it is not granted that the unitarity bound ($\Delta > 1$) holds

- If not, some gauge-invariant operators decouple and other global $U(1)$s emerge: repeat a-maximization [Kutasov, Parnachev, Sahakyan - 2003]

- Even if the unitarity bound holds, it’s not granted the conformal point exist, but there are no clear ‘signs’. The duality with the orientifold of another model make it safe.

Conclusions & Open Problems

- We have found a new mechanism for the orientifold projection that develops a conformal fixed point in the IR

- A web of dualities between orientifold of non-chiral theories ($L^{a,b,a}$) via quadratic marginal deformations

- Together with the authors of [Amariti, Fazzi, Rota, Segati – 2021] we generalize the mechanism with quadratic marginal deformations to the orientifold of chiral theories $(L^{a,b,a})/\mathbb{Z}_2$; the web involves also glide orientifold [García-Valdecasas, Meynet, Pasternak,Tatitscheff - 2021].
A paper will appear soon [Amariti, Bianchi, Fazzi, Mancani, Riccioni, Rota - In progress]

Conclusions & Open Problems

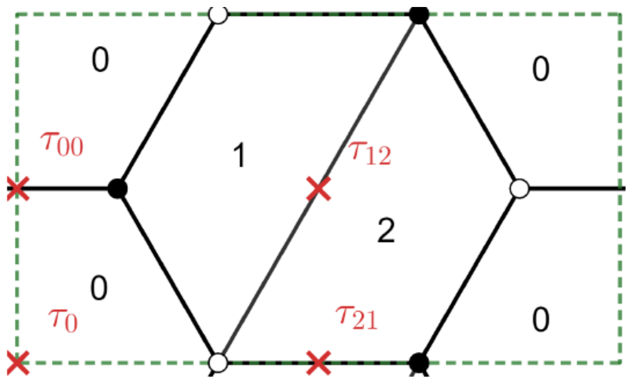
- Only one chiral pair with quintic marginal operator. Can we find others? What is the origin of the duality? [Antinucci, SM, Riccioni - 2020]

- Dual pair must be connected to dual geometries. But before the orientifold projection they are different. How?

Thank you

Backup slides

When operators decouple: orientifold of SPP



$$W^{\Omega}_{\rm SPP} = -\phi_0 X_{01} X_{10} + X_{12} X_{21} X_{10} X_{01}$$

$$r_{00}\,(N_0 - N_1 + 2\tau_0) = -\,(N_0 - N_1 - 2\tau_0)\ ,$$
$$r_{00}\,(N_0 - N_1 - 2\tau_{12}) = -\,(N_0 - N_1 + 2\tau_{12})\ .$$

$$r_{00} = -\frac{p - 2\tau_0}{p + 2\tau_0} = -\frac{p + 2\tau_{12}}{p - 2\tau_{12}}\ ,$$
$$r_{01} = -\frac{2\tau_0}{p + 2\tau_0}\ , \qquad r_{12} = -\frac{p}{p + 2\tau_0}$$

$$\mathcal{O}_{0,j} = \mathrm{Tr}\,\phi_0^j\ , \quad j > 1\ ,$$
$$\mathcal{M}_m = (X_{12}X_{21})^m\ , \quad m \geq 1\ ,$$
$$\widetilde{\mathcal{M}}_{0,lk} = \phi_0^l\,(X_{01}X_{10})^k\ , \quad l \geq 0\ , \quad k \geq 1$$

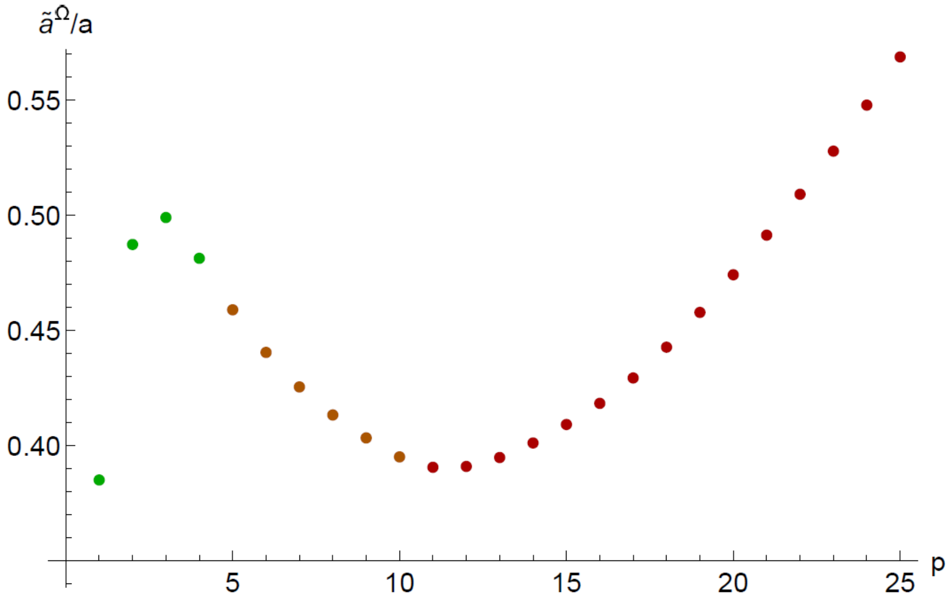


Figure 12: The ratio $\widetilde{a}^{\Omega}_{\rm SPP}/a_{\rm SPP}$ vs $p = N_0 - N_1$. The green points signal that there are no correction to the central charge, on orange points $(X_{12}X_{21})^m$ becomes free, while on red ones operators $\mathrm{Tr}\,\phi_0^j$ start to decouple.

Volume of the horizon

[Gubser -1999]: $\mathrm{Vol}(\mathrm{H}^5) = \frac{\pi^3}{4}\frac{N^2}{a}$

Compare volumes of parent and unoriented at the same radius R:

$R \propto$ unit of 5- form flux $\begin{cases} \propto N & \text{, in parent} \\ \propto \frac{N}{2} & \text{, in presence of O-planes} \end{cases}$

$$\mathrm{Vol}(\mathrm{H}^5) \propto \frac{N^2}{a} \quad \rightarrow \quad Vol^{\Omega} \propto \left(\frac{N}{2}\right)^2 \frac{1}{a^{\Omega}} \propto \frac{1}{4}\frac{N^2}{a^{\Omega}} \quad \rightarrow \quad \frac{Vol^{\Omega}}{Vol} \propto \frac{1}{4}\frac{a}{a^{\Omega}} \begin{cases} = \frac{1}{2} & \text{, in first scenario} \\ > \frac{1}{2} & \text{, in third scenario} \end{cases}$$

Volume of the horizon

Contribution to the superconformal index from matter fields:

$$i_X(t,s) = \sum \frac{t^{R_{ab}}\chi_{\rho_{ab}} - t^{2-R_{ab}}\chi_{\bar{\rho}_{ab}}}{(1-ts)(1-\frac{t}{s})}$$

Fields with $R = 1$:

- Adjoints
- Symmetric or Antisymmetric of real groups
- Symmetric or Antisymmetric in conjugated pairs of unitary groups